

# Semantic 3D Modeling

## CVPR 2016 Tutorial

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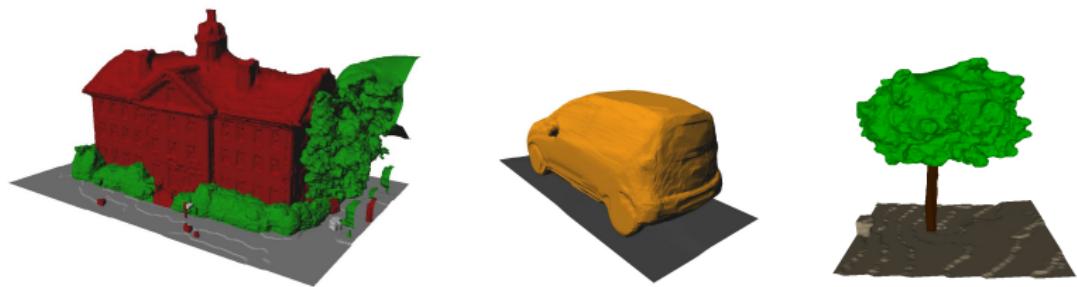
# Semantic 3D Modeling

- Semantic 3D Modeling
  - Dense 3D reconstruction
  - Semantic segmentation
- Joint formulation is beneficial
- Richer representation
  - Volume of just the building?
  - Remove vegetation
  - Where are the streets?
  - Are there cars?



# Outline

- 1 Volumetric 3D Reconstruction
- 2 Convex Optimization
- 3 Convex Continuous Multi-Label Formulation
- 4 Joint 3D Scene Reconstruction and Class Segmentation
- 5 Class Specific 3D Object Shape Priors
- 6 Segment Based 3D Object Shape Priors
- 7 Articulated and Restricted Motion
- 8 Depth Estimation with a Surface Normal Classifier



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# Motivation

- Input: Set of images



Figure : Some example images

- Goal: Dense 3D reconstruction

# Sparse 3D Reconstruction

- Camera locations
- Set of sparse 3D points

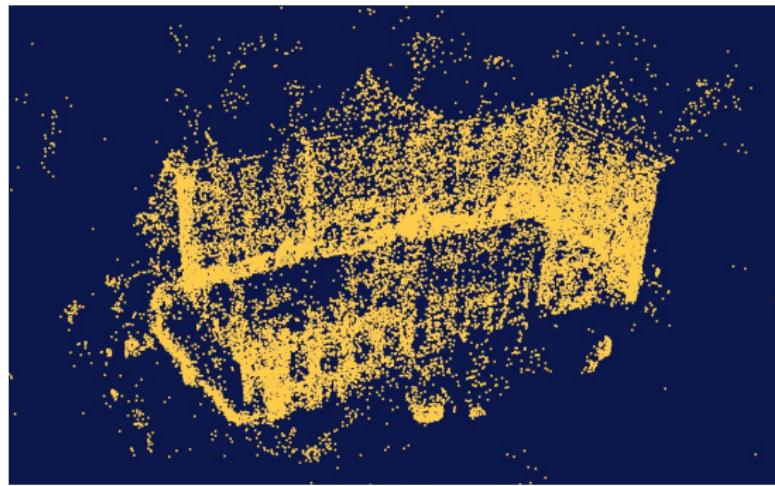


Figure : Sparse point cloud

# Dense Stereo Matching

- Depth map for each of the images
- Set of nearby images for matching

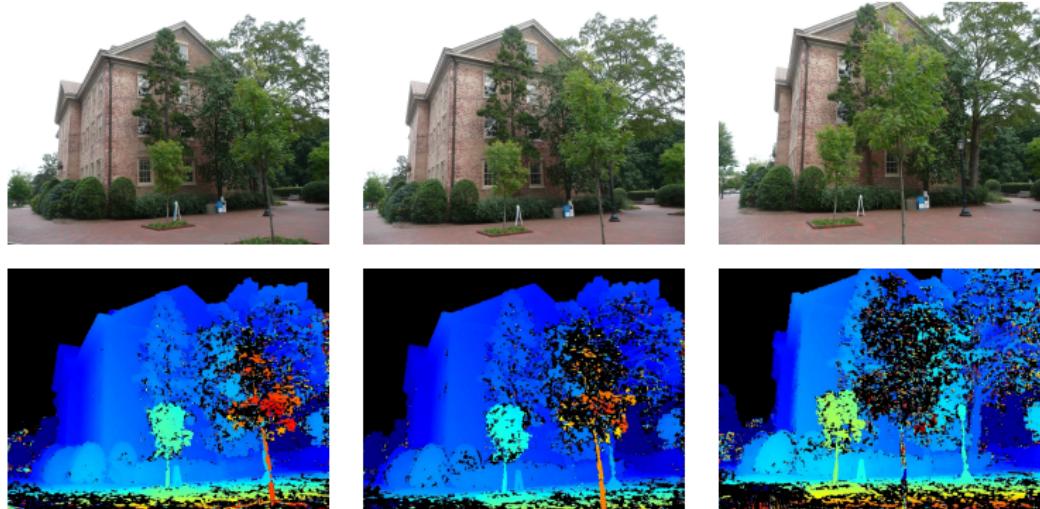
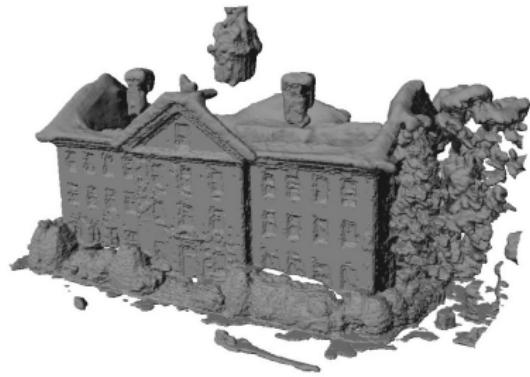
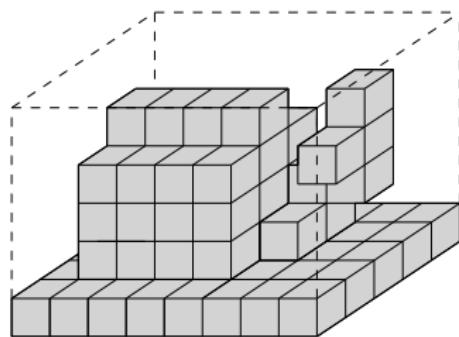


Figure : Some example depth maps (red close, blue far)

# Volumetric Depth Map Fusion

- Segmentation of a voxel space  $\Omega$  into free and occupied space:  $u(s) \in [0, 1]$   
[Curless and Levoy 1996]
- TV-Flux Fusion [Lempitsky and Boykov 2007, Zach 2008]
- Idea: convex energy
  - Unary term: Computed from depth maps
  - Smoothness term: Penalization of surface area



# TV-Flux Fusion: Energy

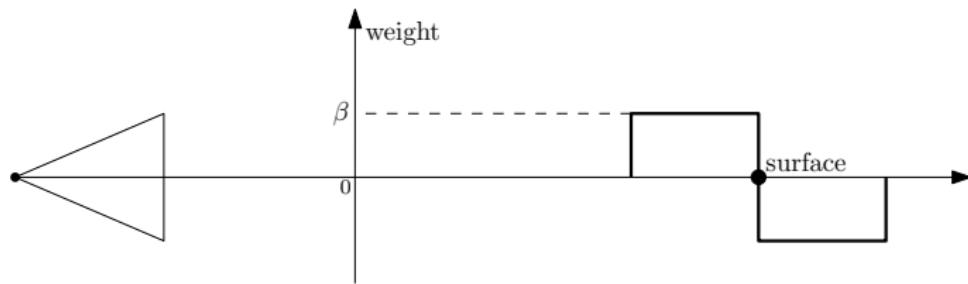
## Energy Minimization

$$\begin{aligned} E(u) &= \int_{\Omega} (\rho(s)u(s) + \|(\nabla u)(s)\|_2) ds \\ \text{s.t. } u(s) &\in [0, 1] \end{aligned}$$

- $u$  indicates occupied space
- Total variation (TV)  $\|(\nabla u)(s)\|_2$ 
  - Penalizes surface area [e.g. Chambolle et al. 2010]
- Unary data term:  $\rho(s)$
- Optimization: next part of the tutorial

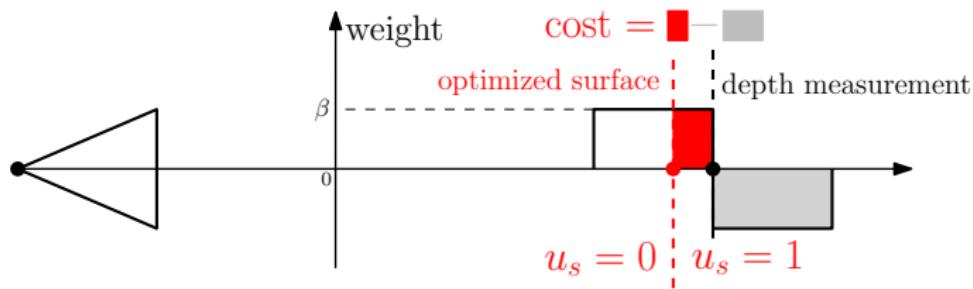
# TV-Flux Fusion: Data Term

- Computed from depth maps
- $\rho_s$ : sum of all the weights per voxel



# TV-Flux Fusion: Data Term

- Computed from depth maps
- $\rho_s$ : sum of all the weights per voxel



# TV-Flux Fusion: Some Results



Figure : Some volumetric fusion results

# Anisotropic Minimal Surfaces [Kolev et al. 2010]

- Surface normals estimated from the data used for regularization
- Helps to preserve surface details

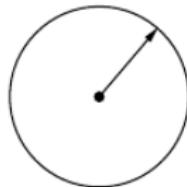


Pictures taken from [K. Kolev, T. Pock and D. Cremers ECCV 2010]

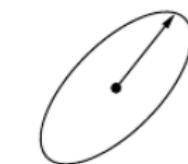
# Anisotropic Regularization: Energy [Kolev et al. 2010]

- Isotropic smoothness changed to anisotropic version

$$\|(\nabla u)(s)\|_2 \quad \rightarrow \quad \sqrt{(\nabla u)(s)^T D(s) (\nabla u)(s)}$$



isotropic metric



anisotropic metric

Figure taken from [K. Kolev, T. Pock and D. Cremers ECCV 2010]

# Anisotropic Regularization: Results [Kolev et al. 2010]



Figure taken from [K. Kolev, T. Pock and D. Cremers ECCV 2010]

# Volumetric 3D Reconstruction Recap

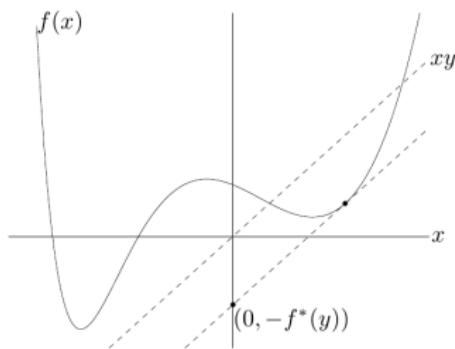
- Pose 3D reconstruction as volumetric segmentation problem
  - Free and occupied space
- Solve optimization problem
- Different data and regularization terms
  - Two examples presented
  - TV-Flux [Zach 2008]
  - Anisotropic regularization [Kolev et al. 2010]
- Many more volumetric reconstruction formulations exist
  - Without regularization:
    - Laser scans [Curless and Levoy 1996]
    - Kinect depth maps [Newcombe et al. 2011]
  - Data adaptive discretization of space [Labatut et al. 2007, Vu et al. 2009]

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- Convex conjugate

- $F : \mathbb{R}^n \rightarrow \mathbb{R} \cup \{\infty\}$
- $F^* : \mathbb{R}^n \rightarrow \mathbb{R} \cup \{\infty\}$ , convex conjugate of  $F$
- $F^*(y) = \sup_y \{y^T x - F(x)\}$
- $F^{**} = F$  iff  $F$  is convex and lower semi-continuous  
(Fenchel-Moreau Theorem)



Boyd and Vandenberghe 2004: "The conjugate function  $f^*(y)$  is the maximum gap between the linear function  $yx$  and  $f(x)$ , as shown by the dashed line in the figure."

Figure from [Boyd and Vandenberghe 2004]

- Convex conjugate

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(Fenchel-Moreau Theorem)

- Indicator function

- Let  $C$  be a nonempty subset of  $\mathbb{R}^n$

$$\iota_C(x) = \begin{cases} 0, & \text{if } x \in C \\ \infty, & \text{if } x \notin C \end{cases}$$

- Proximity operator (generalization of projection)

$$\text{prox}_{\tau F}(y) = \arg \min_x \left\{ \frac{\|x - y\|_2^2}{2\tau} + F(x) \right\}$$

- Let  $D$  be a nonempty closed convex subset of  $\mathbb{R}^n$
- $\text{prox}_{\iota_D}$ , projection to  $D$

# First-Order Primal-Dual Algorithm I

[Chambolle and Pock 2010, Pock and Chambolle 2011]

- Optimization problems of the form (primal)

$$\min_{z \in \mathbb{R}^n} F(Kz) + G(z)$$

- $K : \mathbb{R}^n \rightarrow \mathbb{R}^m$ , linear operator
- $G : \mathbb{R}^n \rightarrow \mathbb{R} \cup \{\infty\}$ , convex function
- $F : \mathbb{R}^m \rightarrow \mathbb{R} \cup \{\infty\}$ , convex function

- Primal-dual form

$$\min_{z \in \mathbb{R}^n} \max_{\xi \in \mathbb{R}^m} \langle Kz, \xi \rangle + G(z) - F^*(\xi)$$

- $\langle \cdot, \cdot \rangle$ , inner product
- $F^*$ , convex conjugate of  $F$

# First-Order Primal-Dual Algorithm II

[Chambolle and Pock 2010, Pock and Chambolle 2011]

- Optimization Problem

$$\min_{z \in \mathbb{R}^n} \max_{\xi \in \mathbb{R}^m} \langle Kz, \xi \rangle + G(z) - F^*(\xi)$$

- Initialization

$$(z^0, \xi^0) \in \mathbb{R}^n \times \mathbb{R}^m, \quad \bar{z}^0 = z^0, \quad \tau, \sigma > 0, \quad \theta \in [0, 1]$$

- Iteration

$$\xi^{n+1} = \text{prox}_{\sigma F^*}(\xi^n + \sigma K \bar{z}^n)$$

$$z^{n+1} = \text{prox}_{\tau G}(z^n - \tau K^T \xi^{n+1})$$

$$\bar{z}^{n+1} = z^{n+1} + \theta(z^{n+1} - z^n)$$

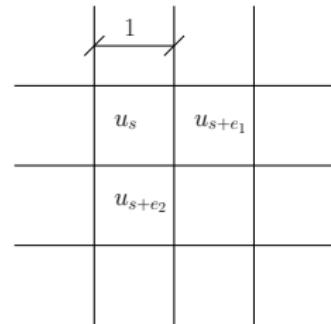
- Convergence:

- Globally convergent if  $\theta = 1$  and  $\tau \sigma \|K\|^2 < 1$

# Application to Volumetric 3D Reconstruction

- Discretize space into regular grid  
Cell length = 1
- Gradient: forward differences

$$(\nabla u)_s = \begin{pmatrix} u_{s+e_1} - u_s \\ u_{s+e_2} - u_s \\ \vdots \end{pmatrix} \quad e_1 = \begin{pmatrix} 1 \\ 0 \\ \vdots \end{pmatrix} \quad e_2 = \begin{pmatrix} 0 \\ 1 \\ \vdots \end{pmatrix}$$



- Discretized energy

$$E(u) = \sum_{s \in \Omega} \left\{ \|(\nabla u)_s\|_2 + \rho_s u_s + \iota_{[0,1]}(u_s) \right\}$$

- Rewrite as primal-dual saddle point problem → primal-dual formulation for TV

$$\|(\nabla u)_s\|_2 = \max_{\|p_s\|_2 \leq 1} \langle (\nabla u)_s, p_s \rangle$$

# Final Primal-Dual Saddle Point Problem

- General primal-dual problem

$$\min_{z \in \mathbb{R}^n} \max_{\xi \in \mathbb{R}^m} \langle Kz, \xi \rangle + G(z) - F^*(\xi)$$

- Primal-dual problem for TV-Flux

$$\min_u \max_p \sum_{s \in \Omega} \left\{ \langle (\nabla u)_s, p_s \rangle + \underbrace{\rho_s u_s + \iota_{[0,1]}(u_s)}_G - \underbrace{\iota_{\{\leq 1\}}(\|p_s\|_2)}_{F^*} \right\}$$

- Due to the structure prox operators can be applied point-wise
- $\text{prox}_{\tau G}(u_s) = \text{clamp}_{[0,1]}(u_s - \tau \rho_s)$
- $\text{prox}_{\sigma F^*(p_s)}$ : Euclidean projection to unit ball

# Example of Update in Code

- Update for  $p$  variables,  $p$  contained in  $\xi^{n+1}$
- $\xi^{n+1} = \text{prox}_{\sigma F^*}(\xi^n + \sigma K \bar{z}^n)$

```
#pragma omp parallel for
for (int z = 0; z < depth; z++)      // loop
    for (int y = 0; y < height; y++)   // over
        for (int x = 0; x < width; x++) // volume
    {
        // xi_n + sigma*K*z_bar
        const int X = x+1 < wdhth ? x+1 : x;
        const int Y = y+1 < height ? y+1 : y;
        const int Z = z+1 < depth ? z+1 : z;
        p_w(x,y,z) += sigma*(u_bar(X,y,z) - u_bar(x,y,z));
        p_h(x,y,z) += sigma*(u_bar(x,Y,z) - u_bar(x,y,z));
        p_d(x,y,z) += sigma*(u_bar(x,y,Z) - u_bar(x,y,z));

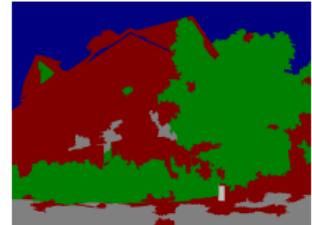
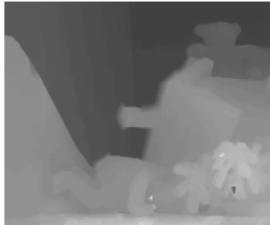
        // prox_sigma*F^*
        project_to_unit_ball(&p_w(x,y,z), &p_h(x,y,z), &p_d(x,y,z));
    }
```

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# Labeling Problems

- Given a set of nodes (pixels, superpixels, voxels)
- Goal: Assign one out of  $L$  labels to each node
- Local preference per node plus regularization
  - Energy minimization problem
- Omni present in computer vision
- Most multi-label ( $L > 2$ ) instances NP-hard



# Approaches

- Different solution approaches
  - Graph-Cuts
  - Belief propagation
  - Convex relaxation
  - ...
- Christian: Convex relaxation
  - Discrete domain (graphical model)
  - Continuous domain
- Lubor: Graph-Cuts

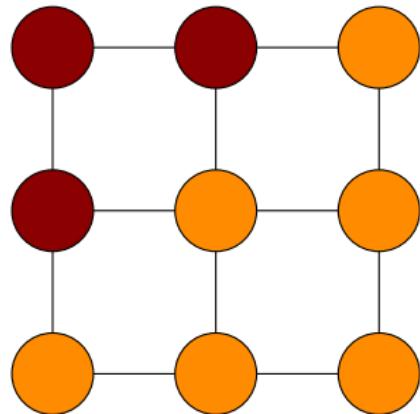
# Default Procedure

- Define a (non-convex) energy that solves the problem
- Relax the problem to a convex one
- In case of a continuous formulation, discretize
- Solution of the relaxed problem is approximation to the original one

# Discrete Domain

- Describe domain by a graph
  - For images, a node per pixel and edges to the neighbors

- Assign one out of  $L$  labels to each node
- $\theta_s^i$ : Cost for assigning label  $i$  at node  $s$
- $\theta_{st}^{ij}$ : Cost for assigning  $i$  at  $s$  and  $j$  at  $t$
- Find assignment that has minimal cost



# LP Relaxation I

- LP relaxation by Schlesinger 1976, review Tomas Werner 2007
- Objective

$$\sum_{s,i} \theta_s^i x_s^i + \sum_{s,t} \sum_{i,j} \theta_{st}^{ij} x_{st}^{ij}$$

- $\theta_s^i$  and  $\theta_{st}^{ij}$  cost for assigning a label or a transition
- $x_s^i \in \{0,1\}$  and  $x_{st}^{ij} \in \{0,1\}$  exact solution but non-convex problem

Example for position  $r$

$$x_r = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \begin{array}{l} \text{label 0} \\ \text{label 1} \\ \text{label 2} \end{array} \quad \rightarrow \quad \sum_i \theta_r^i x_r^i = \theta_r^1$$

- Indicator variables select data cost / smoothness cost

# LP Relaxation II

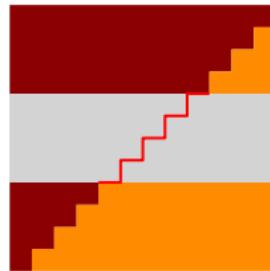
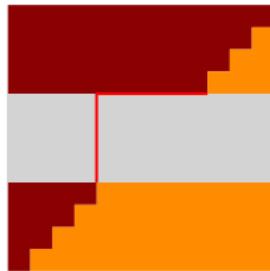
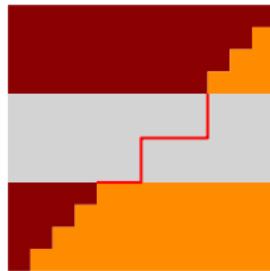
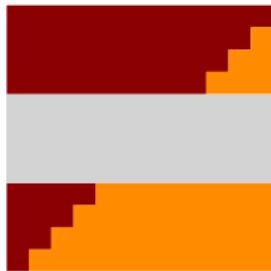
- LP relaxation by Schlesinger 1976, review Tomas Werner 2007

$$\begin{aligned} \min_x \quad & \sum_{s,i} \theta_s^i x_s^i + \sum_{s,t} \sum_{i,j} \theta_{st}^{ij} x_{st}^{ij} \\ \text{s.t. } & x_s^i = \sum_j x_{st}^{ij} \quad x_t^i = \sum_j x_{st}^{ji} \\ & \sum_i x_s^i = 1 \quad x_s^i \geq 0 \quad x_{st}^{ij} \geq 0 \quad \forall s, t, i, j \end{aligned}$$

- $\theta_s^i$  and  $\theta_{st}^{ij}$  cost for assigning a label or a transition
- $x_s^i \in \{0, 1\}$  and  $x_{st}^{ij} \in \{0, 1\}$  exact solution but non-convex problem
- Relaxed to linear program  $x_s^i \in [0, 1]$  and  $x_{st}^{ij} \in [0, 1]$
- Label assignment through thresholding

# Metrication artifacts

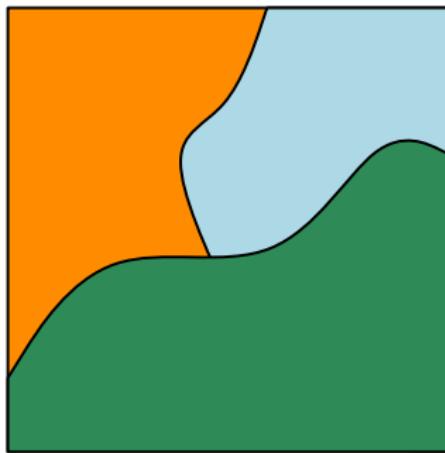
- Grid graph based representation
  - Smoothness cost
    - Measured by crossing edges
  - Inpainting example
  - Multiple equally good solutions



- Penalize true boundary length: Continuous formulation

# Continuous Formulation I

- Domain continuous (e.g. image plane), label space discrete
- Domain segmented into areas that have one out of  $L$  labels assigned



# Continuous Formulation II

- Continuous primal-dual saddle point energy  
[Chambolle et al. 2008, Lellmann and Schnörr 2010]
- Smoothness cost
  - Boundary length between labels  $i$  and  $j$  weighted by  $\theta^{ij}$
  - Needs to form a metric over the label-space



Figure from [Chambolle, Cremers, Pock SIIMS 2012]

# Continuously Inspired Formulation

- Our version: continuously inspired

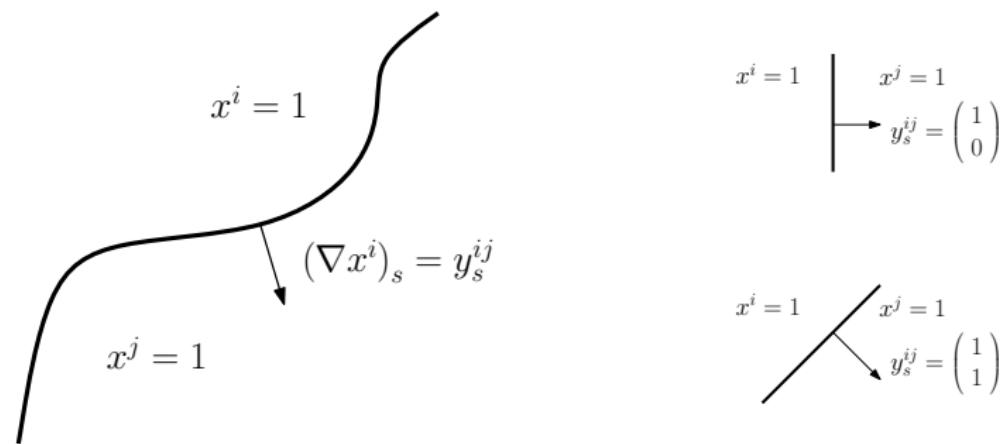
[Zach, Häne, Pollefeys, CVPR 2012, TPAMI 2014]

- Discretized pure primal formulation
- Extended to non-metric and anisotropic smoothness (more later)

$$\begin{aligned} \min_{x,y} \quad & \sum_{s,i} \theta_s^i x_s^i + \sum_s \sum_{i,j:i < j} \theta_s^{ij} \|y_s^{ij}\|_2 \\ \text{s.t.} \quad & (\nabla x^i)_s = \sum_{j:j < i} y_s^{ji} - \sum_{i:j > i} y_s^{ij} \\ & \sum_i x_s^i = 1 \quad x_s^i \geq 0 \quad \forall s, i, j \end{aligned}$$

# Interpretation of the variables $y_s^{ij}$

- Defined in the continuous domain
- Consider the following segmentation result (2D example)



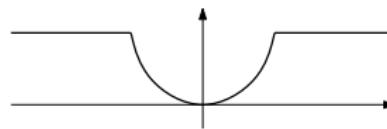
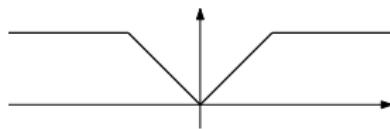
- $y_s^{ij}$  gradient of change from label  $i$  to label  $j$
- Important consequence
  - $y_s^{ij}$  aligned with normal direction of the boundary between  $i$  and  $j$

# Extensions I

- Original continuous formulation only metric smoothness

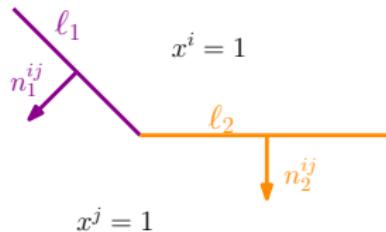
$$\theta_s^{ii} = 0 \quad \theta_s^{ij} \leq \theta_s^{ik} + \theta_s^{kj}$$

- Labels have an order
- Sum of transitions more expensive than direct transition
- Truncated linear is metric
- Truncated quadratic is not metric



- Anisotropic smoothness sometimes desired
  - Aligning segmentation boundary direction with image edges
  - Well known for binary segmentation [Esedoglu and Osher 2004]
  - Can directly be used in the continuous multi-label formulation [Strelakovsky and Cremers 2011]
- Our extended formulation:
  - non-metric and anisotropic smoothness

# Anisotropic Smoothness Term



- Goal: Penalize boundary length  $\ell$  weighted by its direction  $n$
- Exchange  $\theta_s^{ij} \|y_s^{ij}\|_2$  by  $\phi_s^{ij}(y_s^{ij})$  [Eseedoglu and Osher 2004]
- $\phi^{ij}(\cdot): \mathbb{R}^N \rightarrow \mathbb{R}_0^+$  is a convex positively 1-homogeneous function
  - How do we specify such functions? Next slide
- $n_s^{ij}$  normal of boundary between labels  $i$  and  $j$  at position  $s$
- Regularizer penalizes boundary length  $\ell$  weighted by  $\phi^{ij}(n_s^{ij})$

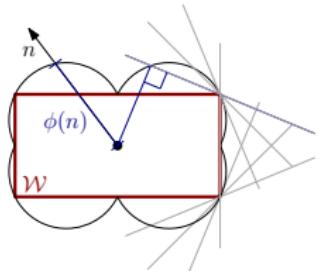
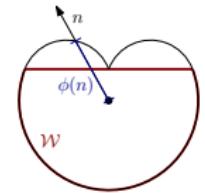
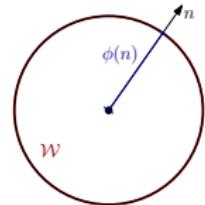
$$\sum_{s \in \Omega} \sum_{k,l:k < l} \phi^{kl}(y_s^{kl}) = \ell_1 \phi^{ij}(n_1^{ij}) + \ell_2 \phi^{ij}(n_2^{ij})$$

# Wulff Shape

- Specifying a function  $\phi(\cdot)$  directly can be hard
- Solution: Wulff shape  $\mathcal{W}_\phi$   
[Eshedoglu and Osher 2004]
  - Convex shape
  - Originates from crystallography [Wulff 1901]
- All possible  $\phi(\cdot)$  can be specified by

$$\phi(y) = \max_{\mu \in \mathcal{W}_\phi} \mu^T y$$

- Defining  $\phi(\cdot)$  through  $\mathcal{W}_\phi$  often easier
- Wulff shapes can be learned from training data



- LP-relaxation does allow for arbitrary smoothness
- Continuously inspired formulation allows only for metrics
- Where is the difference?
  - LP relaxation contains  $x_{st}^{ij}$  variables that have to be non-negative
  - Continuously inspired formulation contains  $y_s^{ij}$  that are in  $[-1, 1]^N$
  - And hence, no non-negative  $x_s^{ii}$
- Fixed by introducing non-negative pseudo-marginals
  - Split positive and negative part of  $y_s^{ij}$  into individual variables
  - $x_s^{ij} := \max\{0, y_s^{ij}\}$  and  $x_s^{ji} := -\min\{0, y_s^{ij}\}$
  - $y_s^{ij} = x_s^{ij} - x_s^{ji}$  still present in the formulation
  - $x_s^{ii}$  and non-negativity constraint added
  - This allows non-metric smoothness for the discretized case

# Final Convex Multi-Label Formulation

- The final formulation allows for non-metric and anisotropic smoothness at the same time

$$\begin{aligned} \min_x \quad & \sum_{s,i} \theta_s^i x_s^i + \sum_s \sum_{i,j:i < j} \phi_s^{ij} (x_s^{ij} - x_s^{ji}) \\ \text{s.t. } & x_s^i = \sum_j (x_s^{ij})_k \quad x_s^i = \sum_j (x_{s-e_k}^{ji})_k \\ & \sum_i x_s^i = 1 \quad x_s^i \geq 0 \quad x_s^{ij} \geq 0 \quad \forall s, i, j, k \end{aligned}$$

- $e_k$ :  $k$ -th canonical basis vector

# Summary

- LP-Relaxation
  - Arbitrary smoothness cost
  - Metrication artifacts
- Continuous formulation
  - Penalizes boundary length
  - Only metric smoothness
- Extensions (continuously inspired formulation)
  - Anisotropic and non-metric smoothness

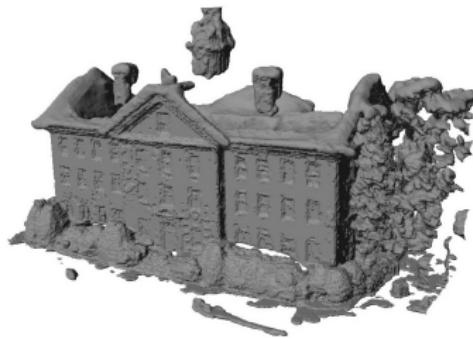
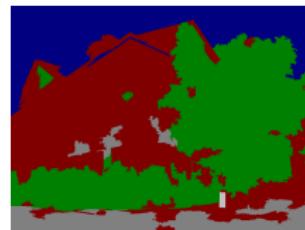
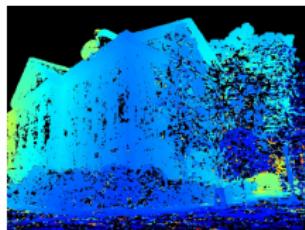
# Outline

- 1 Volumetric 3D Reconstruction
- 2 Convex Optimization
- 3 Convex Continuous Multi-Label Formulation
- 4 Joint 3D Scene Reconstruction and Class Segmentation
- 5 Class Specific 3D Object Shape Priors
- 6 Segment Based 3D Object Shape Priors
- 7 Articulated and Restricted Motion
- 8 Depth Estimation with a Surface Normal Classifier

C. Häne, C. Zach, A. Cohen, R. Angst, M. Pollefeys, **Joint 3D Scene Reconstruction and Class Segmentation**, CVPR 2013

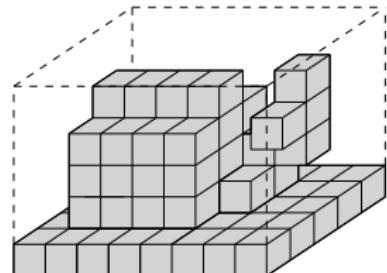
# Idea

- Two challenging problems
  - Image segmentation
  - Dense 3D modeling
- Object category influences desired surface smoothness
- Optimize both jointly

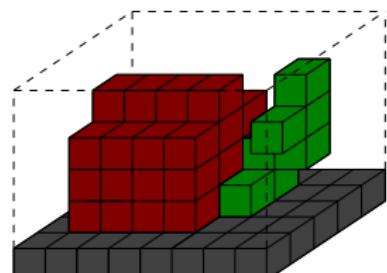


# Formulation

- Baseline Method: Volumetric depth map fusion
  - Segmentation of a voxel space into free and occupied space:  $u_s \in [0, 1]$
- Our joint fusion
  - Labeling of a voxelspace into  $L$  labels:  
 $x_s^i \in [0, 1]$  and  $\sum_i x_s^i = 1$
  - We use: **free space**, **building**, ground, **vegetation**, clutter
- Convex Energy
  - Unary term
    - Connects image appearance and depth maps
  - Smoothness term
    - Dependent on surface orientation and involved labels



Baseline



Proposed

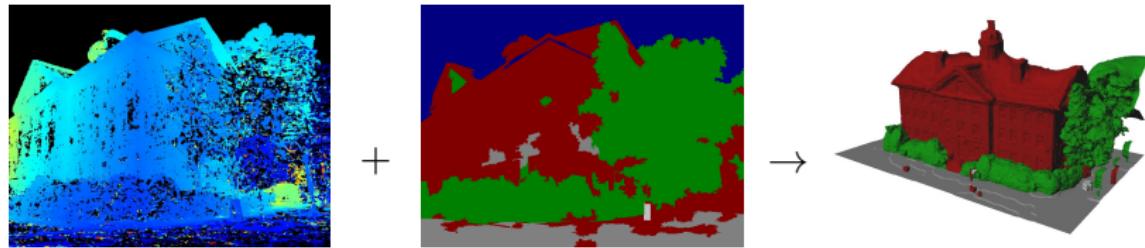
# Joint Fusion: Inference Overview

- Input



- Camera poses
- Vertical direction [Cohen et al. 2012]

- Joint Fusion



## Energy Minimization

$$E(x, y) = \sum_{s \in \Omega} \left( \sum_i \rho_s^i x_s^i + \sum_{i,j:i < j} \phi^{ij}(y_s^{ij}) \right)$$

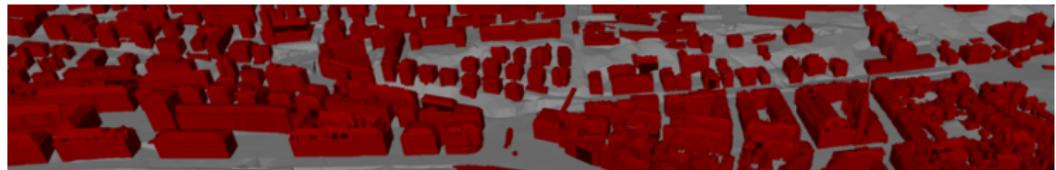
- Subject to marginalization and normalization constraints
- $x_s^i \in [0, 1]$ : indicating whether label  $i$  is chosen at voxel  $s$
- $y_s^{ij} \in [-1, 1]^3$ : label transition gradients
- $\rho_s^i$ : unary term, cost for label  $i$  at location  $s$
- $\phi^{ij}$ : anisotropic surface area penalization between label  $i$  and  $j$
- Optimized using primal-dual algorithm [Chambolle and Pock 2011]

# Input to Energy

- For unary term: image based classifier + depth maps



- For smoothness term: geometric priors (anisotropic smoothness)



# Appearance Likelihoods

- Image based semantic classifier:
  - Boosted decision tree classifier  
[STAIR Vision Library, Gould et al. 2010]
  - Context-based classifier  
[ALE Library, Ladicky et al. ICCV 2009]
- 5 classes: sky, building, ground, vegetation, clutter
- Negative log likelihoods  $\sigma_{\text{class } i}$ , per image pixel

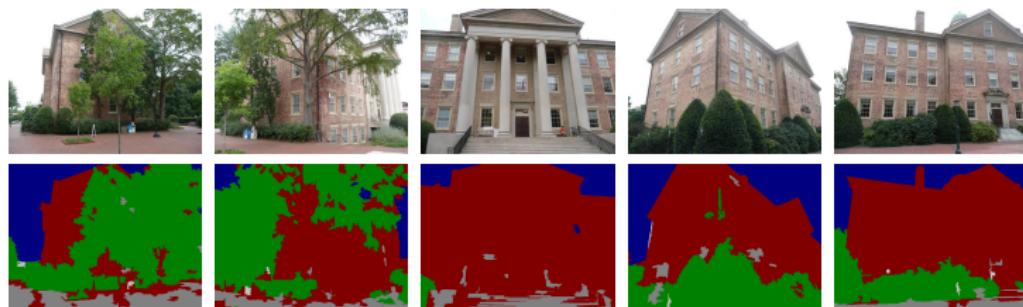
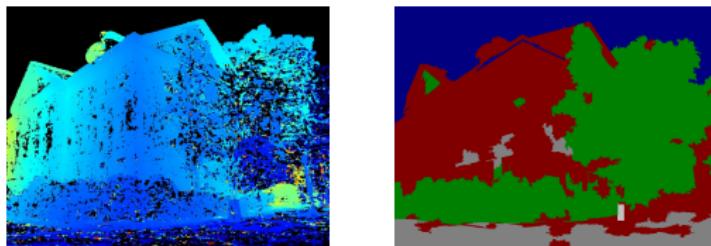
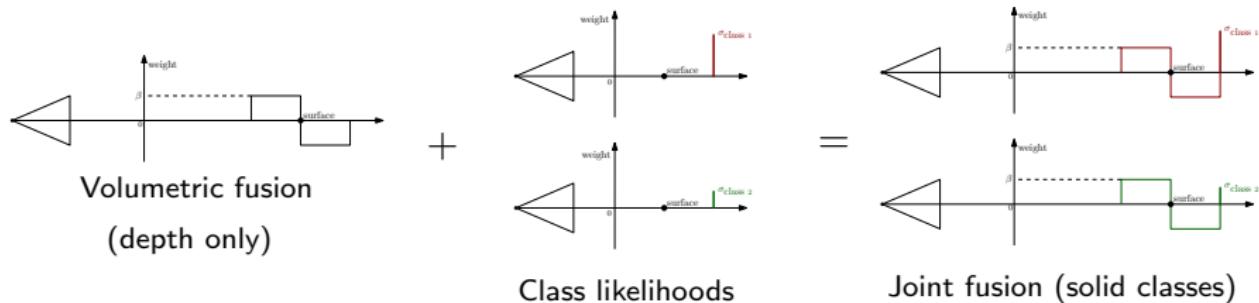


Figure : Best cost labels, STAIR Vision Library

# Unary Term I

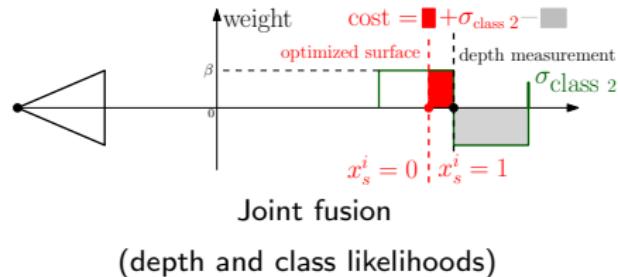
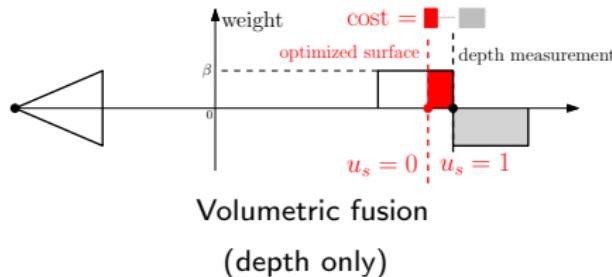


- Computed from depth maps and class likelihoods



# Unary Term II

- Cost induced for a single ray



- Cost for free-space indirect through solid classes
- High likelihood for sky but no depth  $\rightarrow$  free space preference along ray
- Approximates true ray likelihoods faithfully for the important cases
  - Assuming only one transition occurs in the band around the measured depth

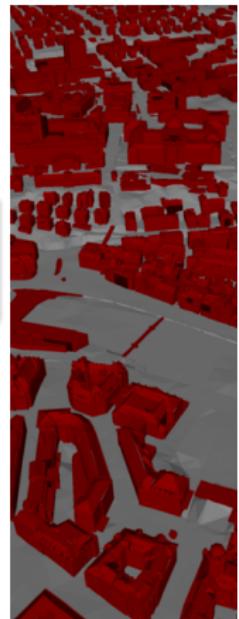
# Penalization According to Normal Orientation

- Vertical ground free space transition penalized differently than e.g. building free space transition

## Convex Smoothness Term

$$\psi^{ij}(\cdot; \theta^{ij}) : \mathbb{S}^2 \rightarrow \mathbb{R}_0^+$$

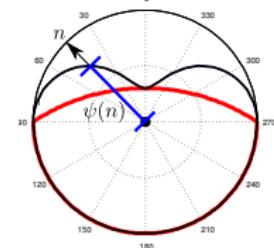
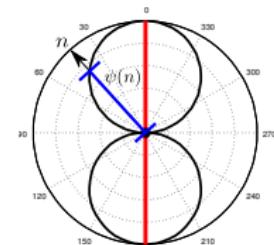
- Direction dependent cost of normal
  - Parameterized according to parameters  $\theta^{ij}$
  - Cadastral 3D city model as training data
  - Maximum-likelihood estimation (MLE)



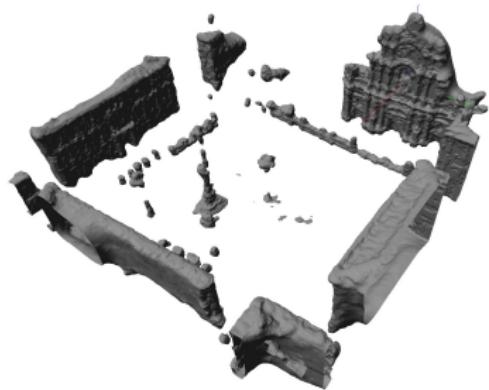
# Smoothness term $\phi^{ij}$

- Split into isotropic and anisotropic part
- Isotropic part: relative frequency of transitions
  - Independent of normal  $n$
  - Determined as constant  $C^{ij}$
- Anisotropic part:  
Selected from a collection of Wulff shapes  $W_{\psi^{ij}}$ 
  - Line Segment
  - Half-sphere plus spherical cap
- Parameters of Wulff shape trained via MLE
  - Which shape
  - Shape parameters: grid search

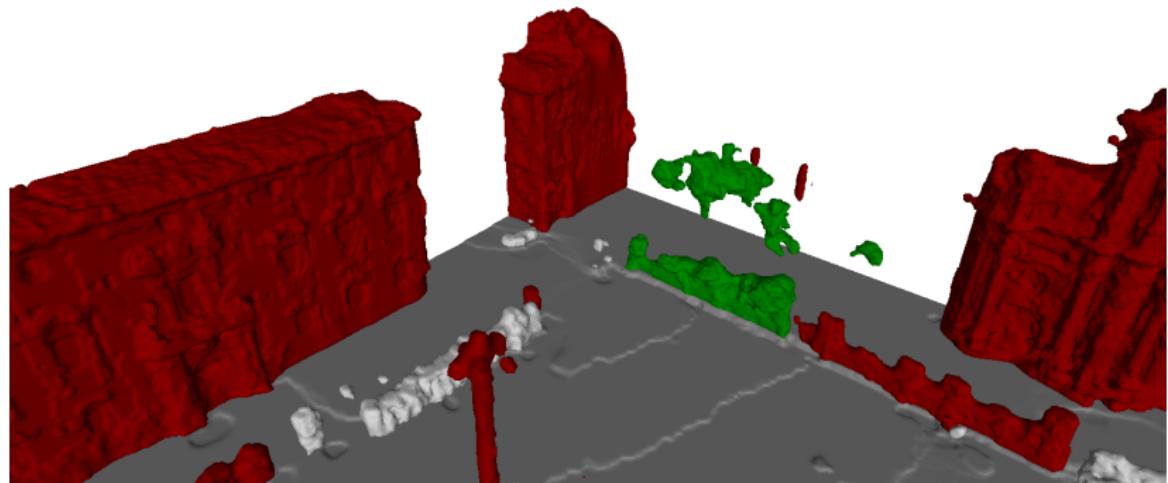
$$\phi^{ij}(n) = \psi^{ij}(n) + C^{ij}$$



# Evolution During Optimization

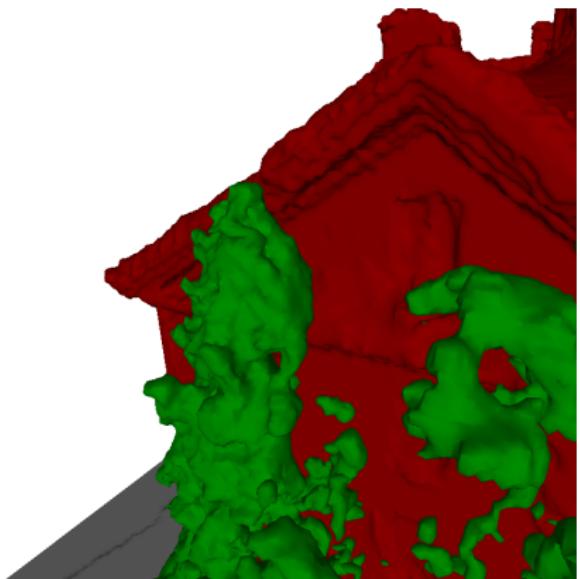
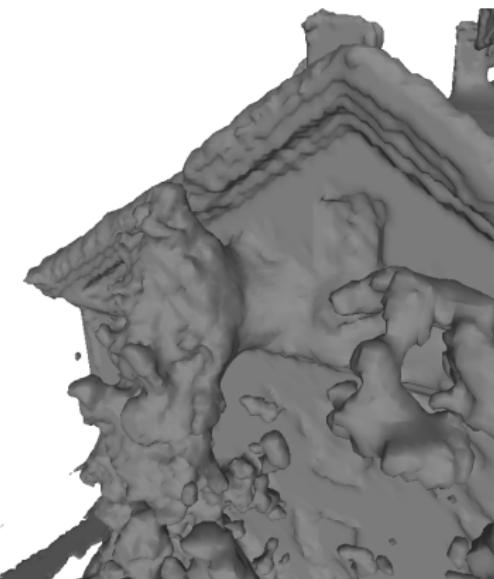


# Weakly Observed Surfaces I



- Ground and buildings correctly reconstructed

## Weakly Observed Surfaces II

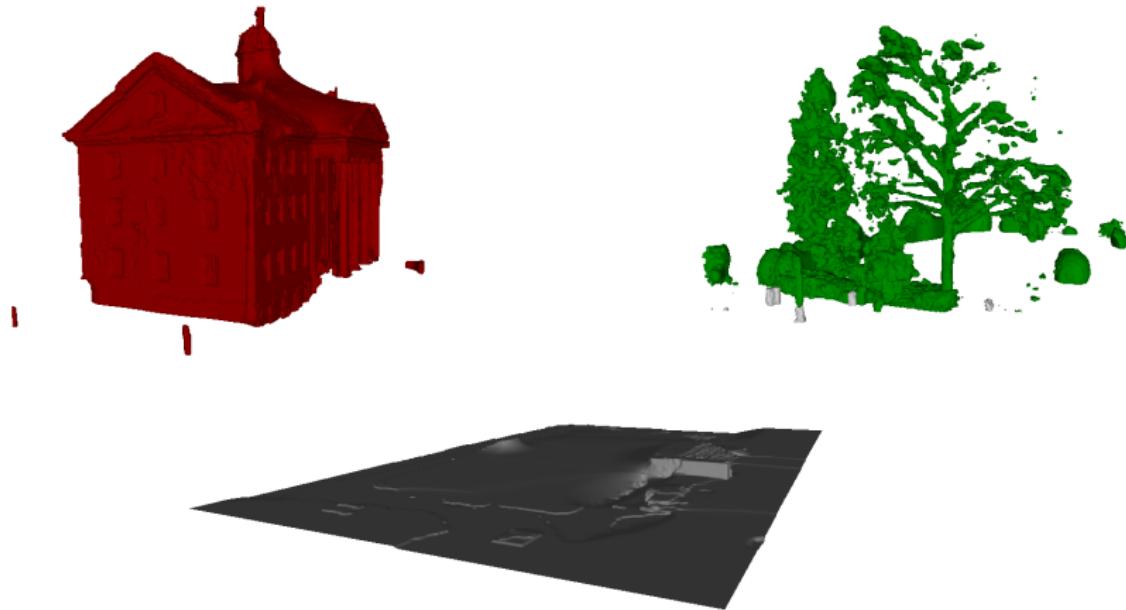


- Vegetation correctly separated from the building

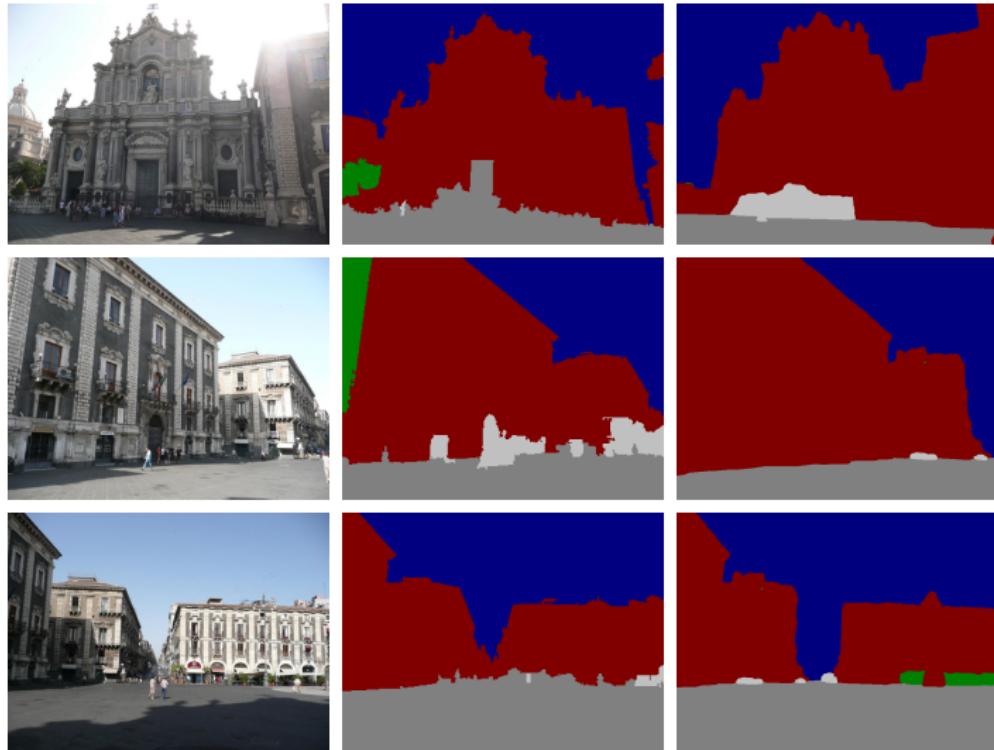
# Unobserved Surfaces



# Unobserved Surfaces



# Advantages to Image Based Segmentation I

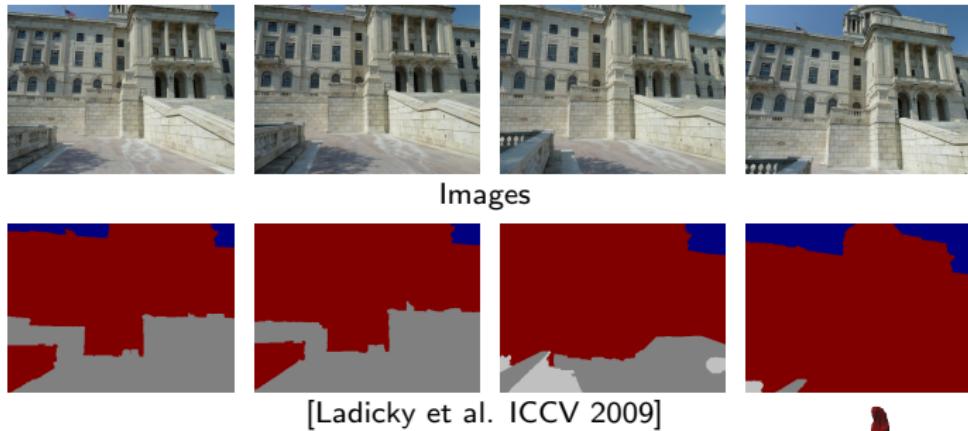


Input Image

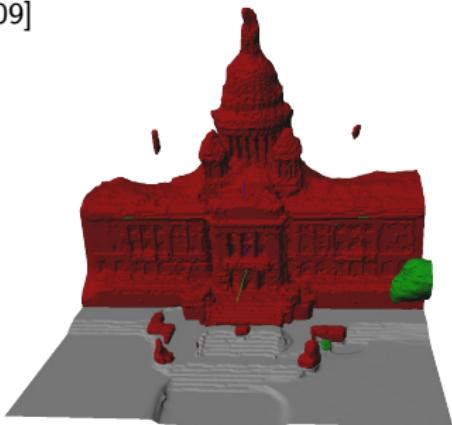
[Ladicky ICCV 2009]

Joint Fusion

# Advantages to Image Based Segmentation II



- Single image segmentations not consistent
  - Ambiguous cases eg. ground or building
- Joint fusion disambiguates



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Joint Fusion

# Strength of the Relaxation

- Relaxation: integral solution not guaranteed
- Experiments show relaxation is strong
  - Similar relaxations experimentally shown to be strong  
[Chambolle et al. 2008]

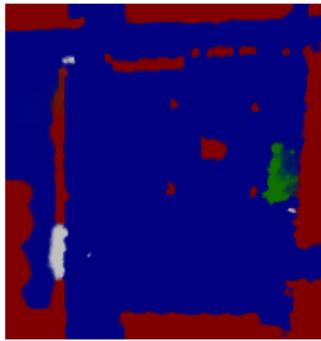


Figure : Slice through volume shows that relaxation is strong

# Comparison to Volumetric Fusion

# Conclusion: Joint Reconstruction and Class Segmentation

- Framework to jointly optimize for reconstruction and segmentation
  - To our knowledge: first volumetric approach
- Geometry and class segmentation coupled tightly
  - Previous joint methods on depth maps use height only  
[Ladicky et al. 2012]
  - We use priors on the normal direction
- Convex energy can be optimized globally
- Relaxation strong
- Geometric priors trained from cadastral 3D model
- Joint formulation improves best cost labeling and geometry

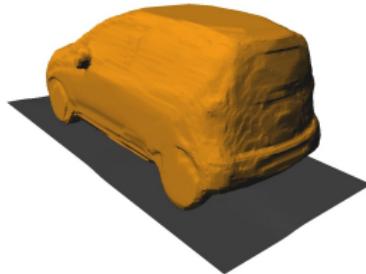
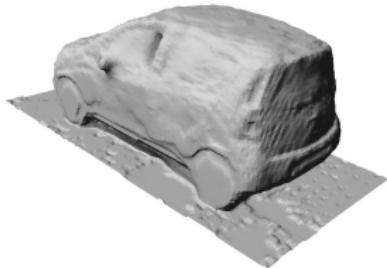
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C. Häne, N. Savinov, M. Pollefeys, **Class Specific 3D Object Shape Priors Using Surface Normals**, CVPR 2014

# Idea

- Some object classes hard to reconstruct
  - Lack of texture
  - Transparency
  - Reflection
- Solution: shape prior
  - Shapes within object class similar
  - Local distribution of surface normals
  - Roof of car always close to horizontal



# Formulation

- Baseline Method: Volumetric depth map fusion
  - Segmentation of a voxel space into free and occupied space
- Shape prior formulation
  - Voxel space **aligned** with object of **known class**
  - 3 labels
    - free space, ground, **object**
- Unary term
  - Computed from depth maps, local preference for solid class
- Smoothness term
  - Dependent on surface orientation, position and involved labels



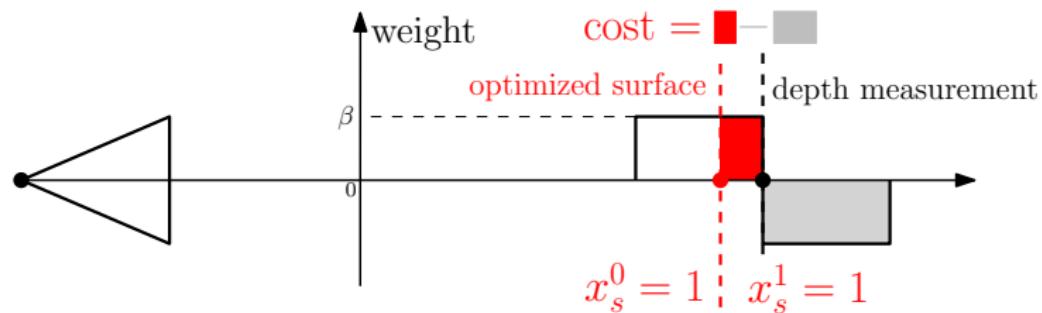
# Energy Formulation

## Minimization

$$E(x, y) = \sum_{s \in \Omega} \left( \sum_i \rho_s^i x_s^i + \sum_{i,j: i < j} \phi_{\textcolor{red}{s}}^{ij}(y_s^{ij}) \right)$$

- Subject to marginalization and normalization constraints
- $x_s^i \in [0, 1]$ : indicating whether label  $i$  is chosen at voxel  $s$
- $y_s^{ij} \in [-1, 1]^3$ : label transition gradients
- $\rho_s^i$ : unary term, cost for label  $i$  at location  $s$
- $\phi_{\textcolor{red}{s}}^{ij}$ : anisotropic surface area penalization between label  $i$  and  $j$

# Unary data term



- Both solid classes use same data cost
- Label chosen solely based on the shape prior

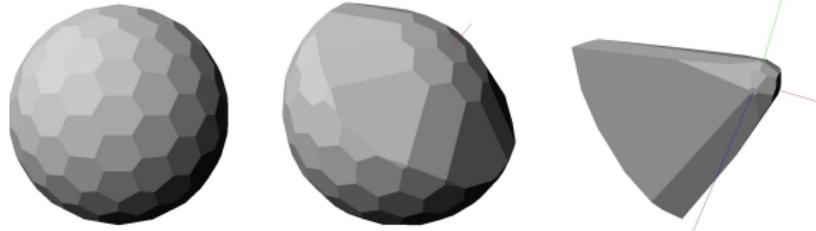
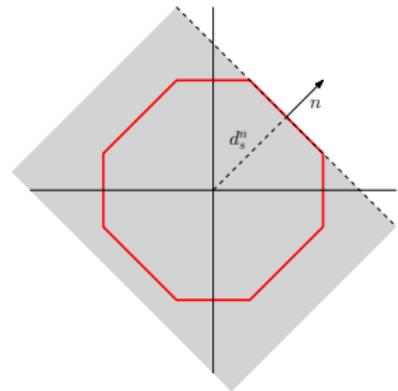
# Smoothness Term (Shape Prior Training)



- Training data aligned
- Per voxel  $s$ 
  - Collect normals of all training samples
  - Generate histogram over normal directions  $\rightarrow P(n_s)$
  - Bins predefined (regular sampling of sphere)
  - Define anisotropic smoothness  $\phi_s$  that reflects the distribution (next slide)
- Transition taken into account
  - free space  $\leftrightarrow$  object
  - ground  $\leftrightarrow$  object

# Discrete Wulff Shape

- Parameterization for  $W_{\phi_s}$ :  
Intersection of half spaces
  - $n$  half space normal
  - $d_s^n$  distance of half-space boundary to origin
- It holds  $\phi_s(n) = d_s^n$  [Eseedoglu and Osher 2004]
- We set  $d_s^n \leftarrow -\log(P_s(n))$

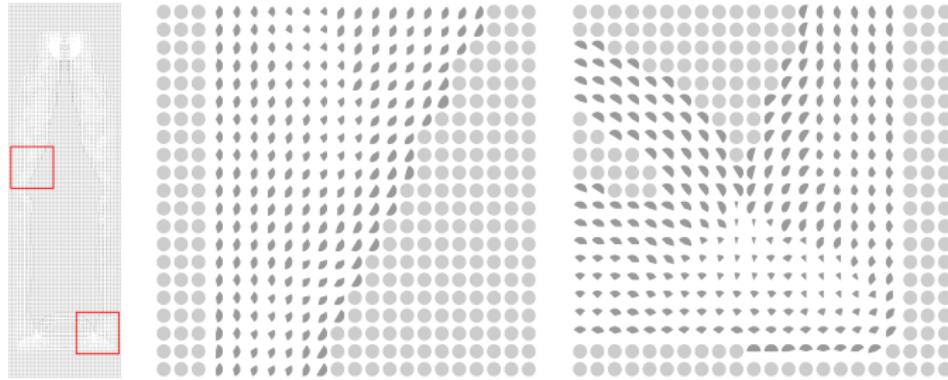


# Visualization of the shape prior

- Training data



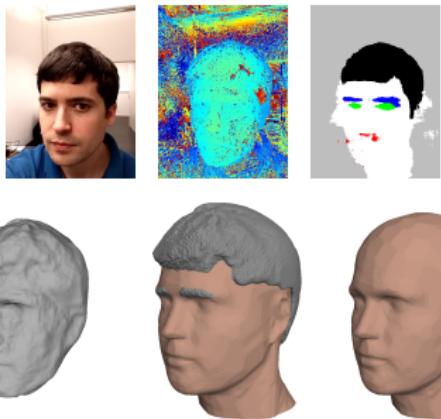
- Vertical cut through the shape prior



# Results

# Semantic Head Reconstruction

- Low resolution images captured on mobile phone with rolling shutter
- Image based classifier necessary
- Manual alignment not possible



F. Maninchedda, C. Häne, B. Jacquet, A. Delaunoy, M. Pollefeys, **Semantic Reconstruction of Heads**, submitted

# Energy Formulation with Non-Convex Alignment

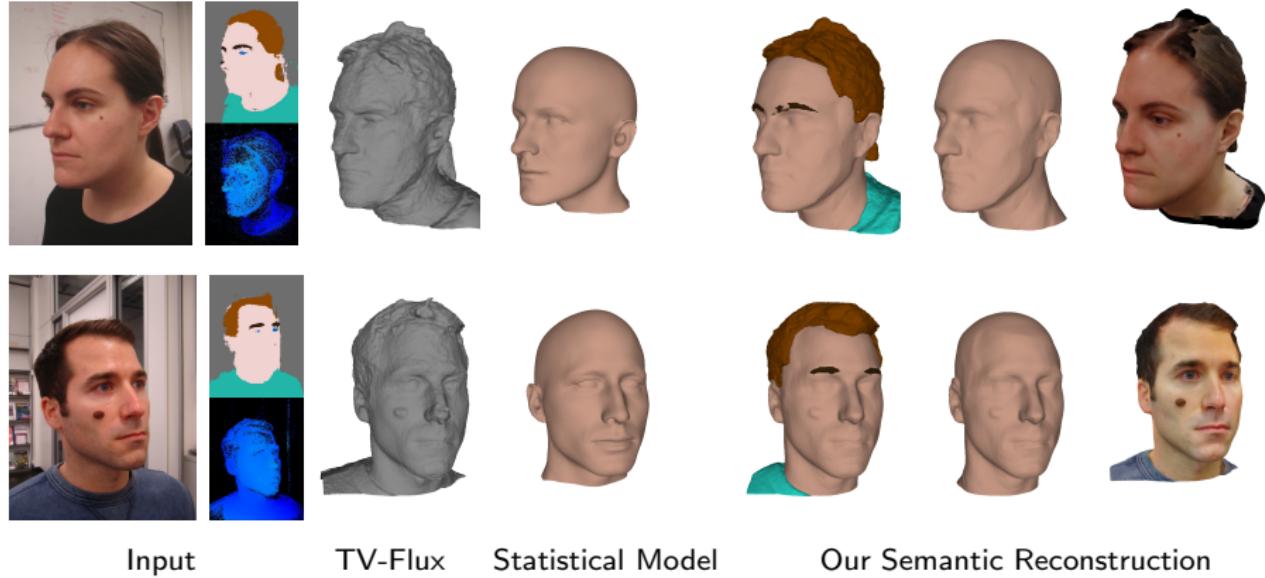
## Minimization

$$E(x, y) = \sum_{s \in \Omega} \left( \sum_i \rho_s^i(\mathcal{T}) x_s^i + \frac{1}{\alpha^2} \sum_{i,j:i < j} \phi_s^{ij}(\mathcal{T}, y_s^{ij}) \right)$$

- Subject to marginalization and normalization constraints
- $x_s^i \in [0, 1]$ : indicating whether label  $i$  is chosen at voxel  $s$
- $y_s^{ij} \in [-1, 1]^3$ : label transition gradients
- $\rho_s^i$ : unary term, cost for label  $i$  at location  $s$
- $\phi_s^{ij}$ : anisotropic surface area penalization between label  $i$  and  $j$
- $\mathcal{T}$ : similarity transform with scaling factor  $\alpha$
- Alternating optimization (reconstruction, alignment)

# Overview of the Method

# Comparisons



Input

TV-Flux

Statistical Model

Our Semantic Reconstruction

# Conclusion: Class Specific 3D Object Shape Priors

- Shape prior based on surface normals
- Allows for shape variation
- Preserves expected surface details
- Recovers hidden surfaces
  - Ground underneath the car
  - Skin underneath hair
- Support points are directly inferred by the optimization
- Shortcomings
  - Alignment between shape prior and input data
  - Different Wulff shape at each voxel

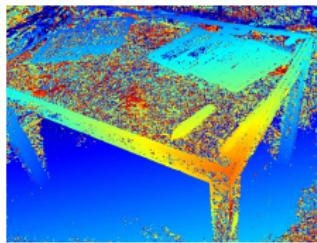
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R. Karimi, C. Häne, M. Pollefeys, **Segment Based 3D Object Shape Priors**, CVPR 2015

# Idea

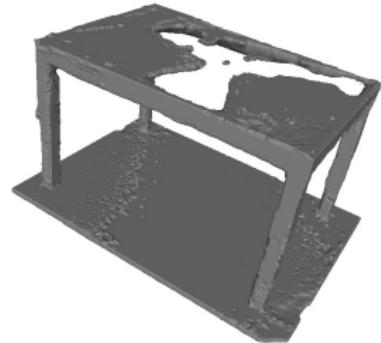
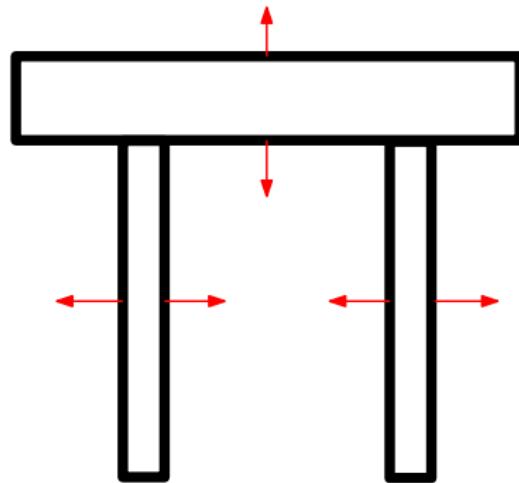
- Idea: Split object into convex segments
- Global (spatially homogeneous) smoothness terms
- Alignment only in terms of rotation



# Splitting

- Why splitting?

- Table top mostly horizontal
- Table legs mostly vertical
- Single anisotropic smoothness not distinctive
- Individual segments very distinctive



# Energy Formulation

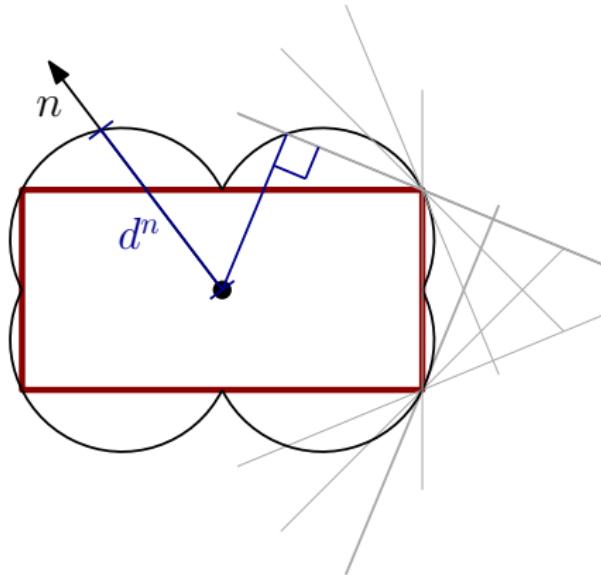
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- $\rho_s^i$ : unary term, cost for label  $i$  at location  $s$
- $\phi^{ij}$ : anisotropic surface area penalization between label  $i$  and  $j$ 
  - For this formulation spatially homogeneous

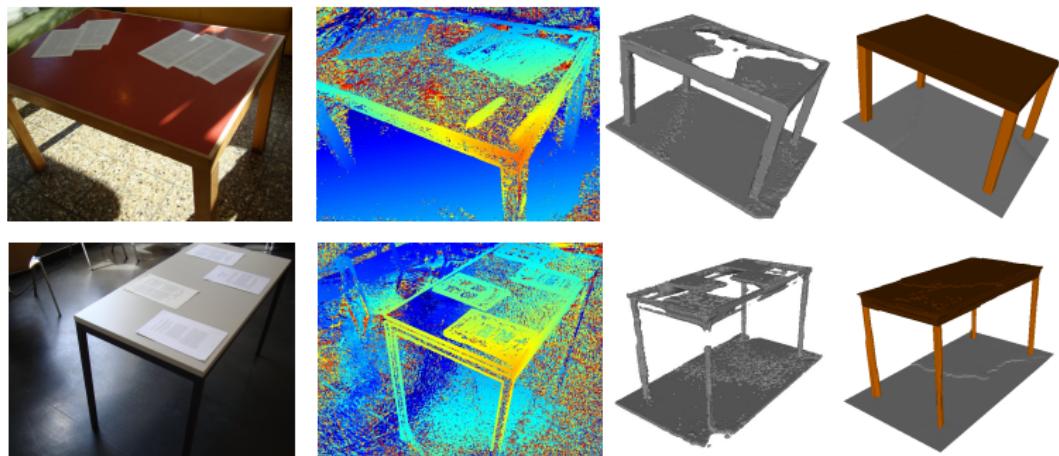
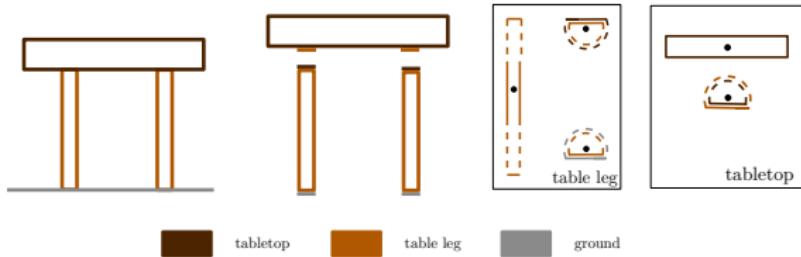
# Smoothness Term

- Reconstructed shape has lowest energy for given volume, if it has the same shape as the Wulff shape

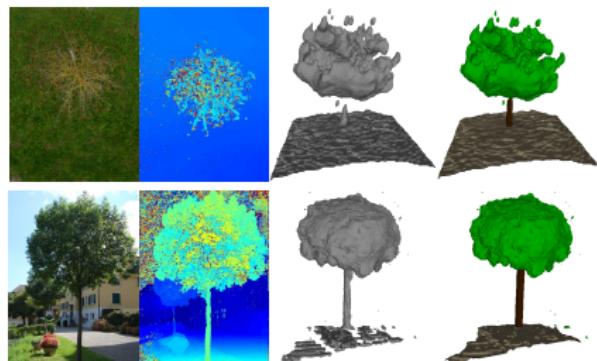
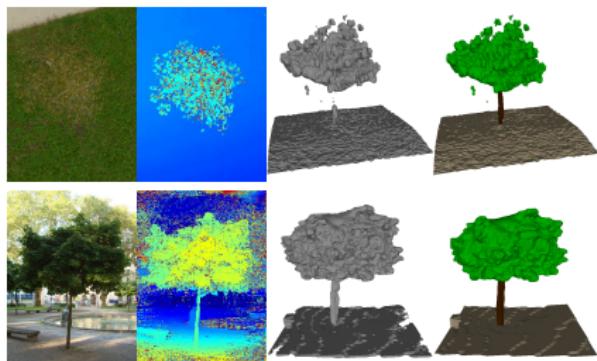
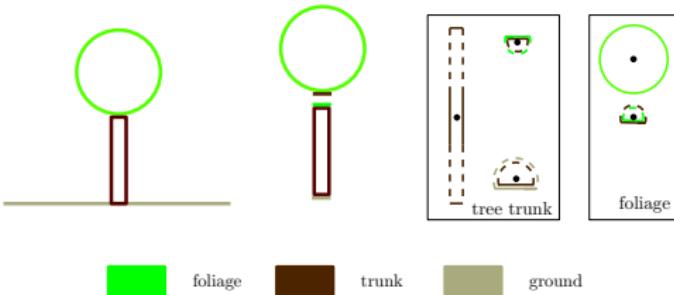


- Multiple transitions per segment
  - E.g. table top connects to leg and free space

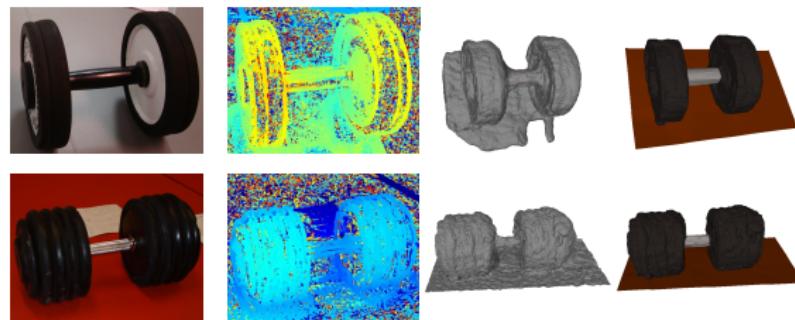
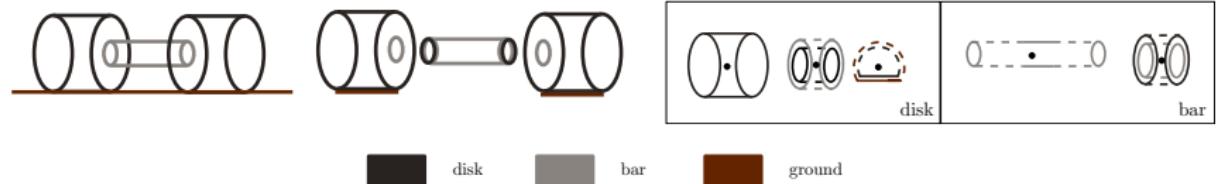
# Results: Table



# Results: Tree

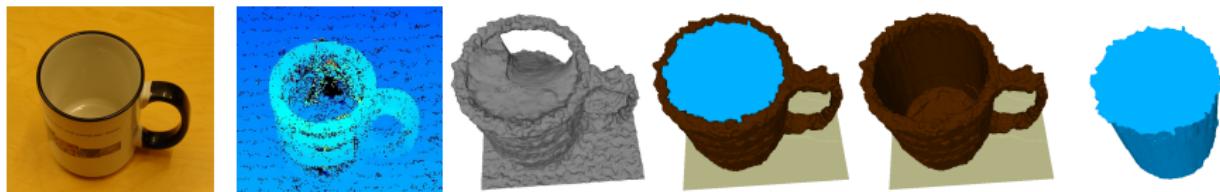
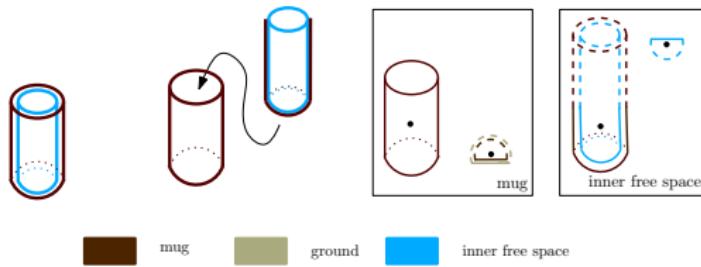


# Results: Dumbbell



# Results: Mug

- Convex segment of free space inside the mug



# Video

# Conclusion: Segment Based 3D Object Shape Priors

- Through splitting object into convex segments
  - Global (spatially homogeneous) smoothness term
  - Alignment only in terms of rotation
- Non-convex objects
  - Splitting free space

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B. Jacquet, C. Häne, R. Angst, M. Pollefeys, **Multi-Body Depthmap Fusion with Non-Intersection Constraints**, ECCV 2014

# Problem Setting

- Rigid objects move around
- Camera poses with respect to both rigid objects can be computed
- Goal: Volumetric reconstruction of both rigid objects

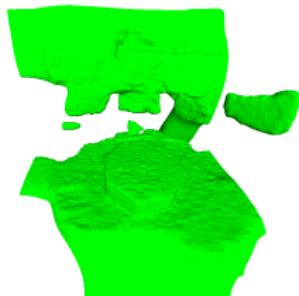


# Volumetric 3D Reconstruction

- Volumetric Reconstruction for each object independently



Background



Box

- Problem: Objects do not separate
- Solution: Enforce that occupied voxels do not intersect

$$\sum_{o \in \mathcal{O}, t \in \mathcal{T}} x_{ts}^o \leq 1$$

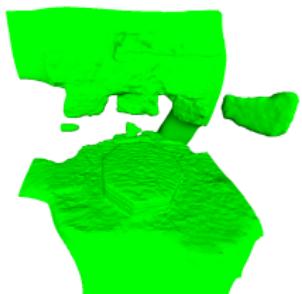
# Result Box



Without constraints



Background

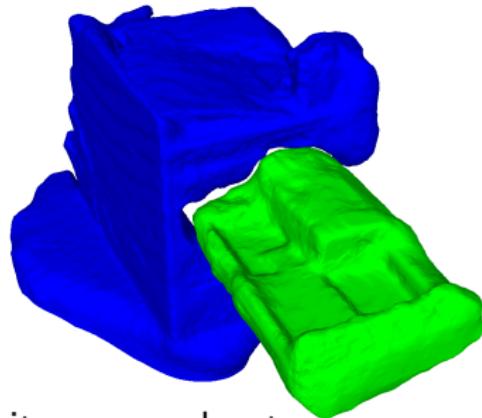


Box



With constraints

# Result Drawer



- Interior of the furniture carved out

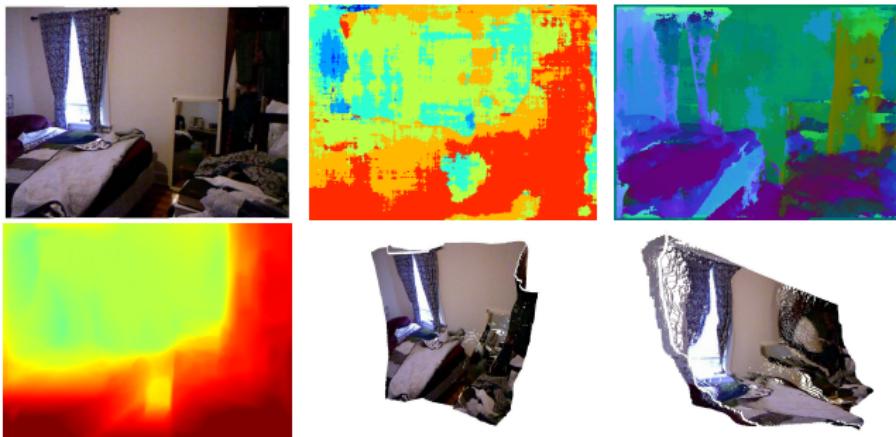
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C. Häne, L. Ladicky, M. Pollefeys, **Direction Matters: Depth Estimation with a Surface Normal Classifier**, CVPR 2015

# Idea

- Idea: Combine single view depth and normal estimates
- Leads to more faithful single view depth estimates

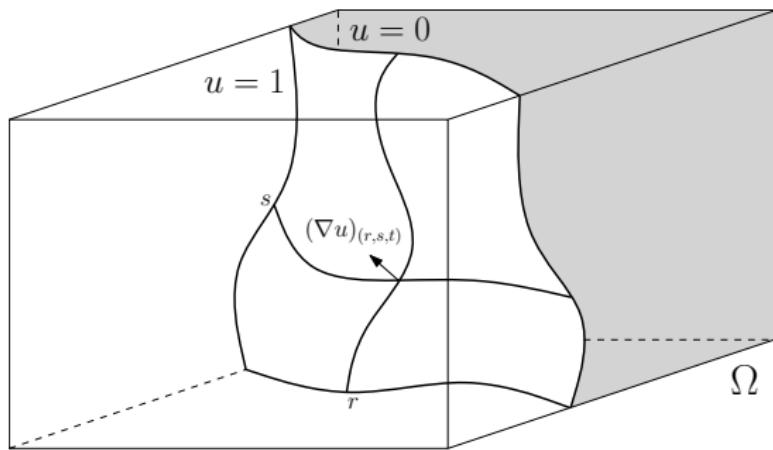


(top row) input to method, (bottom row) results

# Formulation

- Task: Each pixel gets a depth label assigned
- 2D multi-label problem lifted to 3D two-label problem [Roy and Cox, 98]
  - Instead of assigning label  $\ell_{(r,s)}$  to pixel  $(r,s)$
  - Segmenting volume  $\Omega$  into before/behind assigned depth label

$$u_{(r,s,t)} = \begin{cases} 0 & \text{if } \ell_{(r,s)} < t \\ 1 & \text{else,} \end{cases}$$



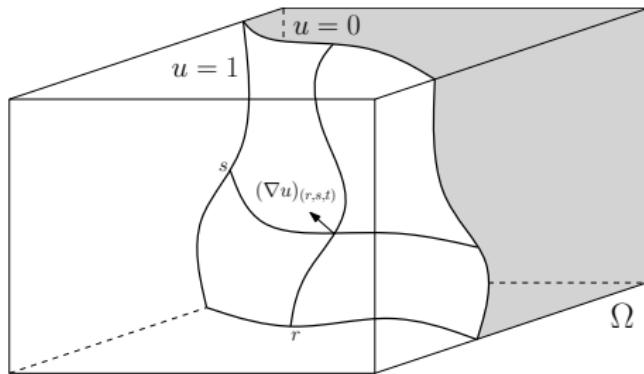
# Convex Energy

## Energy Minimization

$$E(u) = \sum_{r,s,t} \left\{ \rho_{(r,s,t)} |(\nabla_t u)_{(r,s,t)}| + \phi_{(r,s,t)} ((\nabla u)_{(r,s,t)}) \right\}$$

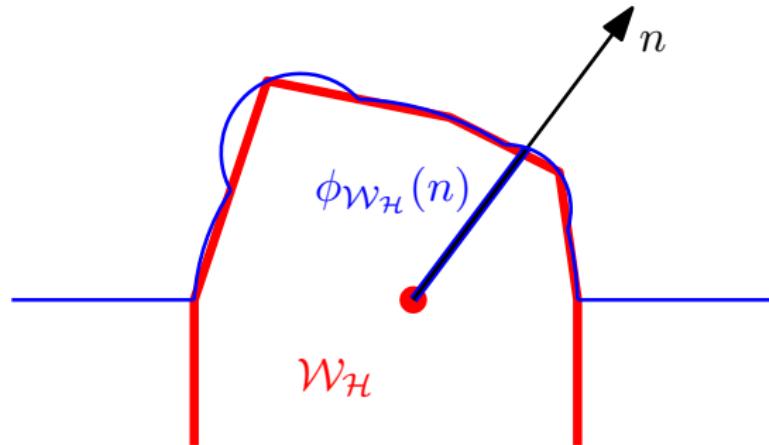
s.t.  $u_{(r,s,0)} = 0 \quad u_{(r,s,L)} = 1 \quad \forall (r, s)$

- $\phi_{(r,s,t)}$ : Anisotropic surface regularizer, convex pos. 1-homogeneous
- $\rho_{(r,s,t)}$ : Cost for assigning label  $t$  to pixel  $(r, s)$

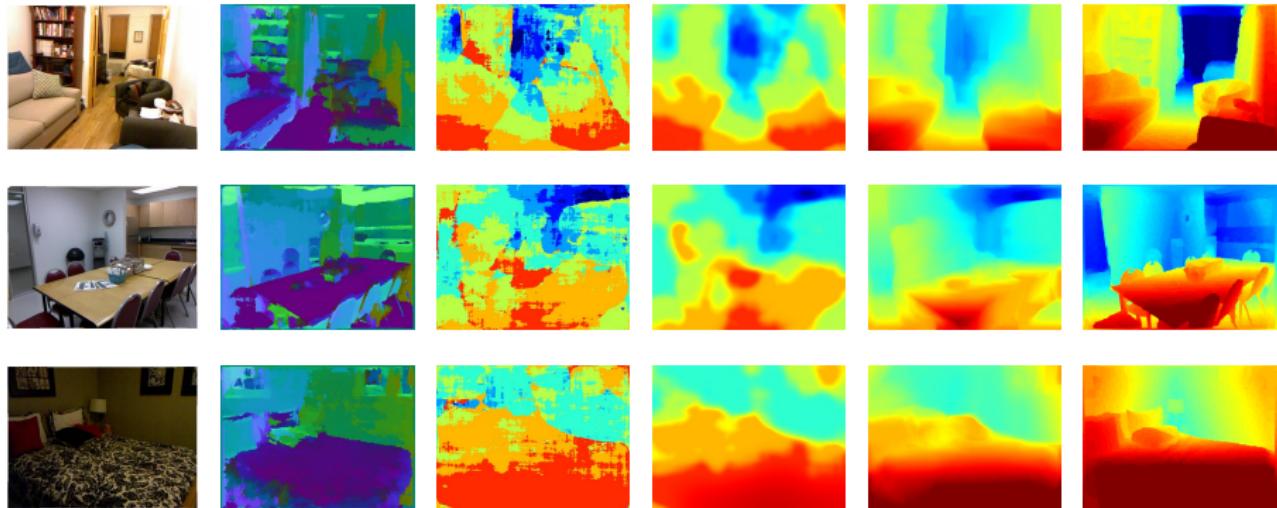


# Regularization Term

- Input: Classifier scores for a discrete set of normal directions
- Goal: Penalizing surface directions according to the classifier scores
- Discrete Wulff shape



# Results



image

normals

[Ladicky ECCV'14]

depth

[Ladicky CVPR'14]

w/o normals

proposed

ground truth

- Conclusion: Normal directions contribute valuable information

- Reasoning about semantics and geometry jointly
  - Improves results
  - Shape prior formulations
  - Leads to more complete understanding of the scene

## Questions ?