In-Context RL

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1 Data Generation

1.1 5-Armed Bandit Problem

```
State space S = \{0,1,2,3,4\}; action space A = \{0,1,2,3,4\} Initialize empty pretraining dataset \mathcal{B} for i in [N] do p_1 \sim \text{Dirchlet distribution}(\mathbb{1}) p_2 \sim \text{point-mass distribution} \omega \sim \text{Unif}(0.1[10]) action distribution p = (1-\omega)p_1 + \omega p_2 action means \mu \sim \text{Unif}[0,1]^5 for h in [H=500] do action a_h \sim p (OHE) reward r_h \sim N(\mu_a,\sigma^2) where \sigma=0.3 goal g_h = (a_h,r_h) + g_{h+1} append a_h,r_h to \mathcal{B}
```

2 Value Function Approximation

```
for i in [#iterations] do sample offline data \{s_t^i, a_t^i, s_{t+1}^i, g_t^i\}_{i=1}^N \sim \mathcal{B}, \{s_0^i\}_{i=1}^M \sim \mu_0 obtain reward \{R(s_t^i; g_t^i)\}_{i=1}^N \sim \mathcal{B} Value objective: L_V(\theta) = \frac{1-\gamma}{M} \sum_{i=1}^M [V_\theta(s_0^i; g_0^i)] + \frac{1}{N} \sum_{i=1}^N \left[f_\star(R_t^i + \gamma V(s_{t+1}^i; g_t^i) - V(s_t^i; g_t^i))\right] update V_\theta \colon V_\theta \leftarrow V_\theta - \alpha_V \nabla L_V(\theta)
```

For bandit data, condition V on a and s instead

3 DT Training

```
for i in [#iterations] do sample offline data \{s_t^i, a_t^i, s_{t+1}^i, g_t^i\}_{i=1}^N \sim \mathcal{B} obtain reward \{R(s_t^i; g_t^i)\}_{i=1}^N \sim \mathcal{B}
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```
Policy objective: L_{\pi}(\phi) = \sum_{i=1}^{N} \left[ \left( f'_{\star} \left( R^i_t + \gamma V_{\theta}(s^i_{t+1}; g^i_t) - V_{\theta}(s^i_t; g^i_t) \right) \log \pi(a \mid s, g) \right] \right] Update \pi_{\phi}: \pi_{\phi} \leftarrow \pi_{\phi} - \alpha_{\pi}
```

For bandit data, condition V on a and s instead

4 Test

4.1 Offline Test

```
\label{eq:bardinger} \begin{split} \# & \mbox{ Bandit version} \\ & \mbox{ subopt = []} \\ & \mbox{ for } i \mbox{ in [500] do} \\ & \mbox{ sample dataset } D \mbox{ with number of } i \mbox{ data} \sim \mathcal{B}_{\text{test}} \\ & s = s_0 \\ & a^* = \arg\max_{a} \mu \\ & \hat{a} = \arg\max_{a \in \mathcal{A}} \pi_\phi(\cdot|s, D) \\ & \mbox{ suboptimality = } \mu_{a^*} - \mu_{\hat{a}} \\ & \mbox{ append suboptimality to subopt} \end{split}
```

4.2 Online Test

```
\label{eq:barder} \begin{split} \# \; & \text{Bandit version} \\ & \text{suboptimality} = 0 \\ & \text{Initialize D} = \big\{ \big\} \\ & \text{for ep in [max\_eps=500] do} \\ & \text{sample dataset } D \sim \mathcal{B}_{\text{test}} \\ & s = s_0 \sim \text{Unif}[0,1] \\ & \hat{a} = \pi_\phi(\cdot|s,D) \\ & \text{suboptimality += } \mu_{a^*} - \mu_{\hat{a}} \\ & \text{add } (a,r) \text{ to D} \end{split}
```