order even with an inadmissible heuristic. The idea of keeping track of the best alternative path appeared earlier in Bratko's (1986) elegant Prolog implementation of A* and in the DTA* algorithm (Russell and Wefald, 1991). The latter work also discusses metalevel state spaces and metalevel learning.

The MA* algorithm appeared in Chakrabarti *et al.* (1989). SMA*, or Simplified MA*, emerged from an attempt to implement MA* as a comparison algorithm for IE (Russell, 1992). Kaindl and Khorsand (1994) have applied SMA* to produce a bidirectional search algorithm that is substantially faster than previous algorithms. Korf and Zhang (2000) describe a divide-and-conquer approach, and Zhou and Hansen (2002) introduce memory-bounded A* graph search and a strategy for switching to breadth-first search to increase memory-efficiency (Zhou and Hansen, 2006). Korf (1995) surveys memory-bounded search techniques.

The idea that admissible heuristics can be derived by problem relaxation appears in the seminal paper by Held and Karp (1970), who used the minimum-spanning-tree heuristic to solve the TSP. (See Exercise 3.30.)

The automation of the relaxation process was implemented successfully by Prieditis (1993), building on earlier work with Mostow (Mostow and Prieditis, 1989). Holte and Hernadvolgyi (2001) describe more recent steps towards automating the process. The use of pattern databases to derive admissible heuristics is due to Gasser (1995) and Culberson and Schaeffer (1996, 1998); disjoint pattern databases are described by Korf and Felner (2002); a similar method using symbolic patterns is due to Edelkamp (2009). Felner *et al.* (2007) show how to compress pattern databases to save space. The probabilistic interpretation of heuristics was investigated in depth by Pearl (1984) and Hansson and Mayer (1989).

By far the most comprehensive source on heuristics and heuristic search algorithms is Pearl's (1984) *Heuristics* text. This book provides especially good coverage of the wide variety of offshoots and variations of A*, including rigorous proofs of their formal properties. Kanal and Kumar (1988) present an anthology of important articles on heuristic search, and Rayward-Smith *et al.* (1996) cover approaches from Operations Research. Papers about new search algorithms—which, remarkably, continue to be discovered—appear in journals such as *Artificial Intelligence* and *Journal of the ACM*.

PARALLEL SEARCH

The topic of **parallel search** algorithms was not covered in the chapter, partly because it requires a lengthy discussion of parallel computer architectures. Parallel search became a popular topic in the 1990s in both AI and theoretical computer science (Mahanti and Daniels, 1993; Grama and Kumar, 1995; Crauser *et al.*, 1998) and is making a comeback in the era of new multicore and cluster architectures (Ralphs *et al.*, 2004; Korf and Schultze, 2005). Also of increasing importance are search algorithms for very large graphs that require disk storage (Korf, 2008).

EXERCISES

3.1 Explain why problem formulation must follow goal formulation.

3.2 Your goal is to navigate a robot out of a maze. The robot starts in the center of the maze

facing north. You can turn the robot to face north, east, south, or west. You can direct the robot to move forward a certain distance, although it will stop before hitting a wall.

a. Formulate this problem. How large is the state space?

b. In navigating a maze, the only place we need to turn is at the intersection of two or more corridors. Reformulate this problem using this observation. How large is the state space now?

c. From each point in the maze, we can move in any of the four directions until we reach a turning point, and this is the only action we need to do. Reformulate the problem using these actions. Do we need to keep track of the robot's orientation now?

d. In our initial description of the problem we already abstracted from the real world, restricting actions and removing details. List three such simplifications we made.

3.3 Suppose two friends live in different cities on a map, such as the Romania map shown in Figure 3.2. On every turn, we can simultaneously move each friend to a neighboring city on the map. The amount of time needed to move from city i to neighbor j is equal to the road distance $d(i, j)$ between the cities, but on each turn the friend that arrives first must wait until the other one arrives (and calls the first on his/her cell phone) before the next turn can begin. We want the two friends to meet as quickly as possible.

a. Write a detailed formulation for this search problem. (You will find it helpful to define some formal notation here.)

b. Let $D(i, j)$ be the straight-line distance between cities i and j. Which of the following heuristic functions are admissible? (i) $D(i, j)$; (ii) $2 \cdot D(i, j)$; (iii) $D(i, j)/2$.

c. Are there completely connected maps for which no solution exists?

d. Are there maps in which all solutions require one friend to visit the same city twice?

3.4 Show that the 8-puzzle states are divided into two disjoint sets, such that any state is reachable from any other state in the same set, while no state is reachable from any state in the other set. (*Hint:* See Berlekamp *et al.* (1982).) Devise a procedure to decide which set a given state is in, and explain why this is useful for generating random states.

3.5 Consider the n-queens problem using the "efficient" incremental formulation given on page 72. Explain why the state space has at least $\sqrt[3]{n!}$ states and estimate the largest n for which exhaustive exploration is feasible. (*Hint*: Derive a lower bound on the branching factor by considering the maximum number of squares that a queen can attack in any column.)

3.6 Give a complete problem formulation for each of the following. Choose a formulation that is precise enough to be implemented.

a. Using only four colors, you have to color a planar map in such a way that no two adjacent regions have the same color.

b. A 3-foot-tall monkey is in a room where some bananas are suspended from the 8-foot ceiling. He would like to get the bananas. The room contains two stackable, movable, climbable 3-foot-high crates.

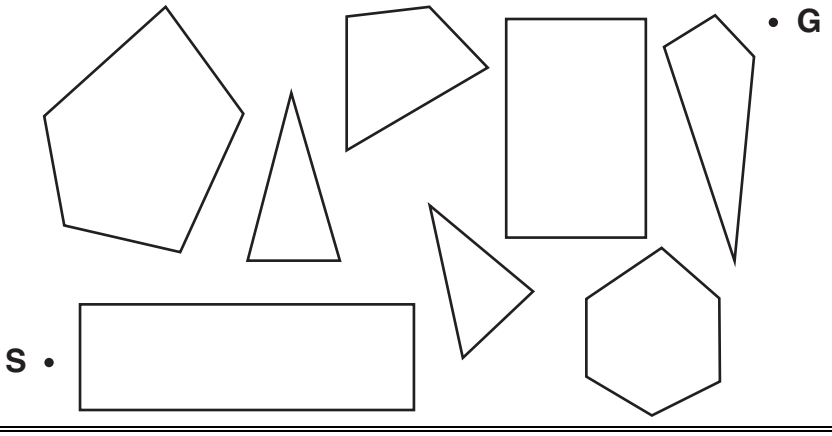
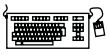


Figure 3.31 A scene with polygonal obstacles. S and G are the start and goal states.

c. You have a program that outputs the message "illegal input record" when fed a certain file of input records. You know that processing of each record is independent of the other records. You want to discover what record is illegal.

d. You have three jugs, measuring 12 gallons, 8 gallons, and 3 gallons, and a water faucet. You can fill the jugs up or empty them out from one to another or onto the ground. You need to measure out exactly one gallon.



3.7 Consider the problem of finding the shortest path between two points on a plane that has convex polygonal obstacles as shown in Figure 3.31. This is an idealization of the problem that a robot has to solve to navigate in a crowded environment.

a. Suppose the state space consists of all positions (x, y) in the plane. How many states are there? How many paths are there to the goal?

b. Explain briefly why the shortest path from one polygon vertex to any other in the scene must consist of straight-line segments joining some of the vertices of the polygons. Define a good state space now. How large is this state space?

c. Define the necessary functions to implement the search problem, including an ACTIONS function that takes a vertex as input and returns a set of vectors, each of which maps the current vertex to one of the vertices that can be reached in a straight line. (Do not forget the neighbors on the same polygon.) Use the straight-line distance for the heuristic function.

d. Apply one or more of the algorithms in this chapter to solve a range of problems in the domain, and comment on their performance.

3.8 On page 68, we said that we would not consider problems with negative path costs. In this exercise, we explore this decision in more depth.

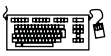
a. Suppose that actions can have arbitrarily large negative costs; explain why this possibility would force any optimal algorithm to explore the entire state space.

b. Does it help if we insist that step costs must be greater than or equal to some negative constant c? Consider both trees and graphs.

c. Suppose that a set of actions forms a loop in the state space such that executing the set in some order results in no net change to the state. If all of these actions have negative cost, what does this imply about the optimal behavior for an agent in such an environment?

d. One can easily imagine actions with high negative cost, even in domains such as route finding. For example, some stretches of road might have such beautiful scenery as to far outweigh the normal costs in terms of time and fuel. Explain, in precise terms, within the context of state-space search, why humans do not drive around scenic loops indefinitely, and explain how to define the state space and actions for route finding so that artificial agents can also avoid looping.

e. Can you think of a real domain in which step costs are such as to cause looping?



3.9 The **missionaries and cannibals** problem is usually stated as follows. Three missionaries and three cannibals are on one side of a river, along with a boat that can hold one or two people. Find a way to get everyone to the other side without ever leaving a group of missionaries in one place outnumbered by the cannibals in that place. This problem is famous in AI because it was the subject of the first paper that approached problem formulation from an analytical viewpoint (Amarel, 1968).

a. Formulate the problem precisely, making only those distinctions necessary to ensure a valid solution. Draw a diagram of the complete state space.

b. Implement and solve the problem optimally using an appropriate search algorithm. Is it a good idea to check for repeated states?

c. Why do you think people have a hard time solving this puzzle, given that the state space is so simple?

3.10 Define in your own words the following terms: state, state space, search tree, search node, goal, action, transition model, and branching factor.

3.11 What's the difference between a world state, a state description, and a search node? Why is this distinction useful?

3.12 An action such as *Go(Sibiu)* really consists of a long sequence of finer-grained actions: turn on the car, release the brake, accelerate forward, etc. Having composite actions of this kind reduces the number of steps in a solution sequence, thereby reducing the search time. Suppose we take this to the logical extreme, by making super-composite actions out of every possible sequence of *Go* actions. Then every problem instance is solved by a single super-composite action, such as *Go(Sibiu)Go(Rimnicu Vilcea)Go(Pitesti)Go(Bucharest)*. Explain how search would work in this formulation. Is this a practical approach for speeding up problem solving?

3.13 Prove that GRAPH-SEARCH satisfies the graph separation property illustrated in Figure 3.9. (*Hint*: Begin by showing that the property holds at the start, then show that if it holds before an iteration of the algorithm, it holds afterwards.) Describe a search algorithm that violates the property.

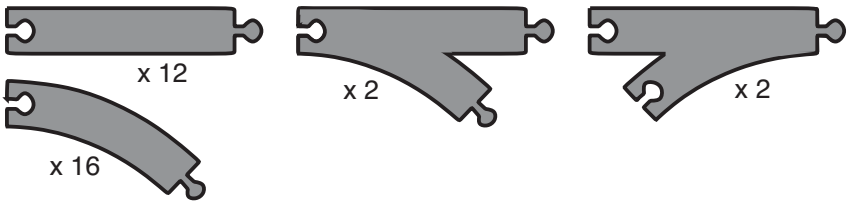


Figure 3.32 The track pieces in a wooden railway set; each is labeled with the number of copies in the set. Note that curved pieces and "fork" pieces ("switches" or "points") can be flipped over so they can curve in either direction. Each curve subtends 45 degrees.

3.14 Which of the following are true and which are false? Explain your answers.

a. Depth-first search always expands at least as many nodes as A* search with an admissible heuristic.

b. $h(n) = 0$ is an admissible heuristic for the 8-puzzle.

c. A* is of no use in robotics because percepts, states, and actions are continuous.

d. Breadth-first search is complete even if zero step costs are allowed.

e. Assume that a rook can move on a chessboard any number of squares in a straight line, vertically or horizontally, but cannot jump over other pieces. Manhattan distance is an admissible heuristic for the problem of moving the rook from square A to square B in the smallest number of moves.

3.15 Consider a state space where the start state is number 1 and each state k has two successors: numbers $2k$ and $2k + 1$.

a. Draw the portion of the state space for states 1 to 15.

b. Suppose the goal state is 11. List the order in which nodes will be visited for breadth-first search, depth-limited search with limit 3, and iterative deepening search.

c. How well would bidirectional search work on this problem? What is the branching factor in each direction of the bidirectional search?

d. Does the answer to (c) suggest a reformulation of the problem that would allow you to solve the problem of getting from state 1 to a given goal state with almost no search?

e. Call the action going from k to $2k$ Left, and the action going to $2k + 1$ Right. Can you find an algorithm that outputs the solution to this problem without any search at all?

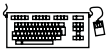
3.16 A basic wooden railway set contains the pieces shown in Figure 3.32. The task is to connect these pieces into a railway that has no overlapping tracks and no loose ends where a train could run off onto the floor.

a. Suppose that the pieces fit together *exactly* with no slack. Give a precise formulation of the task as a search problem.

b. Identify a suitable uninformed search algorithm for this task and explain your choice.

c. Explain why removing any one of the "fork" pieces makes the problem unsolvable.

d. Give an upper bound on the total size of the state space defined by your formulation. (*Hint*: think about the maximum branching factor for the construction process and the maximum depth, ignoring the problem of overlapping pieces and loose ends. Begin by pretending that every piece is unique.)



3.17 On page 90, we mentioned **iterative lengthening search**, an iterative analog of uniform cost search. The idea is to use increasing limits on path cost. If a node is generated whose path cost exceeds the current limit, it is immediately discarded. For each new iteration, the limit is set to the lowest path cost of any node discarded in the previous iteration.

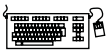
a. Show that this algorithm is optimal for general path costs.

b. Consider a uniform tree with branching factor b, solution depth d, and unit step costs. How many iterations will iterative lengthening require?

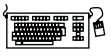
c. Now consider step costs drawn from the continuous range $[\epsilon, 1]$, where $0 < \epsilon < 1$. How many iterations are required in the worst case?

d. Implement the algorithm and apply it to instances of the 8-puzzle and traveling salesperson problems. Compare the algorithm's performance to that of uniform-cost search, and comment on your results.

3.18 Describe a state space in which iterative deepening search performs much worse than depth-first search (for example, $O(n^2)$ vs. $O(n)$).



3.19 Write a program that will take as input two Web page URLs and find a path of links from one to the other. What is an appropriate search strategy? Is bidirectional search a good idea? Could a search engine be used to implement a predecessor function?



3.20 Consider the vacuum-world problem defined in Figure 2.2.

a. Which of the algorithms defined in this chapter would be appropriate for this problem? Should the algorithm use tree search or graph search?

b. Apply your chosen algorithm to compute an optimal sequence of actions for a 3×3 world whose initial state has dirt in the three top squares and the agent in the center.

c. Construct a search agent for the vacuum world, and evaluate its performance in a set of 3×3 worlds with probability 0.2 of dirt in each square. Include the search cost as well as path cost in the performance measure, using a reasonable exchange rate.

d. Compare your best search agent with a simple randomized reflex agent that sucks if there is dirt and otherwise moves randomly.

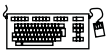
e. Consider what would happen if the world were enlarged to $n \times n$. How does the performance of the search agent and of the reflex agent vary with n?

3.21 Prove each of the following statements, or give a counterexample:

a. Breadth-first search is a special case of uniform-cost search.

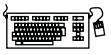
b. Depth-first search is a special case of best-first tree search.

c. Uniform-cost search is a special case of A* search.



3.22 Compare the performance of A* and RBFS on a set of randomly generated problems in the 8-puzzle (with Manhattan distance) and TSP (with MST—see Exercise 3.30) domains. Discuss your results. What happens to the performance of RBFS when a small random number is added to the heuristic values in the 8-puzzle domain?

3.23 Trace the operation of A* search applied to the problem of getting to Bucharest from Lugoj using the straight-line distance heuristic. That is, show the sequence of nodes that the algorithm will consider and the f, g, and h score for each node.



3.24 Devise a state space in which A* using GRAPH-SEARCH returns a suboptimal solution with an $h(n)$ function that is admissible but inconsistent.

HEURISTIC PATH ALGORITHM

3.25 The **heuristic path algorithm** (Pohl, 1977) is a best-first search in which the evaluation function is $f(n) = (2 - w)g(n) + wh(n)$. For what values of w is this complete? For what values is it optimal, assuming that h is admissible? What kind of search does this perform for $w = 0$, $w = 1$, and $w = 2$?

3.26 Consider the unbounded version of the regular 2D grid shown in Figure 3.9. The start state is at the origin, (0,0), and the goal state is at (x, y).

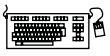
a. What is the branching factor b in this state space?
b. How many distinct states are there at depth k (for $k > 0$)?
c. What is the maximum number of nodes expanded by breadth-first tree search?
d. What is the maximum number of nodes expanded by breadth-first graph search?
e. Is $h = |u - x| + |v - y|$ an admissible heuristic for a state at (u, v)? Explain.
f. How many nodes are expanded by A* graph search using h?
g. Does h remain admissible if some links are removed?
h. Does h remain admissible if some links are added between nonadjacent states?

3.27 n vehicles occupy squares $(1, 1)$ through $(n, 1)$ (i.e., the bottom row) of an $n \times n$ grid. The vehicles must be moved to the top row but in reverse order; so the vehicle i that starts in $(i, 1)$ must end up in $(n - i + 1, n)$. On each time step, every one of the n vehicles can move one square up, down, left, or right, or stay put; but if a vehicle stays put, one other adjacent vehicle (but not more than one) can hop over it. Two vehicles cannot occupy the same square.

a. Calculate the size of the state space as a function of n.
b. Calculate the branching factor as a function of n.
c. Suppose that vehicle i is at (x_i, y_i); write a nontrivial admissible heuristic h_i for the number of moves it will require to get to its goal location $(n - i + 1, n)$, assuming no other vehicles are on the grid.
d. Which of the following heuristics are admissible for the problem of moving all n vehicles to their destinations? Explain.
(i) $\sum_{i=1}^{n} h_i$.
(ii) $\max\{h_1, \ldots, h_n\}$.
(iii) $\min\{h_1, \ldots, h_n\}$.

3.28 Invent a heuristic function for the 8-puzzle that sometimes overestimates, and show how it can lead to a suboptimal solution on a particular problem. (You can use a computer to help if you want.) Prove that if h never overestimates by more than c, A* using h returns a solution whose cost exceeds that of the optimal solution by no more than c.

3.29 Prove that if a heuristic is consistent, it must be admissible. Construct an admissible heuristic that is not consistent.



3.30 The traveling salesperson problem (TSP) can be solved with the minimum-spanning-tree (MST) heuristic, which estimates the cost of completing a tour, given that a partial tour has already been constructed. The MST cost of a set of cities is the smallest sum of the link costs of any tree that connects all the cities.

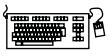
a. Show how this heuristic can be derived from a relaxed version of the TSP.

b. Show that the MST heuristic dominates straight-line distance.

c. Write a problem generator for instances of the TSP where cities are represented by random points in the unit square.

d. Find an efficient algorithm in the literature for constructing the MST, and use it with A* graph search to solve instances of the TSP.

3.31 On page 105, we defined the relaxation of the 8-puzzle in which a tile can move from square A to square B if B is blank. The exact solution of this problem defines **Gaschnig's heuristic** (Gaschnig, 1979). Explain why Gaschnig's heuristic is at least as accurate as h_1 (misplaced tiles), and show cases where it is more accurate than both h_1 and h_2 (Manhattan distance). Explain how to calculate Gaschnig's heuristic efficiently.



3.32 We gave two simple heuristics for the 8-puzzle: Manhattan distance and misplaced tiles. Several heuristics in the literature purport to improve on this—see, for example, Nilsson (1971), Mostow and Prieditis (1989), and Hansson *et al.* (1992). Test these claims by implementing the heuristics and comparing the performance of the resulting algorithms.

Newsletter, published by the Computer Go Association, describes current developments.

Bridge: Smith *et al.* (1998) report on how their planning-based program won the 1998 computer bridge championship, and (Ginsberg, 2001) describes how his GIB program, based on Monte Carlo simulation, won the following computer championship and did surprisingly well against human players and standard book problem sets. From 2001–2007, the computer bridge championship was won five times by JACK and twice by WBRIDGE5. Neither has had academic articles explaining their structure, but both are rumored to use the Monte Carlo technique, which was first proposed for bridge by Levy (1989).

Scrabble: A good description of a top program, MAVEN, is given by its creator, Brian Sheppard (2002). Generating the highest-scoring move is described by Gordon (1994), and modeling opponents is covered by Richards and Amir (2007).

Soccer (Kitano *et al.*, 1997b; Visser *et al.*, 2008) and **billiards** (Lam and Greenspan, 2008; Archibald *et al.*, 2009) and other stochastic games with a continuous space of actions are beginning to attract attention in AI, both in simulation and with physical robot players.

Computer game competitions occur annually, and papers appear in a variety of venues. The rather misleadingly named conference proceedings *Heuristic Programming in Artificial Intelligence* report on the Computer Olympiads, which include a wide variety of games. The General Game Competition (Love *et al.*, 2006) tests programs that must learn to play an unknown game given only a logical description of the rules of the game. There are also several edited collections of important papers on game-playing research (Levy, 1988a, 1988b; Marsland and Schaeffer, 1990). The International Computer Chess Association (ICCA), founded in 1977, publishes the *ICGA Journal* (formerly the *ICCA Journal*). Important papers have been published in the serial anthology *Advances in Computer Chess*, starting with Clarke (1977). Volume 134 of the journal *Artificial Intelligence* (2002) contains descriptions of state-of-the-art programs for chess, Othello, Hex, shogi, Go, backgammon, poker, Scrabble, and other games. Since 1998, a biennial *Computers and Games* conference has been held.

EXERCISES

5.1 Suppose you have an oracle, $OM(s)$, that correctly predicts the opponent's move in any state. Using this, formulate the definition of a game as a (single-agent) search problem. Describe an algorithm for finding the optimal move.

5.2 Consider the problem of solving two 8-puzzles.

a. Give a complete problem formulation in the style of Chapter 3.

b. How large is the reachable state space? Give an exact numerical expression.

c. Suppose we make the problem adversarial as follows: the two players take turns moving; a coin is flipped to determine the puzzle on which to make a move in that turn; and the winner is the first to solve one puzzle. Which algorithm can be used to choose a move in this setting?

d. Give an informal proof that someone will eventually win if both play perfectly.

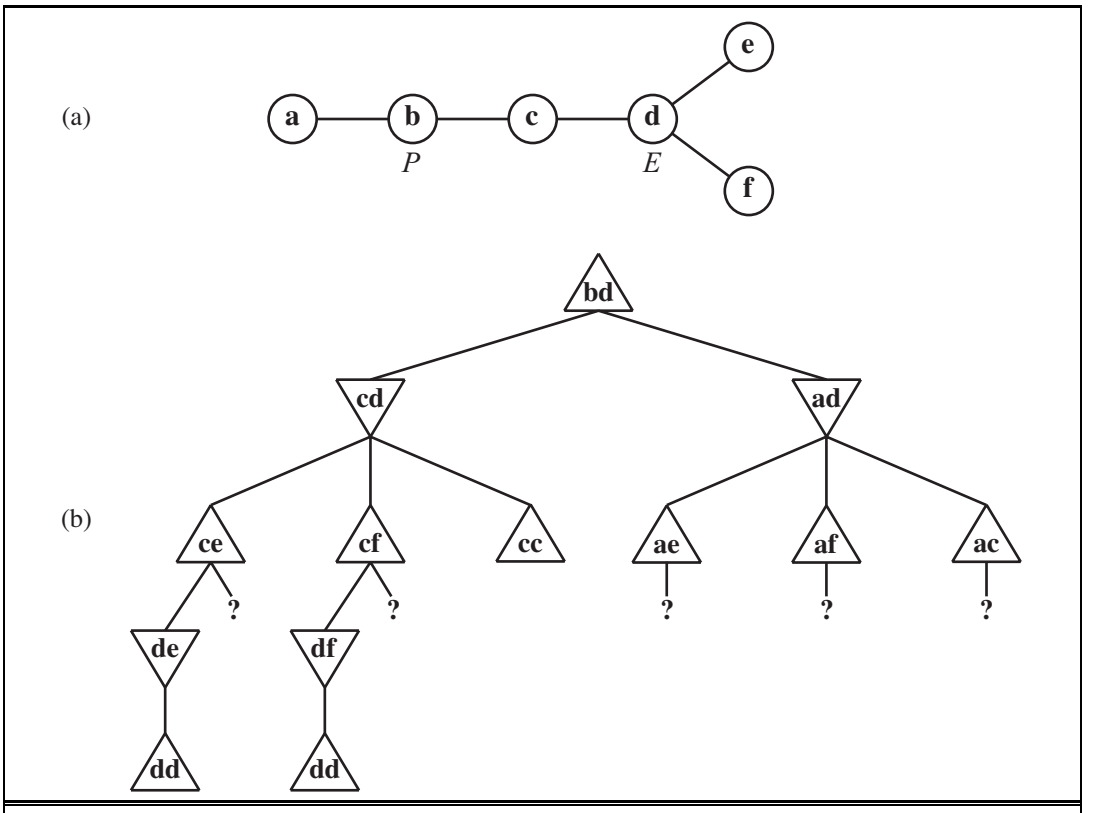


Figure 5.16 (a) A map where the cost of every edge is 1. Initially the pursuer P is at node **b** and the evader E is at node **d**. (b) A partial game tree for this map. Each node is labeled with the P, E positions. P moves first. Branches marked "?" have yet to be explored.

PURSUIT–EVASION

5.3 Imagine that, in Exercise 3.3, one of the friends wants to avoid the other. The problem then becomes a two-player **pursuit–evasion** game. We assume now that the players take turns moving. The game ends only when the players are on the same node; the terminal payoff to the pursuer is minus the total time taken. (The evader "wins" by never losing.) An example is shown in Figure 5.16.

a. Copy the game tree and mark the values of the terminal nodes.

b. Next to each internal node, write the strongest fact you can infer about its value (a number, one or more inequalities such as "≥ 14", or a "?").

c. Beneath each question mark, write the name of the node reached by that branch.

d. Explain how a bound on the value of the nodes in (c) can be derived from consideration of shortest-path lengths on the map, and derive such bounds for these nodes. Remember the cost to get to each leaf as well as the cost to solve it.

e. Now suppose that the tree as given, with the leaf bounds from (d), is evaluated from left to right. Circle those "?" nodes that would *not* need to be expanded further, given the bounds from part (d), and cross out those that need not be considered at all.

f. Can you prove anything in general about who wins the game on a map that is a tree?

5.4 Describe and implement state descriptions, move generators, terminal tests, utility functions, and evaluation functions for one or more of the following stochastic games: Monopoly, Scrabble, bridge play with a given contract, or Texas hold'em poker.

5.5 Describe and implement a *real-time*, *multiplayer* game-playing environment, where time is part of the environment state and players are given fixed time allocations.

5.6 Discuss how well the standard approach to game playing would apply to games such as tennis, pool, and croquet, which take place in a continuous physical state space.

5.7 Prove the following assertion: For every game tree, the utility obtained by MAX using minimax decisions against a suboptimal MIN will be never be lower than the utility obtained playing against an optimal MIN. Can you come up with a game tree in which MAX can do still better using a *suboptimal* strategy against a suboptimal MIN?

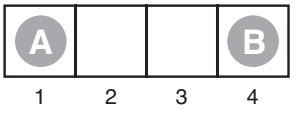


Figure 5.17 The starting position of a simple game. Player A moves first. The two players take turns moving, and each player must move his token to an open adjacent space in either direction. If the opponent occupies an adjacent space, then a player may jump over the opponent to the next open space if any. (For example, if A is on 3 and B is on 2, then A may move back to 1.) The game ends when one player reaches the opposite end of the board. If player A reaches space 4 first, then the value of the game to A is $+1$; if player B reaches space 1 first, then the value of the game to A is -1.

5.8 Consider the two-player game described in Figure 5.17.

a. Draw the complete game tree, using the following conventions:

- Write each state as (s_A, s_B), where s_A and s_B denote the token locations.
- Put each terminal state in a square box and write its game value in a circle.
- Put *loop states* (states that already appear on the path to the root) in double square boxes. Since their value is unclear, annotate each with a "?" in a circle.

b. Now mark each node with its backed-up minimax value (also in a circle). Explain how you handled the "?" values and why.

c. Explain why the standard minimax algorithm would fail on this game tree and briefly sketch how you might fix it, drawing on your answer to (b). Does your modified algorithm give optimal decisions for all games with loops?

d. This 4-square game can be generalized to n squares for any $n > 2$. Prove that A wins if n is even and loses if n is odd.

5.9 This problem exercises the basic concepts of game playing, using tic-tac-toe (noughts and crosses) as an example. We define X_n as the number of rows, columns, or diagonals

with exactly n X's and no O's. Similarly, O_n is the number of rows, columns, or diagonals with just n O's. The utility function assigns $+1$ to any position with $X_3 = 1$ and -1 to any position with $O_3 = 1$. All other terminal positions have utility 0. For nonterminal positions, we use a linear evaluation function defined as $Eval(s) = 3X_2(s)+X_1(s)-(3O_2(s)+O_1(s))$.

a. Approximately how many possible games of tic-tac-toe are there?

b. Show the whole game tree starting from an empty board down to depth 2 (i.e., one X and one O on the board), taking symmetry into account.

c. Mark on your tree the evaluations of all the positions at depth 2.

d. Using the minimax algorithm, mark on your tree the backed-up values for the positions at depths 1 and 0, and use those values to choose the best starting move.

e. Circle the nodes at depth 2 that would *not* be evaluated if alpha–beta pruning were applied, assuming the nodes are generated in the optimal order for alpha–beta pruning.

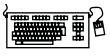
5.10 Consider the family of generalized tic-tac-toe games, defined as follows. Each particular game is specified by a set $\mathcal{S}$ of *squares* and a collection $\mathcal{W}$ of *winning positions.* Each winning position is a subset of $\mathcal{S}$. For example, in standard tic-tac-toe, $\mathcal{S}$ is a set of 9 squares and $\mathcal{W}$ is a collection of 8 subsets of $\mathcal{W}$: the three rows, the three columns, and the two diagonals. In other respects, the game is identical to standard tic-tac-toe. Starting from an empty board, players alternate placing their marks on an empty square. A player who marks every square in a winning position wins the game. It is a tie if all squares are marked and neither player has won.

a. Let $N = |\mathcal{S}|$, the number of squares. Give an upper bound on the number of nodes in the complete game tree for generalized tic-tac-toe as a function of N.

b. Give a lower bound on the size of the game tree for the worst case, where $\mathcal{W} = \{\,\}$.

c. Propose a plausible evaluation function that can be used for any instance of generalized tic-tac-toe. The function may depend on $\mathcal{S}$ and $\mathcal{W}$.

d. Assume that it is possible to generate a new board and check whether it is a winning position in $100N$ machine instructions and assume a 2 gigahertz processor. Ignore memory limitations. Using your estimate in (a), roughly how large a game tree can be completely solved by alpha–beta in a second of CPU time? a minute? an hour?



5.11 Develop a general game-playing program, capable of playing a variety of games.

a. Implement move generators and evaluation functions for one or more of the following games: Kalah, Othello, checkers, and chess.

b. Construct a general alpha–beta game-playing agent.

c. Compare the effect of increasing search depth, improving move ordering, and improving the evaluation function. How close does your effective branching factor come to the ideal case of perfect move ordering?

d. Implement a selective search algorithm, such as B* (Berliner, 1979), conspiracy number search (McAllester, 1988), or MGSS* (Russell and Wefald, 1989) and compare its performance to A*.

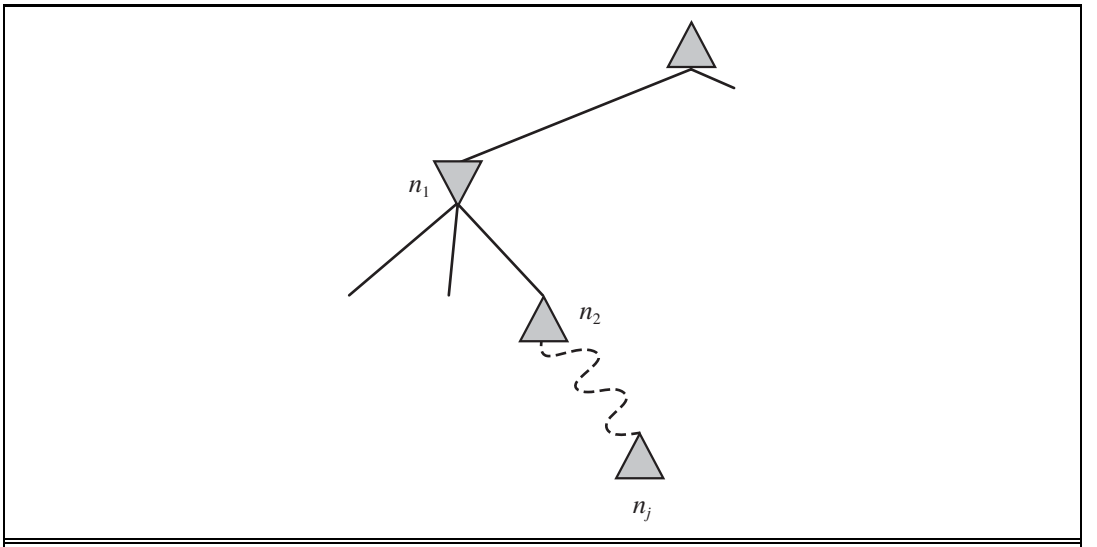


Figure 5.18 Situation when considering whether to prune node n_j.

5.12 Describe how the minimax and alpha–beta algorithms change for two-player, non-zero-sum games in which each player has a distinct utility function and both utility functions are known to both players. If there are no constraints on the two terminal utilities, is it possible for any node to be pruned by alpha–beta? What if the player's utility functions on any state differ by at most a constant k, making the game almost cooperative?

5.13 Develop a formal proof of correctness for alpha–beta pruning. To do this, consider the situation shown in Figure 5.18. The question is whether to prune node n_j, which is a max-node and a descendant of node n_1. The basic idea is to prune it if and only if the minimax value of n_1 can be shown to be independent of the value of n_j.

a. Mode n_1 takes on the minimum value among its children: $n_1 = \min(n_2, n_{21}, \ldots, n_{2b_2})$. Find a similar expression for n_2 and hence an expression for n_1 in terms of n_j.

b. Let l_i be the minimum (or maximum) value of the nodes to the *left* of node n_i at depth i, whose minimax value is already known. Similarly, let r_i be the minimum (or maximum) value of the unexplored nodes to the right of n_i at depth i. Rewrite your expression for n_1 in terms of the l_i and r_i values.

c. Now reformulate the expression to show that in order to affect n_1, n_j must not exceed a certain bound derived from the l_i values.

d. Repeat the process for the case where n_j is a min-node.

5.14 Prove that alpha–beta pruning takes time $O(2^{m/2})$ with optimal move ordering, where m is the maximum depth of the game tree.

5.15 Suppose you have a chess program that can evaluate 10 million nodes per second. Decide on a compact representation of a game state for storage in a transposition table. About how many entries can you fit in a 2-gigabyte in-memory table? Will that be enough for the

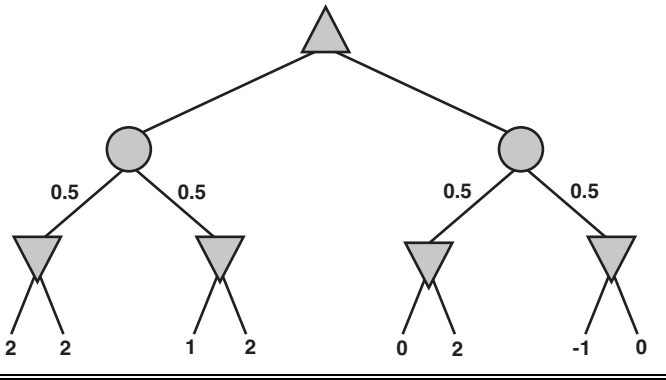


Figure 5.19 The complete game tree for a trivial game with chance nodes.

three minutes of search allocated for one move? How many table lookups can you do in the time it would take to do one evaluation? Now suppose the transposition table is stored on disk. About how many evaluations could you do in the time it takes to do one disk seek with standard disk hardware?

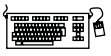
5.16 This question considers pruning in games with chance nodes. Figure 5.19 shows the complete game tree for a trivial game. Assume that the leaf nodes are to be evaluated in left-to-right order, and that before a leaf node is evaluated, we know nothing about its value—the range of possible values is $-\infty$ to ∞.

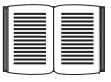
a. Copy the figure, mark the value of all the internal nodes, and indicate the best move at the root with an arrow.

b. Given the values of the first six leaves, do we need to evaluate the seventh and eighth leaves? Given the values of the first seven leaves, do we need to evaluate the eighth leaf? Explain your answers.

c. Suppose the leaf node values are known to lie between –2 and 2 inclusive. After the first two leaves are evaluated, what is the value range for the left-hand chance node?

d. Circle all the leaves that need not be evaluated under the assumption in (c).





5.17 Implement the expectiminimax algorithm and the *-alpha–beta algorithm, which is described by Ballard (1983), for pruning game trees with chance nodes. Try them on a game such as backgammon and measure the pruning effectiveness of *-alpha–beta.

5.18 Prove that with a positive linear transformation of leaf values (i.e., transforming a value x to $ax + b$ where $a > 0$), the choice of move remains unchanged in a game tree, even when there are chance nodes.

5.19 Consider the following procedure for choosing moves in games with chance nodes:

- Generate some dice-roll sequences (say, 50) down to a suitable depth (say, 8).
- With known dice rolls, the game tree becomes deterministic. For each dice-roll sequence, solve the resulting deterministic game tree using alpha–beta.

- Use the results to estimate the value of each move and to choose the best.

Will this procedure work well? Why (or why not)?

5.20 In the following, a "max" tree consists only of max nodes, whereas an "expectimax" tree consists of a max node at the root with alternating layers of chance and max nodes. At chance nodes, all outcome probabilities are nonzero. The goal is to *find the value of the root* with a bounded-depth search. For each of (a)–(f), either give an example or explain why this is impossible.

a. Assuming that leaf values are finite but unbounded, is pruning (as in alpha–beta) ever possible in a max tree?

b. Is pruning ever possible in an expectimax tree under the same conditions?

c. If leaf values are all nonnegative, is pruning ever possible in a max tree? Give an example, or explain why not.

d. If leaf values are all nonnegative, is pruning ever possible in an expectimax tree? Give an example, or explain why not.

e. If leaf values are all in the range $[0, 1]$, is pruning ever possible in a max tree? Give an example, or explain why not.

f. If leaf values are all in the range $[0, 1]$, is pruning ever possible in an expectimax tree?

g. Consider the outcomes of a chance node in an expectimax tree. Which of the following evaluation orders is most likely to yield pruning opportunities?

(i) Lowest probability first
(ii) Highest probability first
(iii) Doesn't make any difference

5.21 Which of the following are true and which are false? Give brief explanations.

a. In a fully observable, turn-taking, zero-sum game between two perfectly rational players, it does not help the first player to know what strategy the second player is using—that is, what move the second player will make, given the first player's move.

b. In a partially observable, turn-taking, zero-sum game between two perfectly rational players, it does not help the first player to know what move the second player will make, given the first player's move.

c. A perfectly rational backgammon agent never loses.

5.22 Consider carefully the interplay of chance events and partial information in each of the games in Exercise 5.4.

a. For which is the standard expectiminimax model appropriate? Implement the algorithm and run it in your game-playing agent, with appropriate modifications to the game-playing environment.

b. For which would the scheme described in Exercise 5.19 be appropriate?

c. Discuss how you might deal with the fact that in some of the games, the players do not have the same knowledge of the current state.

Liberatore (1997). The idea of representing a belief state with propositions can be traced to Wittgenstein (1922).

Logical state estimation, of course, requires a logical representation of the effects of actions—a key problem in AI since the late 1950s. The dominant proposal has been the **situation calculus** formalism (McCarthy, 1963), which is couched within first-order logic. We discuss situation calculus, and various extensions and alternatives, in Chapters 10 and 12. The approach taken in this chapter—using temporal indices on propositional variables—is more restrictive but has the benefit of simplicity. The general approach embodied in the SATPLAN algorithm was proposed by Kautz and Selman (1992). Later generations of SATPLAN were able to take advantage of the advances in SAT solvers, described earlier, and remain among the most effective ways of solving difficult problems (Kautz, 2006).

The **frame problem** was first recognized by McCarthy and Hayes (1969). Many researchers considered the problem unsolvable within first-order logic, and it spurred a great deal of research into nonmonotonic logics. Philosophers from Dreyfus (1972) to Crockett (1994) have cited the frame problem as one symptom of the inevitable failure of the entire AI enterprise. The solution of the frame problem with successor-state axioms is due to Ray Reiter (1991). Thielscher (1999) identifies the inferential frame problem as a separate idea and provides a solution. In retrospect, one can see that Rosenschein's (1985) agents were using circuits that implemented successor-state axioms, but Rosenschein did not notice that the frame problem was thereby largely solved. Foo (2001) explains why the discrete-event control theory models typically used by engineers do not have to explicitly deal with the frame problem: because they are dealing with prediction and control, not with explanation and reasoning about counterfactual situations.

Modern propositional solvers have wide applicability in industrial applications. The application of propositional inference in the synthesis of computer hardware is now a standard technique having many large-scale deployments (Nowick *et al.*, 1993). The SATMC satisfiability checker was used to detect a previously unknown vulnerability in a Web browser user sign-on protocol (Armando *et al.*, 2008).

The wumpus world was invented by Gregory Yob (1975). Ironically, Yob developed it because he was bored with games played on a rectangular grid: the topology of his original wumpus world was a dodecahedron, and we put it back in the boring old grid. Michael Genesereth was the first to suggest that the wumpus world be used as an agent testbed.

EXERCISES

7.1 Suppose the agent has progressed to the point shown in Figure 7.4(a), page 239, having perceived nothing in [1,1], a breeze in [2,1], and a stench in [1,2], and is now concerned with the contents of [1,3], [2,2], and [3,1]. Each of these can contain a pit, and at most one can contain a wumpus. Following the example of Figure 7.5, construct the set of possible worlds. (You should find 32 of them.) Mark the worlds in which the KB is true and those in which

each of the following sentences is true:

α_2 = "There is no pit in [2,2]."
α_3 = "There is a wumpus in [1,3]."

Hence show that $KB \models \alpha_2$ and $KB \models \alpha_3$.

7.2 (Adapted from Barwise and Etchemendy (1993).) Given the following, can you prove that the unicorn is mythical? How about magical? Horned?

> If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

7.3 Consider the problem of deciding whether a propositional logic sentence is true in a given model.

a. Write a recursive algorithm PL-TRUE?(s, m) that returns $true$ if and only if the sentence s is true in the model m (where m assigns a truth value for every symbol in s). The algorithm should run in time linear in the size of the sentence. (Alternatively, use a version of this function from the online code repository.)

b. Give three examples of sentences that can be determined to be true or false in a *partial* model that does not specify a truth value for some of the symbols.

c. Show that the truth value (if any) of a sentence in a partial model cannot be determined efficiently in general.

d. Modify your PL-TRUE? algorithm so that it can sometimes judge truth from partial models, while retaining its recursive structure and linear run time. Give three examples of sentences whose truth in a partial model is *not* detected by your algorithm.

e. Investigate whether the modified algorithm makes TT-ENTAILS? more efficient.

7.4 Which of the following are correct?

a. $\mathit{False} \models \mathit{True}$.
b. $\mathit{True} \models \mathit{False}$.
c. $(A \wedge B) \models (A \Leftrightarrow B)$.
d. $A \Leftrightarrow B \models A \vee B$.
e. $A \Leftrightarrow B \models \neg A \vee B$.
f. $(A \wedge B) \Rightarrow C \models (A \Rightarrow C) \vee (B \Rightarrow C)$.
g. $(C \vee (\neg A \wedge \neg B)) \equiv ((A \Rightarrow C) \wedge (B \Rightarrow C))$.
h. $(A \vee B) \wedge (\neg C \vee \neg D \vee E) \models (A \vee B)$.
i. $(A \vee B) \wedge (\neg C \vee \neg D \vee E) \models (A \vee B) \wedge (\neg D \vee E)$.
j. $(A \vee B) \wedge \neg(A \Rightarrow B)$ is satisfiable.
k. $(A \Leftrightarrow B) \wedge (\neg A \vee B)$ is satisfiable.
l. $(A \Leftrightarrow B) \Leftrightarrow C$ has the same number of models as $(A \Leftrightarrow B)$ for any fixed set of proposition symbols that includes A, B, C.

7.5 Prove each of the following assertions:

a. α is valid if and only if $True \models \alpha$.
b. For any α, $False \models \alpha$.
c. $\alpha \models \beta$ if and only if the sentence $(\alpha \Rightarrow \beta)$ is valid.
d. $\alpha \equiv \beta$ if and only if the sentence $(\alpha \Leftrightarrow \beta)$ is valid.
e. $\alpha \models \beta$ if and only if the sentence $(\alpha \wedge \neg\beta)$ is unsatisfiable.

7.6 Prove, or find a counterexample to, each of the following assertions:

a. If $\alpha \models \gamma$ or $\beta \models \gamma$ (or both) then $(\alpha \wedge \beta) \models \gamma$
b. If $\alpha \models (\beta \wedge \gamma)$ then $\alpha \models \beta$ and $\alpha \models \gamma$.
c. If $\alpha \models (\beta \vee \gamma)$ then $\alpha \models \beta$ or $\alpha \models \gamma$ (or both).

7.7 Consider a vocabulary with only four propositions, A, B, C, and D. How many models are there for the following sentences?

a. $B \vee C$.
b. $\neg A \vee \neg B \vee \neg C \vee \neg D$.
c. $(A \Rightarrow B) \wedge A \wedge \neg B \wedge C \wedge D$.

7.8 We have defined four binary logical connectives.

a. Are there any others that might be useful?
b. How many binary connectives can there be?
c. Why are some of them not very useful?

7.9 Using a method of your choice, verify each of the equivalences in Figure 7.11 (page 249).

7.10 Decide whether each of the following sentences is valid, unsatisfiable, or neither. Verify your decisions using truth tables or the equivalence rules of Figure 7.11 (page 249).

a. $Smoke \Rightarrow Smoke$
b. $Smoke \Rightarrow Fire$
c. $(Smoke \Rightarrow Fire) \Rightarrow (\neg Smoke \Rightarrow \neg Fire)$
d. $Smoke \vee Fire \vee \neg Fire$
e. $((Smoke \wedge Heat) \Rightarrow Fire) \Leftrightarrow ((Smoke \Rightarrow Fire) \vee (Heat \Rightarrow Fire))$
f. $(Smoke \Rightarrow Fire) \Rightarrow ((Smoke \wedge Heat) \Rightarrow Fire)$
g. $Big \vee Dumb \vee (Big \Rightarrow Dumb)$

7.11 Any propositional logic sentence is logically equivalent to the assertion that each possible world in which it would be false is not the case. From this observation, prove that any sentence can be written in CNF.

7.12 Use resolution to prove the sentence $\neg A \wedge \neg B$ from the clauses in Exercise 7.20.

7.13 This exercise looks into the relationship between clauses and implication sentences.

a. Show that the clause $(\neg P_1 \vee \cdots \vee \neg P_m \vee Q)$ is logically equivalent to the implication sentence $(P_1 \wedge \cdots \wedge P_m) \Rightarrow Q$.

b. Show that every clause (regardless of the number of positive literals) can be written in the form $(P_1 \wedge \cdots \wedge P_m) \Rightarrow (Q_1 \vee \cdots \vee Q_n)$, where the Ps and Qs are proposition symbols. A knowledge base consisting of such sentences is in **implicative normal form** or **Kowalski form** (Kowalski, 1979).

IMPLICATIVE NORMAL FORM

c. Write down the full resolution rule for sentences in implicative normal form.

7.14 According to some political pundits, a person who is radical (R) is electable (E) if he/she is conservative (C), but otherwise is not electable.

a. Which of the following are correct representations of this assertion?

(i) $(R \wedge E) \iff C$
(ii) $R \Rightarrow (E \iff C)$
(iii) $R \Rightarrow ((C \Rightarrow E) \vee \neg E)$

b. Which of the sentences in (a) can be expressed in Horn form?

7.15 This question considers representing satisfiability (SAT) problems as CSPs.

a. Draw the constraint graph corresponding to the SAT problem

$$(\neg X_1 \vee X_2) \wedge (\neg X_2 \vee X_3) \wedge \ldots \wedge (\neg X_{n-1} \vee X_n)$$

for the particular case $n = 5$.

b. How many solutions are there for this general SAT problem as a function of n?

c. Suppose we apply BACKTRACKING-SEARCH (page 215) to find *all* solutions to a SAT CSP of the type given in (a). (To find *all* solutions to a CSP, we simply modify the basic algorithm so it continues searching after each solution is found.) Assume that variables are ordered $X_1, \ldots, X_n$ and $false$ is ordered before $true$. How much time will the algorithm take to terminate? (Write an $O(\cdot)$ expression as a function of n.)

d. We know that SAT problems in Horn form can be solved in linear time by forward chaining (unit propagation). We also know that every tree-structured binary CSP with discrete, finite domains can be solved in time linear in the number of variables (Section 6.5). Are these two facts connected? Discuss.

7.16 Explain why every nonempty propositional clause, by itself, is satisfiable. Prove rigorously that every set of five 3-SAT clauses is satisfiable, provided that each clause mentions exactly three distinct variables. What is the smallest set of such clauses that is unsatisfiable? Construct such a set.

7.17 A propositional *2-CNF* expression is a conjunction of clauses, each containing *exactly 2* literals, e.g.,

$$(A \vee B) \wedge (\neg A \vee C) \wedge (\neg B \vee D) \wedge (\neg C \vee G) \wedge (\neg D \vee G) \,.$$

a. Prove using resolution that the above sentence entails G.

b. Two clauses are *semantically distinct* if they are not logically equivalent. How many semantically distinct 2-CNF clauses can be constructed from n proposition symbols?

c. Using your answer to (b), prove that propositional resolution always terminates in time polynomial in n given a 2-CNF sentence containing no more than n distinct symbols.

d. Explain why your argument in (c) does not apply to 3-CNF.

7.18 Consider the following sentence:

$$[(Food \Rightarrow Party) \vee (Drinks \Rightarrow Party)] \Rightarrow [(Food \wedge Drinks) \Rightarrow Party] .$$

a. Determine, using enumeration, whether this sentence is valid, satisfiable (but not valid), or unsatisfiable.

b. Convert the left-hand and right-hand sides of the main implication into CNF, showing each step, and explain how the results confirm your answer to (a).

c. Prove your answer to (a) using resolution.

DISJUNCTIVE NORMAL FORM

7.19 A sentence is in **disjunctive normal form** (DNF) if it is the disjunction of conjunctions of literals. For example, the sentence $(A \wedge B \wedge \neg C) \vee (\neg A \wedge C) \vee (B \wedge \neg C)$ is in DNF.

a. Any propositional logic sentence is logically equivalent to the assertion that some possible world in which it would be true is in fact the case. From this observation, prove that any sentence can be written in DNF.

b. Construct an algorithm that converts any sentence in propositional logic into DNF. (*Hint*: The algorithm is similar to the algorithm for conversion to CNF given in Section 7.5.2.)

c. Construct a simple algorithm that takes as input a sentence in DNF and returns a satisfying assignment if one exists, or reports that no satisfying assignment exists.

d. Apply the algorithms in (b) and (c) to the following set of sentences:

$$A \Rightarrow B$$
$$B \Rightarrow C$$
$$C \Rightarrow \neg A .$$

e. Since the algorithm in (b) is very similar to the algorithm for conversion to CNF, and since the algorithm in (c) is much simpler than any algorithm for solving a set of sentences in CNF, why is this technique not used in automated reasoning?

7.20 Convert the following set of sentences to clausal form.

S1: $A \Leftrightarrow (B \vee E)$.
S2: $E \Rightarrow D$.
S3: $C \wedge F \Rightarrow \neg B$.
S4: $E \Rightarrow B$.
S5: $B \Rightarrow F$.
S6: $B \Rightarrow C$

Give a trace of the execution of DPLL on the conjunction of these clauses.

7.21 Is a randomly generated 4-CNF sentence with n symbols and m clauses more or less likely to be solvable than a randomly generated 3-CNF sentence with n symbols and m clauses? Explain.

7.22 Minesweeper, the well-known computer game, is closely related to the wumpus world. A minesweeper world is a rectangular grid of N squares with M invisible mines scattered among them. Any square may be probed by the agent; instant death follows if a mine is probed. Minesweeper indicates the presence of mines by revealing, in each probed square, the *number* of mines that are directly or diagonally adjacent. The goal is to probe every unmined square.

a. Let $X_{i,j}$ be true iff square $[i, j]$ contains a mine. Write down the assertion that exactly two mines are adjacent to [1,1] as a sentence involving some logical combination of $X_{i,j}$ propositions.

b. Generalize your assertion from (a) by explaining how to construct a CNF sentence asserting that k of n neighbors contain mines.

c. Explain precisely how an agent can use DPLL to prove that a given square does (or does not) contain a mine, ignoring the global constraint that there are exactly M mines in all.

d. Suppose that the global constraint is constructed from your method from part (b). How does the number of clauses depend on M and N? Suggest a way to modify DPLL so that the global constraint does not need to be represented explicitly.

e. Are any conclusions derived by the method in part (c) invalidated when the global constraint is taken into account?

f. Give examples of configurations of probe values that induce *long-range dependencies* such that the contents of a given unprobed square would give information about the contents of a far-distant square. (*Hint*: consider an $N \times 1$ board.)

7.23 How long does it take to prove $KB \models \alpha$ using DPLL when α is a literal *already contained in* KB? Explain.

7.24 Trace the behavior of DPLL on the knowledge base in Figure 7.16 when trying to prove Q, and compare this behavior with that of the forward-chaining algorithm.

7.25 Write a successor-state axiom for the $Locked$ predicate, which applies to doors, assuming the only actions available are $Lock$ and $Unlock$.

7.26 Section 7.7.1 provides some of the successor-state axioms required for the wumpus world. Write down axioms for all remaining fluent symbols.

7.27 Modify the HYBRID-WUMPUS-AGENT to use the 1-CNF logical state estimation method described on page 271. We noted on that page that such an agent will not be able to acquire, maintain, and use more complex beliefs such as the disjunction $P_{3,1} \vee P_{2,2}$. Suggest a method for overcoming this problem by defining additional proposition symbols, and try it out in the wumpus world. Does it improve the performance of the agent?

EXERCISES

8.1 A logical knowledge base represents the world using a set of sentences with no explicit structure. An **analogical** representation, on the other hand, has physical structure that corresponds directly to the structure of the thing represented. Consider a road map of your country as an analogical representation of facts about the country—it represents facts with a map language. The two-dimensional structure of the map corresponds to the two-dimensional surface of the area.

a. Give five examples of *symbols* in the map language.

b. An *explicit* sentence is a sentence that the creator of the representation actually writes down. An *implicit* sentence is a sentence that results from explicit sentences because of properties of the analogical representation. Give three examples each of *implicit* and *explicit* sentences in the map language.

c. Give three examples of facts about the physical structure of your country that cannot be represented in the map language.

d. Give two examples of facts that are much easier to express in the map language than in first-order logic.

e. Give two other examples of useful analogical representations. What are the advantages and disadvantages of each of these languages?

8.2 Consider a knowledge base containing just two sentences: $P(a)$ and $P(b)$. Does this knowledge base entail $\forall\, x\ P(x)$? Explain your answer in terms of models.

8.3 Is the sentence $\exists\, x, y\ \ x = y$ valid? Explain.

8.4 Write down a logical sentence such that every world in which it is true contains exactly one object.

8.5 Consider a symbol vocabulary that contains c constant symbols, p_k predicate symbols of each arity k, and f_k function symbols of each arity k, where $1 \leq k \leq A$. Let the domain size be fixed at D. For any given model, each predicate or function symbol is mapped onto a relation or function, respectively, of the same arity. You may assume that the functions in the model allow some input tuples to have no value for the function (i.e., the value is the invisible object). Derive a formula for the number of possible models for a domain with D elements. Don't worry about eliminating redundant combinations.

8.6 Which of the following are valid (necessarily true) sentences?

a. $(\exists x\ x = x)\ \Rightarrow\ (\forall\, y\ \ \exists z\ y = z)$.

b. $\forall\, x\ \ P(x) \vee \neg P(x)$.

c. $\forall\, x\ \ \mathit{Smart}(x) \vee (x = x)$.

8.7 Consider a version of the semantics for first-order logic in which models with empty domains are allowed. Give at least two examples of sentences that are valid according to the

standard semantics but not according to the new semantics. Discuss which outcome makes more intuitive sense for your examples.

8.8 Does the fact $\neg Spouse(George, Laura)$ follow from the facts $Jim \neq George$ and $Spouse(Jim, Laura)$? If so, give a proof; if not, supply additional axioms as needed. What happens if we use $Spouse$ as a unary function symbol instead of a binary predicate?

8.9 This exercise uses the function $MapColor$ and predicates $In(x, y)$, $Borders(x, y)$, and $Country(x)$, whose arguments are geographical regions, along with constant symbols for various regions. In each of the following we give an English sentence and a number of candidate logical expressions. For each of the logical expressions, state whether it (1) correctly expresses the English sentence; (2) is syntactically invalid and therefore meaningless; or (3) is syntactically valid but does not express the meaning of the English sentence.

a. Paris and Marseilles are both in France.

(i) $In(Paris \wedge Marseilles, France)$.
(ii) $In(Paris, France) \wedge In(Marseilles, France)$.
(iii) $In(Paris, France) \vee In(Marseilles, France)$.

b. There is a country that borders both Iraq and Pakistan.

(i) $\exists\, c \;\; Country(c) \wedge Border(c, Iraq) \wedge Border(c, Pakistan)$.
(ii) $\exists\, c \;\; Country(c) \Rightarrow [Border(c, Iraq) \wedge Border(c, Pakistan)]$.
(iii) $[\exists\, c \;\; Country(c)] \Rightarrow [Border(c, Iraq) \wedge Border(c, Pakistan)]$.
(iv) $\exists\, c \;\; Border(Country(c), Iraq \wedge Pakistan)$.

c. All countries that border Ecuador are in South America.

(i) $\forall\, c \;\; Country(c) \wedge Border(c, Ecuador) \Rightarrow In(c, SouthAmerica)$.
(ii) $\forall\, c \;\; Country(c) \Rightarrow [Border(c, Ecuador) \Rightarrow In(c, SouthAmerica)]$.
(iii) $\forall\, c \;\; [Country(c) \Rightarrow Border(c, Ecuador)] \Rightarrow In(c, SouthAmerica)$.
(iv) $\forall\, c \;\; Country(c) \wedge Border(c, Ecuador) \wedge In(c, SouthAmerica)$.

d. No region in South America borders any region in Europe.

(i) $\neg[\exists\, c, d \;\; In(c, SouthAmerica) \wedge In(d, Europe) \wedge Borders(c, d)]$.
(ii) $\forall\, c, d \;\; [In(c, SouthAmerica) \wedge In(d, Europe)] \Rightarrow \neg Borders(c, d)]$.
(iii) $\neg\forall\, c \;\; In(c, SouthAmerica) \Rightarrow \exists\, d \;\; In(d, Europe) \wedge \neg Borders(c, d)$.
(iv) $\forall\, c \;\; In(c, SouthAmerica) \Rightarrow \forall\, d \;\; In(d, Europe) \Rightarrow \neg Borders(c, d)$.

e. No two adjacent countries have the same map color.

(i) $\forall\, x, y \;\; \neg Country(x) \vee \neg Country(y) \vee \neg Borders(x, y) \vee$
$\neg(MapColor(x) = MapColor(y))$.
(ii) $\forall\, x, y \;\; (Country(x) \wedge Country(y) \wedge Borders(x, y) \wedge \neg(x = y)) \Rightarrow$
$\neg(MapColor(x) = MapColor(y))$.
(iii) $\forall\, x, y \;\; Country(x) \wedge Country(y) \wedge Borders(x, y) \wedge$
$\neg(MapColor(x) = MapColor(y))$.
(iv) $\forall\, x, y \;\; (Country(x) \wedge Country(y) \wedge Borders(x, y)) \Rightarrow MapColor(x \neq y)$.

8.10 Consider a vocabulary with the following symbols:

$Occupation(p, o)$: Predicate. Person p has occupation o.
$Customer(p1, p2)$: Predicate. Person $p1$ is a customer of person $p2$.
$Boss(p1, p2)$: Predicate. Person $p1$ is a boss of person $p2$.
$Doctor$, $Surgeon$, $Lawyer$, $Actor$: Constants denoting occupations.
$Emily$, Joe: Constants denoting people.

Use these symbols to write the following assertions in first-order logic:

a. Emily is either a surgeon or a lawyer.
b. Joe is an actor, but he also holds another job.
c. All surgeons are doctors.
d. Joe does not have a lawyer (i.e., is not a customer of any lawyer).
e. Emily has a boss who is a lawyer.
f. There exists a lawyer all of whose customers are doctors.
g. Every surgeon has a lawyer.

8.11 Complete the following exercises about logical senntences:

a. Translate into *good, natural* English (no xs or ys!):

$$\forall\, x, y, l \;\; SpeaksLanguage(x, l) \wedge SpeaksLanguage(y, l) \Rightarrow Understands(x, y) \wedge Understands(y, x).$$

b. Explain why this sentence is entailed by the sentence

$$\forall\, x, y, l \;\; SpeaksLanguage(x, l) \wedge SpeaksLanguage(y, l) \Rightarrow Understands(x, y).$$

c. Translate into first-order logic the following sentences:

(i) Understanding leads to friendship.
(ii) Friendship is transitive.

Remember to define all predicates, functions, and constants you use.

8.12 Rewrite the first two Peano axioms in Section 8.3.3 as a single axiom that defines $NatNum(x)$ so as to exclude the possibility of natural numbers except for those generated by the successor function.

8.13 Equation (8.4) on page 306 defines the conditions under which a square is breezy. Here we consider two other ways to describe this aspect of the wumpus world.

DIAGNOSTIC RULE

a. We can write **diagnostic rules** leading from observed effects to hidden causes. For finding pits, the obvious diagnostic rules say that if a square is breezy, some adjacent square must contain a pit; and if a square is not breezy, then no adjacent square contains a pit. Write these two rules in first-order logic and show that their conjunction is logically equivalent to Equation (8.4).

CAUSAL RULE

b. We can write **causal rules** leading from cause to effect. One obvious causal rule is that a pit causes all adjacent squares to be breezy. Write this rule in first-order logic, explain why it is incomplete compared to Equation (8.4), and supply the missing axiom.

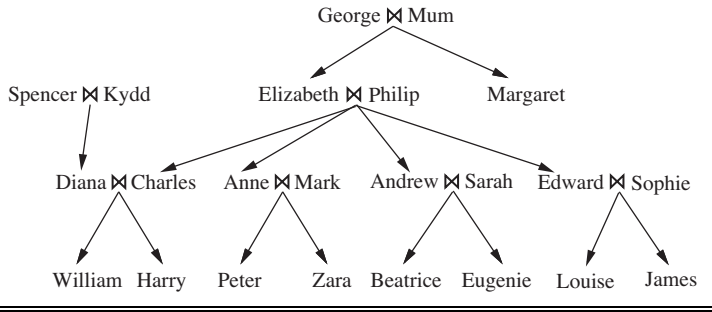
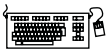


Figure 8.7 A typical family tree. The symbol "⋈" connects spouses and arrows point to children.



8.14 Write axioms describing the predicates $Grandchild$, $Greatgrandparent$, $Ancestor$, $Brother$, $Sister$, $Daughter$, Son, $FirstCousin$, $BrotherInLaw$, $SisterInLaw$, $Aunt$, and $Uncle$. Find out the proper definition of mth cousin n times removed, and write the definition in first-order logic. Now write down the basic facts depicted in the family tree in Figure 8.7. Using a suitable logical reasoning system, TELL it all the sentences you have written down, and ASK it who are Elizabeth's grandchildren, Diana's brothers-in-law, Zara's great-grandparents, and Eugenie's ancestors.

8.15 Explain what is wrong with the following proposed definition of the set membership predicate $\in$:

$$\forall x, s \;\; x \in \{x|s\}$$
$$\forall x, s \;\; x \in s \;\Rightarrow\; \forall y \;\; x \in \{y|s\} \; .$$

8.16 Using the set axioms as examples, write axioms for the list domain, including all the constants, functions, and predicates mentioned in the chapter.

8.17 Explain what is wrong with the following proposed definition of adjacent squares in the wumpus world:

$$\forall x, y \;\; Adjacent([x, y], [x + 1, y]) \wedge Adjacent([x, y], [x, y + 1]) \; .$$

8.18 Write out the axioms required for reasoning about the wumpus's location, using a constant symbol $Wumpus$ and a binary predicate $At(Wumpus, Location)$. Remember that there is only one wumpus.

8.19 Assuming predicates $Parent(p, q)$ and $Female(p)$ and constants $Joan$ and $Kevin$, with the obvious meanings, express each of the following sentences in first-order logic. (You may use the abbreviation $\exists^1$ to mean "there exists exactly one.")

a. Joan has a daughter (possibly more than one, and possibly sons as well).

b. Joan has exactly one daughter (but may have sons as well).

c. Joan has exactly one child, a daughter.

d. Joan and Kevin have exactly one child together.

e. Joan has at least one child with Kevin, and no children with anyone else.

8.20 Arithmetic assertions can be written in first-order logic with the predicate symbol $<$, the function symbols $+$ and $\times$, and the constant symbols 0 and 1. Additional predicates can also be defined with biconditionals.

a. Represent the property "x is an even number."

b. Represent the property "x is prime."

c. Goldbach's conjecture is the conjecture (unproven as yet) that every even number is equal to the sum of two primes. Represent this conjecture as a logical sentence.

8.21 In Chapter 6, we used equality to indicate the relation between a variable and its value. For instance, we wrote $WA = red$ to mean that Western Australia is colored red. Representing this in first-order logic, we must write more verbosely $ColorOf(WA) = red$. What incorrect inference could be drawn if we wrote sentences such as $WA = red$ directly as logical assertions?

8.22 Write in first-order logic the assertion that every key and at least one of every pair of socks will eventually be lost forever, using only the following vocabulary: $Key(x)$, x is a key; $Sock(x)$, x is a sock; $Pair(x, y)$, x and y are a pair; Now, the current time; $Before(t_1, t_2)$, time t_1 comes before time t_2; $Lost(x, t)$, object x is lost at time t.

8.23 For each of the following sentences in English, decide if the accompanying first-order logic sentence is a good translation. If not, explain why not and correct it. (Some sentences may have more than one error!)

a. No two people have the same social security number.

$$\neg\exists\, x, y, n \;\; Person(x) \wedge Person(y) \;\Rightarrow\; [HasSS\#(x, n) \wedge HasSS\#(y, n)].$$

b. John's social security number is the same as Mary's.

$$\exists\, n \;\; HasSS\#(John, n) \wedge HasSS\#(Mary, n).$$

c. Everyone's social security number has nine digits.

$$\forall\, x, n \;\; Person(x) \;\Rightarrow\; [HasSS\#(x, n) \wedge Digits(n, 9)].$$

d. Rewrite each of the above (uncorrected) sentences using a function symbol $SS\#$ instead of the predicate $HasSS\#$.

8.24 Represent the following sentences in first-order logic, using a consistent vocabulary (which you must define):

a. Some students took French in spring 2001.

b. Every student who takes French passes it.

c. Only one student took Greek in spring 2001.

d. The best score in Greek is always higher than the best score in French.

e. Every person who buys a policy is smart.

f. No person buys an expensive policy.

g. There is an agent who sells policies only to people who are not insured.

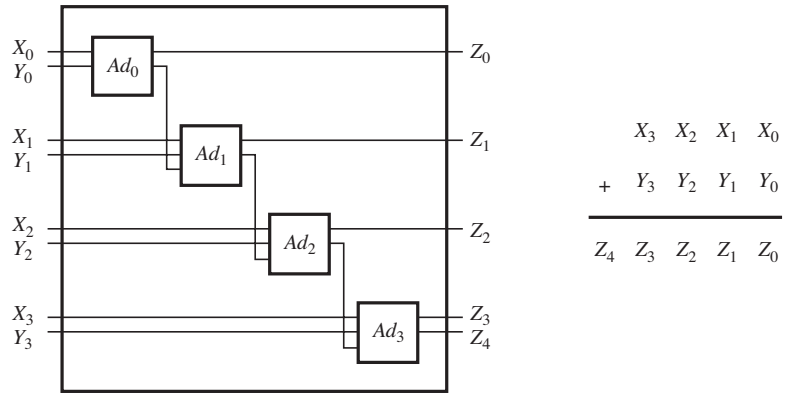


Figure 8.8 A four-bit adder. Each Ad_i is a one-bit adder, as in Figure 8.6 on page 309.

h. There is a barber who shaves all men in town who do not shave themselves.

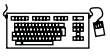
i. A person born in the UK, each of whose parents is a UK citizen or a UK resident, is a UK citizen by birth.

j. A person born outside the UK, one of whose parents is a UK citizen by birth, is a UK citizen by descent.

k. Politicians can fool some of the people all of the time, and they can fool all of the people some of the time, but they can't fool all of the people all of the time.

l. All Greeks speak the same language. (Use $Speaks(x, l)$ to mean that person x speaks language l.)

8.25 Write a general set of facts and axioms to represent the assertion "Wellington heard about Napoleon's death" and to correctly answer the question "Did Napoleon hear about Wellington's death?"



8.26 Extend the vocabulary from Section 8.4 to define addition for n-bit binary numbers. Then encode the description of the four-bit adder in Figure 8.8, and pose the queries needed to verify that it is in fact correct.

8.27 Obtain a passport application for your country, identify the rules determining eligibility for a passport, and translate them into first-order logic, following the steps outlined in Section 8.4.

8.28 Consider a first-order logical knowledge base that describes worlds containing people, songs, albums (e.g., "Meet the Beatles") and disks (i.e., particular physical instances of CDs). The vocabulary contains the following symbols:

$CopyOf(d, a)$: Predicate. Disk d is a copy of album a.
$Owns(p, d)$: Predicate. Person p owns disk d.
$Sings(p, s, a)$: Album a includes a recording of song s sung by person p.
$Wrote(p, s)$: Person p wrote song s.
$McCartney$, $Gershwin$, $BHoliday$, Joe, $EleanorRigby$, $TheManILove$, $Revolver$: Constants with the obvious meanings.

Express the following statements in first-order logic:

a. Gershwin wrote "The Man I Love."

b. Gershwin did not write "Eleanor Rigby."

c. Either Gershwin or McCartney wrote "The Man I Love."

d. Joe has written at least one song.

e. Joe owns a copy of *Revolver*.

f. Every song that McCartney sings on *Revolver* was written by McCartney.

g. Gershwin did not write any of the songs on *Revolver*.

h. Every song that Gershwin wrote has been recorded on some album. (Possibly different songs are recorded on different albums.)

i. There is a single album that contains every song that Joe has written.

j. Joe owns a copy of an album that has Billie Holiday singing "The Man I Love."

k. Joe owns a copy of every album that has a song sung by McCartney. (Of course, each different album is instantiated in a different physical CD.)

l. Joe owns a copy of every album on which all the songs are sung by Billie Holiday.