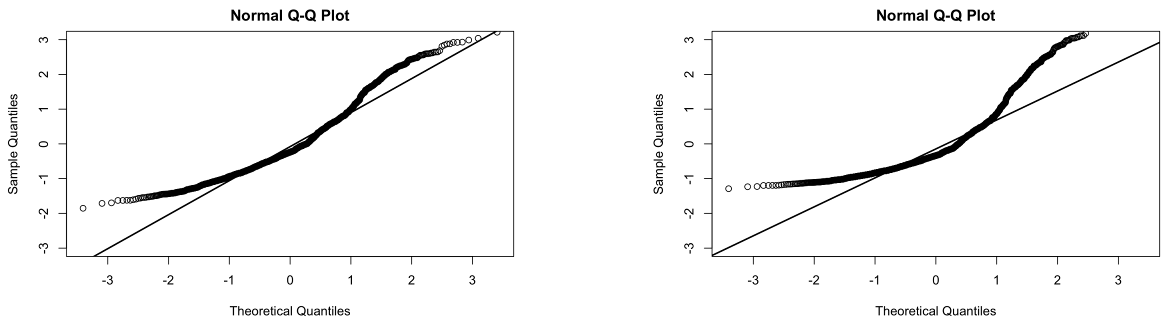
1 Call Price Surface vs. Implied Volatility

The implied volatility series for one specific pair of normalized strike and time to maturity is not normal by Jarque-Bera test, but is still better than the call price surface. The χ^2 score is in average -248 less in the implied volatility surface than in the call price surface. The average skewness and kurtosis for implied volatility are 1.013 and 3.628, while those for call price surface are 1.293 and 4.371. Figure 1 shows the typical qqplot of implied volatility and call price surface. Implied volatility has less skewness and excess kurtosis. Thus, for PLS factors, we use implied volatility as our input data.



(a) Implied Volatility (b) Call Price Surface

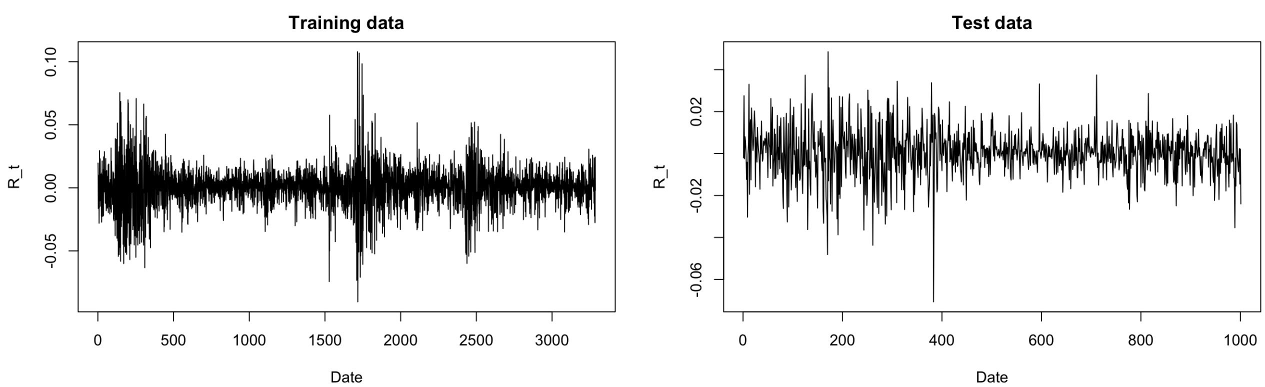
Figure 1: Typical qqplot for implied volatility and call price surface.

2 Cohen's data

2.1 GARCH-X for R

The log return R_t is stationary according to Augmented Dickey-Fuller test. We use GARCH(0,1) without latent factors but with normal distribution as the benchmark model. After that, we add the lag terms inside, which eliminate the autocorrelation of the residuals, but still do not have a good estimation on the mean and variance of R. The results of Neural SDE are also added in the end.

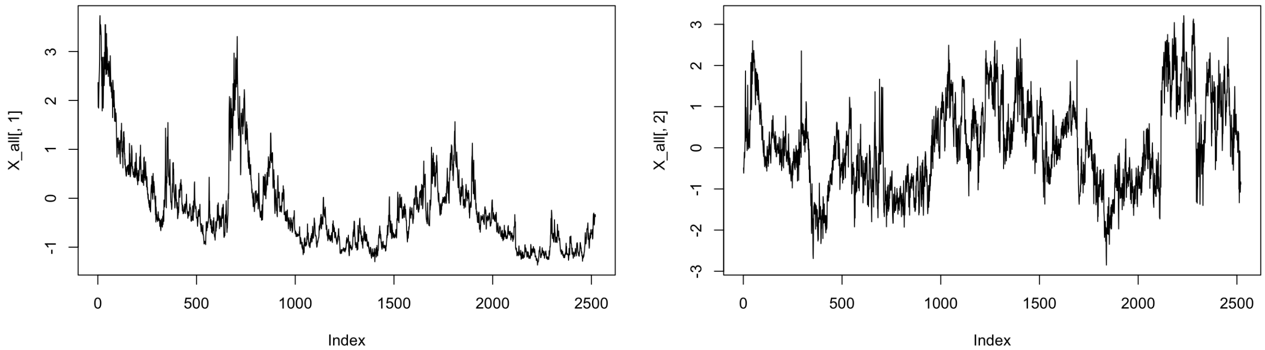
Note that the out-of-sample MSE is better than in-sample MSE since the test data has less unusual change than the training data, as can be seen in figure 2



(a) Training R_t (b) Test R_t

Figure 2: Training data and test data.

The latent factors from Cohen's paper are stationary by the Augmented Dickey-Fuller test. Figure 3 shows the time series of two latent factors.



(a) Factor 1

(b) Factor 2

Figure 3: Series of latent factors from Cohen's paper.

The concluded results are summarized in Table 1. The best model is ARMA(1,1)+eGARCH(2,2) with latent factors. It has the lowest out-of-sample MSE, and is the only one pass the coverage test. However, the only significant variables in the variance model are the lagged one.

$(\times 10^{-4})$	In-Sample MSE	Out-of-Sample MSE	Coverage Test	Coverage Test (95%)	Hypothesis Test
GARCH(0,1) w/o latent factors w/ normal	1.855	1.367	3.7%	Fail	Res: Independent, heteroscedasticity, not normal
GARCH(1,1) w/ latent factors w/ normal	1.855	1.368	5.5%	Pass	Res: Independent, heteroscedasticity, not normal, Insignificant factors
ARMA(1,1)+ eGARCH(2,2) w/ 1-lagged latent factors, w/ student t	1.842	1.291	5.7%	Pass	Res: Independent, heteroscedasticity, significant only in lagged terms in variance model

Table 1: Results using Cohen's factors.

2.2 DCC GARCH for Latent Factors

By SIC, the optimal VAR lag is 4. However, the residuals do not pass the ARCH-LM test. Thus, DCC model is used for latent factors. The residuals for the model pass both Ljung-Box and ARCH-LM test, which gives a base model for factors.

MSE	Factor 1	Factor 2
In-sample	0.0429	0.130
Out-of-sample	0.538	1.239

Table 2: MSE for DCC(2,2)+VAR(4) with each series following eGARCH(1,1)+AR(4) and student t distribution.

3 PLS

3.1 GARCH-X for R

By using the data starting from 2009, the training implied volatility data is stationary at the level of 90% under the Augmented Dickey-Fuller Test. Figure 4 shows one typical IV series which has p-value of 0.073. In general, we can assume that the input data for PLS are stationary, and we can extract PLS factors from them.

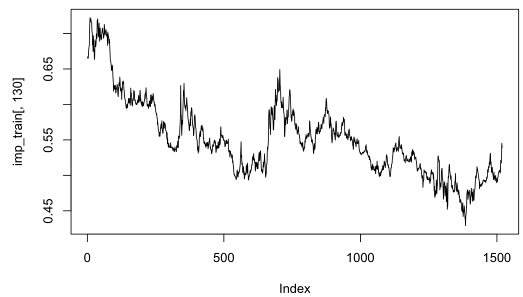
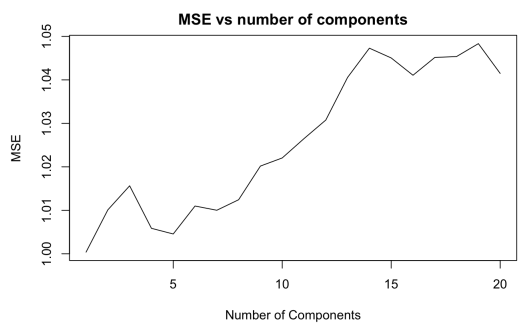
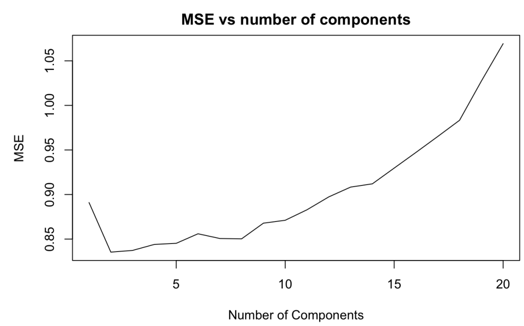


Figure 4: Typical implied volatility series which has high p-value in adf test.

Figure 5 shows that the optimal number of components is 1 for R vs implied volatility and 2 for R^2. We apply latent factors form R or R^2 in GARCH-X model to find the best choice.



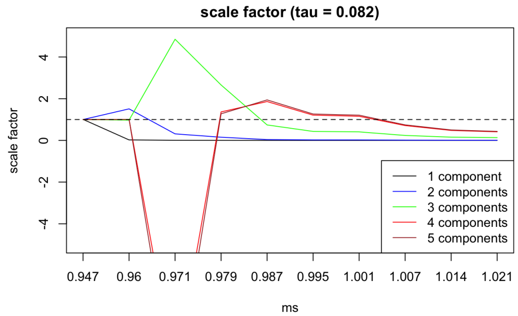
(a) R



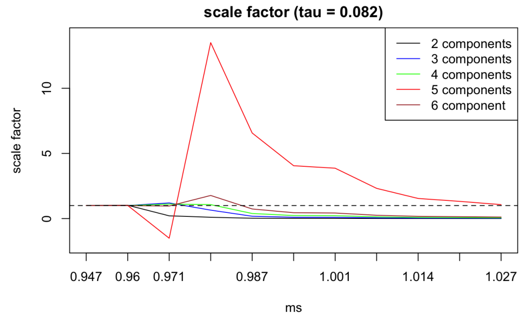
(b) R^2

Figure 5: MSE vs. number of components.

However, figure 6 shows that for R and R^2, 3 and 5 components give more information than PCA. Thus, we will use 3 for PLS of R and 2 and 5 for PLS of R^2 in the following results.



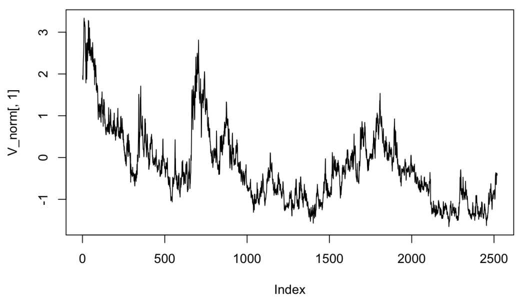
(a) R



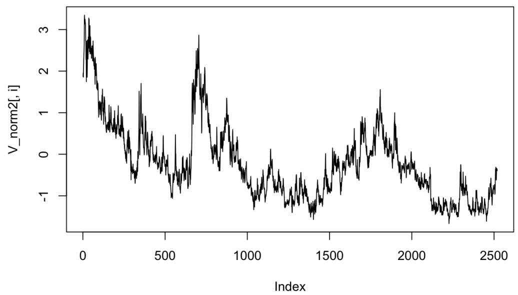
(b) R^2

Figure 6: MSE vs. number of components.

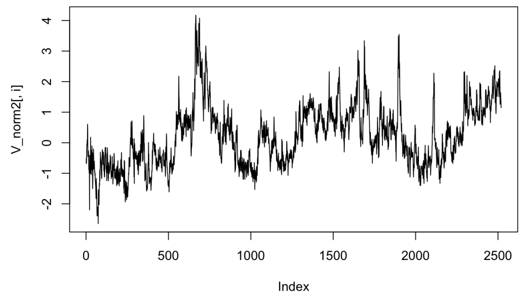
Figure 7 shows the time series for 3 normalized latent factors, including both training and test data, which is normalized through the mean and standard deviation from the training data. Augmented Dickey-Fuller test shows that all 3 series are stationary. Note that the latent factors from R and the first components from R^2 have the correlation of 0.999, and thus, using all 3 factors in GARCH-X model is not necessary. Meanwhile, the first component from PLS and Cohen's results are actually very similar from Figure 3 and Figure 7.



(a) R



(b) R^2



(c) R^2

Figure 7: Series of PLS factors.

The concluded results are summarized in Table 3. The best model is ARMA(1,1)+eGARCH(2,2) with latent factors from R^2. The in-sample and out-of-sample MSEs are similar, but it is the only one which passes the coverage test and has significance of latent factors in variance model.

$(\times 10^{-4})$	In-Sample MSE	Out-of-Sample MSE	Coverage Test (95%)	Coverage Test	Hypothesis Test
GARCH(0,1) w/ 3 PLS latent factors from R w/ normal	1.836	1.367	6.2%	Pass	Res: Independent, heteroscedasticity, not normal, insignificant factors in σ
ARMA(1,1)+ eGARCH(2,2) w/ 3 PLS latent factors from R and their 2 lags w/ ged	1.837	1.240	5.7%	Pass	Res: Independent, heteroscedasticity, significant of 1st components alone in variance model.
ARMA(1,1)+ eGARCH(2,2) w/ 2 PLS latent factors from R^2 and their 2 lags w/ student t	1.847	1.228	5.7%	Pass	Res: Independent, heteroscedasticity, significant factors in variance model
ARMA(1,1)+ eGARCH(2,2) w/ 3 PLS latent factors from R and their 2 lags in the mean model and 5 PLS latent factors from R^2 w/ ged	1.837	1.248	6.1%	Pass	Res: Independent, heteroscedasticity, significant of first 2 components in variance model
ARMA(1,1)+ gjrGARCH(0,1) w/ 3 PLS latent factors from R and their 2 lags in mean model and 5 PLS latent factors from R^2 and their 2 lags w/ ged	1.835	1.173	4.4%	Pass	Res: Independent, heteroscedasticity, significant of factors in both mean and variance model

Table 3: Results using PLS latent factors.

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3.2 DCC GARCH for Latent Factors

By SIC, the optimal VAR lag is 2 for both R and R^2. The residuals still do not pass the ARCH-LM test, so we seek for DCC GARCH model. The residuals pass both Ljung-Box and ARCH-LM test.

	R			R^2				
MSE	Factor 1	Factor 2	Factor 3	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5
In-sample	0.0336	0.0497	0.164	0.0342	0.0806	0.263	0.157	0.105
Out-of-sample	0.00200	0.00647	0.0263	0.00191	0.0130	0.0691	0.0193	0.0109

Table 4: MSE for DCC(1,1)+VAR(2) with each series following eGARCH(1,1)+AR(2) and student t distribution.

References