Asset Pricing with Dynamic Labor Contracts[*]

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ABSTRACT

I study asset prices in a two-agent production economy in which the worker has private information about her labor productivity. The shareholder offers an incentive compatible long-term labor contract, which partially insures the worker against labor income risk. I compare the model's performance to settings with a competitive labor market, and with static labor contracts. My model successfully matches both asset returns data and business-cycle features, including a countercyclical and high equity premium, a low risk-free rate, procyclical labor input, and countercyclical labor share. The results highlight that the dynamic contracting feature in labor relations is quantitatively important in determining asset prices.

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1 Introduction

U.S. workers have an average job tenure of 4.6 years. Why are labor relations long term and how are compensation schemes designed? What are the impacts on asset prices? In this paper, I consider a key observation of the labor market: Certain attributes of a worker, such as ability, passion, and physical condition, largely affect productivity but cannot be directly observed or measured by employers. Although working hours can be specified in the labor contract, effective effort level is beyond the employers' control. To capture such features, I build a model of dynamic labor contracts under private information. Firms use long-term compensation schemes, wage payment and promise of future raises, to better provide insurance as well as incentives to workers. I show that the dynamic contracting feature has unique predictions on the risk-sharing properties of the economy, and is quantitatively important in matching asset prices and busyness cycle facts.

I model a two-agent production economy in which the worker has private information about her labor productivity. The model features heterogeneous preferences and limited stock market participation: the representative shareholder is less risk averse and is better off holding risky assets, whereas the representative worker earns income by supplying labor. The worker's labor productivity risk has two dimensions: one publicly informed and one privately observed by the worker. The shareholder offers an incentive compatible long-term labor contract, which partially insures the worker against her labor income risk.

Following the recursive formulation in Spear and Srivastava (1987), I solve for the consumption allocations and labor input specified by the optimal labor contract. The equity return and risk-free rate are priced using shareholder's consumption stream as pricing kernel. The model is then calibrated with relatively low risk aversions for both agents. I compare the model's performance to settings with a Walrasian competitive labor market, and with static labor contracts. My model successfully matches both asset returns data and business-cycle facts, including a countercyclical and high equity premium, a low risk-free rate, procyclical labor input, and countercyclical labor share.

The model generates an equity premium of 6.13% and a risk-free rate 0.98%, matching the observation in Mehra and Prescott (1985). The calibrated fraction of privately observed labor productivity risk is 0.5. The consumption growth volatility matches Consumer Expenditure

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Survey with that of shareholders within $5.38\% \sim 12\%$, and that of workers within $3\% \sim 5.38\%$.[1] The ratio of the two assumes a value of 1.62, in line with the findings in Mankiw and Zeldes (1991) based on the PSID data. The cyclical feature of macro variables generated in this model are consistent with data.

The model mechanism is as follows. First, the endogenous labor input is procyclical, thus expanding or contracting aggregate production in response to labor productivity shocks. In good economy (e.g. favorable climate, technology) and when being more productive (e.g. smart, energetic), the worker is given incentive to devote higher effort. In the aggregate, hours worked increase when the economy is good. Hence, consumption volatility is endogenously amplified by the incentive compatible labor contracts.[2] Second, the risk-sharing allocation under dynamic contracts endogenously generates a countercyclical labor share. The less risk averse shareholder offers an incentive compatible long-term contract, which partially insures the worker against her labor income risk. Essentially the shareholder bears a larger consumption risk, thus requires to be compensated by a higher equity premium. Finally, the dynamic feature captures the intertemporal risk-sharing in the long-term employment relationship. The promised continuation utility provides shareholders an extra tool to balance insurance and incentive, thus further facilitating risk-sharing. In Section 3, we isolate the effect of intertemporal risk-sharing from intratemporal risk-sharing by observing a higher level of equity premium and risk ratio in the dynamic labor contract model relative to a benchmark static model.

This paper contributes to the large body of literature on equity premium puzzle, which documents the difficulty of using neoclassical models to rationalize the U.S. stock market premium.[3] With plausible constant relative risk aversion, the average consumption growth volatility is too low to rationalize the observed 6.18% equity premium.[4] Constantinides and Duffie (1996) show that it is possible for incomplete market models to explain the asset pricing anomalies in an economy with uninsurable, persistent, and heteroscedastic labor income shocks. Kocherlakota (1996) also argues that modeling incomplete insurance markets where agents fail to fully insure themselves due to private information is a promising direction. As labor income provides the

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[1]See data provided in Malloy, Moskowitz, and Vissing-Jorgensen (2009).

[2]Shorish and Spear (2005) show in their static agency theoretic model that worker's ability to endogenously control output largely affects asset prices.

[3]See Mehra and Prescott (1985) for the puzzle, Kocherlakota (1996) for a survey.

[4]Mehra and Prescott (1985) observe for 1889-1978, market return is 6.98% and risk-free rate is 0.80%.

lion's share of consumption for the working class, partial insurance in labor market as explored here is necessarily relevant.

Shareholder's consumption streams are used as pricing kernel. In this sense, this paper adds to the literature on limited stock market participation.[5] Mankiw and Zeldes (1991) show evidence that only one fourth of US households own stocks and that stockholders' consumption growth is much more volatile than that of non-stockholders. While most papers rely on a fixed cost of participation or borrowing constraints, agents in my model choose their roles to enter the contracts based on heterogeneous risk aversion. Hence, workers have no need to further seek insurance from the asset market because the labor contract has provided them with maximum possible insurance constrained by private information.

Berk and Walden (2013) explore similar idea that investors provide insurance to wage earners who then optimally choose not to participate in the financial markets. They demonstrate that human capital risk is shared in labor markets through bilateral labor contracts, and investors offload the labor market risk they assumed from workers by participating in financial markets. While they focus on the labor contracts based on working flexibility, I differ by building on the incentive compatible contracts in a world with private information.

My paper is closest to Danthine and Donaldson (2002) who show that *operation leverage* by wage claims magnifies the risk of residual payments to firm owners and the required risk premium. However, in their model, the labor supply is fixed and the optimal risk-sharing formula is exogenously specified. This paper differs by solving the endogenous labor supply and limited risk-sharing determined by labor contracts.

The paper proceeds as follows: Section 2 describes the model. Section 3 presents quantitative results on equilibrium allocations and asset pricing implications. Extension on persistent public shocks is presented in Section 4, and Section 5 concludes.

2   Model

In this section, I describe a production economy with heterogeneous agents, private information, and dynamic labor contracts. The goal is to study the risk-sharing properties and asset pricing

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[5]See evidence for example in Mankiw and Zeldes (1991), Basak and Cuoco (1998), Cao, Wang, and Zhang (2005), Berk and Walden (2013).

implications.

2.1 Environment

This economy has a single, non-durable, consumption good. Time is discrete. A continuum of measure α of shareholders invest in the asset market by holding ownership of the firm. A continuum of measure $1 - \alpha$ of workers supply labor in production and earn consumption from labor income. Heterogeneous preferences in risk aversion and labor disutility sort agents into two groups, thus generating limited market participation. Labor productivity risk contains both publicly informed and privately observed components. A labor contract of a horizon of T periods formalizes the employment relationship between a worker and a shareholder.

Workers

Assume workers are all alike, so there exists a representative worker. The worker has additively separable period utility defined over consumption C and labor input L. $U(C_t, L_t)$ is increasing in C_t, decreasing in L_t, twice continuously differentiable and concave in both consumption and labor. Workers discount future utility at constant rate $0 < \beta < 1$. Given a sequence of consumption and labor input $\{C_t, L_t\}_{t=1}^T$, the worker's expected discounted utility over the contract horizon is

$$W\left(\{C_t, L_t\}_{t=1}^T\right) = \mathbb{E}_0 \sum_{t=1}^T \beta^{t-1} U(C_t, L_t), \tag{1}$$

where $\mathbb{E}_0$ denotes expectation at pre-contracting stage.

Shareholders

Shareholders by nature are less risk averse and have stronger labor disutility.[6] The representative shareholder's period utility $V(D_t)$ is also additively separable over time with discount factor β, and is increasing, twice continuously differentiable and concave in consumption D_t.[7] Shareholders hold firm shares S_t and risk-free bonds B_t and obtain dividend income D_t as consumption. Let the equilibrium stock price be P_t and bond price be b_t^{t+1}. Shareholder's budget constraint at time t is

$$D_t + P_t S_{t+1} + B_{t+1} b_t^{t+1} \leq (P_t + D_t) S_t + B_t. \tag{2}$$

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[6] Sources of heterogeneity include age, wealth, social status, etc.

[7] Labor does not enter the utility function directly because shareholders do not directly supply labor in production.

Let ψ be the distribution of shareholders. The asset market clearing condition is given by

$$\int B_t d\psi = 0, \quad \int S_t d\psi = 1. \tag{3}$$

Accordingly, a representative shareholder solves

$$\max_{\{D_t, S_t, B_t\}_{t=1}^T} : \mathbb{E}_0 \sum_{t=1}^T \beta^{t-1} V(D_t), \tag{4}$$

subject to (2) and (3).

Production, Risks, and Information

Since the focus is on labor relations, I fix the capital stock as constant for all periods and states, i.e. $K_t = K, \forall z^t, \theta^t$. There are two dimensions of labor productivity risk $\phi_t(z_t, \theta_t)$: publicly informed z_t and privately observed θ_t.[8] Let $z^t \equiv (z_1, z_2, ..., z_t) \in Z^t$ and $\theta^t \equiv (\theta_1, \theta_2, ..., \theta_t) \in \Theta^t$ denote the histories of the shocks up to period t. $\phi(z_t, \theta_t)$ is increasing with both arguments. Output is given by Cobb-Douglas form with capital share α,

$$Y_t = \phi(z_t, \theta_t) L_t^{1-\alpha} K^\alpha. \tag{5}$$

Private information of a worker concerns with both private labor productivity shock θ_t and the amount of labor input L_t. We make the following assumptions on the distribution of risks.

Assumption 1 *Labor productivity risks satisfy the following.*

(i) The private risk is independent of the public shock.

(ii) The shocks are identically distributed over time and independent of history realizations.

Under Assumption 1, the probability of drawing shocks $\{z_t, \theta_t\}$ is $\Pi(z_t) P(\theta_t)$. Assumption 1.(i) is to isolate the private information nature of the private risks, so that nothing can be inferred from the realization of public risks. We will relax Assumption 1.(ii) by adding persistence to the public shocks in Section 4. Under 1.(ii), the size of the shocks is time-invariant. Following Ales and Maziero (2010), the fraction of privately observed labor productivity risk is given by

$$\Omega = \frac{Var(\theta_t)}{Var(z_t) + Var(\theta_t)} \in [0, 1], \quad \forall t. \tag{6}$$

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[8]θ^t is the systematic component of private labor productivity risk. As in Berk and Walden (2013), the idiosyncratic component of labor productivity risk can be diversified away among workers, even though it is likely to be sizable for an individual worker.

Dynamic Labor Contract

The timeline of the contract is as follows. At $t = 0$, the shareholder and the worker enter a horizon T exclusive labor contract. This employment relationship promises the worker an expected utility above his initial reservation utility W_0. We assume two-sided commitment such that both parties commit to stay in the contract once it is signed. Upon observing the realization of both public and private shocks each period, the worker strategically reports to the shareholder about her private labor productivity and exert effort accordingly. At the end of the period, output is realized and consumption is allocated based on the contract.

I solve for the equilibrium allocations and labor input defined by the optimal contract. The revelation principle ensures that we can restrict to direct mechanisms in which workers truthfully report the private labor productivity. Given a worker's initial reservation utility W_0, the contract specifies consumption allocation, labor input, and required output conditional on the realized history of public shock z^t and the reported history of private shock θ^t, i.e. $\{C(z^t, \theta^t, W_0), Y(z^t, \theta^t, W_0), L(z^t, \theta^t, W_0)\}_{t=1}^T$.

The worker chooses to enter the contract only if the expected discounted utility from the long-term employment is no less than W_0. A contract satisfies *individual rationality (IR)* if the following holds,

$$\sum_{t=1}^T \sum_{z^t, \theta^t} \Pi(z^t) P(\theta^t) \beta^{t-1} U(C(z^t, \theta^t), L(z^t, \theta^t)) \geq W_0. \tag{7}$$

In order to induce truth-telling, the contract is such that there are no gains by deviating from truthfully reporting the privately observed shock state. A contract is *incentive compatible (IC)* if it satisfies the following

$$\sum_{t=1}^T \sum_{z^t, \theta^t} \Pi(z^t) P(\theta^t) \beta^{t-1} \left[U(C(z^t, \theta^t), L(z^t, \theta^t)) - U(C(z^t, \tilde{\theta}^t), \hat{L}(z^t, \tilde{\theta}^t)) \right] \geq 0, \forall \tilde{\theta}^t \in \Theta^t, \tag{8}$$

where the required effort level if lying is

$$\hat{L}(z^t, \tilde{\theta}^t)) = \left[\frac{Y(z^t, \tilde{\theta}^t)}{\phi(z_t, \theta_t) K^\alpha} \right]^{\frac{1}{1-\alpha}} = \left[\frac{\phi(z_t, \tilde{\theta}_t)}{\phi(z_t, \theta_t)} \right]^{\frac{1}{1-\alpha}} L_t(z_t, \tilde{\theta}_t). \tag{9}$$

A contract is *feasible* if it satisfies

$$C(z^t, \theta^t) + D(z^t, \theta^t) + B_{t+1} b_t^{t+1}(z^t, \theta^t) \leq \phi(z_t, \theta_t) L_t^{1-\alpha} K^\alpha + B_t, \ \forall z^t \in Z^t, \theta^t \in \Theta^t. \tag{10}$$

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For a contract satisfying Conditions (8), (9), (10), the worker truthfully reports his private shock states and exerts optimal labor input in exchange for a compensation profile less volatile than his marginal productivity. Thereby, the labor contract provides partial insurance to the worker against her labor income risk. In other words, the existence of private information induces limited risk-sharing between shareholder and worker.

Financial Market

I consider a setting of incomplete financial markets with two assets: the risky asset and the risk-free asset. Risky asset is the dividend claim from holding the firm share, holdings of which are within the shareholders. Allen (1985) shows that incentive compatibility constraint cannot hold if agents are able to hold assets privately. Hence, I make the following assumption

Assumption 2 *Asset holdings of shareholders and workers are public information.*

Based on Assumption 2, denote the bond holding of a worker as B_t^w, then we have the following characterization on worker's consumption.

Lemma 1 *Equilibrium consumption allocation defined by the optimal contract satisfies*

$$C(z^t, \theta^t | B_t^w) = C(z^t, \theta^t | \tilde{B}_t^w), \quad \forall B_t^w \neq \tilde{B}_t^w. \tag{11}$$

Lemma 1 delivers the idea that worker's equilibrium consumption is independent of her bond holdings. When the worker's bond holding is public information, the shareholder will specify labor contract conditional on the bond holdings such that worker's equilibrium consumption stays the same. This way, any potential insurance effect by participating in the financial market is already covered by the labor contract. Therefore, it is without loss of generality that workers are restricted from asset market participation. I further make the following assumption.

Assumption 3 *Risk-free bonds are in zero net supply, i.e. only private bonds are traded. Shareholders are all alike and behave independently and competitively.*

Lemma 2 *The equilibrium allocations specified by the labor contract depend on the shareholder's bond holding. Under Assumption 3, the equilibrium bond holding of a typical shareholder $B_t = 0$.*

Shareholder's Problem

The representative shareholder maximizes his expected discounted utility by designing the optimal dynamic labor contract and choosing the optimal amount of asset holdings. Hence, the representative shareholder solves the following problem subject to Equations (7), (8), (9), and (10).

$$\max_{\{C_t, D_t, L_t, B_t\}_{t=1}^T} : \sum_{t=1}^T \sum_{z^t} \Pi(z^t) \sum_{\theta^t} P(\theta^t) \beta^{t-1} V(D(z^t, \theta^t)). \tag{12}$$

Endogenous Stock Market Participation

For each reservation value W_0, the labor contract specifies a sequence of consumption allocations as a function of the sequence of shocks. The equilibrium reservation value is chosen by shareholders such that workers is better off serving as a worker, that is[9]

$$W_0 \geq \sum_{t=1}^T \sum_{z^t, \theta^t} \Pi(z^t) P(\theta^t) \beta^{t-1} U\left[D(z^t, \theta^t, W_0), L(z^t, \theta^t) = 0\right]. \tag{13}$$

2.2 Recursive Formulation

To solve (12), it is convenient to rewrite the shareholder's problem recursively. Following Spear and Srivastava (1987) and Green (1987), we use the promised utility as a state variable, denoted by W. Provided certain boundary conditions satisfied by our problem, under i.i.d. private shock distribution, temporary incentive compatibility is sufficient to guarantee the general IC condition (8).

The shareholder's problem (12) can be solved by considering two problems separately: (1) *period T problem*, where the shareholder specifies period T consumption and labor, and (2) *period t problem* where the shareholder chooses current consumption, labor, and promised continuation utility. From here onward, we make the additional assumption that both the public and private labor productivity shocks have two state realizations per period, $z_t \in \{z_h, z_l\}, \theta_t \in \{\theta_h, \theta_l\}$, with $z_h > z_l, \theta_h > \theta_l, \forall t$. The seed values $\{\theta_h, \theta_l\}$ are known to all agents.

I consider the relaxed problem: only incentive compatibility constraints to prevent worker

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[9]There are multiple values of W_0 satisfying Condition (13). In the quantitative analysis, I calibrate it to match the level of labor share in dynamic labor contract model with that in the Walrasian RBC model $1 - \alpha = 64\%$.

with high state realization θ_h from lying are considered.[10] The period T problem is

$$S_T(W_T) = \max_{\{C_T, D_T, L_T\}} \sum_{z_T} \Pi(z_T) \sum_{\theta_T} P(\theta_T) V(D_T) \tag{14}$$

subject to

$$\sum_{z_T} \Pi(z_T) \sum_{\theta_T} P(\theta_T) U(C_T, L_T) \geq W_T; \tag{15}$$

$$U[C(z_T, \theta_h), L(z_T, \theta_h)] \geq U\left[C(z_T, \theta_l), \hat{L}(z_T, \theta_l)\right], \quad \forall z_T \in Z_T, \tag{16}$$

where $\hat{L}(z_T, \theta_l) = \left[\frac{\phi(z_T, \theta_l)}{\phi(z_T, \theta_h)}\right]^{\frac{1}{1-\alpha}} L(z_T, \theta_l);$

$$C_T + D_T \leq Y_T, \quad \forall z_T \in Z_T, \theta_T \in \Theta_T. \tag{17}$$

The period t problem is:

$$S_t(W_t) = \max_{\{C_t, D_t, L_t, W_t'\}} \sum_{z_t} \Pi(z_t) \sum_{\theta_t} P(\theta_t) \left[V(D_t) + \beta S_{t+1}(W_t')\right] \tag{18}$$

subject to

$$\sum_{z_T} \Pi(z_T) \sum_{\theta_T} P(\theta_T) \left[U[C_t, L_t] + \beta W_t'\right] \geq W_t; \tag{19}$$

$$U[C(z_t, \theta_h), L(z_t, \theta_h)] + \beta W_t'(z_t, \theta_h) \geq U\left[C(z_t, \theta_l), \hat{L}(z_t, \theta_l)\right] + \beta W_t'(z_t, \theta_l), \quad \forall z_t \in Z_t, \tag{20}$$

where $\hat{L}_t(z_t, \theta_l) = \left[\frac{\phi(z_t, \theta_l)}{\phi(z_t, \theta_h)}\right]^{\frac{1}{1-\alpha}} L_t(z_t, \theta_l).$

$$C_t + D_t \leq Y_t, \quad \forall z_t \in Z_t, \theta_t \in \Theta_t. \tag{21}$$

We adopt the Cobb-Douglas functional form for the worker's utility

$$U(C, L) = \frac{[C^\tau (1-L)^{1-\tau}]^{1-\sigma_w}}{1 - \sigma_w}, \tag{22}$$

where the consumption share is $\tau \in (0, 1)$ and the curvature parameter is $\sigma_w > 1$. Cobb-Douglas utility implies a constant elasticity of substitution between consumption and leisure; hence, the labor input is constant across states when competitive wage is offered at the marginal product of labor. Shareholder has power utility function with constant relative risk aversion σ_s

$$V(C) = \frac{C^{1-\sigma_s}}{1 - \sigma_s}. \tag{23}$$

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[10]The relaxed problem is shown to be equivalent to the original problem in Green and Oh (1991) for a more general setting. In the numerical solutions, I verify that the solution of the relaxed problem satisfies the original problem.
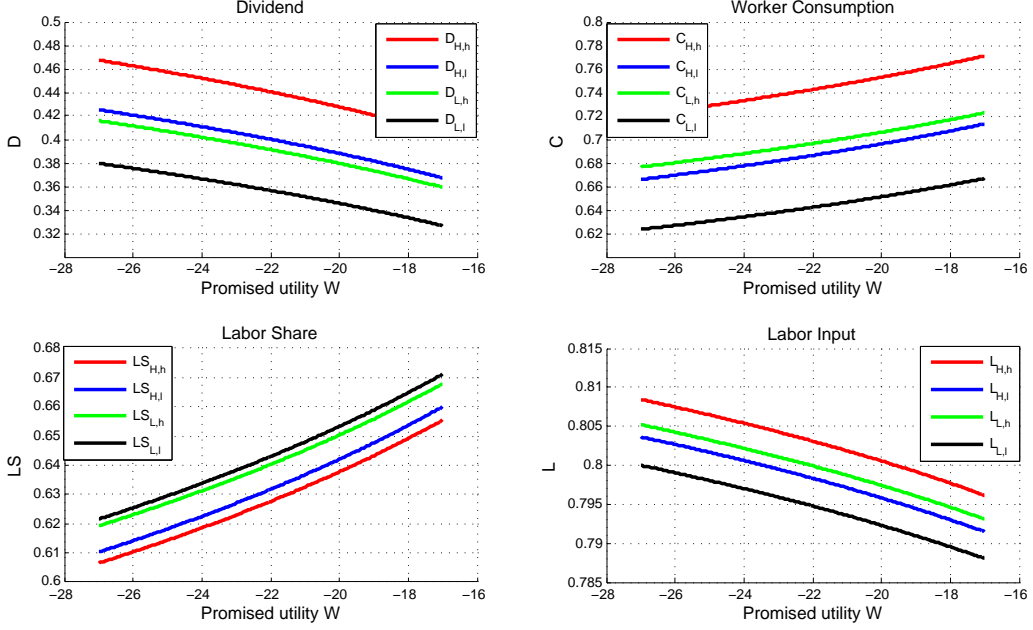
Figure 1. Equilibrium allocations under private information. This figure shows the equilibrium allocations of shareholder dividend, worker consumption, labor share, labor input under private information.

The shareholders are less risk averse than the worker, i.e. $0 < \sigma_s < 1 + \tau(\sigma_w - 1)$. I further assume the technology shock takes the form

$$\phi(z_t, \theta_t) = z_t \theta_t. \tag{24}$$

2.3 Optimality Conditions

I characterize the properties of the equilibrium allocations specified by the dynamic contracts under private information. Figure 1 shows the constrained optimal consumption allocation and labor input at $\Omega = 0.5$. From Panel A and B, both agents' consumption profiles are procyclical with productivity. Shareholder's dividend shows larger percentage deviation than that of the worker. Panel C plots labor share, the percentage of worker's consumption over total output. The countercyclical labor share demonstrates how labor contract provides insurance to the worker. Panel D shows that dynamic labor contract features a procyclical labor input which enlarge the aggregate production risk.

I characterize the optimal solution in the full information case below.

Proposition 1 *The optimal solution under full information features,* $\forall(z, \theta)$,

(i) *Perfect risk-sharing* $\frac{\partial V(z,\theta)}{\partial D(z,\theta)} = \eta_t \frac{\partial U(z,\theta)}{\partial C(z,\theta)}$;

(ii) *Countercyclical labor share* $\frac{C(z,\theta)}{Y(z,\theta)} = \frac{\tau(1-\alpha)}{1-\tau}(\frac{1}{L(z,\theta)} - 1)$;

(iii) *Constant promised continuation utility* $W'(z,\theta) = W$;

(iv) *The first best dynamic contracts can be equivalently implemented by a sequence of static contracts.*

2.4 Asset Prices

The limited market participation implies that equilibrium holding of risky asset is Autarky, i.e. $S_t^* = 1$. Given any state realizations $\{z_t, \theta_t\}_{t=1}^T$ and the equilibrium allocation series $\{D_t, C_t, L_t\}_{t=1}^T$, the price of risky asset is given by

$$P_t = \beta \mathbb{E}_t \left[\frac{V'(D_{t+1})}{V'(D_t)}(P_{t+1} + D_{t+1}) \right] = \mathbb{E}_t \left[\sum_{j=1}^{T-t} \beta^{j-1} \frac{V'(D_{t+j})}{V'(D_t)} D_{t+j} \right]. \tag{25}$$

and $P_T = 0, \quad \forall z_T, \theta_T$.

Implied equilibrium market return from holding the risky asset is

$$R_t = E_t \left[\frac{P_{t+1} + D_{t+1}}{P_t} \right] - 1. \tag{26}$$

Bond price b_t^{t+1} is calculated from the shareholder's intertemporal Euler equation

$$b_t^{t+1} = \beta E_t \left[\frac{V'(D_{t+1})}{V'(D_t)} \right]. \tag{27}$$

The implied equilibrium risk-free rate is

$$r_t^f = \frac{1}{b_t^{t+1}} - 1. \tag{28}$$

Equity premium is the difference between market return and risk-free rate

$$EP_t = R_t - r_t^f. \tag{29}$$

3 Quantitative Results

In this section, I solve the model numerically for equilibrium allocations and asset returns. Then I compare the performance of three different models in matching financial market statistics and macro variable features. The baseline model is the finite horizon dynamic labor contract model under private information. The two reference models are respectively Walrasian real business cycle model (with competitive wage) and a static labor contract model.

Walrasian RBC Model

The canonical Walrasian RBC model features a competitive labor market and is commonly used in the production based asset pricing literature. With a competitive wage $w^* = \frac{\partial Y_{z,\theta}}{\partial L_{z,\theta}} = (1-\alpha)z\theta L_{z,\theta}^{-\alpha}K^{\alpha}$, we get constant labor share: $LS = 1 - \alpha$. Worker has constant labor input $L(z,\theta) = \tau$ and bears her share of income risk $C(z,\theta) = (1-\alpha)z\theta L_{z,\theta}^{1-\alpha}K^{\alpha}$.[11]

Static Agency Model

The second reference model is a static agency model, in which the shareholder and the worker enter a static contract every period. Given the worker's period reservation utility W, the shareholder solves the following problem

$$S(W) = \max_{\{C,D,L\}} \sum_z \Pi(z) \sum_\theta P(\theta)V\left[D(z,\theta)\right] \tag{30}$$

subject to

$$\sum_z \Pi(z) \sum_\theta P(\theta)U\left[C(z,\theta), L(z,\theta)\right] \geq W \tag{31}$$

$$U\left[C(z,\theta_h), L(z,\theta_h)\right] \geq U\left[C(z,\theta_l), \left[\frac{\phi(z,\theta_l)}{\phi(z,\theta_h)}\right]^{\frac{1}{1-\alpha}} L(z,\theta_l)\right] \tag{32}$$

$$C(z,\theta) + D(z,\theta) \leq Y(z,\theta), \quad \forall z \in Z, \theta \in \Theta. \tag{33}$$

As shown in Proposition 1.(iv), the first best dynamic contract can be equivalently implemented by a corresponding sequence of static contract under the same private information structure. This enables us to match the static model with the dynamic contract model to the extent that the optimal allocation under private information of the two models are comparable. Given the worker's initial reservation utility W_0 in the dynamic contract, and the corresponding first best solution of the continuation utility as $\{W'_t(z,\theta)\}_{t=1}^{T-1}$, the equivalent static reservation utility at time t is $\overline{W}_t = W'_{t-1} - \beta W'_t$, as in the proof of Proposition 1. Using this transformation,

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[11]In competitive labor market, individual worker decides how much labor to input. Given the competitive wage level w^*, each worker solves: $\max \sum \Pi_z P_\theta U\left[C(z,\theta), L(z,\theta)\right]$, subject to their budget constraint: $C(z,\theta) \leq w^* L(z,\theta)$. First order conditions yield: $-\frac{\partial U[C(z,\theta),L(z,\theta)]}{\partial C(z,\theta)} w^* = \frac{\partial U[C(z,\theta),L(z,\theta)]}{\partial L(z,\theta)}$. From firm's optimization, $w^* = \frac{\partial Y_{z,\theta}}{\partial L_{z,\theta}} = (1-\alpha)z\theta L_{z,\theta}^{-\alpha}K^{\alpha}$. Combining the two conditions, the equilibrium labor input is: $\frac{\tau}{L_{z,\theta}} = \frac{1-\tau}{1-L_{z,\theta}} \Rightarrow L_{z,\theta} = \tau, \quad \forall z,\theta$.

Table 1. Benchmark Parameter Calibration

| Parameter	Symbol	Value
Contracting time horizon	T	8
Capital share	α	0.36
Constant capital stock	k_0	1.257
Probability of a high publicly observed productivity shock	$\Pi(z = H)$	0.5
Probability of a high privately observed productivity shock	$\Pi(\theta = h)$	0.5
Consumption share in worker's utility	τ	0.8
Time preference (discount rate)	β	0.975
Shareholder's risk aversion	σ_s	3.5
Worker's utility function curvature	σ_w	8
Worker's risk aversion	$\tau(\sigma_w - 1) + 1$	6.6
Worker's lifetime reservation utility	W_0	-78
Fraction of labor productivity risk due to private information	Ω	0.5
Total risk of productivity	$Var(z) + Var(\theta)$	0.0033

the static first best problem and dynamic first best problem *match exactly* in their equilibrium allocations and asset prices.

The importance of noncompetitive labor market and labor contract framework is shown by comparing labor contract models with the Walrasian RBC model. The comparison of models with dynamic contracts versus models with static contracts shows the intertemporal incentive effect by promised continuation utility. This allows us to separate intertemporal risk-sharing from intratemporal risk-sharing.

3.1 Parameter Calibration

The parameter calibration is shown in Table 1. The model is calibrated at an annual frequency. The contracting time horizon T is parametrized with both empirical and computational considerations. Despite the lack of formal statistics, several studies have shown that people change jobs every 3 to 10 years in relatively large businesses. The model generated moments tend to be numerically more stable the longer horizon we take. As computation time increases exponentially with the time horizon, $T = 8$ is chosen as a compromise.

Capital share $\alpha = 0.36$ is set following the convention of real business cycle models. Constant capital K^α in the production function is set to 1.257. Both public and private shocks are i.i.d. in the benchmark model with $\Pi(z = H) = \Pi(z = L) = 0.5$, $P(\theta = h) = P(\theta = l) = 0.5$.
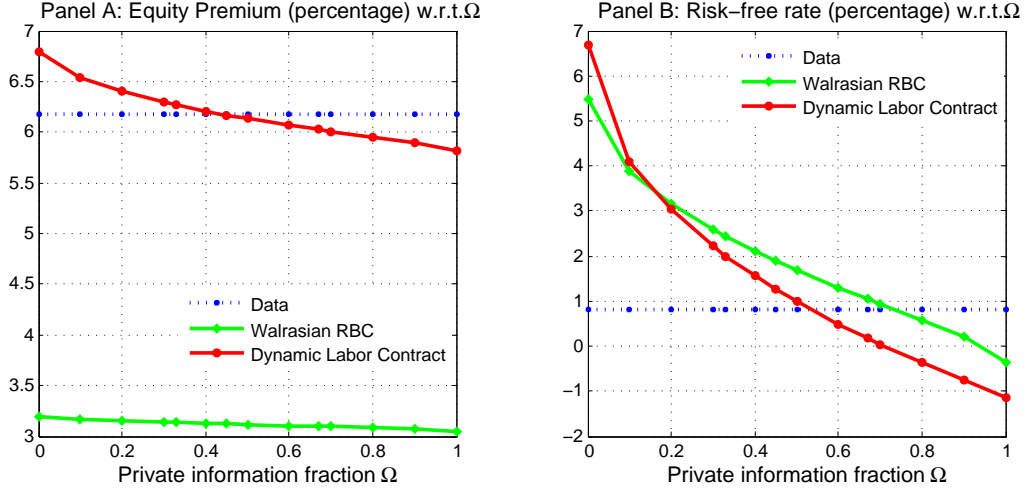
Figure 2. Equity Premium and Risk-free Rate w.r.t. Ω**.** The figure plots the equity risk premium and risk free rate (in percentage) when we vary the fraction of labor productivity risk due to private information Ω.

The consumption share in worker's utility τ, agents' subjective time preference β, and curvature parameters σ_s, σ_w are free parameters calibrated to match the target moments of asset returns and consumption.[12] We set the shareholder's risk aversion $\sigma_s = 3.5$, and set $\sigma_w = 8$ to yield workers' risk aversion as $\tau(\sigma_w - 1) + 1 = 6.6$, a commonly adopted value in the equity premium literature.[13]

Since the worker's reservation utility W_0 directly determines her bargaining power in the contracting process and her welfare, we calibrate it to match the level of labor share in the dynamic labor contract model with that in the Walrasian RBC model $1 - \alpha = 64\%$.

The fraction of labor productivity risk due to private information Ω is calibrated as $\Omega = 0.5$ in the benchmark model. In Figure 2, we see how the level of equity premium and risk-free rate change in the Walrasian RBC and the dynamic labor contract model when we vary Ω in the range of $[0, 1]$ while keeping the total risk of productivity $Var(z) + Var(\theta)$ fixed. Both the equity premium and risk-free rate decrease as the fraction of private information Ω increases. The dynamic labor contract model with value $\Omega = 0.5$ gives the two moments in the right ballpark, while Walrasian RBC has equity premium way lower than data.

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[12]Ales and Maziero (2010) estimate in their benchmark model $\tau = 0.69$ using CEX and PSID data.

[13]Guvenen (2009) uses 7, Gomes and Michaelides (2008) select 5, and Danthine and Donaldson (2002) adopt a value around $4 \sim 7$.

Table 2. Comparing Model Generated Moments with Data

Moments	Data	Dynamic Labor Contract	Walrasian RBC	Static Labor Contract
Asset returns				
Equity premium	6.18%	6.13%	3.12%	4.19%
Market return	6.98%	7.12%	4.80%	7.11%
Risk-free rate	0.80%	0.98%	1.69%	2.92%
$\sigma(EP_t)$	16.67%	2.59%	1.09%	1.51%
$\sigma(R_t)$	16.54%	27.03%	17.75%	21.54%
$\sigma(r_t^f)$	5.67%	24.84%	16.85%	20.29%
Consumption risks				
$\sigma(\Delta D_{t+1})$	5.38% - 12%	7.84%	5.66%	7.82%
$\sigma(\Delta C_{t+1})$	3% - 5.38%	4.84%	5.66%	5.95%
$\frac{\sigma(\Delta D_{t+1})}{\sigma(\Delta C_{t+1})}$	1.6	1.6	1	1.3
Average labor share	0.64	0.64	0.64	0.64
Correlations				
$\mathrm{Corr}(EP_t, z_t)$	-	-0.41	-0.41	-0.62
$\mathrm{Corr}(EP_t, \theta_t)$	-	-0.56	-0.53	-0.31
$\mathrm{Corr}(L_t, z_t)$	+	0.35	0	-0.12
$\mathrm{Corr}(L_t, \theta_t)$	+	0.68	0	0.95
$\mathrm{Corr}(D_t, z_t)$	+	0.56	0.54	0.85
$\mathrm{Corr}(D_t, \theta_t)$	+	0.52	0.54	0.12
$\mathrm{Corr}(C_t, z_t)$	+	0.52	0.54	0.23
$\mathrm{Corr}(C_t, \theta_t)$	+	0.56	0.54	0.79
$\mathrm{Corr}(\frac{C_t}{Y_t}, z_t)$	-	-0.60	0	-0.88
$\mathrm{Corr}(\frac{C_t}{Y_t}, \theta_t)$	-	-0.45	0	0.80

Note: Models are compared under the same sequence of state realizations. Sources of data include: (1) Mehra and Prescott (1985); (2) CEX 1980-2005 Q1, see Mankiw and Zeldes (1991).

3.2 Comparison of Model Performance

With the benchmark model parametrized as above, we numerically compare the model generated moments of equilibrium allocation for consumption, labor input, labor share, and the asset returns in the dynamic labor contract model and the two reference models.

Table 2 shows the comparison of model generated *unconditional moments* with real data. historical asset returns data varies greatly depending on the time window. I take the commonly cited data from Table 1 of Mehra and Prescott (1985). For the benchmark calibration with *i.i.d.* shocks and $\Omega = 0.5$, the dynamic labor contract model generates a level of equity premium 6.13% and a risk-free rate 0.98%, which is close to the empirical average during $1889-1978$. The

dynamic labor contract model clearly performs better in generating higher risky asset returns and lower risk-free rate over the Walrasian RBC and the static contract model.

While the first moments of asset returns and the volatility of market return match the empirical targets, the unconditional volatility of risk-free rate is too high and that of equity premium is too low. This stems from the assumption that shareholders are all alike and their equilibrium bond holdings are zero. In real world, bond trading helps to stabilize risk-free rate. When the volatility of risk-free rate is lowered, we would get a more volatile equity premium.

Using the CEX data available from 1980 to the first quarter of 2005, we get the standard deviation of households' annual (four quarters rather than annualized quarterly data) consumption growth as 3.00%, 5.38%, and 12% respectively for non-shareholders, shareholders, and top shareholders (definitions see Malloy, Moskowitz, and Vissing-Jorgensen (2009)). As our model assumes shareholders do not work and workers do not hold shares, the consumption growth volatility of the two groups of agents should fall within the range of 3% ∼ 5.38% and 5.38% ∼ 12%. The dynamic labor contract model generates the shareholder's consumption growth volatility as 7.84% and that of worker as 4.84%, matching the empirical observations.

Define *risk-ratio* to be the ratio of shareholders' consumption growth volatility to that of non-shareholders as a risk-sharing indicator in the economy. Mankiw and Zeldes (1991) document the ratio to be 1.6 using PSID data, although their consumption measure consists of only food expenditures rather than the nondurable goods. The same ratio assumes a value of 1.6 in our benchmark dynamic labor contract model. In comparison, risk-ratio equals 1 in Walrasian RBC model, which means agents in the economy bear risk equally. Under the competitive labor market setting, no risk is shared between agents with different risk aversion. The static labor contract model yields a ratio of 1.3, which indicates risk is not shared enough through the static labor contract.

Figure 3 shows the countercyclical equity premium and labor share, and procyclical labor input.[14] As in Guvenen (2009), the countercyclical equity premium has been difficult to generate in standard production based asset pricing models with constant labor share. Nonetheless, the countercyclical movement of labor share over the business cycle has been put forth as part of

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[14]To highlight the comovement between series, each data series is detrended, rescaled by its standard deviation. I use log real GDP as output.
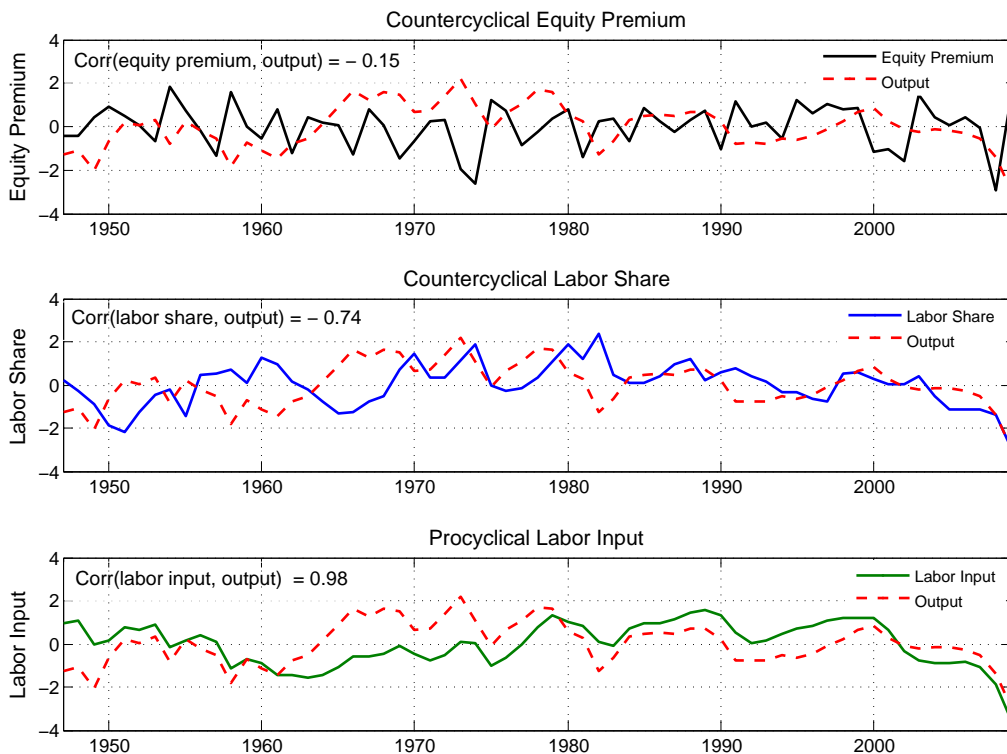
Figure 3. Cyclical Features of Macro Variables. This figure plots the empirical patterns of countercyclical equity premium, countercyclical labor share, and procyclical labor input. The data sources include BLS 1947-2010 and CRSP for asset returns.

an optimal risk-sharing arrangement between firms and workers.[15] The dynamic labor contract model captures this feature and generates equity premium and labor share negatively correlated with labor productivity shocks. As is shown in the lower half of Table 2, the cyclical features of all macro variables in the model with dynamic labor contracts are consistent with data.

To better understand the risk-sharing properties in long-term employment relationship, and the advantage of dynamic contract model over static ones, we construct the equivalent static contract by matching its first best solution with that of the dynamic case. The key difference between dynamic and static labor contract models is the adoption of promised continuation utility as an instrument to provide incentive and facilitate risk-sharing. The comparison allows us to isolate the effect of intertemporal risk-sharing from intratemporal risk-sharing.

Compare Column 3 and 5 in Table 2. Dynamic labor contract model generates a comparable market return but a much lower risk-free rate, compared with static contract model. Not only is the level of equity premium higher, but also is its volatility and countercyclical variation. Labor

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[15]See for example Gomme and Greenwood (1995), Boldrin and Horvath (1995), Gomme and Rupert (2004).
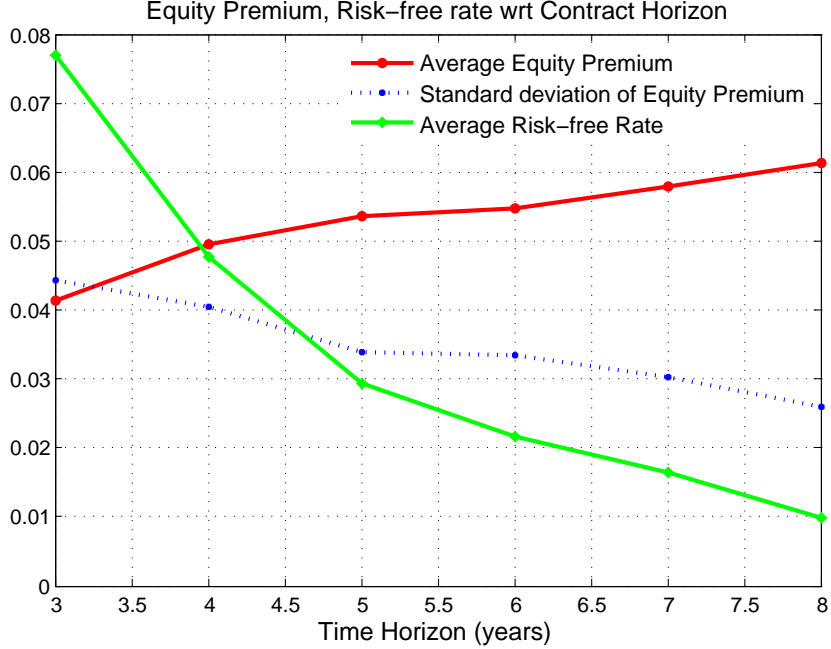
Figure 4. Horizon of Dynamic Labor Contact. This figure plots the asset returns as a function of the horizon of the dynamic labor contract.

input is strongly procyclical and labor share is strongly countercyclical in dynamic model, while both the two moments present ambiguous cyclical features in the static model. We observe that $Corr(L_t, \theta_t)_{\text{dynamic}} < Corr(L_t, \theta_t)_{\text{static}}$. This is because part of the second best distortion is transformed to the spread out of continuation utility in the dynamic contract. The risk-sharing property is summarized by an increase in $\sigma(\Delta D_{t+1})$, a decrease in $\sigma(\Delta C_{t+1})$, and a negative $Corr(\frac{C_t}{Y_t}, \theta_t)$ in the dynamic model. Hence we conclude that the intertemporal incentive from dynamic contract facilitates risk-sharing, evidenced from the higher risk ratio, and contributes an additional increase in the equity premium by 1.94%.

To further study how the dynamic effect changes with the contract horizon, in Figure 4 we present the asset returns with respect to parameter T (the rest parameter values are set as in Table 1). We get relatively more plausible and stable asset returns as T increases. In our benchmark case when $T = 8$, levels of equity premium and risk-free rate become closer to data. Besides, the computation results is more stable with respect to state realizations as can be seen by a lower unconditional standard deviation of the excess return.

Simulation results in Figure 5 show that equity premium in dynamic labor contract model is countercyclical, and is significantly higher than the other models. Shareholder's consumption
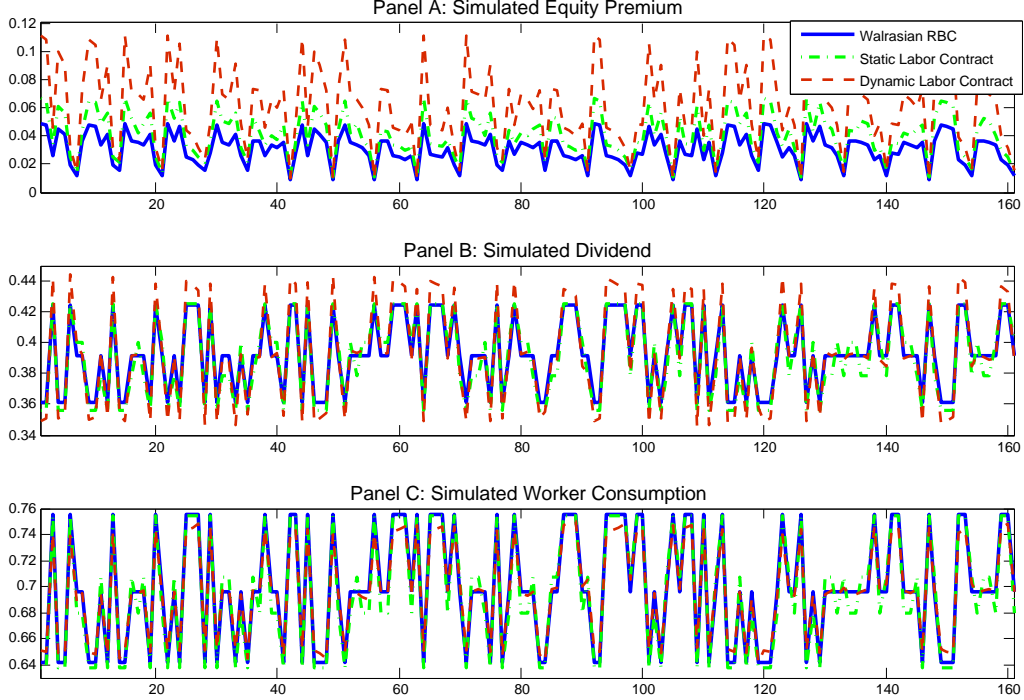
Figure 5. Model Simulated Moments. This figure plots model simulated series of equity premium, dividend (shareholder consumption), and worker consumption for the dynamic labor contract model (red dashed), the static labor contract model (green dot dashed), and a competitive labor market model (green solid). The model is simulated 1000 times and randomly plotted for 160 years of horizon.

growth volatility is amplified procyclically, while worker's consumption is smoothed under the labor contract.

4 Persistence of Public Shock

So far the shocks are assumed to be identically distributed, independent of history realizations, and with an equal probability for high and low state. In this section, we relax this assumption by looking at the effect of persistent public shocks.

The persistence of business cycle risk is well understood in the real business cycle literature. As examples of publicly observed common shocks, global climate and technological developments happen gradually. We model the distribution of public shock as a two state first-order Markov process. The transition matrix $\Pi = \begin{bmatrix} \Pi_{HH} & \Pi_{HL} \\ \Pi_{LH} & \Pi_{LL} \end{bmatrix}$ is public information to both agents. Persistence is governed by $\Pi_{HH} \in (0.5, 1)$, and $\Pi_{LH} \in (0, 0.5)$.

The deviation from i.i.d. shock distribution results a difference in the recursive formulation

Table **3**. Model Generated Moments under Persistent Public Shock

Moments	Data	Dynamic Labor Contract	Walrasian RBC	Static Labor Contract
Asset returns				
Equity premium	6.18%	5.75%	2.91%	4.37%
Market return	6.98%	6.54%	4.60%	7.00%
Risk-free rate	0.80%	0.79%	1.69%	2.63%
$\sigma(EP_t)$	16.67%	2.28%	0.98%	1.51%
$\sigma(R_t)$	16.54%	23.29%	15.42%	19.63%
$\sigma(r_t^f)$	5.67%	21.47%	14.67%	18.49%
Consumption risks				
$\sigma(\Delta D_{t+1})$	5.38% - 12%	8.59%	6.20%	9.55%
$\sigma(\Delta C_{t+1})$	3% - 5.38%	5.22%	6.20%	5.57%
$\frac{\sigma(\Delta D_{t+1})}{\sigma(\Delta C_{t+1})}$	1.6	1.6	1	1.7
Average labor share	0.64	0.64	0.64	0.64
Correlations				
$\text{Corr}(EP_t, z_t)$	-	-0.40	-0.40	-0.60
$\text{Corr}(EP_t, \theta_t)$	-	-0.51	-0.49	-0.25
$\text{Corr}(L_t, z_t)$	+	0.58	0	0.04
$\text{Corr}(L_t, \theta_t)$	+	0.46	0	0.89
$\text{Corr}(D_t, z_t)$	+	0.77	0.74	0.95
$\text{Corr}(D_t, \theta_t)$	+	0.26	0.30	-0.10
$\text{Corr}(C_t, z_t)$	+	0.71	0.74	0.43
$\text{Corr}(C_t, \theta_t)$	+	0.34	0.30	0.63
$\text{Corr}(\frac{C_t}{Y_t}, z_t)$	-	-0.83	0	-0.88
$\text{Corr}(\frac{C_t}{Y_t}, \theta_t)$	-	-0.13	0	0.79

Note: Models are compared under the same sequence of state realizations. Sources of data include: (1) Mehra and Prescott (1985); (2) CEX 1980-2005 Q1, see Mankiw and Zeldes (1991).

and the numerical procedure. Besides the promised utility W, we now need an extra state variable z_{-1}: the previous period public shock realization. The mapping from W to $S(W)$ is now conditional on z_{-1} since we have different implied shock distributions in the optimization problem depending on whether the previous public realization is z_H or z_L. We compute both sets of conditional mappings and plug in the corresponding mapping according to the previous state realization.

To construct the equivalent static contract problem, notice that the static reservation utility is also conditional on the previous public state realization z_{-1}. Define $\bar{W}_t/z_{t-1} = W_t/z_{t-1} - \beta \sum_z \Pi_{/z_{t-1}} \sum_\theta P_\theta W_t'(z, \theta), \quad \forall t = 1, 2, ...T$, where $W_1/z_0 = W_1 = W_0$, $\Pi_{/H} = \Pi_{/L} = \Pi$ at $t = 1$
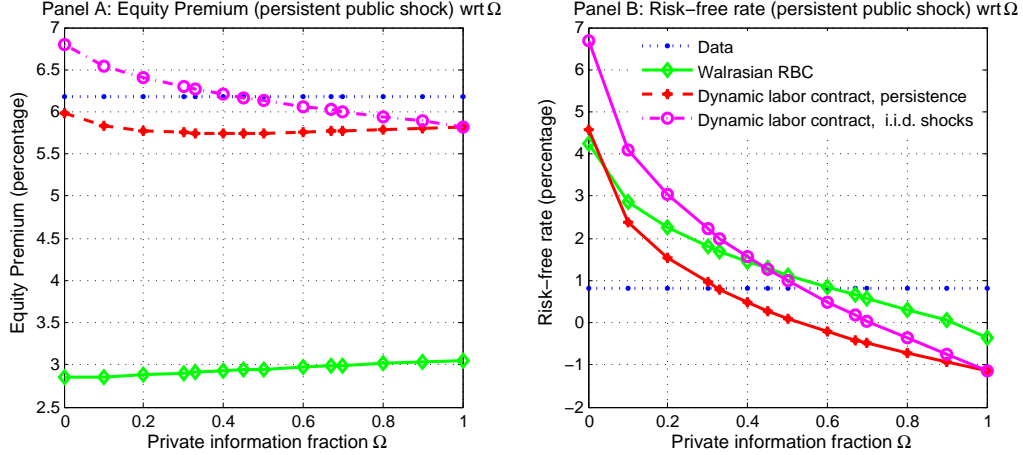
Figure 6. Equity Premium and Risk-free Rate under Persistent Public Shocks. This figure plots the equity risk premium and risk free rate (in percentage) under persistent public shocks when we vary the fraction of labor productivity risk due to private information Ω.

(with no prior) and $W'_T(z, \theta) = 0$. The static problem with conditional static reservation utility $\bar{W}_t/z_{t-1}$ matches with the dynamic contract problem in their first best solution. A detailed derivation can be found in the Appendix, proof of Proposition 1.

We parametrize the transition matrix as $\Pi_{HH} = \Pi_{LH} = 0.5$ for $t = 1$, and $\Pi_{HH} = 0.6, \Pi_{LH} = 0.4$, for $t \geq 2$. This corresponds to a quarterly persistence of 0.88 from high state to high state. We keep the other parameter values fixed as in Table 1, and vary Ω. Figure 6 shows how the level of equity premium and risk-free rate change with private shock component. We also plot the historical data and i.i.d. case as references.

First, asset returns are lower under persistent public shocks relative to the i.i.d. shock case. This is because being aware of the shock persistence, agents expect a smoother economy. Shareholders now put a smaller probability weight to the event which contributes a larger consumption growth volatility, thus require to be compensated by smaller returns. As Ω increases, equity premium tend to decrease since risk-sharing becomes more limited. Meanwhile, a decrease of the public shock component weakens the persistence effect, which drives the equity premium up. As a result, we observe a first downward and then slight upward trend of the equity premium. At $\Omega = 1$, the persistence effect of public shocks disappears. This is where the results of the two cases coincide.

At the value of $\Omega = \frac{1}{3}$, both levels of equity premium and risk free rate are the closest to their

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empirical counterparts. Table 3 demonstrates that the advantage of dynamic contract models are robust under persistent public shocks. The size and allocation of consumption risk remains the same, since the persistent shock distribution only alters agents' expectations. Cyclical features of model generated moments are also robust.[16]

5   Conclusion

Frictions in labor relations are important for understanding risks in the financial markets. In this paper, I demonstrate the potential of modeling labor contracts in a dynamic general equilibrium model to reconcile both financial market and business cycle facts. The model has shown satisfactory quantitative performance in two dimensions. First, with consumption allocations matched to CEX data, the model generates equity premium and risk-free rate comparable with historical average. Second, regarding the cyclical features, this model successfully produces procyclical consumption and labor input, countercyclical equity premium and labor share.

This study adds to the literature by connecting the financial markets with labor market frictions. First, the model generates procyclical labor supply and countercyclical labor share, which are considered challenging for standard production-based asset pricing models. Second, I deviate from previous literature on perfect risk-sharing in labor relations by analyzing the limited risk-sharing implications due to private information. Finally, workers in my model endogenously choose not to participate in the financial markets because the labor contract has insured them to the maximum extent against labor income risk. This rationale provides new insights on the limited asset market participation.

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[16]Although adding history dependent private shock is interesting, complexities arise because the shareholder lacks the common prior on the current period shock distribution, which is conditional on the previous privately observed state realization θ_{-1}. Worker may become better off by deviating from truth-telling in the current period in order to send wrong message about the next period's shock distribution. Fernandes and Phelan (2000) show that *temporary incentive compatibility constraint* plus *threat keeping constraint* is equivalent to the inventive compatibility condition. The *threat keeping constraint* is defined as $U[C_t(z,\theta_h), L_t(z,\theta_h)] + \beta W_t'(z,\theta_h|p_h) \geq U[C_t(z,\theta_l), \tilde{L}_t(z,\theta_h)] + \beta \hat{W}_t^{'}(z,\theta_l|p_h)$. where $\tilde{L}_t(z,\theta_h) = \left[\frac{\phi(z,\theta_l)}{\phi(z,\theta_h)}\right]^{\frac{1}{1-\alpha}} L_t(z,\theta_l)$, and $\hat{W}_t'(z,\theta_l|p_h)$ is the promised continuation utility to the worker if he lies under θ_h.

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Appendix

A   Proofs

A.1   Proof of Lemma 1

We first show that bond holding is not a state variable in the recursive formulation. Suppose for contradiction that the recursive problem has two state variables: W and B, where B is the equilibrium holding of bonds traded between shareholders and workers. At period T, the shareholder solves

$$S_T(W, B) = \max_{\{C_T, D_T, L_T\}} \sum_{z_T} \Pi(z_T) \sum_{\theta_T} P(\theta_T) V[D_T - B] \tag{34}$$

subject to

$$(C_T + B) + (D_T - B) \leq Y_T, \tag{35}$$

$$\sum_{z_T} \Pi(z_T) \sum_{\theta_T} P(\theta_T) U[C_T + B, L_T] \geq W, \quad \text{(IR)} \tag{36}$$

$$U[C(z_T, \theta_h) + B, L(z_T, \theta_h)] \geq U[C(z_T, \theta_l) + B, \tilde{L}(z_T, \theta_h)], \quad \text{(IC)} \tag{37}$$

where $\tilde{L}(z_T, \theta_h) = \left[\frac{\phi(z, \theta_l)}{\phi(z, \theta_h)}\right]^{\frac{1}{1-\alpha}} L(z_T, \theta_l)$. Now fix W, $\forall \hat{B} \neq B$, the FOCs of this static problem (34) stay the same. Hence, $\hat{D}_T - \hat{B} = D_T - B$, $\hat{C}_T + \hat{B} = C_T + B$, $\hat{L} = L$. Essentially, $S_T(W, B) = S_T(W, \tilde{B})$, $\forall B, \tilde{B}$. Therefore, bond holding B is not a state variable for problem (34), i.e. bond holding will not affect the final consumption of both agents at period T.

Back to period $T-1$, if bond holding in period T is irrelevant, then $B'_{z,\theta} = 0$. Then apply the same argument as above, we conclude B is irrelevant as a state variable.

$$S_{T-1}(W, B) = \max_{\{C_{T-1}, D_{T-1}, L_{T-1}, W'\}} \sum_{z_{T-1}} \Pi(z_{T-1}) \sum_{\theta_{T-1}} P(\theta_{T-1})[V(c_{z,\theta} - B + B'b) + \beta S_T(W')] \tag{38}$$

subject to

$$(D_{T-1} - B + B'b) + (C_{T-1} + B - B'b) \leq Y_{T-1} \tag{39}$$

$$\sum_{z_T} \Pi(z_T) \sum_{\theta_T} P(\theta_T) \left(U[C_{T-1} + B - B'b, L] + \beta W'\right) = W, \quad \text{(IR)} \tag{40}$$

$$U(C_{z,h} + B - B'_{z,h} b_{z,h}, L_{z,h}) + \beta W'_h > U(C_{z,l} + B - B'_{z,l} b_{z,l}, \tilde{L}_{z,l}) + \beta W'_l, \quad \text{(IC)} \tag{41}$$

The Lemma is established by applying the same argument through all periods. Q.E.D.

A.2 Proof of Lemma 2

By assumption 2, shareholders are all alike. Hence at equilibrium, we must have $B_i = B_j, \quad \forall i, j$. Because bonds are in zero net supply, equilibrium bond market clearing condition requires: $\int B_{i,t} dB = 0$. Therefore, $B_i = B_j = 0 \ \forall i, j$.

Now suppose we don't have the condition: $B_i = B_j = 0 \ \forall i, j$. Following the trading protocol in Atkeson and Lucas (1992), at $t = T$ type W_0 investor solves the problem

$$S_T(W = W_0) = \max_{\{C, D, L\}} \sum_{z_T} \Pi(z_T) \sum_{\theta_T} P(\theta_T) V[D_T - B]$$

subject to the IR, IC, and feasibility constraints.

$$L = \sum \Pi(z_T) \sum P(\theta_T) V[D_T - B] - \lambda_{z,\theta}(C_T + D_T - Y_T) +$$
$$\eta \left\{ \sum \Pi(z_T) \sum P(\theta_T) U(C_{z,\theta}, L_{z,\theta}) - W \right\} + \delta \left\{ U[C_{z,h}, L_{z,h}] - u[C_{z,l}, \tilde{L}_{z,h}] \right\}. \quad (42)$$

Bond market clearing must satisfy: $\sum B(W_0) = 0$.

Compared to the previous problem (34) when $B = 0$. The two problems will only coincide if $B_T = 0$; iterate backward, there must be $B_t(W_0) = 0, \forall t, W_0$. Hence, we establish that any nontrivial bond holdings of shareholders will change the original labor contracts and the equilibrium allocations. Q.E.D.

A.3 Proof of Proposition 1

Proposition 1.(i), 1.(ii), 1.(iii) are derived from the first order conditions with respect to $C_{z,\theta}$, $L_{z,\theta}$ and $W'_{z,\theta}$ in the first best problem.

Next we prove 1.(iv). Given the initial reservation utility for $t = 1$ is as W_1, the first best problem for the dynamic contract at $t = 1, 2, ..T - 1$ is:

$$S_t(W_t) = \max_{\{C_t, D_t, L_t, W'_t\}} \sum_z \Pi_z \sum_\theta P_\theta [V(D_{z,\theta}) + \beta S_{t+1}(W'_t)]$$

subject to feasibility and the individual rationality constraint. As $W'_t(z, \theta)$ is independent of state (z, θ) for the i.i.d. shocks from 1.(iii), the IR constraint reduces to

$$\sum_z \Pi_z \sum_\theta P_\theta U[C_t, L_t] \geq W_t - \beta W'_t. \quad (43)$$

Solving the first best dynamic contract problem backward, we obtain $\{W_t\}_{t=1}^T$. Redefine: $\bar{W}_t = W_t - \beta W_{t+1}, \forall t = 1, 2, ..., T - 1$, and $\bar{W}_T = W_T$. We get the equivalent first best static

contract problem $\forall t = 1, 2, ..., T$:

$$S_t(W) = \max_{\{C_t, D_t, L_t\}} \sum_z \Pi_z \sum_\theta P_\theta V(D_{z,\theta})$$

subject to the feasibility constraint and an equivalent static IR constraint

$$\sum_z \Pi_z \sum_\theta P_\theta U[C_t, L_t] \geq \bar{W}_t.$$

The equivalent static contract under persistent public shocks Under persistent public shocks, mappings $W \to S(W)$ is conditional on the previous public state realization z_{-1}. The first order condition shows $S'_{/z}(W') + \eta = 0$, where η is the Lagrangian multiplier on the IR constraint.

Given that $S'(.)$ is monotonic, $S'_{/z=H} \neq S'_{/z=L}$ and $S'_{/z=H}(W'_{H,\theta}) = S'_{/z=L}(W'_{L,\theta}) = -\eta$ jointly imply that $W'_{H,\theta} \neq W'_{L,\theta}$. The IR constraint is also conditional on previous public shock realization. For

$$\sum_z \Pi(z/z_{-1} = H) \sum_\theta P_\theta \left\{ U[C_t, L_t] + \beta W'_t \right\} \geq W_t(z_{-1} = H) = W'_{t-1}(H, \theta); \quad (44)$$

$$\sum_z \Pi(z/z_{-1} = L) \sum_\theta P_\theta \left\{ U[C_t, L_t] + \beta W'_t \right\} \geq W_t(z_{-1} = L) = W'_{t-1}(L, \theta). \quad (45)$$

Hence the conditional IR constraint reduces to

$$\sum_z \Pi(z/z_{-1} = H) \sum_\theta P_\theta U[C_t, L_t] \geq W_t(z_{-1} = H) - \beta \sum_z \Pi(z/z_{-1} = H) \sum_\theta P_\theta W'_t; \quad (46)$$

$$\sum_z \Pi(z/z_{-1} = H) \sum_\theta P_\theta U[C_t, L_t] \geq W_t(z_{-1} = L) - \beta \sum_z \Pi(z/z_{-1} = H) \sum_\theta P_\theta W'_t. \quad (47)$$

Redefine

$$\bar{W}_t/z_{t-1} = W_t/z_{t-1} - \beta \sum_z \Pi(z/z_{-1}) \sum_\theta P_\theta W'_t, \quad \forall t = 1, 2, ..., T, \quad (48)$$

where $W_1/z_0 = w_1 = w_0$, $\Pi_H = \Pi_L = \Pi$ at $t = 1$ (with no prior) and $W'_T(z, \theta) = 0$. Hence, by defining the conditional static reservation utility $\bar{W}_t/z_{t-1}$, we construct the equivalent static first best problem, of which the equilibrium allocation and asset returns coincide with those of the dynamic first best contract problem. Q.E.D.

B Numerical Procedure

To solve the model numerically, we adopt the numerical method as in Ales and Maziero (2010).[17] With the recursive formulation of the contract problem, we break the optimization algorithm into end period T problem and period $T-1$ problem, and solve it using backward induction.

1. Solve the dynamic contracting problem

   (a) In each period, the state variable W (promised utility) is discretized on the appropriate time-variant support interval with grid step 0.05. Given a set of first order equations, we solve the first best problem using Newton's method. Setting the first best solution as the initial guess for the corresponding second best problem greatly helps to improve the computation efficiency and convergence stability.

   (b) Having solved for the optimal policy function, we compute the value function (expected utility) of the shareholder and its numerical derivatives. The first order derivatives are computed using two-sided difference formula, second derivatives using a three-point formula. We repeat the above procedure for period t problem except that the state-contingent promised continuation utility $W'(z, \theta)$ is added to the system of first order equations.

   (c) As the mappings from W to $S(W)$ and its derivatives are well behaved in our setting (see Figure 10), we use cubic spline interpolation as the numerical approximation over the interval of state variable support. The entire dynamic first best and second best mappings are solved by integrating the procedure back to $t = 1$.

2. Simulate the allocations

   (a) Given the period value functions computed above, we now solve the equilibrium allocation forward from $t = 1, 2, , , T$ given the initial reservation utility W_0 and a state realization history (z^T, θ^T).

   (b) W_0 is the promised utility in period $t = 1$ problem. Pick the promised continuation utility according to the state realization (z_1, θ_1) as the promised utility for period $t = 2$, and repeat till $t = T$. This procedure gives us the sequences of the worker's required labor input and consumption streams of both agents in a dynamic contract.

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[17]I extend their framework to a risk averse shareholder. As a result, I solve for the maximization of expected utility rather than cost minimization.

3. Solve the equivalent static contract problem.

   (a) Take the sequence of promised utility solved in the dynamic contract problem $\{W_t\}_{t=1}^T$ and $\{W_t'\}_{t=1}^{T-1}$ and transform to the redefined static promised utility: $\{\overline{W}_t\}_{t=1}^T = \{W_t - \beta W_t'\}_{t=1}^T$, with $W_T' = 0$.

   (b) Take $\overline{W}_t$ as promised utility for each period t and use the same procedure as in Step 1, period T problem, we get the sequences of the worker's required labor input and consumption streams of both agents in the equivalent static contract.

4. Solve the asset prices.

   (a) With a finite horizon setting tailored to accommodate the dynamic labor contract, the price of risky asset at time t depends on the shareholder's consumption stream from the time t node to all the following possible realizations till end period T. We solve for the contract in all possible state realizations in the event tree, which amounts to a number of 4^T sets of equilibrium allocations.

   (b) Start from the end of the event tree, we calculate period $T-1$ asset price with the Euler equation (we have 4^{T-1} of them).

   (c) Iterate backward till $t = 1$, we get all the asset prices at each node.

   (d) Picking the sequence of realized asset prices according to the specific state realization (z^T, θ^T) gives equilibrium prices and returns of risky asset.

   (e) Similar procedures are applied to the pricing of bonds.