- Distribution of stocks' daily (log-)returns: not Gaussian

Volatility clustering provides one mechanical reason why the distribution of daily returns would not be Gaussian.

- Some returns from a sample come from high volatility periods, other returns come from low volatility periods.
- The full sample is a *mixture* of these two distributions.

The *mixture* of two Gaussian distributions with different variances has a larger kurtosis than a Gaussian distribution.

- Rationale for Gaussianity comes from the Central Limit Theorem
    - The sum of a bunch of (roughly) *i.i.d.* random variables is approximately Gaussian
    - The individual RV's don't need to be Gaussian
        - E.g., consider a single coin-flip (+1 for heads, -1 for tails)
        - Compare to the sum of many coin-flips...

Linear combinations of Gaussian RV's are also Gaussian

- Linear combinations of Gaussians are also Gaussian, so returns on a portfolio of stocks will be Gaussian

Univariate Gaussian fully described by two parameters (μ and σ)

- Multivariate Gaussian distns fully described by two analogous objects

The only features of our portfolio's returns that we can adjust are the mean, the variance, and the covariances with other assets' respective returns

Justifying/motivating mean-variance optimization

Gaussianity implies that the only features of our portfolio's returns that we can adjust are the mean, the variance, and the covariances with other assets' respective returns

Basic assumptions about preferences:

- Holding variability constant, we prefer higher expected returns
- Holding expected returns constant, we prefer lower variability (i.e., we're at least slightly risk-averse.)

These are pretty innocuous assumptions: we're not requiring that utility functions have a specific functional form

Mean-variance optimization is pretty mechanical

- Solve a constrained quadratic optimization problem to find optimal portfolio weights, w (a vector):

$$\begin{aligned} \min_w \quad w'\Sigma w \quad & \text{such that} \\ w'\mathbf{1} \quad = \quad & 1 \\ w'\bar{r} \quad = \quad & \bar{r}_p \end{aligned}$$

- Note that $w'\Sigma w = \sigma_p^2$, i.e., portfolio variance

- Two-fund theorem:
    - Assume short-sales are allowed, all assets are risky, and all investors have the same estimates of means, variances, and covariances.
    - ***Result:*** *Investors seeking minimum variance portfolios need only invest in combinations of two minimum variance portfolios.*

Let (w^1,λ^1,μ^1) and (w^2,λ^2,μ^2) be solutions corresponding to $\bar{r}_p^1$ and $\bar{r}_p^2$, respectively. Then the solution to the mean-variance problem for $\bar{r}_p^3$ is:

$$(w^3,\lambda^3,\mu^3) = \alpha(w^1,\lambda^1,\mu^1) + (1-\alpha)(w^2,\lambda^2,\mu^2)$$

where α solves:

$$\bar{r}_p^3 = \alpha\bar{r}_p^1 + (1-\alpha)\bar{r}_p^2$$

- Now introduce a risk-free asset, and the two-fund theorem reduces to the one-fund theorem:
    - *There is a single fund F of risky assets such that any efficient portfolio can be constructed as a combination of the fund F and the risk-free asset.*
    - Implies that every M-V efficient portfolio holds the risky assets in the same relative proportions (unless it holds no risky assets at all).
- The adding-up constraint, from aggregating over all investors' portfolios, lets us infer from this proportionality result that the "one fund" must be the market portfolio

Two-fund theorem (similarly, one-fund theorem) implies that every investor will hold risky assets in the same proportions

- One-fund theorem leads directly to CAPM
- CAPM tells us what each asset's risk premium should be (holding covariances fixed) to make the market portfolio M-V efficient
    - $\bar{r}_i - r_f = \beta_i(\bar{r}_M - r_f)$, where $\beta_i = \frac{\sigma_{iM}}{\sigma_M^2} = \frac{\mathrm{cov}(r_i,r_M)}{\mathrm{var}(r_M)}$
- The economic content of the CAPM is that the market portfolio is a mean-variance efficient portfolio.

- Empirical evidence indicates that the CAPM does have explanatory power, but that the model doesn't really work very well

- Consider the statistical "market model" representation of i's excess returns:

$$r_i - r_f = \alpha + \beta_i(r_m - r_f) + \varepsilon_i$$

- The variance of the excess return can be decomposed into two components

$$\sigma_i^2 = \beta_i^2\sigma_m^2 + \mathrm{var}(\varepsilon_i)$$

- $\beta_i^2\sigma_m^2$: systematic/market/non-diversifiable risk of i (determined by β_i).
- $\mathrm{var}(\varepsilon_i)$: nonsystematic/idiosyncratic/diversifiable risk of i

- CAPM states/predicts that only market risk is "priced" (earns risk premium)
- Let's return, at last, to the "how you value a payoff in state ω" thing
- We built up to the CAPM in steps:
    - CLT $\rightarrow$ Gaussianity $\rightarrow$ M-V optimization $\rightarrow$ one-fund thm. $\rightarrow$ CAPM
- The "Gaussianity $\rightarrow$ M-V optimization" step ignored the state-dependent thing
- A rational investor might care about the covariance between his portfolio returns and some outside factor
    - E.g., ACJ cares about his portfolio returns' covariance with:
        - The market price of lamotrigine
        - Meteorite impacts
    - Lamotrigine risk: mostly idiosyncratic to ACJ; few people need to be compensated for bearing this risk
    - Meteorite risk: affects everyone, can't be diversified away. This kind of risk should command a premium.
- Extend the CAPM notion of "priced risk-factors" to include things other than just market risk
    - Reasonable to imagine that a risk which affects everyone, and can't be diversified away, will be priced
    - Formal/conceptual extension of CAPM to a multi-factor model is easy

- Grossman-Stiglitz paradox regarding informational efficiency

If markets are efficient then there is no incentive to gather and/or analyze information, since prices fully reflect all available information

- Instead of doing a bunch of research to figure out what the price *should* be, you can just look at what the price actually *is*

But how can the market include all of that information if nobody bothers to gather it?
Noise to the rescue...

Introduce some noise into the model

- E.g., set supply $= 1+\varepsilon$, where ε is random and unobservable to investors
- Now the REE price is no longer fully revealing

Improved investment performance from using private info offsets the cost of gathering that info

But investors only gather information until the marginal benefit equals the marginal cost

As a result the more costly information is to obtain the less the current price will reflect this information.

Unlikely that markets are fully efficient...

- EMH: weak-form, semi-strong form, strong form

Weak Form: A capital market is said to be weakly efficient if it fully incorporates the information in past stock prices. This means that it is not possible to generate excess returns by simply studying the history of the share price.

If markets were not weakly efficient then it would be possible to spot patterns in prices and be able to make money from these patterns.

Semi-strong form: A capital market is said to display semi-strong efficiency if prices reflect all publicly available information such as published accounting statements as well as historical price information. This means that it is not possible to generate excess returns by poring over the press and company reports.

If markets were not semi-strong efficient, then investors would be able to make better returns than the market by studying earnings reports etc.

Strong form: the strong form of market efficiency says that prices reflect all information whether public or private. This says that it is impossible to generate excess returns even if you have access to insider information.

If markets are not strongly efficient then people with inside information will be able to make larger returns than the market.

- Thus, RWH is defined by two statements about the stochastic process $\{r_t\}$ that provides observed returns:
  - The mean is stationary, $E[r_t] = \mu$
  - Returns are uncorrelated, $\text{corr}(r_t, r_{t+\tau}) = 0, \tau \neq 0$

If the RWH hypothesis is true:

- The latest return and all previous returns are irrelevant if we attempt to predict future returns using linear predictors.
- There is no reason to believe that non-linear predictors are useful.
- The variables $\varepsilon_t = r_t - \mu$ can be assumed to be a "martingale difference" for practical purposes.
- The expected returns do not depend on the history of time series information.

Any tests we develop of the RWH can provide insights into market efficiency.

Rejection of the RWH is not sufficient to reject the weak form of the efficient market hypothesis.

- A trading rule is a method that translates a price history into investment decisions, thus:
  - Price history $I_t = p_t, p_{t-1}, p_{t-2}, \ldots,$ (or any other accounting information) at time t.
  - Investment quantity q_{t+1} for the period from the time of price observation t until the time of price observation $t+1$.
  - q_{t+1} is a function of I_t.

A trading rule is uninformative about expected returns if $E[r_{t+1}|q_{t+1}]$ is the same number for all possible q_{t+1}. In other words, returns do not depend upon your decision to buy/sell on a given day.

A trading rule is informative about expected returns if $E[r_{t+1}|q_{t+1}]$ is not always the same number.

Evidence that a rule was informative would imply:

- The random walk hypothesis is probably false
- If expected returns are stationary with mean μ then excess returns, $r_t - \mu$, are not a martingale difference.

Efficient market hypothesis

NO trading rule has an **expected, risk-adjusted, net** return greater than that provided by risk-free investment.

Formalization of (weak-form) EMH in terms of trading rules

- Trading rules can be informative without violating EMH
- Tons of trading rules are informative; few (or none) violate EMH

- Roll critique
  - CAPM assumes that "market portfolio" is mean-variance efficient
  - (Can prove that CAPM holds if and only if "market portfolio" is M-V efficient)
  - True market portfolio is unobservable

  - We therefore should not expect the CAPM to correctly predict asset returns if we use covariance with returns of the portfolio of marketable equities as our proxy for exposure to undiversifiable risk.

- Can we beat the market?
- At the very least, can tie a "better" market
  - Roll critique of CAPM
  - Yale endowment, diversification using non-traditional asset-classes

- Asset-classes
  - Rationale for grouping assets into "classes"
  - Implementation considerations

Does this mean that we should expect to get a positive α on an individual non-(easily)-marketable asset?"

NO.

- The ostensible α arises from the additional opportunities for diversification.
- I.e., you can reduce the amount of uncompensated, idiosyncratic risk that you face

One successful approach: divide the universe of assets into "classes" on the basis of assets' characteristics, and function in the context of a portfolio

Asset classes offer a framework for (approximately) optimizing portfolio allocations when the joint distributions of returns are difficult to summarize and/or directly estimate

- Delegated money-management
  - Rationale for delegation
  - Monitoring, performance evaluation
  - Principal-agent problem; aligning incentives

Benefits of delegating individual-asset-level investment decisions to highly specialized fund-managers:

- Investment opportunities in obscure assets may require dedicated, specialized infrastructure
- Specialized fund-managers have competitive advantages in obtaining valuable, asset-specific information

$F_{t0} = S_{t0}(1 + riskfree^{(t)})$

If the forward price is too high then we short the forward, borrow the cash and go long in the underlying.

- This will lock in the high price at which we can sell the underlying

If the forward price is too low, we go long the forward, short the asset and invest the cash.

- This will lock in the low price at which we can buy the underlying.

Put-call parity

$C(t) - P(t) = S_t - e^{-r(T-t)}K$

$C(t) - P(t) = S_t - PV(divs.) - e^{-r(T-t)}K, \;=\; e^{-r(T-t)}[F_t - K]$

Using put, stock, bond to create a call

stock + put - bond = call

Straddle: bought put and call on w/same strike (written straddle is written put and call)

Strangle: bought put and call w/different strikes.

- The coefficients 0.6 and 0.4 are called "risk-neutral" probabilities.
  - Such probabilities always exist, if there are no arbitrage opportunities (This can be proved using theorems about the existence of solutions to systems of linear inequalities.)
  - They are not necessarily the actual or "physical" probabilities.
  - They do not necessarily correspond to the beliefs of any actual investor.
  - They are probabilities in the sense that they satisfy the minimal mathematical requirements of probabilities.

The q's are the probabilities that make the expected returns on all assets equal to the riskless rate:

$$q = \frac{e^{rh} - d}{u - d} ; B = e^{-rh}\frac{uC_d - dC_u}{u - d} \qquad \Delta = \frac{C_u - C_d}{S(u - d)}$$

If the underlying asset (perhaps a stock index) pays a flow of dividends at a constant rate δ, then:

$$V(S,t) = Se^{-\delta(T-t)}N(d_1) - e^{-r(T-t)}KN(d_2)$$

where now

$$d_1 = \frac{\ln(S/K) + (r - \delta + \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{(T-t)}}$$

$$d_2 = \frac{\ln(S/K) + (r - \delta - \frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{(T-t)}} = d_1 - \sigma\sqrt{T-t}$$

As it must, Black-Scholes formula has a probabilistic interpretation.

$e^{-r(T-t)}$ is the discount.

$S_t e^{r(T-t)} N(d_1)$ is the risk-neutral expected value of S_T, conditional on $S_T > K$.

$-K$ is the amount that you pay.

$N(d_2)$ is the risk-neutral probability that $S_T > K$.

We have seen that if we assume that stock returns are normally distributed (and so stock prices are lognormally distributed) then under the assumption of risk-neutrality we can value European call and put options analytically using the Black-Scholes formula.

We saw that the limit of the binomial model when we have large numbers of time steps is also the Black-Scholes price and so we can use the binomial model to price derivatives whose value cannot be calculated via analytic integration.

American option pricing

$P = \max\left(e^{-rh}[qP_u + (1-q)P_d], K - S\right)$

Real Options

Delay helps avoid unfavorable investment that you will regret given the new information

Hysteresis: waiting has value when

- There is uncertainty in payoff of investment
- You can learn in the future by delaying
- You can delay the investment
- Investment is irreversible or costly reversible

The value is called *option value*

- Much like financial option value
  - Example: call option: opportunity to invest in year two
  - Value is \$30
- Investment now kills this option
- Invest now only if ENPV ≥ OV, or if the benefit can cover both the cost and the OV

The Dixit-Pindyck Framework

Basic Idea: McDonald and Siegel (1986)

- An investment project whose value V_t follows geometric Brownian motion:

$$dV_t = \alpha V_t dt + \sigma V_t dz_t$$

Two Solution Methods:

- Contingent claims analysis
  - Similar to valuation of financial options: another version of Black and Scholes
  - Applicable when the risk dz_t can be spanned by existing assets in financial markets: *rich* set of assets
  - Market has to be in equilibrium: no arbitrage
  - Can value F without any assumption about the discount rate or the investor's risk attitude (without knowing ρ):
    - The price of the *option* is *relative to* other assets that are traded in the market
- Dynamic programming, or optimal stopping
  - Has to assume a discount rate
  - Applicable to many environmental problems

III. Solution method: DP

- Bellman equation for F(V)

$$F(V(t)) = \max\{V(t) - I, e^{-\rho dt}E[F(V(t+dt))]\}$$

Not straightforward to solve: discrete decision

- Trick: transform into optimal stopping
  - Exists a critical value V^* so that
    - Continuation region: wait if $V<V^*$
      $$F(V(t)) = e^{-\rho dt}E[F(V(t+dt))]$$
    - Stopping region: invest if $V \geq V^*$
      $$\Omega(V(t)) = V(t) - I$$
    - At V^* (due to max{•, •})
      $$\text{Value matching} : F(V^*) = \Omega(V^*)$$
      $$\text{Smooth Pasting} : F_V(V^*) = \Omega_V(V^*)$$

Interpretation of the results

$$V^* = \frac{\beta_1}{\beta_1 - 1}I$$

- Hysteresis: $V^* > I$
  - More reluctant to invest, compared with neoclassical investment rule ($V^* = I$)
    - Don't want to jump as V may rise further
    - VMC $V^*=I+F(V^*)$: return from investment has to overcome both cost I and option value F
  - Investment barrier increases
    - As uncertainty rises: V^* increasing in σ^2
    - As ρ decreases: cost of waiting goes down
- Investment barrier vs. probability of investment
  - Move in same direction if exogenous changes do not affect the distribution of V_t
  - As σ^2 rises, investment prob may rise or fall (Sarkar, 2000)

Real Options Implications

- In the asset market, the uncertainty is only generated from the future price movement
- In the physical investment project, the uncertainty is not that simple, steaming from uncountable factors.

Demand uncertainty Policy Uncertainty Supply Uncertainty

Market-Making

Buy-side quote is called a "bid"
Sell-side quote is called an "ask"
The ask price is higher than the bid price

Informed traders trade in the same direction as expected future price movements

"Adverse selection" The times when informed traders want to trade are the worst times to trade

An informed aggressor will only buy if his expectation of "fundamental value" exceeds the best ask

$\mathbb{E}[FV|buy] = \mathbb{E}[FV|FV \geq \textit{best ask}] \geq \textit{best ask}$

No trading occurs if $\mathbb{E}[FV|buy] > \textit{best ask}$...

If only **some** aggressors are informed, still have $\mathbb{E}[FV|buy] > \mathbb{E}[FV]$

But it is not necessarily true that $\mathbb{E}[FV|buy] \geq \textit{best ask}$

optimal-stopping problem

- Due to the finite horizon ($N < \infty$), the cutoffs depend on "time" (n)
- $\frac{\partial}{\partial n}(c_n) < 0$, i.e., option-value decreases as you approach "expiration"

Strong similarities to a real-option problem

- Formal similarities to American option-pricing
- Due to the structure of discrete stopping-time problems, backward induction is very efficient solution method

The Greeks: motivation

The Black-Scholes formulae require five input parameters: $S_0, K, T-t, r$ and σ.

Once you have purchased the option the K remains fixed but each of the other parameters will change.

Time to maturity $T-t$ will change in a predictable manner but the other parameters may change somewhat unpredictably.

As with our bond portfolios we will be concerned about the sensitivity of our option to changes in these parameters and we may want to hedge ourselves against these risks

$V(S)-V(S_0) \approx \Delta(S-S_0)+\frac{1}{2!}\Gamma(S-S_0)^2 \quad (r-r_0)\rho+(\sigma-\sigma_0)\text{vega}+(t-t_0)\Theta$

Put option thetas are nearly always negative, although if it is very far in the money then the time decay reverses and the option price increases as time increases.

Theta can be useful when changes in delta and/or gamma happen over long time periods as you also have to consider how the option price is changing relative to the movements in time.

the combined Delta is a weighted average of the individual deltas

$\Delta_C = \frac{\partial C}{\partial S} = N(d_1) \quad \Gamma = \frac{\partial^2 V}{\partial S^2} = \frac{N'(d_1)}{S\sigma\sqrt{T-t}} \quad N'(d_1) = \frac{1}{\sqrt{2\pi}}e^{-{d_1}^2/2}$

Name	Symbol	Equation	Explanation
Delta	Δ	$\Delta = \frac{\partial V}{\partial S} \approx \frac{\Delta V}{\Delta S}$	Sensitivity of the option price to share price movements.
Gamma	Γ	$\Gamma = \frac{\partial^2 V}{\partial S^2} \approx \frac{\Delta\Delta}{\Delta S}$	Sensitivity of the option price to changes in the Delta.
Vega	$\mathscr{V}$	$\mathscr{V} = \frac{\partial V}{\partial \sigma} \approx \frac{\Delta V}{\Delta \sigma}$	Sensitivity of the option price to changes in volatility.
Rho	ρ	$\rho = \frac{\partial V}{\partial r} \approx \frac{\Delta V}{\Delta r}$	Sensitivity of the option price to changes in the risk-free rate.
Theta	Θ	$\Theta = \frac{\partial V}{\partial t} \approx \frac{\Delta V}{\Delta t}$	Sensitivity of the option price to time movement
Elasticity	ε	$\varepsilon = \frac{S}{V}\frac{\partial V}{\partial S} = \frac{\Delta V/V}{\Delta S/S}$	% change in option price per unit of % change in stock price

If a call option can be purchased for less than its theoretical Black-Scholes price then you can hedge to lock in this difference in price by purchasing the underpriced option and selling the replicating portfolio ($\Delta S + B$).

The problem here is that unlike futures arbitrage positions, the value of Δ is continuously changing which means that you have to continually readjust your hedging portfolio.

In general, gamma is best used when the option is near the money and there are potential large changes in delta, or when there are potentially large movements in the underlying (which again gives large changes in delta).

Making a portfolio gamma-neutral is not free and so you have to sacrifice some of the potential profit from your delta neutral strategy, so there is clearly a trade-off.

Note that gamma is the same for both call and put options and is obviously highest when delta is steepest or the slope of the option price exhibits more curvature.

If $\Delta = 0, \Gamma > 0$ (first portfolio), then stock price move in either direction results in profit then it must be that no stock price move results in loss.

Θ must be $< rV = r$ (it tells us what happens if a little bit of time passes, but stock price stays the same; recall value of portfolio V = 1)

If $\Delta = 0, \Gamma < 0$ (second portfolio): then stock price move in either direction results in loss then it must be that no stock price move results in profit.

Θ must be $>$r (it tells us what happens if a little bit of time passes, but stock price stays the same)

$r_{0.25}(0.25, 0.50)$ is for period 0.25 to 0.5, but is quoted at time 0.25; thus, given the information available at time 0, rate $r_{0.25}(0.25, 0.50)$ is **random**.
$f_0(0.25, 0.5)$ is also for period from 0.25 to 0.5, but is rate quoted at 0; thus, given the information available at time 0 the rate $f_0(0.25, 0.50)$ is **known** and **fixed**.

$(1+0.25r_0(0, 0.25))(1+0.25f_0(0.25, 0.5)) = 1+0.5r_0(0, 0.5)$

The two FRAs are equivalent as they have the same value and we can convert one to the other by borrowing/lending the cash flow at LIBOR.

Time:	0	0.25	0.5
FRA	0	0	$0.25[r_{0.25}(0.25, 0.5)-0.08]N$
FRA	0	$\frac{0.25[r_{0.25}(0.25,0.5)-0.08]N}{1+0.25r_{0.25}(0.25,0.5)}$	0

So the value of the FRA = PV(forward loan) + PV(interbank lending), but PV(interbank lending) = 0 and so:

$$FRA = \frac{N}{1+r_0(0,t_1)t_1} + \frac{-[1+\tau k]N}{1+r_0(0,t_2)t_2} = \frac{N\tau(f_0(t_1,t_2)-k)}{1+r_0(0,t_2)t_2}$$

"Horizontal" decomposition

Decompose swap into floating and fixed-rate notes:

time t	0.25	0.5
floating RN	$0.25r_0(0, 0.25)$	$1+0.25r_{0.25}(0.25, 0.5)$
fixed RN	$-0.25(0.08)$	$-1-0.25(0.08)$

$$Swap(t_0) = N\left(1 - s_{t0}\tau\sum_{i=1}^{n}P(t_0,t_i) - P(t_0,t_n)\right)$$

To value a swap

Build the term structure: compute forward rates for all maturities and zero-coupon rates for all maturities.

Replace floating payments with fixed payments based on forward rates.

Discount the resulting net cash flow for each date and add them up.

Note: This procedure only works for the most common timing convention, when the cash flow at t = 0.5 is based on $r_{0.25}(0.25.0.5)$.

The **yield-to-maturity**, y, is implicitly defined by the equation $B = \sum_{i=1}^{n}\frac{C}{(1+y/g)^i}$
Definition: Macaulay Duration:

$$\begin{aligned} D &= \frac{PV(t_1)t_1 + PV(t_2)t_2 + \cdots + PV(t_n)t_n}{PV_{total}} \\ &= w_{t_1}t_1 + w_{t_2}t_2 + \cdots + w_{t_n}t_n \end{aligned}$$

Let P be the price of the bond. Then the modified duration, D_{mod}, of the bond is:

$$D_{mod} = -\frac{1}{P(y_0)}\frac{dP(y)}{dy}\Big|_{y=y_0}$$

The quantity $D_{mod} \times P$ is often called the dollar duration.

$$\text{Dollar duration} = D_{mod} \times P = -\frac{1}{P(y_0)}\frac{dP(y)}{dy}\Big|_{y=y_0} \times P(y_0) \approx -\frac{\Delta P}{\Delta y}$$

We use the concept of "duration" to try to generalize the idea that for a zero coupon bond the time to maturity captures its sensitivity to yield.

Change in bond prices $\quad \Delta P \approx -D_{mod} \times P \times \Delta y + \frac{1}{2}\mathscr{C} \times P \times (\Delta y)^2$

Sensitivity to Spot (zero-coupon) Rates

$$D_Q = \frac{1}{PV}\sum_{i=1}^{n}\frac{ix_i/m}{(1+r(t_i)/m)^{i+1}}$$

The **credit spread** is the spread over the risk-free rate such that if you discount the *promised* cash flows using the sum of the risk-free rate and the credit spread, their value equals V:

Credit spread $\approx$ expected loss rate *only if investors are risk-neutral*

Crucial Points about Credit Ratings

1. Ratings reflect (estimated) **PHYSICAL** probabilities of default
2. Ratings for individual bonds do not provide any information about correlations of default

If we are currently at time 0, and we know that we want to move cash from time 1 to time 2, we have two alternatives:

Commit today (time 0) to invest at the forward rate; or

Wait until time 1, and then invest at the spot rate $r_1(1,2)$

Expectations hypothesis is plausible *if* uncertainty about interest rates is not important, either because there isn't much uncertainty about interest rates or because investors don't care about it.

liquidity preference hypothesis $E[r_u(u,v)] = \lambda + f_0(u,v)$

where λ is the liquidity premium.

- Note that λ can vary, depending on the maturity of the forward rate.
- Here "liquidity" here means the cost of having an investment for such a long period of time that you do not have the money available for other investments.