ASTR 405
Planetary Systems
<u>Microlensing</u>

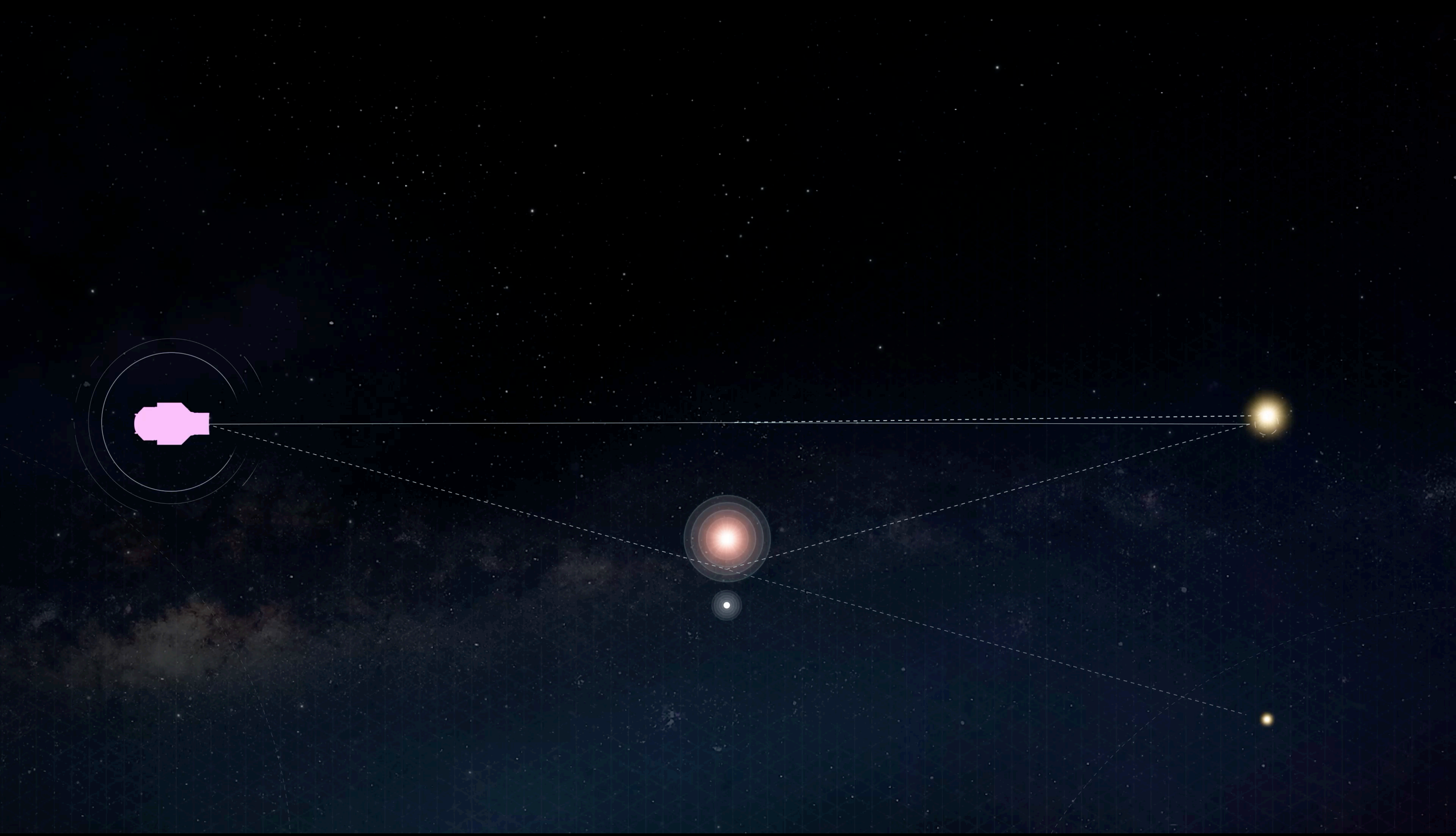
Fall 2025

Prof. Jiayin Dong

Module I: Exoplanet Detection Methods

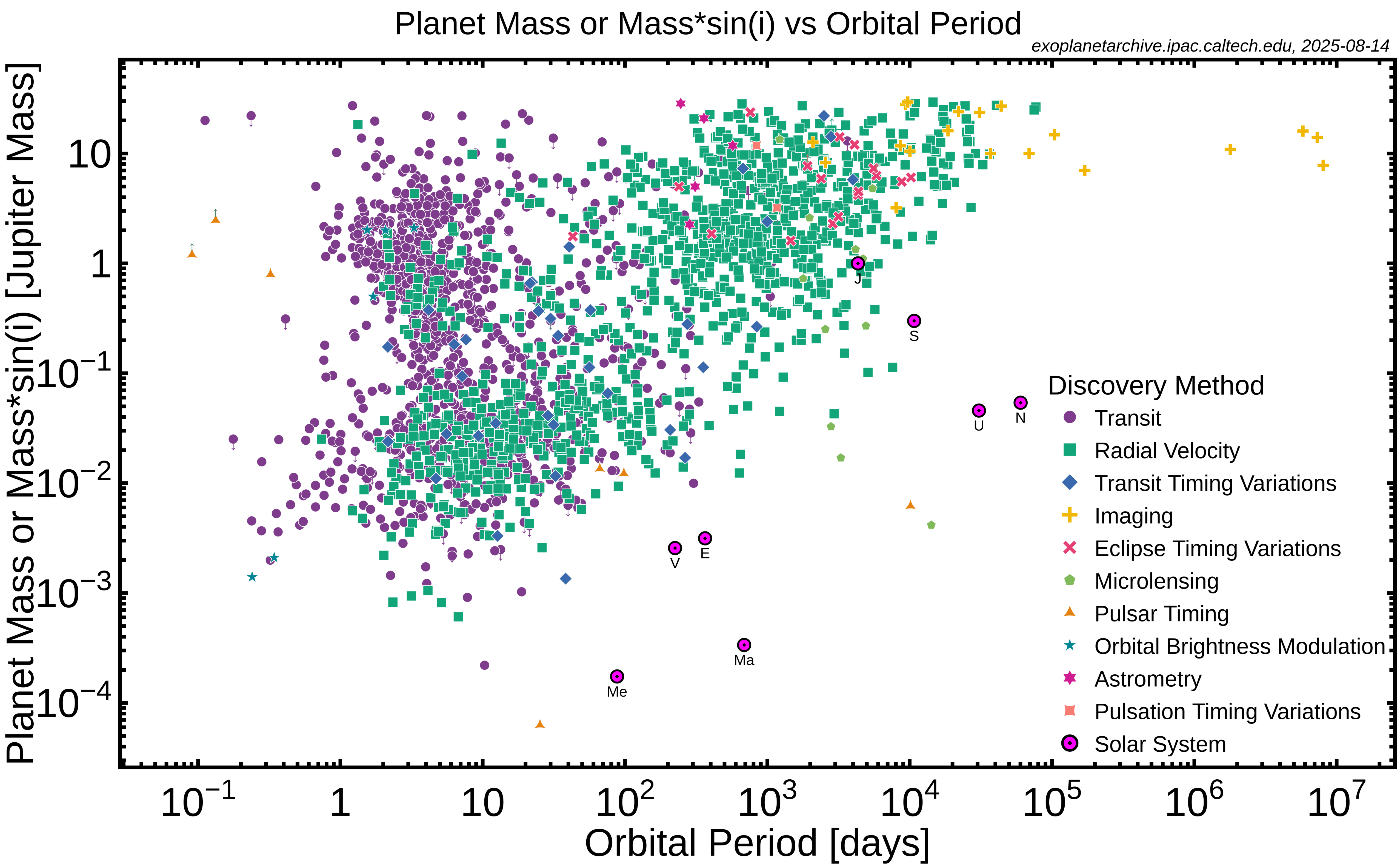
- Radial Velocity: detecting exoplanets by measuring Doppler shifts from a star's radial reflex motion along our line of sight
- Astrometry: detecting exoplanets by measuring tiny changes in a star's sky position from its tangential reflex motion
- Transit: detecting exoplanets by observing the dimming of a star's light when a planet passes in front of it
- Microlensing: observing the **brightening of a background star** caused by the gravity of a **planet-hosting foreground star** acting as a lens
- Direct Imaging

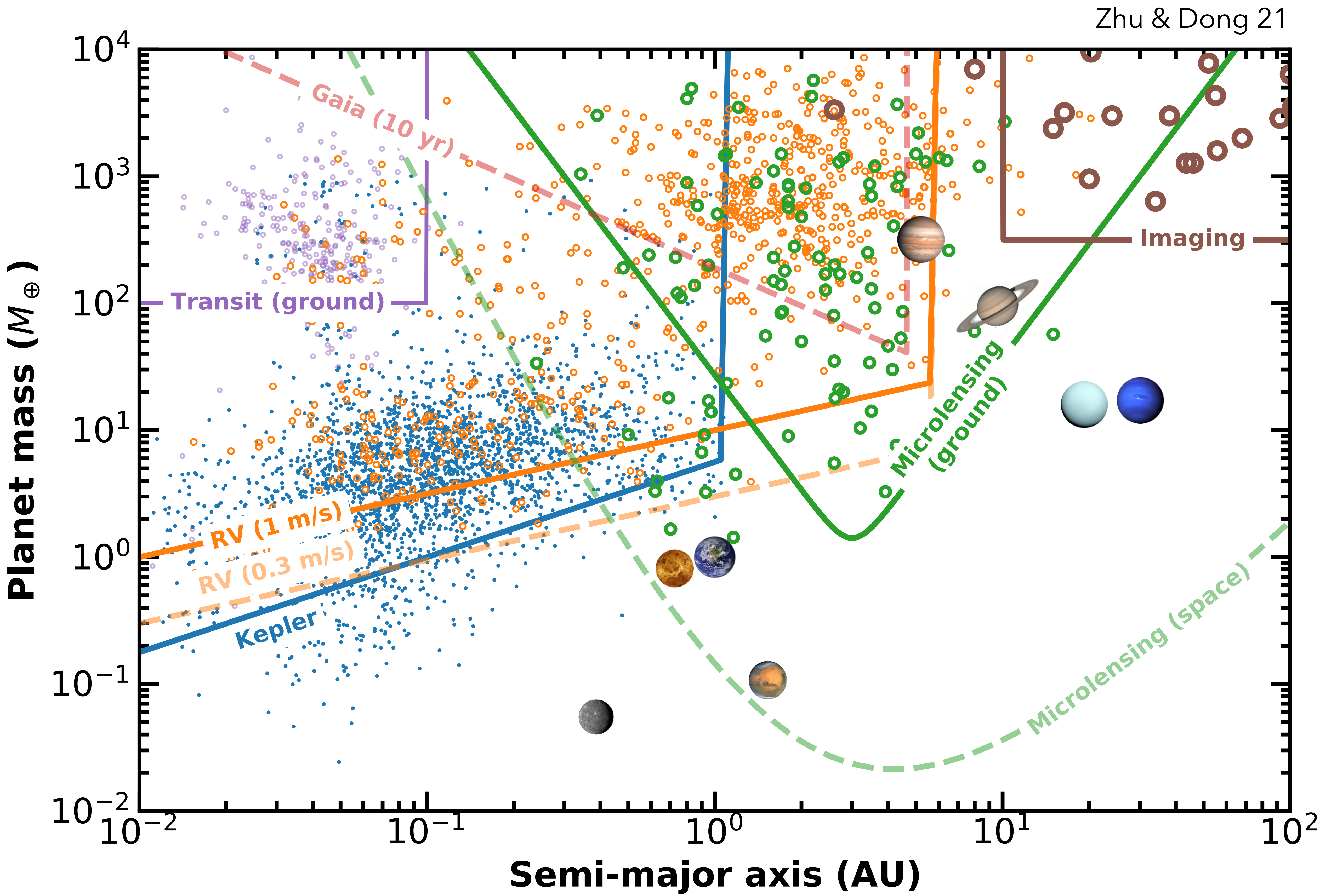
NASA Exoplanet Exploration Program
https://exoplanets.nasa.gov/resources/2168/gravitational-microlensing/



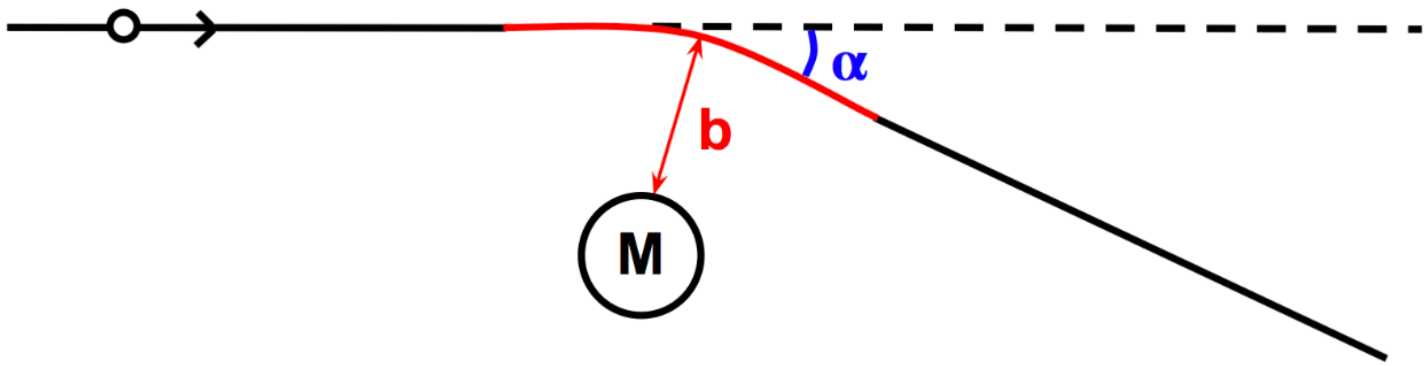
Roman Space Telescope Microlensing Animation
https://science.nasa.gov/mission/roman-space-telescope/microlensing/

Exoplanet Mass–Period Distribution




Zhu & Dong 21
Planet mass ($M_\oplus$)
Semi-major axis (AU)
Gaia (10 yr)
Transit (ground)
Imaging
Microlensing (ground)
Microlensing (space)
RV (1 m/s)
RV (0.3 m/s)
Kepler

Deflection of Light

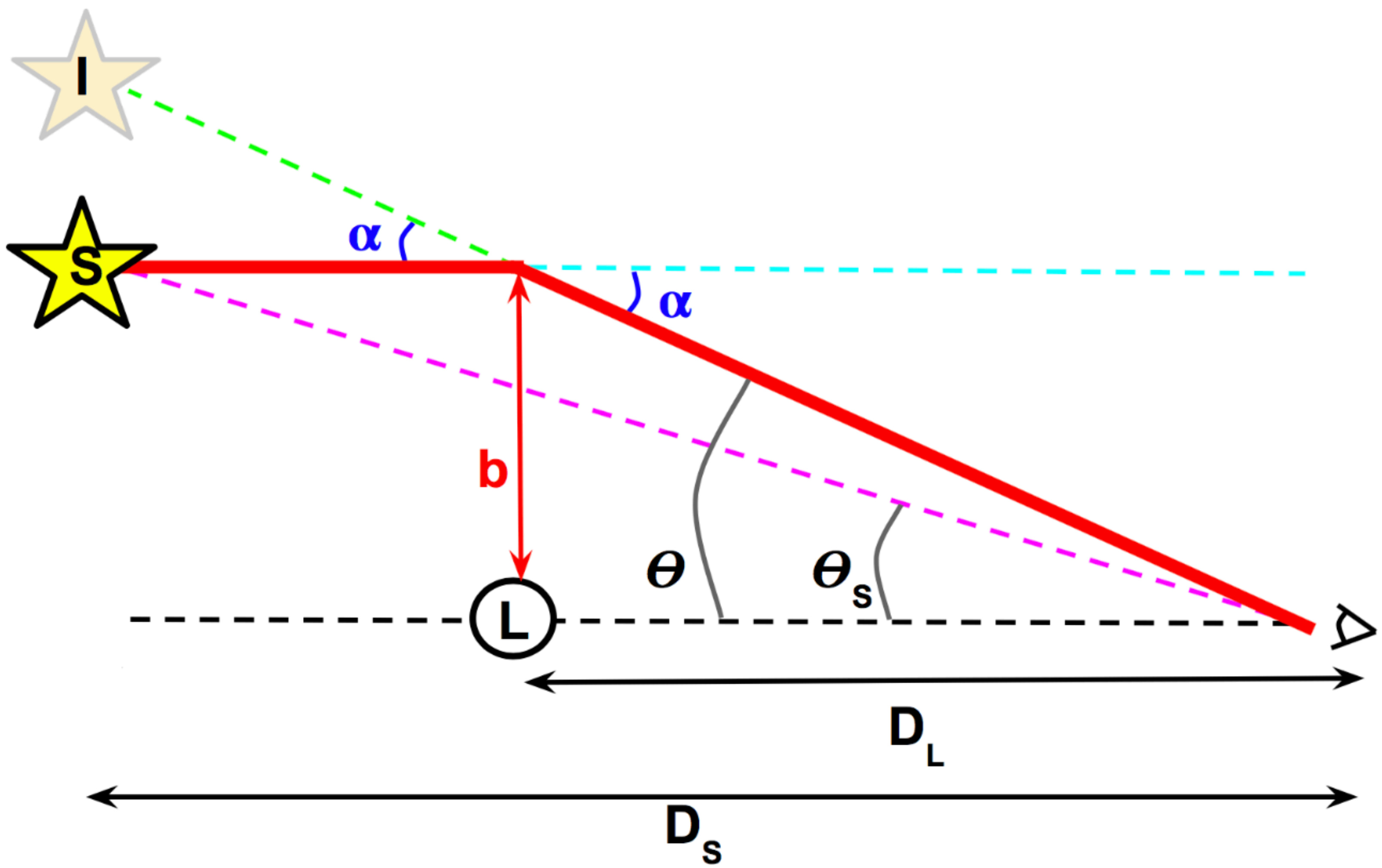


- α = deflection angle
- M = mass of the lens
- b = impact parameter (distance of closest approach)

Deflection angle $\boxed{\alpha = \frac{4GM}{c^2 b}}$

- More massive lenses (i.e., larger M), larger α
- Light passing closer to the lens M (i.e., smaller b), larger α

Lens Equation



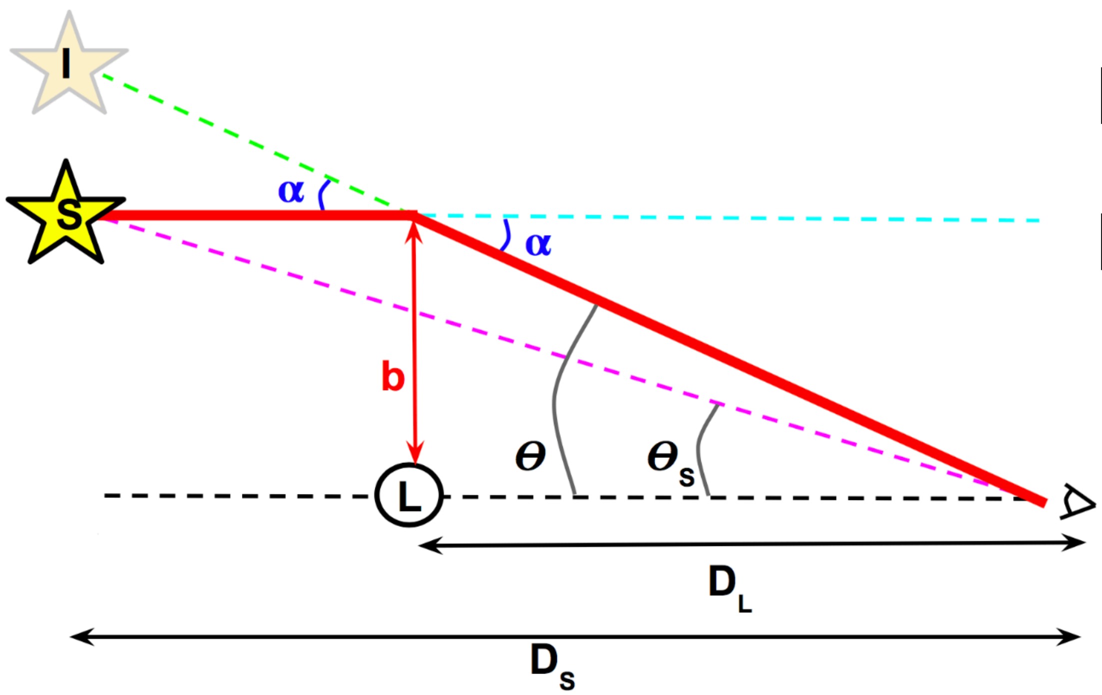
Credit: Gaudi & Stephan

- **S**: background source
- **I**: Image of background source
- **L**: foreground lens star

- D_L = distance to lens
- D_S = distance to source
- b = impact parameter

- θ = angular position of image
- θ_S = angular position of background source
- $\theta = \alpha$ = deflection angle

Lens Equation



Credit: Gaudi & Stephan

How does θ change as a function of θ_S?

Knowns: D_L, D_S, M (mass of lens)

$$b = \theta_S D_S = \theta D_L = \frac{4GM}{c^2\theta}$$

The lens equation $\boxed{\theta^2 - \theta_S\theta - \theta_E^2 = 0}$,

where θ_E is the Einstein ring $\theta_E = \sqrt{\frac{4GM}{c^2}\frac{D_S - D_L}{D_S D_L}}$.

In-Class Activity

Solving the Lens Equation

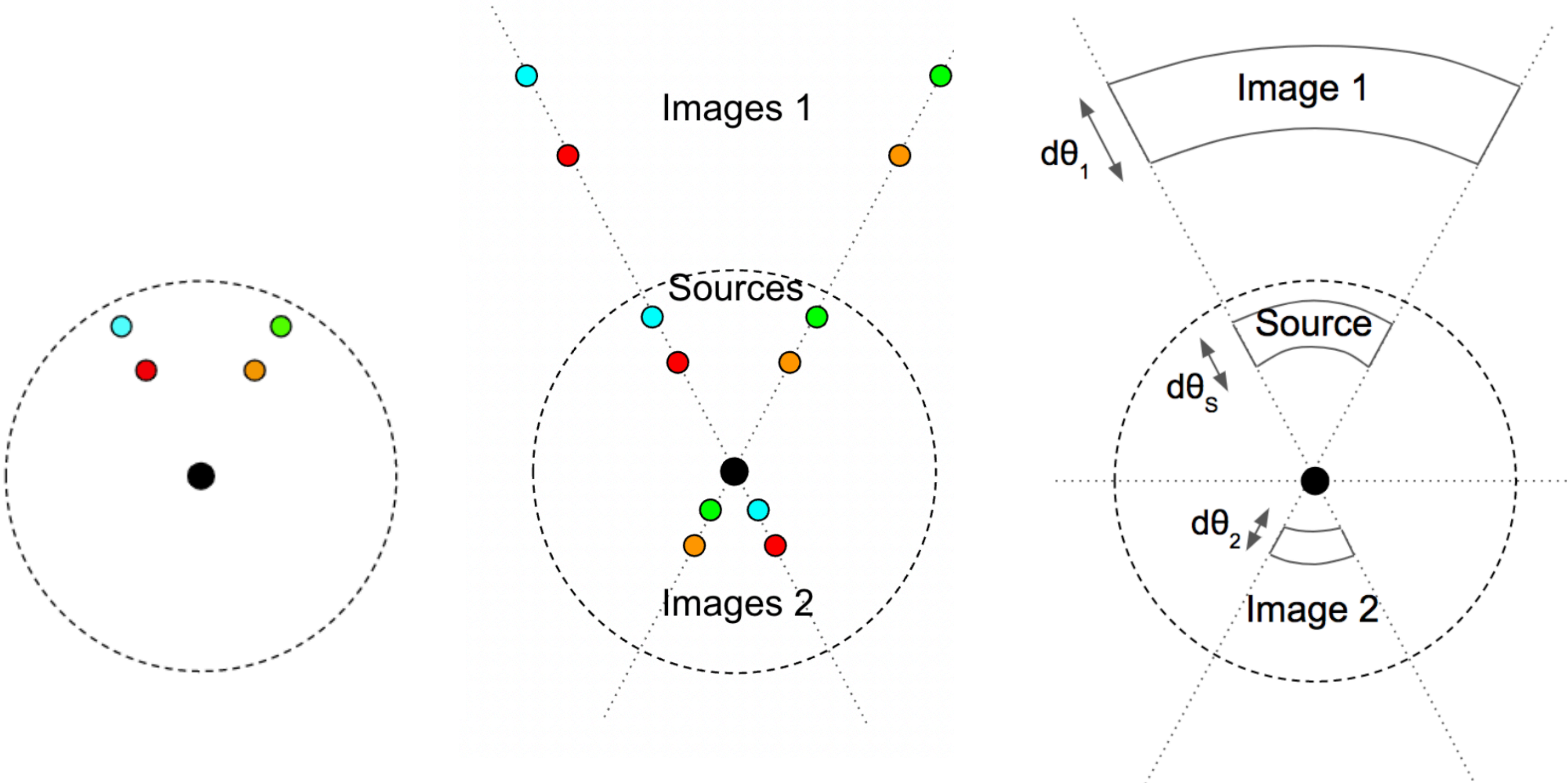
Why called “Microlensing”

The angular separation between these images at the time of closest alignment is $\sim 2\,\theta_E$.

Thus for typical lens masses (0.1–1 $M_\odot$) and lens and source distances (1–10 kpc), $\theta_E \lesssim 1\,\mathrm{mas}$ and so the images are not resolved, which led to the nomenclature “microlensing”.

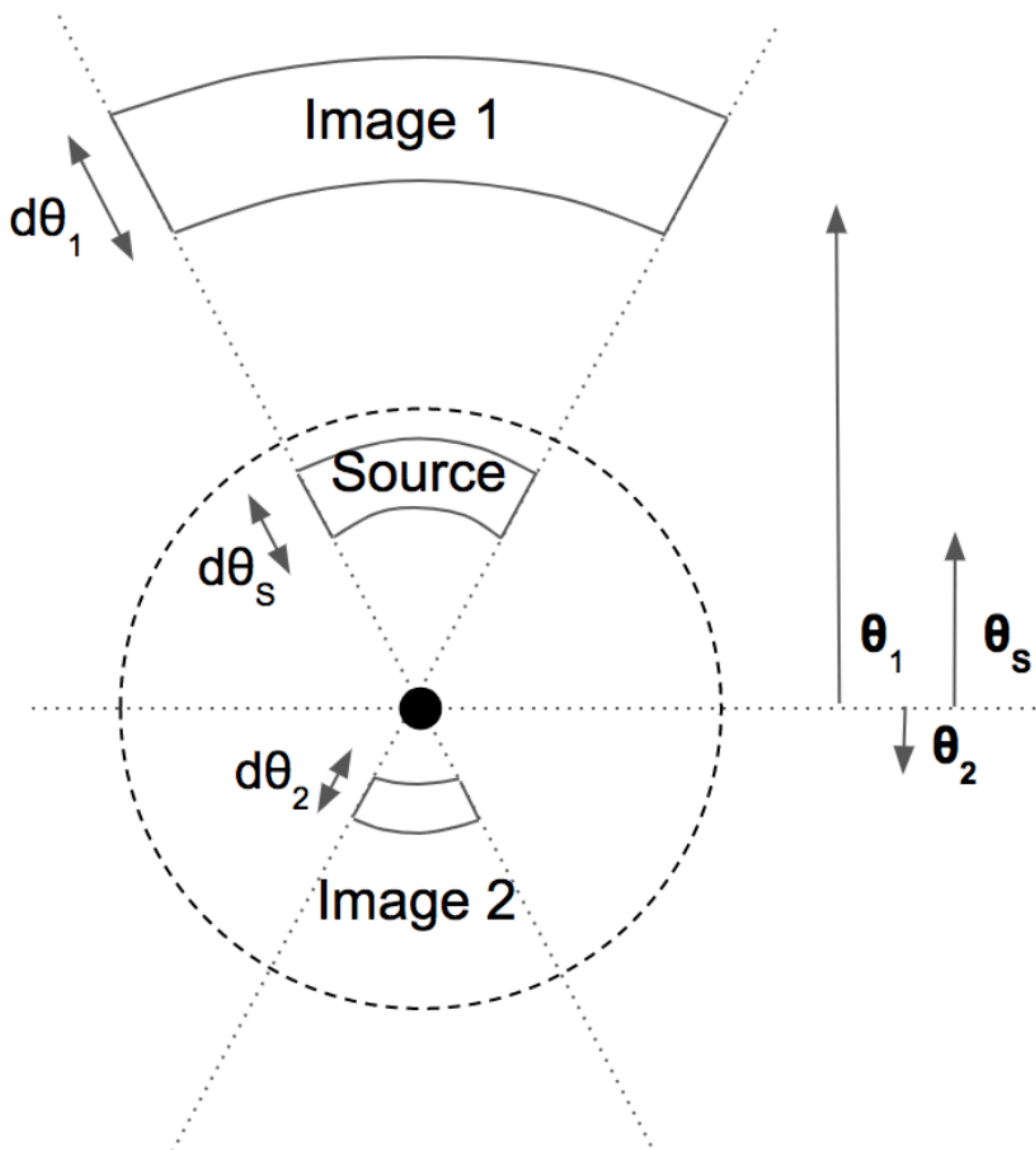
However, the images are also distorted, and since surface brightness is conserved, this implies they are also magnified.

Microlensing Magnification



Credit: Lam

Microlensing Magnification



Credit: Lam

Magnification of Image 1

$$A_1 = \left| \frac{\theta_1}{\theta_S} \frac{d\theta_1}{d\theta_S} \right|$$

Magnification of Image 2

$$A_2 = \left| \frac{\theta_2}{\theta_S} \frac{d\theta_2}{d\theta_S} \right|$$

Total magnification $A = A_1 + A_2$

Microlensing Magnification

From the lens equation $\theta_{1,2} = \dfrac{\theta_S \pm \sqrt{\theta_S^2 + 4\theta_E^2}}{2}$ and $A_{1,2} = \left| \dfrac{\theta_{1,2}}{\theta_S} \dfrac{d\theta_{1,2}}{d\theta_S} \right|$,

we get the magnification amplitude

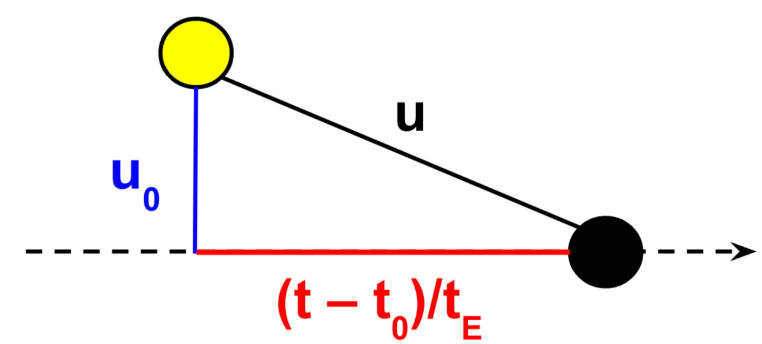
$$\boxed{A_{1,2} = \frac{u^2 + 2}{2u\sqrt{u^2 + 4}} \pm 0.5},$$

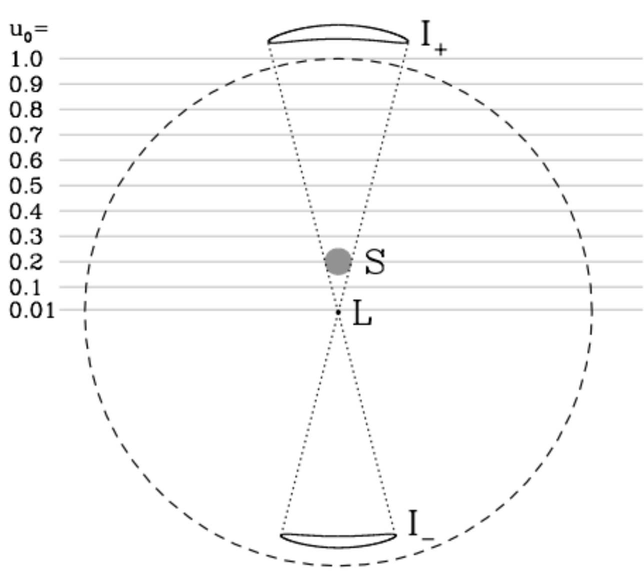
where u is the distance of the source from the lens in units of the Einstein radius $u \equiv \dfrac{\theta_S}{\theta_E}$.

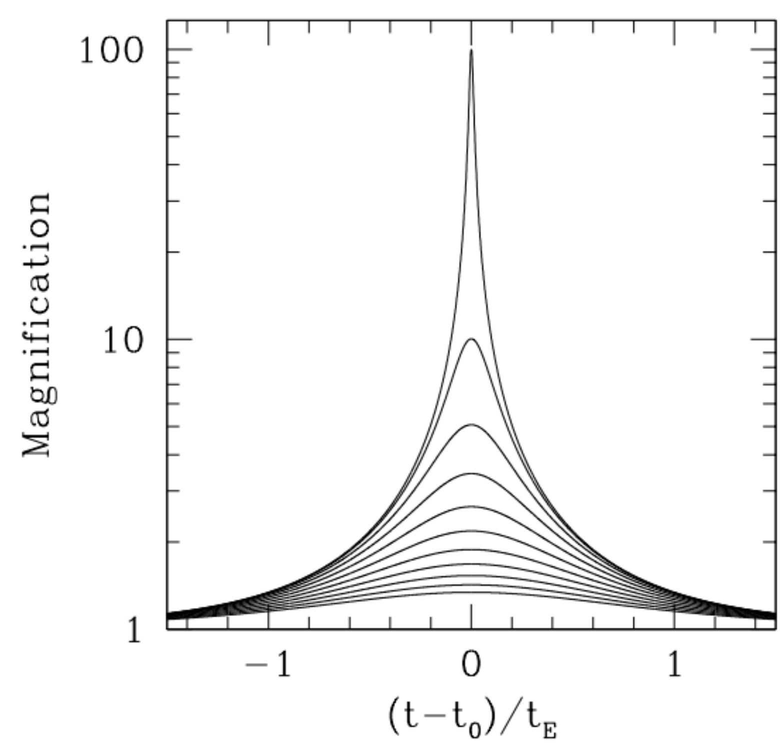
Microlensing Lightcurve

$A(u)$ describes how magnification changes with the dimensionless source-lens projected separation u. To make a lightcurve we want to know how u as a function of time t:

$$u(t) = \sqrt{u_0^2 + \left(\frac{t - t_0}{t_E}\right)^2}$$



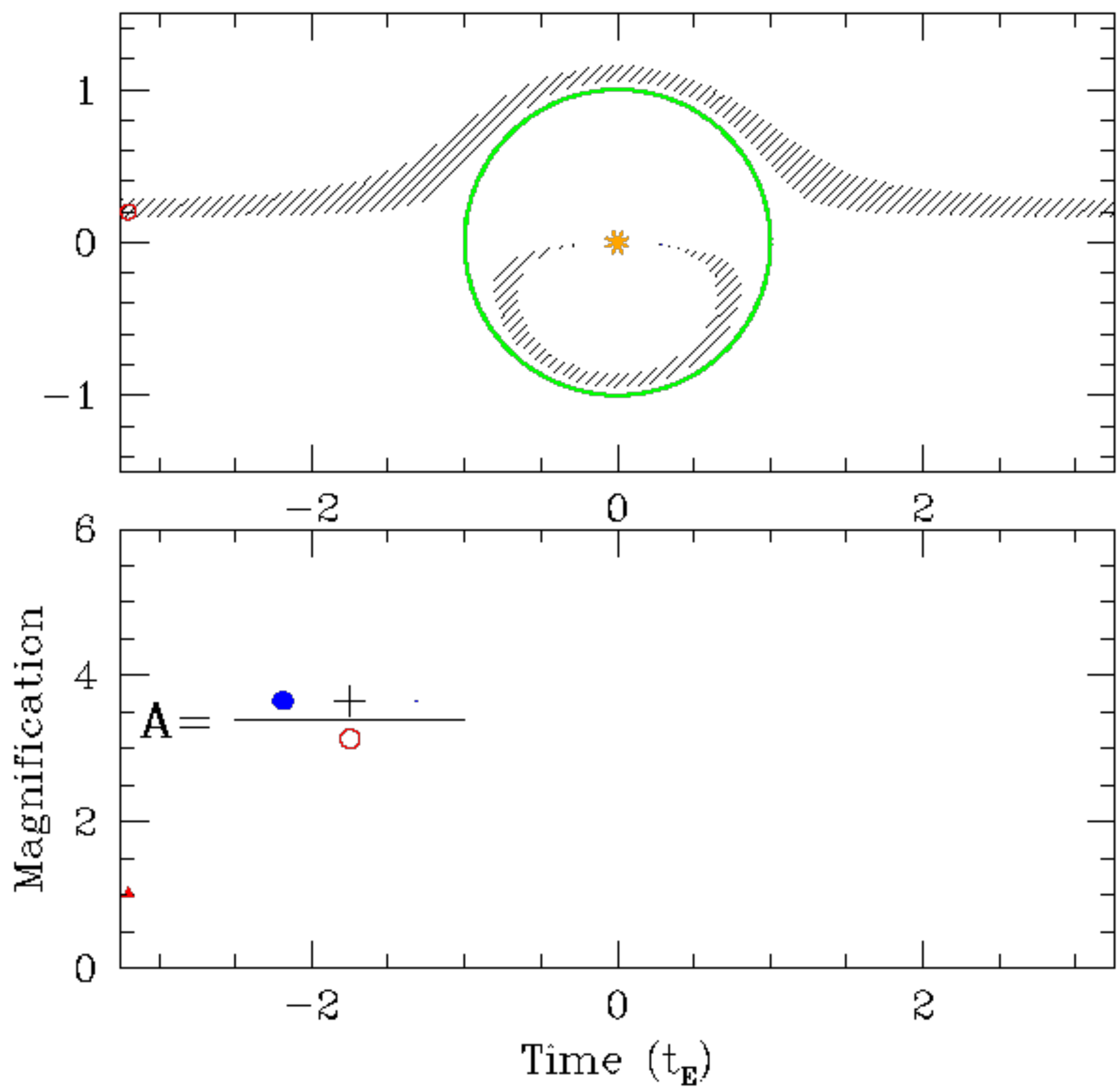




- t_E = Einstein crossing time (time it takes for the source to move the distance of the Einstein radius)
- t_0 = time of closest approach
- u_0 = impact parameter (distance of closest approach)

Gaudi 10

Microlensing lightcurve: star only



Credit: Gaudi

Event Length

Recall that $t_E = R_E / v_t$, where v_t is the tangential velocity of the star. Using the expression for the Einstein ring radius $R_E = \theta_E D_L$, we get

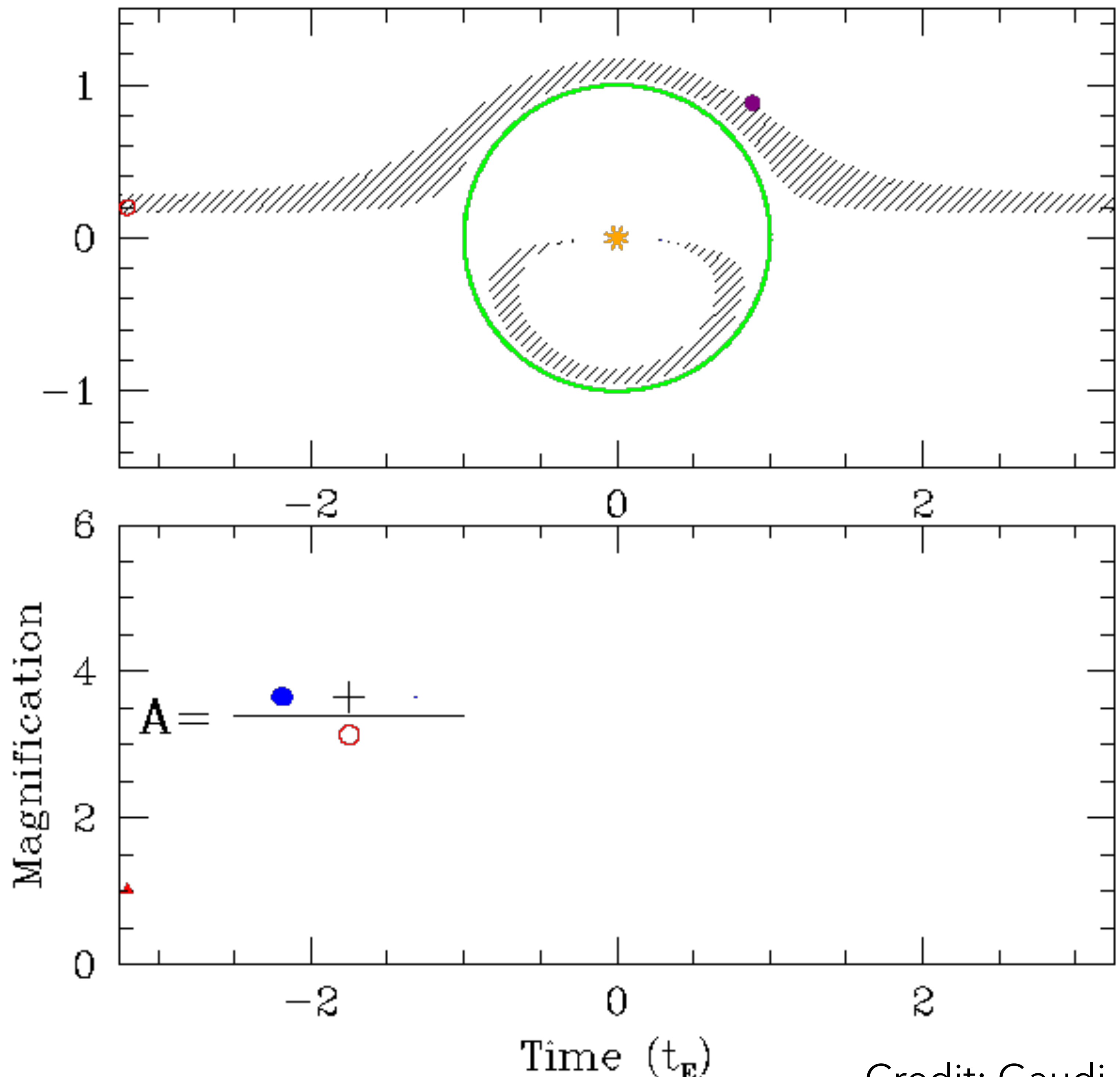
$$t_E = 0.214 \text{ yr} \left(\frac{M}{M_\odot}\right)^{1/2} \left(\frac{D_L}{10 \text{ kpc}}\right)^{1/2} \left(1 - \frac{D_L}{D_S}\right)^{1/2} \left(\frac{200 \text{ km s}^{-1}}{v_t}\right) .$$

In-Class Activity

Microlensing Optical Depth

Microlensing lightcurve: star + planet

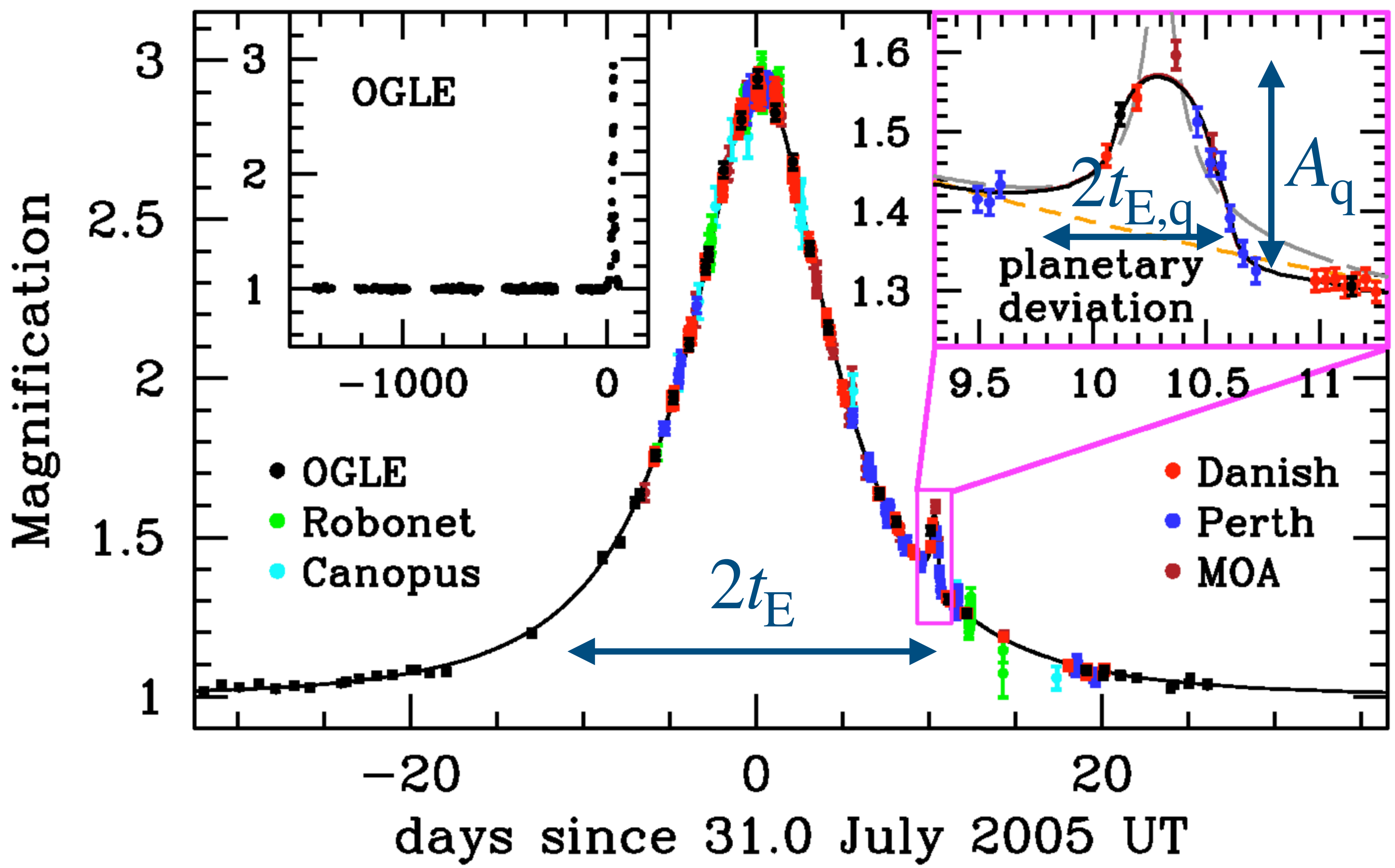
microlensing detectability max at $a \sim R_E$



Credit: Gaudi

Planetary Properties from Microlensing

- Planet-star mass ratio $q = \dfrac{M_p}{M_\star} \sim \left(\dfrac{t_{\mathrm{E,q}}}{t_\mathrm{E}}\right)^2$
- Magnification $A_\mathrm{q} \sim a_\perp / R_\mathrm{E}$



Beaulieu+06

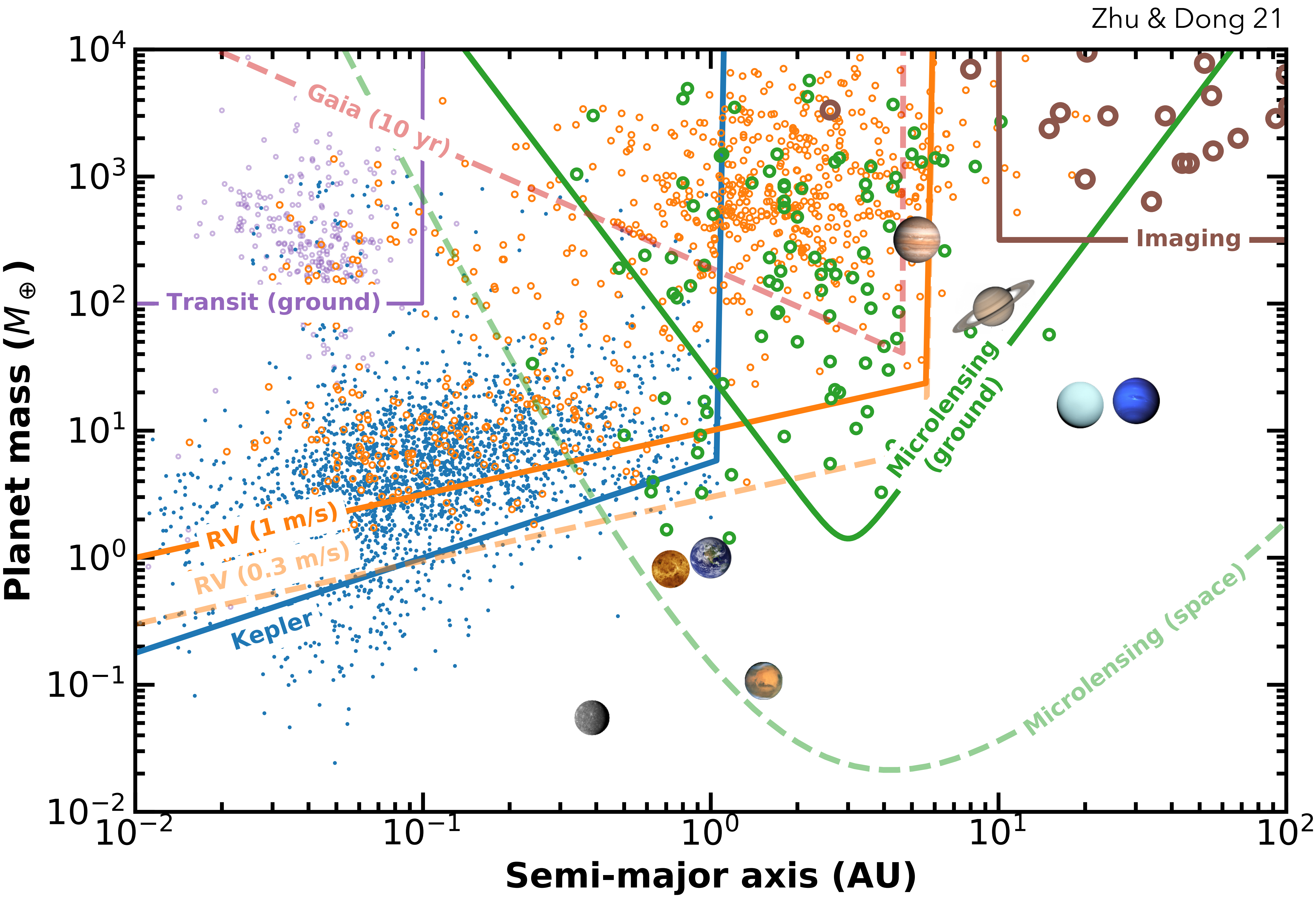
Microlensing Planet Discoveries

Ground-Based

- 253 planets (as of Sep 2025; all ground-based)
- Main surveys: **OGLE** (1992–present), **MOA** (2006–2014), **KMT** (2009–present) –> pointing at galactic bulge
- First planet (OGLE-2003-BLG-235L b) in 2004 (Bond+2004)
- Also hints of free-floating planets (~1–2 per star)

Space-Based

- Mainly follow-up, no large independent samples
- **Nancy Grace Roman (2027)**: 2.4 m, wide-field (0.28 deg^2)
- Expected yield: >1500 planets


Zhu & Dong 21
Gaia (10 yr)
Transit (ground)
Imaging
Microlensing (ground)
Microlensing (space)
RV (1 m/s)
RV (0.3 m/s)
Kepler
Planet mass ($M_\oplus$)
Semi-major axis (AU)
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}