ASTR 405
Planetary Systems
Terrestrial Planet Formation

Fall 2025

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Supplementary Readings: **formation.pdf Section III B & C** on Canvas

Lecture Notes on the Formation and Early Evolution of Planetary Systems by Armitage

Module II: Exoplanet Demographics and Planet Formation

- **Protoplanetary Disks:** Gas–dust disks around young stars; evolve on Myr timescales, set the initial conditions for planet formation

- **Dust, Pebbles, and Planetesimals:** Dust grains stick → pebbles (mm–cm); rapid drift & instabilities lead to km-scale planetesimals

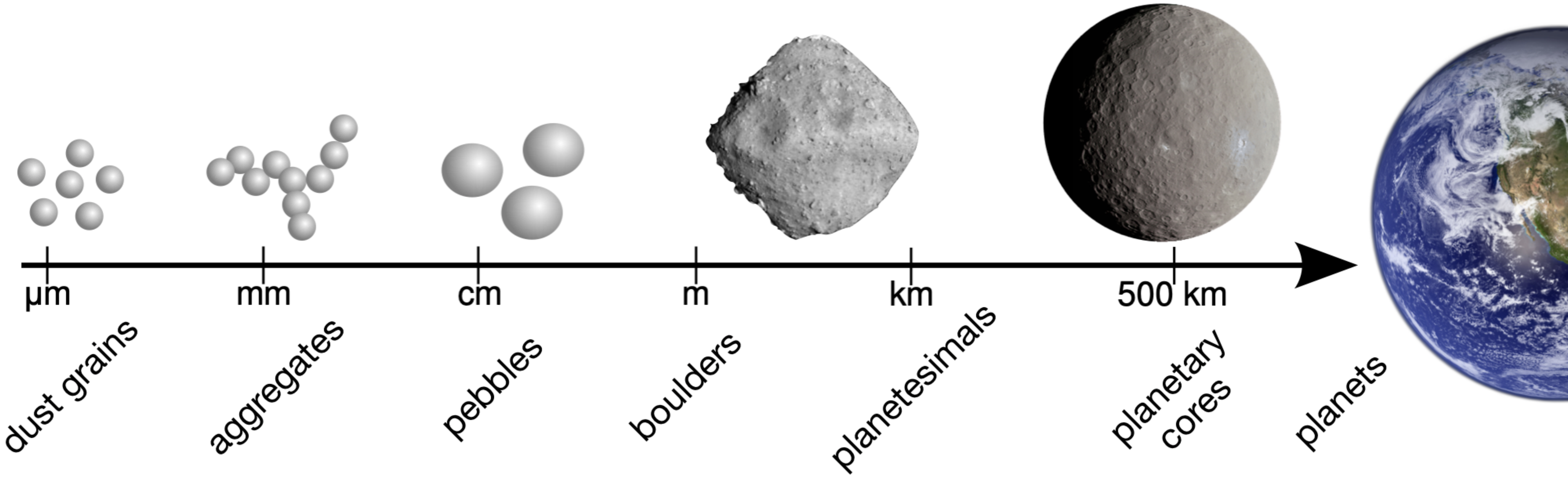
- **Planet Formation: Terrestrial and Giant Planets**

  Terrestrials: runaway/oligarchic growth → embryos → giant impacts

  Giants: ~10 $M_\oplus$ cores accrete gas before disk dispersal or via disk instability

- **Evolution of Planetary Systems:** Migration, resonances, and instabilities sculpt exoplanet architectures

Phases of Planet Formation



Credit: J. Drazkowska

3. Runaway & Oligarchic Growth

- The largest planetesimals undergo **runaway growth**, becoming planetary embryos.
- Growth slows in the **oligarchic phase**, until each embryo reaches its **isolation mass**—having consumed all material in its feeding zone.

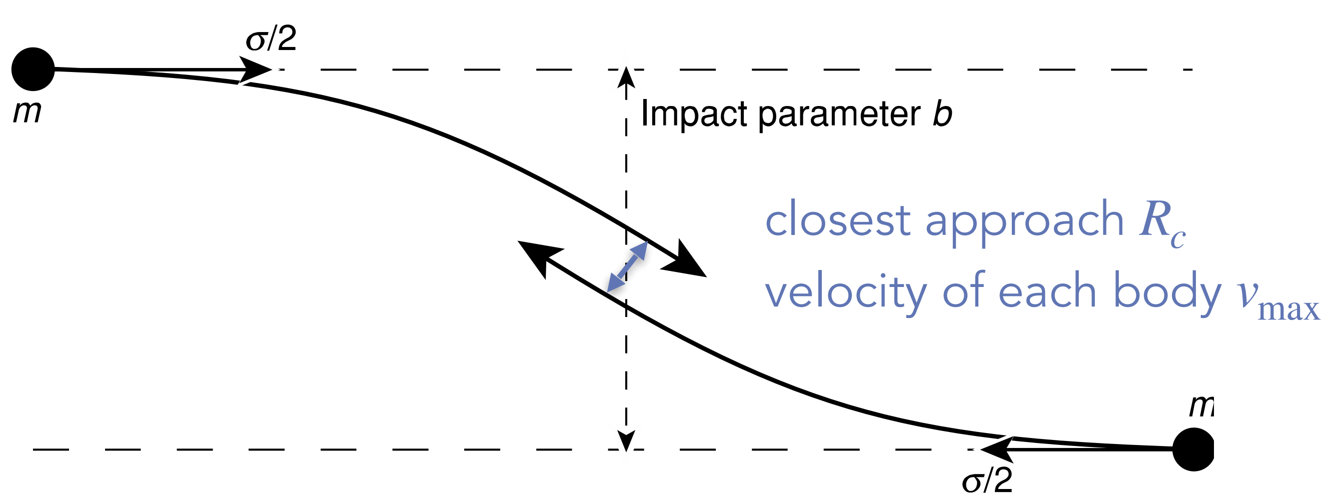
4. Giant Impacts → Final Planets

- Collisions between embryos merge them into the final terrestrial planets.
- The last **giant impact** marks the end of formation and the start of the planet's long-term evolution.

Accretion of Planetesimals

Gravitational Focusing

Gravity deflects planetesimal trajectories. Effective collision cross-section is **much larger** than the physical cross-section of the colliding planetesimals.



Assume we have

- Two planetesimals each of mass m
- Relative velocity at infinity σ
- Impact parameter b

The effective cross-section for collisions is

$$\Gamma = \pi R_s^2 \left(1 + \frac{v_{\mathrm{esc}}^2}{\sigma^2} \right),$$

where R_s is the sum of the radii of the two planetesimals and v_{esc} is the **mutual** escape velocity of the two planetesimals when they contact (i.e., $v_{\mathrm{esc}}^2 = 4Gm/R_s$).

Accretion of Planetesimals

Runaway Growth

Starting with a disk of planetesimals, the growth rate of protoplanet is

$$\frac{dM}{dt} \propto \pi R_s^2 \left(1 + \frac{v_{\rm esc}^2}{\sigma^2} \right)$$

With the assumption of $v_{\rm esc} \gg \sigma$ (i.e., low velocity dispersion),

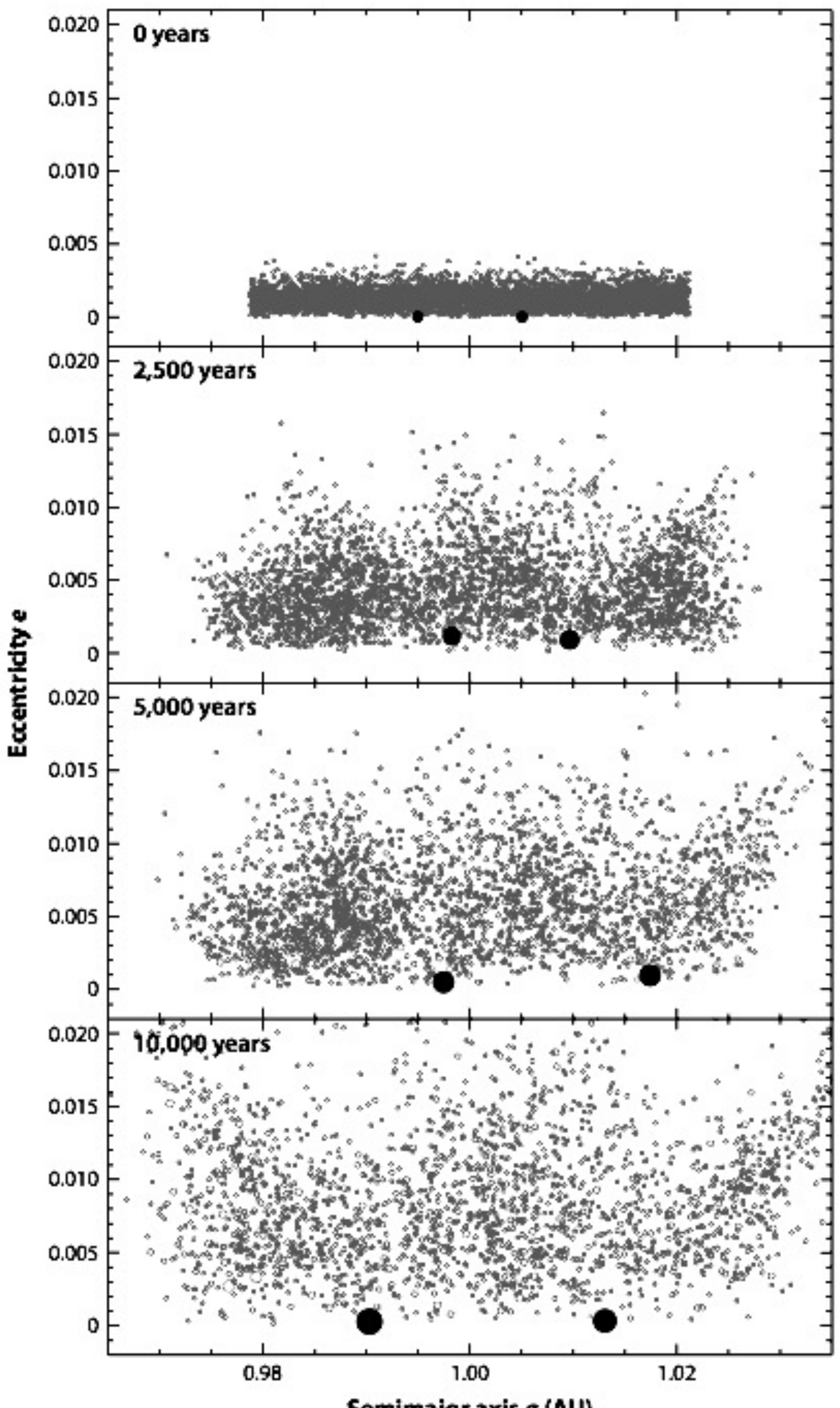
$$\frac{dM}{dt} \propto \pi R_s^2 \frac{v_{\rm esc}^2}{\sigma^2} \propto M R_s \propto M^{4/3}$$

- Larger bodies grow faster by efficiently sweeping up smaller ones, creating a *positive* feedback loop.
- Result: **a few massive protoplanets rapidly outgrow the rest** – this is called **runaway growth**.

Accretion of Planetesimals

Oligarchic Growth

Protoplanets perturb planetesimals and excite their eccentricities over time.



After runaway accretion,

- Large protoplanets dominate their feeding zones and **grow more slowly** as they stir up planetesimals, increasing the velocity dispersion $\sigma \sim eV_K$.
- Higher velocities reduce gravitational focusing, limiting collision rates.
- Multiple protoplanets grow concurrently but maintain spacing, forming a stable system of "**oligarchs**."
- This phase sets the stage for later giant impacts and planet assembly.

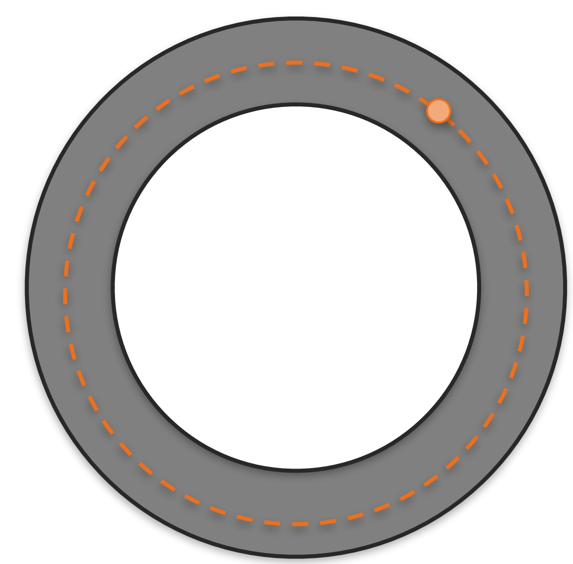
Isolation Mass

The **isolation mass** is the mass a protoplanet reaches after it has accreted all the solid material in its local feeding zone.

Suppose the feeding zone has width $\Delta a = C\, r_H$, where $r_H = \left(\frac{M_p}{3M_\star}\right)^{1/3} a$ is the Hill radius and C is a constant with a typical value of $C = 2\sqrt{3}$.

Mass of planetesimals in the feeding zone:

$$M_{\rm iso} = 2\pi a \cdot 2\Delta a \Sigma_p = 4\pi a^2 C \left(\frac{M_{\rm iso}}{3M_\star}\right)^{1/3} \Sigma_p$$



Solving for the isolation mass, we find:

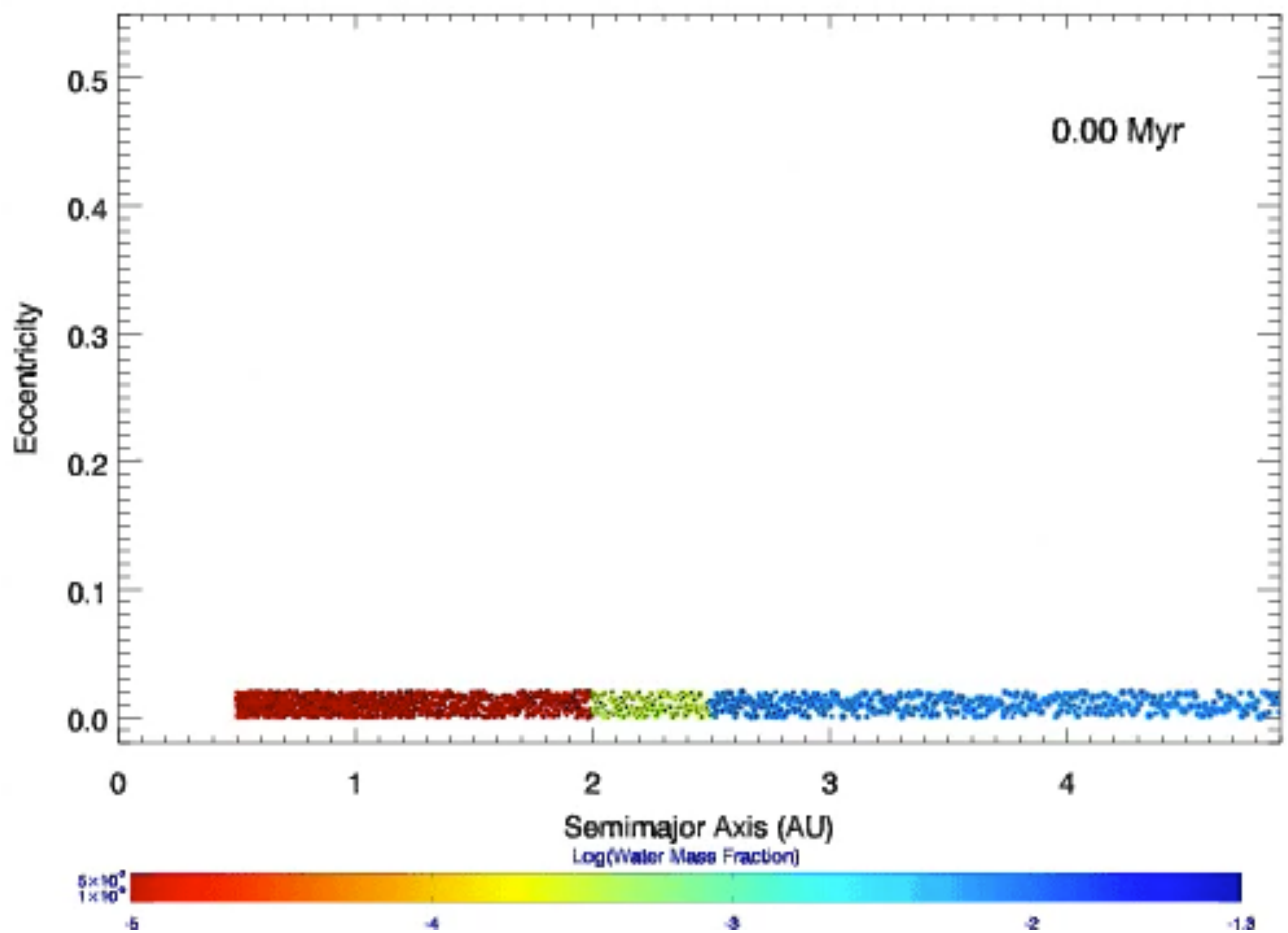
$$M_{\rm iso} = \frac{8}{\sqrt{3}} \pi^{3/2} C^{3/2} M_\star^{-1/2} \Sigma_p^{3/2} a^3$$

In-Class Activity

Estimating the Isolation Mass

Final Assembly

Collisions between a population of planet embryos with masses ~Moon or Mars and separation of ~10 Hill radii



Credit: S. Raymond

Summary of Terrestrial Planet Formation

The formation of terrestrial planets can be separated into five main stages:

1. The settling and agglomeration of small (starting with sub-micron-sized) dust particles to form cm-m sized pebbles. The coagulation of dust is mediated by electrostatic forces, which allows dust to grow to pebbles via pairwise collisions.

2. The growth of dust and pebbles to planetesimals. This requires a bypass of the meter-sized barrier, which likely occurs through some form of gravitational instability of solid material in the disk, perhaps mediated by the streaming instability. Once these planetesimals form, they can rapidly grow through accretion of pebbles due to radial drift and gravitational focusing.

3. Runaway growth of the largest planetesimals to become planetary embryos, with a resulting phase of oligarchic growth where embryos grow more slowly until they reach the isolation mass. At this point, each embryo has accreted all material in its feeding zone.

4. Collisions between planetary embryos result in growth of planets to their final masses. The final giant impact between planet and embryo is the point at which the formation of the planet has ceased, and evolution has begun.