ASTR 405
Planetary Systems
<u>Protoplanetary Disks</u>

Fall 2025

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Supplementary Readings: **formation.pdf Section II** on Canvas

Lecture Notes on the Formation and Early Evolution of Planetary Systems by Armitage

Module II: Exoplanet Demographics and Planet Formation

- **Protoplanetary Disks:** Gas–dust disks around young stars; evolve on Myr timescales, set the initial conditions for planet formation

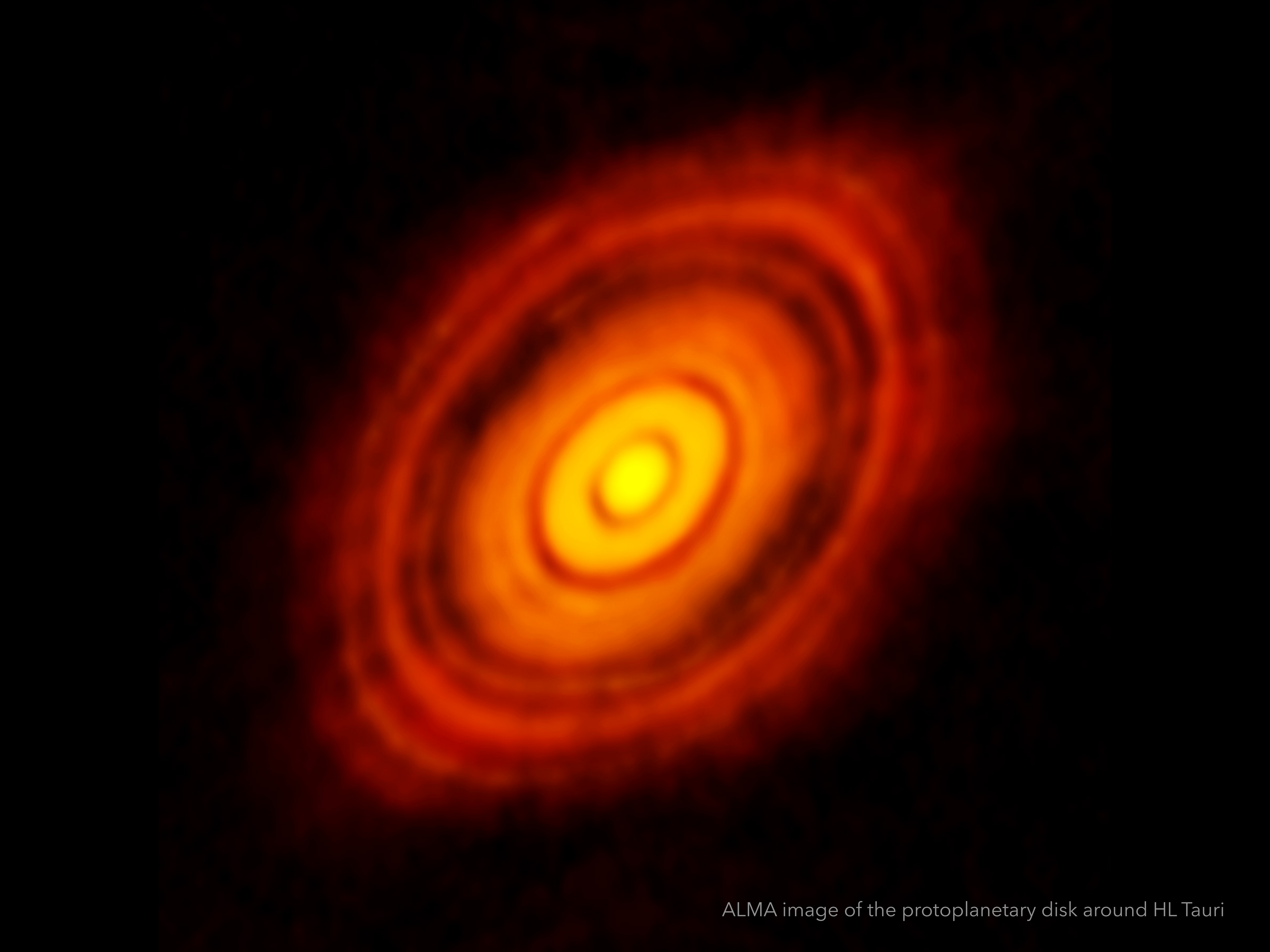
- **Dust, Pebbles, and Planetesimals:** Dust grains stick → pebbles (mm–cm); rapid drift & instabilities lead to km-scale planetesimals

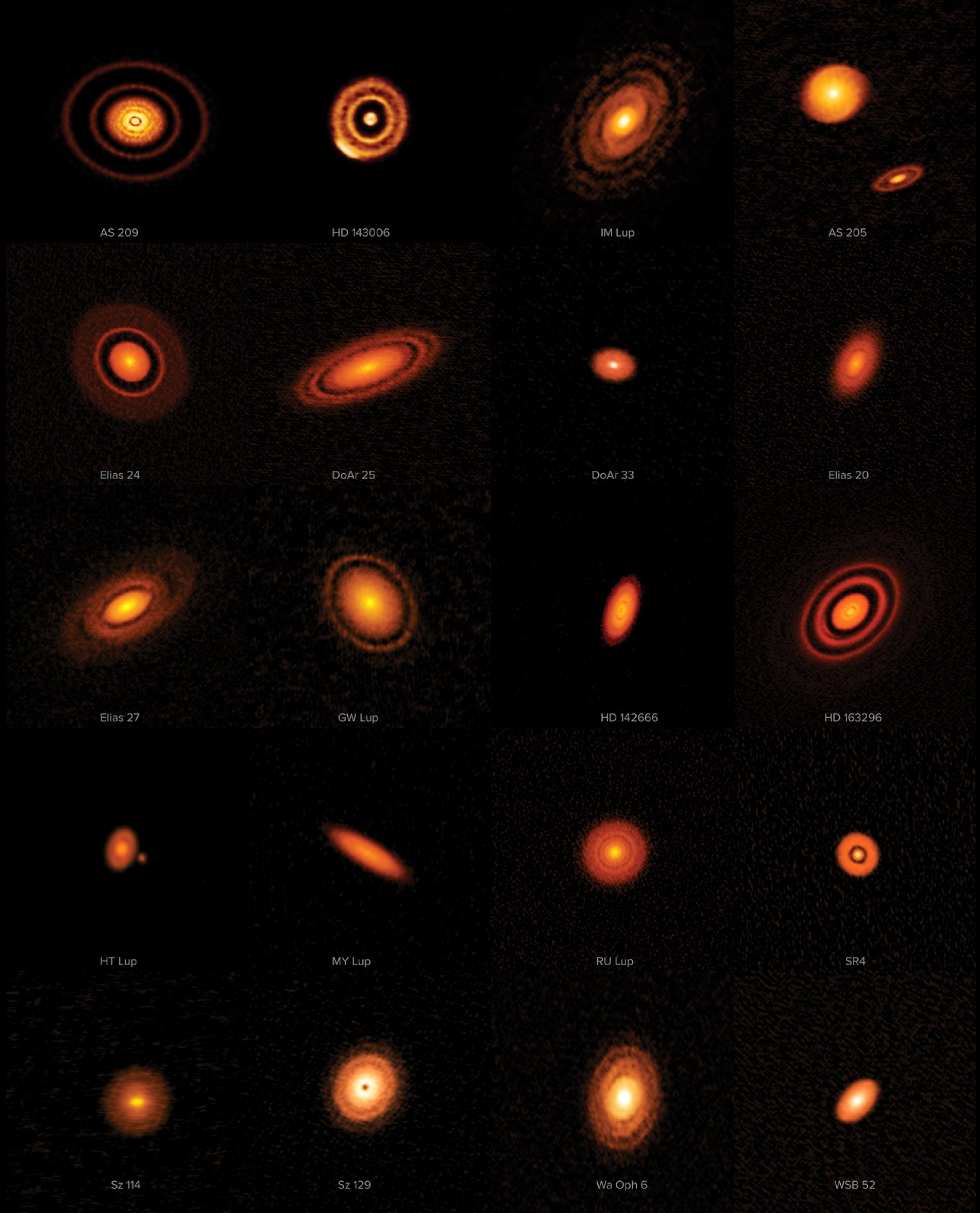
- **Planet Formation: Terrestrial and Giant Planets**

  Terrestrials: runaway/oligarchic growth → embryos → giant impacts

  Giants: ~10 $M_{\oplus}$ cores accrete gas before disk dispersal or via disk instability

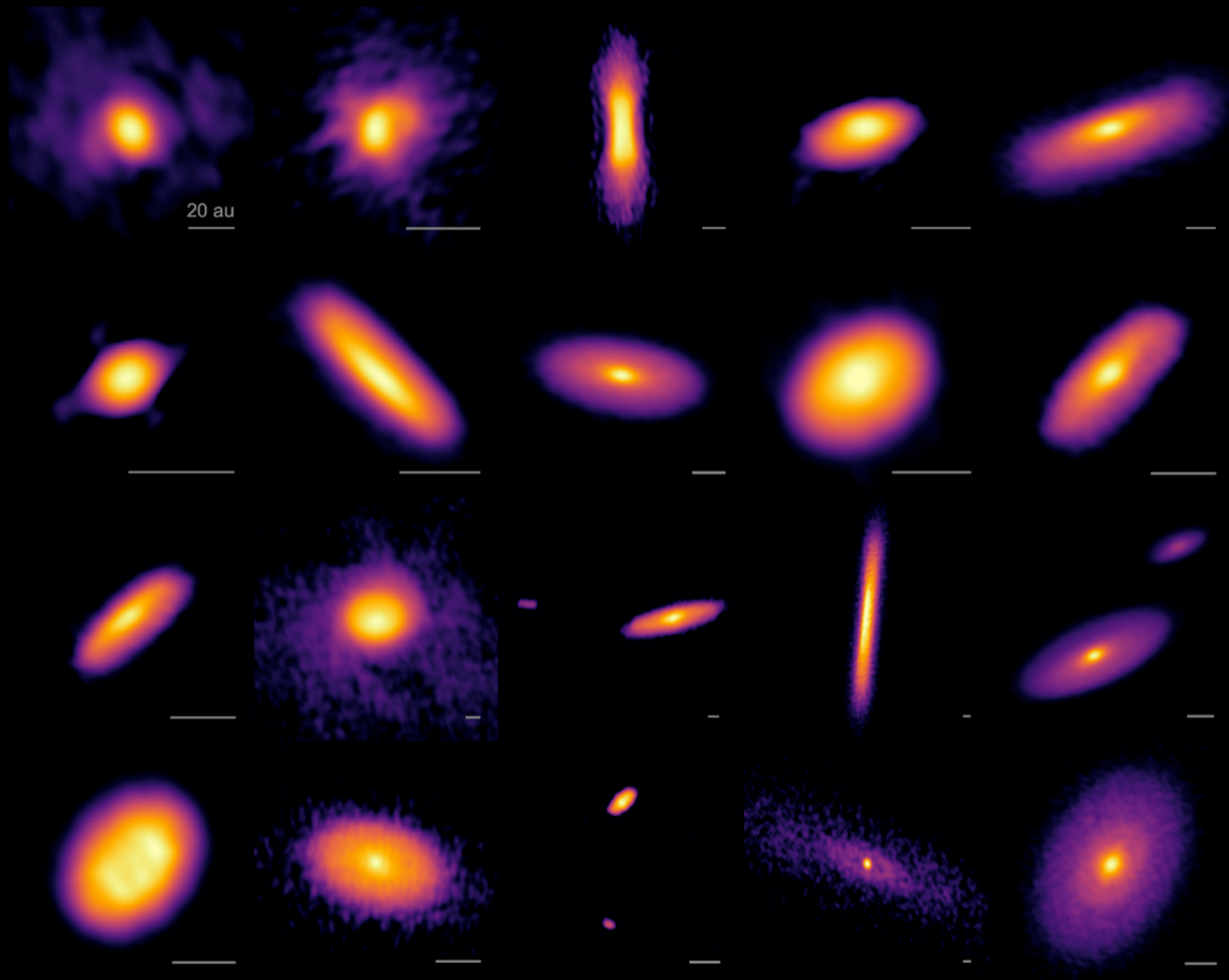
- **Evolution of Planetary Systems:** Migration, resonances, and instabilities sculpt exoplanet architectures


ALMA image of the protoplanetary disk around HL Tauri



ALMA DSHARP Survey on Class II Young Stellar Objects

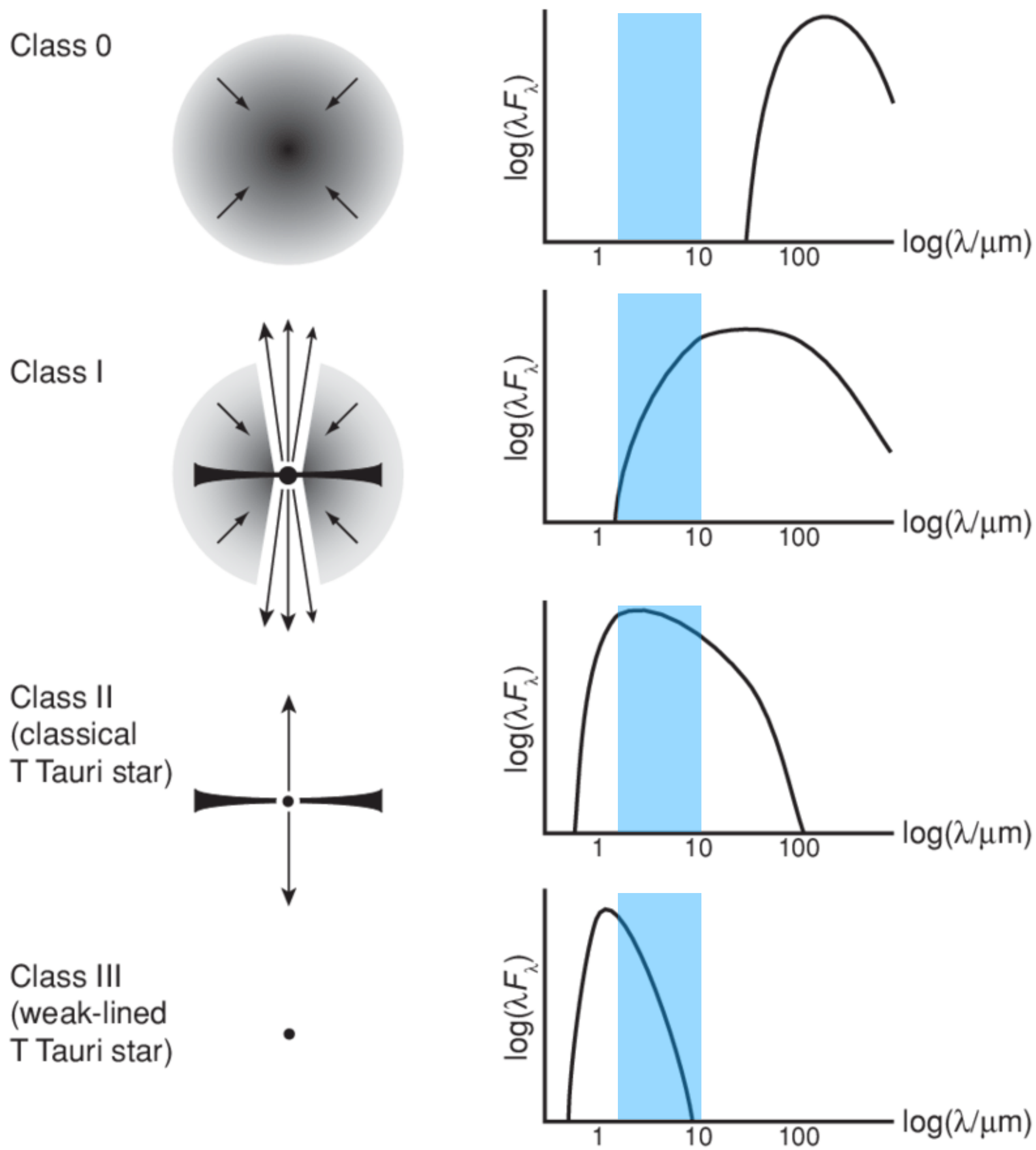
Credit: ALMA (ESO/NAOJ/NRAO), S. Andrews et al.; NRAO/AUI/NSF, S. Dagnello



ALMA eDisks Survey on Class 0/I Young Stellar Objects

Credit: ALMA (ESO/NAOJ/NRAO), N. Ohashi et al.

Young Stellar Object (YSO) Classification
Class 0, I, II, and III

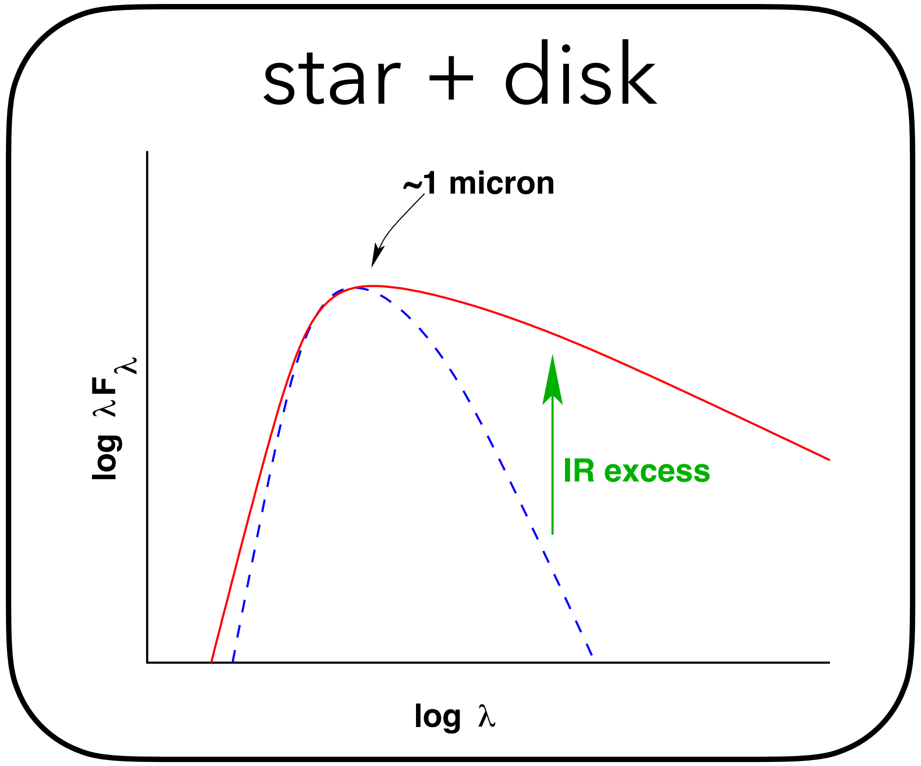


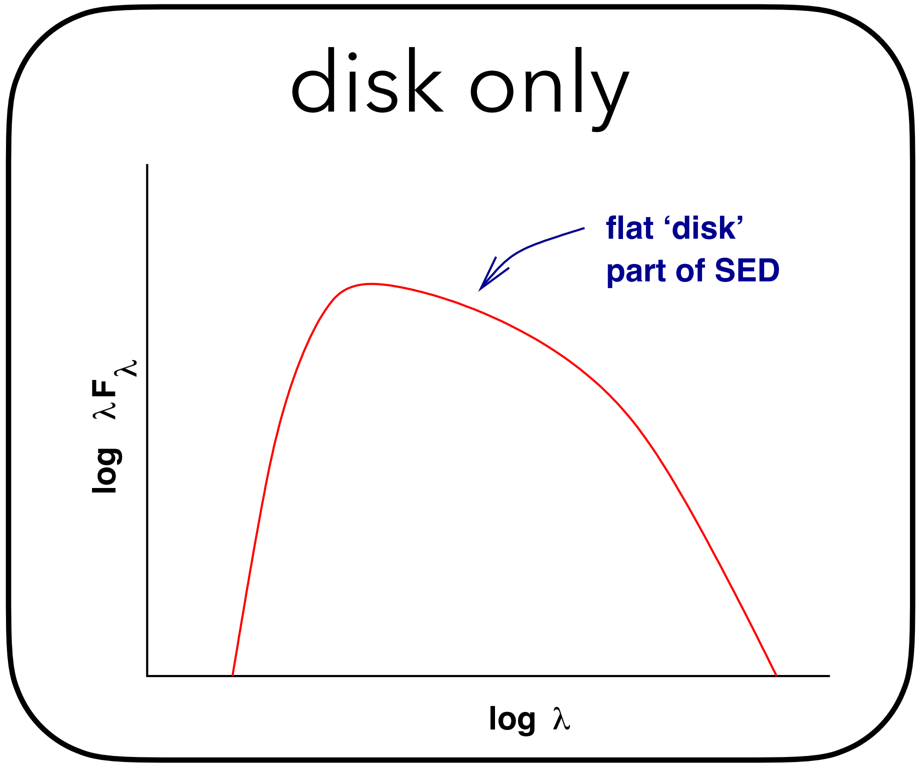
The slope of infrared (IR) emission from K band (at ~2 μm) to N band (at ~10 μm)

$$\alpha_{\rm IR} = \frac{\Delta \log \lambda F_\lambda}{\Delta \log \lambda}$$

- Class 0: Spectral energy distribution (SED) peaks in the far-IR; no flux in near-IR
- Class I: $\alpha_{\rm IR} > 0$
- Class II: $-1.5 < \alpha_{\rm IR} < 0$
- Class III: No IR excess

Spectral Energy Distribution (SED) of a Passive Disk





A disk emits as the sum of blackbody radiation from concentric annuli, each at local temperature $T(r)$

Integrate over the full disk:

$L_\lambda = \pi \int_{r_{\rm in}}^{r_{\rm out}} 2\pi r\, B_\lambda[T(r)]\, dr$, where B_λ is the Planck function: $B_\lambda(T) = \dfrac{2hc^2}{\lambda^5} \dfrac{1}{e^{hc/\lambda kT} - 1}$ and that $F_\lambda = \pi B_\lambda$

Three cases:

$$\lambda F_\lambda \propto \begin{cases} \lambda^{-4} e^{-hc/\lambda k T_{\rm in}}, & \text{short wavelengths (Wien limit)} \\ \lambda^{-4/3}, & \text{intermediate region} \\ \lambda^{-3}, & \text{long wavelengths (Rayleigh-Jeans limit)} \end{cases}$$

Disk Thermal Structure

How does local temperature $T(r)$ vary?

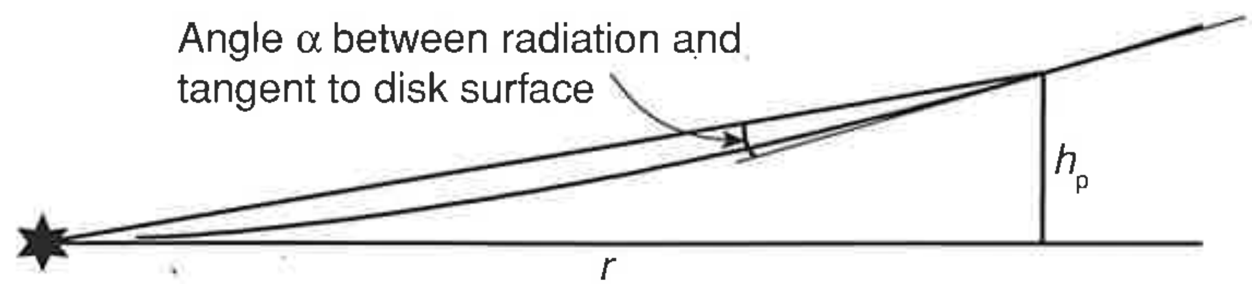


Fig. 2.4. Geometry for calculation of the radial temperature profile of a flared protoplanetary disk. At distance $r \gg R_*$, radiation from the star is absorbed by the disk at height h_p above the mid-plane. The angle between the tangent to the disk surface and the radiation is α.

Assuming that the disk is locally in radiative equilibrium with incoming starlight onto an area A and that the flaring angle α is small,

$$\frac{L_\star}{4\pi r^2} A\sin\alpha = A\sigma T_d^4$$

$$\frac{L_\star}{4\pi r^2}\alpha = \sigma T_d^4 \ ,$$

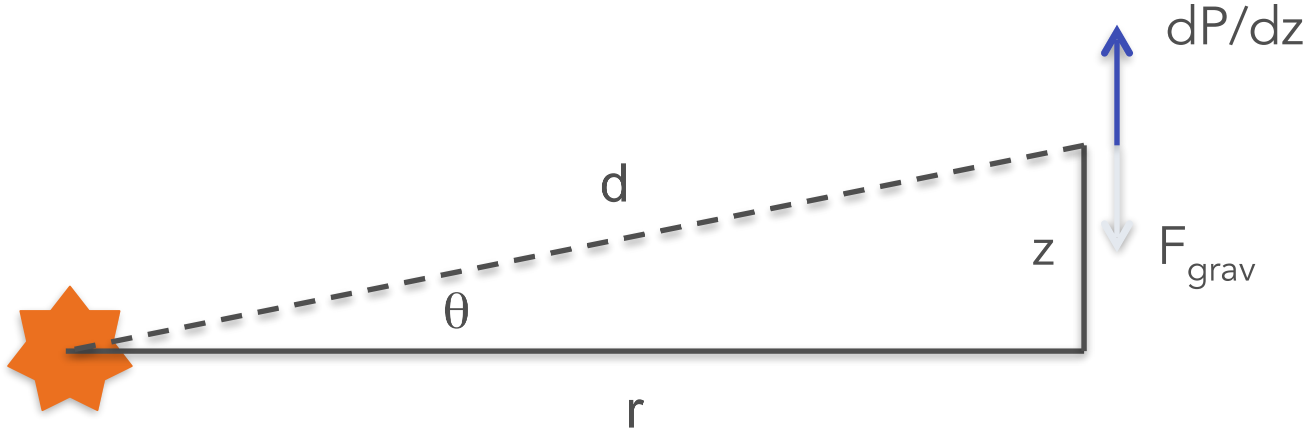
Using the Stefan-Boltzmann law to substitute $L_\star = 4\pi R_\star^2 \sigma T_\star^4$,

the temperature of the disk is $\boxed{T_d = \sqrt{\frac{R_\star}{r}}\alpha^{1/4} T_\star}$.

In-Class Activity

Condensation Points and Ice Lines

Vertical Disk Density Structure



Vertical hydrostatic equilibrium:

$\boxed{\dfrac{dP}{dz} = -\rho\, g_z}$, where ρ is the gas density and $g_z = g\ \sin\theta = \dfrac{GM_\star}{d^3} z$

Use the ideal gas law to relate pressure and density:

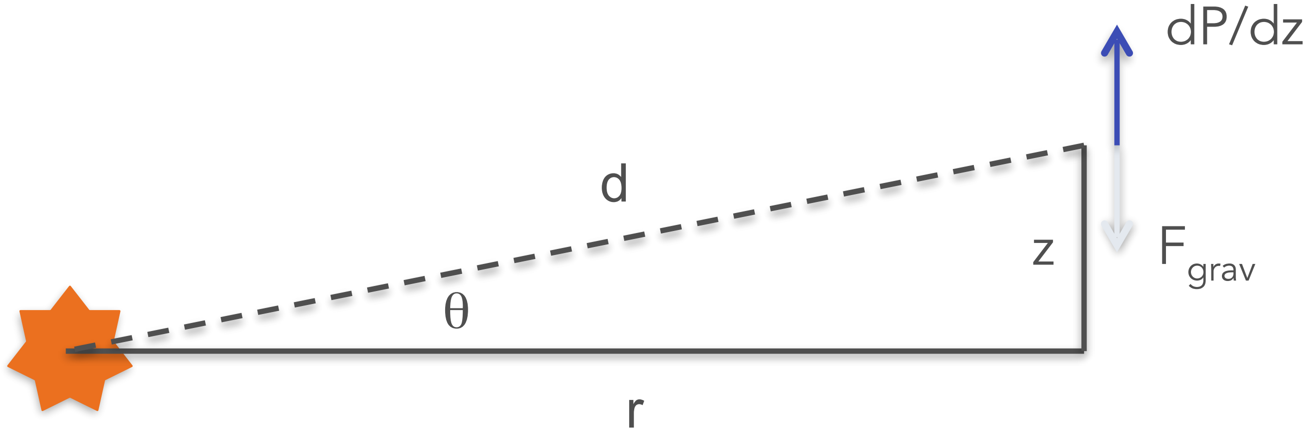
$\boxed{P = \rho c_s^2}$, where $c_s^2 = \dfrac{k_B T}{\mu m_p}$ is the isothermal sound speed

$\mu \approx 2.3$ is the mean molecular weight and m_p is the proton mass

In-Class Activity

Deriving Disk Vertical Density Profile

Vertical Disk Density Structure



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Disk vertical density profile

$\boxed{\rho = \rho_0 e^{-z^2/2h^2}}$, where $h = \dfrac{c_s}{\Omega}$ is the disk vertical scale height and Ω is the angular frequency, and

$$\rho_0 = \frac{1}{\sqrt{2\pi}} \frac{\Sigma}{h}$$

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Minimum Mass Solar Nebula (MMSN)

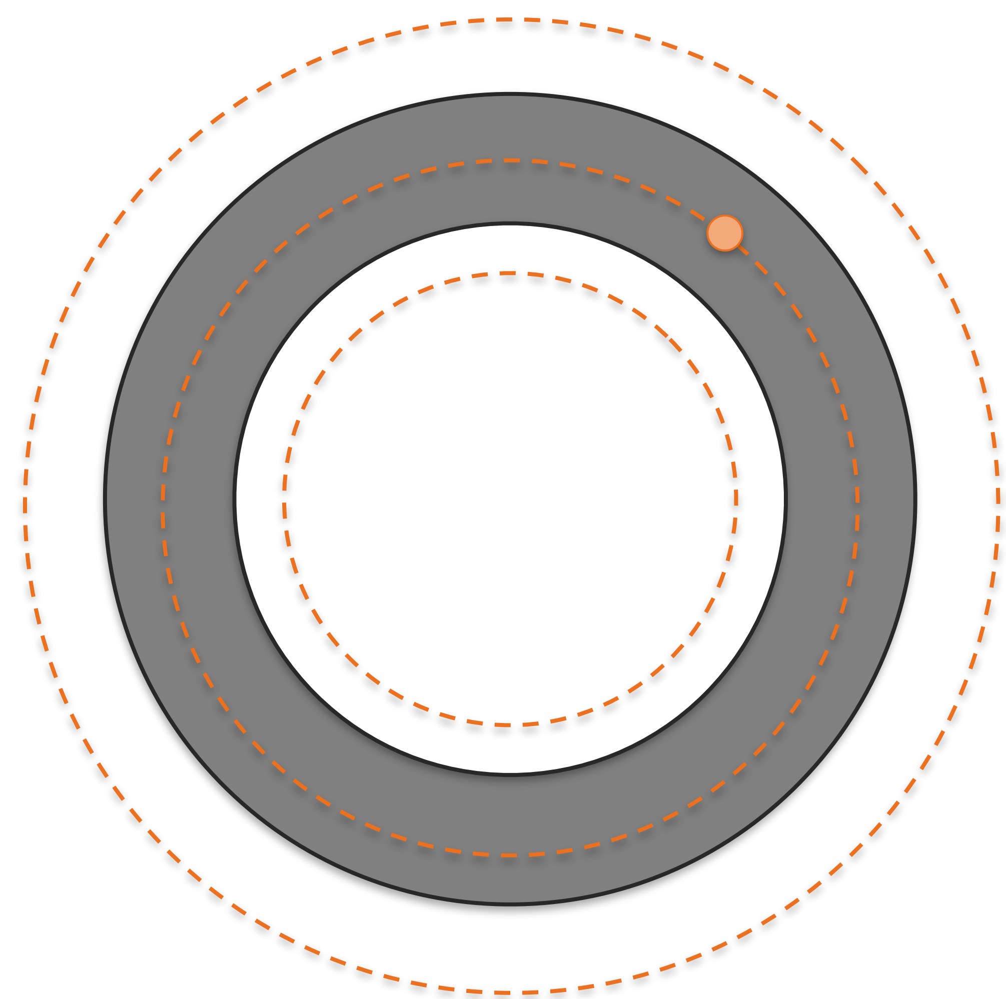
- Vertical gas density profile is a gaussian
- Radial profile of surface density is not determined by any simple consideration

Minimum mass Solar Nebula is an attempt to estimate $\Sigma(r)$ using observed properties of Solar System planets

(1) Divide Solar System into annuli around each planet's orbit

(2) Compute area: For each planet, take measured heavy element mass, calculate augmented mass if the planet had Solar composition

Estimate $\Sigma(r_{\text{planet}})$ = augmented mass / A



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Weidenschilling 1977

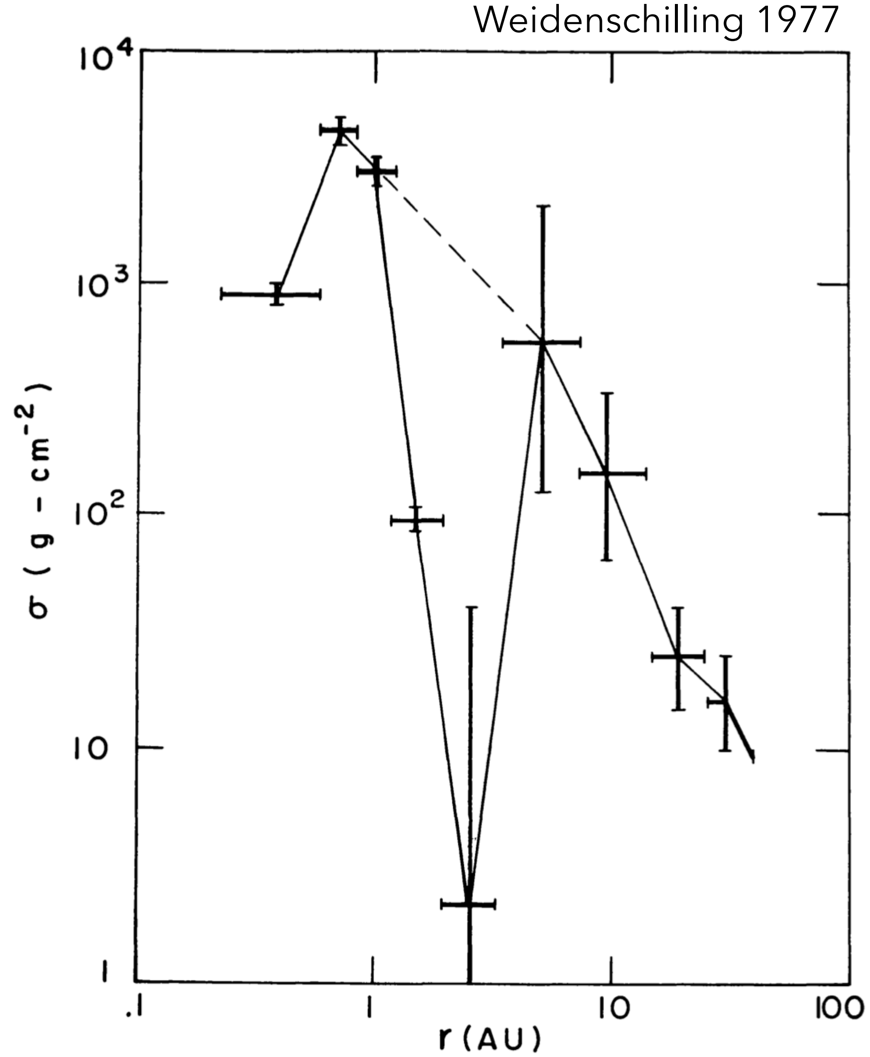

Fig. 1. Surface densities, σ, obtained by restoring the planets to solar composition and spreading the resulting masses through contiguous zones surrounding their orbits. The meaning of the 'error bars' is discussed in the text.

Disk Dynamics

The radial force balance for the gas in a protoplanetary disk is largely between three forces: the centrifugal force, pressure gradients, and gravity.

$$\frac{v_\phi^2}{r} = \frac{1}{\rho}\frac{dp}{dr} + \frac{GM_\star}{r^2}$$

Assuming that the pressure follows $p = p_0(r/r_0)^{-n}$, we get

$$v_\phi = v_K\sqrt{1 - n\frac{h^2}{r^2}}$$

For $dp/dr < 0$, the velocity of the gas will be **sub-Keplerian**. This is critical for determining **the motions of dust grains in the disk**, as they move at Keplerian speeds while the gas is slightly sub-Keplerian.