ASTR 405
Planetary Systems
Radial Velocity

Fall 2025

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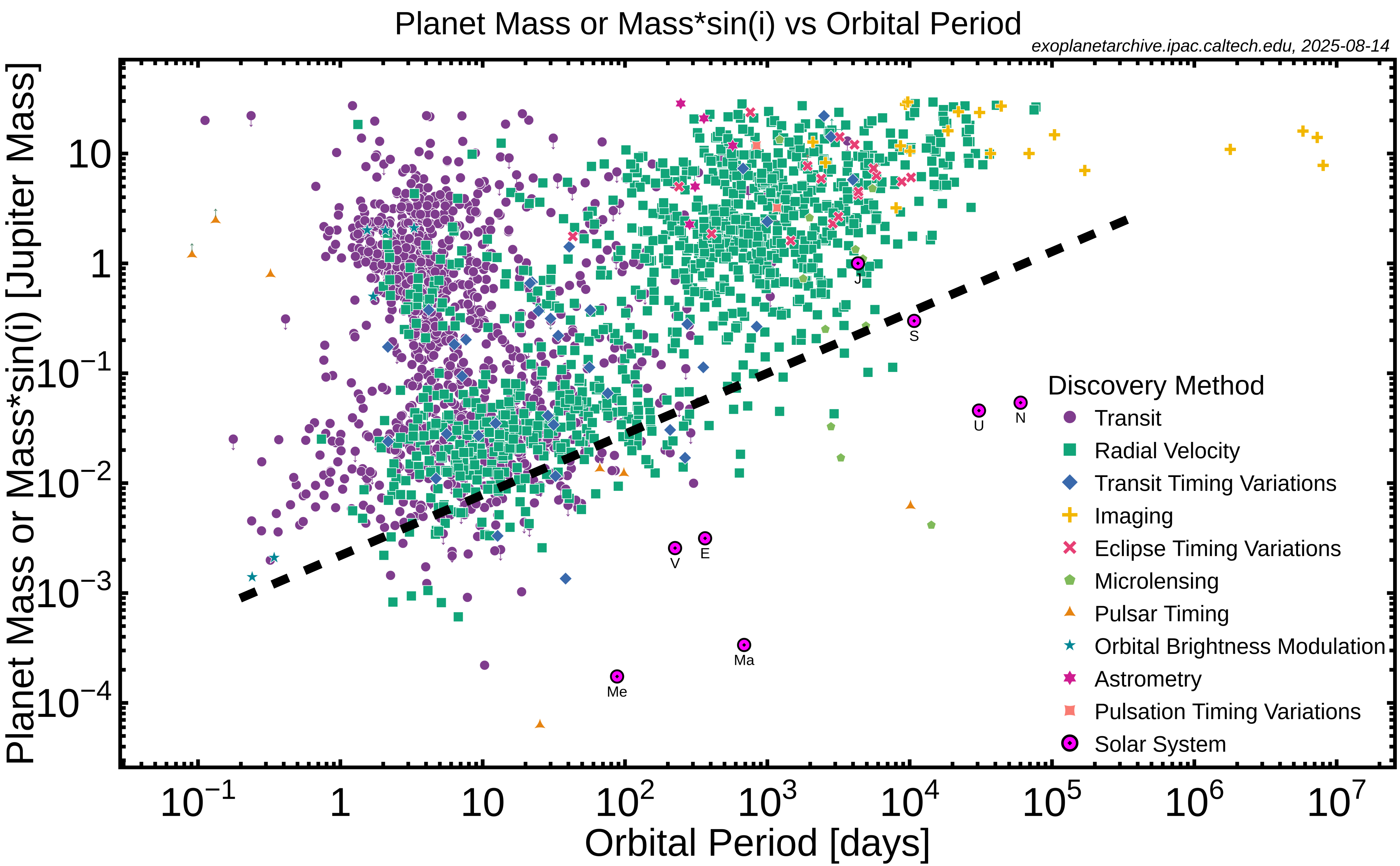
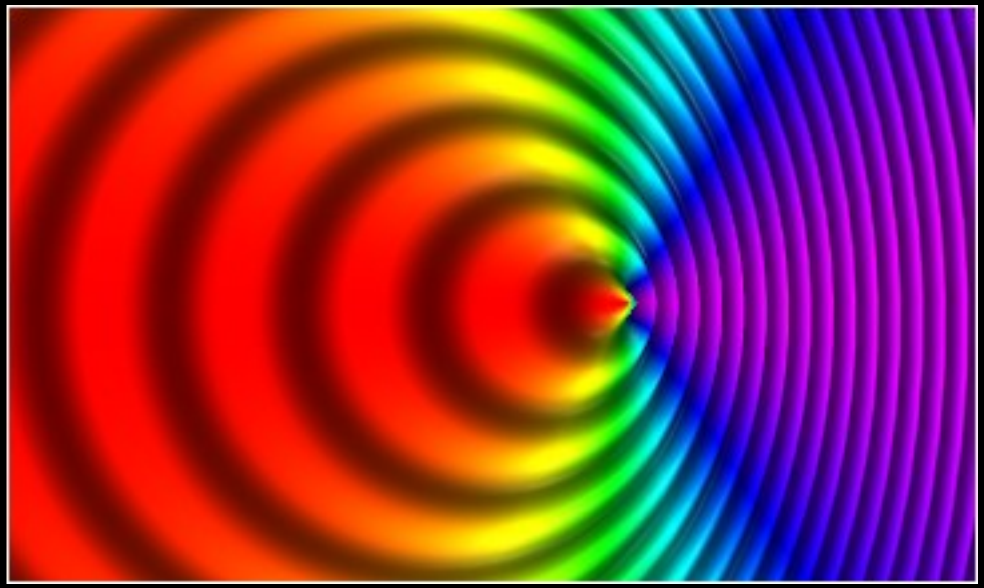
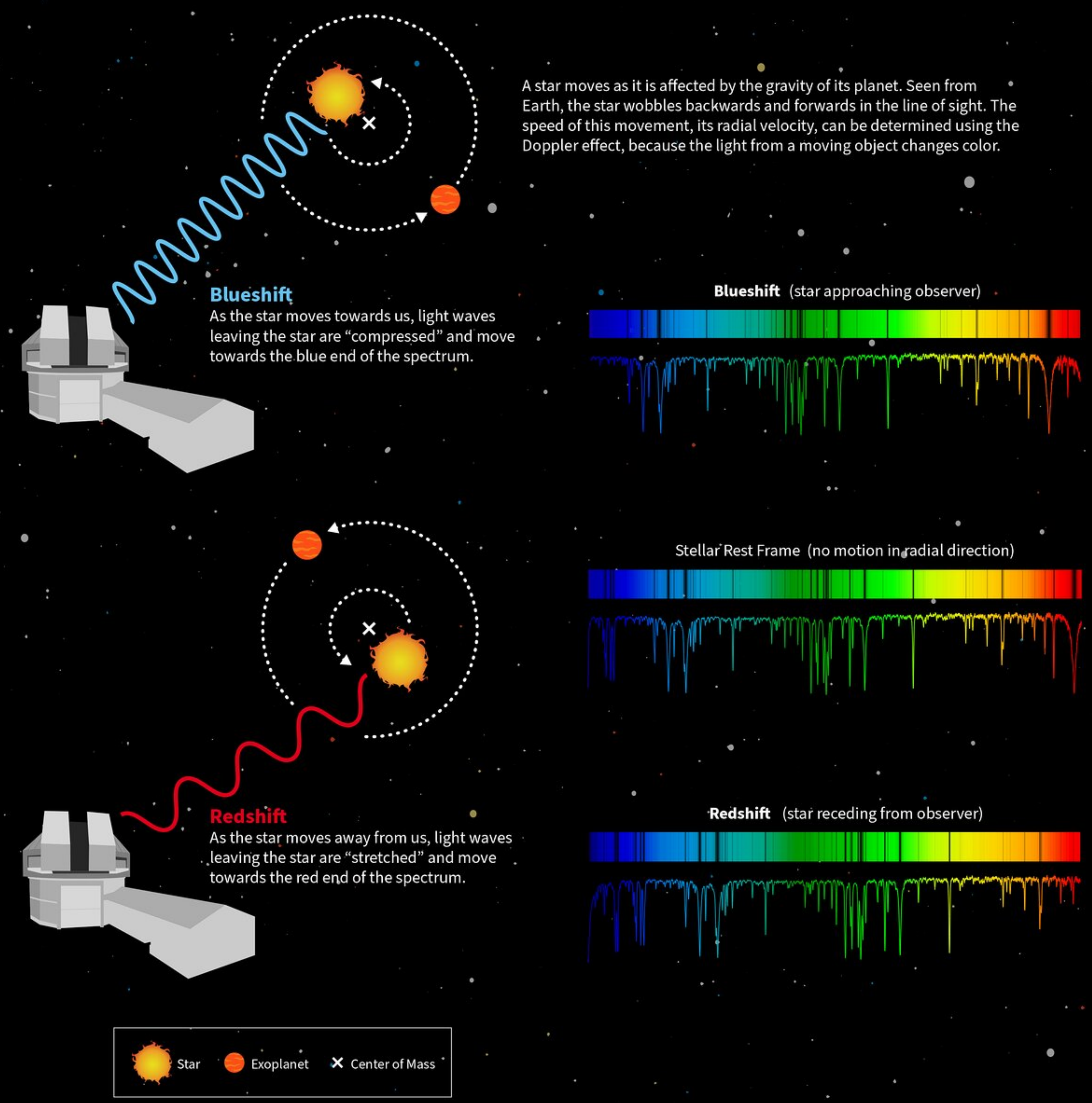
Exoplanet Mass–Period Distribution

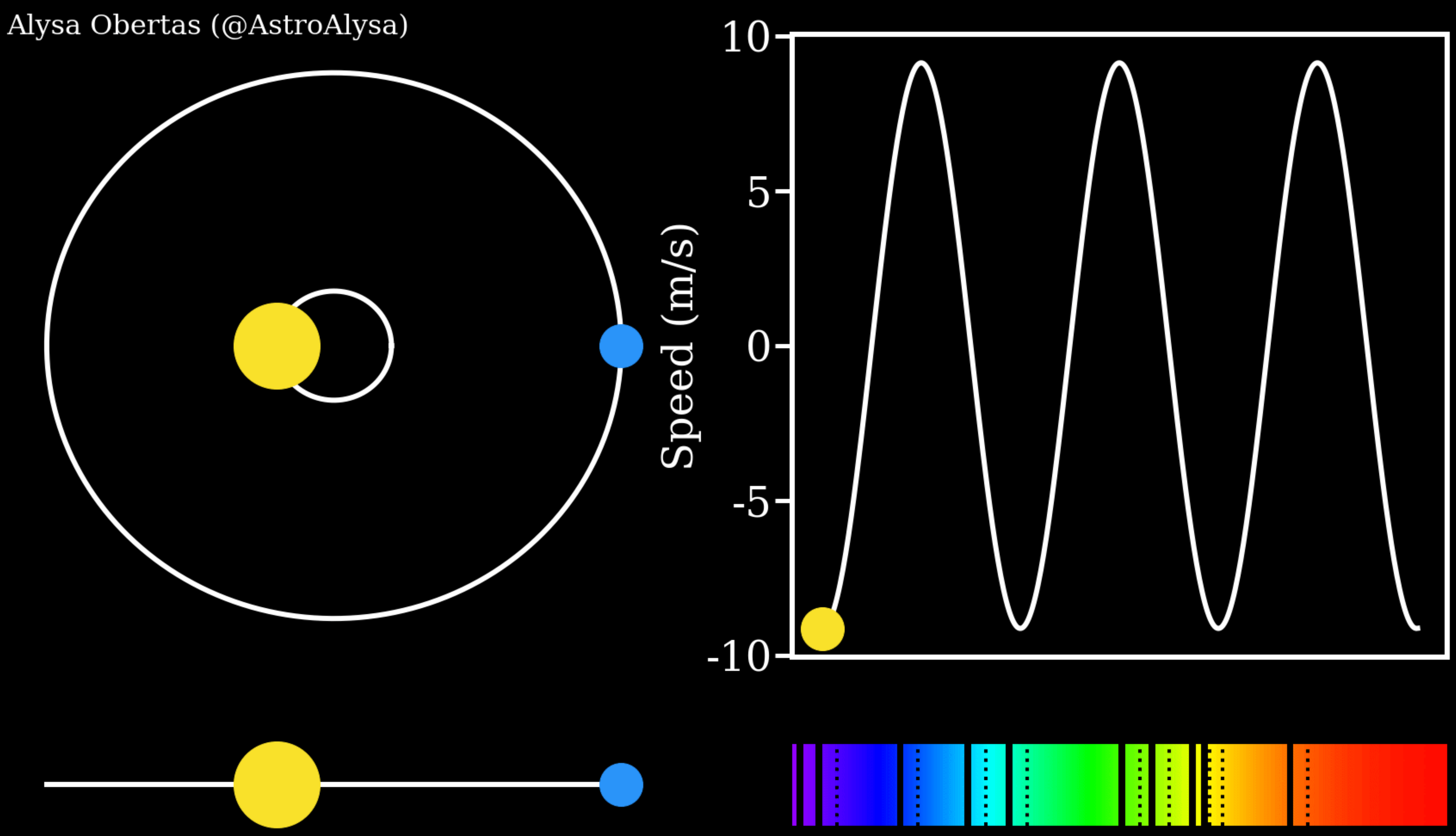


Doppler Effect



A source of light waves moving to the right, relative to observers, with velocity 0.7c. The frequency is higher for observers on the right, and lower for observers on the left. *Wikipedia*


A star moves as it is affected by the gravity of its planet. Seen from Earth, the star wobbles backwards and forwards in the line of sight. The speed of this movement, its radial velocity, can be determined using the Doppler effect, because the light from a moving object changes color.
Blueshift
As the star moves towards us, light waves leaving the star are "compressed" and move towards the blue end of the spectrum.
Blueshift (star approaching observer)
Stellar Rest Frame (no motion in radial direction)
Redshift
As the star moves away from us, light waves leaving the star are "stretched" and move towards the red end of the spectrum.
Redshift (star receding from observer)
Star
Exoplanet
Center of Mass



Alysa Obertas (@AstroAlysa)
Speed (m/s)
10
5
0
-5
-10

The Doppler Effect for Stellar Motion

λ_{obs} - observed wavelength from star
λ_{rest} - laboratory (rest) wavelength
v_r - radial velocity (line-of-sight motion)
θ - angle between star's motion and line of sight
c - speed of light

- Wavelength Difference: $\Delta\lambda = \lambda_{\text{obs}} - \lambda_{\text{rest}}$
- Full Relativistic Doppler Shift:

$$\lambda_{\text{obs}} = \lambda_{\text{rest}} \frac{1 + \frac{v}{c}\cos\theta}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

- Non-Relativistic Approximation (v/c << 1): $\lambda_{\text{obs}} = \lambda_{\text{rest}} \left(1 + \frac{v}{c}\cos\theta\right)$
- Radial Velocity: $v_r = \frac{\Delta\lambda}{\lambda_{\text{rest}}} c$

In-Class Activity

Deriving the radial velocity semi-amplitude for circular orbits

$$K = \left(\frac{2\pi G}{P}\right)^{1/3} \frac{M_p \sin(i)}{(M_\star + M_p)^{2/3}}$$

Rearranging for P and subbing for a, we find

$$P^2 = \frac{4\pi^2}{G}\frac{a^3}{M_\star + M_p} = \frac{4\pi^2}{G(M_\star + M_p)}\left(\frac{P}{2\pi}\right)^3\left(v_p + v_\star\right)^3 = \frac{1}{2\pi G}\frac{P^3\left(v_p + v_\star\right)^3}{\left(M_\star + M_p\right)}.$$

Now we can replace $v_p = v_\star M_\star / M_p$:

$$P^2 = \frac{1}{2\pi G}\frac{P^3}{\left(M_\star + M_p\right)}\left(v_\star + v_\star\frac{M_\star}{M_p}\right)^3 = \frac{1}{2\pi G}\frac{P^3}{\left(M_\star + M_p\right)}v_\star^3\left(1 + \frac{M_\star}{M_p}\right)^3.$$

Let's now re-arrange for $v_\star$:

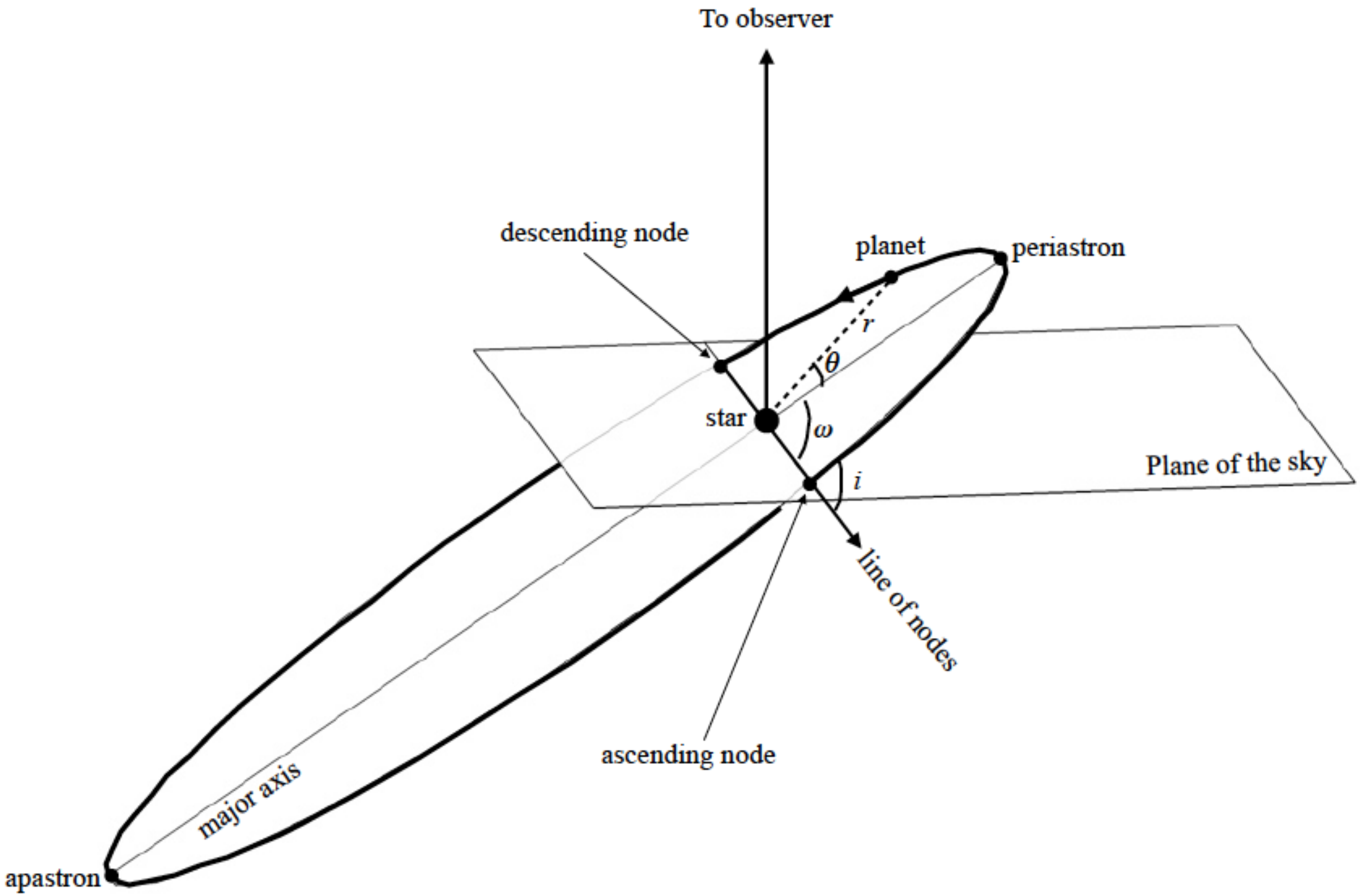
$$v_\star^3 = \frac{2\pi G}{P}\left(M_\star + M_p\right)\left(1 + \frac{M_\star}{M_p}\right)^{-3}.$$

Now we take the cube root and re-arrange:

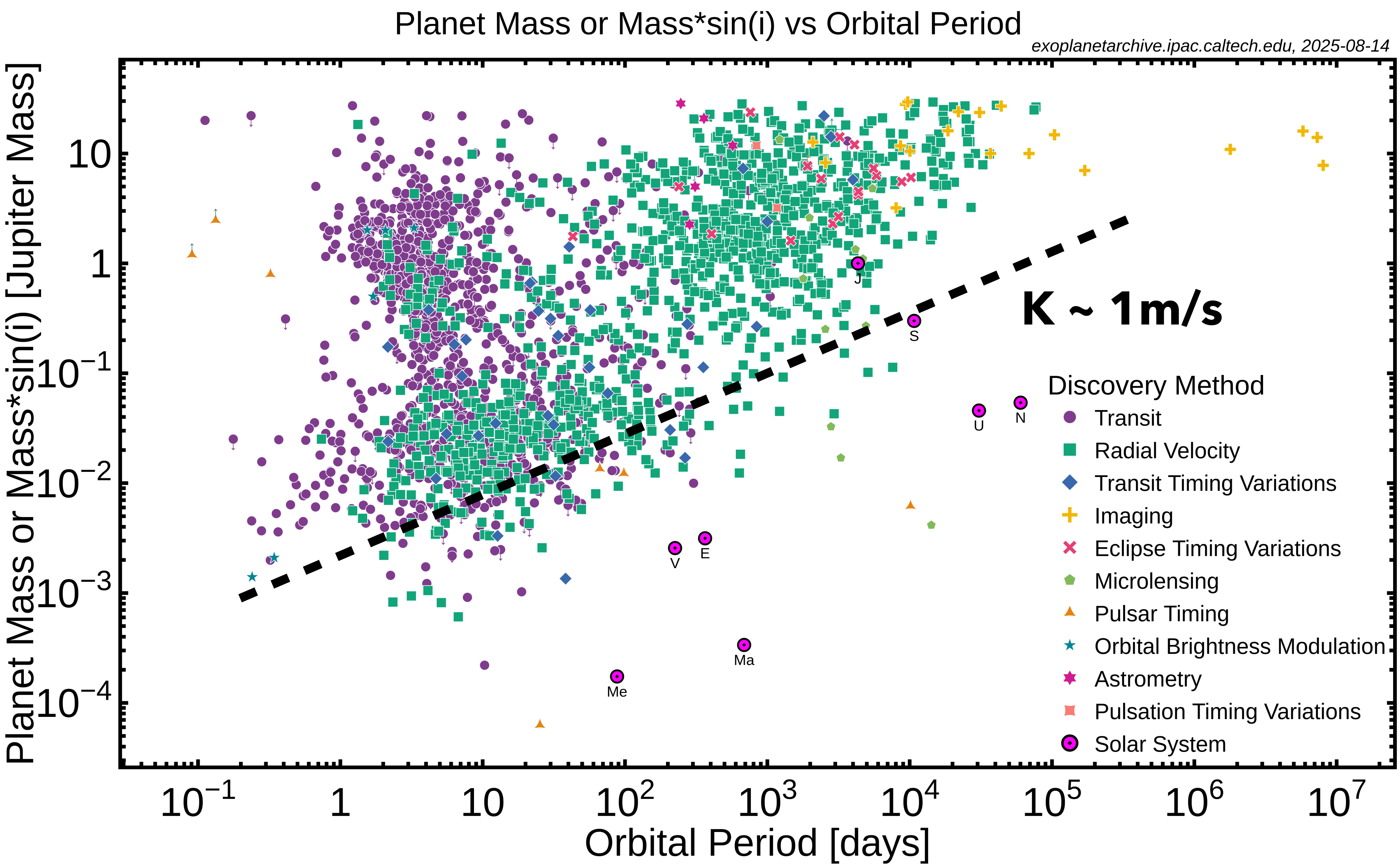
$$v_\star = \left(\frac{2\pi G}{P}\right)^{1/3}\frac{(M_\star + M_p)^{1/3}}{(1 + M_\star/M_p)} = \left(\frac{2\pi G}{P}\right)^{1/3}\frac{M_p}{(M_\star + M_p)^{2/3}}.$$

$$\boxed{K = \left(\frac{2\pi G}{P}\right)^{1/3}\frac{M_p \sin(i)}{(M_\star + M_p)^{2/3}}\frac{1}{\sqrt{1 - e^2}}}$$

Geometry of a 3D Orbit

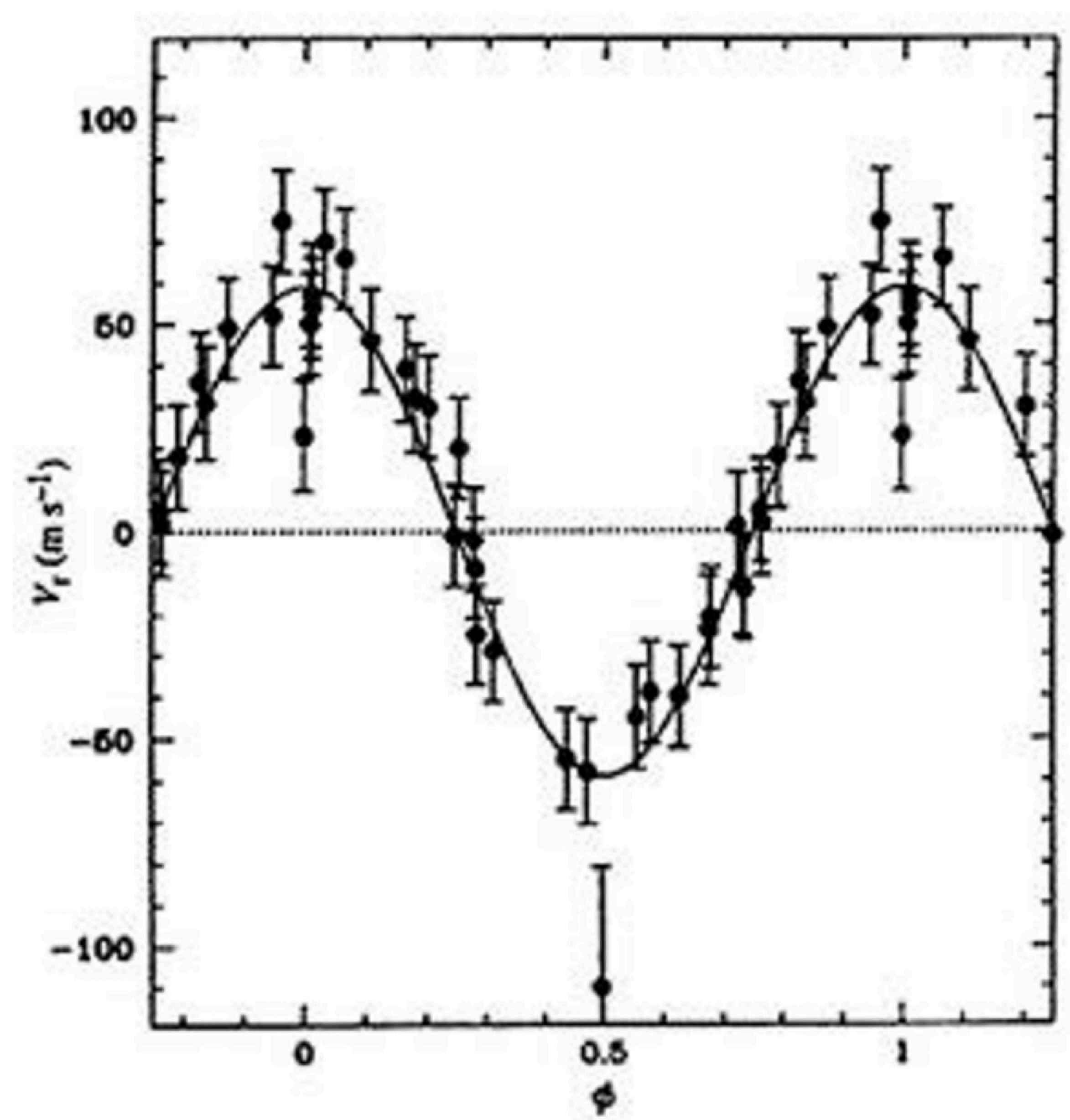


Exoplanet Mass—Period Distribution



Detecting Exoplanets via RV In Practice

- The radial velocity method discovered the first exoplanet around a Sun-like star, 51 Pegasi b (Mayor & Queloz 1995)
- The orbital period of 51 Pegasi b to be 4.23 days and the **minimum mass** of the planet to be 0.468 Jupiter masses
- Note that radial velocity method alone does not allow a mass to be measured directly, it only places a **lower limit** on the mass



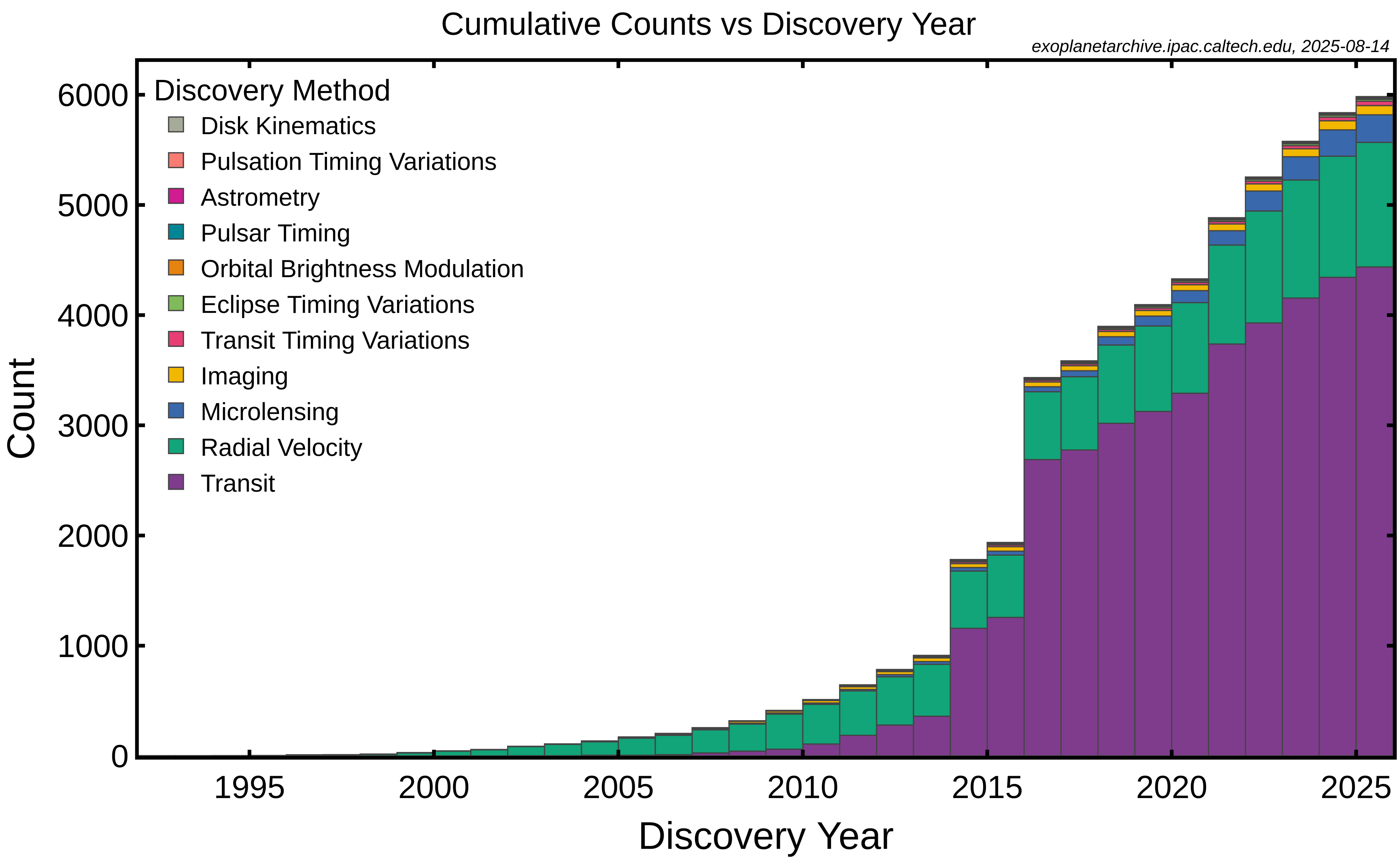
Mayor & Queloz 1995

In-Class Activity

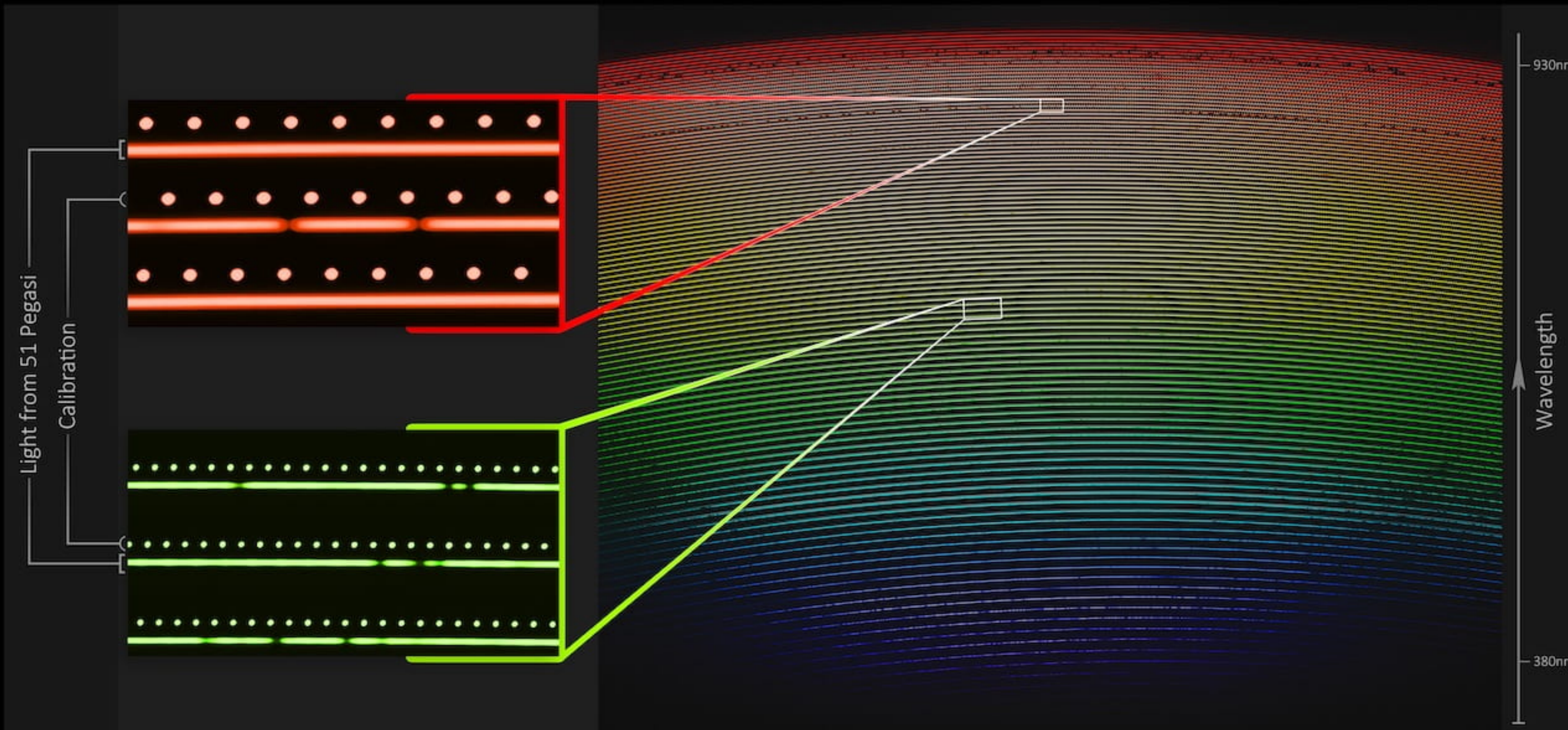
Radial Velocity In Practice

$$\boxed{K = \left(\frac{2\pi G}{P}\right)^{1/3} \frac{M_p \sin(i)}{(M_\star + M_p)^{2/3}} \frac{1}{\sqrt{1 - e^2}}}$$

Cumulative Exoplanet Discoveries



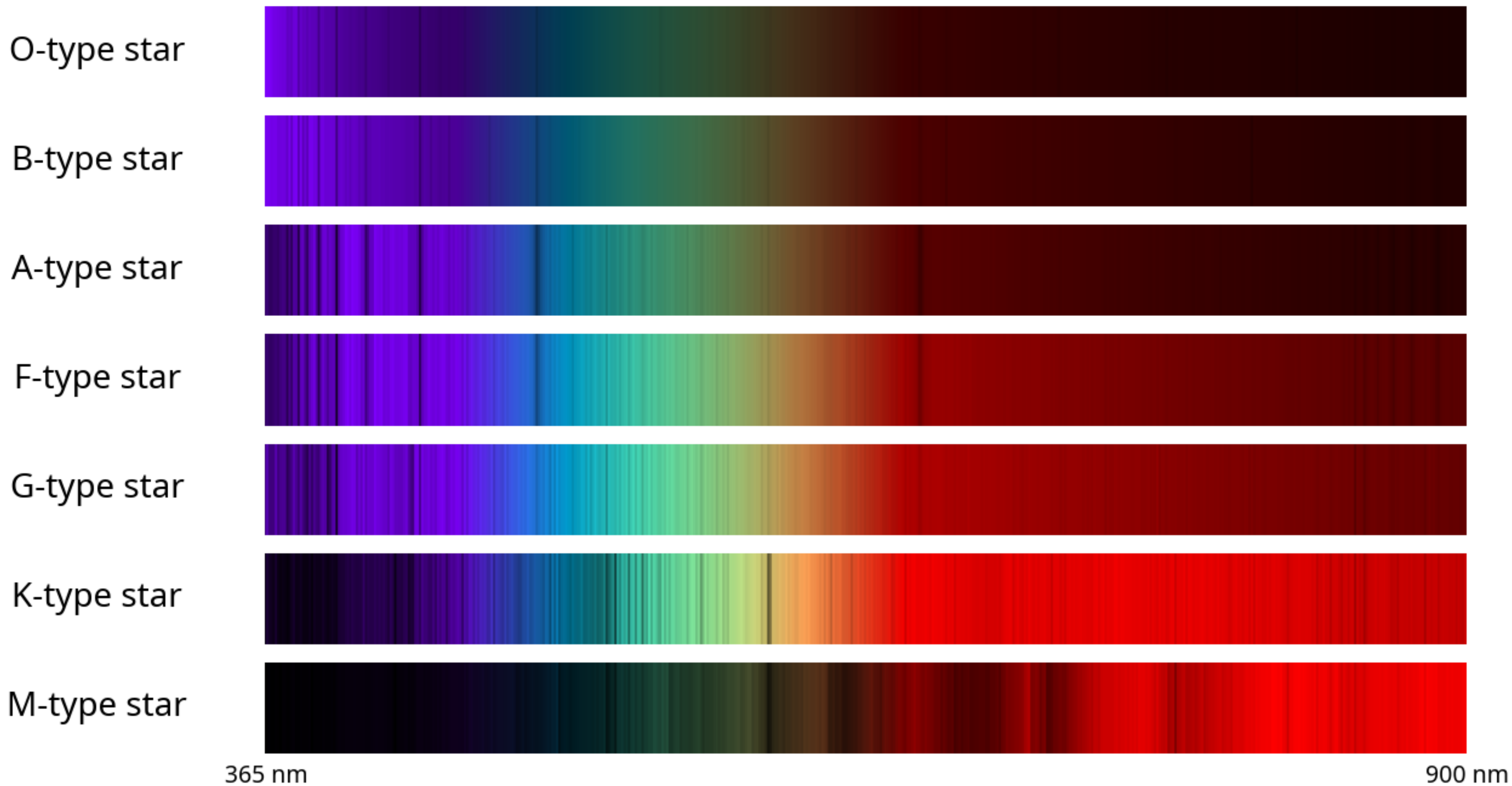
Modern spectrographs can achieve < 1 m/s RV precision



First light spectrum of 51 Pegasi as captured by NEID on the WIYN telescope with blowup of a small section of the spectrum. The right panel shows the light from the star, highly dispersed by NEID, from short wavelengths (bluer colors) to long wavelengths (redder colors). The colors shown, which approximate the true color of the starlight at each part of image, are included for illustrative purposes only. The region in the small white box in the right panel, when expanded (left panel), shows the spectrum of the star (longer dashed lines) and the light from the wavelength calibration source (dots). Deficits of light (dark interruptions) in the stellar spectrum, are due to stellar absorption lines – "fingerprints" of the elements that are present in the atmosphere of the star. By measuring the subtle motion of these features, to bluer or redder wavelengths, astronomers can detect the "wobble" of the star produced in response to its orbiting planet.

Credit: Guðmundur Kári Stefánsson/Princeton University/Penn State/NSF's National Optical-Infrared Astronomy Research Laboratory/KPNO/AURA

Radial Velocity Limitations



Hot Stars (O, B, A)

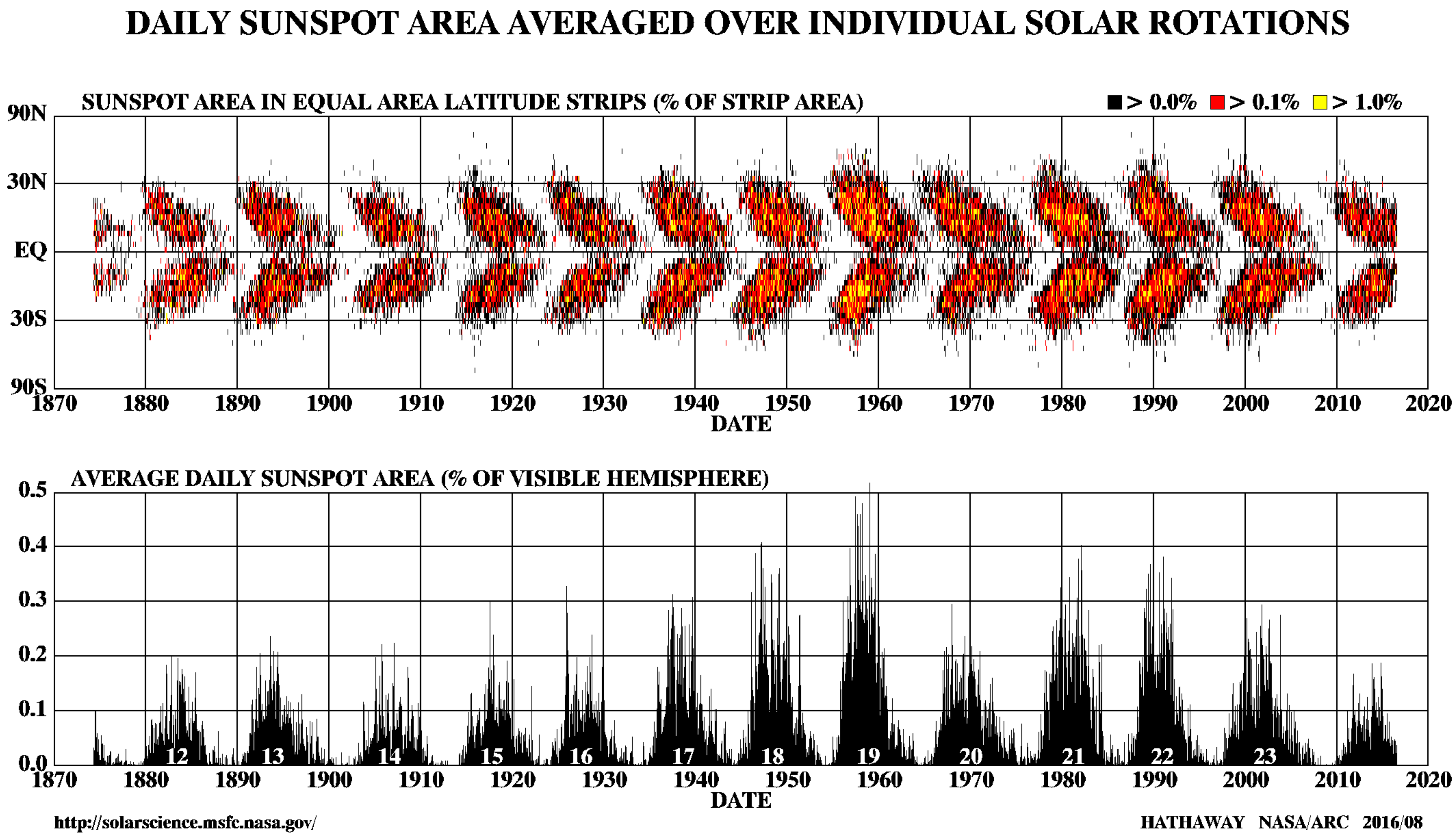
- Higher temperatures → fewer absorption lines in spectra
- Limits RV precision (less Doppler information)

Cool Stars (M)

- Richer in absorption lines (good for RV)
- But fainter → lower signal-to-noise for RV measurements

Know Thy Star, Know Thy Planet

Butterfly plot showing the Solar activity cycle every 11 years



Source of Stellar Activity

Star spots and faculae

- Cool (spots) and hot (faculae) surface regions cause significant RV jitter.
- Amplitudes can reach up to ~100 m/s in active stars.
- Activity indicators (e.g., Ca II H & K lines) used to assess stellar activity.
- Stars (like the Sun) undergo long-term activity cycles, with starspot migration following a "butterfly" pattern (~ m/s RV variation).

Granulation

- Caused by small-scale convection cells at the stellar surface.
- Induces radial velocity (RV) variability of ~ m/s.

Stellar oscillations (P-modes)

- Due to pressure waves propagating inside stars.
- Cause RV variations of ~ m/s.
- Typically mitigated by integrating over oscillation periods (tens of minutes).
  - Timescale scales as $\tau \propto \sqrt{\rho}$ (ρ = density).
  - Lower-mass stars → longer oscillation periods → longer required integrations.