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MA 681
HW5.

1. small sample => t-test.
$\bar{X}_1$ = 2000 yr, $\bar{X}_2$ = 2001 yr
$\bar{X}_1 = 368.9475$, $\bar{X}_2 = 288.9265$, $\hat{\delta} = \bar{X}_1 - \bar{X}_2 = 80.021$

$H_0: \delta = 0$, $H_1: \delta \neq 0$, $\delta = \mu_1 - \mu_2$

$\hat{se} = \sqrt{\frac{s_1^2}{m} + \frac{s_2^2}{n}} = \sqrt{\frac{405.5926^2}{20} + \frac{157.7778^2}{50}} = 93.3978$

$w = \frac{\hat{\delta} - 0}{\hat{se}} = \frac{80.021}{93.3978} = 0.8568$, $df = 21.339$

p-value = 0.4011 > 0.05 $\Rightarrow$ fail to reject H_0.

There is not enough evidence to reject $\mu_1 = \mu_2$, so the average price of a home in Denver did not change from 2000 to 2001.

2. $m = 26$, $n = 26$.
$H_0: \delta = 0$, $H_1: \delta \neq 0$, $\delta = \mu_1 - \mu_2$ => t-test

$\hat{\delta} = \bar{x}_1 - \bar{x}_2 = \bar{X}_{unseeded} - \bar{X}_{seeded} = 164.5885 - 441.9846 = -277.3961$

$\hat{se} = \sqrt{\frac{s_1^2}{m} + \frac{s_2^2}{n}} = \sqrt{\frac{278.43^2}{26} + \frac{650.79^2}{26}} = 138.8199$

$w = \frac{\hat{\delta} - 0}{\hat{se}} = \frac{-277.3961}{138.8199} = -1.9982$, $df = 33.855$

p-value = 0.05377 > 0.05, $\Rightarrow$ fail to reject H_0

There is not enough evidence to reject $\mu_1 = \mu_2$, so the difference in mean precipitation between the two groups is almost zero

3. ① Chlopromazine vs. Placebo

$H_{01}: \delta = 0$, $H_{11}: \delta \neq 0$. $\delta = p_1 - p_2$

$\hat{\delta} = \hat{p}_1 - \hat{p}_2 = \frac{26}{75} - \frac{45}{80} = -0.2158$

$\hat{se} = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{m} + \frac{\hat{p}_2(1-\hat{p}_2)}{n}} = \sqrt{\frac{0.3467 \cdot 0.6533}{75} + \frac{0.5625 \cdot 0.4375}{80}} = 0.078$

$w = \frac{\hat{\delta}}{\hat{se}} = \frac{-0.2158}{0.078} = -2.7644$, p-value$_1$ = 0.0057 < 0.05

$\Rightarrow$ reject H_{01}, so the effectiveness of Clorpromazine and

Placebo is different.

- Bernferroni: $p_1 = 0.0057 < \frac{\alpha}{m} = \frac{0.05}{4} = 0.0125 \Rightarrow$ reject H_{01}

② Dimenhydrinate vs. Placebo

- $H_{02}: \delta = 0$, $H_{12}: \delta \neq 0$

$\hat{\delta} = \hat{p}_1 - \hat{p}_2 = 0.0493$, $\hat{se} = 0.0766$

$W = \frac{\hat{\delta}}{\hat{se}} = 0.6429$, $p\text{-value}_2 = 0.5203 > 0.05$

$\Rightarrow$ fail to reject H_{02}, so the effectiveness of Dimenhydrinate and Placebo is similar.

- Bernferroni: $p_2 = 0.15203 > 0.0125 \Rightarrow$ fail to reject H_{02}

③ Pentobarbital (100mg) vs. Placebo

- $H_{03}: \delta = 0$, $H_{13}: \delta \neq 0$

$\hat{\delta} = -0.0401$, $\hat{se} = 0.0825$, $W = \frac{\hat{\delta}}{\hat{se}} = -0.4864$

$p\text{-value}_3 = 0.6267 > 0.05$

$\Rightarrow$ fail to reject H_0. so the effectiveness of Pentobarbital (100mg) and Placebo is similar

- Bernferroni: $p_3 = 0.627 > 0.0125 \Rightarrow$ fail to reject H_{03}.

④ Pentobarbital (150mg) vs. Placebo

- $H_{04}: \delta = 0$. $H_{14}: \delta \neq 0$.

$\hat{\delta} = -0.1272$. $\hat{se} = 0.0773$. $W = \frac{\hat{\delta}}{\hat{se}} = -1.6456$

$p\text{-value} = 0.0996 > 0.05$

1150 mg

⇒ fail to reject H_0, so the effectiveness of Pentobarbital and Placebo is similar

- Benferroni: $p_4 = 0.0996 > 0.0125 \Rightarrow$ fail to reject H_{04}.

FDR method: BH procedure: find the largest i $\wedge P_i < \frac{i\alpha}{m}$

$P_1 = 0.0057$, $P_2 = 0.5203$, $P_3 = 0.6267$, $P_4 = 0.0996$

$\frac{i\alpha}{m} = \frac{1 \cdot 0.05}{4} = 0.025$, $\frac{2 \cdot 0.05}{4} = 0.025$, $\frac{3 \cdot 0.05}{4} = 0.0375$, $\frac{4 \cdot 0.05}{4} = 0.05$

$i = 1$. Thus we reject only the first hypothesis.

⇒ The result is consistent with what we get with the Benferroni method.

4. ① Wald test: t-test.

$H_0: \delta = 0$, $H_1: \delta \neq 0$, $\delta = p_1 - p_2$

$\hat{\delta} = \hat{p}_1 - \hat{p}_2 = \frac{0.225 + \cdots + 0.247}{8} - \frac{0.209 + \cdots + 0.201}{10} = 0.2319 - 0.2097 = 0.0222$

$\hat{se} = \sqrt{\frac{0.01456^2}{8} + \frac{0.0097^2}{10}} = 0.006$

$W = \frac{\hat{\delta}}{\hat{se}} = \frac{0.0222}{0.006} = 3.7036$, $df = 11.671$

p-value $= 0.003156 < 0.05 \Rightarrow$ reject H_0

Therefore, the mean proportion of three letter words is not the same. So the ten essays were not actually written by Mark Twai.