HW 1

HW1A.8 - Orthogonal Plane

HW1A.8. Orthogonal plane

Consider the vector $\mathbf{v} = \begin{bmatrix} 3 \\ 0 \\ 4 \end{bmatrix}$.

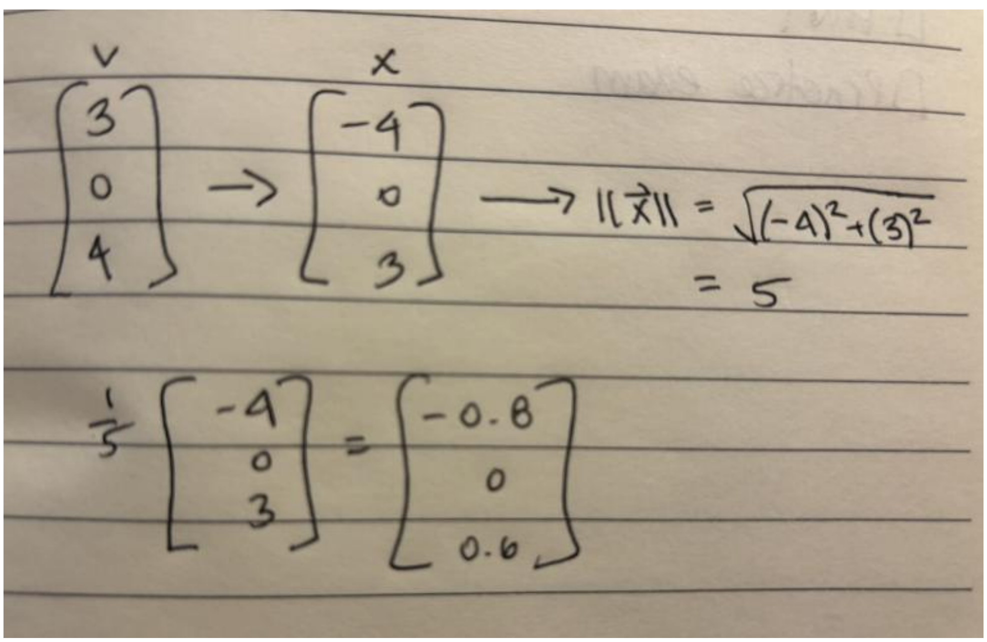
We will find a set of orthogonal unit vectors whose linear combinations will fill the plane that is orthogonal to the vector $\mathbf{v}$.

First, give a unit vector $\mathbf{x}$ that is *perpendicular* to $\mathbf{v}$.

$\mathbf{x} = \begin{bmatrix} -0.8 \\ 0 \\ 0.6 \end{bmatrix}$ ✓100%

Now, give a unit vector $\mathbf{y}$ that is *perpendicular* to *both* $\mathbf{v}$ and $\mathbf{x}$.

$\mathbf{y} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ ✓100%



- Perpendicular = orthogonal

- For perpendicular to both, y vector can be combo of 1s and 0s where the 1s match 0s from vector v and any non-zero digits from vector v would be 0 in vector y

HW1A.14 - Finding q value given the rank of a Matrix

HW1A.14. Finding q value given the rank of a Matrix

Consider the matrix $A = \begin{bmatrix} 2 & 4 & 4 \\ 2 & 6 & q \\ 1 & 4 & 5 \end{bmatrix}$

Find q such that A has rank 2.

$q =$ 7 ✓100%

1A.14

*rank 2 = 2 linear indep. rows or cols

1. $R_2 \to R_2 - R_1$

$$\begin{bmatrix} 2 & 4 & 4 \\ 0 & 2 & q-4 \\ 1 & 4 & 5 \end{bmatrix}$$

2. $R_3 \to R_3 - \frac{1}{2} R_1$

$$\begin{bmatrix} 2 & 4 & 4 \\ 0 & 2 & q-4 \\ 0 & 2 & 3 \end{bmatrix}$$

3. $R_3 \to R_3 - R_2$

$$\begin{bmatrix} 2 & 4 & 4 \\ 0 & 2 & q-4 \\ 0 & 0 & 3-(q-4) \end{bmatrix}$$

4. Simplify:

$$\begin{bmatrix} 2 & 4 & 4 \\ 0 & 2 & q-4 \\ 0 & 0 & 7-q \end{bmatrix}$$

5. For rank = 2, zero out a row

To zero out third row, $q = 7$

because: $7 - q = 0$

HW 2

HW2A.5 - Rank 1 matrix factorization

HW2A.5. Rank 1 matrix factorization

Consider the **rank 1** matrix $A = \begin{bmatrix} 4 & 16 & 16 \\ 14 & 56 & 56 \\ 8 & 32 & 32 \end{bmatrix}$. Give matrices C and R such that $A = CR$.

$C =$	[[4],[14],[8]]	✓100%
$R =$	[1,4,4]	✓100%

- C → take first column of A
  - If two columns of A are not identical, need to take 2 columns of A (instead of one) to satisfy rank 1 condition
- R → find values that when multiplied give A

HW2A.6 - Matrix factorization 3

HW2A.6. Matrix factorization 3

Given the matrices $A = \begin{bmatrix} 18 & 17 & -4 & -9 \\ 10 & 21 & -8 & -5 \\ 16 & 18 & -5 & -8 \end{bmatrix}$ and $C = \begin{bmatrix} -9 & -4 \\ -5 & -8 \\ -8 & -5 \end{bmatrix}$, give the matrix R such that $A = CR$

.

$$R = \begin{bmatrix} -2 & -1 & 0 & 1 \\ 0 & -2 & 1 & 0 \end{bmatrix}$$

✓100%

$$A = \begin{bmatrix} 18 & 17 & -4 & -9 \\ 10 & 21 & -8 & -5 \\ 16 & 18 & -5 & -8 \end{bmatrix} \qquad C = \begin{array}{c} \begin{bmatrix} -9 & -4 \\ -5 & -8 \\ -8 & -5 \end{bmatrix} \\ \quad x_1 \quad\;\; x_2 \end{array}$$

$$R = \begin{bmatrix} a & b & c & d \\ e & f & g & h \end{bmatrix} = \begin{bmatrix} -2 & -1 & 0 & 1 \\ 0 & -2 & 1 & 0 \end{bmatrix}$$

$x_1 a + x_2 e \Rightarrow$ creates col #1 of A

- Linear combinations of C with R that result in A

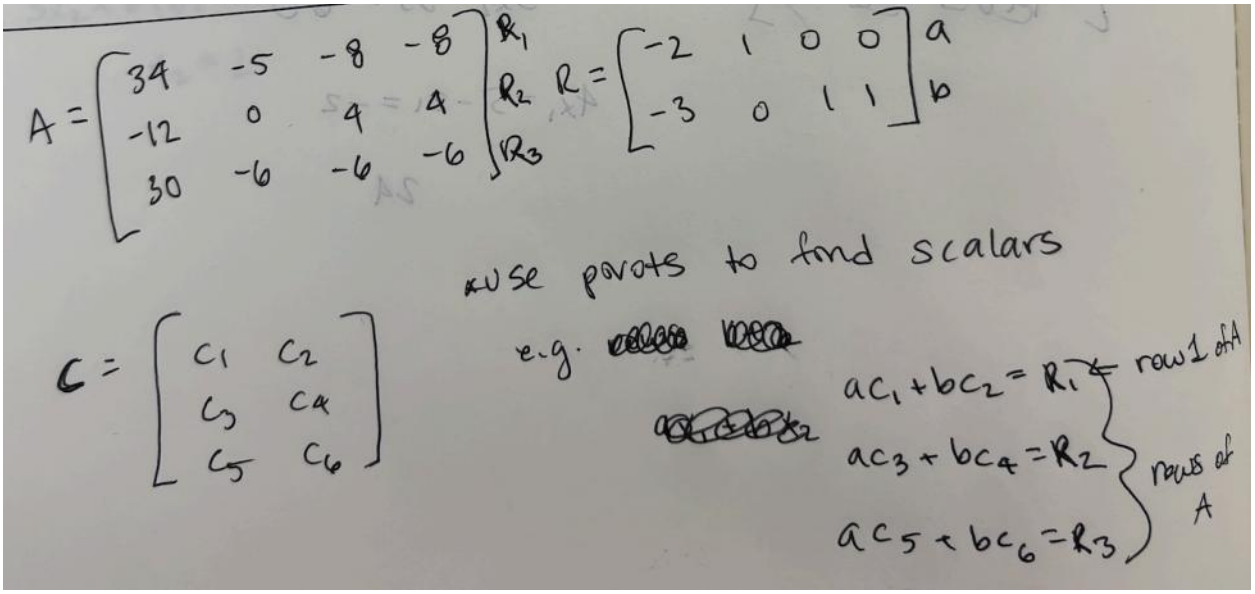
HW2A.7. Matrix factorization 5

HW2A.7. Matrix factorization 5

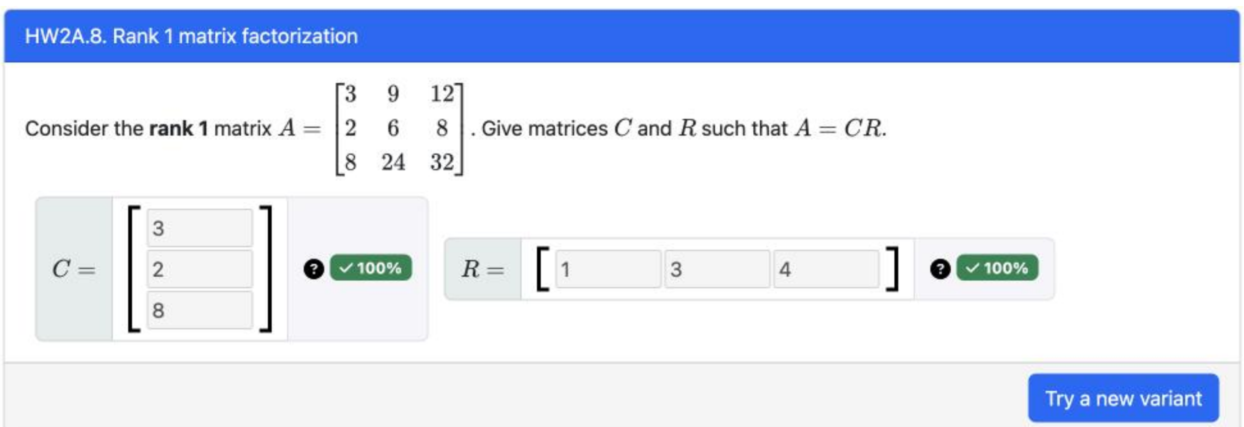
Given the matrices $A = \begin{bmatrix} 34 & -5 & -8 & -8 \\ -12 & 0 & 4 & 4 \\ 30 & -6 & -6 & -6 \end{bmatrix}$ and $R = \begin{bmatrix} -2 & 1 & 0 & 0 \\ -3 & 0 & 1 & 1 \end{bmatrix}$, give the matrix C such that $A = CR$.

$$C = \begin{bmatrix} -5 & -8 \\ 0 & 4 \\ -6 & -6 \end{bmatrix}$$

? ✓ 100%



HW2A.8 - Rank 1 matrix factorization


HW2A.8. Rank 1 matrix factorization

Consider the **rank 1** matrix $A = \begin{bmatrix} 3 & 9 & 12 \\ 2 & 6 & 8 \\ 8 & 24 & 32 \end{bmatrix}$. Give matrices C and R such that $A = CR$.

$C = \begin{bmatrix} 3 \\ 2 \\ 8 \end{bmatrix}$ ✓ 100%

$R = \begin{bmatrix} 1 & 3 & 4 \end{bmatrix}$ ✓ 100%

Try a new variant

- For C: Take smallest column of A
- For R: Find which multiples of C result in columns of A

HW2A.9 - Matrix construction 4

HW2A.9. Matrix construction 4

Give a 4×4 matrix A of rank 2 and matrices C and R such that $A = CR$.

$$A = \begin{bmatrix} 11 & 14 & 17 & 20 \\ 23 & 30 & 37 & 44 \\ 35 & 46 & 57 & 68 \\ 47 & 62 & 77 & 92 \end{bmatrix}$$ ✓100%

$$C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \\ 7 & 8 \end{bmatrix}$$ ✓100%

$$R = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{bmatrix}$$ ✓100%

- Need linearly independent cols and rows for C and R respectively
    - Use 1s and 0s for easy of multiplication
- To find A, calculate CR

HW2A.12 - Transform into upper triangular matrix

HW2A.12. Transform into upper triangular matrix

Given the matrix $A = \begin{bmatrix} 1 & 5 & 8 \\ -1 & -1 & 0 \\ 2 & 10 & 18 \end{bmatrix}$, transform A into an upper triangular matrix B using elimination.

$$B = \begin{bmatrix} 1 & 5 & 8 \\ 0 & 4 & 8 \\ 0 & 0 & 2 \end{bmatrix}$$ ✓100%

$$\begin{bmatrix} 1 & 5 & 8 \\ -1 & -1 & 0 \\ 2 & 10 & 18 \end{bmatrix}$$

1. $R_2 \to R_1 + R_2$ 2. $R_3 \to -2R_1 + R_3$

$$\begin{bmatrix} 1 & 5 & 8 \\ 0 & 4 & 8 \\ 2 & 10 & 18 \end{bmatrix} \qquad \begin{bmatrix} 1 & 5 & 8 \\ 0 & 4 & 8 \\ 0 & 0 & 2 \end{bmatrix}$$

HW2A.13 - Solving a linear system

HW2A.13. Solving a linear system

Given the matrix $A = \begin{bmatrix} 1 & 6 & 3 \\ 2 & 14 & 12 \\ 5 & 34 & 28 \end{bmatrix}$ and vector $\mathbf{b} = \begin{bmatrix} -33 \\ -62 \\ -154 \end{bmatrix}$, give the upper triangular matrix U obtained from A via elimination and the vector $\mathbf{x}$ such that $A\mathbf{x} = \mathbf{b}$.

$U = \begin{bmatrix} 1 & 6 & 3 \\ 0 & 2 & 6 \\ 0 & 0 & 1 \end{bmatrix}$ ✓ 100%

$\mathbf{x} = \begin{bmatrix} 0 \\ -7 \\ 3 \end{bmatrix}$ ✓ 100%

$$\begin{bmatrix} 1 & 6 & 3 \\ 2 & 14 & 12 \\ 5 & 34 & 28 \end{bmatrix}$$

1. $R_2 \to -2R_1 + R_2$

$$\begin{bmatrix} 1 & 6 & 3 \\ 0 & 2 & 6 \\ 5 & 34 & 28 \end{bmatrix}$$

2. $R_3 \to -5R_1 + R_3$

$$\begin{bmatrix} 1 & 6 & 3 \\ 0 & 2 & 6 \\ 0 & 4 & 13 \end{bmatrix}$$

3. $R_3 \to -2R_2 + R_3$

$$\begin{bmatrix} 1 & 6 & 3 \\ 0 & 2 & 6 \\ 0 & 0 & 1 \end{bmatrix}$$

4. transform b

$$\begin{bmatrix} -33 \\ -62 \\ -154 \end{bmatrix} \to \begin{bmatrix} -33 \\ 4 \\ -154 \end{bmatrix} \to \begin{bmatrix} -33 \\ 4 \\ 3 \end{bmatrix}$$

5. Solve $Ax=b$

$\boxed{x_3 = 3}$

$2x_2 + 18 = 4$

$\boxed{x_2 = -7}$

$-33x_1 + 6(-7) + 3(3) = -33$

$-33x_1 - 33 = -33$

$x_1 = 0$

HW2A.14 - Coefficient that forces row exchange and result in missing pivot

HW2A.14. Coefficient that forces row exchange and result in missing pivot

Consider the system of equations with an unknown coefficient m (watch out for the order of the variables):

$$\begin{aligned} 4x_1 + 4x_2 + 3x_3 &= 0 \\ 8x_3 + 12x_1 + mx_2 &= 3 \\ 3x_2 - x_3 &= 2 \end{aligned}$$

Give a value m_r for m that forces a row exchange.

$m_r =$ 12 ✓100%

Give a value m_s for m that makes transformation into an upper triangular matrix impossible.

$m_s =$ 15 ✓100%

Mr = (R2x1/R1x1) x R1x2
Ms = Mr + R3x2

For the above examples:
(12/3)*4 = 12
12+ 3 = 15

HW2A.15. Elimination roadblocks

HW2A.15. Elimination roadblocks

Consider the following system of equations:

$$ax + 18y = -72$$
$$2x + 6y = 24$$

Give a number a_p for a such that this system cannot be transformed into an upper triangular matrix.

$a_p =$ 6 ✓100%

Give a number a_t for a such that this system can be transformed into an upper triangular matrix via elimination, but after performing a row exchange.

$a_t =$ 0 ✓100%

Using a_t from above for a, solve the resulting system for x and y after performing the necessary row exchange.

$x =$ 24 ✓100%

$y =$ -4 ✓100%

- A is scalar of the other equation
  - 18/6 = 3
  - 3 * 2 = 6
- At always = 0
- Set a=0 and solve y
  - -72/18 = -4
- Plug y into second equation and find x
  - 2x-24=24
  - x=24

HW2A.19 - Elimination matrix inverses

HW2A.19. Elimination matrix inverses

Consider the 4 by 4 matrix $E_{2,1}$ that subtracts -8 times row 1 from row 2. Give the **inverse** matrix for this row operation.

Note. For this question, entries left blank will be interpreted as 0.

$$\mathbf{E}_{2,1} = \begin{bmatrix} 1 & & & \\ -8 & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix}$$

✓ 100%

- Inverse means positive becomes negative and vice versa

HW2A.21 - Row Operation Matrices

HW2A.21. Row Operation Matrices

Let A be a 3×3 matrix.

Give the matrix E_a that adds row 1 of A to row 2 of A and *at the same time* adds row 2 of A to row 1 of A.

$$E_a = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

✓ 100%

Give the matrix E_b that adds row 1 of A to row 2 of A *and then* adds row 2 to row 1 of the result.

$$E_b = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

✓ 100%

HW 3

HW3A.1 - Solve with LU

- First swap A so smallest numbers are top row and largest numbers bottom row; generally focus on first column of A
- Adjust P according to the row swaps
- L = E-1 (L is inverse of E)

HW3A.2 - Solving linear systems 4

HW3A.2. Solving linear systems 4

Suppose the matrix A has an LU factorization into $L = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ and

$U = \begin{bmatrix} 4 & 1 & 6 \\ 0 & 6 & 5 \\ 0 & 0 & 4 \end{bmatrix}$.

Given $\mathbf{b} = \begin{bmatrix} 0 \\ -11 \\ 9 \end{bmatrix}$, solve $A\mathbf{x} = \mathbf{b}$.

$\mathbf{x} = \begin{bmatrix} -6 \\ -6 \\ 5 \end{bmatrix}$? ✓ 100%

$$\begin{bmatrix} 4 & 1 & 6 \\ -4 & 5 & -1 \\ 0 & 6 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{matrix} 0 \\ -11 \\ 9 \end{matrix}$$

1. $R_2 \rightarrow R_1 + R_2$

$$\begin{bmatrix} 4 & 1 & 6 \\ 0 & 6 & 5 \\ 0 & 6 & 9 \end{bmatrix}$$

2. $R_3 \rightarrow -R_2 + R_3$

$$\begin{bmatrix} 4 & 1 & 6 \\ 0 & 6 & 5 \\ 0 & 0 & 4 \end{bmatrix}$$

3. transform b

$$\begin{bmatrix} 0 \\ -11 \\ 9 \end{bmatrix} \rightarrow \begin{bmatrix} 0 \\ -11 \\ 20 \end{bmatrix}$$

$$4x_1 + x_2 + 6x_3 = 0$$
$$6x_2 + 5x_3 = -11$$
$$4x_3 = 20$$

$$x = \begin{bmatrix} -6 \\ -6 \\ 5 \end{bmatrix}$$

$$6x_2 + 25 = -11$$
$$6x_2 = -36$$

$$4x_1 - 6 + 30 = 0$$
$$4x_1 = -24$$

HW3A.4 - L and elimination

HW3A.4. L and elimination

Given the matrix $L = \begin{bmatrix} 1 & 0 & 0 \\ -5 & 1 & 0 \\ -68 & 12 & 1 \end{bmatrix}$ such that $A = LU$ for some invertible matrix A, give the elimination matrix $E = L^{-1}$.

$E = \begin{bmatrix} 1 & 0 & 0 \\ 5 & 1 & 0 \\ 68 & -12 & 1 \end{bmatrix}$? × 0%

** L31 should = 8

$L = \begin{bmatrix} 1 & 0 & 0 \\ -5 & 1 & 0 \\ -68 & 12 & 1 \end{bmatrix}$

$(L21)(L32) - L31 = x$

$(-5)(-12) - 68 = 8$

$$\begin{matrix} 1 & 0 & 0 \\ 5 & 1 & 0 \\ x & -12 & 1 \end{matrix}$$

HW3A.8 - Permutation inverses 6

HW3A.8. Permutation inverses 6

Consider the 5 by 5 matrix $\mathbf{P}$ that exchanges columns 3 and 5, then columns 5 and 1.

Give the **inverse** of $\mathbf{P}$.

Note. For this question, entries left blank will be interpreted as 0.

$$\mathbf{P}^{-1} = \begin{bmatrix} & & 1 & & \\ & 1 & & & \\ & & & & 1 \\ & & & 1 & \\ 1 & & & & \end{bmatrix}$$

? ✓100%

- The trick is for inverse P, if it says columns swap rows, and vice versa

HW3A.13 - Planes and subspaces

HW3A.13. Planes and subspaces

Let P be the plane in $\mathbb{R}^3$ with equation $x - 3y - z - 4 = 0$. Show that P is not a subspace by finding two different vectors $\mathbf{v}$ and $\mathbf{w}$ in P such that $\mathbf{v} + \mathbf{w}$ is not in P.

$\mathbf{v} = \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix}$? ✓100% $\mathbf{w} = \begin{bmatrix} 8 \\ 1 \\ 1 \end{bmatrix}$? ✓100%

Another reason P is not a subspace is that $\mathbf{0}$ is not in P. Describe a plane P' using an equation in x, y, z that is *parallel* to P but *is* a subspace.

$0 =$ x-3y-z ? ✓100%

$x-3y-z-4=0$

$$v=\begin{bmatrix}4\\0\\0\end{bmatrix}$$

$w \to$ set $y=1$ & $z=1$ and solve for x

equation: drop the constant

HW3A.15 - Column spaces and solvability

HW3A.15. Column spaces and solvability

Consider the matrix $A = \begin{bmatrix} -4 & 1 & -4 & 1 & -12 \\ -8 & 2 & -8 & 2 & -24 \\ 0 & 0 & 0 & 0 & 0 \\ -16 & 4 & -16 & 4 & -48 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

Suppose we add an extra $\mathbf{b}$ to A. Does the column space of the new matrix $B = [A \quad \mathbf{b}]$ get larger?

Give an example of a vector $\mathbf{b}_>$ as a single column vector such that B has a *larger* column space than A.

$\mathbf{b}_> =$ [[1],[0],[0],[0],[0]] ✓ 100%

Give an example of a non-zero vector $\mathbf{b}_=$ as a single column vector that is not already a column of A such that B has the *same* column space as A.

$\mathbf{b}_= =$ [[-4],[-8],[0],[-16],[0]] ✓ 100%

- b> should be a linearly independent column (can always do 1 with 0s)
- b= should be linearly dependent, so you can always copy one of the existing columns

HW 4

HW4A.1. Special solutions to Rx = 0

HW4A.1. Special solutions to Rx = 0

Give a 2×4 matrix R that results in special solutions $\mathbf{s}_1 = \begin{bmatrix} 18 \\ -5 \\ 1 \\ 0 \end{bmatrix}$ and $\mathbf{s}_2 = \begin{bmatrix} 16 \\ -13 \\ 0 \\ 1 \end{bmatrix}$ for $R\mathbf{x} = \mathbf{0}$.

$R = \begin{bmatrix} 1 & 0 & -18 & -16 \\ 0 & 1 & 5 & 13 \end{bmatrix}$ ✓ 100%

free vars.

$$A = \begin{bmatrix} a & b & c & d \\ e & f & g & h \end{bmatrix} \quad \begin{bmatrix} 18 \\ -5 \\ 1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 16 \\ -13 \\ 0 \\ 1 \end{bmatrix}$$

$18a - 5b + c = 0 \qquad 18e - 5f + g = 0$

$16a - 13b + d = 0 \qquad 16e - 13f + h = 0$

set $a = 1, b = 0 \qquad$ set $e = 0, f = 1$

$18 + c = 0 \qquad -5 + g = 0$

$c = -18 \qquad g = 5$

$16 + d = 0 \qquad -13 + h = 0$

$d = -16 \qquad h = 13$

- A trick to solving this faster is setting 0,1 and 1,0 to the free variable columns, then flipping the signs for the other variables

HW4A.2. Compute RREF

HW4A.2. Compute RREF

Let $A = \begin{bmatrix} -12 & -21 & 75 & -120 \\ 6 & 12 & -42 & 66 \end{bmatrix}$. Compute its row-reduced echelon form R.

$R = \begin{bmatrix} 1 & 0 & -1 & 3 \\ 0 & 1 & -3 & 4 \end{bmatrix}$ ✓ 100%

Try a new variant

6	12	-42	66
-12	-21	75	-120
1	2	-7	11
-12	-21	75	-120
1	2	-7	11
0	3	-9	12
1	2	-7	11
0	1	-3	4
1	0	-1	3
0	1	-3	4

HW4A.3. Solving Rx = 0

HW4A.3. Solving Rx = 0

Let $R = \begin{bmatrix} 1 & -6 & 2 & 6 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ be a matrix in row-reduced echelon form.

Find the special solutions for $R\mathbf{x} = \mathbf{0}$. Submit the special solutions as columns of a single matrix N.

$N =$ [[6,-2,-6],[1,0,0],[0,1,0],[0,0,1],[0,0,0]] ✓ 100%

$$x_1 - 6x_2 + 2x_3 + 6x_4 = 0$$
$$x_5 = 0$$

x_2	x_3	x_4
1	0	0
0	1	0
0	0	1

3 special solutions

$$\begin{bmatrix} 6 & -2 & -6 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

HW4A.5. Special Solutions

Suppose the only special solution of $A\mathbf{x} = \mathbf{0}$ for a 3×4 matrix A is $\begin{bmatrix} -4 \\ 6 \\ 7 \\ 1 \end{bmatrix}$.

What is the *rank* of A?

3 ? ✓100%

What is the row reduced echelon form of A?

$$\begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -6 \\ 0 & 0 & 1 & -7 \end{bmatrix}$$

? ✓100%

- rank(A) = # of pivot columns
- Last column is all the values flipped (i.e., neg/pos) except for the last value

HW4A.6. Pivot and nullity

HW4A.6. Pivot and nullity

Consider a 4×5 matrix A with 0 pivot columns.

What is the minimum number of vectors needed to span the null space of A?

5 ✓100%

Give an example of A.

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

✓100%

- Except for the pivot columns, you can do all zero vectors

HW4A.7. Row-reduced echelon forms 4

HW4A.7. Row-reduced echelon forms 4

Consider the matrix $R = \begin{bmatrix} 1 & 0 & 0 & 17 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 5 \end{bmatrix}$, which is in row-reduced echelon form.

Show that two different matrices other than R can arrive at the same row-reduced echelon form matrix by giving two different matrices that result in R after performing elimination.

$$A = \begin{bmatrix} 2 & 0 & 0 & 34 \\ 0 & 2 & 0 & 16 \\ 0 & 0 & 2 & 10 \end{bmatrix}$$

✓100%

$$B = \begin{bmatrix} 3 & 0 & 0 & 51 \\ 0 & 3 & 0 & 24 \\ 0 & 0 & 3 & 15 \end{bmatrix}$$

✓100%

- Just scale the original matrix…douh

HW4A.10. Filling 1s given pivot columns

HW4A.10. Filling 1s given pivot columns

Note. For this question, entries left blank will be interpreted as 0.

Put as many 1's as possible in a 5×8 upper triangular matrix U whose pivot columns are 2, 3, 5.

Correct answer

Note. For this question, entries left blank will be interpreted as 0.

$$U = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

HW4A.12. Solvable systems 3

HW4A.12. Solvable systems 3

Consider the system of linear equations $\begin{bmatrix} 5 & 4 & 4 \\ -10 & -8 & -8 \\ 10 & 8 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$.

You will give conditions on b_1, b_2, b_3 for the system to be solvable.

Give a condition on b_1 in terms of b_2 for the above system to be solved.

$b_1 =$ -b2/2 ? ✓100%

Give a condition on b_1 in terms of b_3 for the above system to be solved.

$b_1 =$ b3/2 ? ✓100%

- Find equation to get R1x1

HW4A.13. Solving Ax = b completely

HW4A.13. Solving Ax = b completely

Let $A = \begin{bmatrix} 3 & 2 & -13 & -9 \\ 6 & 7 & -14 & 0 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} -33 \\ -75 \end{bmatrix}$. Consider the linear equation $A\mathbf{x} = \mathbf{b}$.

Give the particular solution $\mathbf{x}_p$ to this equation.

$$\mathbf{x}_p = \begin{bmatrix} -9 \\ -3 \\ 0 \\ 0 \end{bmatrix}$$

✓ 100%

Find the special solutions $\mathbf{x}_n$ for this equation. Submit the special solutions as columns of a single matrix.

$\mathbf{x}_n =$ [[7,7],[-4,-6],[1,0],[0,1]]

✓ 100%

				b
3	2	-13	-9	-33
6	7	-14	0	-75
1	0.6666667	-4.3333333	-3	-11
6	7	-14	0	-75
1	0.6666667	-4.3333333	-3	-11
0	3	12	18	-9
1	0.6666667	-4.3333333	-3	-11
0	1	4	6	-3
1	0	-7	-7	-9
0	1	4	6	-3

$$\begin{array}{cccc|c} & & A & & \vec{b} \end{array}$$
$$\left[\begin{array}{cccc|c} 3 & 2 & -13 & -9 & -33 \\ 6 & 7 & -14 & 0 & -75 \end{array}\right]$$

$$\begin{array}{ccccc} 1 & 0 & -7 & -7 & -9 \\ 0 & 1 & 4 & 6 & -3 \end{array}$$

free vars

$$x_1 - 7x_3 - 7x_4 = -9$$
$$x_2 + 4x_3 + 6x_4 = -3$$

$$x_1 = -9$$
$$x_2 = -3$$

- First row reduce (RREF)
- Particular solution is where free variables both = 0

HW4A.14. Computing rank

Let $A = \begin{bmatrix} 18 & 54 & 0 & -3 \\ -5 & -15 & 0 & 1 \\ -8 & -24 & 1 & 0 \end{bmatrix}$.

Compute the rank of A.

$\text{rank}(A) =$ 3 ✓ 100%

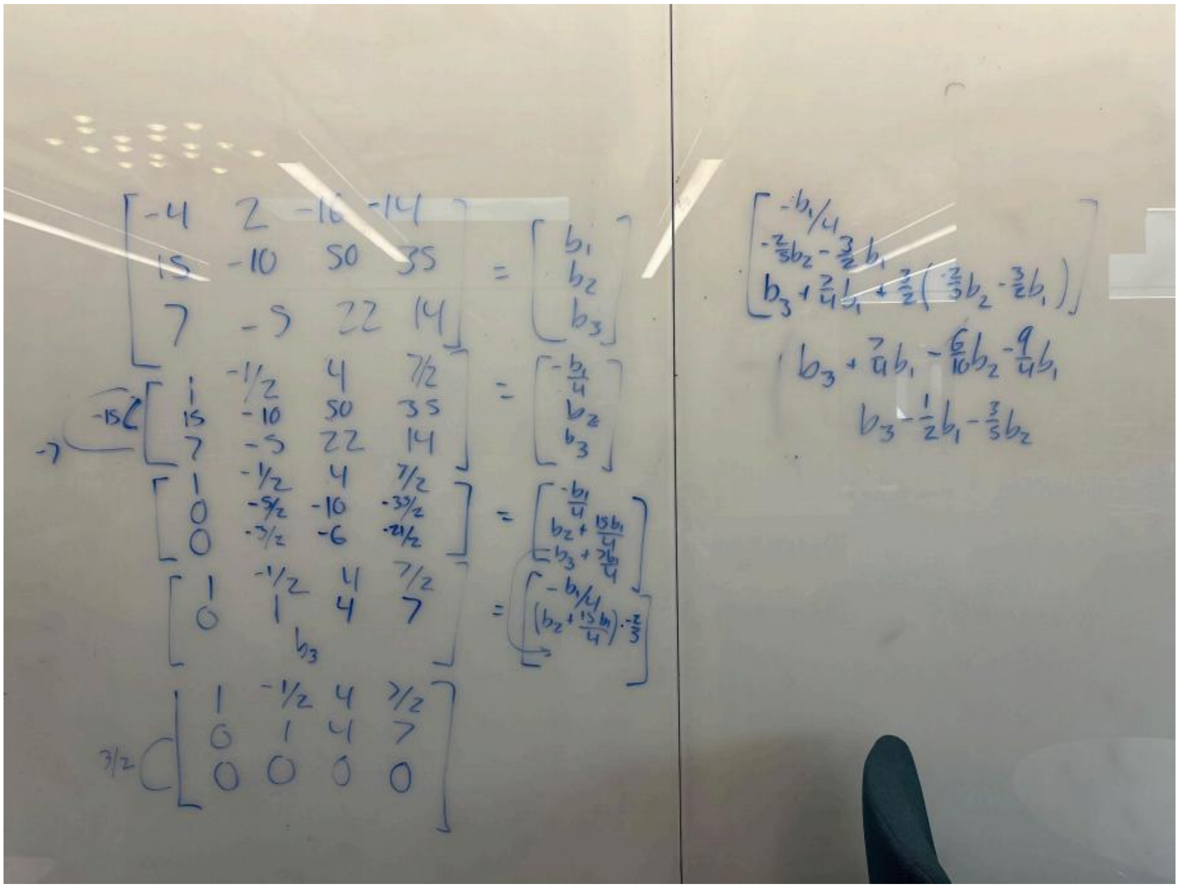
- min(# of LI rows or cols)
- Divide the rows by scalars to reduce the #s → helps find linear dependence
    - You can divide each row by a different scalar; don't need to apply scalar operations to all rows

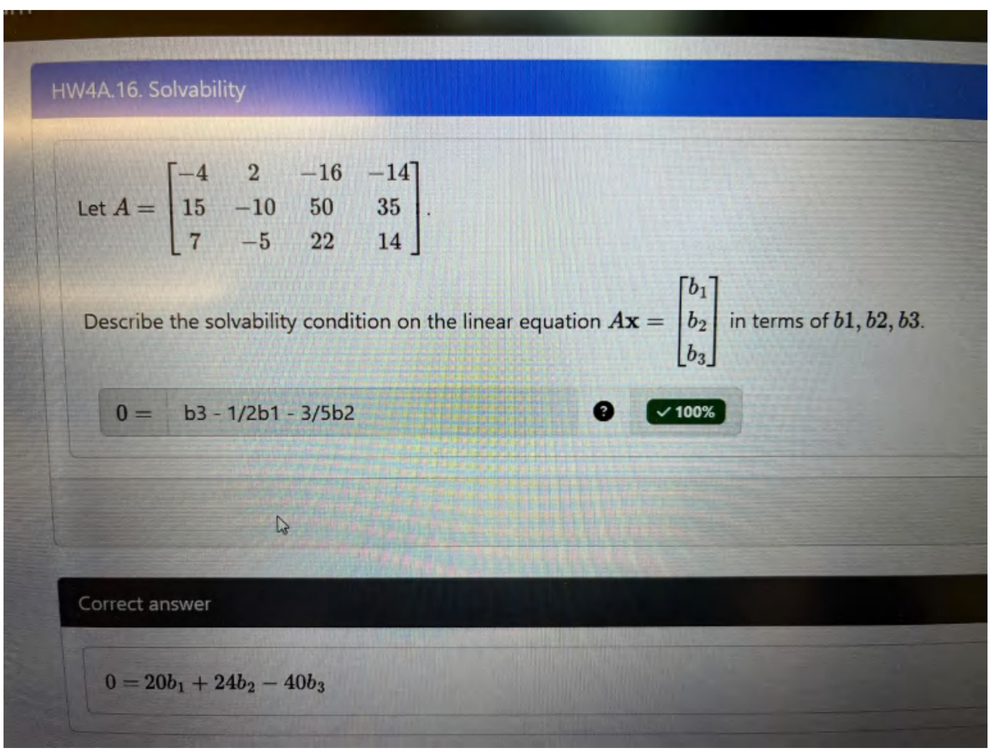
HW4A.15. Solving Rx = b

- For particular solution, set all free variables = 0
- For special solutions, use a combinations of 0s and 1s
- # special solutions = # free variables

HW4A.16 Solvability

- Do basic row reduction (NOT RREF) and record what you do to each row separately. Get it to b3 at the bottom.
- The biggest challenge here is keeping everything straight




HW4A.16. Solvability
Let $A = \begin{bmatrix} -4 & 2 & -16 & -14 \\ 15 & -10 & 50 & 35 \\ 7 & -5 & 22 & 14 \end{bmatrix}$.
Describe the solvability condition on the linear equation $A\mathbf{x} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ in terms of $b1, b2, b3$.
0 = b3 - 1/2b1 - 3/5b2
✓ 100%
Correct answer
$0 = 20b_1 + 24b_2 - 40b_3$

HW4A.17. Solving Ax = b with some solutions

HW4A.17. Solving Ax = b with some solutions

Let $A = \begin{bmatrix} 3 & 27 & -18 & 6 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} -57 \\ -16 \\ 7 \end{bmatrix}$. Consider the linear equation $A\mathbf{x} = \mathbf{b}$.

Solve for $\mathbf{x}$ and give two different vectors $\mathbf{x}_1$ and $\mathbf{x}_2$ that satisfy the equation.

$\mathbf{x}_1 = \begin{bmatrix} -3 \\ 0 \\ 5 \\ 7 \end{bmatrix}$? ✓ 100%

$\mathbf{x}_2 = \begin{bmatrix} -12 \\ 1 \\ 5 \\ 7 \end{bmatrix}$? ✓ 100%

				b
3	27	-18	6	-57
0	0	1	-3	-16
0	0	0	1	7
divide top row by 3				
1	9	-6	2	-19
0	0	1	-3	-16
0	0	0	1	7
zero out R1,3				
1	9	0	-16	-115
0	0	1	-3	-16
0	0	0	1	7
zero out last column above 1s				
1	9	0	0	-3
0	0	1	0	5
0	0	0	1	7

- Steps are:
  - Row reduce
  - Plug in values for free variables and solve

HW 5

HW5A.1. Dimension

HW5A.1. Dimension

Consider the matrix $A = \begin{bmatrix} 1 & 0 & 2 & -5 \\ 0 & 1 & 6 & -7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

What is the dimension of the column space of A?

$\dim \mathbf{C}(A) =$ 2 ✓100%

What is the dimension of the row space of A?

$\dim \mathbf{C}(A^T) =$ 2 ✓100%

What is the dimension of the null space of A?

$\dim \mathbf{N}(A) =$ 2 ✓100%

- dimC(A) = # of pivot cols
- dimC(A) = dimC(AT)
- dimN(A) = total cols - # pivot cols

HW5A.2. Rectangular matrix properties

HW5A.2. Rectangular matrix properties

Consider a 4×5 matrix A of maximal rank.

The largest possible rank of A is

4 ✓100%

Then for echelon form matrix U and row-reduced echelon form matrix R,

- (a) there is a pivot in every row ✓
- (b) there is a pivot in every column
- (c) there is a pivot for every free variable
- (d) there is a pivot for every special solution

✓100%

For any vector $\mathbf{b}$, the solution to $A\mathbf{x} = \mathbf{b}$:

- (a) may exist
- (b) is unique
- (c) does not exist
- (d) always exists ✓

✓100%

The column space of A is

- (a) a plane
- (b) a line
- (c) a 5D space
- (d) a 4D space ✓

✓100%

Give an example of A.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix}$$

? ✓ 100%

HW5A.4. Basis for a plane

HW5A.4. Basis for a plane

Consider the plane $-6x + 3y + 8z = 0$. Give a basis for the vectors in this plane as the columns of a single matrix.

[[0.5, 1.33],[1,0],[0,1]]

? ✓ 100%

Try a new variant

- Set y=1 and z=0 and solve for x
- Set y=0 and z=1 and solve for x

HW5A.5. Span dimension

HW5A.5. Span dimension

Consider the vectors $\begin{bmatrix} 0 \\ -14 \\ 2 \\ -6 \\ -8 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -7 \\ 1 \\ -3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -2 \end{bmatrix}.$

What is the dimension of the vector space spanned by these vectors?

3 ? ✓100%

The span of the vectors is a subspace of which vector space?

$\mathbb{R}^n, n =$ 5 ? ✓100%

- Dimension = rank
- Rn, n = # rows

HW5A.6. Span basis

HW5A.6. Span basis

Consider the vectors $\begin{bmatrix} -2 \\ 3 \\ 1 \\ -12 \\ 3 \end{bmatrix}, \begin{bmatrix} -6 \\ 9 \\ 3 \\ -36 \\ 9 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$

Give a basis for the subspace spanned by these vectors as the columns of a single matrix. Fill in any extra columns as the zero vector.

$$\begin{bmatrix} 1 & -2 & 0 \\ 0 & 3 & 0 \\ 0 & 1 & 0 \\ 0 & -12 & 0 \\ 0 & 3 & 0 \end{bmatrix}$$

? ✓100%

- Basis = linearly independent columns
- Set all other columns = 0

HW5A.8. Matrix subspace bases: A and R

- C(A) = pivot cols corresponding from R
- C(R) = pivot cols
- C(AT) = C(RT) = non-zero rows of R
- N(A) = N(R) = special solutions

HW5A.11. Subspace dimensions

HW5A.11. Subspace dimensions

Consider the matrix $A = \begin{bmatrix} 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & -28 \\ 0 & 0 & 9 & 45 \\ 1 & 0 & 0 & -7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

What is the dimension of the column space of A?

$\dim \mathbf{C}(A) =$ 3 ✓100%

What is the dimension of the row space of A?

$\dim \mathbf{C}(A^T) =$ 3 ✓100%

What is the dimension of the null space of A?

$\dim \mathbf{N}(A) =$ 1 ✓100%

What is the dimension of the left null space of A?

$\dim \mathbf{N}(A^T) =$ 4 ✓100%

What is the rank of A?

$\operatorname{rank} A =$ 3 ✓100%

- C(A) = C(AT) = # LI rows or cols (take the min)
- N(A) = # cols - C(A)
- N(AT) = # rows - C(AT)
- Rank A = C(A)

HW5A.13. Vectors in space

HW5A.13. Vectors in space

Consider the matrix $A = \begin{bmatrix} 1 & -7 & 0 & 0 & -5 & -7 & 7 & -3 & 7 \\ 0 & 0 & 1 & 0 & -5 & 3 & -8 & -4 & -2 \\ 0 & 0 & 0 & 1 & -4 & 4 & 7 & 3 & 7 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

Give a nonzero vector that belongs to the column space of A and is not a column of A.

[[1],[0],[0],[0],[0]] ✓100%

Give a nonzero vector that belongs to the row space of A and is not a row of A.

[[0],[0],[0],[2],[-8],[8],[14],[6],[14]] ✓100%

Give a nonzero vector that belongs to the null space of A and is not a special solution.

[[14],[2],[0],[0],[0],[0],[0],[0],[0]] ✓100%

Give a nonzero vector that belongs to the left null space of A and is not a special solution.

[[0],[0],[0],[0],[1]] ✓100%

1) First value always 1 followed by 0s
2) Scale any row by a constant
3) Take a pivot column and non-pivot column and find values that would make the two columns equal to 0
    a) E.g. if we multiplied col 2 by 2, we get -14, so col 1 would need to be multiplied by 14 to get 0 → 14 - 14 = 0
4) Always 0s ending with the last row as 1

HW 6

HW6A.1. Projecting a vector onto a line

HW6A.1. Projecting a vector onto a line

Let $\mathbf{b}_1 = \begin{bmatrix} -8 \\ 8 \\ -5 \end{bmatrix}$ and $\mathbf{a}_1 = \begin{bmatrix} -2 \\ -4 \\ -5 \end{bmatrix}$.

Python

```
import numpy as np

b1 = np.array([[-8], [ 8], [-5]])
a1 = np.array([[-2], [-4], [-5]])
```

copy this text

Compute the projection $\mathbf{p}_1$ of $\mathbf{b}_1$ on the line passing through $\mathbf{a}_1$.

$\mathbf{p}_1 = \begin{bmatrix} -0.4 \\ -0.8 \\ -1 \end{bmatrix}$? ✓ 100%

Give error vector $\mathbf{e}_1$ for this projection.

$\mathbf{e}_1 = \begin{bmatrix} -7.6 \\ 8.8 \\ -4 \end{bmatrix}$? ✓ 100%

Give the length $||\mathbf{e}_1||$ of the error for this projection.

$||\mathbf{e}_1|| =$ 12.3 ? ✓ 100%

$\boxed{\text{HW 6}}$

6A.3 Same as 6A.1

6A.1 $p_1 = \dfrac{b_1 \cdot a_1}{a_1 \cdot a_1} a_1$ $\quad e_1 = b_1 - p_1$

$\|e_1\| = \sqrt{e_{1,1}^2 + e_{1,2}^2 + e_{1,3}^2}$

$c = \dfrac{b_1 \cdot a_1}{a_1 \cdot a_1}$

$p_1 = c a_1$

ex. $b_1 = \begin{bmatrix} 0 \\ 4 \\ 1 \end{bmatrix}$ $\quad a_1 = \begin{bmatrix} 1 \\ -4 \\ -9 \end{bmatrix}$

$b_1 \cdot a_1 = (0 \times 1) + (4 \times -4) + (1 \times -9) = -7$

$a_1 \cdot a_1 = (1 \times 1) + (-4 \times -4) + (-9 \times -9) = 98$

$c = \dfrac{-7}{98} = -\dfrac{1}{14}$

$p_1 = -\dfrac{1}{14} \begin{array}{c} \begin{bmatrix} 0 \\ -4 \\ -9 \end{bmatrix} \end{array} \begin{array}{l} k_1 \\ k_2 \\ k_3 \end{array}$

$e_1 = b_1 - p_1$

$e_1 = \begin{bmatrix} 0 \\ 4 \\ 1 \end{bmatrix} - \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}$

* different problem but same process
* def do in excel

HW6A.2. Projecting a vector onto a line E

HW6A.2. Projecting a vector onto a line E

Let $\mathbf{b} = \begin{bmatrix} -2 \\ -8 \\ 8 \end{bmatrix}$ and $\mathbf{a} = \begin{bmatrix} -6 \\ 6 \\ -3 \end{bmatrix}$. We will compute the projection of the vector $\mathbf{b}$ onto the line through $\mathbf{a}$.

Give $\hat{\mathbf{x}}$, where $\mathbf{p} = \hat{\mathbf{x}}\mathbf{a}$.

$\hat{\mathbf{x}} = \begin{bmatrix} -0.74 \end{bmatrix}$ ✓ 100%

Give the projection vector $\mathbf{p}$, for $\mathbf{b}$ on the line going through $\mathbf{a}$.

$$\mathbf{p} = \begin{bmatrix} 4.44 \\ -4.44 \\ 2.22 \end{bmatrix}$$

✓ 100%

Give the projection matrix P for the line going through $\mathbf{a}$.

$$P = \begin{bmatrix} 0.44 & -0.44 & 0.22 \\ -0.44 & 0.44 & -0.22 \\ 0.22 & -0.22 & 0.11 \end{bmatrix}$$

✓ 100%

6A.2 Given $\vec{b}$ and $\vec{a}$, find $\hat{x}$, p, and P where $p = \hat{x}a$

same as: 6A.4

$\hat{x}$ = the scalar = $\frac{b \cdot a}{a \cdot a}$ projection matrix P:

$p = \hat{x}\vec{a}$ $P = \frac{aa^T}{a \cdot a}$

ex. $b = \begin{bmatrix} 4 \\ 6 \\ 1 \end{bmatrix}$ $a = \begin{bmatrix} -6 \\ 4 \\ 0 \end{bmatrix}$ $\hat{x} = \frac{b \cdot a = (4 \times -6) + (6 \times 4) + (1 \times 0) = 0}{a \cdot a = (-6 \times -6) + (4 \times 4) + (0 \times 0) = 52}$

$p = 0 \cdot \vec{a} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $\hat{x} = \frac{0}{52} = 0$

$P: aa^T = \begin{bmatrix} -6 \\ 4 \\ 0 \end{bmatrix} \begin{bmatrix} -6 & 4 & 0 \end{bmatrix} = \begin{bmatrix} 36 & -24 & 0 \\ -24 & 16 & 0 \\ 0 & 0 & 0 \end{bmatrix} \times \frac{1}{52}$ ← a·a

*divide each value in matrix by 52

* Different Q same process
* Solution in excel below

b	a		aT		
-2	-6		-6	6	-3
-8	6				
8	-3				
ba	-60				
aa	81				
x hat	-0.7407407				
p	4.4444444				
	-4.4444444				
	2.2222222				
aat	36	-36	18		
	-36	36	-18		
	18	-18	9		
	0.4444444	-0.4444444	0.2222222		
	-0.4444444	0.4444444	-0.2222222		
	0.2222222	-0.2222222	0.1111111		

HW6A.7. Projection onto a plane 3

HW6A.7. Projection onto a plane 3

Consider the plane $x + 8y - z = 0$. We will compute the projection of the vector $\mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$ onto the plane.

Give the projection vector $\mathbf{p}$.

$$\mathbf{p} = \begin{bmatrix} -0.075757 \\ 0.393939 \\ 3.0757575 \end{bmatrix}$$

✓100%

Give the projection matrix P.

$$P = \begin{bmatrix} 0.984848 & -0.1212121 & 0.0151515 \\ -0.1212121 & 0.030303 & 0.1212121 \\ 0.0151515 & 0.1212121 & 0.984848 \end{bmatrix}$$

✓100%

- Equations needed to solve this problem:

$$x - y + 7z = 0$$

$$n = \begin{bmatrix} 1 \\ -1 \\ 7 \end{bmatrix} \qquad \text{proj}_n(b) = \frac{bn}{nn} n$$

$$p = b - \text{proj}_n(b)$$

$$P = I - \frac{nn^T}{nn}$$

b	n		nT			
0	1		1	8	-1	
1	8					
3	-1					
bn	5					
nn	66					
x hat	0.0757576					
projn(b)	0.0757576	p	-0.0757576			
	0.6060606		0.3939394			
	-0.0757576		3.0757576			
nnt	1	8	-1			
	8	64	-8			
	-1	-8	1			
	0.0151515	0.1212121	-0.0151515			
	0.1212121	0.969697	-0.1212121			
	-0.0151515	-0.1212121	0.0151515			
I	1	0	0			
	0	1	0			
	0	0	1			
P	0.9848485	-0.1212121	0.0151515			
	-0.1212121	0.030303	0.1212121			
	0.0151515	0.1212121	0.9848485			

HW6A.10. Orthogonal plane using projections

HW6A.10. Orthogonal plane using projections

Consider the vectors $\mathbf{v}_1 = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 7 \\ 2 \\ 1 \end{bmatrix}$ and $\mathbf{v}_3 = \begin{bmatrix} 7 \\ 1 \\ 3 \end{bmatrix}$.

Decompose $\mathbf{v}_3$ into two components: one parallel to $\mathbf{v}_1$ and $\mathbf{v}_2$, and the other orthogonal to $\mathbf{v}_1$ and $\mathbf{v}_2$.

Give the component that is *parallel* to $\mathbf{v}_1$ and $\mathbf{v}_2$.

$$\mathbf{v}_{3_{par}} = \begin{bmatrix} 7 \\ 2 \\ 1 \end{bmatrix}$$

? ✓ 100%

Give the component that is *perpendicular* to $\mathbf{v}_1$ and $\mathbf{v}_2$.

$$\mathbf{v}_{3_{perp}} = \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}$$

? ✓ 100%

	v1	v2							
A	**2**	**7**		**AT**	**2**	**2**	**1**	**v3**	**7**
	2	**2**			**7**	**2**	**1**		**1**
	1	**1**							**3**
ATA	9	19							
	19	54							
det	54	-19		125					
	-19	9							
ATA-1	0.432	-0.152							
	-0.152	0.072							
A(ATA)-1	-0.2	0.2							
	0.56	-0.16							
	0.28	-0.08							
A(ATA)-1AT	1	0	0						
	0	0.8	0.4						
	0	0.4	0.2						
A(ATA)-1ATv3	7								
aka v3 par	2								
	1								
v3perp	0								
	-1								
	2								

- V3perp = v3-v3par

HW6A.11. Projection onto a plane D

HW6A.11. Projection onto a plane D

Consider the plane $x - 2y + z = 0$. We will compute the projection matrix P for the plane.

Give a vector $\mathbf{e}$ that is perpendicular to the plane.

$$\mathbf{e} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

? ✓ 100%

Compute the matrix $Q = \frac{\mathbf{e}\mathbf{e}^T}{\mathbf{e}^T\mathbf{e}}$.

$$Q = \begin{bmatrix} 0.1666666 & -0.333333 & 0.1666666 \\ -0.333333 & 0.6666666 & -0.333333 \\ 0.1666666 & -0.333333 & 0.1666666 \end{bmatrix}$$

? ✓ 100%

Give the projection matrix $P = I - Q$.

$$P = \begin{bmatrix} 0.833333 & 0.333333 & -0.166666 \\ 0.333333 & 0.333333 & 0.333333 \\ -0.166666 & 0.333333 & 0.833333 \end{bmatrix}$$

? ✓ 100%

e						
1			**1**	**-2**	**1**	
-2						
1						
eTe	6		eeT	1	-2	1
				-2	4	1
				1	-2	1
	1	-2	1			
	-2	4	-2			
	1	-2	1			
Q	0.16666667	-0.3333333	0.16666667			
	-0.3333333	0.66666667	-0.3333333			
	0.16666667	-0.3333333	0.16666667			
I	1	0	0			
	0	1	0			
	0	0	1			
P	0.83333333	0.33333333	-0.1666667			
	0.33333333	0.33333333	0.33333333			
	-0.1666667	0.33333333	0.83333333			

HW6A.14. Least Squares Approximations

HW6A.14. Least Squares Approximations

Suppose we want to construct a line $b = C + Dt$ that goes through

1. $b = 3$ at $t = -3$,
2. $b = 2$ at $t = 0$,
3. $b = 1$ at $t = 6$.

Find the line that minimizes error from the given points by finding the least squares solution for C and D

$C =$	2.21	?	✓ 100%
$D =$	-0.21	?	✓ 100%

A			b		AT		
1	-3		3		1	1	1
1	0		2		-3	0	6
1	6		1				
ATA	3	3					
	3	45					
	45	-3		126	det(A)		
	-3	3		0.008	<-- 1/det(A)		
ATA-1	0.3571	-0.02381					
	-0.024	0.0238095					
ATb	6						
	-3						
C	2.2143	<-- (ATA)-1*Atb					
D	-0.214						

HW6A.20. Orthogonality 7

HW6A.20. Orthogonality 7

Consider the pair of vectors $\mathbf{v}_1 = \begin{bmatrix} 1.25 \\ 1.00 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 25.25 \\ -29.00 \end{bmatrix}$. Which of the following best describes them?

Note: Due to rounding issues, orthogonal computations might not be exactly zero.

- ○ Orthonormal
- ◉ Independent ✓
- ○ Orthogonal
- ○ Dependent

✓ 100%

check dot product	
v1	v2
1.25	25.25
1	-29
2.5625	<-- if not 0, then not orthogonal nor orthonormal
check independence (are they scalars?)	
20.2	
-29	

HW6A.21. Orthonormal Vectors in a Plane

HW6A.21. Orthonormal Vectors in a Plane

Find two orthonormal vectors in the plane $x + 6y + z = 0$.

$\mathbf{v}_1 = \begin{bmatrix} -0.962250 \\ 0.1924500 \\ -0.192450 \end{bmatrix}$? ✓100%

$\mathbf{v}_2 = \begin{bmatrix} -0.218536 \\ -0.124878 \\ 0.967805 \end{bmatrix}$? ✓100%

Try a new variant

transform equation into vector						
1						
6						
1						
pick y=1, z=-1	pick y=0, z=1		orthogonlize v2 against u1			
x+6-1=0	x+1=0		v2u2	0.76980036		
x=-5	x=-1		u1u1	1	<- bc normal vector	
v1	v2		proj_u1(v2)	-0.74074074	<--v2u2(u1)	
-5	-1			0.14814815		
1	0			-0.14814815		
-1	1					
			subtract above from v2 to get orthogonal vector			
orthonormalize using gram-schmidt				-0.25925926		
||v1||	5.19615242			-0.14814815		
				1.14814815		
normalize						
1/||v1||	0.19245009		normalize			
			length	1.18634203		
u1	-0.96225045			0.84292723		
	0.19245009					
	-0.19245009			-0.21853669		
				-0.12487811		
				0.96780534		

HW6A.24. Orthonormal Basis K

HW6A.24. Orthonormal Basis K

Consider the plane spanned by $\mathbf{a}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ and $\mathbf{a}_2 = \begin{bmatrix} -2 \\ 2 \\ 2 \\ -2 \end{bmatrix}$.

Find orthonormal vectors $\mathbf{q}_1, \mathbf{q}_2$ in the plane spanned by $\mathbf{a}_1$ and $\mathbf{a}_2$.

$\mathbf{q}_1 = \begin{bmatrix} 0.7071067 \\ 0 \\ 0.7071067 \\ 0 \end{bmatrix}$ ✓ 100%

$\mathbf{q}_2 = \begin{bmatrix} -0.5 \\ 0.5 \\ 0.5 \\ -0.5 \end{bmatrix}$ ✓ 100%

Which vector $\mathbf{v}$ on the plane is closest to $\mathbf{b} = \begin{bmatrix} 3 \\ -4 \\ -1 \\ 3 \end{bmatrix}$?

$\mathbf{v} = \begin{bmatrix} 3.75 \\ -2.75 \\ -1.75 \\ 2.75 \end{bmatrix}$ ✓ 100%

a1	a2	b			
1	-2	3			
0	2	-4			**compute v**
1	2	-1		bq1	1.4142136
0	-2	3		projq1(b)	1
					0
||a1||	1.41421356				1
1/||a1||	0.70710678				0
q1	0.70710678			bq2	-5.5
	0			Projq2(b)	2.75
	0.70710678				-2.75
	0				-2.75
					2.75
Project a2 onto q1					
a2q1	0	<-- a2 is already orthogonal			
				v	3.75
normalize					-2.75
||a2||	4				-1.75
1/||a2||	0.25				2.75
q2	-0.5				
	0.5				
	0.5				
	-0.5				

Equations to solve this problem:

$$q_1 = \frac{a_1}{\|a_1\|}$$

We first make a_2 orthogonal to q_1 by subtracting its projection:

$$v_2 = a_2 - \mathrm{Proj}_{q_1}(a_2)$$

The projection formula:

$$\mathrm{Proj}_{q_1}(a_2) = \left(\frac{a_2 \cdot q_1}{q_1 \cdot q_1}\right) q_1$$

$$q_2 = \frac{a_2}{\|a_2\|}$$

We need to compute the projection of b onto the plane:

$$v = \text{Proj}_{q_1}(b) + \text{Proj}_{q_2}(b)$$

Given:

$$b = \begin{bmatrix} 3 \\ -4 \\ -1 \\ 3 \end{bmatrix}$$

Projection onto q_1

$$\text{Proj}_{q_1}(b) = (b \cdot q_1)q_1$$

Projection onto q_2

$$\text{Proj}_{q_2}(b) = (b \cdot q_2)q_2$$

$$\begin{aligned} v &= \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 2.75 \\ -2.75 \\ -2.75 \\ 2.75 \end{bmatrix} \\ &= \begin{bmatrix} 3.75 \\ -2.75 \\ -1.75 \\ 2.75 \end{bmatrix} \end{aligned}$$