1 (a): C.O.G

The approximated equation listed in Draine is:

$$W \simeq \sqrt{\pi}\frac{b}{c}\frac{\tau_0}{1+\tau_0/(2\sqrt{2})}, \tag{1}$$

for $\tau_0 < 1.25393$. And:

$$W \simeq \sqrt{(\frac{2b}{c})^2 \ln\frac{\tau_0}{\ln 2} + \frac{b}{c}\frac{\gamma_{\ell e}\lambda_{\ell u}}{c}\frac{(\tau_0 - 1.25393)}{\sqrt{2}}}, \tag{2}$$

for $\tau_0 > 1.25393$.

The question nastily only gives us W_λ^{rest} with a unit, so we have to convert it to the dimensionless W. The conversion should be given in eq (9.4) in Draine:

$$W_\lambda = \int d\lambda(1 - e^{-\tau_\nu}) \simeq \lambda_0 W,$$

so ideally we have $W \simeq W_\lambda/\lambda_0$.

- Fe II: $W \simeq W_\lambda^{\mathrm{r}est}/\lambda_{\mathrm{rest}} = 0.051/2382.7642 = 2.140 \times 10^{-5}$
- Fe II: $W \simeq W_\lambda^{\mathrm{r}est}/\lambda_{\mathrm{rest}} = 0.0047/2249.8768 = 2.089 \times 10^{-6}$
- C II: $W \simeq W_\lambda^{\mathrm{r}est}/\lambda_{\mathrm{rest}} = 0.060/1334.5323 = 4.496 \times 10^{-5}$

Now the strategy is to determine which equation to use. Since we consider ranges $\log N_{\mathrm{Fe\,II}} \sim \mathrm{U}(12, 16)$ and $\log N_{\mathrm{C\,II}} \sim \mathrm{U}(13, 17)$. Consider the definition of τ_0 as function of N_ℓ:

$$\begin{aligned}\tau_{0,\tilde{N}_\ell}(N_\ell) &= \sqrt{\pi}\frac{e^2}{m_e c}N_\ell f_{\ell u}\lambda_{\ell u}\frac{1}{b}(1 - \frac{N_u/g_u}{N_\ell/g_\ell}) \\ &= 1.497 \times 10^{-2}\frac{\mathrm{cm}^2}{s}\frac{f_{u\ell}\lambda_{\ell u}}{b}N_\ell,\end{aligned} \tag{3}$$

After plugging the numbers into eq 3, unfortunately the values of τ_0 is not falling just one-side of the $\tau_0 = 1.25393$:

$$\tau_{0,\tilde{N}_\ell}(N_\ell, b = 10, f = 0.320, \lambda_{\mathrm{rest}} = 2382.7642) = (0.11, 1141.44).$$

We thus have to consider both equations.

We clearly know the form of τ_0, so it's a matter of making decisions for plotting. The way I implemented is estimating τ_0 first based on given b and N_ℓ, and using the value of τ_0 to determine which equation we are going to use to compute W. Finally, convert W to W_λ using $\lambda_0 W$.

Here are examples of dimensionless W C.O.G:

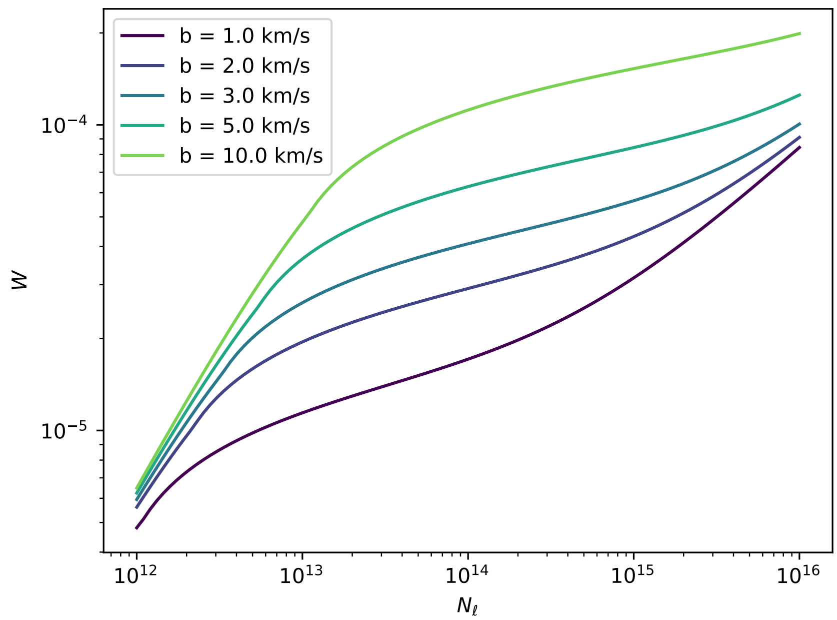

Figure 1: Fe II, $\lambda_{\rm rest} = 2382.7642$ Å

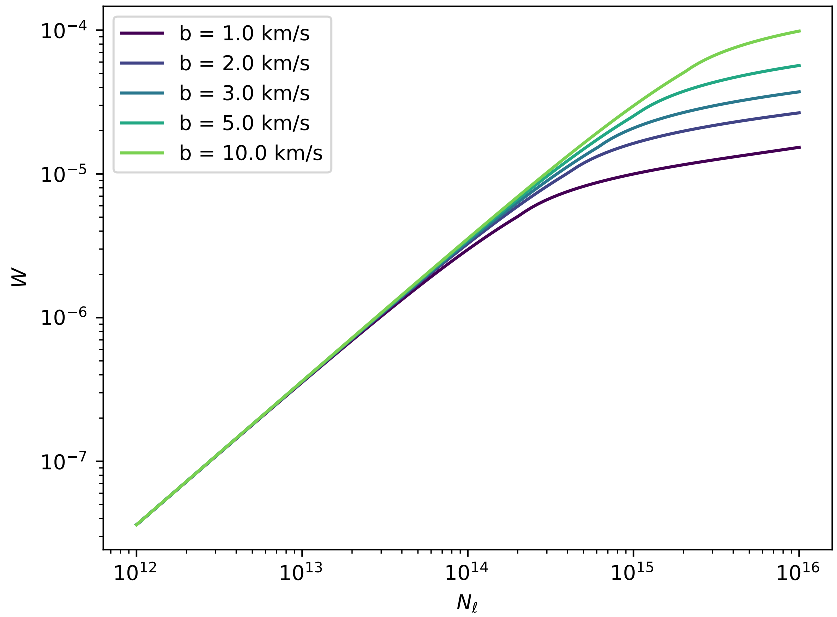

Figure 2: Fe II, $\lambda_{\mathrm{rest}} = 2249.8768$ Å

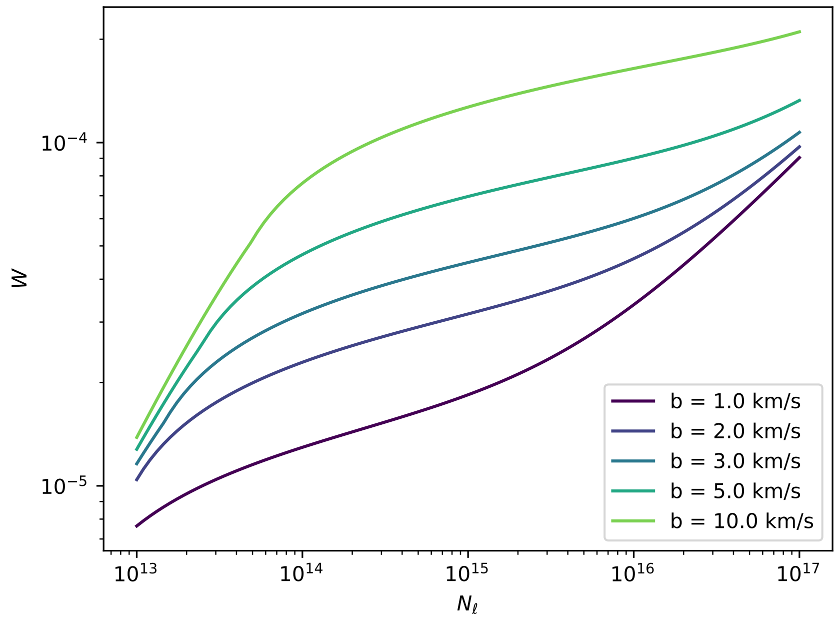

Figure 3: C II, $\lambda_{\mathrm{rest}} = 1134.5323$ Å

Here are examples of W_λ^{rest} C.O.G:

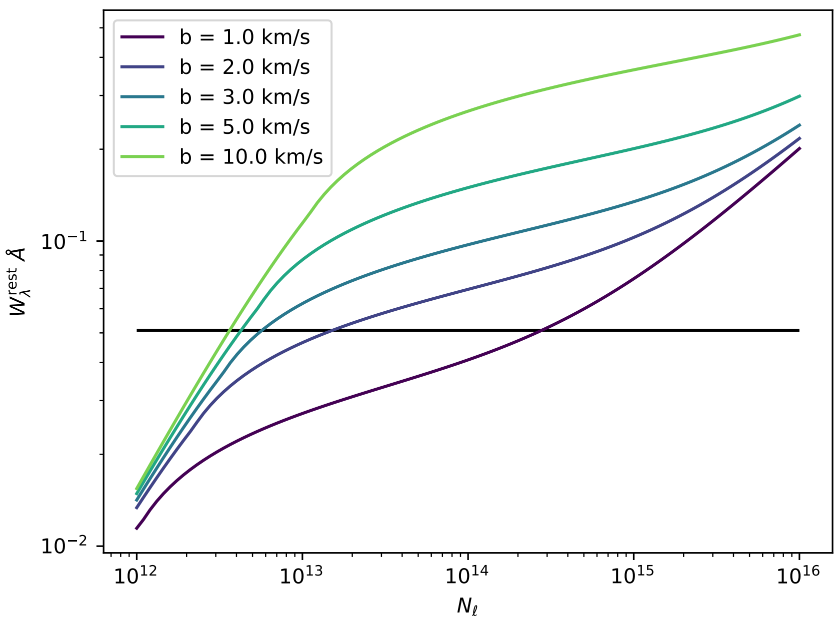

Figure 4: Fe II, $\lambda_{\rm rest} = 2382.7642$ Å

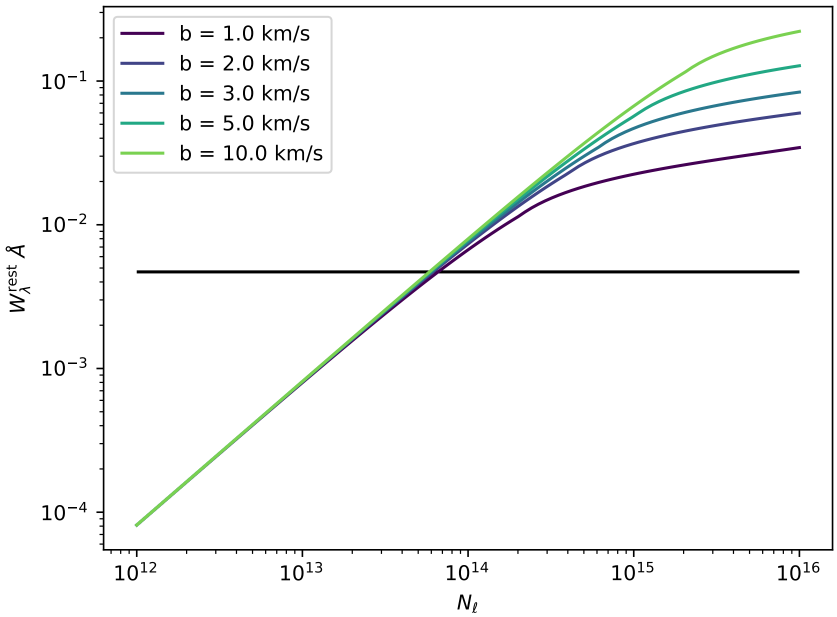

Figure 5: Fe II, $\lambda_{\text{rest}} = 2249.8768$ Å

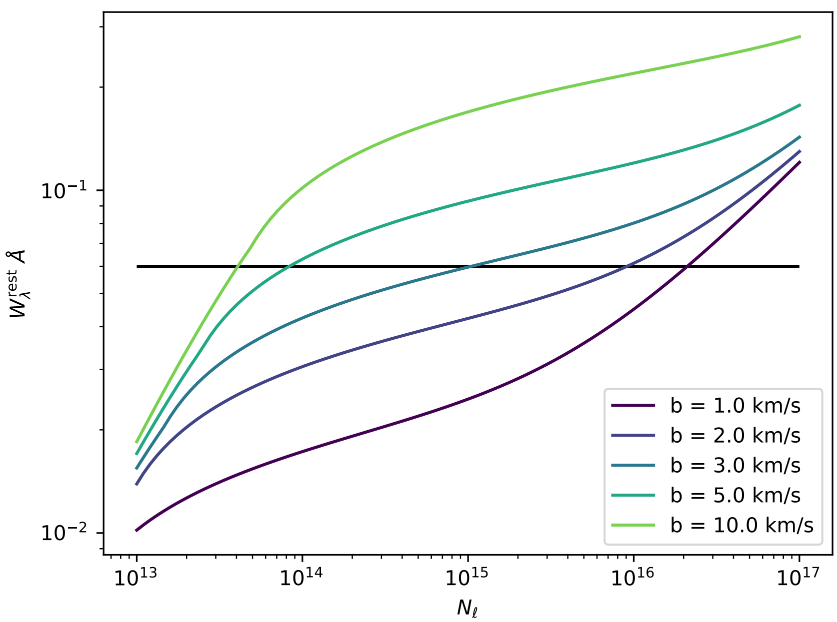

Figure 6: C II, $\lambda_{\text{rest}} = 1134.5323$ Å

The black lines I plotted there are the observed $W_\lambda^{\mathrm{rest}}$ of these lines. Previously (in the previous version), I set a wrong $c = 10^{11} cm/s$, which should be corrected as $c = 10^{10} cm/s$. Here the black lines are nicely crossing the COG.

1 (b): Fe II Metallicity

The question asks us to identify the value of b parameters based on the plots we plotted in the question (a). We probably could solve it analytically, but I think the question asked us to find the value by eyes.

Based on the curves, we see only Fig 5 has a stable solution for N_ℓ, which gives $N_\ell \simeq 10^{13.6} cm^{-2}$. This column density could be used to solve the b for FeII with 2382.7642 Å. The argument is both FeII lines should have the same N_ℓ. By squinting Fig 4, we roughly get $b \sim 1.5\,\mathrm{km/s}$.

Now it's the matter of converting b to $(\Delta v)_{\mathrm{FWHM}}$. With the help from Draine:

$$(\Delta v)_{\mathrm{FWHM}} = 2\sqrt{\ln 2}\, b. \tag{4}$$

You can think this is the property of a normal distribution, that the FWHM has a nice relation with the standard deviation. For this particular line FeII 2382.7642 Å, :

$$(\Delta v)_{\mathrm{FWHM}} = 2\sqrt{\ln 2}\, 1.5\,\mathrm{km/s} \simeq 2.5\,\mathrm{km/s},$$

The thing is we should expect the velocity being stretched by the redshifting of the universe by a factor of $(1+z)$:

$$(\Delta v)^{obs}_{\mathrm{FWHM}} = (1+z) \times 17 \simeq 7.5\,\mathrm{km/s} \tag{5}$$

The thing is the resolution of HIRES is around $7 - 9\,\mathrm{km/s}$. So it's hard to tell wether it is resolvable or not.

We now are supposed to calculate the metallicity:

$$[Fe/N] = \log\left(N_{Fe}/N_H\right) - \log\left(N_{Fe}/N_H\right)_\odot. \tag{6}$$

Now we steal the table from Asplund et al. 2009, ARAA, 47, 481, which is suggested by the instructor:

Z	Element	Photosphere	Meteorites	Z	Element	Photosphere	Meteorites
1	H	12.00	8.22 ± 0.04	44	Ru	1.75 ± 0.08	1.76 ± 0.03
2	He	[10.93 ± 0.01]	1.29	45	Rh	0.91 ± 0.10	1.06 ± 0.04
3	Li	1.05 ± 0.10	3.26 ± 0.05	46	Pd	1.57 ± 0.10	1.65 ± 0.02
4	Be	1.38 ± 0.09	1.30 ± 0.03	47	Ag	0.94 ± 0.10	1.20 ± 0.02
5	B	2.70 ± 0.20	2.79 ± 0.04	48	Cd		1.71 ± 0.03
6	C	8.43 ± 0.05	7.39 ± 0.04	49	In	0.80 ± 0.20	0.76 ± 0.03
7	N	7.83 ± 0.05	6.26 ± 0.06	50	Sn	2.04 ± 0.10	2.07 ± 0.06
8	O	8.69 ± 0.05	8.40 ± 0.04	51	Sb		1.01 ± 0.06
9	F	4.56 ± 0.30	4.42 ± 0.06	52	Te		2.18 ± 0.03
10	Ne	[7.93 ± 0.10]	−1.12	53	I		1.55 ± 0.08
11	Na	6.24 ± 0.04	6.27 ± 0.02	54	Xe	[2.24 ± 0.06]	−1.95
12	Mg	7.60 ± 0.04	7.53 ± 0.01	55	Cs		1.08 ± 0.02
13	Al	6.45 ± 0.03	6.43 ± 0.01	56	Ba	2.18 ± 0.09	2.18 ± 0.03
14	Si	7.51 ± 0.03	7.51 ± 0.01	57	La	1.10 ± 0.04	1.17 ± 0.02
15	P	5.41 ± 0.03	5.43 ± 0.04	58	Ce	1.58 ± 0.04	1.58 ± 0.02
16	S	7.12 ± 0.03	7.15 ± 0.02	59	Pr	0.72 ± 0.04	0.76 ± 0.03
17	Cl	5.50 ± 0.30	5.23 ± 0.06	60	Nd	1.42 ± 0.04	1.45 ± 0.02
18	Ar	[6.40 ± 0.13]	−0.50	62	Sm	0.96 ± 0.04	0.94 ± 0.02
19	K	5.03 ± 0.09	5.08 ± 0.02	63	Eu	0.52 ± 0.04	0.51 ± 0.02
20	Ca	6.34 ± 0.04	6.29 ± 0.02	64	Gd	1.07 ± 0.04	1.05 ± 0.02
21	Sc	3.15 ± 0.04	3.05 ± 0.02	65	Tb	0.30 ± 0.10	0.32 ± 0.03
22	Ti	4.95 ± 0.05	4.91 ± 0.03	66	Dy	1.10 ± 0.04	1.13 ± 0.02
23	V	3.93 ± 0.08	3.96 ± 0.02	67	Ho	0.48 ± 0.11	0.47 ± 0.03
24	Cr	5.64 ± 0.04	5.64 ± 0.01	68	Er	0.92 ± 0.05	0.92 ± 0.02
25	Mn	5.43 ± 0.04	5.48 ± 0.01	69	Tm	0.10 ± 0.04	0.12 ± 0.03
26	Fe	7.50 ± 0.04	7.45 ± 0.01	70	Yb	0.84 ± 0.11	0.92 ± 0.02

To compute the solar metallicity, we grab the photosphere values from H and Fe:

$$\log\left(N_{Fe}/N_H\right)_\odot = 7.5 - 12 = -4.5$$

Thus,

$$[Fe/H] = \log\left(N_{Fe}/N_H\right) - \log\left(N_{Fe}/N_H\right)_\odot = \log\left(10^{13.6}/10^{20.3}\right) - (-4.5) = 13.6 - 20.3 + 4.5 = -2.2.$$

1 (c): CII Metallicity

I did not know this before I read the solution. But for turbulent broadening, all ions of the system have the same bs. So CII will have $b \sim 1.5\,\mathrm{km/s}$, the same as FeII 2382 Åin the previous question.

By squinting the Fig 6, we roughly get $N_\ell \sim 10^{16.5} cm^{-2}$.

Now we carry out the metallicity:

$$\begin{aligned}[C\,II/H] &= \log\left(N_{CII}/N_H\right) - \log\left(N_{CII}/N_H\right)_\odot = \\ &= \log\left(10^{16.5}/10^{20.3}\right) - (8.43 - 12) \approx 16.5 - 20.3 + 3.57 = -0.23.\end{aligned}$$

Comparing to FeII,

$$[C\,II, Fe\,II] = \log\left(N_{CII}/N_{FeII}\right) - \log\left(N_{CII}/N_{FeII}\right)_\odot = 16.5 - 13.6 - (8.43 - 7.5) = 1.97.$$

1 (d): Thermally broadening

We know the thermal velocity is also a Gaussian like distribution, so the FWHM could be written as:

$$(\Delta v)^{\text{thermal}}_{\text{FWHM}} = 2\sqrt{\ln 2}\left(\frac{kT}{M}\right)^{1/2} = 2.15\left(\frac{T/100\,\mathrm{K}}{M/m_H}\right)^{1/2}\ \mathrm{km/s}, \tag{7}$$

where we know $M_{Fe}/m_H \simeq 56$. We would thus be able to infer the value of T if we assume thermal velocity contribute to the b we observe from the plots.

The least condition for these lines being dominated by thermal broadening is that the $(\Delta v)^{\text{thermal}}_{\text{FWHM}} > (\Delta v)_{\text{FWHM}}$.

$$\begin{aligned}&(\Delta v)^{\text{thermal}}_{\text{FWHM}} > (\Delta v)_{\text{FWHM}} \\ &\Rightarrow 2.15\left(\frac{T/100\,\mathrm{K}}{M/m_H}\right)^{1/2}\ \mathrm{km/s} > (\Delta v)_{\text{FWHM}} \\ &\Rightarrow T/100\,\mathrm{K} > 1/(2.15)^2 * \frac{M}{m_H}(\Delta v)^2_{\text{FWHM}} \\ &\Rightarrow T > 100/2.15^2 * 56 * 2.5^2 K \simeq 7571.7\,K\end{aligned}$$

1(e) Infer b for thermal broadening

Since we have Eq 7, we thus know $(\Delta v)_{\text{FWHM}}^{\text{thermal}} \propto \frac{1}{\sqrt{M_X}}$, where $X \in \{\text{ions}\}$. By the linear relationship between $(\Delta v)_{\text{FWHM}}^{\text{thermal}}$ and b, we have:

$$b \propto \frac{1}{\sqrt{M_X}},$$

where $X \in \{\text{ions}\}$. Thus b for CII is estimated by $\sqrt{\frac{M_{FeII}}{M_{CII}}} b_{FeII} = 5.4\,\text{km/s}$. The corresponding N_ℓ would be $N_\ell = 10^{12.3} cm^{-2}$ by squinting.

The value of metallicity would be:

$$\begin{aligned}[C\,II/H] &= \log\left(N_{CII}/N_H\right) - \log\left(N_{CII}/N_H\right)_\odot = \\ &= \log\left(10^{12.3}/10^{20.3}\right) - (8.43 - 12) \approx 12.3 - 20.3 + 3.57 = -4.43.\end{aligned}$$

Comparing to iron:

$$[C\,II, Fe\,II] = \log\left(N_{CII}/N_{FeII}\right) - \log\left(N_{CII}/N_{FeII}\right)_\odot = 12.3 - 13.6 - (8.43 - 7.5) = -2.29.$$

1 (f): Carbon enhanced

Given that carbon rich DLA is rare, it's more possible that thermal broadening is the dominated effect since it gives lower carbon abundance.