Draine 15.4: O Star

Consider a runaway O star (O8V spectral type) traveling through a diffuse region with $n_H \simeq 0.2 \text{ cm}^{-3}$.

(a): Strömgren radius

By equating the rates of photo-ionization and radiative recombination gives the steady state condition for the balance:

$$Q_0 = \frac{4\pi}{3} R_{\mathrm{S0}}^3 \alpha_B n(\mathrm{H}^+) n_e, \tag{1}$$

thus if $n(\mathrm{H}^+) = n_e = n_H$ (according to balance condition)

$$R_{\mathrm{S0}} = \left(\frac{3Q_0}{4\pi n_H^2 \alpha_B} \right)^{1/3} \tag{2}$$

Plugging the numbers:

$$\begin{aligned} Q_0 &= 10^{48.44} \mathrm{s}^{-1} \\ n_H &= 0.2 \text{ cm}^{-3} \\ \alpha_B &\simeq 2.56 \times 10^{-13} T_4^{-0.83} \, \mathrm{cm}^3 \, \mathrm{s}^{-1} \\ T_4 &= 1 \end{aligned} \tag{3}$$

This gives:

$$R_{\mathrm{S0}} = 4 \times 10^{20} \, \mathrm{cm}. \tag{4}$$

(b): travelling star

Time requires traveling Strömgren radiuswith $v_* = 100$ km/s is:

$$t_{\mathrm{travel}} = R_{\mathrm{S0}} / v_*. \tag{5}$$

The timescale for recombination is:

$$t_{rec} = \frac{1}{\alpha_B n(\mathrm{H}^+)} = \frac{\frac{4}{3}\pi R_{\mathrm{S0}}^3 n(\mathrm{H}^+)}{Q_0} \tag{6}$$

The comparison can be done by taking the ratio of recombination timescale and traveling timescale:

$$\text{ratio} = \frac{t_{travel}}{t_{rec}} = \frac{R_{\mathrm{S0}}/v_*}{\frac{\frac{4}{3}\pi R_{\mathrm{S0}}^3 n(\mathrm{H}^+)}{Q_0}} = \frac{Q_0}{v_* \frac{4}{3}\pi R_{\mathrm{S0}}^2 n(\mathrm{H}^+)} = 205. \tag{7}$$

(c): implication of the comparison in item (b)

Ionization responds on a timescale short compared to the star travelling time. But if we have low hydrogen density, it could help to decrease the ratio, which means low density system has comparable ionization time and star travelling time.

2: UVB background (produced by stars and SGN)

(a): Photo-ionization rate

Relevant given values:

$$\begin{aligned} z &= 3 \\ \mathbb{E}I_\nu(\nu_0) &= 10^{-21}\mathrm{erg\,cm^{-2}s^{-1}Hz^{-1}sr^{-1}} \\ h\nu_0 &= 13.6\,\mathrm{eV} \end{aligned} \tag{8}$$

The intensity at given frequency:

$$I_\nu = I_\nu(\nu_0 = 13.6\,\mathrm{eV}/h)\nu^{-1.5}/\nu_0^{-1.5}. \tag{9}$$

Based on the values given, obviously we can only compute photo-ionization rate averaged on a surface (q_0):

$$Q_0 = q_0/N_{\mathrm{HI}},$$

with N_{HI} in unit of cm^{-2} for the surface density.

Photo-ionization rate is the integral:

$$\begin{aligned} q_0 &= \int_{\nu_0}^{\infty} \frac{I_\nu}{h\nu} d\nu = \frac{I_\nu(\nu_0)}{h\nu_0^{-1.5}} \int_{\nu_0}^{\infty} \nu^{-2.5} d\nu = \frac{I_\nu(\nu_0)}{h\nu_0^{-1.5}} \times \frac{1}{1.5}\nu_0^{-1.5} = \frac{I_\nu(\nu_0)}{1.5\,h} \\ &= 10^{-21}\mathrm{erg\,cm^{-2}s^{-1}Hz^{-1}sr^{-1}} \frac{1}{1.5\,h} = 0.66 \times 10^{-21}\mathrm{erg\,cm^{-2}s^{-1}Hz^{-1}sr^{-1}}h^{-1}. \end{aligned} \tag{10}$$

(b): Strömgren radius (or length in this case)

Some given values:

$$\begin{aligned} n(\mathrm{H}^+) &\simeq 0.1\ \mathrm{cm}^{-3} \\ T_4 &= 1. \end{aligned} \tag{11}$$

Do the same thing as problem one:

$$Q_0 = \pi R^2 H_{S0}\,\alpha_B n(\mathrm{H}^+) n_e, \tag{12}$$

where H_{S0} is the height of the disk since we are physicists we only know spheres, cylinders, and boxes.

With a little bit massage, assuming q_0 is the photo-ionization rate from the surface of the disk:

$$q_0 = H_{S0}\alpha_B n(\mathrm{H}^+) n_e = H_{S0}\alpha_B n_H^2, \tag{13}$$

where the same argument applied for $n(\mathrm{H}^+) = n_e = n_H$.

Thus:

$$H_{S0} = \frac{q_0}{\alpha_B n_H^2} \tag{14}$$

By the same assumptions:

$$\begin{aligned} n_H &= 0.1 \text{ cm}^{-3} \\ \alpha_B &\simeq 2.56 \times 10^{-13} T_4^{-0.83}\,\mathrm{cm}^3\,\mathrm{s}^{-1} \\ T_4 &= 1, \end{aligned}$$

which gives:

$$H_{S0} = 3.89 \times 10^{19} \text{ cm}. \tag{15}$$

(c) maximum total column density

Naive guess would be just the average column density in the ionized disk if we assume light is coming into the face-on disk:

$$N_{\mathrm{HI}} = H_{S0} \times n_H = 3.89 \times 10^{19} \times 0.1 \text{ cm}^{-2} = 3.89 \times 10^{18} \text{ cm}^{-2}. \tag{16}$$