18.2: line ratio

(a): T and n

I think the question is asking us to draw a horizontal line on Fig 18.2, which means, for O III,

- $T \sim 10^4\,K$ if $n = 10^6$ cm^{-3}
- $T \sim 1.6 * 10^4\,K$ if $n = 10^5$ cm^{-3}
- $T \sim 1.8 * 10^4\,K$ if $n = 10^4$ cm^{-3}
- $T \sim 2 * 10^4\,K$ if $n = 10^3$ cm^{-3}

For O II, based on Fig 18.4,

$$n_e T_4^{-1/2} \simeq 2 * 10^2 \text{ cm}^{-3}. \tag{1}$$

So it seems $T \sim 2 * 10^4\,K$ if $n = 10^3$ cm^{-3} makes more sense.

(b): reddening

The reddenning is $A(4364.4)\text{Å} - A(5008.2\text{Å}) = 0.31\,\text{mag}$. The line ratio contribution from dust is $10^{0.31} = 2.04$. The new line ratio for O III is:

$$\frac{I([OIII]4364.4)}{I([OIII]5008.2)} = 0.003/2.04 = 0.0015. \tag{2}$$

For Fig 18.2, we have:

- $T \sim 8 * 10^3\,K$ if $n = 10^6$ cm^{-3}
- $T \sim 1.1 * 10^4\,K$ if $n = 10^5$ cm^{-3}
- $T \sim 1.3 * 10^4\,K$ if $n = 10^4$ cm^{-3}
- $T \sim 1.4 * 10^4\,K$ if $n = 10^3$ cm^{-3}

So we only have upper limits: $n_e < 10^3$ cm^{-3}; and $T > 1.4 * 10^4\,K$.

Heating and Cooling

Given:

- $T = 32\,000\,K$.

(a): center, balance photo/cooling

Since it is in the center, we should choose $\langle\psi\rangle$ since almost all photons will produce photoionizations.

Heating rate:

$$\Gamma(H \rightarrow H^+) \simeq \alpha_B n_H n_e \psi k T_c \tag{3}$$

Cooling rate:

$$\Lambda_{rr} = \alpha_B n_e n(H^+) \langle E_{rr} \rangle \tag{4}$$

Known:

$$\begin{aligned} T_c &= 32\,000\,K \\ \langle\psi\rangle &= 1.380 \end{aligned} \tag{5}$$

Assume $n(H^+) \simeq n(H)$. Equate (3) and (4):

$$\langle E_{rr} \rangle = \psi k T_c = 1.380 \times k \times 32\,000$$

From (27.21), we have a suspicious expressions for kinetic energy:

$$\langle E_{rr} \rangle = (2 + \gamma) k T, \tag{6}$$

where γ depends on case A and case B and roughly to be negative unity (technically ~ 0.7), so $\langle E_{rr} \rangle \sim kT$.

Thus,

$$T \sim 1.38 \times 32000 = 44\,160.$$

(b): mass-weighted average temperature

So we should take into account the cross-section at this time. Since there would only have hydrogen, we could imagine mass is proportional to number of particles, and number of particles linearly related to cross-section.

The choice of ψ will switch to $\psi_0 = 0.864$, which means (based on the same calculations in (a)) $T \sim 0.86/0.7 \times 32000 \sim 32000$.