

Problem 1

So let me list some of the values here:

- $(R_* | X = 1, m = 20, MK) = 1890 \text{ e}^-/\text{s}$
- $(m_{sky} | MK, m = m_{sky}, r_{pix}) = 20.3 \text{ mag}/\text{arsec}^2$
- $(r_{pix} | MK) = 0.135 \text{ arcsec}/\text{pix}$
- $(RN^2 | MK) = 5 \text{ e}^-$
- $(G | MK) = 1.2 \text{ e}^-/\text{DN}$

Here are the questions,

1. R_{sky} of detected in R-band?

We knew,

$$(R_* | X = 1, m = 20, MK) = 1890 \text{ e}^-/\text{s}. \quad (1)$$

Now we took off the condition on R magnitude,

$$(R_* | X = 1, m, MK) = 1890 \times 10^{-\frac{m-20}{2.5}} \quad (2)$$

since $R_* \propto 10^{-\text{mag}/2.5}$.

Condition on pixel size to get R_{sky}

$$\begin{aligned} (m_{sky} | MK, m = m_{sky},) &= (m_{sky} | MK, m = m_{sky}, r_{pix}) - 2.5 \log_{10}(r_{pix} | MK)^2 \\ &= 20.3 - 5 \times \log_{10} 0.135 \\ &= 24.6 \text{ mag}/\text{pixel} \end{aligned} \quad (3)$$

Use the information of sky magnitude on a pixel to get the rate of detection on a pixel, which is

$$\begin{aligned} (R_* | X = 1, m = m_{sky}, MK) &= 1890 \times 10^{-\frac{m_{sky}-20}{2.5}} \\ &= 1890 \times 10^{-(24.6-20)/2.5} = 26.13 \text{ e}^-/\text{pixel}/\text{s} \quad \square \end{aligned} \quad (4)$$

2. Find $R_* | m = 26, X = 1.2$.

As suggested by the instructor of fundamental astrophysics- never use magnitude,

$$\begin{aligned} (R_* | X = 1.2, m = 26) &\propto 10^{-\frac{26-20}{2.5}} \times \exp(-cX) \\ &\propto 10^{-\frac{26-20+kX}{2.5}} \\ &\propto 10^{-\frac{26-20+0.1*1.2}{2.5}}. \end{aligned} \quad (5)$$

Remember to bring back your constant

$$(R_* | X = 1.2, m = 26) = 1890 \times 10^{-\frac{26-20+0.1*1.2}{2.5}} = 6.74 \text{ e}^-/\text{s}. \quad (6)$$

3. What's the time range for sky-dominated? Condition on radius of 7 pixels aperture.

Since we have more electrons from sky, we need to wait for enough time to reach source-dominated. The effect of sky decrease as $t^{-1/2}$ because there's a t in the nominator in S/N equation while there's only $t^{1/2}$ in the denominator.

The uncertainty term came from Poisson distribution, which is

$$\sigma = \sqrt{R_*t + R_{sky}n_{pix}t + (RN)^2n_{pix} + DN \times n_{pix}t} \quad (7)$$

Though the description in the question is unclear about '7' pixels, (Does it mean it contains 7 pixels or its radius has a range of 7 pixelscales?) I tried both cases and I thought it should be $n_{pix} = 49$.

Therefore,

$$\begin{aligned} \sigma &= \sqrt{R_*t + R_{sky}n_{pix}t + (RN)^2n_{pix} + DN \times n_{pix}t} \\ &= \sqrt{6.74t + 26.13 * 49 * t + 5 * 49 + 1.2 * 49 * t}. \end{aligned} \quad (8)$$

What we want is sky-dominated, so $\sigma > R_*t = \text{signal}$. Allow me to make some sloppy approximations,

$$\begin{aligned} \sigma &> 6.7t \\ \sqrt{6.74t + 26.13 * 49 * t + 5 * 49 + 1.2 * 49 * t} &> \\ \sqrt{7t + 25 * 50 * t + 250 + 70 * t} &\gtrsim \\ \sqrt{1327 * t + 250} &\gtrsim \end{aligned} \quad (9)$$

Square both sides,

$$\begin{aligned} 1327t + 250 &\gtrsim 40t^2 \\ \Rightarrow 6 &\gtrsim t^2 - 30t = (t - 15)^2 - 225 \\ \Rightarrow 230 &\gtrsim (t - 15)^2. \end{aligned}$$

So roughly before 15 + 15 seconds is sky-dominated,

$$t \lesssim 30 \text{ s}. \quad (10)$$

4. Explain how S/N scale with seeing?

Seeing is caused by the turbulence in the atmosphere. Therefore, the effect of seeing will contribute to the width of PSF. To a first order approximation, let's say seeing $\propto \sigma_{PSF}$.

So that we arrive this scaling relation,

$$n_{pix} \propto r_{measure}^2 \propto \sigma_{PSF}^2 \propto \text{seeing}^2. \quad (11)$$

Recall the equation of S/N,

$$S/N = \frac{R_* t}{\sqrt{R_* t + R_{sky} n_{pix} t + (RN)^2 n_{pix} + DN \times n_{pix} t}} \quad (12)$$

Let's make some assumptions and drop those small terms, e.g., assume the time is long enough to drop RN and we don't care DN.

So,

$$\begin{aligned} S/N &\propto \frac{R_*}{\sqrt{R_* + R_{sky} n_{pix}}} \\ &\propto \frac{1}{\sqrt{1 + \frac{R_{sky}}{R_*} \text{seeing}^2}}. \end{aligned} \quad (13)$$

For our case, S/N roughly scales as

$$\begin{aligned} (S/N \mid X = 1.2, m_* = 26, m_{sky} = 20.3, MK) \\ \propto \frac{1}{\sqrt{1 + \frac{1}{4} \text{seeing}^2}}. \end{aligned} \quad (14)$$

5. What is the exposure time to reach SN = 20?

Before solving $S/N \mid X = 1.2, m_* = 26, m_{sky} = 20.3$, we need to know $R_{sky} \mid X = 1.2$.

$$(R_{sky} \mid X = 1.2) = 1890 * 10^{-\frac{20.3-20+0.12}{2.5}} \times 0.135^2 = 23.395 \text{ e}^-/\text{s}. \quad (15)$$

So the problem is just solving,

$$\begin{aligned} S/N = 20 &= \frac{6.74t}{\sqrt{6.74t + 23.395 * 49t + 5 * 49 + 1.2 * 49t}} \\ &\sim \frac{7t}{\sqrt{7t + 1000t + 250 + 70t}} \\ &\Rightarrow 1077t + 250 \sim 49/400t^2 \sim 1/8t^2 \end{aligned} \quad (16)$$

$$\begin{aligned} &\Rightarrow t^2 - 8600t - 2000 \sim 0 \\ &\Rightarrow (t - 4300)^2 - 4300^2 \sim 2000 \\ &\Rightarrow t \sim 4300 + \sqrt{4300^2 + 2000} \sim 9000. \end{aligned}$$

So it's roughly $t \sim 9\,000$ seconds. I carefully solved it numerically,

Snippet 1: Solve t

```
import sympy as sp

# construct the signal to noise equation using lambda
sn = lambda R_s, R_sky, RN, DN, npix, t :
    R_s * t /
    ( R_s * t + R_sky * t * npix + RN**2 * npix + DN * t * npix )**(1/2)

# solve polynomial with sympy.solve
expr = sn(6.74, 23.395, 5**(1/2), 1.2, 7**2, t)
sp.solve(sp.Eq(expr, 20), t)
```

and I found $t = 10\,671$ seconds.

6. Find the exposure time on the HST.

I checked this filter and thought it's close enough to R band we used: http://svo2.cab.inta-csic.es/svo/theory/fps3/index.php?id=HST/WFC3_UVIS2.F625W

After plugging those numbers, I got $t = 2059.6179 \sim 2\,000$ seconds.

7. Try out some parameters you are interested.

- $0.4''$: $\sim 9\,500$ s
- $4''$: $\sim 700\,000$ s
- F600LP : $\sim 1\,000$ s
- 850LP : $\sim 5\,000$ s
- Narrow 631N : $\sim 120\,000$ s
- Medium F21M : $\sim 5\,000$ s