Structural Equation Modeling

Julian Ibarguen

12/09/2022

Contents

0.1 Introduction

We aim to fit a Structural Equation Model (SEM) following the exercise proposed by Petri Nokelainen from the Research Centre for Vocational Education, University of Tampere, Finland. Data and documentation for this exercise can be found here. The exercise was carried using R and the lavaan package. All the scripts for this exercise can be found here

We used the proposed data set with 447 rows and 27 columns. The respondents in both samples are staff members of Finnish polytechnic institute for higher education. The original measurement instrument (Growth-oriented Atmosphere Questionnaire, GOAQ) has 13 factors and 92-items (Ruohotie, 1996; Ruohotie, Nokelainen & Tirri, 2002), but for the purposes of this exercise Nokelainen selected 13 factors and 27 items, all consisting of a five–point Likert scale from 1 (totally disagree) to 5 (totally agree) was applied (Nokelainen, n.d.)

0.2 Evaluation of assumptions

0.2.1 Univariate distribution

Although data is ordinal, thus non-normal in nature, we check for normality assumption. We will apply z re-scaling to see how they approximate to mean 0. Will also check kurtosis and skewnewss, and plot the histograms. We reject applying Saphiro test or other normality test, as being only 5 points scale ordinal is unlikely will provide any meaningful result.

Given a normally distribute variable would have a skewness of 0 and kurtosis of 3. For this purpose we calculated a synthetic measure for skewness and kurtosis as the average between the skewness and excess kurtosis ($kurtosis - 3$). Under this synthetic measure, a perfect normal distribution would have a value of 0. For our purpose, we select those variables whose deviance from the normal distribution is lower than 0.4, according to the synthetic measure create. The selected variables were further confirmed with the histogram.

Observing Table 1 and Figure 1, we assumed normality for the following variables: v2, v3, v4, v15, v31, v33. The remaining variables were assumed non-normally distributed

Table 1: Descriptive statistics to assess approximation to normal distribution

variable	skewness	kurtosis	kurtosis_excess	avg_skew_kurt
v1	-0.072	2.239	-0.761	0.417
v2	0.082	2.350	-0.650	0.366
v3	-0.014	2.318	-0.682	0.348
v4	-0.203	2.592	-0.408	0.306
v5	-0.812	2.698	-0.302	0.557
v6	-0.743	2.521	-0.479	0.611
v7	-0.496	2.600	-0.400	0.448
v8	-0.419	2.368	-0.632	0.526
v9	-0.382	2.454	-0.546	0.464
v12	0.607	2.431	-0.569	0.588
v13	0.555	2.531	-0.469	0.512
v14	-0.271	2.333	-0.667	0.469
v15	0.022	2.291	-0.709	0.366
v16	-0.578	2.295	-0.705	0.641
v17	-0.372	2.243	-0.757	0.565
v18	-0.164	1.980	-1.020	0.592
v29	0.042	2.084	-0.916	0.479
v30	-0.107	2.141	-0.859	0.483
v31	-0.198	2.447	-0.553	0.375
v33	-0.610	2.876	-0.124	0.367
v34	-0.806	3.277	0.277	0.542
v42r	-0.026	2.137	-0.863	0.444
v43	-0.716	2.875	-0.125	0.420
v44	-0.341	2.133	-0.867	0.604
v45	0.522	2.299	-0.701	0.612
v46	0.558	2.384	-0.616	0.587
v47	0.596	2.271	-0.729	0.663

0.2.2 Bivariate distribution

Based on the univariate analysis, we select three different variable combinations: 1) both are assumed normally distributed; 2) one is assumed normally distributed, but the other one no; 3) neither fo them is assumed normally distributed.

In Figure 2, we can observe the linear relationship between the 3 different combination. When we assume normality in both variables, we observe linear relationship, which decreases for the second combination (one variable assumed normal and the other no), and the relationship disappear for the third combination

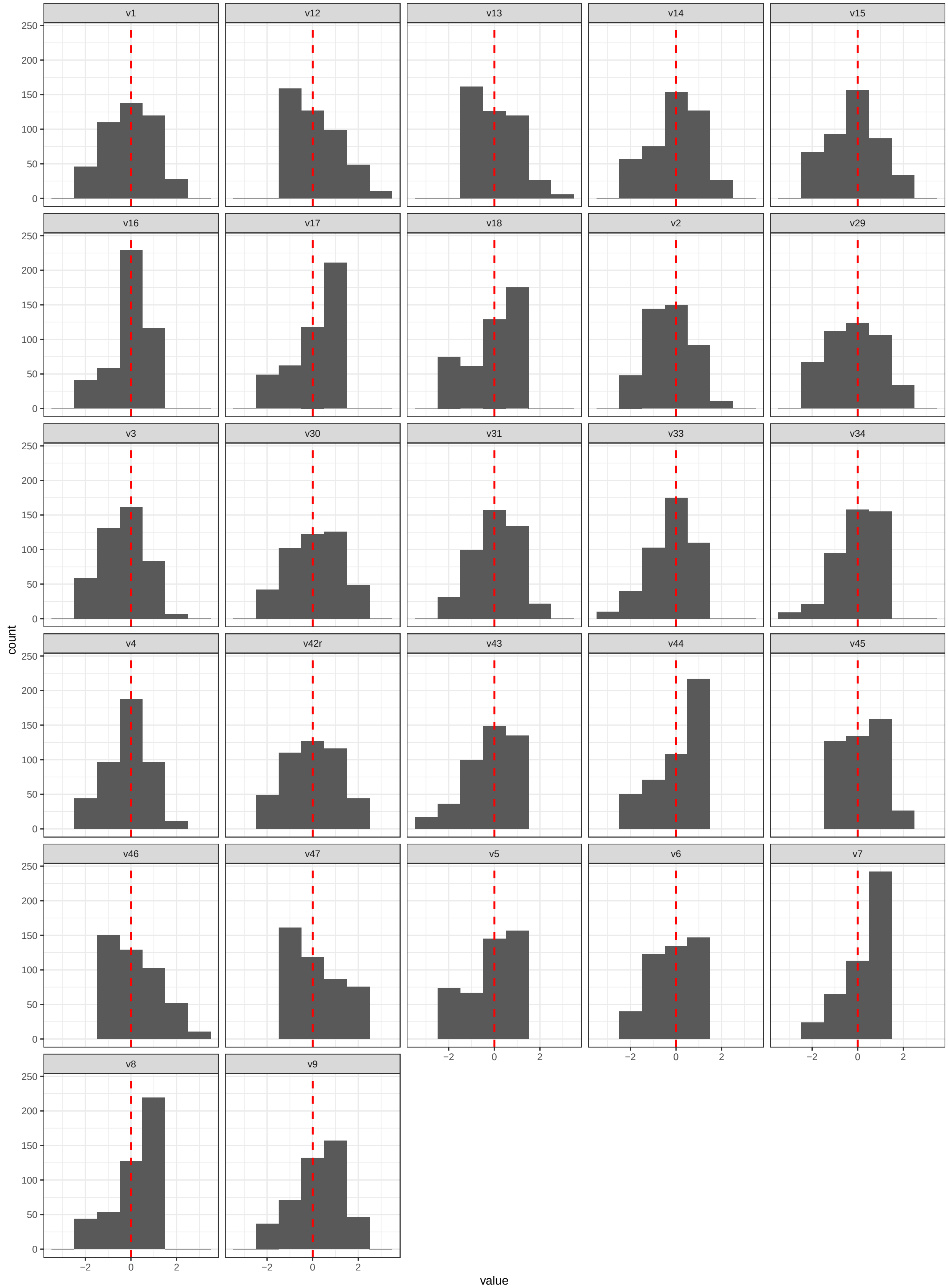
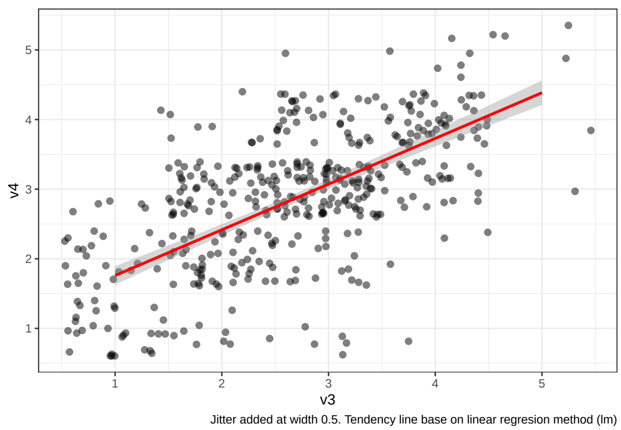
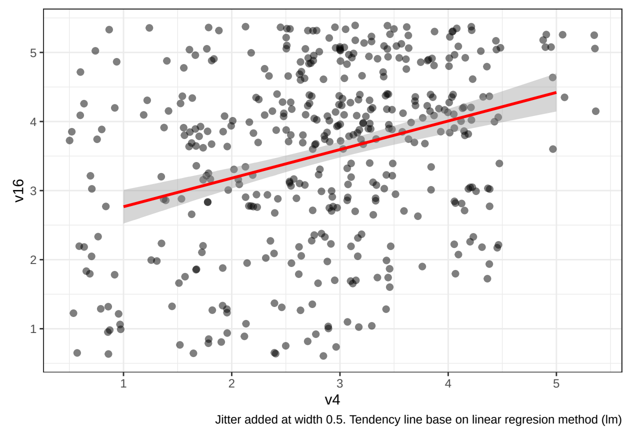


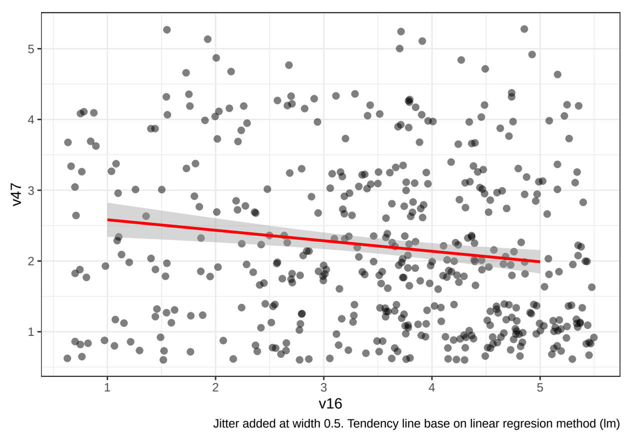
Figure 1: Histograms



(a) Two variables assumed normally distributed



(b) One variable assumed normally distributed



(c) No variable assumed normally distributed

Figure 2: Bivariate distributions

0.2.3 Correlation matrix

We observe now the correlation matrix between all 27 variables (Figure 3). For SEM and other factorial type of analysis, the correlation coefficient should be between ± 0.3 and ± 0.8. Too low correlations indicate weak inter-item dependency, too high correlations might indicate multicollinearity (Nokelainen, n.d.).

The highest correlation coefficient (r) in absolute terms was betwee v18, v17 with \$r = \$0.8344792 and R^2 = `rmax_r2. The lowest correlation coefficient ($r$) was between v45, v7 with $r = $-0.3372552 and $R^2 =$r` min_r2. Therefore, we cna conlcude that our variables are suitable for a SEM model

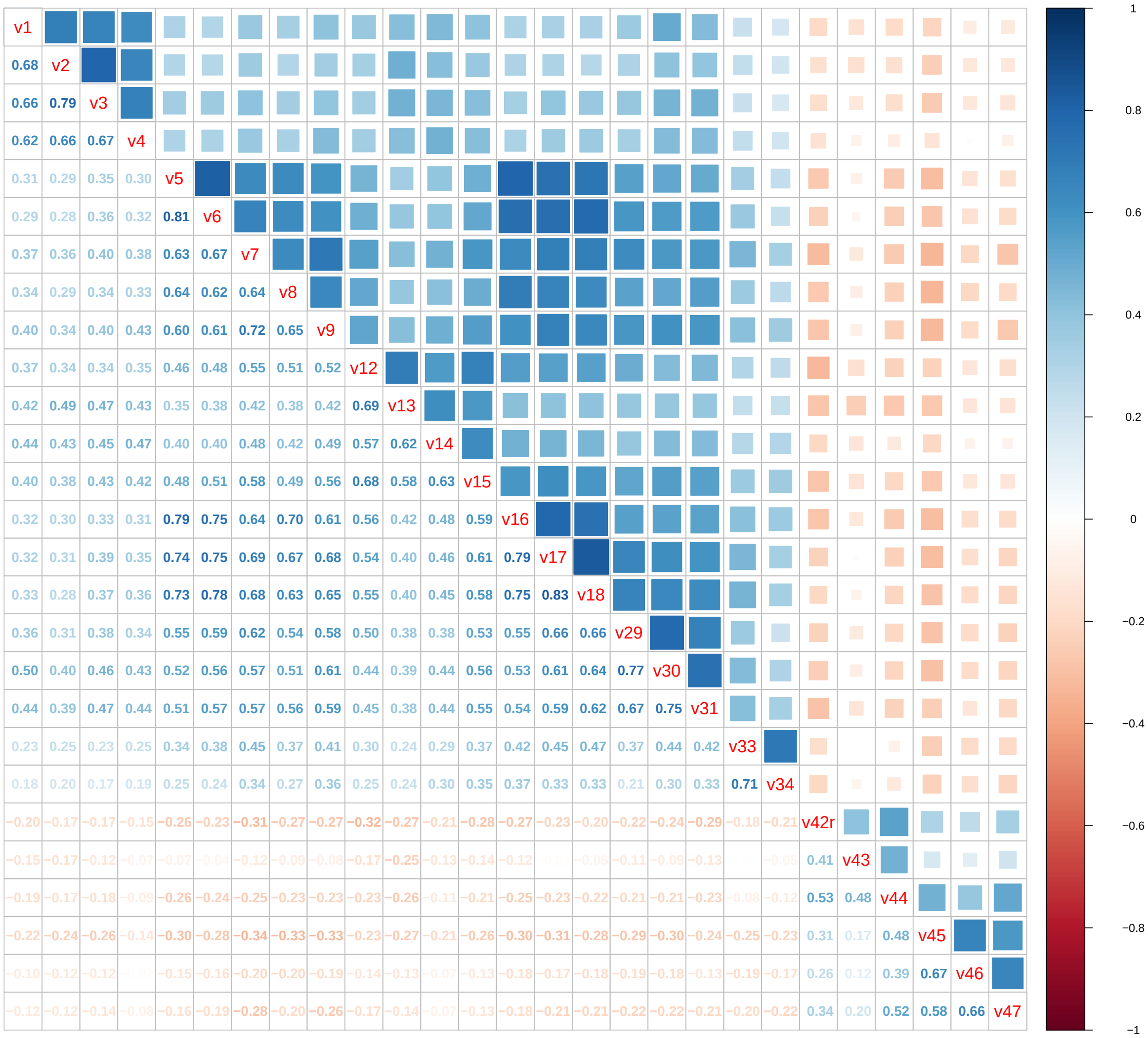


Figure 3: Correlation matrix

0.3 Path Analysis

0.3.1 Model 1

To perform the path analysis we make use of the 13 factors available in our data. The data consists of the thirteen growth-oriented atmosphere factors. The sample is the same as in data1, 447 staff members of Finnish polytechnic institute for higher education. The sample was collected in 2000. Missing values were estimated with maximum likelihood.

The model that we build here examines four predictors (IV's) of valuation of the work (DV). The predictors are: Encouraging leadership, Know-how rewarding, Incentive value of the job, Clarity of the job.

$$ta\_val7 = \beta_0 + \beta_1 kj\_enc1 + \beta_2 op\_rew3 + \beta_3 tk\_inv5 + \beta_4 ts\_cla + \varepsilon_{ta\_val7}$$

where (Table2):

Table 2: Labels for factor variables

variable_label	**variable_name**
1. Encouraging leadership	kj_enc1
2. Strategic leadership	sj_str2
3. Know-how rewarding	op_rew3
4. Know-how developing	ok_dev4
5. Incentive value of the work	tk_inv5
6. Clarity of the work	ts_cla6
7. Valuation of the work	ta_val7
8. Relationship-based learning	vk_rel8
9. Team spirit	rh_tes9
11. Psychical stress of the work	tr_psy11
12. Build-up of work requirements	tv_bui12
13. Commitment to work and organization	si_com13

The following model was fitted (Figure 4):

From the results of the model we obtain the following conclusions:

- How much dependent variable variance the four independent variables predict? $R^2 = 0.527$.
- Order the IVs in the following rows (best predictor comes first):
  1. The first (strongest) predictor for Valuation of the work is *1. Encouraging leadership* $r = 0.45$
  2. The second predictor for Valuation of the work is *5. Incentive value of the work* $r = 0.22$
  3. The third predictor for Valuation of the work is *3. Know-how rewarding* $r = 0.09$
  4. The fourth predictor for Valuation of the work is *6. Clarity of the work* $r = 0.09$
- Select Unstandardized estimates and complete the following sentences:
  - When *Encouraging leadership* goes up by 1, *Valuation of the work* goes up by 0.38.
  - When *Know-how rewarding* goes up by 1, *Valuation of the work* goes up 0.09.

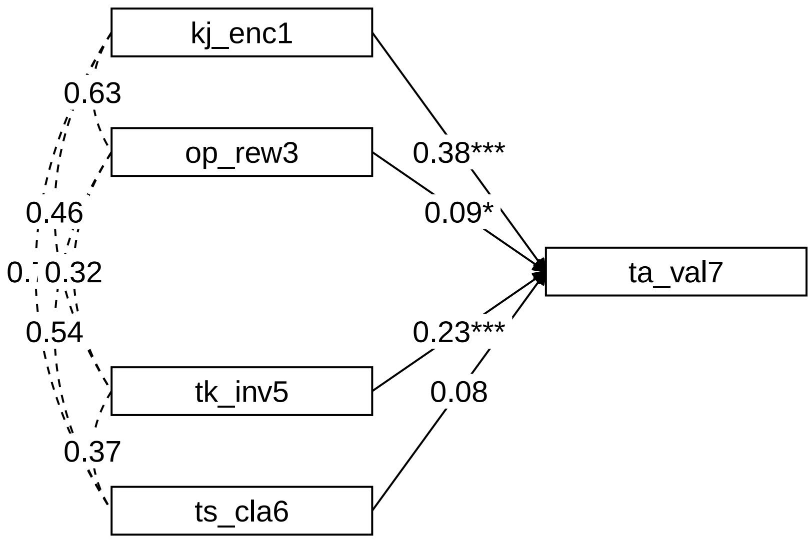


Figure 4: Measurment model

0.3.2 Model 2: adding mediation effect

We modify the model 1 to account for an indirect effect from *Know-how rewarding* (op_rew3) via *Incentive value of of the work* (tk_inv5) to *Valuation of the work* (ta_val7).

$$ta\_val7 = \beta_0 + \beta_1 kj\_enc1 + \beta_2 op\_rew3 + \beta_3 ts\_cla + \beta_4 op\_rew3 * tk_i nv5 + \varepsilon_{ta\_val7}$$

By accounting for the mediation effect, we obtain the following result (Figure 5):

And we can extract the following conclusions:

- How much DV's variance the four IV's predict? Model 1 $R^2 = 0.527$. Compared to Model 2 with mediation 0.65 %.
- How does the indirect path affect the regression model? controlling for the mediation effect have improved the effect of Clarity of the work (ts_cla6) and *Know-how rewarding* (op_rew3). On the contrary have decreased the effect of *Encouraging leadership* (kj_enc1). However, the new model seems to explain better the *Valuation of the work* (ta_val7),a s per $R2$ metric.

0.4 Latent model

0.4.1 Model 1

The model examines how encouraging leadership and build-up of work requirements together affect on psychical stress of the work. The data file (n = 447) is collected in 2000 from Finnish polytechnic for higher education staff and includes their answers to 22 growth-oriented atmosphere questions. Missing values were estimated with maximum likelihood.

While fitting the model (Figure 6), the two latent variables were allowed to correlate in the model. This decision was based on a theoretical assumption. We created error variables for each observed variable setting regression weight of one.

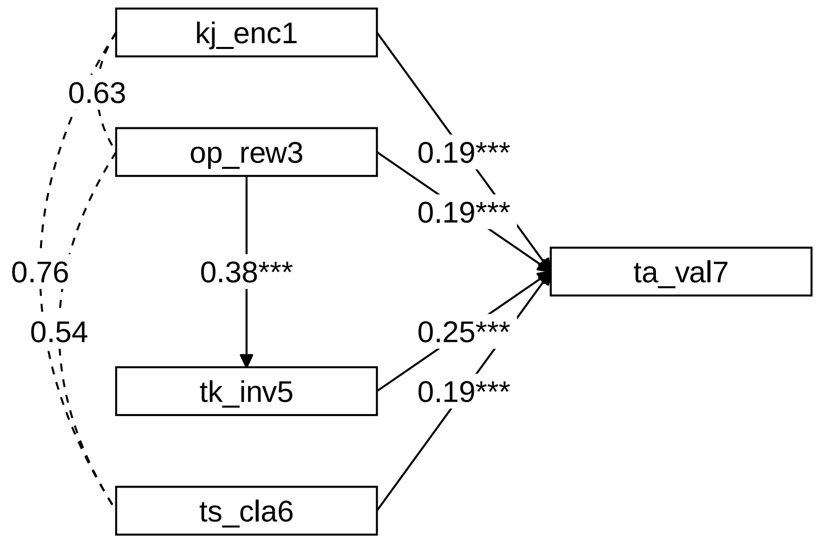


Figure 5: Fitted model with mediation

Furthermore, we set at least one arrow departing from each variable in the model with a regression weight of 1. This procedure helps the model to become identifiable and indicates which one of the observed variables is the ”number one operationalization” of the latent variable.

We extracted the following conclusions form the model:

Consider, on the basis of the standardized estimates of the model, how the two IV’s explain the DV, *psychic stress of the work*:

- Best predictor for *Psychic stress of the work* is 12. Build-up of work requirements $r = 0.53$
- Second best predictor for *Psychic stress of the work* is 1. Encouraging leadership $r = -0.01$

How well the two IV’s predict the DV? $R^2 = 0.64$

0.4.2 Model 2: adding rewarding of know-how

We added one predictor to the model, namely *Rewarding of know-how.* At the same time we remove one variable from *Encouraging leadership*, mainly becasue the excersise was proposed to be completed with AMOS, which in its student version limits the numebr of variabels that can be used.

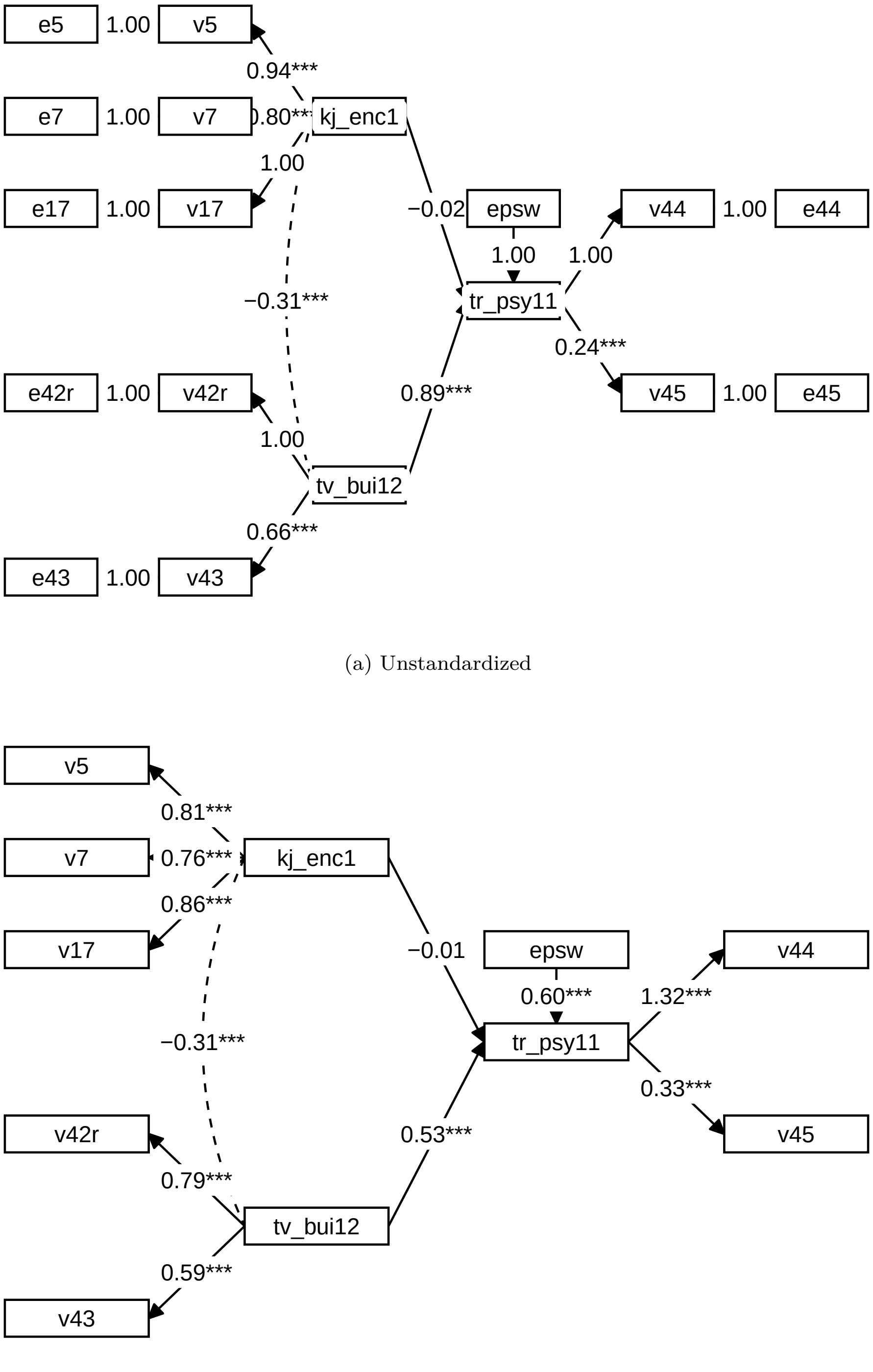
All in all, we obtain the following path diagram for the new model (Figure 7):

How do the three IV’s differ as predictors for the DV?

- The first (strongest) predictor for *Psychic stress* of the work is 12. Build-up of work requirements $r = 0.58$
- The second predictor for *Psychic stress of the work* is 3. Know-how rewarding $r = 0.13$
- The third predictor for *Psychic stress of the work* is 1. Encouraging leadership $r = -0.10$

How well the three IV’s predict the DV? $R^2 = 0.66$

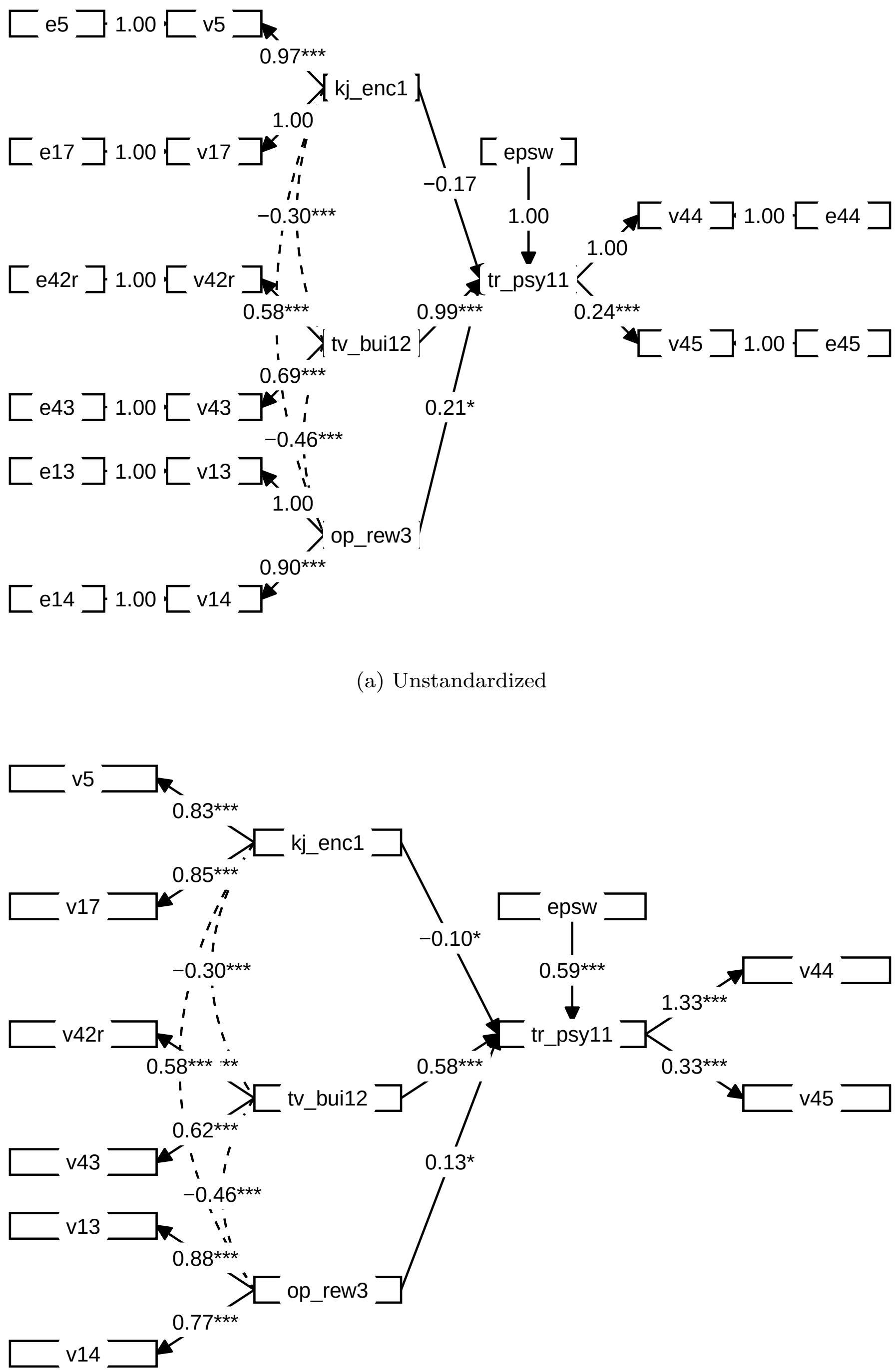
- When *Encouraging leadership* goes up by 1, *Psychic stress of the work* goes down by -0.17.
- When *Build-up of work requirements* goes up by 1, *Psychic stress of the work* goes up 0.99.
- When *Rewarding of know-how* goes up by 1, *Psychic stress of the work* goes up 0.21.



(a) Unstandardized

(b) Standardized

Figure 6: Path diagram for model 1



(a) Unstandardized

(b) Standardized

Figure 7: Path diagram for model 2

0.5 Model comparision and Goodness of Fit

Once we have the two models, we proceed to compare them to see which one offer a better goodness of fit to the data. Table 3 provides the Goodness of Fit measures to compare between model 1 and model 2.

As we can observe, model 1 perform better in all fitness measures, with a lower X^2, lower RMSEA, and higher NFI, CFI and TLI. Although added complexity to the model tend to bring the fitness measures down, the difference is significant as to attribute only to a greater complexity of the second model.

Table 3: Table

measure	**model_1**	**model_2**
chisq	109.118000	487.31900
df	14.000000	25.00000
chisq_df	7.794143	19.49276
pvalue	0.000000	0.00000
rmsea	0.123000	0.20400
rmsea.ci.lower	0.102000	0.18800
rmsea.ci.upper	0.145000	0.22000
nfi	0.903000	0.68200
cfi	0.914000	0.69100
tli	0.871000	0.55500