

Analyzing clickstream sessions as little directed graphs is a powerful way to capture transition-based behaviors—and then comparing graph-level features (or even subgraph patterns) between your satisfied vs. dissatisfied groups can reveal what “paths” are most diagnostic of frustration. Below are a few approaches you might try, ordered roughly from simplest to more advanced.

1. Global Markov-Chain Comparison

1. Build two transition matrices

- One for all satisfied sessions, one for all dissatisfied.
- Each matrix T_{ij} counts transitions from page i to page j , normalized to probabilities.

2. Highlight “hot” edges

- Compute the difference $\Delta T = T^{\text{diss}} - T^{\text{sat}}$.
- Sort edges by $|\Delta T_{ij}|$ to find transitions much more (or less) common in dissatisfied users.

3. Statistical testing

- For top candidate transitions, do a chi-square or proportion-test to confirm significance.

This gives you a ranked list of “most distinctive” page-to-page moves in each group.

2. Per-Session Directed-Graph Features

Treat each session as its own directed graph $G=(V,E)$ where

- V = the set of pages visited,
- E = directed edges for each successive hit (with possible edge weights = counts).

For each session-graph compute simple graph metrics, for instance:

- **Number of unique nodes** (breadth of exploration)
- **Total edges** (path length)
- **Average out-degree** (how “branchy” a session is)
- **Diameter** or **longest shortest-path** (how deep/deviated a session goes)
- **Proportion of self-loops** (repeated page hits)
- **Page-rank** or **in-/out-centrality** of high-traffic nodes

Then compare the distributions of these metrics between satisfied vs. dissatisfied (e.g., via boxplots or a Mann-Whitney U test). Features that differ most can pinpoint behavioral signatures (e.g., dissatisfied users have higher average out-degree, suggesting more aimless clicking).

3. Mining Subgraph/Motif Patterns

Rather than mining sequential patterns with PrefixSpan, you can:

- **Enumerate small k-node subgraphs** (e.g., 2- or 3-step paths) using a graph-mining library (like NetworkX’s `triadic_census` or custom code).
 - **Count motif frequencies per session**, then do a differential analysis (e.g., t-tests) on motif counts between the two groups.
 - This often scales better than full sequence mining, since you limit motif size.
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4. Sequence-Embedding + Clustering

1. **Embed each session sequence** into a fixed-length vector:
 - Use n-gram TF-IDF on page bigrams/trigrams.

- Or train a simple sequence embedding (Word2Vec on pages + average, or doc2vec per session).
 - 2. **Cluster embeddings** (e.g., k-means) and label clusters by their satisfaction rate.
 - Clusters dominated by dissatisfied sessions reveal common patterns.
 - 3. **Interpret via top n-grams** or representative sequences per cluster.
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5. Differential N-Gram Ranking

If full PrefixSpan is too slow, fall back to:

- **Counting all 2- and 3-grams** of pages (sliding window of size 2 or 3) via a single pass through your data.
 - For each n-gram compute support in both groups and rank by difference in support.
 - This is $O(NL)O(NL)$ (where LL is average session length) and much faster than mining arbitrary-length patterns.
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6. Session Graph Embeddings (Graph2Vec)

- Represent each session-graph as a point in embedding space using Graph2Vec (or similar).
 - Train a simple classifier (e.g., logistic regression) on the embeddings.
 - Use the classifier's coefficients or SHAP values to identify which graph "structures" contribute most to predicting dissatisfaction.
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7. Putting It All Together: Sample Workflow in Python

Here's a high-level sketch of how you might code approach (2) with NetworkX:

```

import networkx as nx
import pandas as pd
from scipy.stats import mannwhitneyu

# assume df has columns ['session_id', 'page_sequence', 'satisfaction']
features = []
for sid, group in df.groupby('session_id'):
    seq = group['page_sequence'].iat[0]      # a list of pages
    sat = group['satisfaction'].iat[0]
    G = nx.DiGraph()
    # build edges
    for u, v in zip(seq, seq[1:]):
        if G.has_edge(u, v):
            G[u][v]['weight'] += 1
        else:
            G.add_edge(u, v, weight=1)
    # compute metrics
    num_nodes = G.number_of_nodes()
    num_edges = G.number_of_edges()
    avg_out_deg = sum(d for _, d in G.out_degree()) / num_nodes
    diameter = nx.diameter(G.to_undirected()) if num_nodes > 1 else 0
    features.append({
        'session_id': sid,
        'satisfaction': sat,
        'num_nodes': num_nodes,
        'num_edges': num_edges,
        'avg_out_deg': avg_out_deg,
        'diameter': diameter
    })

feat_df = pd.DataFrame(features)

# Compare distributions
metrics = ['num_nodes', 'num_edges', 'avg_out_deg', 'diameter']
for m in metrics:
    sat_vals = feat_df.loc[feat_df.satisfaction==1, m]
    dis_vals = feat_df.loc[feat_df.satisfaction==0, m]
    stat, p = mannwhitneyu(sat_vals, dis_vals, alternative='two-sided')
    print(f'{m}: U={stat:.1f}, p={p:.3g}')

```

◆ From here you'd pick the metrics with the most significant p-values as your key behavioral differences.

Recommendations

- **Start simple:** try the global Markov-chain edge-difference method (Section 1). It's fast, highly interpretable, and often surfaces the biggest “aha” transitions.
- **Augment with graph features** (Section 2) to capture broader navigation behaviors.
- If you need more nuance, dive into motif mining or embedding-based clustering.

By combining these techniques you'll not only avoid the scalability issues of PrefixSpan on long sessions, but also gain richer, graph-based insights into what frustrated users do differently.