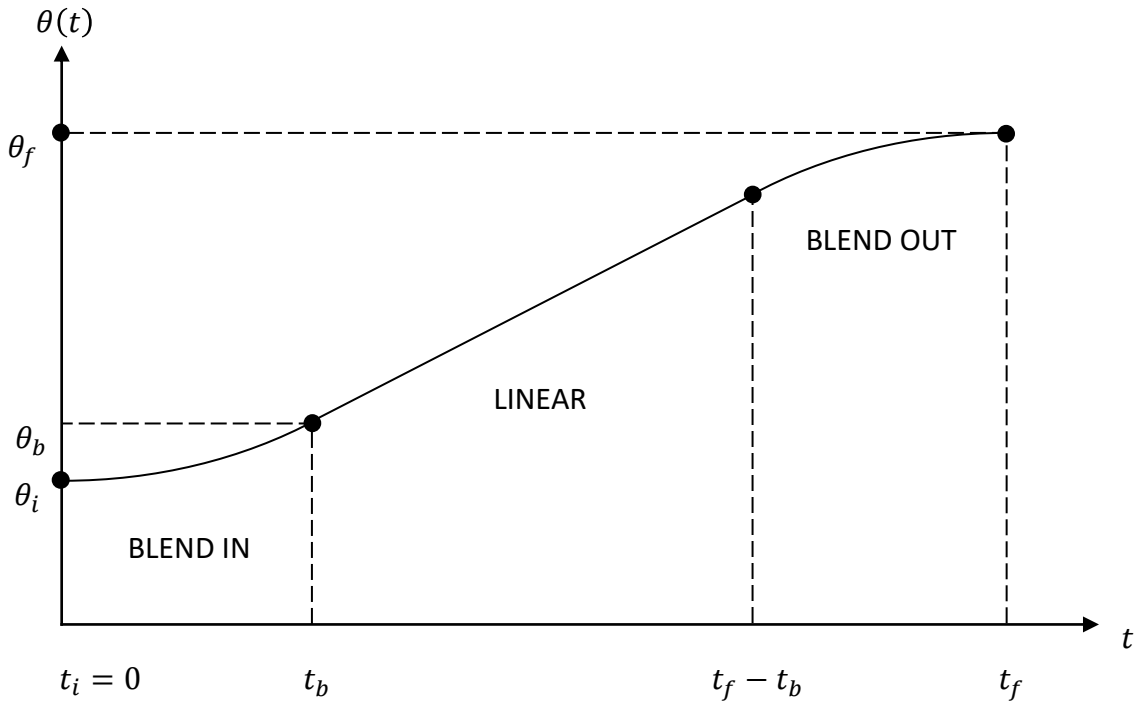
TRAPEZOIDAL (LSPB) FUNCTIONS



BLEND IN SECTION

Equations

$$position \rightarrow \theta(t) = at^2 + bt + c$$

$$velocity \rightarrow \dot{\theta}(t) = 2at + b$$

$$acceleration \rightarrow \ddot{\theta} = acc = 2a$$

Constraints

$$\theta(0) = \theta_i$$

$$\dot{\theta}(0) = 0$$

Coefficients

$$a = \frac{acc}{2}$$

$$b = 0$$

$$c = \theta_i$$

Solution

$$\theta(t) = \frac{acc}{2}t^2 + \theta_i$$

$$\dot{\theta}(t) = acc\ t$$

$$\ddot{\theta} = acc$$

LINEAR SECTION

Equations

$$position \rightarrow \theta(t) = at + b$$

$$velocity \rightarrow \dot{\theta}(t) = a$$

$$acceleration \rightarrow \ddot{\theta} = 0$$

Constraints

$$\dot{\theta}(t_c) = acc\ t_b = a$$

$$\theta(t_c) = \frac{acc}{2} t_b^2 + \theta_i = acc\ t_b^2 + b$$

Coefficients

$$a = acc\ t_b$$

$$b = \theta_i - \frac{acc}{2} t_b^2$$

Solution

$$\theta(t) = acc\ t_b\ t + \theta_i - \frac{acc}{2} t_b^2$$

$$\dot{\theta}(t) = acc\ t_b$$

$$\ddot{\theta} = 0$$

BLEND OUT SECTION

Equations

$$\theta(t) = at^2 + bt + c$$

$$\dot{\theta}(t) = 2at + b$$

$$\ddot{\theta} = 2a$$

Constraints

$$\theta(t_f) = \theta_f$$

$$\dot{\theta}(t_f) = 0$$

Coefficients

$$a = -\frac{acc}{2}$$

$$b = acc\ t_f$$

$$c = \theta_f - \frac{acc}{2} t_f^2$$

Solution

$$\theta(t) = -\frac{acc}{2} t^2 + acc\ t_f\ t + \theta_f - \frac{acc}{2} t_f^2$$

$$\dot{\theta}(t) = -acc\ t + acc\ t_f$$

$$\ddot{\theta} = -acc$$

FINDING Tb AND MINIMUM Acc

$$\theta(t_f - t_b)$$

$$=$$

$$acc\ t_b\ (t_f - t_b) + \theta_i - \frac{acc}{2} t_b^2$$

$$=$$

$$-\frac{acc}{2}(t_f - t_b)^2 + acc\ t_f(t_f - t_b) + \theta_f - \frac{acc}{2} t_f^2$$

$$t_b = \frac{t_f}{2} - \frac{\sqrt{acc^2 t_f^2 - 4\ acc\ (\theta_f - \theta_i)}}{2\ acc}$$

$$acc \geq 4\frac{(\theta_f - \theta_i)}{t_f^2}$$

FINAL EQUATIONS

Blend In

$0 \le t \le t_b$

$$\theta(t) = \frac{acc}{2} t^2 + \theta_i$$
$$\dot{\theta}(t) = acc\ t$$
$$\ddot{\theta} = acc$$

Where

$$t_b = \frac{t_f}{2} - \frac{\sqrt{acc^2 t_f^2 - 4\ acc\ (\theta_f - \theta_i)}}{2\ acc}$$

Linear

$t_b < t < t_f - t_b$

$$\theta(t) = acc\ t_b\ t + \theta_i - \frac{acc}{2} t_b^2$$
$$\dot{\theta}(t) = acc\ t_b$$
$$\ddot{\theta} = 0$$

and

$$acc \ge 4 \frac{(\theta_f - \theta_i)}{t_f^2}$$

Blend Out

$t_f - t_b \ \le t \le t_f$

$$\theta(t) = -\frac{acc}{2} t^2 + acc\ t_f\ t + \theta_f - \frac{acc}{2} t_f^2$$
$$\dot{\theta}(t) = -acc\ t + acc\ t_f$$
$$\ddot{\theta} = -acc$$