Shaping More Than Prices: Corrective Taxes with Product Reformulation

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Abstract

How do corrective taxes work when firms can reformulate products to avoid them? I develop an equilibrium model of product reformulation to study product responses to the 2018 UK Soft Drinks Industry Levy, a multi-tiered tax targeting excessive sugar content. The model isolates the role of reformulation by using interactive fixed effects to account for multiple endogenous unobserved product characteristics, allowing counterfactuals that revert products to their pre-reformulation attributes. I find the levy reduced sugar sales by 22% and led firms to reformulate more than one-third of products, cutting average sugar content by 40% while lowering product quality, differentiation and tax liabilities. Reformulation benefits nearly all consumers, with gains concentrated among lower-income households and modest losses at the top. Larger firms reformulate a greater share of their products and protect profits more effectively than smaller ones. Overall, reformulation reduced sugar intake relative to a no-policy baseline but also constrained the tax's ability to further curb consumption. My results show product responses are first-order for welfare and harm reduction, and that multi-tier taxes leverage them more effectively than the non-tiered taxes commonly applied to sugar-sweetened beverages.

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I Introduction

How firms change their products in response to corrective taxes can matter as much for reducing harm as how consumers respond to higher prices. When firms reformulate their products by adjusting their characteristics and composition, they can reduce the targeted harm while easing the tax burden on consumers and themselves. These adjustments ripple through the market; changing prices, costs, product quality, and the underlying harms that prompted regulation in the first place. Yet most models of optimal tax design still account only for price adjustments, treating products as fixed (See Allcott et al., 2019).

The effects of reformulation, however, are ambiguous and empirically difficult to measure. Consider a tax change that induces soft drinks manufacturers to cut sugar to qualify for a lower tax tier. Such reformulation may make each drink less harmful yet temp consumers to drink more due to lower prices, potentially undoing any health benefits. Weighting these effects requires separating the impact of prices from that of reformulation, as well as accounting for other demand changes occurring at the same time. The task is hard because many product changes are not fully visible. When a firm cuts sugar content, it is likely to change other product characteristics to compensate, some of which are not easily observed. Firms usually adjust multiple product characteristics jointly, not one at a time. So, observed and unobserved changes move together creating a complex endogeneity problem. Changing consumer preferences add yet another layer of noise; blurring efforts to isolate the impact of reformulation.

This paper makes three contributions to understanding how firms' reformulation choices shape the effects of corrective taxes. First, it provides new evidence by documenting large-scale reformulation following the 2018 UK Soft Drinks Industry Levy, a multi-tiered tax targeting excessive sugar content. After the levy was introduced, more than one-third of products were reformulated, cutting average sugar content by 40%. Second, the paper develops and estimates an equilibrium model of product reformulation that accounts for multiple endogenous unobserved characteristics and time-varying aggregate preferences, allowing to separate the effects of changes in preferences, prices and product characteristics. This setup allows counterfactual simulations that assess how reformulation affects both market outcomes and policy performance. I find that, relative to a no-policy counterfactual, the UK Soft Drinks Industry Levy reduced sugar sales by 22%. My results show that tax induced reformulation directly reduced sugar intake but also constrained the potential for further reductions by limiting price increases, preserving both consumer surplus and firm profits. I also find firms adopt new technologies, allowing to add novel ingredients to reformulate their products. Third, the paper uses the model to examine the distributional consequences of reformulation and the role of tax design. Reformulation benefits nearly all consumers, with gains concentrated among lower-income households and modest losses at the top. Larger firms reformulate a greater share of their products and preserve profits more effectively than smaller ones. By contrast, I find that a non-tiered tax, similar to those introduced in other countries, fails to induce meaningful product responses, achieving larger reductions in sugar intake but at a much higher welfare cost.

Taken together, these findings highlight the limits of the most common corrective policy on sugar-sweetened beverages: non-tiered taxes. These taxes offer firms little incentive to make products healthier unless they can eliminate excessive sugar entirely. In contrast, multi-tiered taxes are better aligned with Sandmo's (1975) targeting principle. By linking tax liability to the degree of harm, multi-tiered taxes leverage firms' product responses to encourage innovation, reduce harm at its source, and lessen the burden on both consumers and firms. In a sense, multi-tiered taxes combine Baumol's (1972) idea of acceptable-harm standards with the menu-of-contracts approach from optimal regulation theory (Laffont & Tirole, 1986, 1993), offering a more flexible framework for firms to self-select the nature of their corrective efforts.

The UK Soft Drinks Industry Levy had a profound impact on the non-alcoholic beverage market. Using descriptive evidence, I show that firms adjusted along two key margins. First, they raised prices on both taxed and untaxed products, consistent with strategic complementarities in pricing, and exhibited near-complete pass-through of the tax. Second, they reformulated more than one-third of all products, reducing sugar content to just below the levy's thresholds. Reformulation coincided with the tax's introduction and accelerated an existing downward trend in sugar content. The data reveal that these changes lowered tax liabilities, narrowed product differentiation, intensified price competition, and reduced market concentration. The magnitude of these effects suggests that reformulation is a first-order determinant of how the levy shaped market and welfare outcomes, motivating a structural model to isolate its role.

I develop and estimate an equilibrium model of product reformulation that accounts for changes in unobserved product characteristics using interactive fixed effects. Ignoring these changes would bias estimates, since they are likely correlated with firms' sugar decisions and thus endogenous. Modeling these characteristics also allows me to separate shifts in consumer preferences from reformulation-driven changes. My model then enables me to run counterfactual simulations that hold preferences fixed while reverting product characteristics to their pre-reformulation levels, thereby isolating the role of reformulation in shaping policy outcomes. It also enables me to study the distributional consequences of reformulation by examining welfare changes among consumers with differing price sensitivities, which can be seen as their marginal utility of money.

My results show that the levy reduced total sugar sales by 22%. Firms reformulated roughly 40% of products, cutting their sugar content by an average of 43% and clustering just below the levy's thresholds. Reformulation also prevented an additional 6% increase in prices, allowing a larger volume of drinks to be sold. As a result, sugar sales were higher than they would have been under a no-reformulation scenario under the same tax. At the same time, reformulation preserved consumer surplus and firm profits, while still delivering a substantial reduction in sugar intake relative to a no-policy baseline.

Reformulation has important distributional consequences. For consumers, it benefits nearly everyone, with gains concentrated among lower-income households. The only group left worse off is high-income consumers above the 80th percentile, who place greater weight on product quality than on price. For firms, reformulation allows larger producers to protect profits by adjusting roughly half of their taxable products. Smaller firms reformulate less and are therefore more exposed to the tax's effects, leaving them relatively more affected than they would have been under the same tax without reformulation.

Marginal cost estimates reveal systematic differences in cost structures between reformulated and non-reformulated products. These differences reveal that firms adopted new production technologies to manufacture reformulated goods, suggesting that the tax was effective in spurring industry-wide innovation to support its reformulation efforts. Evidence from ingredient data supports this interpretation: the number of unique ingredients used in the production of non-alcoholic beverages increased, driven partly by the adoption of alternative sweeteners.

My analysis introduces several new methods to address the challenges of isolating the effects of reformulation and ensuring computational feasibility. First, I control for multiple endogenous unobserved product characteristics using Interactive Fixed Effects (factor models) within a discrete choice demand framework, following Moon et al. (2018). This approach exploits the panel structure of the data to correct for endogeneity that arises when observed and unobserved characteristics are correlated. Second, for identification, I connect the latent factors and loadings to economic fundamentals using micro-founded moments, following insights from Hansen and Singleton (1982) and Petrin et al. (2022). This strategy, similar in spirit to the one in Cunha et al. (2010), combines statistical techniques with economic theory to deal with unobserved heterogeneity. The final innovation is computational. I adapt the Minimum Distance–Least Squares estimator proposed by Moon and coauthors (2018) to handle unbalanced panels, which naturally arise when products are reformulated. Rather than re-estimating factors at every parameter value, I use the two-stage expectation–maximization procedure of Norkutė et al. (2021). This method delivers equivalent estimates more efficiently and avoids the bias corrections required by Bai (2009) and Moon and Weidner (2015).

Firms' product responses represent a margin of adjustment that has been largely overlooked in the corrective tax literature since Pigou's (1920) original analysis of externalities. Early studies focused primarily on consumer responses and tax pass-through under uniform commodity taxes. Later research examined optimal corrective taxation under broader forms of market failure: First, emphasizing heterogeneity in consumers' exposure to externalities (Diamond, 1973), and later incorporating consumers' behavioral biases (Bernheim & Rangel, 2004; Farhi & Gabaix, 2020; Gruber & Köszegi, 2001). More recently, Allcott et al. (2019) propose a unified framework that integrates externalities, internalities, and distributive motives within a single model of optimal tax design. Yet this extensive body of work has paid little attention to firms' product decisions as an additional channel through which corrective taxes influence welfare and the correction of market failures.

The importance of firms' product responses for health outcomes from food policies has been highlighted before (Griffith et al., 2017). Such responses can either reinforce or undermine the effectiveness of these policies. Existing studies have focused mainly on changes in product characteristics that are externally observable. Consequently, much less is known about how reformulation affects unobserved product dimensions that shape consumer choices but are not easily measured, such as product taste or quality. Cross-country differences in food consumption patterns suggest that these unobserved factors may play an important role (Dubois et al., 2014), yet most empirical analyses treat them as fixed. For instance, Barahona et al. (2023) assume that unobserved quality remains unchanged after reformulation in response to Chile's front-of-package labeling law, effectively treating it as independent from products' sugar, salt and fat content. Similarly, Griffith et al. (2017) document that UK consumers substituted toward saltier products following voluntary salt-content regulations but do not explore the underlying drivers of this behavior.

In contrast, the framework I develop in this paper allows for simultaneous changes in both observed and unobserved product characteristics. Moreover, these changes need not be independent, and may be freely correlated. This feature helps explain why manufacturers may use potentially harmful ingredients for reasons beyond consumers' explicit preferences for them. Consumers often derive utility from the sensory effects these ingredients produce (such as sweetness, mouthfeel, or texture) rather than from the ingredients themselves. Well-designed policies can therefore encourage firms to provide these sensory experiences through healthier means. In the soft drinks industry, for example, firms can replicate the sweet appeal of sugar by using alternative sweeteners. The results also show that policy design influences how effectively firms can make this substitution, by encouraging the adoption of new production technologies that enable more successful reformulation.

Shifting attention to supply-side dynamics raises questions about the role of concentration and competition in shaping tax effectiveness. I show that a tiered corrective tax can reshape market competition by reducing product differentiation and shifting market shares away from dominant products, thereby lowering concentration and potentially weakening market power. Although concentration and market power are not synonymous, they often move together, so changes in concentration can still have policy relevance. As O'Connell and Smith (2020) note, weaker market power tends to lower prices and expand output. In the absence of reformulation, this could reduce welfare by increasing consumption and amplifying existing biases and externalities. However, my results also indicate that larger firms are more likely to reformulate, which strengthens the policy's health effects. The net welfare implications of reduced concentration are therefore ambiguous.

The paper also contributes to the growing literature on endogenous product offerings.

Crawford (2012) organizes this research into two strands: studies of *whether* firms choose to offer certain products and studies of *where* they choose to position them. My paper belongs to the latter category. Related work includes Fan (2013), who examines newspapers' repositioning following mergers, and Crawford et al. (2019), who analyze welfare effects of ownership structure using a one-dimensional measure of endogenous unobserved quality in the U.S. cable television market. More recently, Petrin et al. (2022) propose an alternative approach to address the endogenous provision of quality. This paper extends that line of research by modeling firms' choices over a multi-dimensional measure of quality, capturing how multiple unobserved product characteristics interact with consumer preferences to jointly determine perceived product quality. It also relaxes a classical assumption in the study of differentiated product markets that unobserved quality is independent of all other observed characteristics (Berry et al., 1995; Nevo, 2001; Petrin, 2002).

Finally, the paper contributes to the literature on factor models for large panel data (Ahn et al., 2013; Ahn et al., 2001; Bai, 2009; Moon & Weidner, 2015, 2017) by proposing an alternative identification strategy based on moments derived from the underlying economic model rather than on arbitrary statistical normalizations. This approach resolves potential inconsistencies between normalization conditions and the economic interpretation of the factors and loadings. By linking latent variables to market fundamentals, it extends the use of factor models to counterfactual simulation and, ultimately, to policy evaluation.

The rest of the paper is organized as follows. Section II describes the UK non-alcoholic beverages market, introduces the Soft Drinks Industry Levy, and presents the data. Section III provides descriptive evidence on the levy's impacts and documents the resulting large-scale product reformulation. Section IV presents the model, and Section V discusses its identification and estimation. Section VI presents the results of the structural estimation. Section VII develops counterfactual simulations to assess the roles of reformulation and tax design. Section VIII concludes and discusses implications for the design of corrective taxes and food policies.

II Market Setting and Data

The UK's non-alcoholic beverage market records annual sales of about £22.3 billion[1] (approximately $30 billion) and includes carbonates, fruit juices, dilutables, bottled waters, and sports and energy drinks. Although it accounts for only 6.9% of the overall UK take-home food and drink market, it is one of the main contributors to national sugar intake. In 2015, juices and soft drinks were the single largest source of dietary sugar for both adults and children, providing up to 39% of intake among children aged 11 to 18.[2] Adults showed similar, though somewhat lower, figures. High sugar consumption from sugar-sweetened beverages is also associated with socioeconomic deprivation: the National Diet

[1]British Soft Drinks Association - 2025 Annual Report

[2]Public Health England (2015) Sugar Reduction: The evidence for action.

and Nutrition Survey reports that adults in the lowest income group consume more sugar than those in higher income groups, and that intake of sugary soft drinks is particularly high among both adults and teenagers in the lowest income group.

Contrary to common perception, non-alcoholic beverages are highly localized in both production and product variety. The British Soft Drinks Association counts more than 100 members, ranging from small and medium-sized producers to large multinationals. Most firms develop locally tailored versions of their concentrated syrups, which are then distributed to domestic bottling and packaging facilities. Only the largest companies operate their own bottling plants, though even they often adapt product formulations to local demand. This production structure makes most products sold in the UK distinct from those available in Europe or the United States. Consequently, firms can adjust their offerings in response to UK-specific policies without these changes spilling over to other markets.

Soft Drinks Industry Levy

The UK Soft Drinks Industry Levy (SDIL) was announced in the 2016 March budget and implemented in April 2018. It applies to non-alcoholic beverages containing added sugar or sugar-containing substances,[3] but excludes sugar substitutes such as natural and artificial sweeteners. The levy is recognized for introducing a novel multi-tiered design which links liability to sugar content by imposing higher tax rates on more sugary drinks while exempting diet and low-sugar beverages.

The levy amount varies with sugar concentration. Drinks containing less than 5 grams of sugar per 100 ml are untaxed. Products with sugar concentrations between 5 and 8 grams per 100 ml are taxed at 18 pence per litre, while those with 8 grams or more per 100 ml are taxed at 24 pence per litre. At the time of the announcement, most sugary products fell in the highest tier. The tiered structure was explicitly designed to encourage reformulation, while the two-year gap before implementation gave firms time to adjust their products in advance of the tax taking effect.

Sugar per 100 ml	Tax (per liter)
Less than 5 grams	£0.00
Between 5 and 8 grams	£0.18
More than 8 grams	£0.24

Table 1: Levy rates based on sugar concentration

Several categories of drinks are exempt from the tax. These include beverages with at least 75% milk or milk substitutes, alcohol replacements, and drinks made solely from fruit or vegetable juice without added sugar. Other exemptions apply to liquid flavourings,

[3] This includes sucrose, glucose, fructose, lactose, and galactose.

powdered drinks, and beverages prepared on site and served in open containers. Infant and follow-on formula, baby foods, and formulated foods intended for total diet replacement or special medical purposes are also excluded.

Since 2018, the levy has raised about £340 million per year, with 97% of revenues coming from products in the highest tier.[4] This is well below the £520 million originally projected when the policy was announced. [5] Projections were later revised downward *"to reflect a judgement that producers will reformulate a higher proportion of their products towards lower sugar content"* than initially expected.[6] This adjustment makes clear that the government expected reformulation to be a central channel of the levy's impact and anticipated it would reduce tax revenues.

Finally, although levy revenues were initially intended for childhood obesity programs, they have been absorbed into the general tax pot since the policy's first year. The government has since announced that, beginning in 2025, the levy will be gradually increased over five years to account for accumulated inflation to preserve its value in real terms.

Data sources

This paper draws on three primary data sources to study market responses to the UK Soft Drinks Industry Levy. The focus lies on how firms adjusted product characteristics and pricing following the policy, as well as how consumers responded across regions and time. All data span the period from January 2010 to December 2023.

The first dataset is a panel of brand-level characteristics I construct drawing primarily from Numerator. I distinguish across brand product lines (e.g., regular, diet, flavored) and for each brand-month I collect information on product type (e.g., cola, lemonade, ginger ale), private-label status, manufacturer, and a comprehensive set of nutritional attributes.

I link this brand-level information to regional purchase data from *Worldpanel* by Numerator's Take Home panel. The dataset covers 11 regions defined by the UK's NUTS1 (ITL1) administrative boundaries. Each observation reflects month-region-product line and includes total monthly expenditure and liters purchased by survey respondents. Additionally, I observe the number of households with beverage purchases by region and month, as well as the total number of households purchasing any beverages during the year. I use these to compute adjusted market shares that include non-participation. That is, households that do not purchase packaged beverages in a given month but did so that year.

To capture cost variation and demand shifters, I collect data from the UK Office for National Statistics (ONS) and other national regulatory bodies. This includes regional wholesale prices for water and prices of key inputs such as sugar and coffee, as well

[4] HM Revenue & Customs, Soft Drinks Industry Levy Statistics Commentary 2024
[5] HM Treasury, Budget 2016 Policy Costings, March 2016, p. 12
[6] Office for Budget Responsibility (OBR), Economic and Fiscal Outlook 2017

as logistical inputs like fuel and vehicle oil. I also incorporate regional demographic information and monthly weather conditions, including temperature and rainfall. All monetary values are deflated using national price indices from the ONS and expressed in 2018 prices.

Brand characteristics

Soft drinks are highly differentiated products, and their appeal cannot be fully captured by observable characteristics. Nonetheless, prior research shows that consumers do respond to information on nutritional content, although there is ongoing debate about the most effective way to present such information.[7]

Guided by this evidence, I collect product-level information on sugar, salt, fat, protein, fibre, and sodium per 100 milliliters. I exclude calorie content due to its near-perfect correlation with sugar (98%), which makes it difficult to separately identify their effects. Using this nutritional information, I compute the implied tax liability for each product under the SDIL's tiered structure. I then construct an indicator for reformulation based on changes in a product's tax liability over time. Finally, I augment this dataset with brand metadata, including the manufacturer's identity, the holding company, and whether the product is a private label owned by a retailer.

Household Purchase Data

The Worldpanel survey (formerly Kantar Worldpanel) provides comprehensive panel data on fast-moving consumer goods purchased by approximately 35,000 households across the United Kingdom over a 15-year period. Households use handheld barcode scanners to record their purchases and are incentivized through non-monetary rewards to ensure high participation and avoid affecting purchasing behavior.

I aggregate monthly sales data at the regional level, excluding smaller regions to focus on the main markets in England, Scotland, and Wales. This yields brand-level sales and volume information for 11 geographic regions spanning the 2010–2023 period. I define a brand as a uniquely identifiable set of products sharing the same formulation or recipe. For example, Pepsi and Pepsi Max are treated as distinct brands, as are Coca-Cola, Coca-Cola Zero, and Diet Coke. I compute prices per litre and calculate market shares based on litres sold. To account for the outside option of not purchasing packaged beverages (e.g., tap water consumption), I adjust market shares by the category's household penetration in each region and month.

Two major changes occurred in the market's product offering during the period (Table 2, Panel B). First, a large number of sugary products were reformulated, particularly those initially in the highest tax tier, many of which moved into the exempt or standard tiers. Second, there was both entry and exit of brands. Together, products that entered the market after 2018 account for less than 9% of total sales.

[7] See Barahona et al. (2023) and Rønnow (2020) for a discussion on food labeling policies.

The resulting dataset includes 985 unique products from 301 manufacturers. This rich panel enables a detailed analysis of consumer substitution patterns and firm pricing and product strategies before and after reformulation.

				Average over each period		
	25%	75%	Max	2010-2015	2016-2017	2018-2023
Panel A - Product Characteristics[a] *(grams per 100 ml)*						
Sugars	1.87	9.04	21.60	6.12	6.64	5.25
Fibre	0.01	0.04	10.40	0.04	0.07	0.07
Fat	0.00	0.01	4.04	0.02	0.02	0.02
Sodium	0.01	0.02	11.70	0.07	0.03	0.02
Protein	0.01	0.05	5.05	0.06	0.06	0.07
				Average over each period		
	25%	75%	Max	2010-2015	2016-2017	2018-2023
Panel B - Count of product offering by type[b]						
All products	417	456	480	443	456	430
Private label	154	201	211	202	178	148
Reformulated	44	133	141	42	74	133
Sugary (Taxed)	207	265	281	246	271	193
Highest tier (£0.24)	101	224	239	211	230	100
Standard tier (£0.18)	37	85	118	35	41	93
Exempt	191	222	273	196	185	236
				Average over each period		
	25%	75%	Max	2010-2015	2016-2017	2018-2023
Panel C - Inflation-Adjusted Prices (Base Year = 2018)						
Price per litre	0.45	1.80	6.22	1.12	1.27	1.39

[a] Excluding entries with no nutritional content, such as bottled water.
[b] Aggregated across all markets within each year.

Table 2: Summary Statistics

Cost and Demand Shifters

In estimating demand, I treat both prices and observed product characteristics as potentially endogenous. To address this, I construct instruments for retail prices and brand attributes using variation in firms' input costs. Specifically, I exploit regional and temporal variation in the prices of key inputs to beverage production and distribution, which serve as cost shifters plausibly exogenous to demand shocks.

Input price data are sourced from the UK's Office for National Statistics (ONS), which publishes the price quotes underlying the Consumer Price Index (CPI) as well as industry-level prices from the Producer Price Index (PPI) series. I include input costs for major beverage components[8] (e.g., sugar, water, coffee) and logistical inputs (e.g., vehicle fuel, motor oil). These cost variables vary across regions and over time, providing exogenous variation for identification of the demand parameters.

Additionally, I include information about two sets of demand shifters. First, weather variables including the monthly regional averages for temperature and rainfall. This information comes from the UK's Meteorological Office (Met Office) and helps capture seasonal and climatic variation in beverage consumption patterns. I also incorporate regional measures of wealth, proxied by average house prices, which enter the specification interacted with the price coefficient to capture differences in price sensitivity across wealthier and poorer regions.

III Descriptive analysis

The Soft Drinks Industry Levy had a substantial impact on consumers' sugar consumption. As Figure 1 shows, average sugar content in soft drinks was already declining modestly prior to the policy's announcement in 2016. In contrast, when the levy took effect in 2018, the average sugar content per item sold declined sharply by roughly 40 percent, marking a clear structural break from the earlier trend.

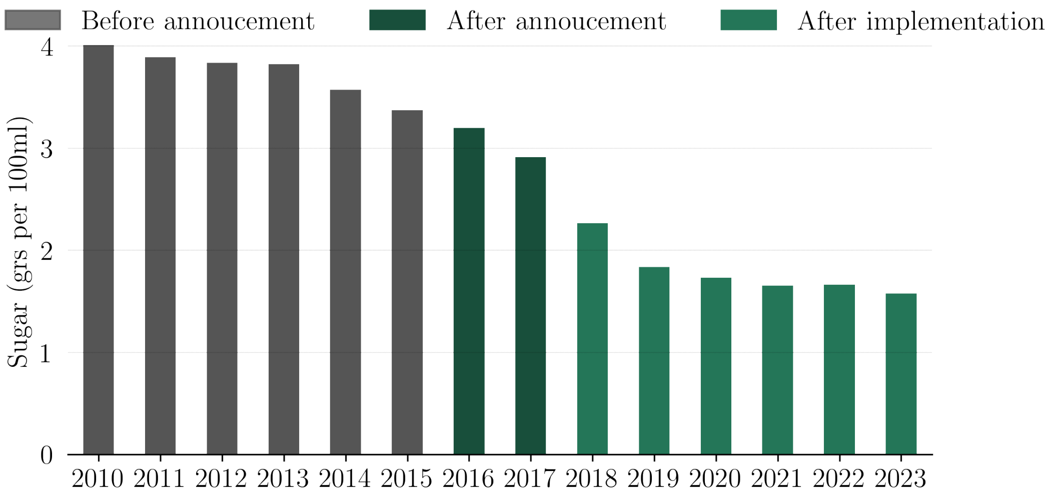


Figure 1: Volume-weighted sugar per product sold.

This change in sugar intake can't be explained by changes in relative prices alone. Figure 2 shows that the prices of both sugary and non-sugary products rose with the introduction of the levy and continued to rise thereafter. This pattern is consistent with strategic complementarity in pricing. That is, firms appear to have adjusted both taxed and untaxed products in order to preserve relative price ratios across their product portfolios.

[8] The choice of variables was guided by discussions with industry experts and the availability of regional-level data.

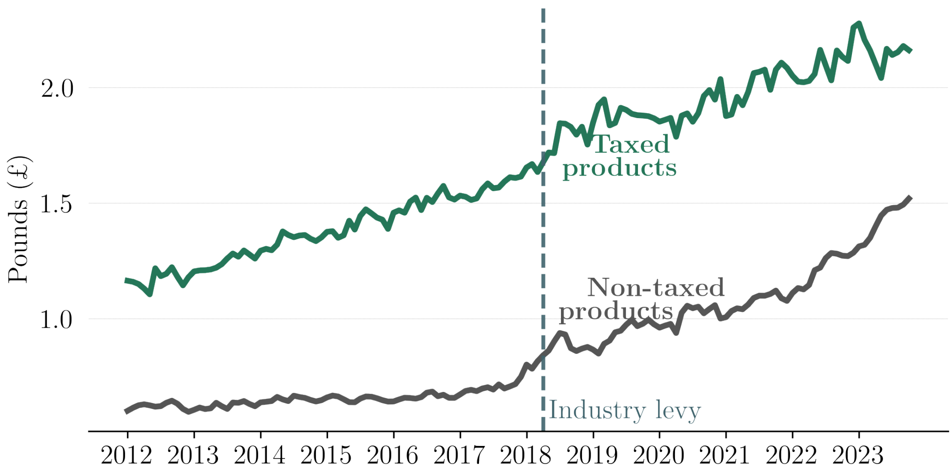


Figure 2: Price of taxed and non-taxed products

A more compelling explanation for the decline in sugar intake lies in changes in products' sugar content. Before the tax, sugary drinks were concentrated at the upper end of the sugar-content distribution, often well above the 8 grams per 100 ml threshold of the levy's highest tier (see Figure 3). After the levy was introduced, most sugary products clustered just below the tax thresholds. This change implies that between 2016 and 2018 firms reformulated their products in direct response to the levy's design. In doing so, they optimized along the intensive margin of product characteristics, adjusting formulations to fully or partially avoid the tax.

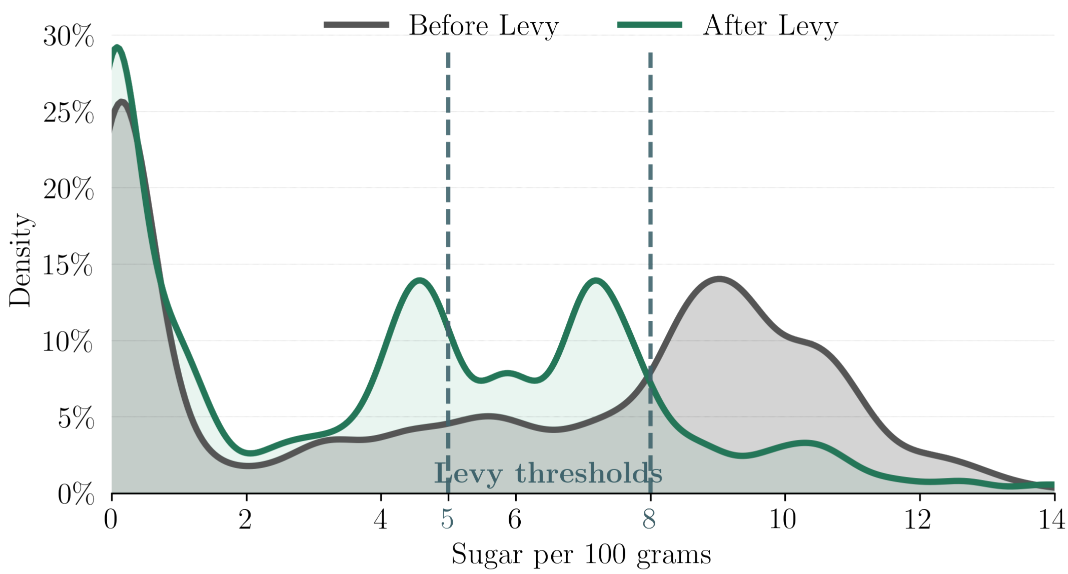


Figure 3: Sugar-content per 100 mls (grams)

The timing of this reformulation further underscores its connection to the policy. While the industry had previously experimented with reducing the sugar content of less popular products, it was only after the levy's announcement in 2016 that firms began to reformulate their more popular items. As shown in Figure 4, these efforts accelerated following the announcement and spiked in 2018, when the levy came into effect.

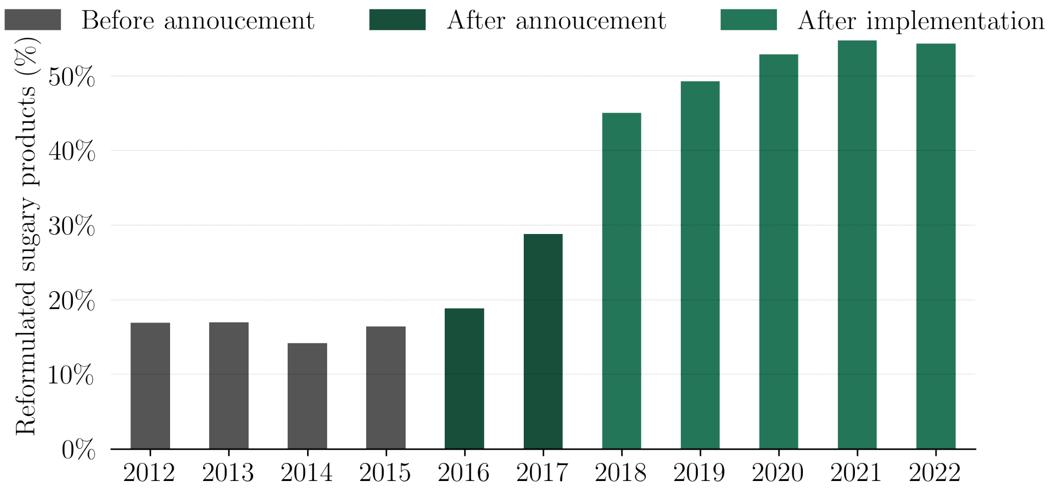


Figure 4: Share of reformulated products per period

Reformulation appears to have influenced not only product composition, but also the broader competitive dynamics of the market. This effect is particularly evident in the reduced price dispersion across taxed products (Figure 5), which suggests heightened competitive pressure on prices. Notably, the most pronounced reduction in price variability occurred within the mid or standard rate tax tier. By contrast, non-taxed products showed an opposite trend, with price dispersion widening as these items claimed a larger share of the market.

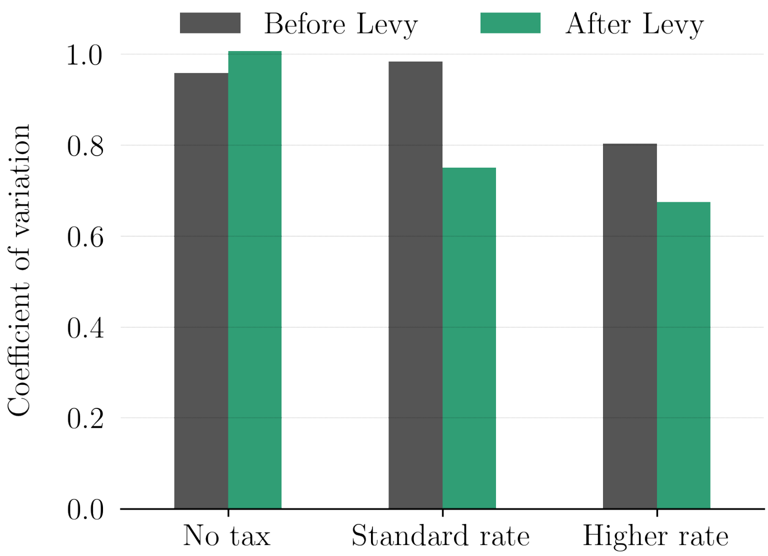


Figure 5: Price dispersion across tax brackets

The change in the market's competitive dynamics is further underscored by the noticeable decrease in market concentration following the tax implementation. As revealed in Figure 6, the product-level Herfindahl-Hirschman Index (HHI) experienced a downward trend across markets, signaling a more evenly distributed competitive landscape. Moreover, not only did the average HHI decline, but its variability also diminished, suggesting a stabilization in market concentration levels.

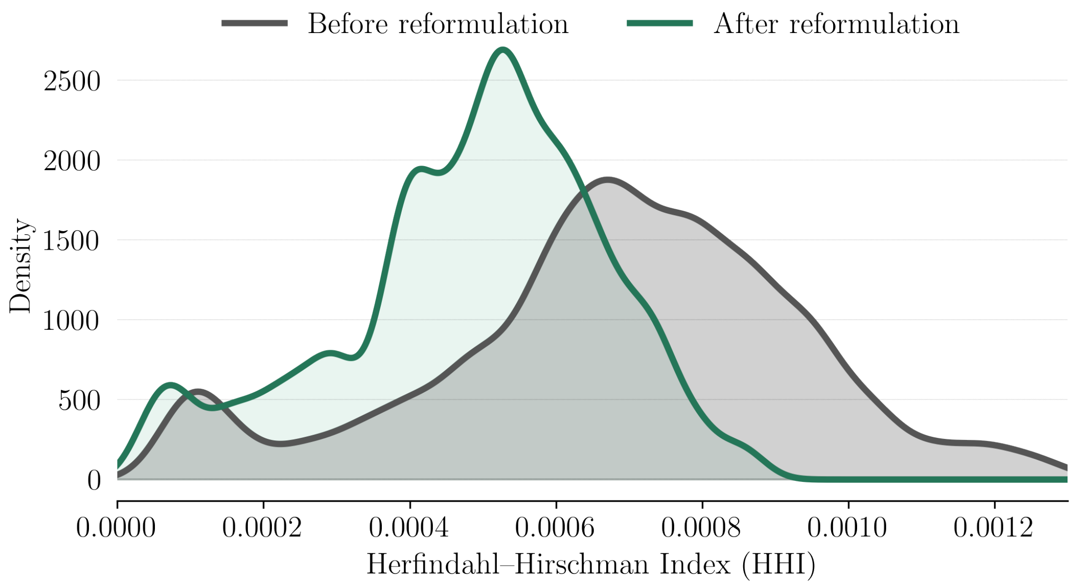


Figure 6: Product-level concentration before and after the policy

These changes in the competitive market dynamics show that products became more homogeneous after reformulation. The shrinkage of the product space may have arisen from reductions in sugar content, particularly if these changes narrowed the range of unobserved quality available in the market. Although this interpretation is consistent with the data, it does not constitute definitive evidence. An alternative explanation for the increased homogeneity is that production costs converged, narrowing price differences among firms.

A structural model is needed to examine these changes further and to quantify the impact of reformulation and price increases on welfare and on the reduction of sugar intake.

IV Model

This section develops an equilibrium model in which firms choose whether and how to reformulate their products in response to the tax policy by adjusting both observed and unobserved characteristics. Firms select these characteristics jointly, implying that choices across dimensions are correlated. Movements in observed characteristics therefore signal movements in unobserved ones, providing a source of identification from variation in observable characteristics. Demand is modeled as a mixed multinomial logit with interactive fixed effects, following Moon et al. (2018), which allows utility from unobserved characteristics to be correlated with that from observed ones. Consumers are assumed to be fully informed about all product characteristics and to exhibit heterogeneous preferences that vary across markets and idiosyncratically in their sensitivity to prices.

The timing of the model unfolds as follows: At the beginning of each period, firms observe realizations of cost shocks (such as changes in input prices) and decide whether to reformulate their products relative to the previous period's offerings. If they reformulate, they incur a per-product sunk cost each time a product is changed. They then observe their rivals' product portfolios and the market-specific demand shifters, and set prices optimally under Nash–Bertrand competition. Firms solve this problem by working backward from the pricing stage: they compute the equilibrium profits that would arise under any possible set of product offerings and then choose the product characteristics that maximize those profits. The econometrician mirrors this structure, solving the problem in the same way.

Demand

Let a market be defined by a region-month-year combination, then the indirect utility that consumer i derives from product j in market t is determined by its preferences over prices (p_{jt}) and observed product characteristics (x_{jt}), together with an unobserved utility component (ξ_{jt}) and an idiosyncratic logit error term (ϵ_{ijt});

$$u_{ijt} = (\alpha_t + \tilde{\alpha}_i)p_{jt} + \beta_t \cdot x_{jt} + \xi_{jt} + \epsilon_{ijt} \tag{1}$$

Market shares arise from aggregating consumers' discrete choices among differentiated products. The share of product j in market t depends on three factors:[9] (i) its mean utility in that market (δ_{jt}), (ii) idiosyncratic consumer-specific shocks to price sensitivity, modeled as a random coefficient ($\tilde{\alpha}_i$), and (iii) the set of competing products available. The random coefficient $\tilde{\alpha}_i$ is assumed to follow a normal distribution with mean zero and variance Σ. This term plays a central role in evaluating the welfare and distributional effects of the corrective tax because it captures heterogeneity in consumers' price sensitivity. Such heterogeneity, in turn, serves as a proxy for differences in the marginal utility of money across consumers within a market.

$$s_{jt}(\delta, X_t, p_t \mid \Sigma) = \int_{-\infty}^{\infty} \frac{e^{\delta_{jt} + \tilde{\alpha}_i p_{jt}}}{1 + \sum\limits_{k \in \mathcal{J}_t} e^{\delta_{kt} + \tilde{\alpha}_i p_{kt}}} dF(\tilde{\alpha}_i \mid \Sigma) \quad \text{for all } j \text{ and } t \tag{2}$$

Products' mean utilities depend on prices, the vector x_{jt} of observable characteristics of dimension $M \times 1$, and the unobserved component ξ_{jt} that captures the utility contribution of unobserved product characteristics. Observable characteristics include nutritional information and brand-related features, summarized in Table 2.

$$\delta_{jt}(x_j, f_j) = \alpha_t \cdot p_{jt} + \beta_t \cdot x_{jt} + \xi_{jt} \tag{3}$$

Just as consumers derive utility from observed product characteristics, they are also assumed to value unobserved ones. I capture this intuition by decomposing the utility component ξ_{jt} into the interaction of two elements: product-specific factors (f_j), which

[9] Berry (1994)

represent unobserved characteristics of each beverage, and market-specific loadings (λ_t), which capture how average preferences for these attributes vary across markets. Formally, this structure follows the *Interactive Fixed Effects* framework of Bai (2009). An additional error term (e_{jt}) accounts for any remaining demand shocks specific to a product and market.

$$\xi_{jt} = \lambda_t \cdot f_j + e_{jt} \tag{4}$$

The factors and loadings are treated as non-random parameters to be estimated from a large panel of market data. The unobserved factors f_j form an $R \times 1$ vector of product characteristics for which relevant data are missing (e.g., distribution intensity) or that are inherently difficult to quantify (e.g., product taste). Such unobserved characteristics may also include information on undisclosed ingredients or proprietary manufacturing methods that give a product its distinctive flavor or experience, akin to Nevo's famous *"mushiness"* in ready-to-eat cereal (Nevo, 2001). Similarly, the loadings λ_t constitute an $R \times 1$ vector that captures how the average valuation of the unobserved attributes f_j varies across markets.

Allowing the fixed-effect vectors to be freely correlated with both prices and observed characteristics is a distinctive feature of my model. This flexibility relaxes the standard exogeneity assumption used since Berry et al. (1995), which requires unobserved utility components to be independent of observed product characteristics. By permitting such correlation, the model addresses the endogeneity that arises when firms choose all product characteristics jointly rather than in isolation, and it helps distinguish demand shifts driven by consumer preferences from those resulting from product reformulation. This condition can be expressed formally as:

$$\mathbb{E}[\xi_{jt} \mid x_{jt}, p_{jt}] \neq 0$$

Intuitively, the unobserved interactive fixed effects are identified from the principal components (PC) of the residual covariance matrix of unobserved utilities within a large-panel framework. The joint growth of N and T is crucial for identification. It transforms what would otherwise be a fixed dimension setting, where the incidental parameters problem induces inconsistency; into one where the factor space can be consistently estimated. In this environment, the least-squares estimator effectively controls for unobserved factors that generate cross-sectional and temporal correlations in the data. I then use moments derived from the economic model to resolve the rotational indeterminacy of the factors, ensuring they have a clear structural interpretation.

$$F = PC\left(\frac{\xi\xi'}{T}\right) \tag{5}$$

Supply side

Firms engage in a two-stage competitive process. First, they all establish their offerings simultaneously by choosing both observed and unobserved product characteristics. Then, after reviewing the availability of products in the market, they engage in price competition following a Nash-Bertrand game. Thus, profits of firm f depends on products prices, marginal costs, and the full set of available products (J_t), as shown below.

$$\Pi_{ft}(J_t, \{p_{jt}\}_{j \in J_t}) = \sum_{j \in J_{ft}} (p_{jt} - c_{jt} - \tau_{jt}) \times s_{jt}(\delta_t, p_t) \tag{6}$$

Where the product's marginal cost c_{jt} depends on its observed and unobserved characteristics.

A product's industry tax τ_{jt} is calculated based on sugar concentration, measured in grams per 100 milliliters. Because sugar concentration is directly observable from the product's nutritional label, it is included in x_{jt}. Denoting sugar concentration by g, the total tax on a product (expressed in 2018 values) is given in Equation 7. It is important to note that the tax rate is fixed over time and therefore erodes in real value each year due to inflation.[10]

$$\tau_{jt}(g_{jt} \in x_{jt}) = \begin{cases} 0 & \text{if } g_{jt} < 5 \\ 0.18 & \text{if } 5 \leq g_{jt} < 8 \\ 0.24 & \text{if } g_{jt} \geq 8 \end{cases} \tag{7}$$

Firms set their product offering common to all markets within a given period. This involves defining the complete set of characteristics of each product. Thus, the firm aims to set its product offering to maximize the expected market profits conditional on its information set, denoted as I_f. In doing so, the firm faces a reformulation cost (κ) to alter the characteristics of one item on its menu. This reflects the fixed costs of adjusting production lines and the logistical challenges necessary to deploy the changed product in the market. As a result, the firms' problem can be written as:

$$\Pi_f = \max_{J_f = \{x_j, f_j\}} \max_{\{p_{jt}\}_{j \in J_{ft}}} \mathbb{E}\left[\Pi_{ft} \mid I_f\right] - \kappa(J_{ft-1}, J_{ft}) \tag{8}$$

Following Petrin et al. (2022), the information set varies across firms and may or may not include details on cost shifters, information about its own and others' product offerings, and signals related to these variables. Because of the uncertainty on the market conditions, firms may sometimes make mistakes on their product offering. However, under rational expectations, firms anticipate future events accurately, resulting in errors that, on average, cancel each other out (Hansen and Singleton, 1982). As a consequence, the first-order conditions of profit maximization can be expected to hold on average.

[10] In 2025, the government announced its intention to raise the tax to preserve its real value.

$$\mathbb{E}\left[\frac{\partial \Pi_f}{\partial x_j} \mid I_f\right] = 0, \quad \mathbb{E}\left[\frac{\partial \Pi_f}{\partial f_j} \mid I_f\right] = 0 \tag{9}$$

These conditions can then be used to break down ξ_{jt} and identify the unobserved product characteristics (f_j) and consumers preferences over those characteristics (λ_t).

Cost parameters

I assume costs are based on the industry's manufacturing process. Typically, soft drinks companies create a concentrated syrup that defines the drink's taste, flavor, and nutritional content. This syrup is sent to processing facilities, that add water and CO_2 as specified by the syrup makers and package the final product into cans, bottles, and other formats. The packaged drinks are then distributed to wholesalers or retailers for sale. Therefore, each beverage can be seen as the combination of multiple inputs (W_z) that collectively determine its observed and unobserved characteristics as outputs.

I assume that for any given set of output characteristics, firms select the combination of inputs that minimizes the products marginal costs, given the prevailing input prices (p_w). Consequently, the marginal cost function takes the form:

$$c_{jt} = \min_{\{W_z\}} C\big(x_j, f_j, p_w \mid \rho\big) \tag{10}$$

Where ρ is the vector of parameters of the marginal cost function.

Capturing unobserved reformulation

Observed changes in sugar content are likely correlated with other changes in product characteristics, since firms choose all attributes jointly. Sugar is a key ingredient for manufacturers because it shapes several dimensions of a beverage. Most directly, it determines sweetness, but it also affects calorie content and the drink's texture.[11] These are characteristics that consumers value highly but that are not solely determined by sugar content, making its overall role difficult to measure directly. When firms respond to the tax by reformulating their products to reduce sugar content and thereby lessen their tax burden, they are also likely to alter other product characteristics. These changes are especially likely to affect unobserved characteristics that are difficult or impossible for the econometrician to measure directly.

To capture these unobserved changes within the model, I treat each reformulated product as a distinct entity in the model, requiring the estimation of a new vector of latent factors for its post-reformulation version. Let j' denote the reformulated version of product j, with unobserved characteristics represented by the latent factors $f_{j'}$. The unobserved component of the reformulated product's mean utility is then defined as:

[11] For a discussion of the use of sugar in food and beverages, see Koivistoinen and Hyvönen (1985)

$$\xi_{jt} = \begin{cases} \lambda_t \cdot f_j + e_{jt} & \textit{before reformulation} \\ \lambda_t \cdot f_{j'} + e_{jt} & \textit{after reformulation} \end{cases} \tag{11}$$

This structure allows the model to separate unobserved changes in demand from unobserved changes in supply. Variation in λ_t captures shifts in consumers' preferences for unobserved product characteristics across markets and over time, while the change from f_j to $f_{j'}$ reflects firms' endogenous adjustments to those characteristics through reformulation. In the data, significant reductions in sugar content mark the moment of reformulation. More precisely, I use changes in a product's tax liability as an indicator of reformulation, allowing unobserved characteristics to vary in tandem with changes in sugar content and other observed attributes.

V Identification and Estimation Strategy

My identification and empirical strategy proceeds sequentially: each stage uses parameters recovered in earlier steps until the full set of model parameters is identified. Table 3 below offers an overview of the whole process.

Parameters		Approach
Mean utilities	$\delta^{(\Sigma)}$	Demand inversion
Demand parameters	$\Sigma, \alpha, \beta, \xi_{jt}$	Factor model
Marginal Costs	c_{jt}	Pricing FOC
Cost parameters	ρ_w, ρ_x	OLS Regression
Factors & Loadings	λ_t, f_j	Supply moments
Reformulation costs	κ	Partial identification

Table 3: Overview of the identification process

Demand side parameters

The identification of demand-side parameters follows Moon et al. (2018), who separate them into two groups. The nonlinear parameters capture how consumer preferences vary across individuals (the variance of random coefficients, Σ) and appear only inside the integral in Equation 2. The linear parameters affect products' mean utilities directly (Equation 3). The nonlinear parameters are identified using exogenous variation from a valid set of instruments (z_{jt}), for which I use information on cost shifters. The linear parameters are then estimated through a factor regression on the mean product utilities.

Key to this identification strategy is that, once the nonlinear parameters (Σ) are known, market shares uniquely determine products' mean utilities through a one-to-one mapping (Berry, 1994; Berry et al., 2013). In practice, this involves solving the nonlinear system described in Equation 2 for all products in each market, a procedure known as *demand inversion.* Intuitively, once we know each product's characteristics and how preferences vary across consumers within the market, shares tell us how attractive each product must be on average to rationalize those choices.

$$\delta_t^{(\Sigma)} = s^{-1}(s_t|\Sigma)$$

The identification argument proceeds in two steps:

First, conditional on the nonlinear parameters, the remaining identification problem for $\delta_{jt}^{(\Sigma)}$ becomes linear and can be addressed using a factor structure with interactive fixed effects (Bai, 2009; Moon & Weidner, 2015). The regression for the linear parameters (Equation 3) is then augmented with a set of valid instruments (z_{jt}), which enter as auxiliary regressors (Equation 12). The parameters are identified as the solution to the the corresponding least-squares problem. At this stage, it is possible to identify the coefficients on prices, observed characteristics, instruments, and the utility component associated with unobserved product characteristics, ξ_{jt}, which consists of the joint product $\lambda_t \cdot f_j$ and the idiosyncratic error term (Equation 4).

$$\delta_{jt}^{(\Sigma)} = \alpha_t \cdot p_{jt} + \beta_t \cdot x_j + \lambda_t \cdot f_j + \gamma \cdot z_{jt} + e_{jt} \tag{12}$$

Second, the exclusion restrictions implied by the instruments (Equation 13) are used to validate the assumed value of the nonlinear parameters, ensuring that the correct Σ is identified. Moon et al. (2018) show that, under valid instruments, the coefficient on the instruments, γ, must be zero if the exclusion restriction holds. They further prove that this condition is satisfied only at the true value of the nonlinear parameters. Hence, the estimated coefficient on the auxiliary regressors provides a test of the assumed Σ. The intuition is that, with valid instruments, the exclusion restriction holds only when the recovered $\delta_t^{(\Sigma)}$ corresponds to the true mean utilities implied by the data.

$$\hat{\gamma}(\Sigma) = 0 \iff \Sigma = \Sigma_o \tag{13}$$

Where γ is the least-square solution to the augmented mean utilities regression (Equation 14).

$$\gamma(\Sigma) = \underset{\gamma}{\arg} \min_{\alpha_t, \beta_t, \gamma} \mathbb{E}\left[\left(\delta_t^{(\Sigma)} - (\alpha_t \cdot p_{jt} + \beta_t \cdot x_j + \lambda_t \cdot f_j + \gamma \cdot z_{jt}) \right)^2 \right] \tag{14}$$

For estimation, Moon et al. (2018) propose a two-step estimator referred to as *Least Squares—Minimum Distance (LSMD)*, that mirrors the identification argument. In the first step, the estimator uses a factor regression to solve the least-squares minimization of the residuals of Equation 12. This step in nested in the second one, which minimizes the distance between the estimated coefficients on the the instruments and those implied by the exclusion restriction.

In practice, the LSMD estimator proposed by Moon et al. (2018) is computationally efficient but only applicable to balanced panels. Previous work has extended linear regressions with interactive fixed effects to unbalanced panels (e.g., Bai et al., 2015; Bai, 2009 Supplemental material), but these methods rely on a nested expectation–maximization (EM) algorithm for each set of parameters evaluated during estimation. This nested structure can become computationally infeasible in real-world applications with unbalanced panels, and computing time grows with the number of missing observations in the panel. The issue is particularly relevant in my context, where product reformulation generates long gaps in the panel surrounding reformulation events.

To address this, I compute the interactive effects regression in an unbalanced panel by solving the least-squares stage using the approach proposed by Norkutė et al. (2021). This method delivers equivalent estimates more efficiently and scales well with missing data. Their two-step procedure replaces the nested EM algorithms with only two EM iterations to remove the influence of common latent factors in the regressors and outcomes.

In the first step, factors are estimated from the equilibrium observed characteristics function (Equation 15) and used to remove their linear projection on the factors, a process referred to as *defactorization* of the regressors. After defactorization, the regressors are no longer endogenous because the common factor component has been purged. However, at this stage, measurement error in the estimated factors, captured in ζ_{jt}, may still be correlated with the residuals e_{jt}, since both are estimated using the same source of identification: the regressors.

$$x^m_{jt}(f_j) = \mathcal{T}^m_t \cdot f_j + \zeta_{jt} \tag{15}$$

The equilibrium relationship linking observed and unobserved characteristics represents a reduced-form equation rather than a structural one, analogous to a hedonic price function that holds only in equilibrium. From this relationship, the factors and loadings are estimated in the usual way, allowing the projection of the observed characteristics onto the latent factors to be removed from them (Equation 16). This process effectively removes the endogenous part of observed regressors.

$$\tilde{x}_{jt} = x_{jt} - \hat{\mathcal{T}}^m_t \cdot \hat{f}^{(1)}_j \tag{16}$$

The first-stage estimates $\hat{\alpha}^{(1)}$, $\hat{\beta}^{(1)}$ and $\hat{\gamma}^{(1)}$ are obtained by using the defactorized regressors as instruments to estimate the coefficients in Equation 12.

$$\delta^{(\Sigma)}_{jt} - \hat{\alpha}^{(1)} p_{jt} - \hat{\beta}^{(1)} \cdot x_{jt} - \hat{\gamma}^{(1)} \cdot z_{jt} = \lambda_t \cdot f_j + e_{jt} \tag{17}$$

In the second step, the factors and loadings are re-estimated from the first-stage residuals of the outcome equation (Equation 17). These new estimates are then used to defactorize the mean utilities, removing the common factor component from the outcomes and completing the defactorization process.

$$\tilde{\delta}_{jt}^{(\Sigma)} = \delta_{jt}^{(\Sigma)} - \hat{\lambda}_t^{(2)} \cdot \hat{f}_j^{(2)} \tag{18}$$

The final unbiased estimates of the linear parameters are obtained through a standard OLS regression of the defactorized mean utilities (Equation 18) on the defactorized regressors (Equation 16). At this point, the regressors are no longer endogenous, and the measurement errors and regression residuals are uncorrelated, as they are now identified using different sources.

$$\mathbb{E}[\zeta_{jt} e_{jt}] = 0$$

Finally, the nonlinear parameters Σ are estimated by minimizing the distance between $\hat{\gamma}$ and its expected value under the exclusion restriction (zero), as shown in Equation 19.

$$\hat{\Sigma} = \arg\min_{\Sigma} \left|\hat{\gamma}(\Sigma)\right| \tag{19}$$

Marginal Costs and Cost Parameters

Firms' markup equations provide the basis for identifying marginal costs (c_{jt}) as a function of the data and the estimated demand parameters (Berry et al., 1995). For illustration, equation 20 presents one such equation for a single-product firm, although the model accommodates multi-product firms.

$$\frac{\partial \Pi_{ft}}{\partial p_j} = 0 \rightarrow c_{jt} + \tau_{jt} = p_{jt} + \frac{s_{jt}}{\partial s_{jt}/\partial p_{jt}} \tag{20}$$

I assume the solution the the production cost minimization problem (Equation 10) follows the functional form described in equation 21, with ι_{jt} being a normally distributed error term. As a result, the cost parameters can be identified by estimating the marginal cost regression using OLS.

$$\begin{aligned} c_{jt} = &\sum_x \rho_x log(x_{jt}) + \rho_{xx} log(x_{jt})^2 \\ &+ \sum_w \rho_w log(p_{wt}) + \sum_w \rho_{ww} log(p_{wt})^2 \\ &+ \sum_x \sum_w \rho_{xw} log(x_{jt}) log(p_{wt}) + \iota_{jt} \end{aligned} \tag{21}$$

Factors and loadings

The joint identification of the interactive term ($\lambda_t f_j$), offered by the LSMD estimator, is not sufficient for the purposes of my analysis. Joint identification allows consistent estimation of mean utilities, demand parameters, and marginal costs, but it does not disentangle whether observed variation in market outcomes arises from shifts in consumer preferences or from changes in product characteristics. In the context of product reformulation, this distinction is critical: a policy may induce firms to alter product characteristics, consumers to adjust their preferences, or both. Without separating the factors (f_j) and loadings (λ_t), these mechanisms are observationally equivalent, making it impossible to interpret the estimated effects or to perform counterfactual exercises that isolate the effects of product reformulation.

The standard approach in the literature for identifying factors and loadings relies on statistical normalizations. Most commonly, this means assuming the orthogonality among factors and independence across loadings. Although convenient, these assumptions constrain the economic interpretation of the parameters and may be inconsistent with the theoretical framework that motivates their use. In a product reformulation setting such as mine, these conditions would imply that firms choose unobserved product characteristics entirely in isolation once the observed ones have been determined. This assumption conflicts with the economic behavior captured in my model, where firms jointly determine all product characteristics and thereby create the endogeneity that the factors are meant to address. Moreover, these normalization provide only weak identification, since the factor structure is invariant to arbitrary scaling and rotation. To see this, let Ξ denote the $J \times T$ matrix of unobserved utilities and rewrite equation 4 in matrix form. As shown below, any invertible linear transformation H applied to the factors and loadings also satisfies the factor structure.

$$\Xi = \Lambda F + E = (\Lambda H^{-1})(HF) + E$$

To overcome these limitations, I introduce additional structure by using moments derived from the supply side of the market. This approach ensures that observed demand shifts can be attributed unambiguously to either shifts in preferences or product changes, which is essential for interpreting policy effects and isolating the role of reformulation. The intuition is that firms' optimal choices of product characteristics contain information about how unobserved product attributes and consumer preferences interact in equilibrium. This enables the use of moments derived from firms' first-order conditions (FOCs) to identify f_j and λ_t separately.

The moments I use build on the identification assumption in Petrin et al. (2022) that firms' ex-post optimization errors are conditionally mean independent of everything known to them at the time they choose product characteristics. This assumption allows any variable known to firms when making these choices to serve as an instrument in Hansen and Singleton (1982) generalized instrumental variables framework with the moments in equation 22. The key difference between Petrin and coauthors' approach and mine is that they use these moments to address the endogeneity in the mean-utility regression, while I employ them solely to disentangle the sources of unobserved heterogeneity, conditional on all other parameters. As a result, their framework delivers consistent estimates of the demand parameters but cannot separate the sources of unobserved heterogeneity, and therefore cannot be used to run counterfactual simulations that isolate the role of product reformulation.

$$\mathbb{E}\left[\frac{\partial \Pi_f}{\partial f_j} \cdot z_j\right] = 0 \tag{22}$$

As a result, I am able to identify the factors and loadings up to a scaling transformation, as stated in the following identification theorem.

Theorem 1. *The set $\{f_j, \lambda_t\}$ satisfies equations 4 and 22 if and only if it is a scalar transformation of the true model parameters $\{f_0, \lambda_0\}$.*

Proof. **See Appendix A1**

□

Computing the moments in Equation 22 requires the partial derivatives of equilibrium prices with respect to the unobserved factors, which are unknown during estimation. These derivatives capture how a firm's optimal pricing responds to changes in its product characteristics, linking supply-side behavior to the unobserved factors that also shape demand. For reference, Equation 23 presents the first-order condition (FOC) for a single-product firm.

$$\frac{d\Pi_{ft}}{df_j} = \overbrace{(p_{jt} - c_{jt})\frac{\partial s_{jt}}{\partial f_j^m}}^{\text{Direct effect on market shares}} + \overbrace{\left(\frac{\partial p_{jt}}{\partial f_{jt}^m} - \frac{\partial c_{jt}}{\partial f_j^m}\right) s_{jt}}^{\text{Direct effect on margins}} + \overbrace{(p_{jt} - c_{jt})\sum_{l \in \mathcal{G}_f} \frac{\partial s_j}{\partial p_l}\frac{\partial p_{lt}}{\partial f_{jt}^m}}^{\text{Indirect effect on shares through prices}} \tag{23}$$

During the estimation, I compute this derivatives following an approach analogous to the one used in Villas-Boas (2007), who derives the partial derivatives of retailer prices with respect to wholesale prices by implicitly differentiating the pricing first-order conditions. Similarly, implicitly differentiating the system of pricing FOCs in equation 20 yields a system of linear equations that I solve to obtain the required partial derivatives.

Number of factors

A common concern in the identification of factor models is determining the appropriate number of factors to include. The flexibility offered by the multiplicative structure of interactive fixed effects places this responsibility on the econometrician. In theory, the number of factors can be consistently estimated from the data (Moon and Weidner, 2015; Bai, 2009 Supplemental material). In practice, however, empirical applications often face difficulties in implementing these procedures (Ahn et al., 2013; Onatski, 2010), and my product reformulation model is no exception.

Rather than relying solely on statistical selection criteria, I adopt what I refer to as an *econometric criterion*, which is grounded in the role of the estimated factors. The key objective is to include enough factors to adequately address endogeneity between observed and unobserved characteristics, which is the primary motivation for incorporating interactive effects in the model. This approach follows Moon and Weidner (2015), who show that the limiting distribution of the estimated coefficients from the interactive fixed effect estimator is invariant to the number of factors used, provided that this number is not underestimated. Hence, while the exact count of factors is not critical, ensuring that it is not underestimated is essential for obtaining reliable estimates.

Accordingly, I set the number of factors R to ensure stability. That is, I choose the smallest R such that adding another factor does not materially affect the estimated coefficients. This strategy provides a principled way to select the number of factors, balancing theoretical guidance with empirical robustness, as expressed in Equation 24.

$$\begin{aligned} \hat{\alpha}^{(R+1)} &= \hat{\alpha}^{(R)} \\ \hat{\beta}^{(R+1)} &= \hat{\beta}^{(R)} \end{aligned} \tag{24}$$

Reformulation costs

Firms' reformulation decisions provide valuable information about the costs they face and can be used to bound these costs structurally. In particular, by comparing counterfactual profits under the tax with and without the reformulated product j', one can infer the implicit cost threshold at which reformulation becomes a profitable strategy. A firm that chooses to reformulate reveals that the expected gains exceed the associated costs. Conversely, if a firm opts not to reformulate, the implied costs must outweigh the expected benefits. By systematically analyzing these choices across firms and products, I can identify a range within which reformulation costs are likely to lie, providing structural basis to the interpretation of firms' strategic responses to the tax.

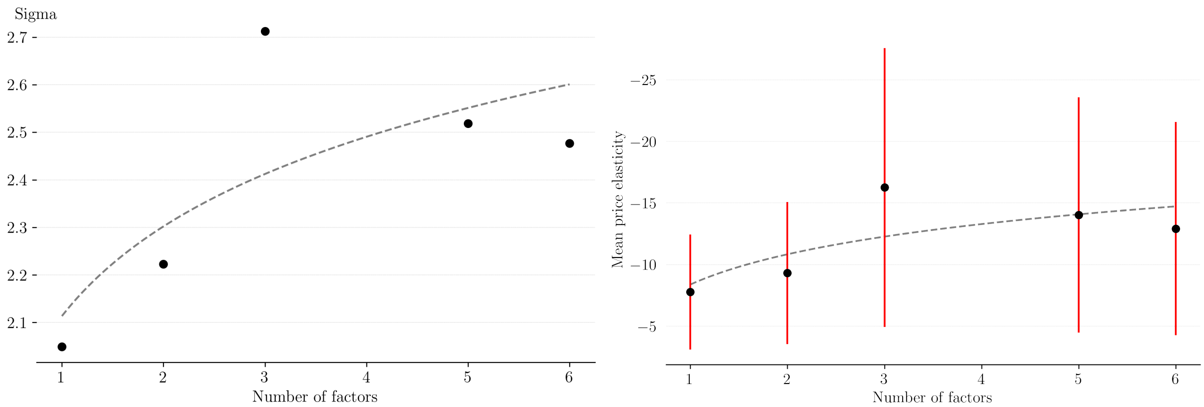
$$\max_{j\text{:}\textit{Not Reformulated}} \Pi_f(j') - \Pi_f(j) \leq \kappa \leq \min_{j\text{:}\textit{Reformulated}} \Pi_f(j') - \Pi_f(j) \tag{25}$$

VI Results

Number of Factors

The number of factors (R) in the estimation determines how much unobserved heterogeneity in product utilities the model can capture. Consequently, the resulting estimates vary with this specification choice. Nonetheless, as the number of factors increases, the estimates converge, indicating that the model effectively captures the main sources of unobserved heterogeneity in the data (Figure 7). This convergence provides empirical support for the econometric criterion proposed for determining the number of factors.

My analysis centers on the two economic dimensions embedded in the factor structure: variation across products and variation across consumers. The factors capture product heterogeneity, summarized by the distribution of own-price elasticities, while the loadings capture consumer heterogeneity, summarized by the variance of the random coefficients. As additional factors are included, these components increasingly mitigate the endogeneity arising from the correlation between unobserved utility and observed product characteristics until it is fully accounted for.



(a) Variance of the random coefficient

(b) Distribution of demand price elasticities

Figure 7: Estimates under alternative numbers of factors

Ignoring the correlation between product characteristics and unobserved utility leads to biased estimates due to endogeneity. I show this by comparing my results to those from the standard model of Berry et al. (1995), which assumes that unobserved utility is mean independent of the observed characteristics and thus rules out endogeneity by construction. Table 4 shows that the estimates from this restricted model differ systematically from those obtained when such correlation is allowed. This comparison highlights the empirical relevance of this endogeneity in the beverages market and validates the use of a factor structure to address it.

Price elasticities estimated from the standard model are substantially lower than those from the one with interactive effects. This pattern aligns with Petrin et al. (2022), who show that accounting for endogeneity arising from the correlation between observed and unobserved characteristics increases estimated elasticities in the automobile market. The lower price sensitivity in the standard model likely reflects its more limited ability to capture product quality, whereas the model with interactive effects provides a more accurate comparison once utility from unobserved characteristics is properly accounted for. Furthermore, because marginal costs and markups are inferred from firms' pricing conditions, the lower elasticities in the standard model imply lower estimated costs and higher markups. The resulting average markup of roughly 70% appears excessively high relative to typical margins in retail markets.

Demand

The demand estimates reported in Table 5 are obtained from the specification with six latent factors. This number provides sufficient flexibility to capture higher-dimensional

	LSMD ($R = 6$)	BLP
Price Elasticity	-12.92	-3.37
	(8.66)	(3.03)
Markups	0.22	0.70
	(0.79)	(0.83)
Marginal Cost	0.95	0.72
	(0.91)	(0.94)

Table 4: Estimates across models

product and consumer heterogeneity, allowing the model to address the endogeneity between observed and unobserved characteristics discussed earlier. Increasing the number of factors beyond six does not materially affect the results. For instance, the estimated variance of the random coefficients changes by less than 2% of the price coefficient when moving from three to six factors.

Deriving analytical standard errors for the model coefficients is nontrivial given the several innovations introduced in the estimation procedure. Such an effort falls beyond the scope of this paper. Instead, I report bootstrap-based standard errors, which indicate that the coefficients are estimated with a high degree of precision.

Overall, the estimates appear reasonable and yield sensible implied costs and markups. The estimated price elasticities are high relative to industry benchmarks, which is expected once the model allows for correlation between observed and unobserved product characteristics (Petrin et al., 2022). Consumers display a strong negative sensitivity to price, partially offset by the positive interaction with market-level house prices, indicating that demand is less elastic in wealthier regions. The positive coefficient on Private Label products suggests that consumers perceive them as offering good value, although other unobserved attributes may limit their market share relative to branded alternatives. Among observed characteristics, beverages with higher sugar content and lower protein levels are more highly valued, consistent with consumer preferences for sweetness in this category. Seasonal effects are modest: demand for Festive and Winter beverages is slightly below average, while Lemonade products are substantially more popular than the baseline category of Colas. This difference is not surprising, as the high market share of a few leading cola brands reflects brand-specific popularity rather than a general preference for colas as a category.

Costs

The cost parameters from the marginal cost function are precisely estimated (Appendix A2). In addition to product characteristics and input prices, I include an interaction term between reformulation status and sugar prices to capture potential changes in production technology across firms. The reasoning behind this inclusion is that a production tech-

	Estimate	Std Error[a]
Constant	12.96	-
Price	-17.92	-
Private Label	5.99	-
Sugars	0.07	-
Protein	-2.93	-
Sodium	0.60	-
Festive	-1.21	-
Winter	-0.83	-
Ginger Beer	-0.94	-
Lemonade	2.20	-
Other	0.75	-
Tonic Water	0.08	-
Price x Market House Prices	0.10	-

[a] Standard errors are being computed using bootstrap.

Table 5: Demand Estimates ($R = 6$)

nology can be represented by a minimum cost function (McFadden, 1978). Differences in cost sensitivity to input prices between reformulated and non-reformulated products would therefore indicate that firms employ distinct production technologies for reformulated goods. In this case, the tax would have not only encouraged firms to reformulate but also pushed them to develop or adopt new production technologies.

The positive and significant coefficient on the interaction between sugar prices and reformulation status is comparable in magnitude to the coefficient on sugar prices alone, indicating that the marginal costs of reformulated products are almost twice as sensitive to changes in wholesale sugar prices. This finding supports the interpretation that firms adopted different production technologies for reformulated products. Among these new technologies are natural and artificial sweeteners that substitute for sugar. These sweeteners were not used in soft drinks prior to the policy's announcement and had been approved only a few years earlier; for example, Stevia (2011), Advantame (2013), or Erythritol (2015).

Factors

The model provides a clear economic interpretation of the estimated factor structure: the factors represent unobserved product characteristics chosen by firms, and the loadings capture average consumer preferences for those characteristics within each market. Because these variables are latent, their dimensions cannot be described exhaustively in semantic terms. I therefore validate this interpretation through an information-recovery exercise in which I deliberately remove certain product information from the data, re-estimate the factors and loadings, and test whether the omitted information can be recovered from the estimated factors. If the factors truly capture unobserved product heterogeneity, they should contain enough information to reconstruct the missing characteristics.

I choose to conceal the private-label status of products for my information-recovery exercise. As shown in Table 5, consumers exhibit strong preferences for this attribute, making it likely to influence their choices. I then attribute its effect to the unobserved utility and re-estimate the factors and loadings following the same procedure described above. Finally, I apply a t-SNE dimensionality reduction (Maaten & Hinton, 2008) to the estimated multi-dimensional factors to visualize the results of this exercise in just two dimensions (Figure 8).

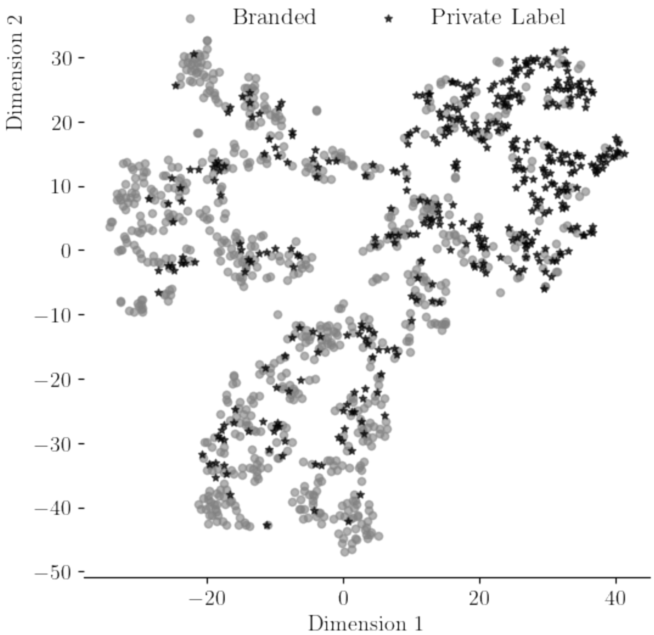


Figure 8: Dimensionality reduction of branded and private label products

The (re)estimated factors successfully recover the private-label status of products. As shown in the figure above, most private-label products cluster in the upper-right quadrant, indicating that the factors capture similarities among these products in the underlying characteristic space. This pattern confirms that the factors encode unobserved information about products and supports their proposed economic interpretation, allowing a deeper analysis of the original ones.

The factors and loadings help address potential endogeneity arising from correlation between observed and unobserved product characteristics. To assess how pervasive this issue is, I regress each latent factor on all observed characteristics. Table 6 shows that such correlations are substantial in the data. Moreover, the correlation patterns differ across factors, indicating that each factor captures distinct information about products. This explains why ignoring this potential source of endogeneity leads to different estimates (Table 4).

	Factor 1	Factor 2	Factor 3	Factor 4	Factor 5	Factor 6
Constant	1.32***	0.0	-0.08***	0.11***	0.05***	-0.01**
	(0.0)	(0.01)	(0.01)	(0.01)	(0.0)	(0.0)
Price	-0.62***	-0.05***	0.01***	-0.09***	0.03***	-0.15***
	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
Private Label	0.74***	0.33***	-0.05***	0.08***	-0.1***	0.11***
	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
Sugars	-0.02***	0.01***	0.01***	-0.01***	-0.0***	-0.01***
	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
Protein	0.0	-0.17***	-0.13***	0.26***	0.09***	0.13***
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Sodium	0.01***	0.04***	-0.04***	0.04***	-0.01***	-0.02***
	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
Festive	-0.0	-0.02***	-0.0	-0.0	-0.0	0.0
	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
Winter	-0.0*	0.02***	-0.0	0.0	-0.01*	-0.0
	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
Ginger Beer	-0.13***	0.07***	0.11***	-0.13***	0.04***	0.02**
	(0.0)	(0.01)	(0.01)	(0.01)	(0.01)	(0.0)
Lemonade	0.24***	0.01	-0.02***	0.12***	-0.03***	0.03***
	(0.0)	(0.01)	(0.0)	(0.0)	(0.0)	(0.0)
Other	-0.03***	0.01*	0.05***	0.0	0.05***	0.05***
	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
Tonic Water	0.02***	-0.04***	0.06***	-0.02***	-0.06***	0.02***
	(0.0)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)

Table 6: Coefficients of regressing factors on observables

Reformulation

The shrinkage of the product space is also reflected in the distribution of the latent factors. Table 7 reports dispersion measures for reformulated products before and after reformulation. The range of all factors, that is, the difference between their maximum and minimum values, declined following reformulation. Similarly, the standard deviation fell in four of the six product dimensions, indicating a contraction in the overall diversity of the unobserved product characteristics.

	Std Dev		Range	
	Original	Reformulated	Original	Reformulated
Factors 0	0.99	1.02	4.33	4.09
Factors 1	0.90	0.71	7.55	5.71
Factors 2	0.63	0.83	14.56	11.76
Factors 3	0.74	0.56	12.55	8.67
Factors 4	0.63	0.60	15.38	9.78
Factors 5	0.61	0.44	9.75	9.50

Table 7: Factor dispersion before and after reformulation

VII Counterfactual Policy Analysis

The Role of Reformulation

Understanding the role of product reformulation in response to the Soft Drinks Industry Levy requires comparing outcomes across alternative scenarios, each offering an additional layer of insight into the policy's effects. My analysis focuses on three key mechanisms: tax pass-through to prices, strategic complementarities in firms' pricing behavior, and product adjustments arising from reformulation.

A *baseline scenario* serves as the benchmark against which all alternative ones are compared. It represents a world in which no corrective policy on soft drinks was implemented. The product set and its characteristics are fixed at their March 2016 levels, while consumer preferences are allowed to evolve as they did in the real world. This setup is enabled by the estimated factors and loadings, which together account for unobserved changes in market conditions. The estimated loadings capture overlooked shifts in consumer preferences toward unobserved product characteristics, while reverting the factors to their pre-reformulation values restores the unobserved characteristics of products to their original state.

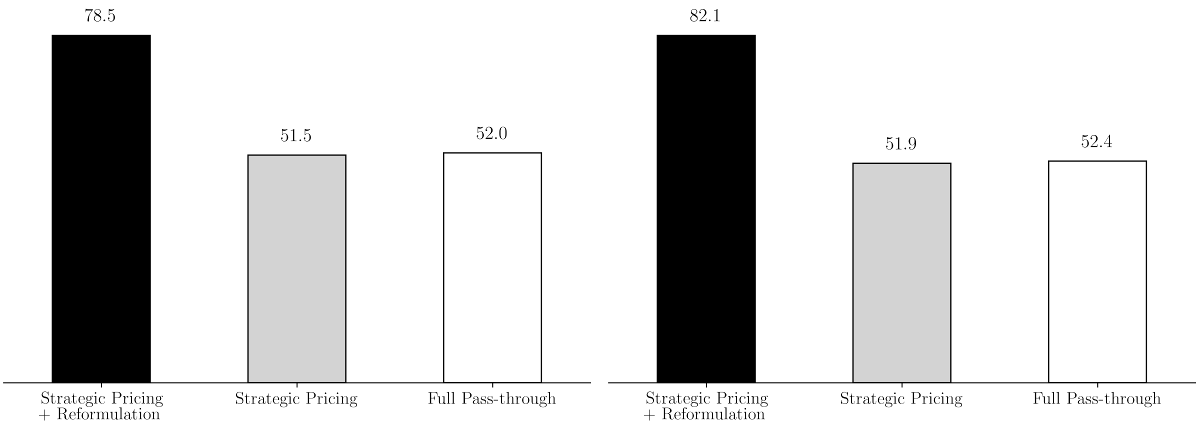
The second scenario assumes that the levy is implemented with the same tax structure and firms fully pass the levy onto consumers, exhibiting *complete tax pass-through* with no strategic pricing or product responses. This scenario therefore isolates the direct price effect of the tax, without allowing firms to adjust the prices of untaxed products or absorb part of the levy. Similar full pass-through behavior has been documented in previous studies of soft drink taxes (e.g. Aguilar et al., 2021; Capacci et al., 2019; Rojas and Wang, 2017; Seiler et al., 2021). The third scenario introduces *strategic pricing* behavior, allowing firms to adjust the prices of both taxed and untaxed products. This setup captures potential complementarities across products and firms' competitive adjustments to the new post-tax price structure.

Finally, the *policy scenario* reflects the world as it unfolded, in which firms not only adjusted the prices of taxed and untaxed products but also reformulated to avoid tax liabilities. In this setting, firms modify both the observed and unobserved characteristics of their products, the latter being captured in the model through changes in the estimated factors.

The market outcomes for each scenario are obtained as follows. First, I compute equilibrium prices under the given consumer preferences and product offerings using the fixed-point algorithm proposed by Morrow and Skerlos (2011). Next, I use Equation 2 to calculate each product's market share. Finally, I compare welfare outcomes across scenarios, examining the distributional impact of reformulation by comparing consumer surplus losses from the tax across the income distribution between the policy scenario with reformulation and the scenario featuring only strategic pricing responses.

Effects on Sugar Consumption

The policy had a substantial impact on sugar consumption, reducing sugar intake from sugar-sweetened beverages by 21.5% (Figure 9.a). However, both pricing-only response scenarios would have produced considerably larger reductions; each leading to roughly a 50% decline in sugar consumption. The reason reformulation results in higher sugar sales relative to pure price responses lies in trade volumes. The policy scenario has sales falling by only 17.9%, whereas both pricing-only scenarios reduce sales volumes by about half (Figure 9.b) Without product responses, there is nearly a one-to-one relationship between reductions in sugar and sales volume. Reformulation improves this ratio by about 17%, yielding roughly a six-to-five relationship between sugar reduction and volume loss.



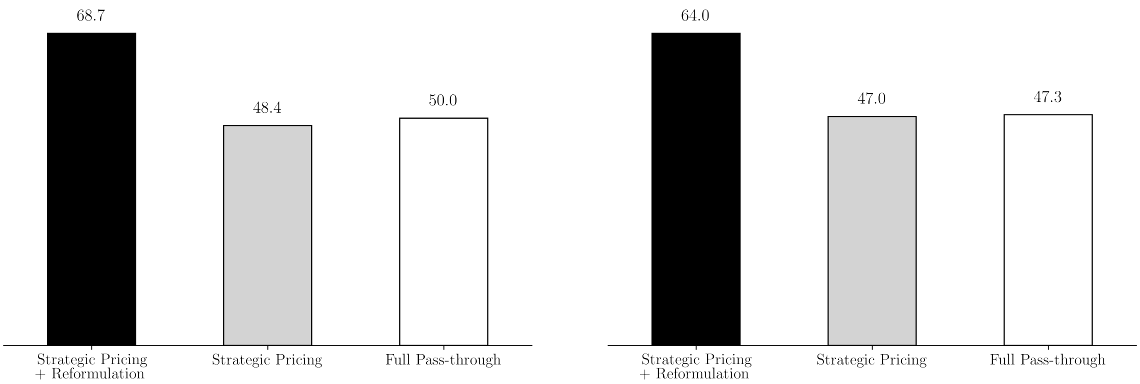
(a) Sugar sold (b) Total volume sold

Figure 9: Policy outcomes as percentage change relative to the Baseline scenario

These effects are substantial but not unheard of. While some authors find relatively small impacts from similar policies, others document much larger effects. Among the former, Aguilar et al. (2021) report only a 2.7% reduction in calories from soft drinks following a 12% non-tiered volume-based tax in Mexico (which raised prices by 9.7%), and Bollinger and Sexton (2023) find little evidence of reduced soft drink purchases in Berkeley after another modest single-tiered volume-based tax. Among those reporting significant effects, Seiler et al. (2021), for instance, find a 34% price increase and a 45% drop in demand in response to Philadelphia's 35% no-exceptions volume tax, although part of this effect is offset by cross-shopping in untaxed jurisdictions. Most importantly, my estimates align closely with those reported by Dubois et al. (2020), who study the same UK policy but focus on the on-the-go market using rich individual-level data. By contrast, my work uses aggregated data from bring-home purchases, which is more readily available and accounts for roughly half of the sugar intake from soft drinks (Dubois et al., 2018).

Effects on Welfare

I then focus my attention on market participants' choice utility (Figure 10, above), meaning the welfare of those within the market while excluding the effects of potential externalities and consumer biases that might distort socially optimal market outcomes. The results show that reformulation largely allowed firms to prevent further losses and significantly improve consumer welfare. Although the policy still reduces industry profits by 30%, without reformulation firms could have seen their profits cut in half. Consumers also benefit from reformulation, as it prevents additional price increases and allows more consumers to remain in the market, increasing their surplus by 17 percentage points.



(a) Firm profits (b) Consumer surplus

Figure 10: Choice utility compared to the Baseline scenario

Where does the additional welfare in the policy scenario come from? Mostly lower prices. Figure 11 shows that the full policy raises industry prices by 4.4%, whereas the pricing-only scenarios would have increased prices by about 10%, more than double the observed effect with reformulation. Importantly, these are average prices across the entire market, with larger increases for products that were or remain sugary compared to diet alternatives such as bottled water.

Firms' strategic pricing behavior appears to plays only a marginal role in shaping outcomes. In terms of prices, the additional increase relative to full tax pass-through is less than one percentage point. Moreover, the similarity in outcomes across both scenarios indicates that the small price rise among untaxed products has negligible effects on either policy effectiveness or welfare.

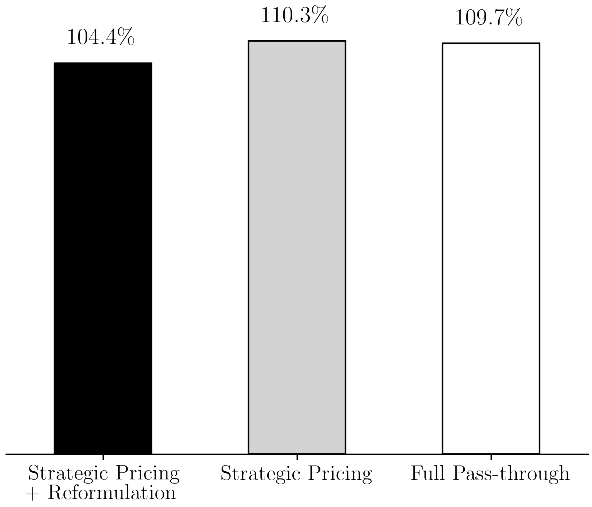


Figure 11: Price relative to the baseline

Meanwhile, reformulation reduced product quality. I compare the utility from unobserved product characteristics before and after reformulation while holding consumer preferences fixed (Figure 12). This comparison is feasible because the estimated factors and loadings allow me to control for potential unobserved shifts in preferences between the pre-policy and post-reformulation periods. I find that reformulation lowered both the level and dispersion of unobserved product utility, implying that reformulated products are of lower multidimensional quality and more similar to one another than they used to be. The resulting shrinkage of the product space intensified price competition and reduced opportunities for differentiation, as Figure 5 initially suggested.

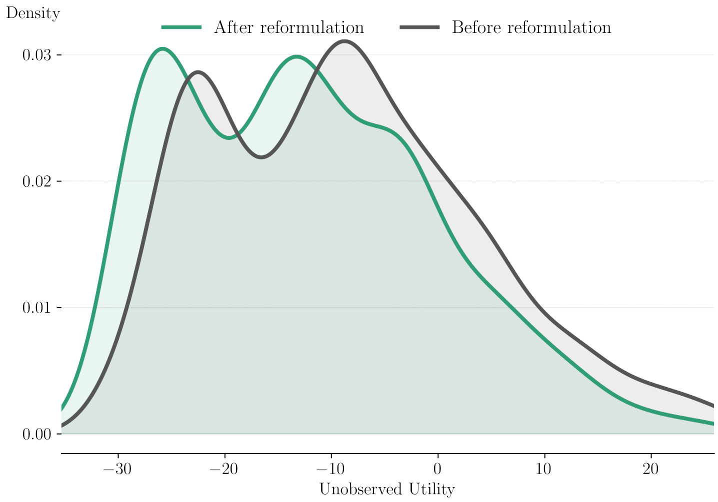


Figure 12: Distribution of unobserved utilities before and after reformulation

More broadly, the lesson is clear: tax design matters, and it involves a trade-off between policy effectiveness and efficiency Different levy structures generate distinct market responses, shaping market and welfare outcomes differently. In my strategic-pricing scenario, the observed behavior corresponds to a policy that fixes each product's tax at its pre-announcement tier, without re-assessment if firms later alter sugar content. Such a design delivers larger reductions in sugar but at greater cost to consumer and producer welfare. By contrast, a regime that updates taxes when sugar levels change encourages reformulation and improves efficiency, though at some loss in effectiveness. These results reveal a clear trade-off that policymakers must weigh according to their objectives and priorities.

Distributional Impacts

On Consumers

The random coefficient on price allows me to examine the distributional effects of the policy. This coefficient captures consumers' heterogeneous price sensitivities and, indirectly, their marginal utility of money. Assuming a monotonic inverse relationship between income and price sensitivity, I simulate welfare changes for consumers at each income percentile under different scenarios. The results of this exercise are shown in Figure 13.

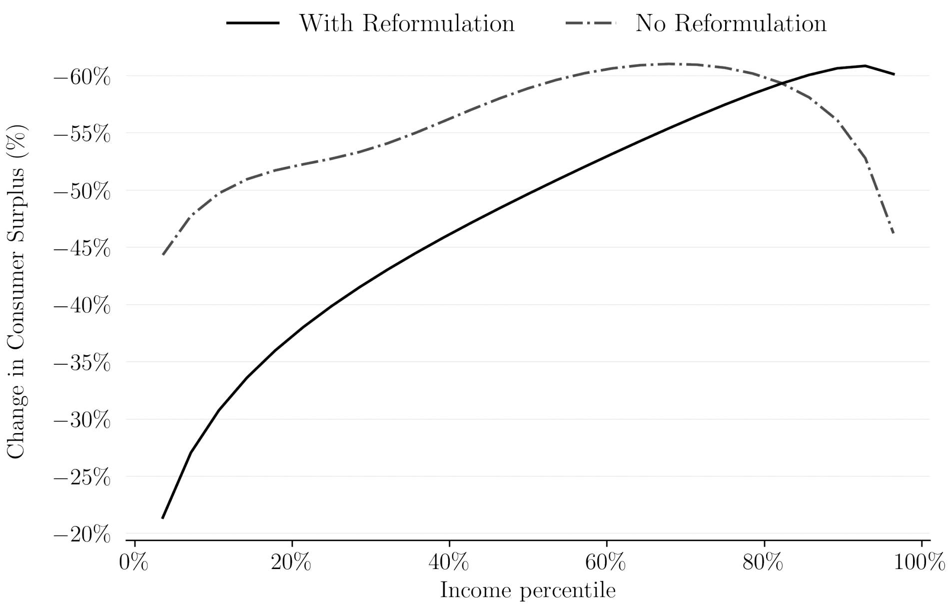


Figure 13: Change in consumer surplus across scenarios

Reformulation primarily benefits poorer consumers, who are most sensitive to higher prices. Without product responses, the Soft Drinks Industry Levy would have a larger overall effect on consumer welfare, but this effect would be distributed more evenly across the income distribution. When firms reformulate, however, the welfare loss among poorer households falls by nearly half due to lower prices. The only group worse off relative to the no-reformulation scenario are households above the 80th income percentile, who are less affected by price increases but more concerned about the decline in product quality resulting from reformulation.

Despite the common belief that soft drink taxes disproportionately affect poorer consumers, the simulation shows that even without reformulation the poorest are slightly less affected than those in middle–high income groups. One possible explanation is that the

Soft Drinks Industry Levy allows consumers to switch to lower-sugar alternatives, reducing their overall tax burden. This pattern may arise if poorer consumers are more likely to substitute toward less sugary products, while middle–high-income consumers maintain their choices because they place greater weight on product quality. Such flexibility is absent under non-tiered taxes, for which this belief has been shown to hold.

On Firms

The impact of the policy varies by firm size. I group firms based on the number of distinct brands the own Figure 14 shows that smaller firms are the most affected under all scenarios, and under the reformulation scenario, they are even more affected in relative terms. This is primarily because they are more exposed to the tax, having fewer products in their portfolios to offset its effects. In contrast, larger firms can mitigate the impact by keeping a significant share of their offerings outside the tax's reach. Interestingly, reformulation appears to have been particularly beneficial for medium and large firms, helping them preserve revenues even relative to the baseline scenario with no tax.

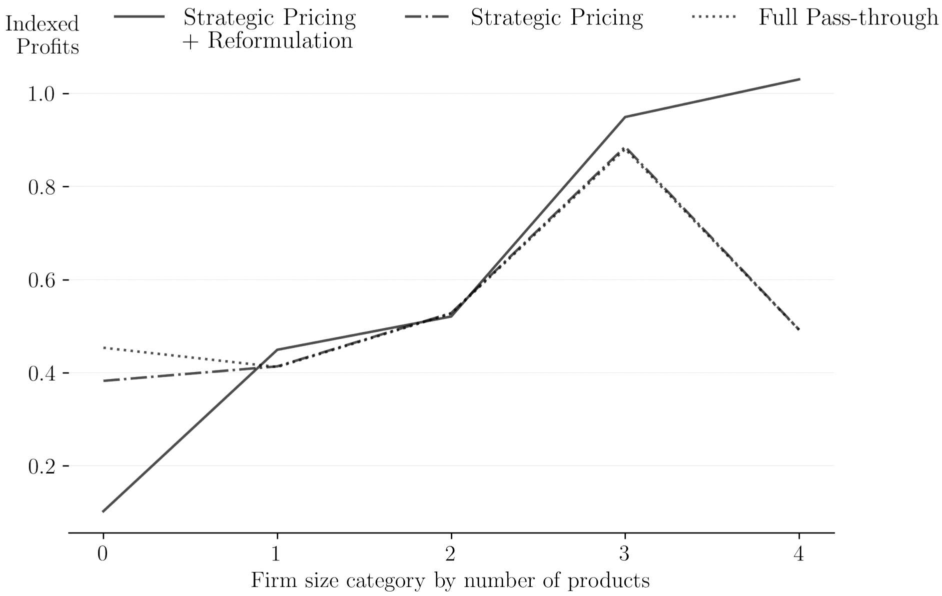


Figure 14: Profits relative to the Baseline, by firm size bin

Moreover, revenue loss is closely linked to firms' reformulation efforts. Figure 15 shows that larger firms reformulated a greater share of their products and generally achieved better market outcomes as a result. This reinforces the idea that product adaptation played a key role in mitigating the tax's effects. However, it also highlights the potential unequal capacity of firms to adjust, as smaller firms may face higher barriers to reformulation due to resource constraints or limited access to reformulation technologies. It is also possible that the smaller sales volumes of these firms do not justify the high costs associated with effectively reformulating their products.

The disparity in firm responses suggests that if governments aim to protect smaller and independent firms, additional policy interventions may be needed. For instance, reformulation subsidies or technical support could help smaller firms adapt while maintaining the policy's objective of reducing sugar consumption. Supporting firms through this transition would mitigate revenue losses, limit potential employment effects, and help preserve market competition.

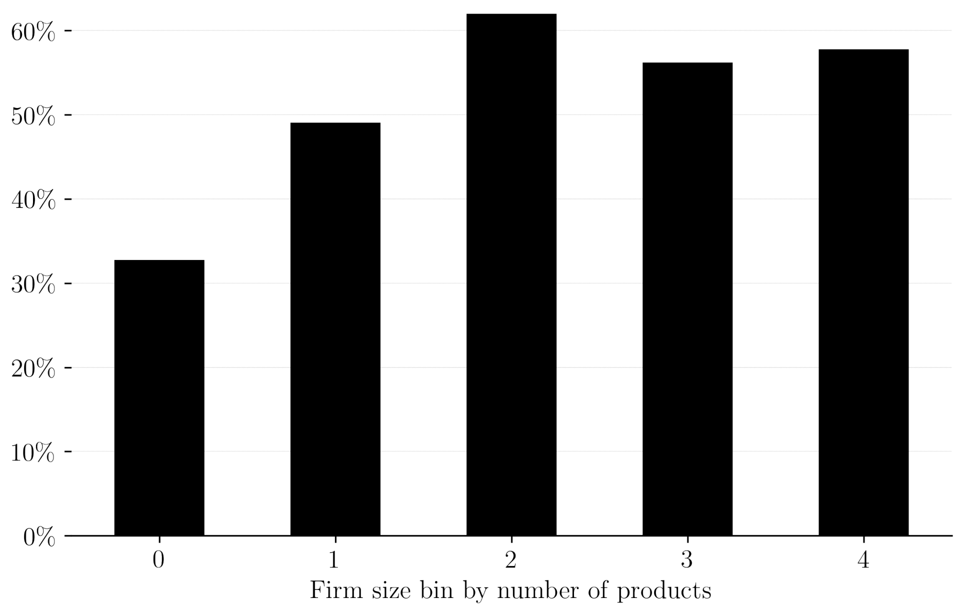


Figure 15: Share of reformulated products, by firm size bin

The Role of Tax Design

Comparing the UK's tiered levy with an alternative tax design provides a clean test of how tax structure shapes market and policy outcomes. To study this, I simulate the policy under a non-tiered tax: the most common design for soft drink levies internationally (WHO, 2023). Under this policy, all beverages containing sugar face the same tax liability per unit of volume. As is standard in such taxes, the policy specifies a threshold for what qualifies as a sugary drink; I assume that any product with more than 1 gram of sugar per 100 milliliters is taxed. This counterfactual isolates the role of tax design in determining firms' reformulation incentives and the resulting effects on prices and welfare.

A non-tiered tax provides no direct incentive for firms to reformulate existing products. International evidence suggests that, under such policies, firms typically introduce low-sugar variants of their flagship brands while retaining high-sugar versions in the market. This does not preclude reformulation altogether, but it does suggests a smaller role for this margin of adjustment. The picture is further complicated by uncertainty about how non-tiered taxes influence firms' adoption of new technologies. My analysis shows that the UK's multi-tiered levy prompted firms to invest in new production technologies to reduce sugar content and reformulate their products more effectively. No such response has not been documented under non-tiered regimes. Hence, the approach to this exercise is not obvious.

Rather than imposing a single assumed response from firms, whose validity would be difficult to justify; I simulate alternative reformulation scenarios. In the first, I model a non-tiered tax equivalent to the highest tier of the UK levy, applied uniformly to all sugary drinks, with *no reformulation* response from firms. In the second, I allow only those products that fully eliminated sugar under the tiered levy to do so under the non-tiered system. This represents a *partial reformulation* scenario, where only certain products reformulate, while those that moved near the tier thresholds remain unchanged, as partial reductions in sugar content would yield no tax benefit and incur in a reformulation cost for the firm.

Non-tiered taxes have a pronounced impact on market outcomes. Table 8 presents equilibrium results for the UK's tiered levy and two alternative reformulation scenarios under a non-tiered tax of £0.24 per liter, each reported relative to a no-policy baseline. The near-identical results for the non-tiered scenarios with and without reformulation indicate that product adjustments play only a minor role under this design. Because a non-tiered tax provides firms no incentive to lower sugar content, firms respond primarily through price increases rather than reformulation, leading to larger welfare losses and sharper contractions in sales.

	Prices	Sugar Sold	Volume Sold	Profits	Consumer Surplus
Full policy	104.4%	78.5%	82.1%	68.7%	64.0%
Non-tiered* (No reformulation)	114.2%	48.9%	49.4%	48.0%	43.9%
Non-tiered* (Partial reformulation)	114.1%	49.0%	49.7%	47.4%	43.1%

* Non-tiered tax equivalent to £0.24 per litre on all sugary drinks

Table 8: Alternative policy designs against no-policy baseline

When compared to the tiered-design, the non-tiered taxes generates substantially larger price increases, about 15 percent above baseline compared with the 4 percent under the tiered policy. It leads to sharp contractions in sales, profits, and consumer surplus. Sugar sales fall by roughly 51 percent, but at a considerably higher welfare cost: consumer surplus and firm profits decline by more than 50 percent each. In contrast, the tiered structure achieves sizable sugar reductions with smaller losses in welfare and market activity, illustrating that differentiated tax designs can curb sugar consumption more efficiently by leveraging firms' reformulation responses rather than relying solely on price adjustments.

Optimal Tax Design

The results above show that tax design plays a crucial role in shaping firms' product responses, which in turn influence both market and policy outcomes. Therefore, the analysis of an optimal corrective tax must consider not only the optimal rate but also the optimal structure, meaning how the tax is levied and on what basis.

The model developed here is flexible enough to evaluate alternative tax designs and simulate their full market implications, including the distributional consequences of each policy. The central input for this exercise is the tax schedule, which can depend on any combination of observed and unobserved product characteristics; and take various functional forms such as tiered, continuous, or nonlinear designs. Given a tax schedule, it is possible to use the model to solve for firms' optimal product positioning while accounting for reformulation costs, and then simulates the resulting equilibrium prices. Finally, using the equilibrium products, their characteristics, and market prices, the model computes the corresponding market outcomes, welfare effects, and distributional impacts.

One key element remains outside the scope of the model: a measure of the harm from excessive sugar consumption. Quantifying such harm, which includes both externalities and internalities, is essential for the optimal tax analysis. However, existing approaches can be applied for this purpose. Externalities can be valued through their fiscal costs, as in Allcott et al. (2019), and internalities can be assessed using the behavioral public finance framework of Farhi and Gabaix (2020). However, no such estimates are currently available for the United Kingdom.

Conclusions

This paper examines how firms' product reformulation choices shape the effectiveness and distributional consequences of corrective taxation, using the UK Soft Drinks Industry Levy as a case study. The central insight is that corrective taxes shape more than prices: the design of the tax structure determines how firms adjust products and if they align with policy goals. Harnessing these product responses introduces a trade-off between effectiveness in reducing excessive sugar consumption and efficiency in maintaining welfare and equity. Reformulation helps preserve both consumer surplus and firm profits, with the largest gains accruing to lower-income households and only modest losses among those at the top of the income distribution. Larger firms reformulate a greater share of their product portfolios and thus protect profits more effectively, while smaller firms remain more exposed to the tax's impact than in the absence of product adjustments.

Four key insights emerge from my analysis. First, tax design matters: distinct levy structures lead to distinct market and policy outcomes, shaping welfare, and the policy effectiveness and efficiency. Second, firms respond strategically to protect profits by adjusting both prices and product characteristics. Third, while reformulation weakens the direct deterrent effect of the tax by keeping more consumers in the market, it simultaneously preserves welfare by limiting price increases and sustaining competition. Finally, the levy spurred technological innovation, as firms were pushed to adopt new manufacturing processes and alternative ingredients to meet reformulation needs.

To quantify these mechanisms, I develop and estimate an equilibrium model of demand and supply with interactive fixed effects that capture unobserved product characteristics and preference shifts. The model reveals that the policy achieved a 22% reduction in sugar sales from soft drinks; reformulation lowered average sugar content by 40%, prevented an additional 6% increase in prices, and narrowed the product quality distribution, reducing product differentiation and intensifying price competition. Larger firms adapted more effectively, reformulating a greater share of their portfolios and protecting profits, while smaller firms bore a relatively higher burden of adjustment.

Counterfactual simulations break down the role of reformulation, showing that the levy achieved substantial reductions in sugar consumption with limited welfare costs. These results underscore that tax design is crucial for steering firms' product decisions toward policy objectives while mitigating potential distributional concerns about the burden on more vulnerable households.

Together, the findings highlight the limitations of the most common corrective policy on sugar-sweetened beverages: flat-rate taxes. Such taxes rarely elicit meaningful product reformulation because they provide no direct incentive for firms to make products healthier. By contrast, the multi-tier design of the Soft Drinks Industry Levy offered firms flexibility to adjust their portfolios and avoid higher tax liabilities, leading to both innovation and greater effectiveness. This supports the use of multi-tier tax structures as a closer approximation to the targeting principle central to optimal tax theory.

More broadly, the analysis highlights a central trade-off in corrective policy between effectiveness, efficiency and equity. Taxes that fix liabilities ex ante achieve greater harm reduction but at higher welfare cost; those that adjust dynamically encourage innovation and reformulation, improving efficiency and reducing distributional impacts, but modestly reducing effectiveness. Recognizing and managing this trade-off is essential for aligning public health goals with market incentives.

As governments confront the global rise in obesity and diet-related disease, designing taxes that target the harmful effects of consumption while encouraging healthier production will be increasingly important. Future research should incorporate firms product responses to a framework of optimal corrective taxation. Also, further research needs to study other ways to harness these product responses and compare it to a direct tax on sugar content. Finally, exploring how such policies affect market entry, innovation, and long-run industry structure will be key to understanding how to use fiscal policy not only to correct consumption externalities and internalities but also to foster innovation that reduces harm at its source.

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Appendix A1: Mathematical Proofs

Theorem 1. Any set $\{f_j, \lambda_t\}$ that satisfies the factor equation $\xi^0_{jt} = \lambda_t f_j + e_{jt}$ and the firm's first-order conditions of the product definition stage for all j and t must be a scalar multiple of the true model parameters $\{f^0_j, \lambda^0_t\}$.

Proof. I show the factor equation and first-order-conditions of the product definition stage (FOC_1) are enough to identify the model parameters up to a scaling factor. To do so, I follow a proof by contradiction strategy.

First, let $\{f^0_j, \lambda^0_t\}$ be the true model parameters. Now consider an alternative set of parameters $\{f_j, \lambda_t\}$ that satisfies both the factor-model equation and the first-order conditions for the product-definition stage game, but that is not a scaled version of the true parameters. In other words:

$$\begin{gathered} \text{There is no } \kappa \in \mathbb{R} \text{ such that } \lambda_{t,r} = \kappa \lambda^0_{t,r} \text{ for all } t \in \{1, ..., T\} \\ \text{and no } a \in \mathbb{R} \text{ such that } f_{j,r} = a f^0_{j,r} \text{ for all } j \in \{1, ..., J\} \end{gathered} \tag{26}$$

Since $\{f_j, \lambda_t\}$ satisfies the first-order-conditions (FOC_1) of the product definition stage of firm h, it must be that for the r-th factor of product k manufactured by firm h (for simplicity, the t dimension has been ommited)

$$FOC_1 : \frac{d\pi_h}{df_{kr}} = 0 = \sum_{j \in \mathcal{G}} \Omega_{hj} \left\{ \left(\frac{\partial p_j}{\partial f_{kr}} - \frac{\partial c_j}{\partial f_{kr}} \right) S_j + (p_j - c_j) \left[\frac{\partial S_j}{\partial f_{kr}} + \sum_{l \in \mathcal{G}_t} \frac{\partial S_j}{\partial p_l} \frac{\partial p_l}{\partial f_{kr}} \right] \right\}$$

This expression can be written in matrix form using the notation introduced in Villas-Boas (2007). To do this, I define the product ownership matrix Υ, whose (i, j) element is equal to 1 if both products i and j are manufactured by the same firm, and 0 otherwise. Δ_x refers to the partial derivative operator with respect to variable x and $\circ$ denotes the element-wise matrix multiplication.

$$FOC1 : (\Upsilon \circ \Delta_{f_r}[P - C])\, S + (\Upsilon \circ (\Delta_{f_r}[S] + \Delta_P[S]\Delta_{f_r}[P])\, (P - C) = 0 \tag{27}$$

The alternative set also satisfies the factor equation, so the following equality holds:

$$\lambda^0_t f^0_j = \lambda_t f_j \quad \text{for all } j \text{ and } t$$

Note that this implies an infinite number of solutions to the factor equation. This becomes evident in its matrix form, where any reversible linear transformation of the true factors and loadings via an invertible matrix O will also satisfy the factor structure:

$$\Lambda^0 F^0 = (\Lambda^0 O)(O^{-1} F^0) = \Lambda F$$

The factor equation also implies the following equalities for the partial derivatives with respect to a given factor, while holding all other product variables constant:

$$\begin{aligned}
\frac{\partial S_{jt}}{\partial f_{kr}}(\lambda_t f_j,\cdot) &= \frac{\partial S_{jt}}{\partial u_{jt}}\frac{\partial u_{jt}}{\partial f_{kr}} \times \frac{\partial u_{jt}/\partial f^0_{kr}}{\partial u_{jt}/\partial f^0_{kr}} &&= \left(\frac{\lambda_{tr}}{\lambda^0_{tr}}\right)\frac{\partial S_{jt}}{\partial f^0_{kr}}(\lambda^0_t f^0_j,\cdot) \\
\frac{\partial^2 S_{jt}}{\partial f_{lr}\partial p_k}(\lambda_t f_j,\cdot) &= \frac{\partial^2 S_{jt}}{\partial u_{jt}\partial p_k}\frac{\partial u_{jt}}{\partial f_{lr}} \times \frac{\partial u_{jt}/\partial f^0_{lr}}{\partial u_{jt}/\partial f^0_{lr}} &&= \left(\frac{\lambda_{tr}}{\lambda^0_{tr}}\right)\frac{\partial^2 S_{jt}}{\partial f^0_{lr}\partial p_k}(\lambda^0_t f^0_j,\cdot)
\end{aligned} \tag{28}$$

And also that the price derivatives across parameter sets are the same:

$$S(\lambda_t f_j) = S_j(\lambda^0_t f^0_j), \quad \Delta_P[S](\lambda_t f_j) = \Delta_P[S](\lambda^0_t f^0_j), \quad \Delta^2_P[S](\lambda_t f_j) = \Delta^2_P[S](\lambda^0_t f^0_j) \tag{29}$$

$$\begin{aligned}
S_j(\lambda_t f_j,\cdot) &= S_j(\lambda^0_t f^0_j,\cdot) \\
\frac{\partial S_j}{\partial p_k}(\lambda_t f_j,\cdot) &= \frac{\partial S_j}{\partial p_k}(\lambda^0_t f^0_j,\cdot) \\
\frac{\partial^2 S_j}{\partial p_l \partial p_k}(\lambda_t f_j,\cdot) &= \frac{\partial^2 S_j}{\partial p_l \partial p_k}(\lambda^0_t f^0_j,\cdot)
\end{aligned} \tag{30}$$

We could use (3) and (4) to rewrite (2) in terms of the true model parameters, but we would still have to find an expression for $\frac{\partial p_i}{\partial f_{kr}}$. So now I turn to finding a relationship between the derivatives of the equilibrium prices with respect to the true and alternative factors using the implicit function theorem on the first-order-condition of the pricing stage (FOC_2).

Recall that the first-order-conditions of the pricing game is given by

$$FOC_2 : \frac{\partial \pi_h}{\partial p_k} = 0 = \Omega_{hk} s_k + \sum_{l\in \mathcal{J}} \Omega_{hl}(p_l - c_l)\frac{\partial s_l}{\partial p_j} = 0$$

Which can be written in matrix form as:

$$FOC2 : S + (\Upsilon \circ \Delta_p[S])(P - C) = 0 \tag{31}$$

Under the alternative parameter set, differentiating FOC_2 for the corresponding firm with respect to the r-th unobserved characteristic of each product yields a system of equations for the derivatives of the equilibrium prices. This system can be written in matrix form as:

$$(\Upsilon \circ \Delta_p[S])\,\Delta_{f_r}[P - C] = \Delta_{f_r}[S] - \frac{\partial}{\partial f_r}\left[(\Upsilon \circ \Delta_p[s])\right]^k (P - C)$$

Here, $[A]^k$ denotes the k^{th} column of the matrix A. Note that the coefficients of $\Delta_p[S]$ correspond to the price derivatives of the market shares which are the same for the true and alternative parameter sets.

Solving the system of equations and using the equalities from (3) and (4) shows that the derivatives of the equilibrium prices under the alternative parameter set are an affine transformation of those under the true model parameters, and are given by:

$$\Delta_{f^r}[P] = \left(\frac{\lambda_{tr}}{\lambda^0_{tr}}\right) \Delta_{f^0_r}[P] - \left(\frac{\lambda_{tr}}{\lambda^0_{tr}}\right) \Delta_{f^0_r}[C] + \Delta_{f_r}[C] \tag{32}$$

By substituting equation (6) into equation (2) and performing some algebra, we obtain:

$$\left(\frac{\lambda_{tr}}{\lambda^0_{tr}}\right) \left[\left(\Upsilon \circ \Delta_{f^0_r}[C]\right) S - \left(\Upsilon \circ \Delta_{f^0_r}[S]\right) (P - C)\right] - \left(\Upsilon \circ \Delta_{f^r}[C]\right) S + \left[T \circ \left(\Delta_{f^r}[S] + \Delta_{f^r}[C]\right)\right] (P - C) = 0$$

Which implies

$$\left(\frac{\lambda_{tr}}{\lambda^0_{tr}}\right) = \frac{\left(\Upsilon \circ \Delta_{f_r}[C]\right) S + \left[T \circ \left(\Delta_{f_r}[S] + \Delta_{f_r}[C]\right)\right] (P - C)}{\left[\left(\Upsilon \circ \Delta_{f^0_r}[C]\right) S - \left(\Upsilon \circ \Delta_{f^0_r}[S]\right) (P - C)\right]}$$

That is, the r-th loading of the alternative parameter set is a scaled-up version of the true loading parameter, and by the factor equation, the same holds for the factors themselves.

This contradicts our initial statement in equation (1) and proves the theorem.

□