LINEAR ALGEBRA

linear Algebra is a branch of mathematics that study vectors, matrices, linear transformation and systems of linear equations.

↳ It gives techniques to encrypt the data
↳ It gives tool to handle large data
↳ It helps in graphics.

Linear Equation:

An eq. is linear if

1) All variables appear in that eq. have power 1.
2) variable does not appear in product form.
3) variable can't appear as trignometric, hyperbolic and exponential function.

e.g

$x^2 + y = 5$ ✗ ; $x + 5y = 3$ ✓

$9e^x + \sin x = 4$ ✗ ; $x_1 + 2x_2 - 3x_3 = 0$ ✓

$x^{1/3} + \sqrt{2}\, y = 1$ ✗ ; $\frac{1}{2}x - y + 3z = 1$ ✓

Elementary Row Operation:

1. Interchange any two rows
2. Add / Sub any two rows

A

...thematics
...ansformation
data
...a
...ue power 1.
...form.
...ic, hyperbolic
✓
✓

3. Mul / Div any zero scalar with a zero.

Echelon form : / Row Echelon form / REF

1. Below ~~[illegible]~~ leading '1' all elements must be zero.
2. All zero rows must be bottom.

If these cond. satisfies then we can identify the variables values.

leading elem.

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1^{L.E} & 5 \\ 0 & 0 & 1^{LE} \end{bmatrix} \checkmark \qquad \begin{bmatrix} 0 & 1^{LE} & 3 & 2 \\ 0 & 0 & 1^{LE} & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \checkmark$$

$$\begin{bmatrix} 1^{LE} & 2 & 3 & 0 \\ 0 & 0 & 1^{LE} & 2 \end{bmatrix} \checkmark \qquad \begin{bmatrix} 1^{LE} & 1/2 & 3/2 & 5/2 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 3 & 0 \end{bmatrix} \times$$

$$x + 2y = 5$$
$$3x + 4y = 6$$

$$\underbrace{\left[\begin{array}{cc|c} \underbrace{\begin{matrix} 1 & 2 \\ 3 & 4 \end{matrix}}_{A} & & \begin{matrix} 5 \\ 6 \end{matrix} \end{array}\right]}_{\text{Augmented matrix}}$$

$$2i + 3j \rightarrow \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

Reduced Raw Echelon Form (RREF)

1. matrix is in RREF
2. All leading entries are '1'
3. All entries other than leading entry in a leading column are zero.

$$\left[\begin{array}{cc|c} 1 & 0 & 6 \\ 0 & 1 & 4 \end{array}\right] \qquad \left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 2 \end{array}\right]$$

Ques: convert the following into Echelon form.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 5 & 2 \\ 5 & 10 & 8 \end{bmatrix}$$

$R_2 \rightarrow 3R_1 - R_2$

$R_3 \rightarrow 5R_1 - R_3$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 7 \\ 0 & 0 & 7 \end{bmatrix} \xRightarrow[R_3 \rightarrow R_3/7]{} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 7 \\ 0 & 0 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 2 & 3 & 5 \\ 2 & 4 & 6 & 10 \\ 3 & 6 & 5 & 2 \end{bmatrix}$$

$R_2 \rightarrow 2R_1 - R_2$

$R_3 \rightarrow 3R_1 - R_3$

$R_3 \rightarrow R_3/4$

$$\begin{bmatrix} 1 & 2 & 3 & 5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \underline{4} & 13/4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 3 & 5 \\ 0 & 0 & 1 & 13/4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Interchanging R_2 & R_3

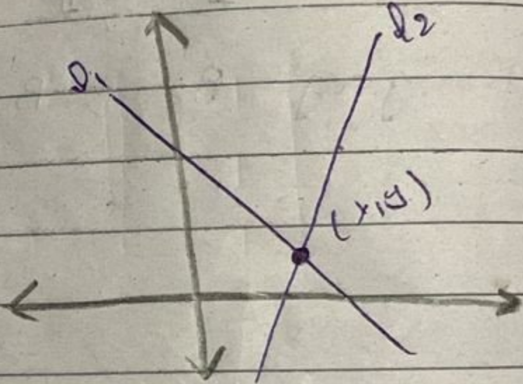
Sytem of Linear Equation :-

(1) $2x + 3y = 5$

$4x - 6y = 8$

$$\begin{array}{r} 4x + 6y = 10 \\ 4x - 6y = 8 \\ \hline 8x = 18 \end{array}$$

$x = 9/4$



$2x + 3y = 5$

$2(9/4) + 3y = 5$

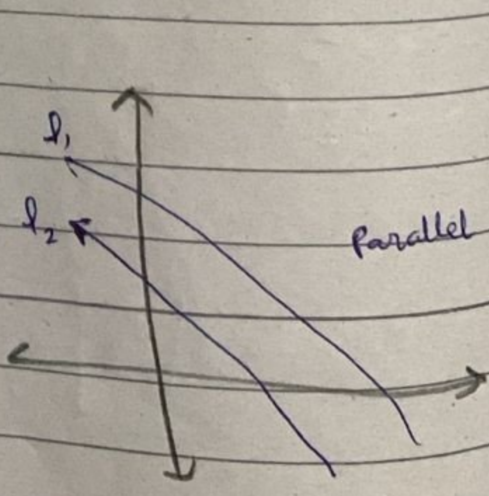
$\boxed{y = 1/6}$

unique solution
consistent.

(2) $2x + 3y = 5$

$4x + 6y = 3$

$$\begin{array}{r} 4x + 6y = 10 \\ \pm 4x \pm 6y = 3 \\ \hline 0x + 0y = 7 \end{array}$$



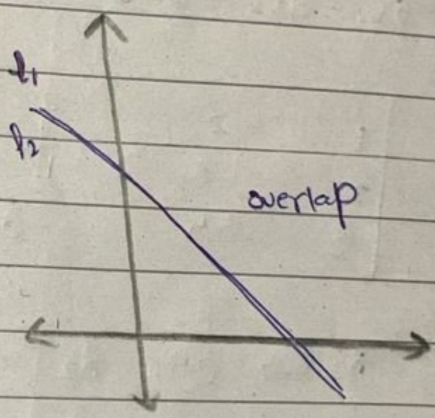
$$\left[\begin{array}{cc|c} 2 & 3 & 5 \\ 4 & 6 & 3 \end{array}\right]$$

$$\begin{array}{cc} x & y \end{array}$$
$$\left[\begin{array}{cc|c} 4 & 6 & 10 \\ 0 & 0 & 7 \end{array}\right]$$

$0x + 0y = 7$ No solution
In consitent

(3) $2x + 3y = 5$
$4x + 6y = 10$

$4x + 6y = 10$
$\pm 4x \pm 6y = 10$
$0x + 0y = 0$



$$\left[\begin{array}{cc|c} 2 & 3 & 5 \\ 4 & 6 & 10 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 4 & 6 & 10 \\ 0 & 0 & 0 \end{array}\right]$$

infinite many solutions
consistent

Three variables ~~dimension~~ means three dimensions each could be a plane.

Possible solution:

* a point
* a line
* a plane
* No solution

Ques:
Solve the system by Their augmented matrix to Echelon form.

$x_1 - 2x_2 - 2x_3 = -1$

$2x_1 + 3x_2 + x_3 = 1$

$5x_1 - 4x_2 - 3x_3 = 1$

$x_1 = 11/9$

$x_2 = -9/19$

$x_3 = 24/19$

$$\left[\begin{array}{ccc|c} 1 & -2 & -2 & -1 \\ 2 & 3 & 1 & 1 \\ 5 & -4 & -3 & 1 \end{array}\right]$$

$R_2 \rightarrow 2R_1 - R_2$

$R_3 \rightarrow 5R_1 - R_3$

$$\left[\begin{array}{cccc} 1 & -2 & -2 & -1 \\ 0 & -7 & - & \\ & & & \end{array}\right]$$

Ques: $x_1 - x_2 + 2x_3 = 0$
$4x_1 + x_2 + 2x_3 = 1$
$x_1 + x_2 + x_3 = -1$

$x = 1$
$x_2 = -1$
$x_3 = -1$

Sol

The augmented matrix is :

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 0 \\ 4 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 \end{array}\right]$$

$R_2 \leftarrow 4R_1 - R_2$
$R_3 \leftarrow R_1 - R_3$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 0 \\ 0 & -5 & 6 & -1 \\ 0 & -2 & 1 & 1 \end{array}\right]$$

last row $\longrightarrow \left[\begin{array}{ccc|c} 0 & 0 & 0 & 3 \end{array}\right]$ ↳ any const

then → 'No Solution'

Ques: Solve the following linear system

$$x_1 - 2x_2 + 3x_3 = -2$$
$$-x_1 + x_2 - 2x_3 = 3$$
$$2x_1 - x_2 + 3x_3 = 1$$

Sol

The augmented matrix is

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & -2 \\ -1 & 1 & -2 & 3 \\ 2 & -1 & 3 & 1 \end{array}\right]$$

$R_2 \leftarrow R_1 + R_2$

$R_3 \leftarrow 2R_1 - R_3$

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & -2 \\ 0 & -1 & 1 & 1 \\ 0 & -3 & 3 & -5 \end{array}\right]$$

$R_3 \leftarrow 3R_2 - R_3$

$$\begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \left[\begin{array}{ccc|c} 1 & -2 & 3 & -2 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & 0 & 8 \end{array}\right] \end{array}$$

↳ No Solution.

Ques: $x_1 - 2x_2 + x_3 + x_4 = 0$

$2x + x_4 = 4$

The augment matrix for this system is :

No. of variables > No. of ~~solutions~~ equations
(infinite many solution)

$$\begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & & \end{array} \\ \left[\begin{array}{cccc|c} 1 & -2 & 1 & 1 & 0 \\ 0 & 2 & 0 & 1 & 4 \end{array}\right] \end{array}$$

$R_2 \leftarrow R_2/2$ does not contain L.E (column)

$$\begin{array}{c} \begin{array}{cccccc} \checkmark & \checkmark & \times & \times & & \\ x_1 & x_2 & x_3 & x_4 & & \end{array} \\ \left[\begin{array}{cccc|c} 1\ \text{L.E} & -2 & 1 & 1 & 0 \\ 0 & 1\ \text{L.E} & 0 & 1/2 & 2 \end{array}\right] \end{array}$$

Suppose,

$$x_3 = r$$
$$x_4 = s$$

Now we have

$$x_2 + 1/2\, x_4 = 2$$
$$x_2 + 1/2\,(s) = 2$$
$$\boxed{x_2 = 2 - 1/2\, s}$$

and

$$x_1 - 2x_2 + x_3 + x_4 = 0$$
$$x_1 - 2(2 - 1/2\, s) + r + s = 0$$
$$x_1 - 4 + s + r + s = 0$$
$$\boxed{x_1 = 4 - 2s - r}$$

Ques: $x_1 - x_2 + 2x_3 + x_4 + 2x_5 = 1$

$2x_3 + x_4 + 3x_5 = 3$

$x_4 = 1$

The augment matrix for the given system is

$$\left[\begin{array}{ccccc|c} 1 & -1 & 2 & 1 & 2 & 1 \\ 0 & 0 & 2 & 1 & 3 & 3 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{array}\right]$$

$R_2 \leftarrow R_2/2$

$$\begin{array}{ccccc} x_1 & \cancel{x_2} & x_3 & x_4 & \cancel{x_5} \end{array}$$
$$\left[\begin{array}{ccccc|c} 1\text{ LE} & -1 & 2 & 1 & 2 & 1 \\ 0 & 0 & 1\text{ LE} & 1/2 & 3/2 & 3/2 \\ 0 & 0 & 0 & 1\text{ LE} & 0 & 1 \end{array}\right]$$

suppose,

$x_2 = s$

$x_5 = r$

Now we have,

$x_4 = 1$

and

$x_3 + 1/2\, x_4 + \frac{3}{2} x_5 = 3/2$

$x_3 + 1/2(1) + 3/2(r) = 3/2$

$x_3 = \frac{3}{2} - \frac{1}{2} - \frac{3r}{2}$

$$\boxed{x_3 = \frac{2-3r}{2}}$$

and

$$x_1 - x_2 + 2x_3 + x_4 + 2x_5 = 1$$

$$x_1 - s + 2\left(\frac{2-3r}{2}\right) + 1 + 2(r) = 1$$

$$x_1 - s + 2 - 3r + 1 + 2r = 1$$

$$x_1 - s + 3 - r = 1$$

$$\boxed{x_1 = s + r - 2}$$

Ques. Find all the values of 'k' for which the system has:

a) unique solution $[0 \ 0 \ 2 \mid 5]$

b) No solution $[0 \ 0 \ 0 \mid 7]$

c) infinitely many solution. $[0 \ 0 \ 0 \mid 0]$

$$x + 2y - z = 1$$

$$2x + 3y + z = 1$$

$$-4x - 5y + (k^2 - 9)z = k + 1$$

Sol

The augmented matrix for this system is

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 2 & 3 & 1 & 1 \\ -4 & -5 & (k^2-9) & k+1 \end{array}\right]$$

$R_2 \leftarrow 2R_1 - R_2$, $R_3 \leftarrow 4R_1 + R_3$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & 1 & -3 & 1 \\ 0 & 3 & -4 + (k^2 + 9) & 5 + k \end{array}\right]$$

$R_3 \leftarrow 3R_2 - R_3$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & 1 & -3 & 1 \\ 0 & 0 & -k^2+4 & -k-2 \end{array}\right]$$

For unique solution:

$$-k^2 + 4 \neq 0$$
$$k^2 \neq 4$$
$$k \neq \pm 2$$

For no ~~unique~~ solution:

if $k = 2$

$$[\begin{array}{ccc|c} 0 & 0 & -(2)^2+4 & -2-2 \end{array}]$$

For infinitely solutions

if $k = -2$

$$[\begin{array}{ccc|c} 0 & 0 & -(-2)^2+4 & 2-2 \end{array}]$$

Ques: $x + z = k^2$

$2x + y + 3z = -3k$

$3x + y + 4z = -2$

Sol

The augmented matrix for this system is

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & k^2 \\ 2 & 1 & 3 & -3k \\ 3 & 1 & 4 & -2 \end{array}\right]$$

$R_2 \leftarrow 2R_1 - R_2$

$R_3 \leftarrow 3R_1 - R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & k^2 \\ 0 & -1 & -1 & 2k^2+3k \\ 0 & -1 & -1 & 3k^2+2 \end{array}\right]$$

$R_3 \leftarrow R_2 - R_3$, $R_2 \leftarrow R_2/-1$

$$\left[\begin{array}{cccc} 1 & 0 & 1 & k^2 \\ 0 & 1 & 1 & -2k^2 - 3k \\ 0 & 0 & 0 & -k^2+3k-2 \end{array}\right]$$

<u>For unique solution:</u>

unique solution does not exist

<u>For no solution:</u>

No solution exist when we use any value of 'k' except 1 & 2

$$-k^2+3k-2=0$$

$$k=1,2$$

<u>For infinitely many solutions:</u>

if $k=1,2$

then,

$$-(1)^2+3(1)-2=0$$

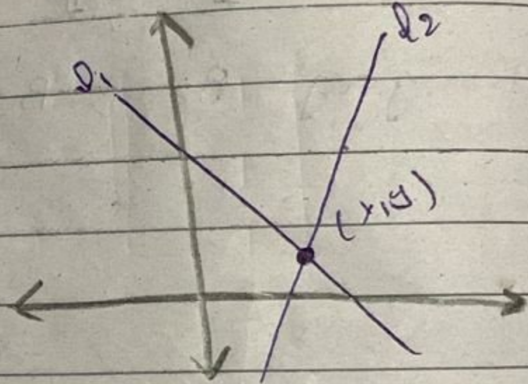
$$-(2)^2+3(2)-2=0$$

Sytem of Linear Equation :-

(1) $2x + 3y = 5$
$4x - 6y = 8$

$$\begin{array}{r} 4x + 6y = 10 \\ 4x - 6y = 8 \\ \hline 8x = 18 \end{array}$$

$x = 9/4$

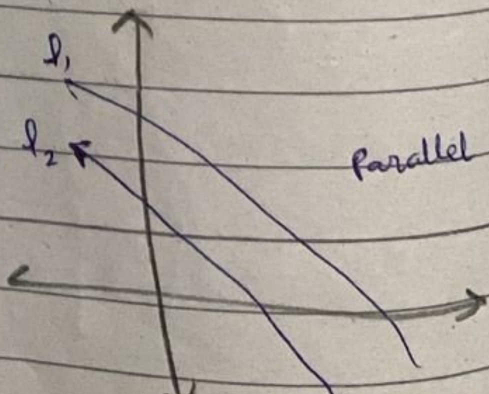


$2x + 3y = 5$
$2(9/4) + 3y = 5$
$\boxed{y = 1/6}$

unique solution
consistent.

(2) $2x + 3y = 5$
$4x + 6y = 3$

$$\begin{array}{r} 4x + 6y = 10 \\ \pm 4x \pm 6y = 3 \\ \hline 0x + 0y = 7 \end{array}$$



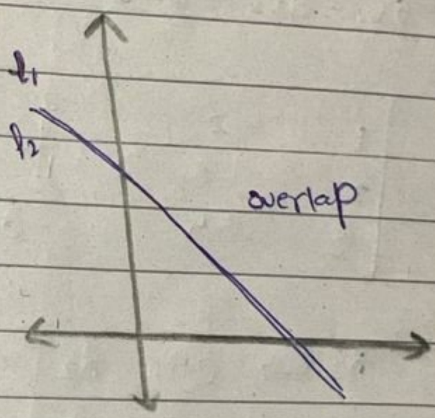
$$\left[\begin{array}{cc|c} 2 & 3 & 5 \\ 4 & 6 & 3 \end{array}\right]$$

$$\begin{array}{cc} x & y \end{array}$$
$$\left[\begin{array}{cc|c} 4 & 6 & 10 \\ 0 & 0 & 7 \end{array}\right]$$

$0x + 0y = 7$ No solution
In consistent

(3) $2x + 3y = 5$
$4x + 6y = 10$

$$4x + 6y = 10$$
$$\underline{\pm 4x \pm 6y = 10}$$
$$0x + 0y = 0$$



$$\left[\begin{array}{cc|c} 2 & 3 & 5 \\ 4 & 6 & 10 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 4 & 6 & 10 \\ 0 & 0 & 0 \end{array}\right]$$

infinite many solutions
consistent

Three ~~dimension~~ variables means three dimensions each could be a plane.

Possible solution:
* a point
* a line
* a plane
* No solution

Ques:
Solve the system by Their augmented matrix to Echelon form.

$x_1 - 2x_2 - 2x_3 = -1$

$2x_1 + 3x_2 + x_3 = 1$

$5x_1 - 4x_2 - 3x_3 = 1$

$x_1 = 11/9$

$x_2 = -9/19$

$x_3 = 24/19$

$$\left[\begin{array}{ccc|c} 1 & -2 & -2 & -1 \\ 2 & 3 & 1 & 1 \\ 5 & -4 & -3 & 1 \end{array}\right]$$

$R_2 \rightarrow 2R_1 - R_2$

$R_3 \rightarrow 5R_1 - R_3$

$$\begin{bmatrix} 1 & -2 & -2 & -1 \\ 0 & -7 & - & \\ & & & \end{bmatrix}$$

Ques : $x_1 - x_2 + 2x_3 = 0$

$4x_1 + x_2 + 2x_3 = 1$ $\qquad$ $x = 1$

$x_1 + x_2 + x_3 = -1$ $\qquad$ $x_2 = -1$

$x_3 = -1$

Sol

The augmented matrix is :

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 0 \\ 4 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 \end{array}\right]$$

$R_2 \leftarrow 4R_1 - R_2$

$R_3 \leftarrow R_1 - R_3$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 0 \\ 0 & -5 & 6 & -1 \\ 0 & -2 & 1 & 1 \end{array}\right]$$

last row $\longrightarrow \left[\begin{array}{ccc|c} 0 & 0 & 0 & 3 \end{array}\right]$

then $\longrightarrow$ 'No Solution' $\hookrightarrow$ any const

Ques: Solve the following linear system

$$x_1 - 2x_2 + 3x_3 = -2$$
$$-x_1 + x_2 - 2x_3 = 3$$
$$2x_1 - x_2 + 3x_3 = 1$$

Sol

The augmented matrix is

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & -2 \\ -1 & 1 & -2 & 3 \\ 2 & -1 & 3 & 1 \end{array}\right]$$

$R_2 \leftarrow R_1 + R_2$

$R_3 \leftarrow 2R_1 - R_3$

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & -2 \\ 0 & -1 & 1 & 1 \\ 0 & -3 & 3 & -5 \end{array}\right]$$

$R_3 \leftarrow 3R_2 - R_3$

$$\begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \left[\begin{array}{ccc|c} 1 & -2 & 3 & -2 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & 0 & 8 \end{array}\right] \end{array}$$

$\hookrightarrow$ No Solution.

Ques: $x_1 - 2x_2 + x_3 + x_4 = 0$

$2x + x_4 = 4$

The augment matrix for this system is :

No. of variables > No. of ~~solutions~~ equations
(infinite many solution)

$$\begin{array}{cccc|c} x_1 & x_2 & x_3 & x_4 & \\ \end{array}$$
$$\left[\begin{array}{cccc|c} 1 & -2 & 1 & 1 & 0 \\ 0 & 2 & 0 & 1 & 4 \end{array}\right]$$

$R_2 \leftarrow R_2/2$ — does not contain L.E (column)

$$\begin{array}{cccc} \checkmark & \checkmark & \times & \times \\ x_1 & x_2 & x_3 & x_4 \end{array}$$
$$\left[\begin{array}{cccc|c} 1\ \text{L.E} & -2 & 1 & 1 & 0 \\ 0 & 1\ \text{L.E} & 0 & 1/2 & 2 \end{array}\right]$$

Suppose,

$$x_3 = r$$
$$x_4 = s$$

Now we have

$$x_2 + 1/2\, x_4 = 2$$
$$x_2 + 1/2\,(s) = 2$$
$$\boxed{x_2 = 2 - 1/2\, s}$$

and

$$x_1 - 2x_2 + x_3 + x_4 = 0$$
$$x_1 - 2(2 - 1/2\, s) + r + s = 0$$
$$x_1 - 4 + s + r + s = 0$$
$$\boxed{x_1 = 4 - 2s - r}$$

Ques: $x_1 - x_2 + 2x_3 + x_4 + 2x_5 = 1$

$2x_3 + x_4 + 3x_5 = 3$

$x_4 = 1$

The augment matrix for the given system is

$$\left[\begin{array}{ccccc|c} 1 & -1 & 2 & 1 & 2 & 1 \\ 0 & 0 & 2 & 1 & 3 & 3 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{array}\right]$$

$R_2 \leftarrow R_2/2$

$$\begin{array}{ccccc} x_1 & \cancel{x_2} & x_3 & x_4 & \cancel{x_5} \end{array}$$
$$\left[\begin{array}{ccccc|c} 1\ \text{LE} & -1 & 2 & 1 & 2 & 1 \\ 0 & 0 & 1\ \text{LE} & 1/2 & 3/2 & 3/2 \\ 0 & 0 & 0 & 1\ \text{LE} & 0 & 1 \end{array}\right]$$

suppose,

$x_2 = s$

$x_5 = r$

Now we have,

$x_4 = 1$

and

$x_3 + 1/2\, x_4 + \frac{3}{2} x_5 = 3/2$

$x_3 + 1/2(1) + 3/2(r) = 3/2$

$x_3 = \frac{3}{2} - \frac{1}{2} - \frac{3r}{2}$

$$\boxed{x_3 = \frac{2-3r}{2}}$$

and

$$x_1 - x_2 + 2x_3 + x_4 + 2x_5 = 1$$

$$x_1 - s + 2\left(\frac{2-3r}{2}\right) + 1 + 2(r) = 1$$

$$x_1 - s + 2 - 3r + 1 + 2r = 1$$

$$x_1 - s + 3 - r = 1$$

$$\boxed{x_1 = s + r - 2}$$

Ques. Find all the values of 'k' for which the system has:

a) unique solution $\quad [0 \ 0 \ 2 \mid 5]$

b) No solution $\quad [0 \ 0 \ 0 \mid 7]$

c) Infinitely many solution. $[0 \ 0 \ 0 \mid 0]$

$$x + 2y - z = 1$$

$$2x + 3y + z = 1$$

$$-4x - 5y + (k^2 - 9)z = k + 1$$

Sol

The augmented matrix for this system is

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 2 & 3 & 1 & 1 \\ -4 & -5 & (k^2-9) & k+1 \end{array}\right]$$

$R2 \leftarrow 2R_1 - R_2 \quad , \quad R_3 \leftarrow 4R_1 + R_3$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & 1 & -3 & 1 \\ 0 & 3 & -4+(k^2+9) & 5+k \end{array}\right]$$

$R_3 \leftarrow 3R_2 - R_3$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & 1 & -3 & 1 \\ 0 & 0 & -k^2+4 & -k-2 \end{array}\right]$$

For unique solution:

$$-k^2 + 4 \neq 0$$
$$k^2 \neq 4$$
$$k \neq \pm 2$$

For no ~~unique~~ solution:

if $k = 2$

$$[\,0 \quad 0 \quad -(2)^2+4 \mid -2-2\,]$$

For infinitely solutions

if $k = -2$

$$[\,0 \quad 0 \quad -(-2)^2+4 \mid 2-2\,]$$

Ques: $x + z = k^2$

$2x + y + 3z = -3k$

$3x + y + 4z = -2$

Sol

The augmented matrix for this system is

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & k^2 \\ 2 & 1 & 3 & -3k \\ 3 & 1 & 4 & -2 \end{array}\right]$$

$R_2 \leftarrow 2R_1 - R_2$

$R_3 \leftarrow 3R_1 - R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & k^2 \\ 0 & -1 & -1 & 2k^2+3k \\ 0 & -1 & -1 & 3k^2+2 \end{array}\right]$$

$R_3 \leftarrow R_2 - R_3$, $R_2 \leftarrow R_2/-1$

$$\left[\begin{array}{cccc} 1 & 0 & 1 & k^2 \\ 0 & +1 & +1 & -2k^2 - 3k \\ 0 & 0 & 0 & -k^2+3k-2 \end{array}\right]$$

<u>For unique solution:</u>

unique solution does not exist

<u>For no solution:</u>

No solution exist when we use any value of 'k' except 1 & 2

$$-k^2 + 3k - 2 = 0$$
$$k = 1, 2$$

<u>For infinitely many solutions:</u>

if $k = 1, 2$

then,

$$-(1)^2 + 3(1) - 2 = 0$$
$$-(2)^2 + 3(2) - 2 = 0$$

$R_3 \leftarrow 3R_2 - R_3$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & 1 & -3 & 1 \\ 0 & 0 & -k^2+4 & -k-2 \end{array}\right]$$

for unique solution:

$$-k^2 + 4 \neq 0$$
$$k^2 \neq 4$$
$$k \neq \pm 2$$

For no ~~unique~~ solution:

if $k = 2$

$$\left[\begin{array}{ccc|c} 0 & 0 & -(2)^2+4 & -2-2 \end{array}\right]$$

For infinitely solutions

if $k = -2$

$$\left[\begin{array}{ccc|c} 0 & 0 & -(-2)^2+4 & 2-2 \end{array}\right]$$

Ques: $x + z = k^2$

$2x + y + 3z = -3k$

$3x + y + 4z = -2$

Sol

The augmented matrix for this system is

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & k^2 \\ 2 & 1 & 3 & -3k \\ 3 & 1 & 4 & -2 \end{array}\right]$$

$R_2 \leftarrow 2R_1 - R_2$

$R_3 \leftarrow 3R_1 - R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & k^2 \\ 0 & -1 & -1 & 2k^2+3k \\ 0 & -1 & -1 & 3k^2+2 \end{array}\right]$$

$R_3 \leftarrow R_2 - R_3$, $R_2 \leftarrow R_2/-1$

$$\left[\begin{array}{cccc} 1 & 0 & 1 & k^2 \\ 0 & 1 & 1 & -2k^2-3k \\ 0 & 0 & 0 & -k^2+3k-2 \end{array}\right]$$

<u>For unique solution:</u>

unique solution does not exist

<u>For no solution:</u>

No solution exist when we use any value of 'k' except 1 & 2

$$-k^2 + 3k - 2 = 0$$

$$k = 1, 2$$

<u>For infinitely many solutions:</u>

if $k = 1, 2$

then,

$$-(1)^2 + 3(1) - 2 = 0$$

$$-(2)^2 + 3(2) - 2 = 0$$