

$$SSE = e^T e = Y^T (I_n - H) Y$$

$$= \text{Tr}(SSE)$$

$$= \text{Tr}(Y^T (I_n - H) Y)$$



$$(\text{tr}(AB) = \text{tr}(BA))$$

$$= \text{Tr}((I_n - H) Y \cdot Y^T)$$

$$E(SSE) = E(\text{Tr}((I_n - H) Y \cdot Y^T))$$

$$= \text{Tr}(E((I_n - H) Y \cdot Y^T))$$

$$= \text{Tr}((I_n - H) \cdot E(Y \cdot Y^T))$$

$$= \text{Tr}((I_n - H) \cdot (\sigma^2 I_n + X\beta\beta^T X^T))$$

$$= \text{Tr}((I_n - H) \cdot \sigma^2 I_n) + \text{Tr}((I_n - H) \cdot X\beta\beta^T X^T)$$

$$= \sigma^2 \text{Tr}(I_n - H)$$

$$= \sigma^2 (n - \text{Tr}(H)) = \sigma^2 (n - 2)$$

$$Y = \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}, \quad EY = X\beta$$

$$\sigma^2 \{Y\} = \sigma^2 I_n$$

$$E(YY^T) = \sigma^2 \{Y\} + [E(Y)][E(Y)]^T$$

$$= \sigma^2 I_n + (X\beta)(X\beta)^T$$

$$= \sigma^2 I_n + X\beta\beta^T X^T$$

$$(I_n - H)X = X - HX$$

$$= X - X = 0$$

$$\text{Tr}(H) = \text{Tr}(X(X^T X)^{-1} X^T)$$

$$= \text{Tr}(\underbrace{(X^T X)^{-1} X^T X}_{I_2}) = \text{Tr}(I_2) = 2$$

$$I_n = \begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}, \quad \text{tr}(I_n) = 1 + 1 + \dots + 1 = n$$