Decision & Regression Trees

CSci 5525: Machine Learning

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Attribute-based representations

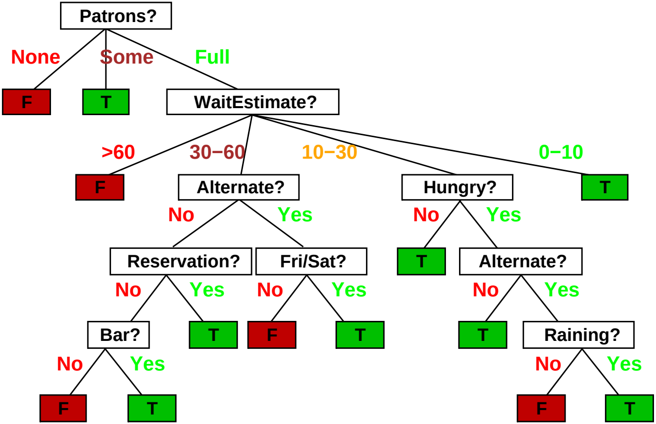
Examples described by attribute values (Boolean, discrete, continuous, etc.)

Example	*Attributes*										*Target*
	Alt	*Bar*	*Fri*	*Hun*	*Pat*	*Price*	*Rain*	*Res*	*Type*	*Est*	WillWait
X_1	T	F	F	T	Some	\$\$\$	F	T	French	0–10	T
X_2	T	F	F	T	Full	\$	F	F	Thai	30–60	F
X_3	F	T	F	F	Some	\$	F	F	Burger	0–10	T
X_4	T	F	T	T	Full	\$	F	F	Thai	10–30	T
X_5	T	F	T	F	Full	\$\$\$	F	T	French	>60	F
X_6	F	T	F	T	Some	\$\$	T	T	Italian	0–10	T
X_7	F	T	F	F	None	\$	T	F	Burger	0–10	F
X_8	F	F	F	T	Some	\$\$	T	T	Thai	0–10	T
X_9	F	T	T	F	Full	\$	T	F	Burger	>60	F
X_{10}	T	T	T	T	Full	\$\$\$	F	T	Italian	10–30	F
X_{11}	F	F	F	F	None	\$	F	F	Thai	0–10	F
X_{12}	T	T	T	T	Full	\$	F	F	Burger	30–60	T

Classification of examples is positive (T) or negative (F)

Decision Tree Example (Restaurant)

One possible representation for hypotheses



Example: Playing Golf

Play golf dataset

Independent variables				Dep. var
OUTLOOK	TEMPERATURE	HUMIDITY	WINDY	PLAY
sunny	85	85	FALSE	Don't Play
sunny	80	90	TRUE	Don't Play
overcast	83	78	FALSE	Play
rain	70	96	FALSE	Play
rain	68	80	FALSE	Play
rain	65	70	TRUE	Don't Play
overcast	64	65	TRUE	Play
sunny	72	95	FALSE	Don't Play
sunny	69	70	FALSE	Play
rain	75	80	FALSE	Play
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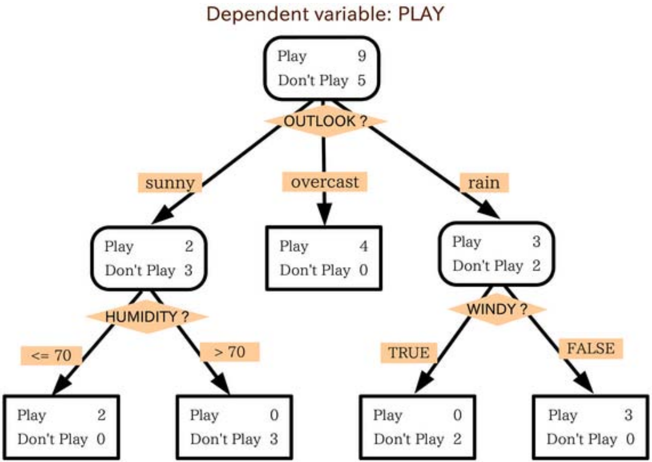
Decision Tree Example (Golf)



Expressiveness

- Decision trees can express any function of the input attributes

A	B	A xor B
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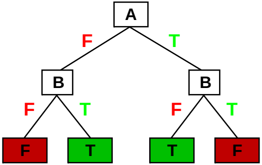
T	T	F



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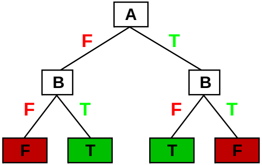

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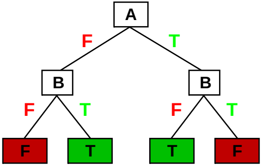

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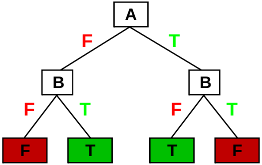

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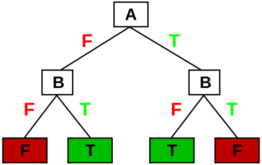

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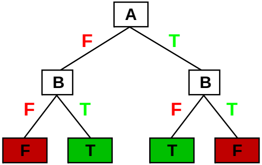

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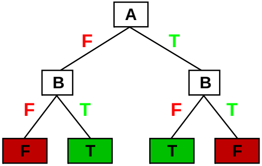

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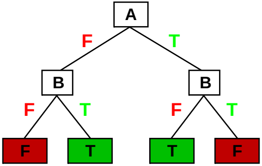

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  - But it probably won't generalize to new examples
- Prefer to find more compact decision trees

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  - Increases number of hypotheses consistent w/ training set
    $\Longrightarrow$ may get worse predictions

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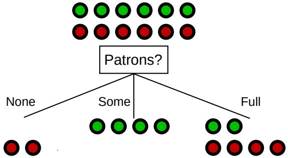

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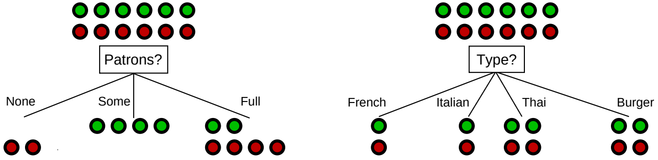

Patrons? is a better choice

—gives *information* about the classification

Information Gain

- The entropy of a r.v. X with distribution $(p(x_1), \ldots, p(x_n))$

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  - For 12 restaurant examples, $p = n = 6$, so we need 1 bit

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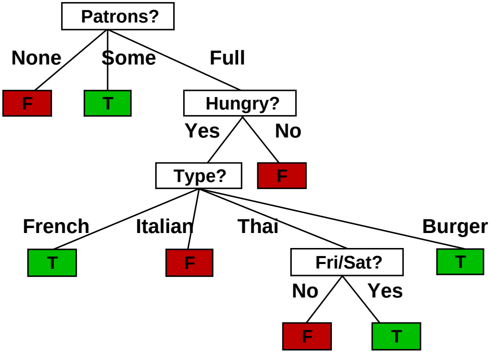
- Choose the attribute that maximizes information gain

A Simple Decision Tree

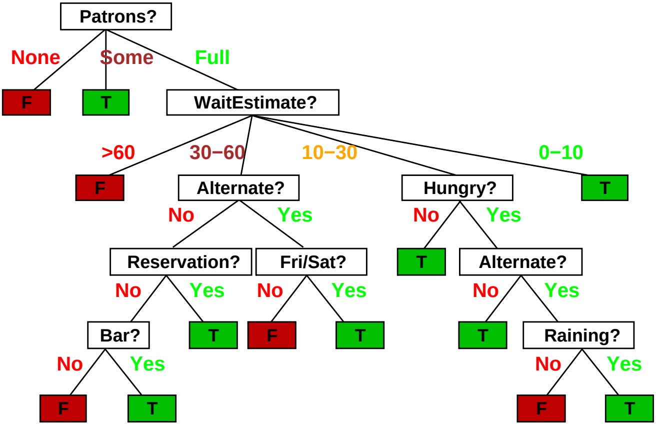
- Decision tree learned from the 12 examples:



- Substantially simpler than “true” tree
  - More complex hypothesis is not justified by a small dataset

The Original Tree

The original (complex) tree with same performance on the training set



Another Example: Edibility

Skin	Color	Thorny?	Flowering?	Edible?
smooth	pink	no	yes	yes
smooth	pink	no	no	yes
scaly	pink	no	yes	no
rough	purple	no	yes	no
rough	orange	yes	yes	no
scaly	orange	yes	no	no
smooth	purple	no	yes	yes
smooth	orange	yes	yes	no
rough	purple	yes	yes	no
smooth	purple	yes	no	no
scaly	purple	no	no	no
scaly	pink	yes	yes	no
rough	purple	no	no	yes
rough	orange	yes	no	yes

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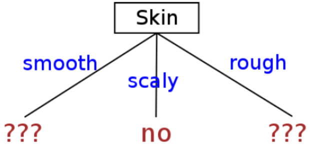

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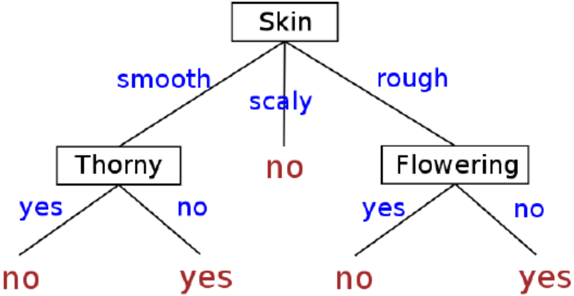
- Consider $Skin = rough$
  - Choose *Flowering*

Edibility Decision Tree



Other Methods for Feature Selection

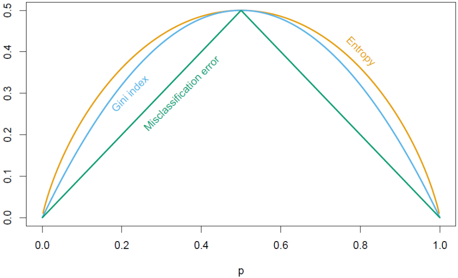
- Issues with Information Gain
- Gain Ratio

$$GainRatio(X|Y) = \frac{IG(X|Y)}{H(Y)}$$

- Gini Index

$$\begin{aligned} Gini(X) &= \sum_{i \neq j} p(i)p(j) \\ Gini(X|Y) &= \sum_{j} p(y_j) Gini(X|y_j) \\ GiniGain(X|Y) &= Gini(X) - Gini(X|Y) \end{aligned}$$

Loss functions for Decision Trees



p = proportion of points in class 2

Entropy has been scaled to pass through $(\frac{1}{2}, \frac{1}{2})$

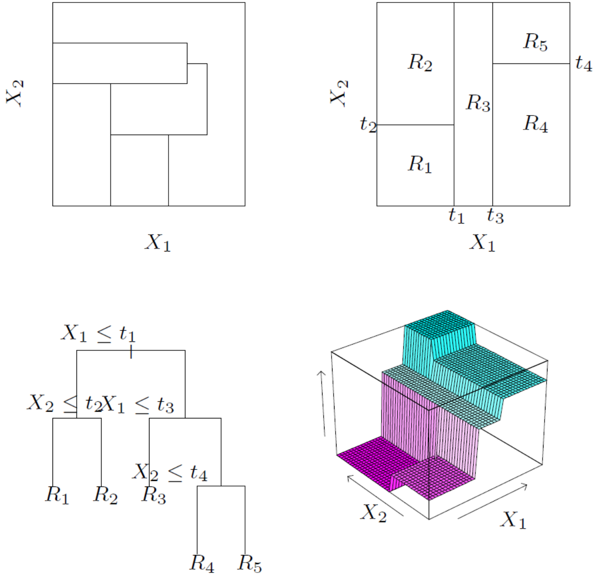
Challenges: Overfitting, Continuous features, Stability

- Overfitting: May generate a large tree
  - Good training set performance, poor test set performance
  - Decision tree pruning: Remove irrelevant attributes
  - Information gain is small after splitting
  - Significance test with null hypothesis of no underlying pattern
- Continuous valued attributes
  - Split into regions, convert to categorical variable
  - Find the split(s) so as to maximize information gain

Temperature	40	48	60	72	80	90
Play Tennis	No	No	Yes	Yes	Yes	No

- Stability
  - Mild change in attributes can change the tree
  - High variance, unstable, but interpretable
  - Ensembles approaches to reduce variance

Regression Trees



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