CSci 5512 – Artificial Intelligence II Jiecao Chen ID:4716311
HOMEWORK #1 DUE: DUE WED, FEB 20

Question 1

- $P(T = true) = \sum_B P(T = true|B)P(B) = 0.97 * 0.98 + 0.02 * 0.02 = 0.951$
- $P(T = false) = 1 - 0.951 = 0.049$
- $P(F = empty, S = no) = \sum_T P(F = empty)P(S = no|T, F = empty)P(T)$
  $= 0.05 * 0.92 * 0.951 + 0.05 * 0.99 * 0.049 = 0.0462$
- $P(S = no) = \sum_{T,F} P(S = no|T, F)P(T)P(F)$
  $= \sum_T P(T) \sum_F P(S = no|T, F)P(F)$
  $= 0.049 * (0.05 * 0.99 + 0.95 * 1.00) + 0.951 * (0.05 * 0.92 + 0.95 * 0.01) = 0.1018$

So $P(F = empty|S = no) = \frac{P(F=empty,S=no)}{P(S=no)} = \frac{0.0462}{0.1018} = 0.4532$

Question 2

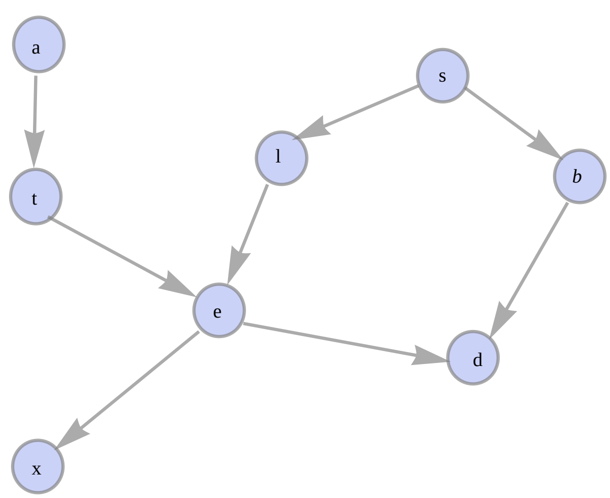

FIGURE 1. The Chest Clinic network

a. State if the following conditional independence are true or false:

i. $t \perp s | d$
Use D-separation or D-connection. In the undirected path $t - e - d - b - s - b - s$, d is a collider, thus t and s are D-connected, thus $t \perp s | d$ is false.

ii. $l \perp b | s$
s is not a collider, thus $t \perp s | d$ is true.

iii. $a \perp s | l, d$
There are only two undirected paths from a to s, one is $a - t - e - d - b - s$, all colliders in this path (i.e. l) is in $\{l, d\}$, and all non-colliders are not in $\{l, d\}$, thus $a \perp s | l, d$ is false.

b. We use bold font to stand fixed variables, thus $P(\mathbf{s}|\mathbf{a}, \mathbf{x}, \mathbf{b}) = \frac{P(\mathbf{s},\mathbf{a},\mathbf{x},\mathbf{b})}{P(\mathbf{a},\mathbf{x},\mathbf{b})}$

$$P(\mathbf{s}, \mathbf{a}, \mathbf{x}, \mathbf{b}) = \sum_{e,t,l} P(\mathbf{s})P(\mathbf{a})P(\mathbf{b}|\mathbf{s})P(\mathbf{x}|e)P(e|t,l)P(t|\mathbf{a})P(l|\mathbf{s})$$
$$= P(\mathbf{s})P(\mathbf{a})P(\mathbf{b}|\mathbf{s}) \sum_t P(t|\mathbf{a}) \sum_l P(l|\mathbf{s}) \sum_e P(\mathbf{x}|e)P(e|t,l)$$

$$P(\mathbf{a}, \mathbf{x}, \mathbf{b}) = \sum_{s,e,t,l} P(\mathbf{a})P(\mathbf{b}|s)P(s)P(\mathbf{x}|e)P(e|t,l)P(t|\mathbf{a})$$
$$= P(\mathbf{a}) \sum_s P(\mathbf{b}|s)P(s) \sum_l P(l|s) \sum_t P(t|\mathbf{a}) \sum_e P(\mathbf{x}|e)P(e|t,l)$$

c. If will know the value of e, then the computation complexity would significantly decrease. See the following formulas:
$P(\mathbf{s}|\mathbf{a}, \mathbf{x}, \mathbf{b}, \mathbf{e}) = \frac{P(\mathbf{s},\mathbf{a},\mathbf{x},\mathbf{b},\mathbf{e})}{P(\mathbf{a},\mathbf{x},\mathbf{b},\mathbf{e})}$

$$P(\mathbf{s}, \mathbf{a}, \mathbf{x}, \mathbf{b}) = \sum_{e,t,l} P(\mathbf{s})P(\mathbf{a})P(\mathbf{b}|\mathbf{s})P(\mathbf{x}|e)P(e|t,l)P(t|\mathbf{a})P(l|\mathbf{s})$$
$$= P(\mathbf{s})P(\mathbf{a})P(\mathbf{b}|\mathbf{s})P(\mathbf{x}|\mathbf{e}) \sum_t P(t|\mathbf{a}) \sum_l P(l|\mathbf{s})P(\mathbf{e}|t,l)$$

$$P(\mathbf{a}, \mathbf{x}, \mathbf{b}) = \sum_{s,e,t,l} P(\mathbf{a})P(\mathbf{b}|s)P(s)P(\mathbf{x}|e)P(e|t,l)P(t|\mathbf{a})$$
$$= P(\mathbf{a})P(\mathbf{x}|\mathbf{e}) \sum_s P(\mathbf{b}|s)P(s) \sum_l P(l|s) \sum_t P(t|\mathbf{a})P(\mathbf{e}|t,l)$$

We no longer need to sum for the variable e.

Question 3

(a). See the Figure 2

(b). See Table 1.

TABLE 1. conditional prob table

T	F_G	$P(G = high \mid T, F_G)$
high	faulty	y
high	work	x
normal	faulty	$1 - y$
normal	work	$1 - x$

(c). See Table 2.

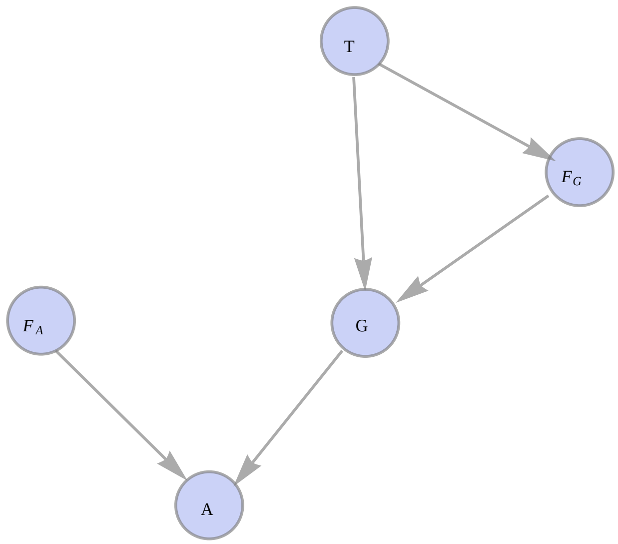

FIGURE 2. Nuclear Temperature System Network

TABLE 2. conditional prob table

G	F_A	$P(A = sound \mid G, F_A)$
high	faulty	0
high	work	1
normal	faulty	0
normal	work	0

(d). we have notations: h-high, w-work, s-sound.
$P(T = h|F_A = w, F_G = w, A = s) = \frac{P(T=h,F_A=w,F_G=w,A=s)}{P(F_A=w,F_G=w,A=s)}$, where
$P(T = h, F_A = w, F_G = w, A = s)$
$= P(T = h)P(F_A = w)P(F_G = w|T = h)$
$\sum_G P(A = s|F_A = w, G)P(G|T = h, F_G = w)$
$P(F_A = w, F_G = w, A = s)$
$= P(F_A = w) \sum_T P(F_G = w|T)P(T) \sum_G P(A = s|F_A = w, G)P(G|T, F_G = w)$

Question 4

(a). We prove $P(X|\{U_i\}, \{Z_j\}) = P(X|\{U_i\})$ by contradiction:
Assume X and $\{Z_i\}$ are D-connected by $\{U_i\}$, then there exist $Z \in \{Z_i\}$ such that there is a undirected path U from Z to X, and all its colliders are in $\{U_i\}$.

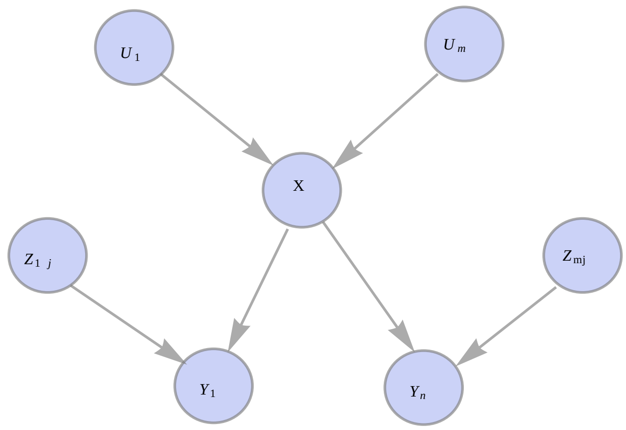

FIGURE 3. Counterexample

case 1 $U \bigcap \{U_i\} = A \neq \emptyset$
Since every node in A is also in $\{U_i\}$, thus they are all non-colliders, in this case, X and $\{Z_i\}$ are D-connected by the path U.

case 2 $U \bigcap \{U_i\} = \emptyset$
Thus $U \bigcap \{Y_i\} = A \neq \emptyset$, from the graph, if there is no collider in A, then all arrows in U are in the same direction (either from $Z \leftarrow X$ or $X \leftarrow Z$), but that's not true based on the graph in the question. So, there is at least one collider in U, but it is not in $\{U_i\}$, which contradicts to the assumption **there exist** $Z \in \{Z_i\}$ **such that there is a undirected path** U **from** Z **to** X**, and all its colliders are in** $\{U_i\}$**.**

We therefore have proved that $P(X|\{U_i\}, \{Z_j\}) = P(X|\{U_i\})$

(b). $P(X|\{U_i\}, \{Y_j\}, \{Z_j\}) = P(X|\{U_i\}, \{Y_j\})$ is no longer true, consider the following counterexample (see Figure 3):

In this case, $\{U_j\} = \emptyset$,Y is a collider, thus Z and X are not conditional independent given Y.

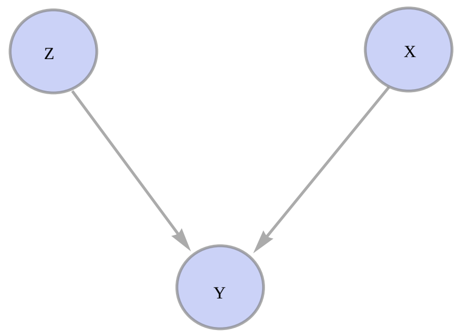

FIGURE 4. Counterexample