

CSCI 503B: HOMEWORK 1

Each question is worth 20 points. Show your work.

1. Let $f(n) = n$ and $g(n) = n^{1+\sin(n)}$. Is $f(n) = O(g(n))$? Is $g(n) = O(f(n))$? Prove your answers.
2. Give an example of a function which is $o(1)$. Just use the definition (or think intuitively), prove your answer.
3. You are told that $f(n) = \Theta(g(n))$. Is it necessarily true that $2^{f(n)} = \Theta(2^{g(n)})$? Argue your answer. Is it true that $(f(n))^2 = \Theta((g(n))^2)$? Argue.
4. Prove that if $f(n) = O(g(n))$ then $f(n) + g(n) = O(g(n))$. If $f(n) = \Omega(g(n))$ then is it true that $f(n) - g(n) = \Omega(g(n))$? Prove.
5. Find what is wrong with this sentence: “the running time of this algorithm is at least $O(n^2)$.” Analyze from a mathematical point of view what the content of this statement is.