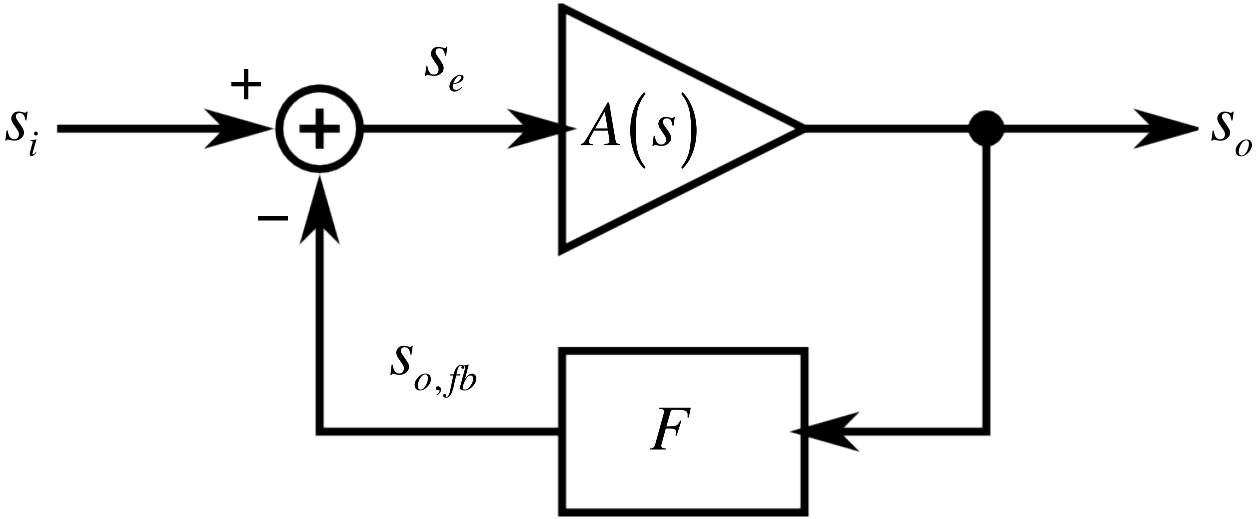
# EEE 51: Second Semester 2017 - 2018

Lecture 20

# Feedback Frequency Response

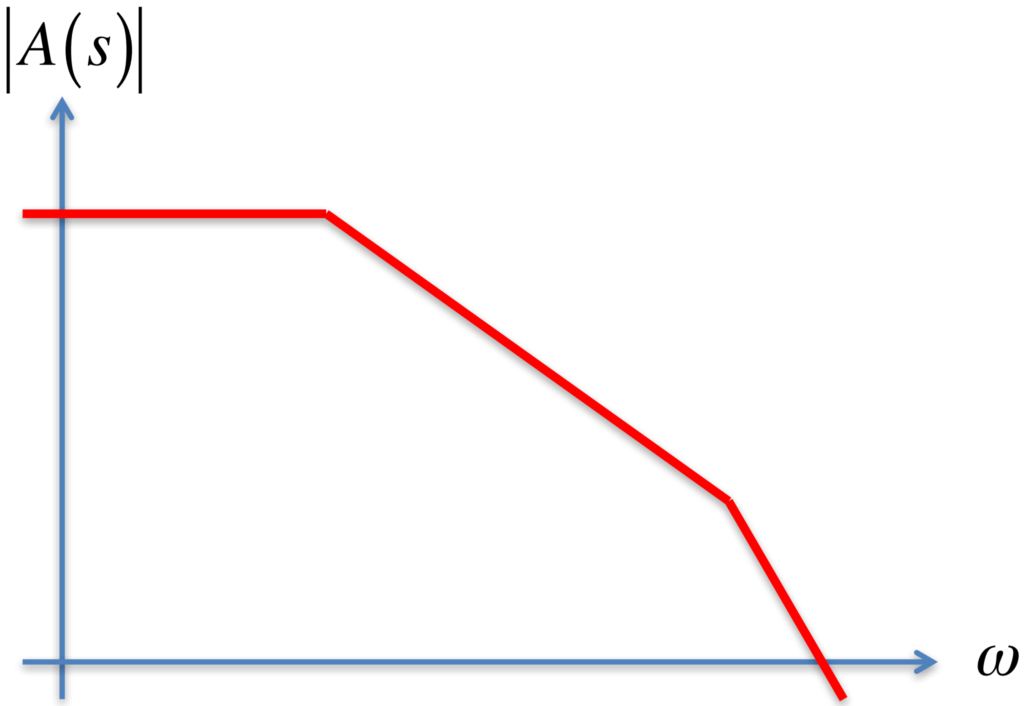
# Feedback Frequency Response




$$A_{CL}(s) = \frac{s_o}{s_i} = ?$$

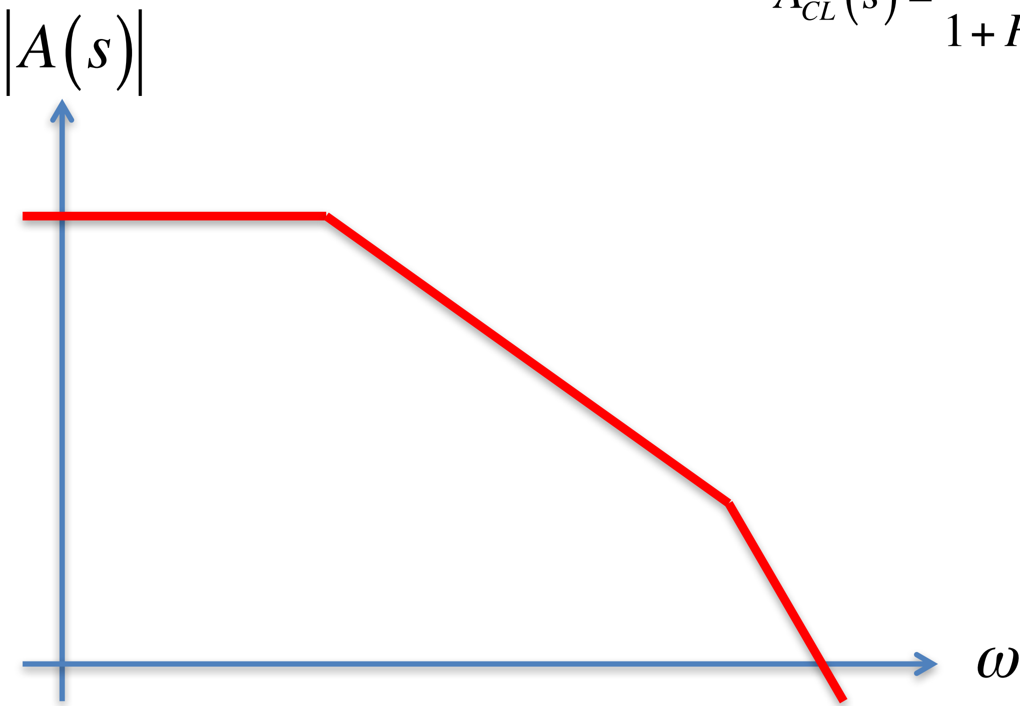
# Loop Gain

$$T(s) = A(s) \cdot F$$

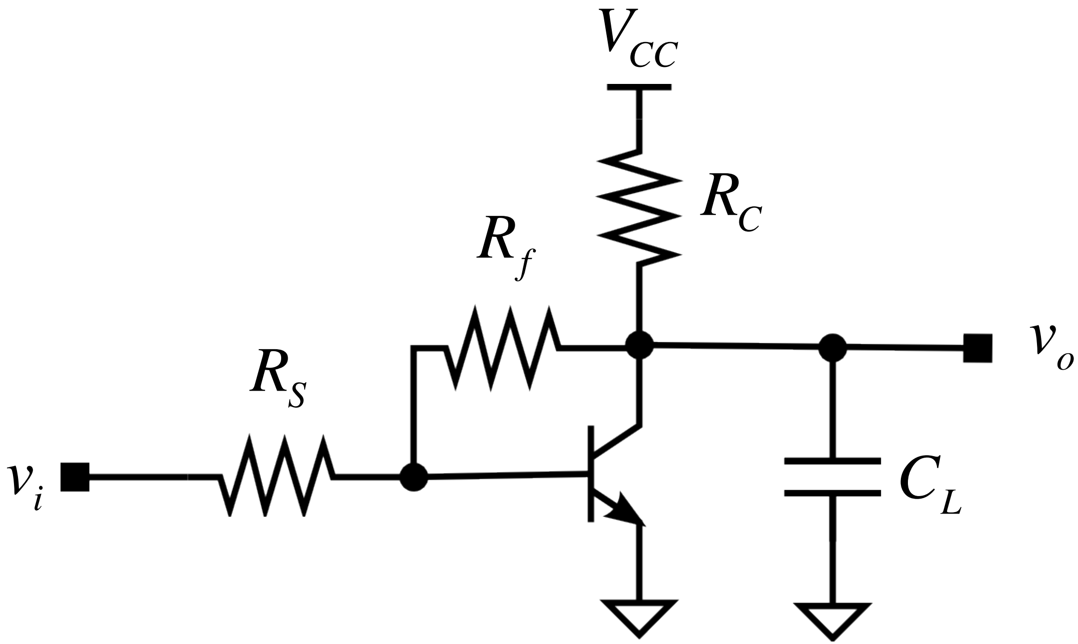


# Closed-Loop Gain

$$A_{CL}(s) = \frac{A(s)}{1+FA(s)} = \frac{A(s)}{1+T(s)} = \frac{1}{F} \cdot \frac{T(s)}{1+T(s)}$$

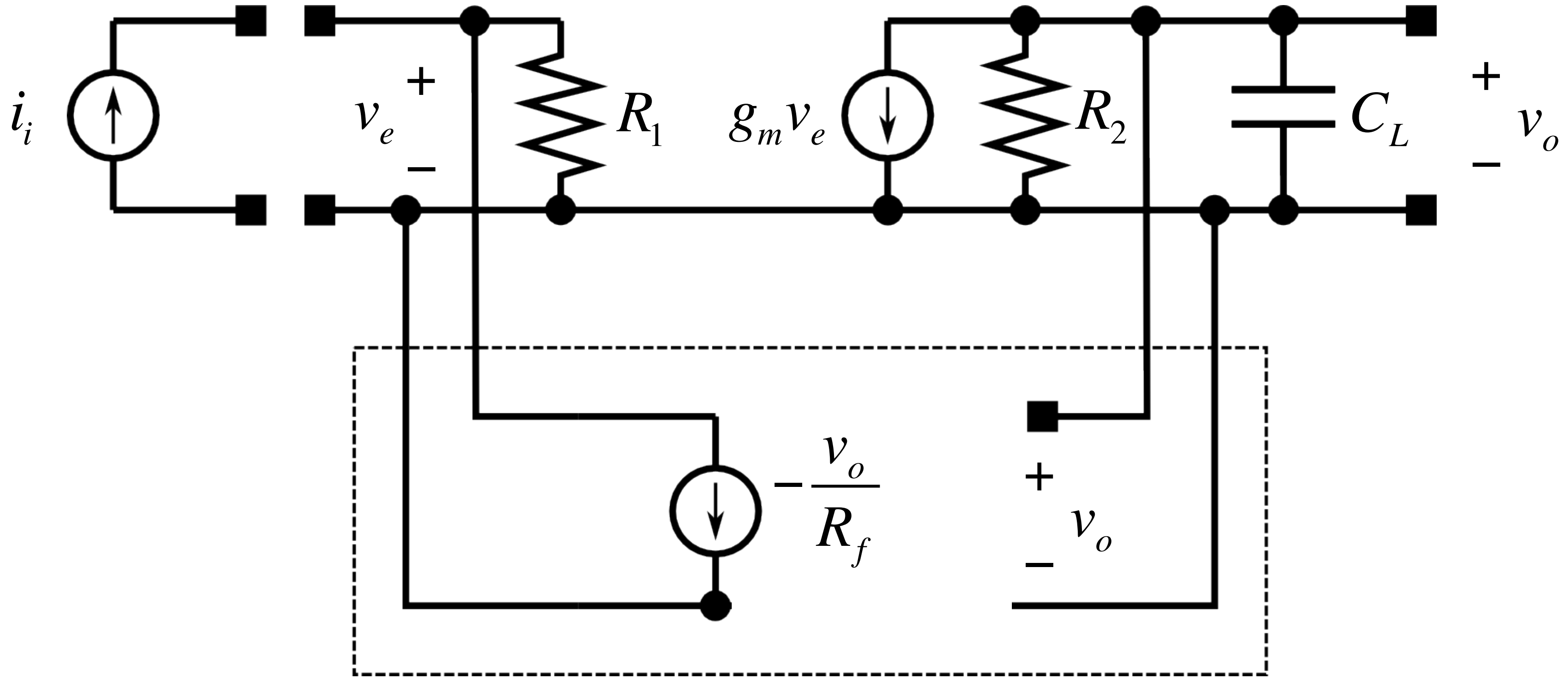


# Frequency Response



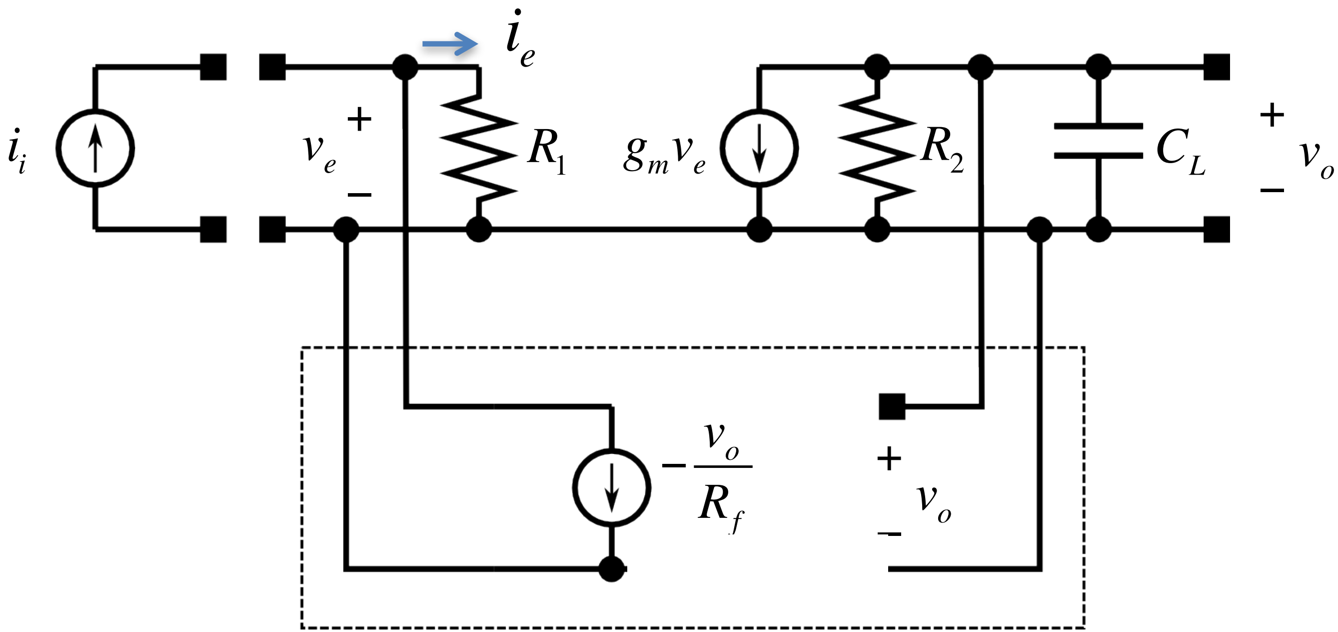

How many poles in the forward gain?
How many poles in the loop gain?

# Shunt-Shunt Small Signal Model




$$R_1 = R_S \parallel R_f \parallel r_\pi \qquad R_2 = R_C \parallel R_f$$

# Shunt-Shunt Small Signal Model




$$\frac{v_o}{i_e} = -\frac{g_m R_1 R_2}{1 + sC_L R_2}$$

$$= -\frac{g_m R_1 R_2}{1 + j\frac{\omega}{\omega_p}}$$

$R_1 = R_S \parallel R_f \parallel r_\pi$ $\quad$ $R_2 = R_C \parallel R_f$

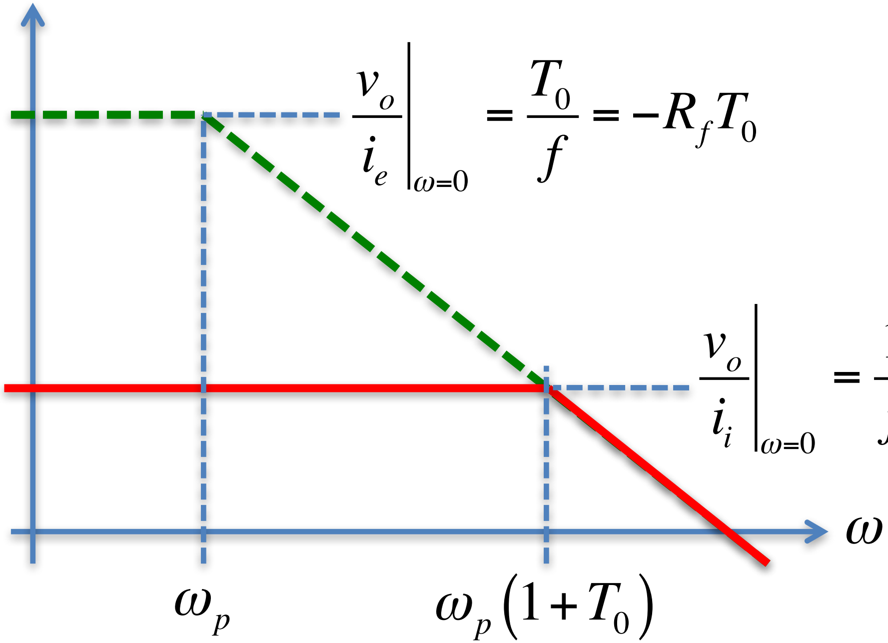
$$f = -\frac{1}{R_f} \qquad T = \frac{g_m}{R_f} \cdot \frac{R_1 R_2}{1 + sC_L R_2} = \frac{T_0}{1 + j\frac{\omega}{\omega_p}} \qquad \omega_p = \frac{1}{C_L R_2}$$

# Closed Loop Gain

$$\frac{v_o}{i_i}=\frac{1}{f}\cdot\frac{T}{1+T}=-R_f\left[\frac{\frac{g_mR_1R_2}{R_f}\left(\frac{1}{1+sC_LR_2}\right)}{1+\frac{g_mR_1R_2}{R_f}\left(\frac{1}{1+sC_LR_2}\right)}\right]=-R_f\left[\frac{\frac{g_mR_1R_2}{R_f}}{1+sC_LR_2+\frac{g_mR_1R_2}{R_f}}\right]$$

$$=-R_f\frac{\frac{g_mR_1R_2}{R_f}}{1+\frac{g_mR_1R_2}{R_f}}\left[\frac{1}{1+s\frac{C_LR_2}{1+\frac{g_mR_1R_2}{R_f}}}\right]=-R_f\frac{T_0}{1+T_0}\left[\frac{1}{1+j\frac{\omega}{(1+T_0)\omega_p}}\right]$$

# Closed Loop Magnitude Response




$$\left.\frac{v_o}{i_e}\right|_{\omega=0} = \frac{T_0}{f} = -R_f T_0$$

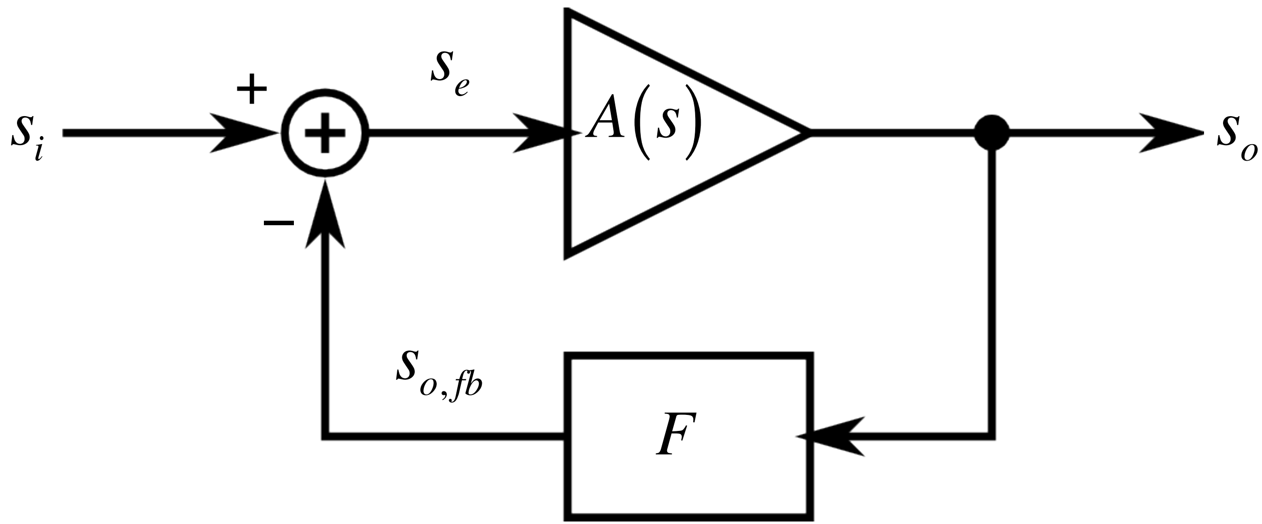
$$\left.\frac{v_o}{i_i}\right|_{\omega=0} = \frac{1}{f}\cdot\frac{T_0}{1+T_0} = -R_f\frac{T_0}{1+T_0}$$

$$\frac{v_o}{i_i} = -R_f\cdot\frac{T_0}{1+T_0}\left[\frac{1}{1+j\dfrac{\omega}{(1+T_0)\omega_p}}\right]$$

$$\frac{v_o}{v_i} = -\frac{R_f}{R_S}\cdot\frac{T_0}{1+T_0}\left[\frac{1}{1+j\dfrac{\omega}{(1+T_0)\omega_p}}\right]$$

Is this circuit stable?

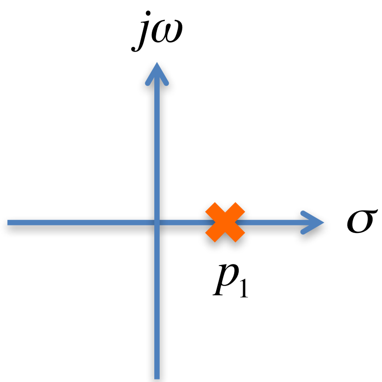
# Feedback and Stability



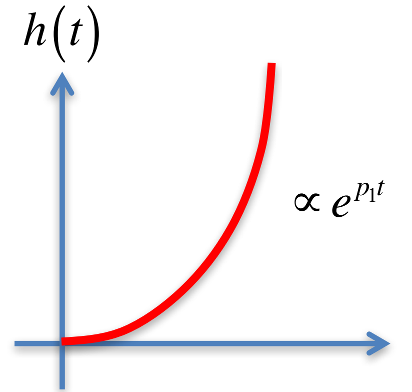

Exponential Stability: For finite input, the output should settle to its final state exponentially

# Frequency Response

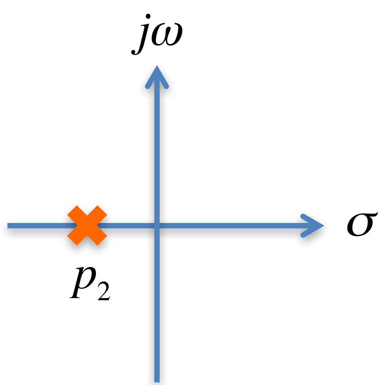
Right-hand plane pole:



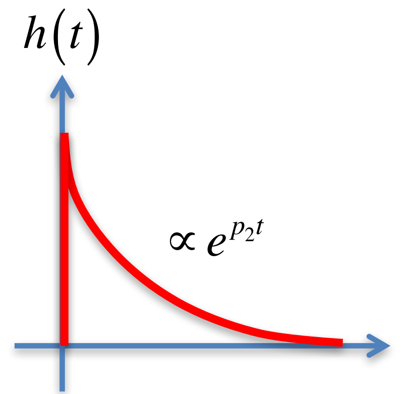

Impulse Response



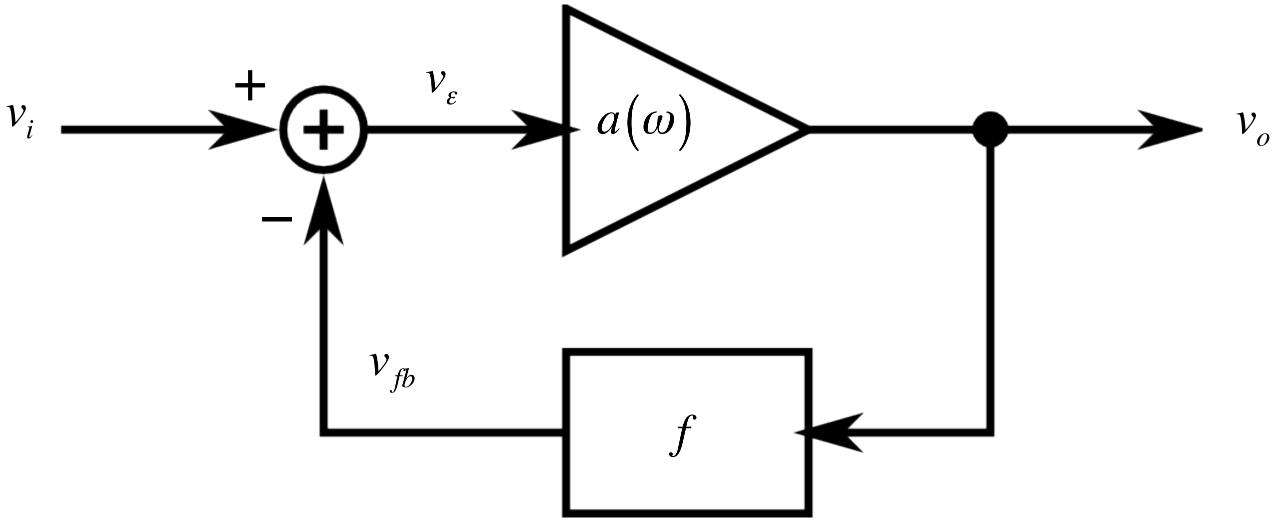

Left-hand plane pole:




Impulse Response



# Effect of Frequency Response




Example: $a(\omega) = \dfrac{a_0}{1 + j\dfrac{\omega}{\omega_p}}$

# Effect of Frequency Response

Forward gain: $a(\omega) = \dfrac{a_0}{1 + j\dfrac{\omega}{\omega_p}}$

Note: $T_0 = a_0 f$

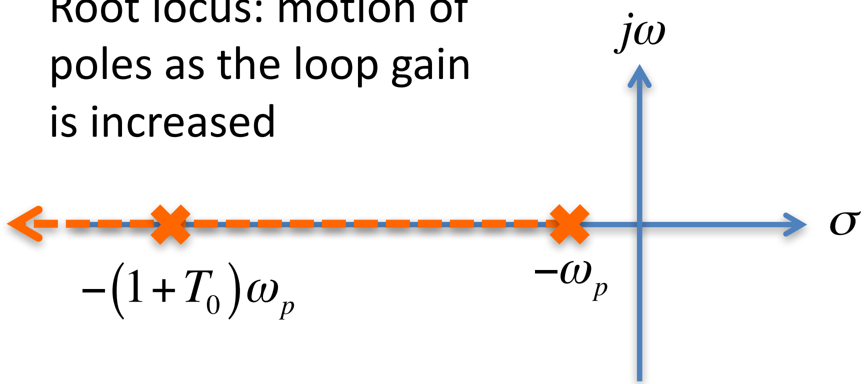
Closed-loop gain: $A_{CL}(\omega) = \dfrac{1}{f} \cdot \dfrac{T(\omega)}{1 + T(\omega)} = \dfrac{\dfrac{a_0}{1 + j\dfrac{\omega}{\omega_p}}}{1 + \dfrac{a_0 f}{1 + j\dfrac{\omega}{\omega_p}}}$

$$= \frac{a_0}{1 + j\dfrac{\omega}{\omega_p} + a_0 f} = \frac{a_0}{1 + a_0 f} \cdot \frac{1}{1 + j\dfrac{\omega}{(1 + a_0 f)\omega_p}}$$

$$= \frac{1}{f} \cdot \frac{T_0}{1 + T_0} \cdot \frac{1}{1 + j\dfrac{\omega}{(1 + T_0)\omega_p}}$$

# Effect of Frequency Response

Root locus: motion of poles as the loop gain is increased



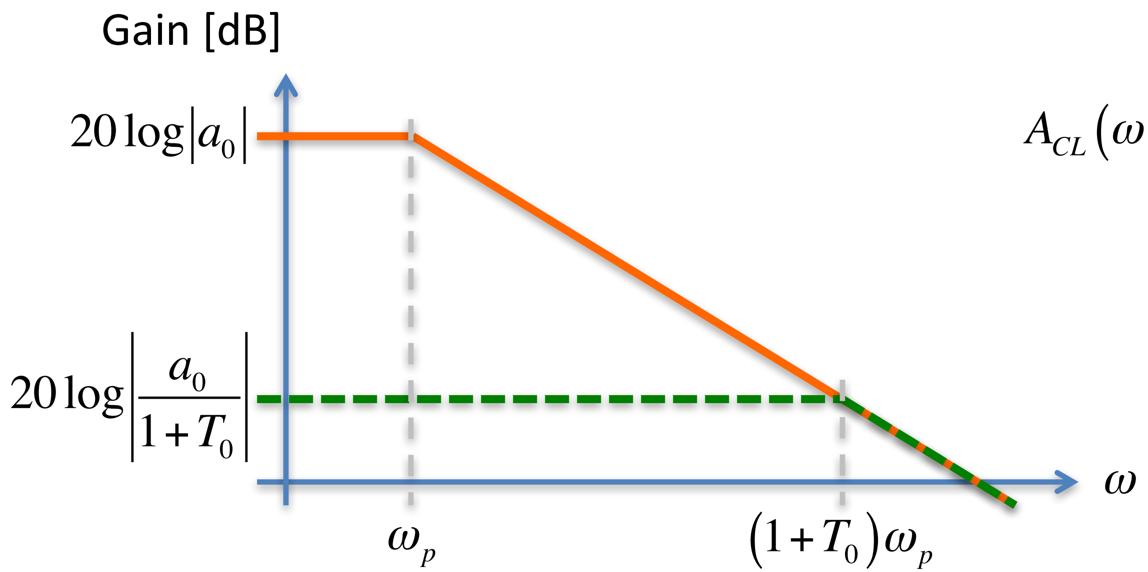

$$a(\omega)=\frac{a_0}{1+j\dfrac{\omega}{\omega_p}}$$

$$T(\omega)=a(\omega)\cdot f$$

$$T_0=a_0 f$$

$$A_{CL}(\omega)=\frac{1}{f}\cdot\frac{T_0}{1+T_0}\cdot\frac{1}{1+j\dfrac{\omega}{(1+T_0)\omega_p}}$$

# Effect of Frequency Response




$$A_{CL}(\omega) = \frac{1}{f} \cdot \frac{T_0}{1+T_0} \cdot \frac{1}{1+j\frac{\omega}{(1+T_0)\omega_p}}$$

$$a(\omega) = \frac{a_0}{1+j\frac{\omega}{\omega_p}}$$

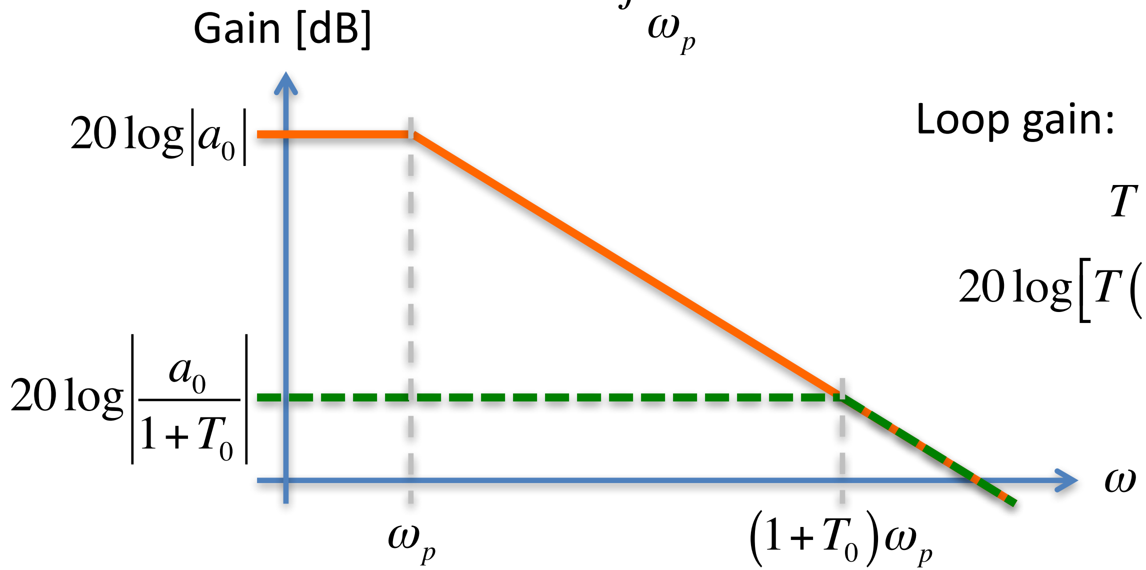
$$T(\omega) = a(\omega) \cdot f$$

$$T_0 = a_0 f$$

Feedback reduces gain by $(1+T_0)$ and increases the bandwidth by $(1+T_0)$

# Effect of Frequency Response

$$a(\omega) = \frac{a_0}{1 + j\dfrac{\omega}{\omega_p}} \qquad T_0 = a_0 f \qquad A_{CL}(\omega) = \frac{1}{f} \cdot \frac{T_0}{1+T_0} \cdot \frac{1}{1 + j\dfrac{\omega}{(1+T_0)\omega_p}}$$



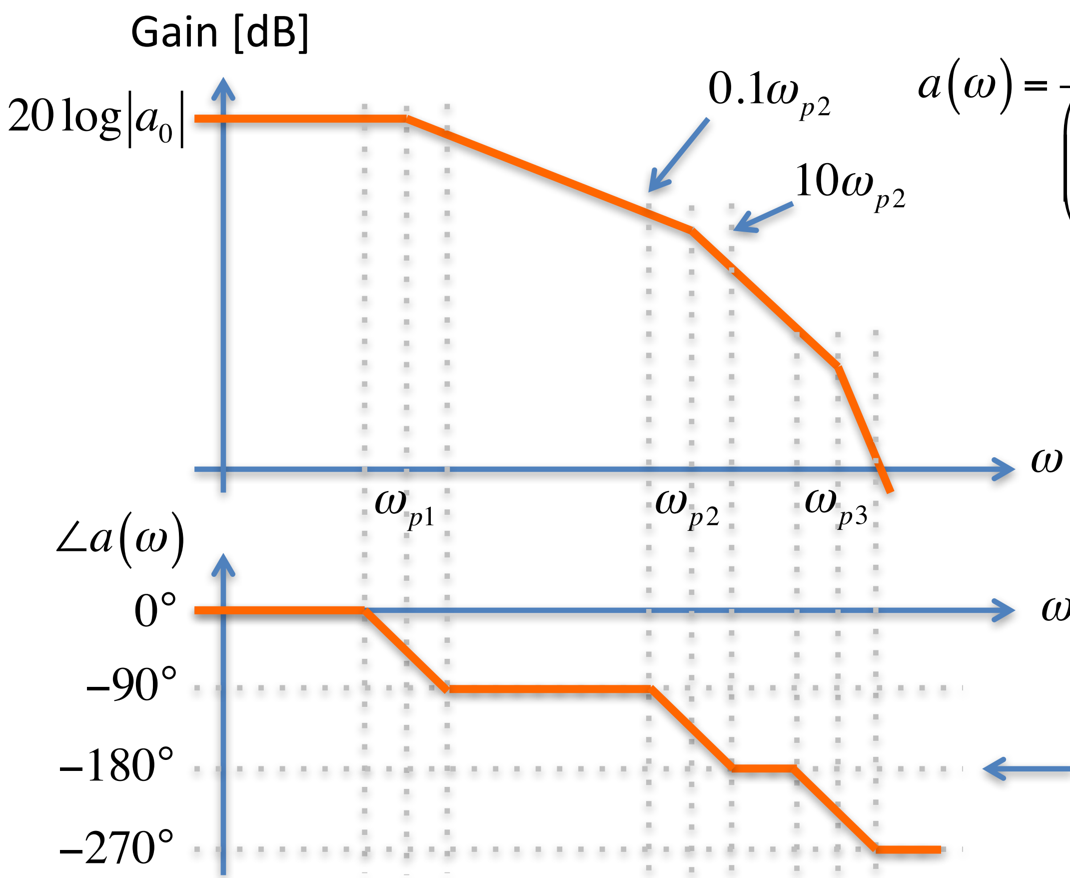

Loop gain:

$$T(\omega) = a(\omega) \cdot f$$

$$\begin{aligned} 20\log[T(\omega)] &= 20\log[a(\omega) \cdot f] \\ &= 20\log[a(\omega)] + 20\log[f] \\ &= 20\log[a(\omega)] - 20\log\left[\frac{1}{f}\right] \end{aligned}$$

Note: $\dfrac{a_0}{1+T_0} = \dfrac{1}{f}\dfrac{T_0}{1+T_0} \approx \dfrac{1}{f}$

# Effect of Frequency Response




$$a(\omega) = \frac{a_0}{\left(1 + j\frac{\omega}{\omega_{p1}}\right)\left(1 + j\frac{\omega}{\omega_{p2}}\right)\left(1 + j\frac{\omega}{\omega_{p3}}\right)}$$

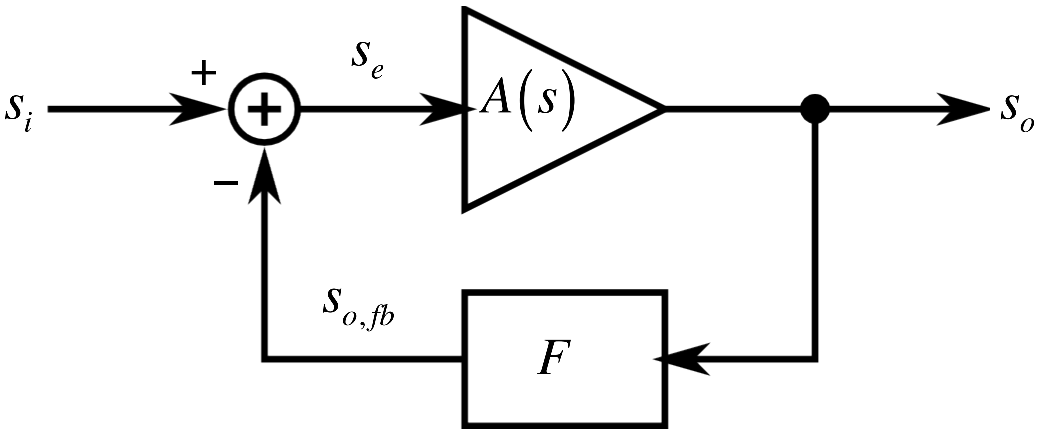
Why not let $T_0 \rightarrow \infty$?

For more than one pole:

- Negative feedback at DC is now positive feedback at $10\omega_{p2}$

sign inversion

# Stability




The loop must attenuate all transients when the input has settled.

$$\begin{cases} \angle T < 180^\circ \\ \angle T \geq 180^\circ \Rightarrow |T| < 1 \end{cases}$$

# Positive Feedback

Forward gain: $$a(\omega) = \frac{a_0}{1 + j\dfrac{\omega}{\omega_p}}$$

Note: $T_0 = a_0 f$

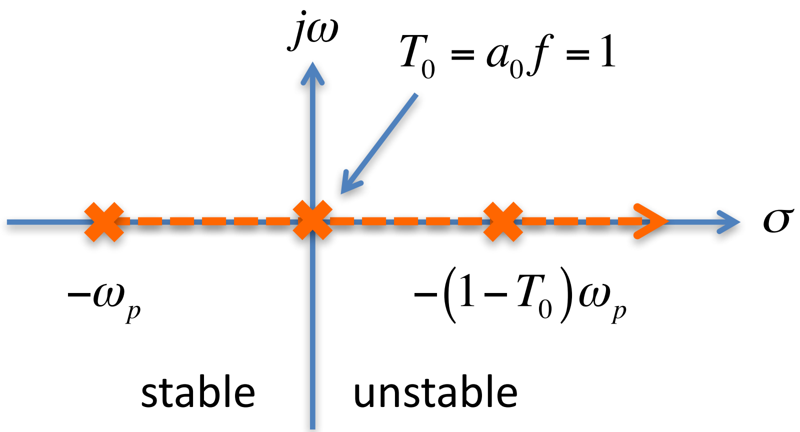
Positive feedback: $$A_{CL}(\omega) = \frac{1}{f} \cdot \frac{T(\omega)}{1 - T(\omega)} = \frac{\dfrac{a_0}{1 + j\dfrac{\omega}{\omega_p}}}{1 - \dfrac{a_0 f}{1 + j\dfrac{\omega}{\omega_p}}}$$

$$= \frac{a_0}{1 + j\dfrac{\omega}{\omega_p} - a_0 f} = \frac{a_0}{1 - a_0 f} \cdot \frac{1}{1 + j\dfrac{\omega}{(1 - a_0 f)\omega_p}}$$

$$= \frac{1}{f} \cdot \frac{T_0}{1 - T_0} \cdot \frac{1}{1 + j\dfrac{\omega}{(1 - T_0)\omega_p}}$$

# Effect of Frequency Response

Positive feedback:




$$a(\omega) = \frac{a_0}{1 + j\frac{\omega}{\omega_p}}$$

$$T(\omega) = a(\omega) \cdot f$$

$$T_0 = a_0 f$$

$$A_{CL}(\omega) = \frac{1}{f} \cdot \frac{T_0}{1-T_0} \cdot \frac{1}{1 + j\frac{\omega}{(1-T_0)\omega_p}}$$

Unstable when

$$1 - T_0 < 0$$

# Effect of Frequency Response

Multi-pole forward gain transfer function: $a(\omega) = \dfrac{a_0}{\left(1 + j\dfrac{\omega}{\omega_{p1}}\right)\left(1 + j\dfrac{\omega}{\omega_{p2}}\right)\cdots\left(1 + j\dfrac{\omega}{\omega_{pn}}\right)}$

Closed loop gain: $A_{CL}(\omega) = \dfrac{1}{f} \cdot \dfrac{T(\omega)}{1 + T(\omega)}$

Nyquist Criteria (simple version):

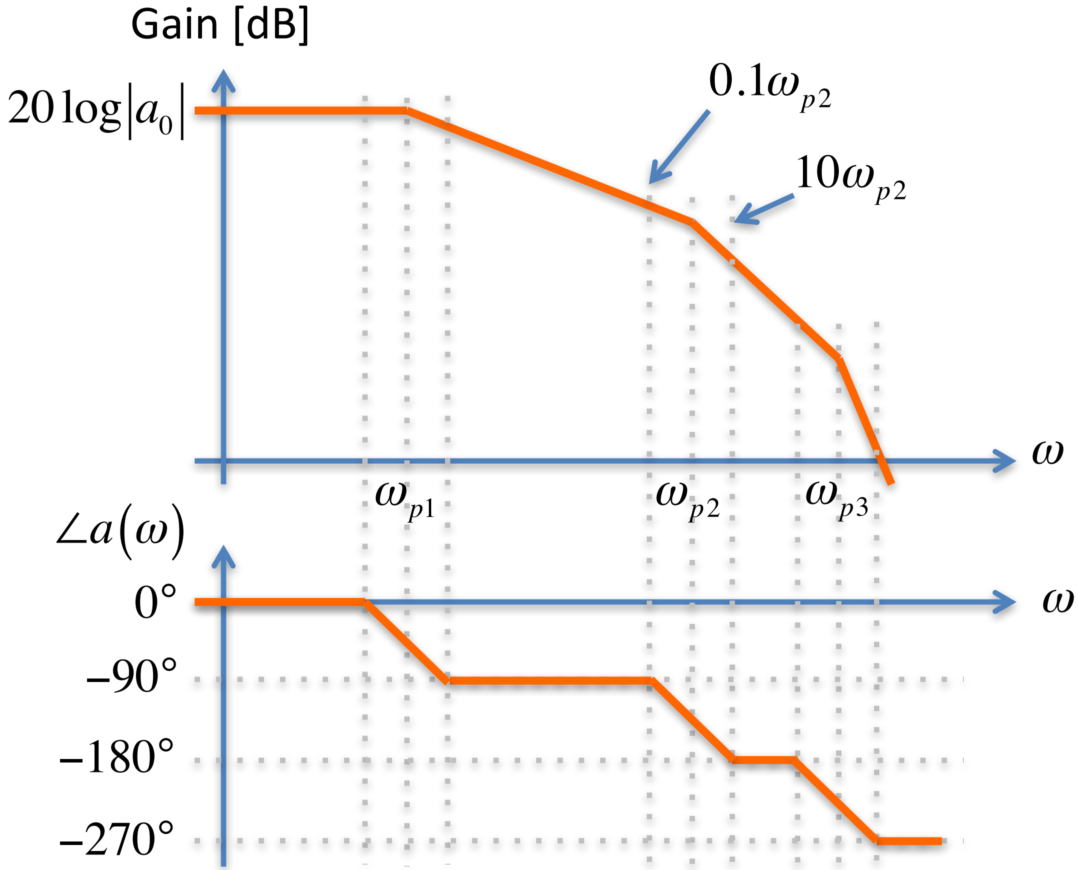
- The circuit is unstable if |T(jω)| > 1 at the frequency where the phase of T(jω) = -180°

$$\angle T(j\omega) = \tan^{-1}\left(\frac{\mathrm{Im}\{T(j\omega)\}}{\mathrm{Re}\{T(j\omega)\}}\right)$$

# Feedback and Stability

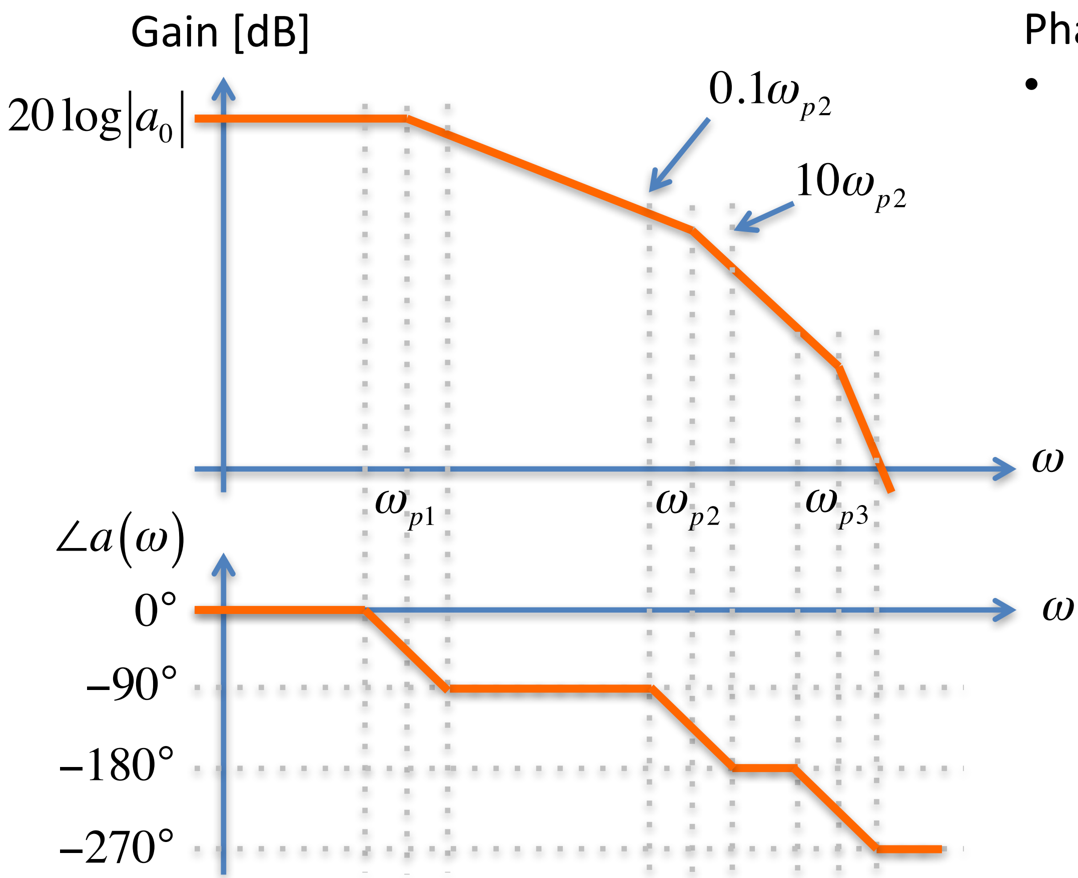
- Metrics
  - Phase margin
  - Gain margin
- Dependent on the loop gain T(s)
  - Forward amplifier second (non-dominant) pole
  - Feedback factor
- Worst-case feedback factor?

# Effect of Frequency Response




Worst-case feedback factor?

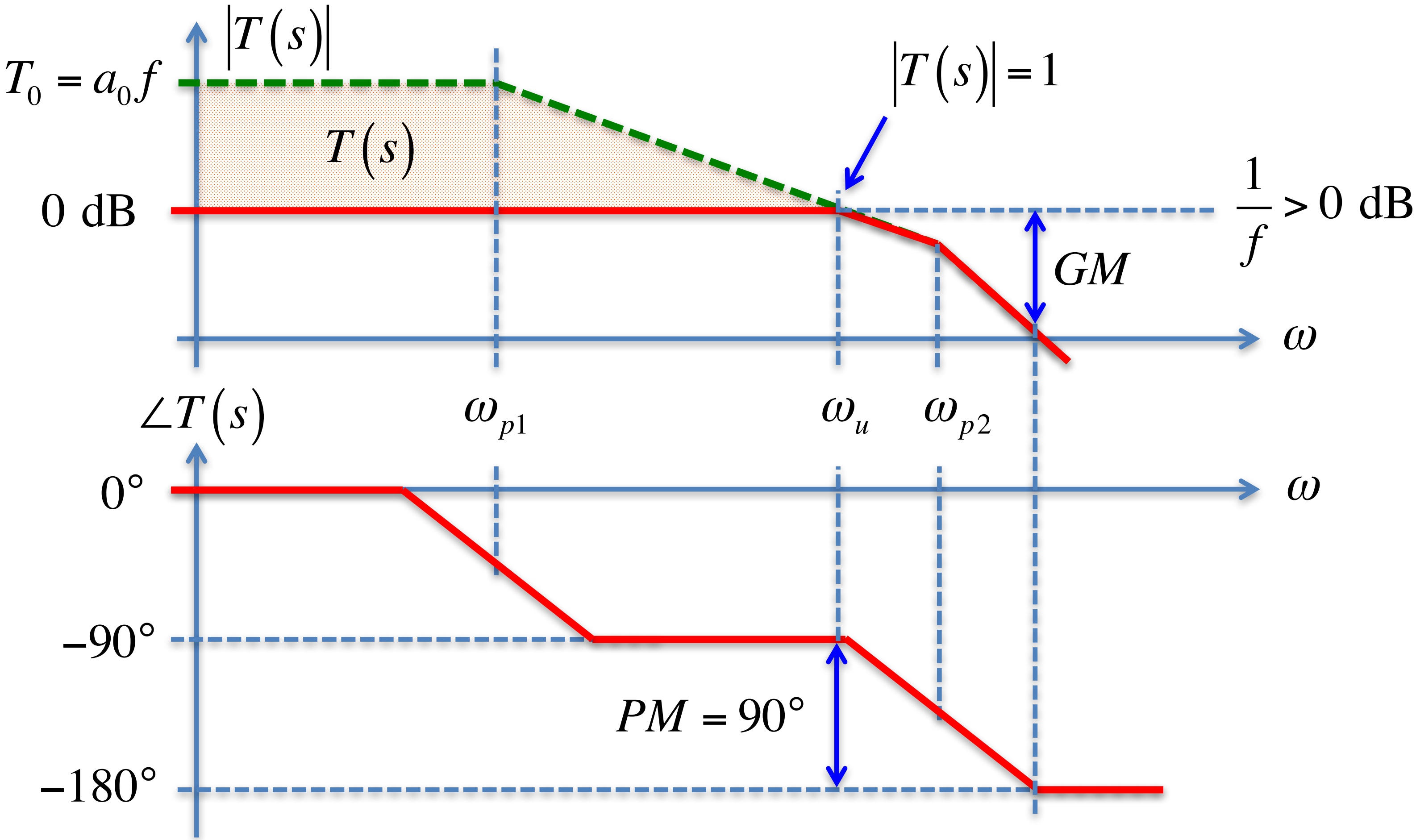
# Phase Margin



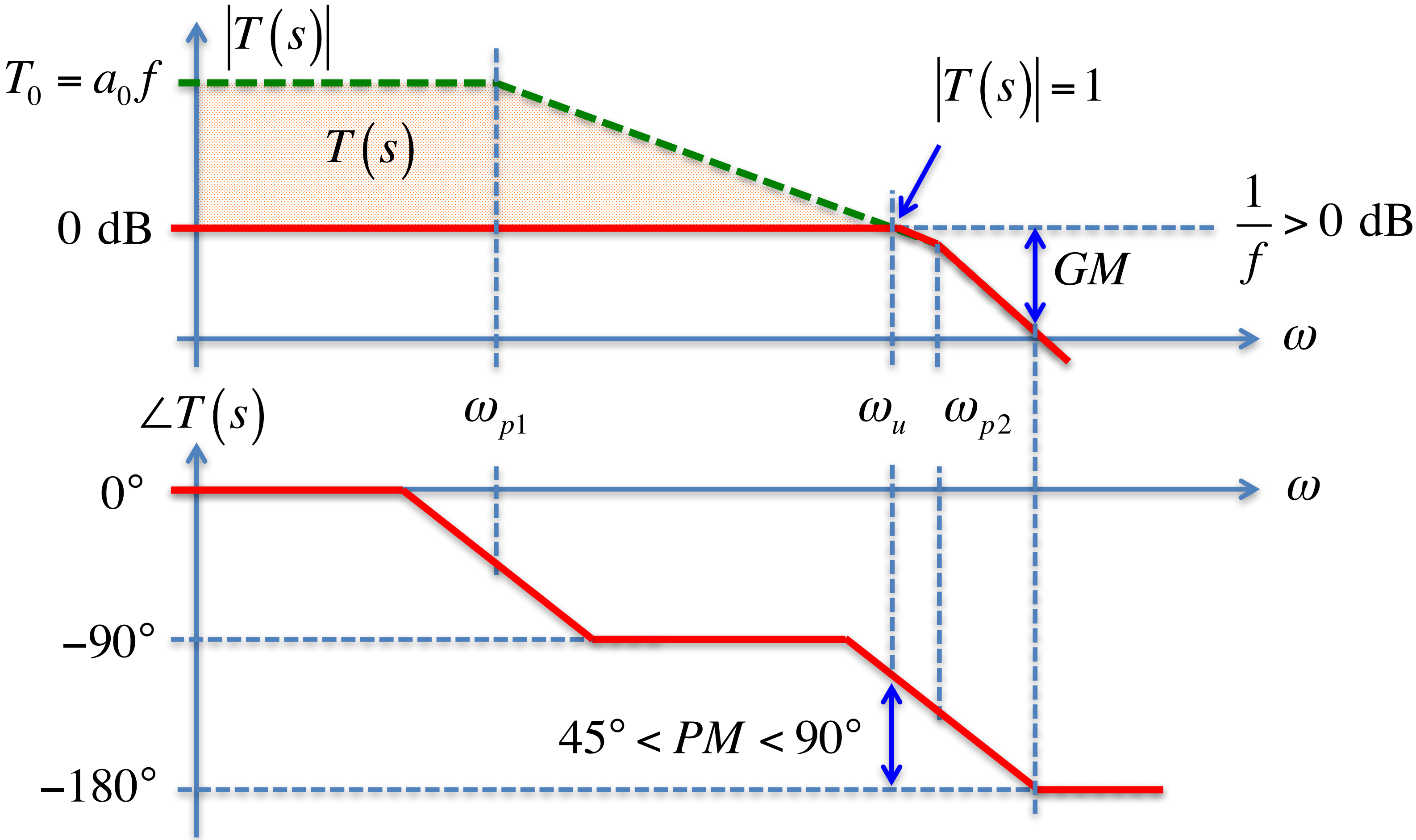

Phase Margin:

- The difference between the actual phase shift and -180° when |T(jω)| = 1

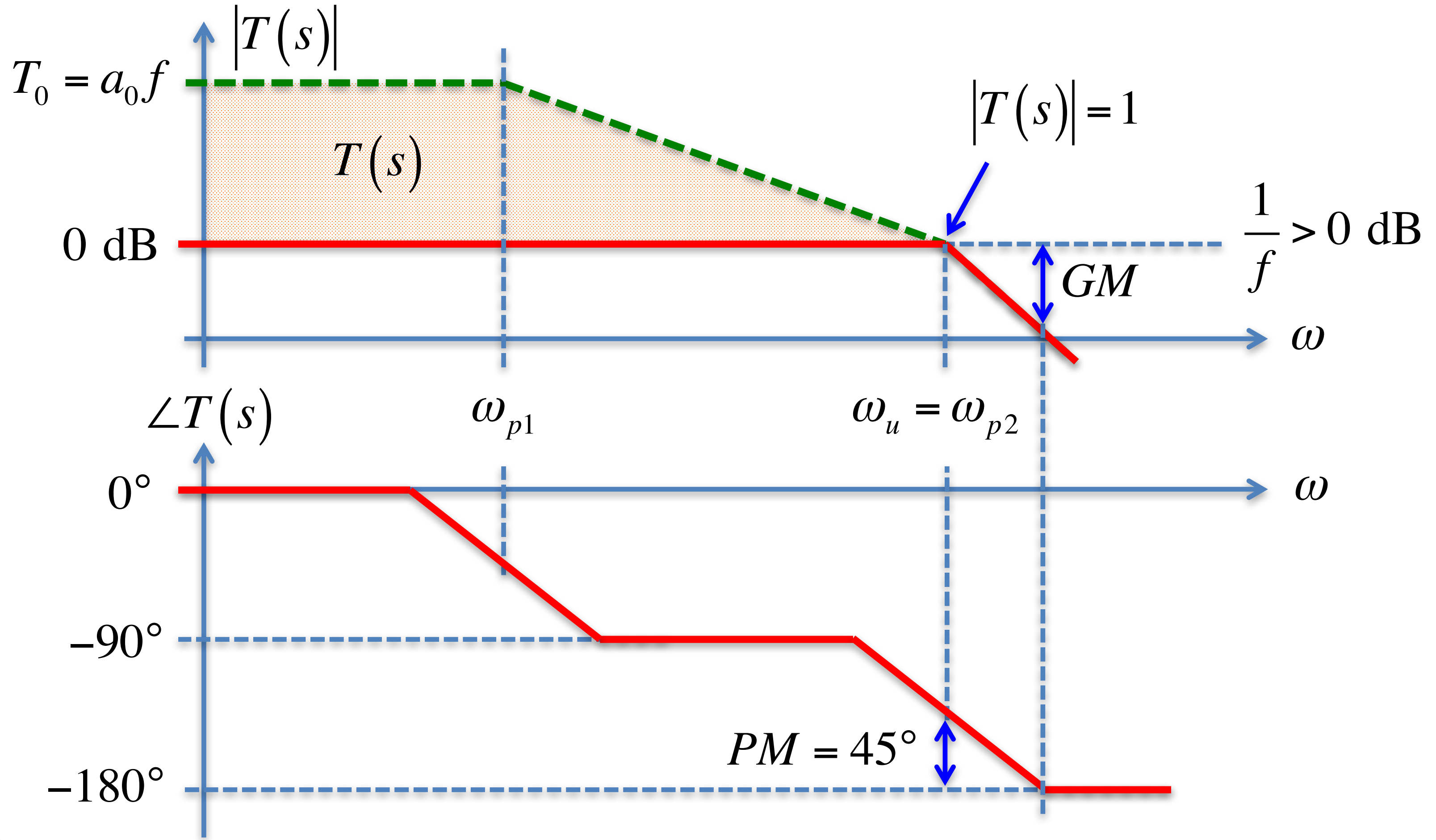
# Phase and Gain Margins



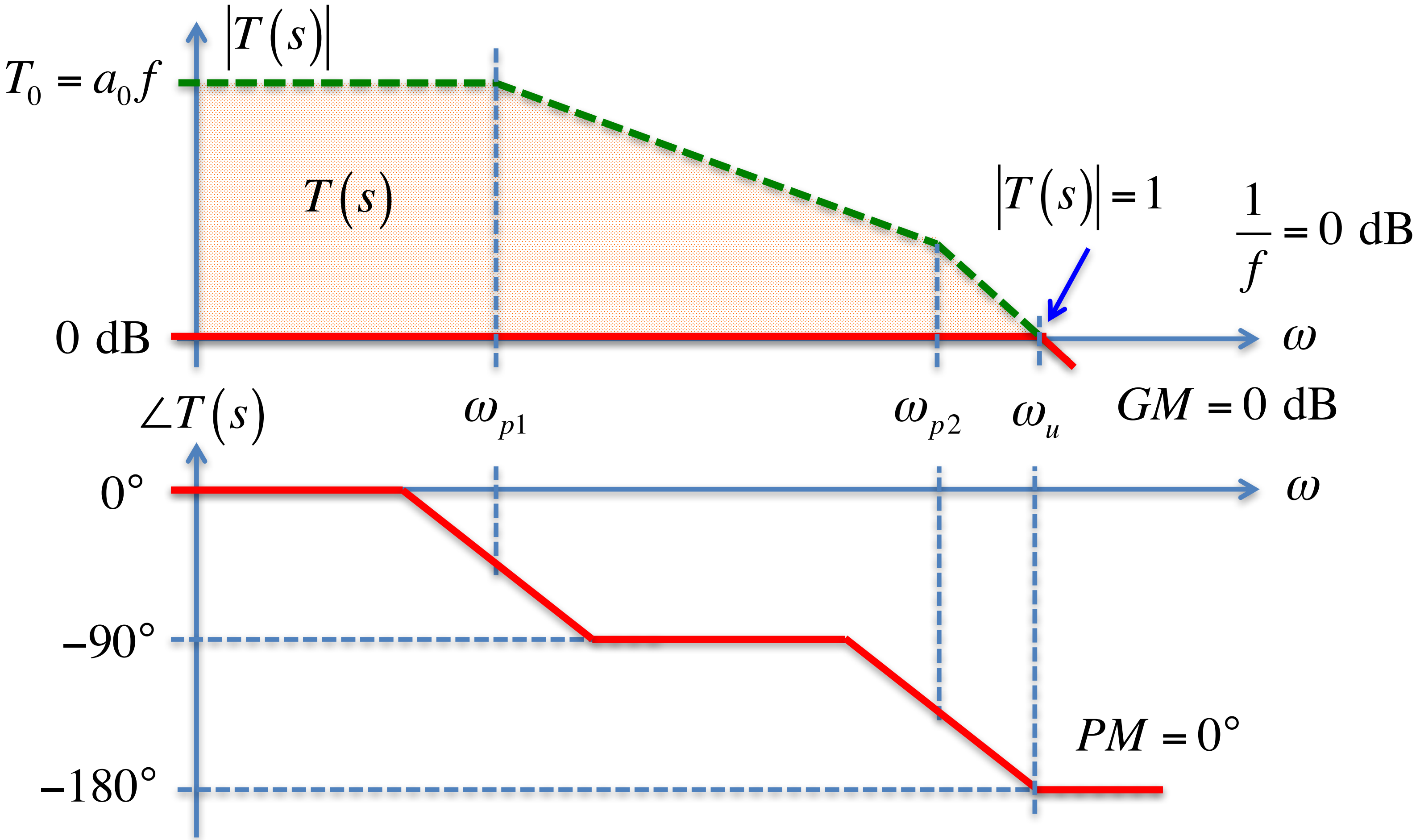
# Phase and Gain Margins



# Phase and Gain Margins



# Phase and Gain Margins



# Next Meeting

- Feedback Amplifiers