Lecture 5
Balanced Three Phase Systems

~~Agenda~~

- Lecture

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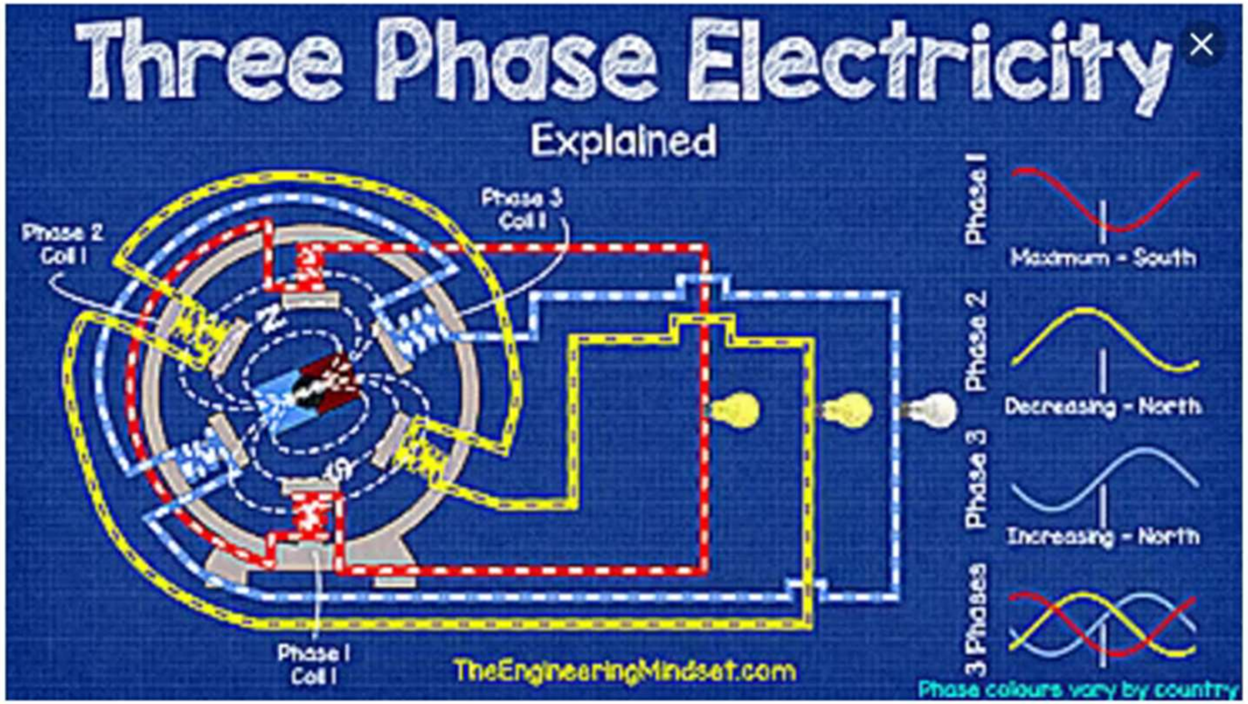
Three Phase Voltages and Current



Lecture Outcomes

at the end of the lecture, the student must be able to …

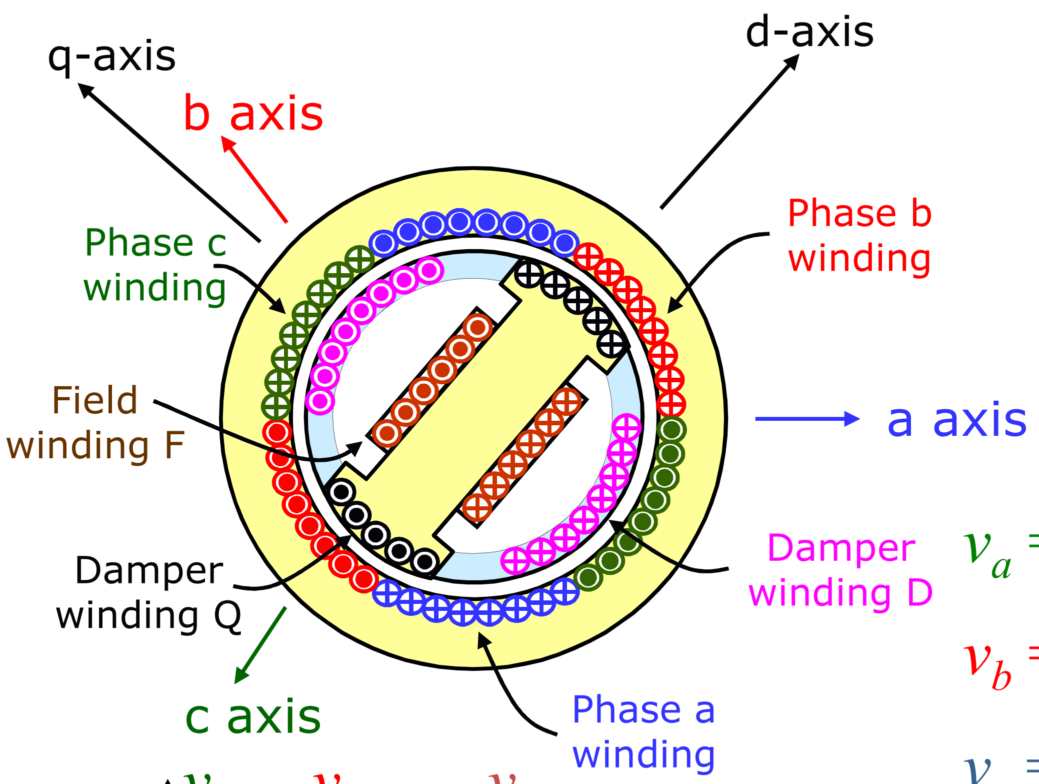
- Identify Balanced Three Phase System Components
- Compute the Voltages and Currents in a balanced three phase system.

Things to Watch out For.

- The Operator “a’
- Neutral Conductor

BALANCED
THREE-PHASE SYSTEMS

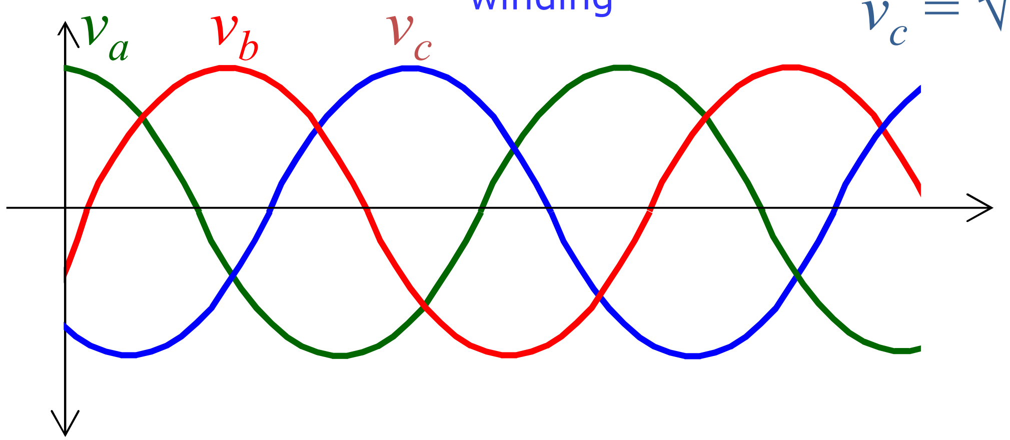
Balanced Three-Phase Voltages



$$v_a = \sqrt{2}\,V_m \cos(\omega t + \theta)$$

$$v_b = \sqrt{2}\,V_m \cos(\omega t + \theta - 120^\circ)$$

$$v_c = \sqrt{2}\,V_m \cos(\omega t + \theta + 120^\circ)$$



Note:
Phase sequence shown is *a-b-c*.

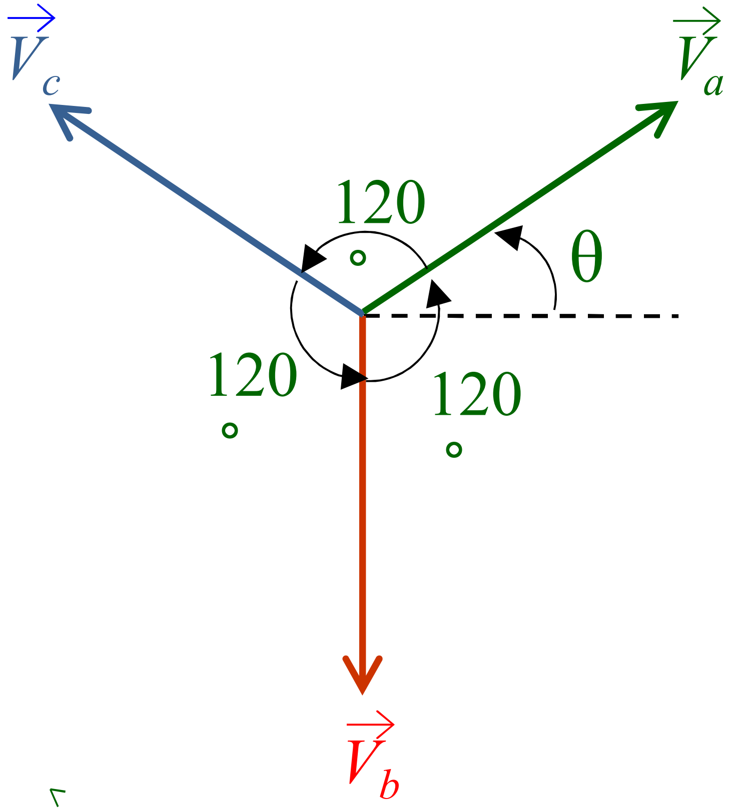
Balanced Three-Phase Voltages

Transforming to phasors, we get

$$\vec{V}_a = V\angle\theta$$

$$\vec{V}_b = V\angle\theta - 120^\circ$$

$$\vec{V}_c = V\angle\theta + 120^\circ$$

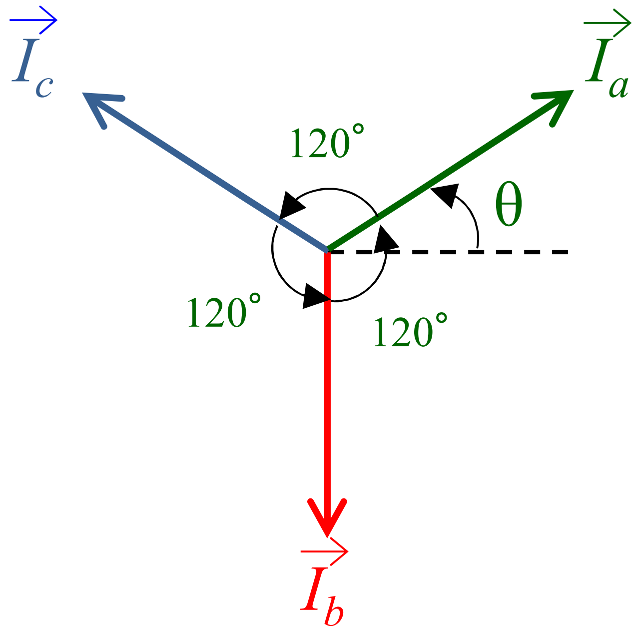


Balanced Three-Phase Currents

The currents

$$i_a = \sqrt{2}\, I_m \cos(\omega t + \theta)$$
$$i_b = \sqrt{2}\, I_m \cos(\omega t + \theta - 120^\circ)$$
$$i_c = \sqrt{2}\, I_m \cos(\omega t + \theta + 120^\circ)$$

are three-phase balanced.



In phasor form, we get

$$\vec{I}_a = I \angle \theta$$
$$\vec{I}_b = I \angle \theta - 120^\circ$$
$$\vec{I}_c = I \angle \theta + 120^\circ$$

Balanced Three-Phase System

A balanced three-phase system consists of :

1. Balanced three-phase sinusoidal sources;
2. Balanced three-phase loads; and
3. The connecting wires have equal impedances.

A balanced three-phase load has:

a) Equal impedances per phase; or
b) Equal P and Q per phase.

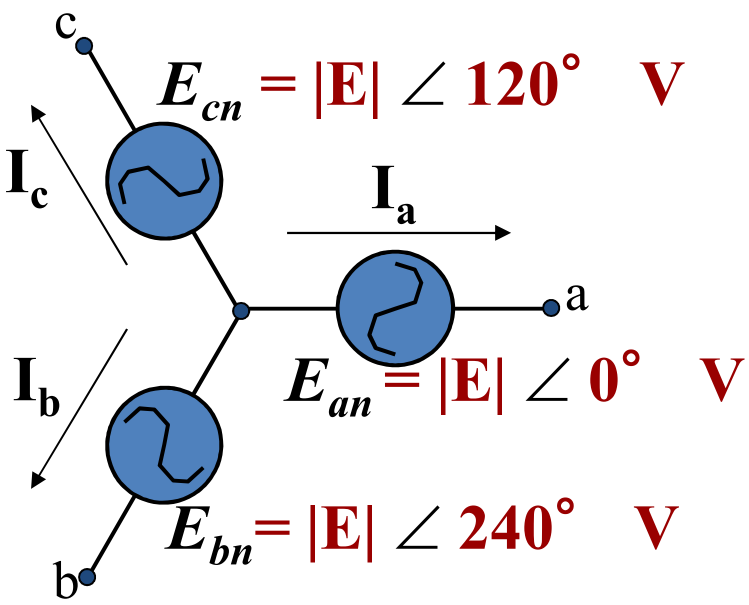
Note: The load may be connected in wye or delta.

Three Phase System Components

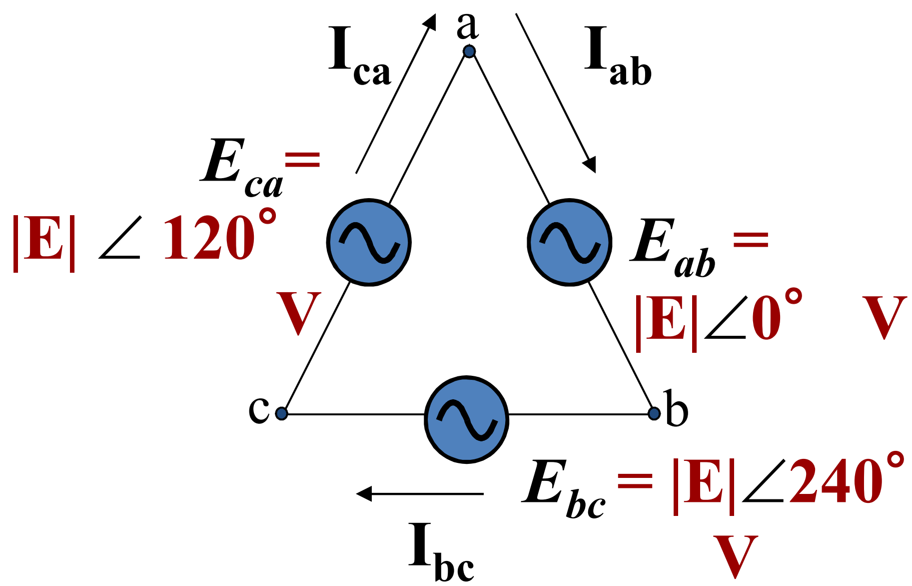
- Sources
- Loads
- Lines

Three-Phase Sources

Wye-Connected Source



Delta-Connected Source

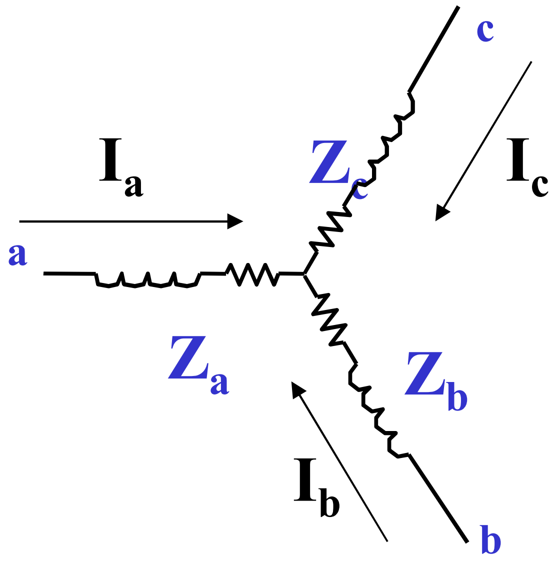


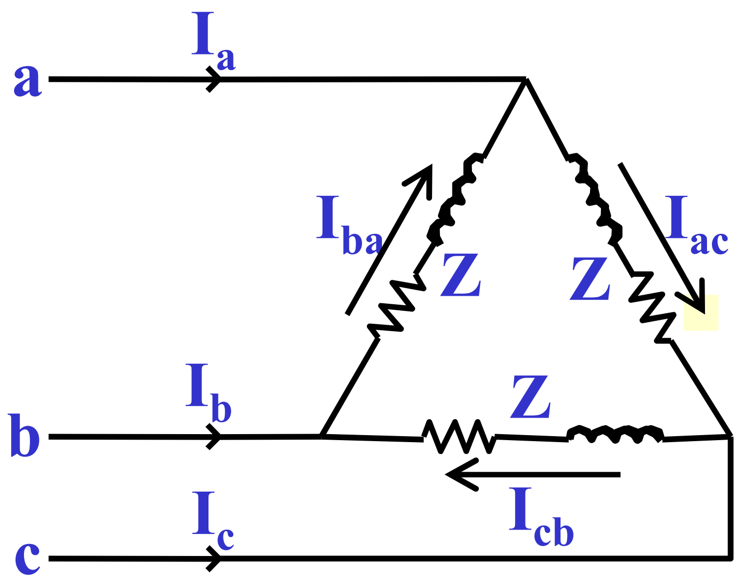
Note 1: The synchronous generator is a three-phase machine designed to generate balanced three-phase voltages.

Note 2: Neutral point *n* exists only for wye-connected systems.

Three-Phase Loads

Wye-Connected Load

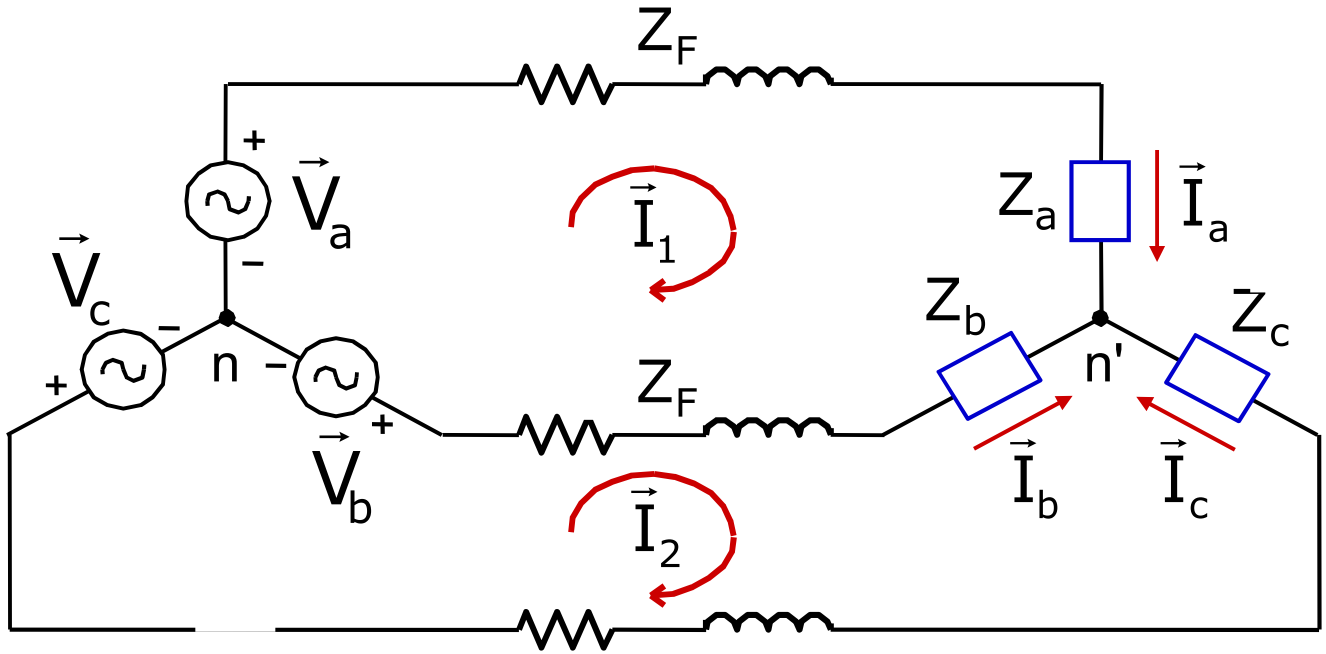




Delta-Connected Load

Note: Neutral point *n'* exists only for wye-connected systems.

Given $V_a = 200\underline{/0^o}$ volts, $V_b = 200\underline{/-120^o}$ volts and $V_c = 200\underline{/120^o}$ volts. Find the phasor currents I_a, I_b and I_c. Also, find P and Q supplied to Z_a, Z_b and Z_c and the load power factor.



$Z_F = 1 + j1\ \Omega \quad Z_a = Z_b = Z_c = 7 + j5\ \Omega$

Mesh equations using loop currents I_1 and I_2.

$$V_a - V_b = 2(Z_f + Z_L)I_1 - (Z_f + Z_L)I_2$$
$$V_b - V_c = -(Z_f + Z_L)I_1 + 2(Z_f + Z_L)I_2$$

Substitution gives

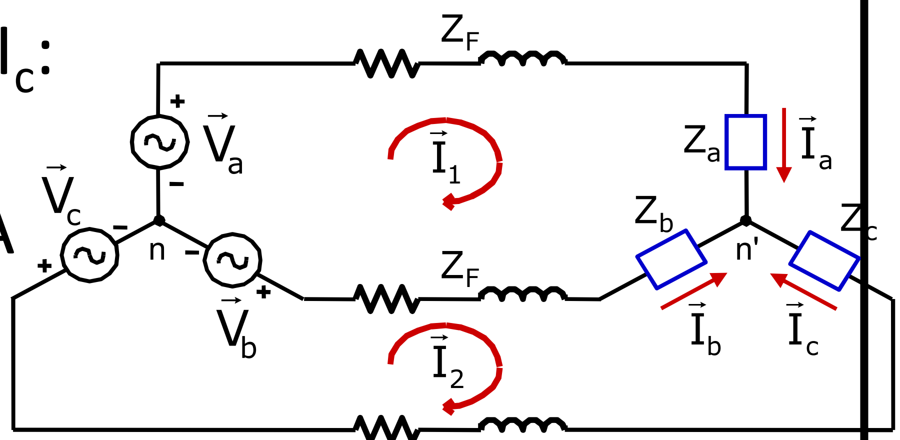
$$300 + j173.2 = (16 + j12)I_1 - (8 + j6)I_2$$
$$-j346.4 = -(8 + j6)I_1 + (16 + j12)I_2$$

Solving simultaneously we get

$$I_1 = 20\angle -36.87^\circ \quad \text{A}$$
$$I_2 = 20\angle -96.87^\circ \quad \text{A}$$

Solving for currents I_a, I_b and I_c:

$$I_a = I_1 = 20\angle -36.87^\circ \quad A$$
$$I_b = I_2 - I_1 = 20\angle -156.87^\circ \quad A$$
$$I_c = -I_2 = 20\angle 83.13^\circ \quad A$$



Note: Currents I_a, I_b and I_c are balanced.
Try to get the sum of the currents. What is it equal to?

Power and Reactive Power supplied to load impedances Z_a, Z_b and Z_c.

$$P_a = P_b = P_c = |I|^2R = (20)^2(7) = 2800 \text{ Watts}$$
$$Q_a = Q_b = Q_c = |I|^2X = (20)^2(5) = 2000 \text{ VARs}$$

Note: These equations for P and Q are correct if R and X are used

The Operator *"a"*

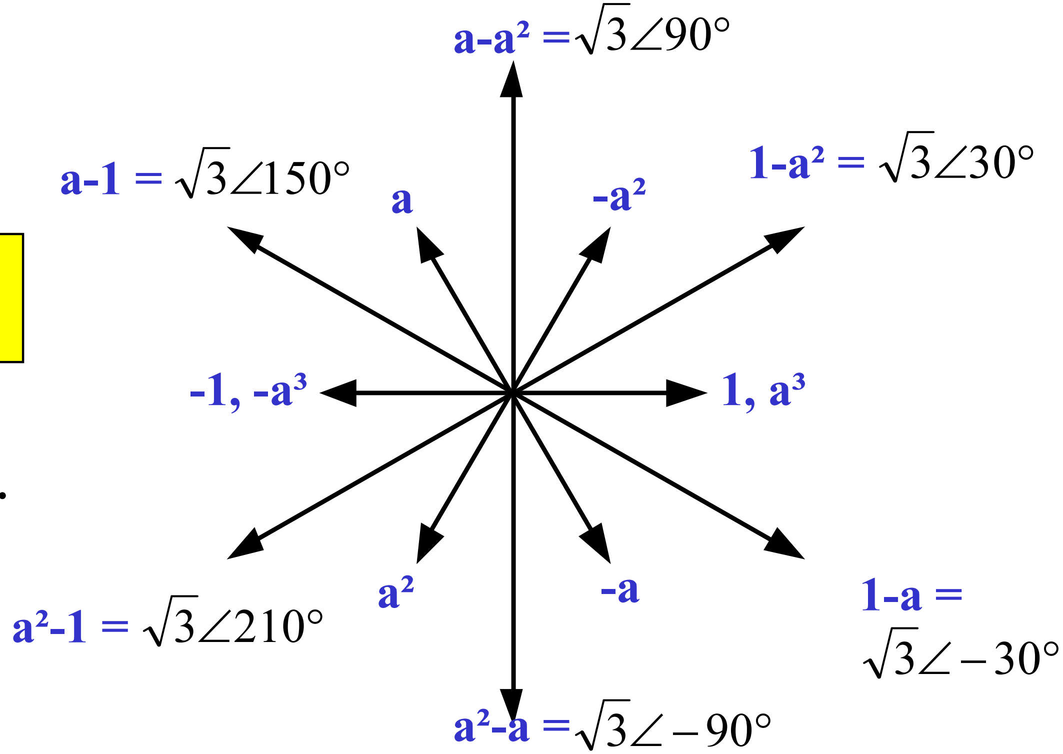
Define: $a = 1 \angle 120^\circ = 1e^{j2\pi/3} = -0.5 + j0.866$

$a^2 = 1 \angle 240^\circ = 1e^{j4\pi/3} = -0.5 - j0.866$

$a^3 = 1 \angle 360^\circ = 1e^{j2\pi} = 1 \angle 0^\circ = 1$

Note: $1 + a + a^2 = 0$

The sum of three-phase balanced phasors is zero.



Definitions: Voltages in Three-Phase Systems

Line-to-Line Voltage (V_{LL})

The voltage across any two line terminals.

Line-to-Neutral Voltage (V_{LN})

The voltage across any one line terminal and neutral.

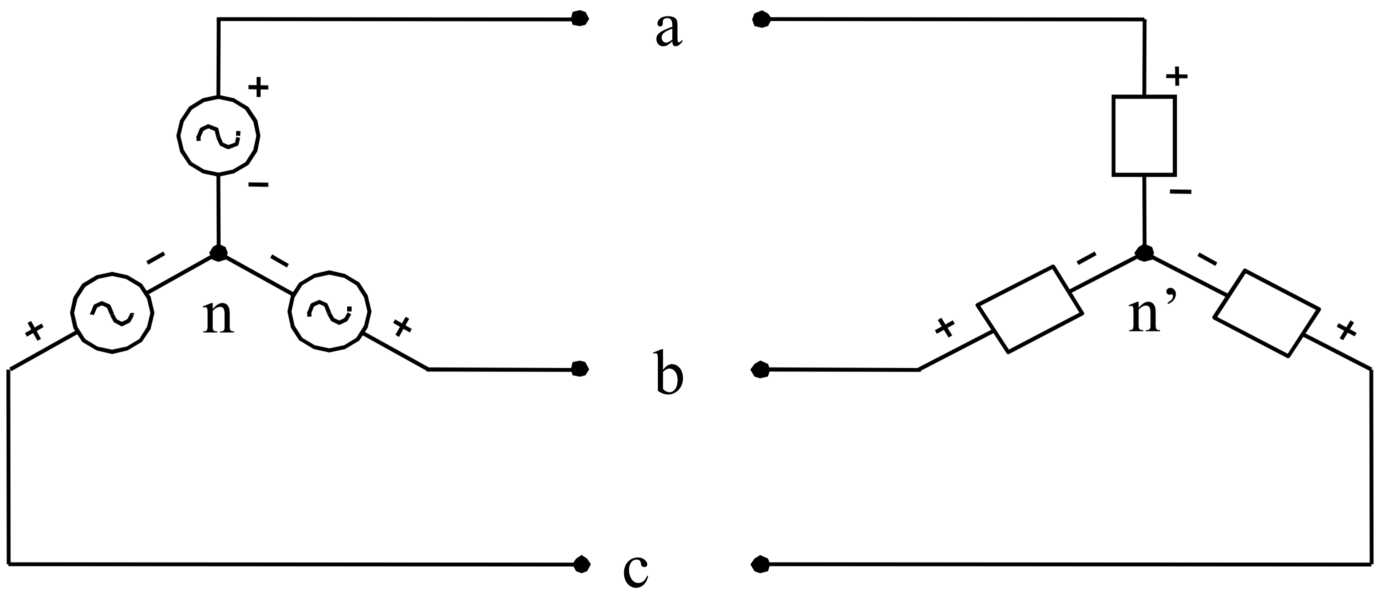
Phase Voltage (V_P)

The voltage across any one leg of a 3-phase system.

- Wye-connected: $V_P = V_{LN}$
- Delta-connected: $V_P = V_{LL}$

Line-to-Line, Line-to-Neutral, and Phase Voltages

Consider a three-phase wye-connected generator or a three-phase wye-connected load.



$\vec{V}_{an}$, $\vec{V}_{bn}$ and $\vec{V}_{cn}$ are line-to-neutral voltages V_{LN}

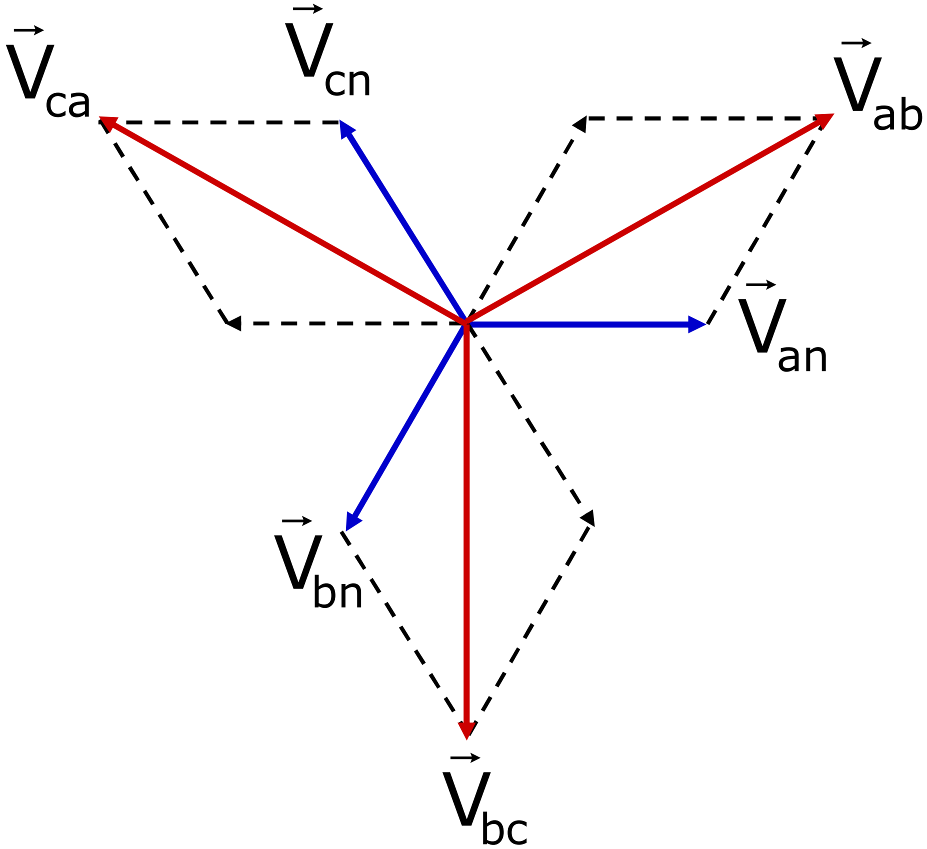
$\vec{V}_{ab}$, $\vec{V}_{bc}$ and $\vec{V}_{ca}$ are line-to-line voltages V_{LL}

For a Delta Connected System?

For a balanced WYE-connected three-phase system, from KVL:

$$V_{ab} = V_{an} + V_{nb}$$
$$= V_{an} - V_{bn}$$
$$= V\angle 0^\circ - V\angle -120^\circ$$
$$= V(1 - a^2)$$
$$V_{ab} = V(\sqrt{3}\angle 30^\circ)$$

$$\left|V_{LL}\right| = \sqrt{3}\left|V_{LN}\right|$$



Definitions: Currents in Three-Phase Systems

Line Current (I_L)

The current flowing through lines from source to load.

Phase Current (I_P)

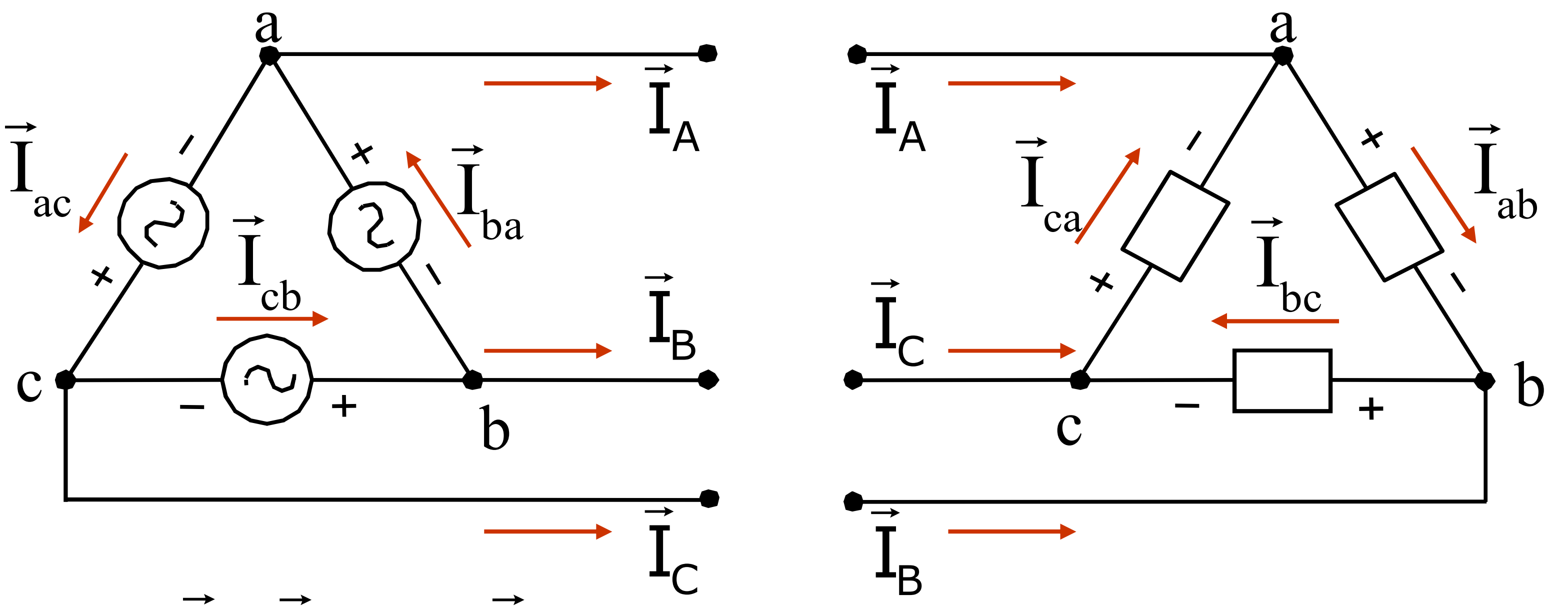
The current flowing through any one leg of the system.

- Wye connection: $I_P = I_L$
- Delta connection: $I_P = \frac{I_L}{\sqrt{3}}$

Note: There is no definition for "Line-to-Line Current."

Line and Phase Currents

Consider a three-phase delta-connected generator or a three-phase delta-connected load.



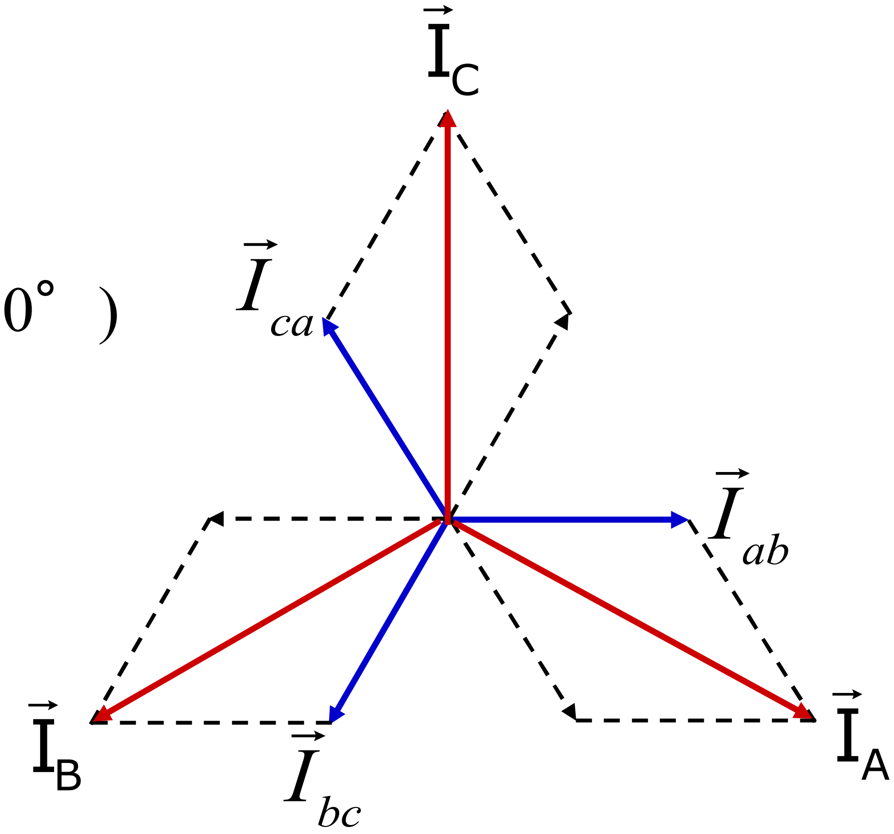
$\vec{I}_{ab}$, $\vec{I}_{bc}$ and $\vec{I}_{ca}$ are phase currents I_ϕ or I_P

$\vec{I}_A$, $\vec{I}_B$ and $\vec{I}_C$ are line currents I_L

For the balanced DELTA-connected three-phase load, from KCL:

$$
\begin{aligned}
I_A &= I_{ab} - I_{ca} \\
&= I \angle 0^\circ - I \angle 120^\circ \\
&= I\,(1 \angle 0^\circ - 1 \angle 120^\circ) \\
&= I\,(1 - a) \\
I_A &= I\,(\sqrt{3} \angle -30^\circ)
\end{aligned}
$$

$$\boxed{|I_L| = \sqrt{3}\,|I_P|}$$



Voltage and Current Equations

For balanced three-phase systems:

Wye - connected

$$\left|V_{LL}\right| = \sqrt{3}\left|V_{LN}\right|$$

$$I_p = I_L$$

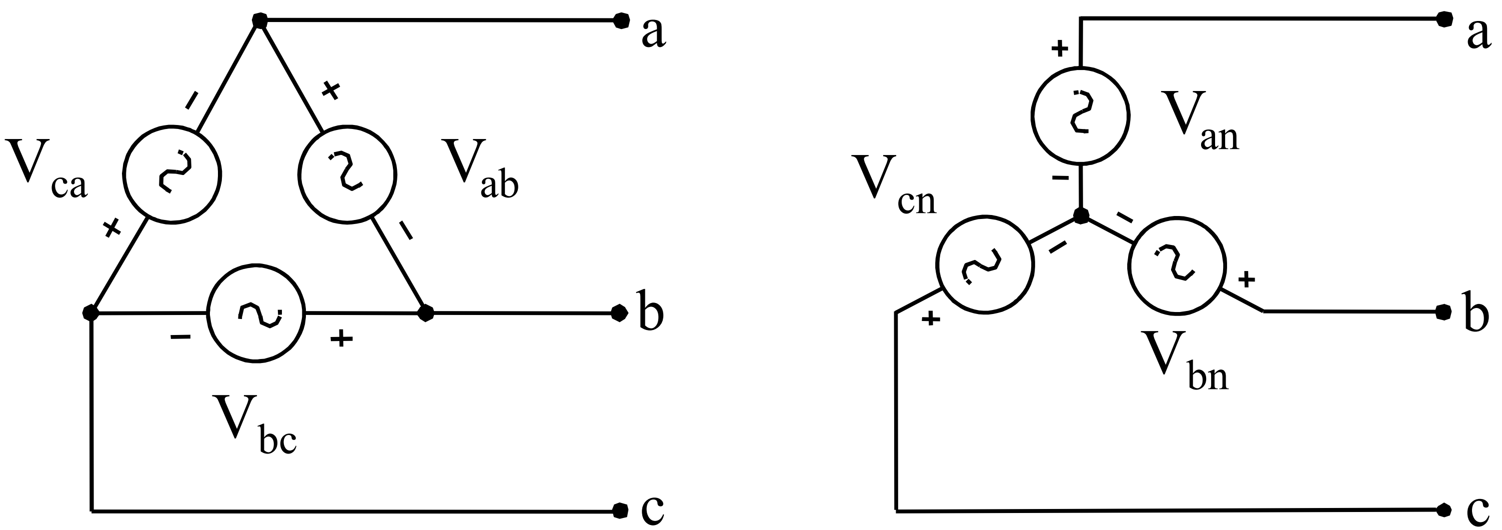
Delta - connected

$$V_P = V_{LL}$$

$$I_p = \frac{I_L}{\sqrt{3}}$$

Δ-Y Conversion for Generators

Given a balanced 3-phase delta connected generator, what is its equivalent wye ?

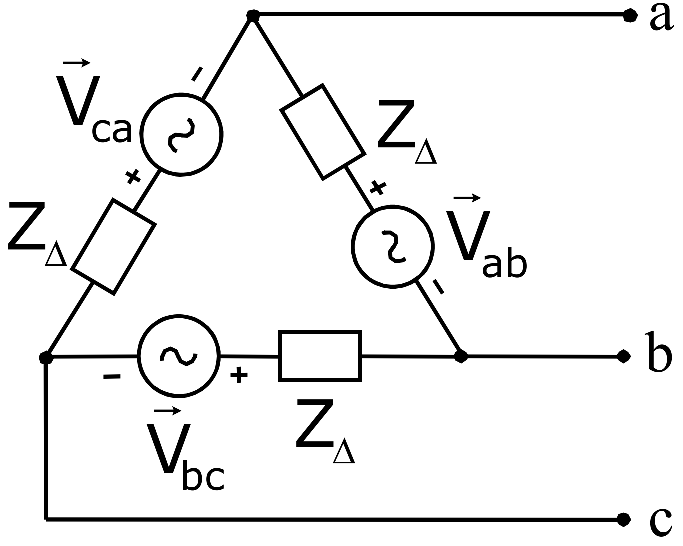


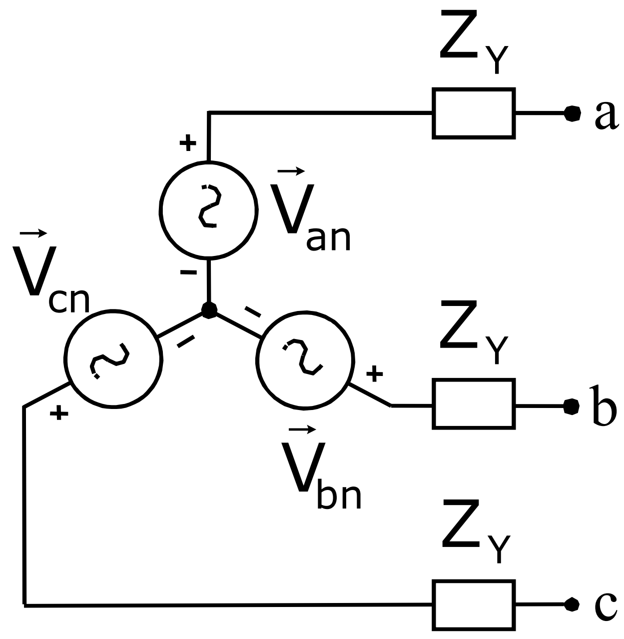
Note: The line-to-line voltages must be the same.

If $V_{ab} = V_L \angle \alpha$, then $V_{an} = (1/\sqrt{3})\, V_L \angle \alpha\text{-}30°$

Conversion for a Generator with Z

Given $\vec{V}_{ab}$ and Z_Δ, find $\vec{V}_{an}$ and Z_Y





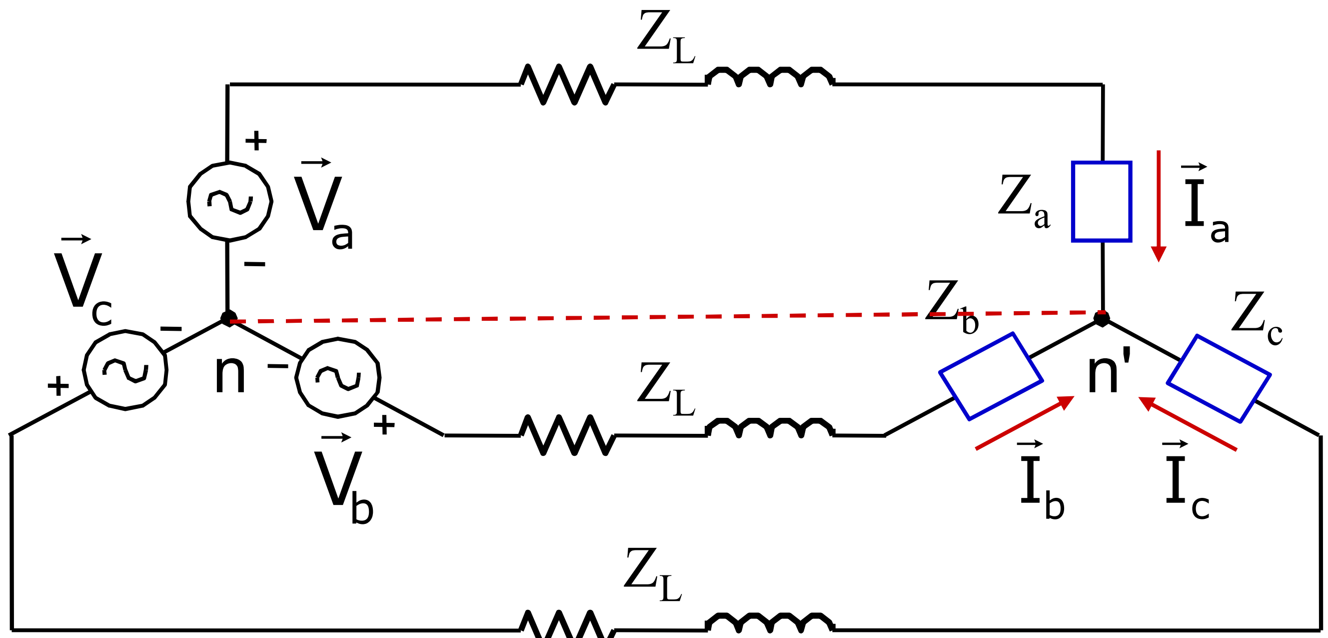
We get

$$\vec{V}_{an} = \left(\tfrac{1}{\sqrt{3}}\angle -30^\circ\right)\vec{V}_{ab}$$

$$Z_Y = \tfrac{1}{3} Z_\Delta$$

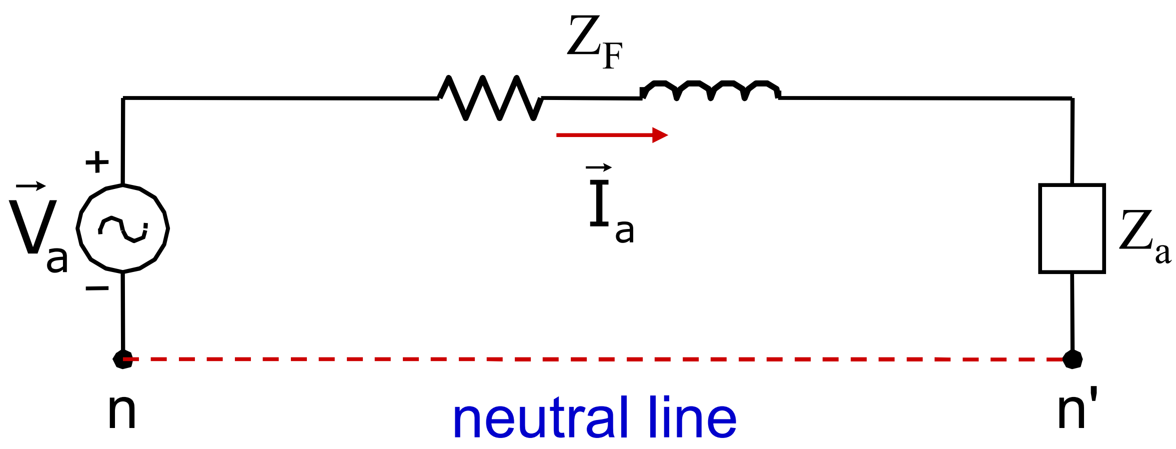
The Neutral Conductor

Consider a balanced three-wire three-phase system with wye-connected generator and loads.



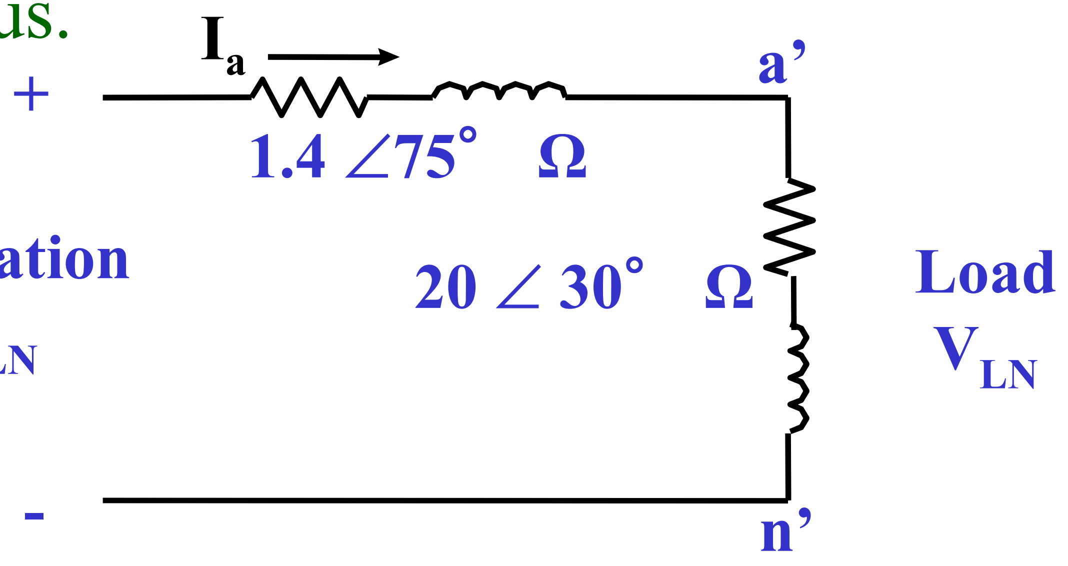
The Neutral Conductor

1. The sum of balanced phasors is zero.
2. If a neutral conductor is connected between n and n’, no current will flow through the neutral conductor.
3. The nodes n and n’ are at the same potential!
4. □ We can analyze the circuit using single-phase analysis.



Example

The terminal voltage of a wye-connected load consisting of three equal impedances of $20 \angle 30^\circ\ \Omega$ is 4.4 kV line-to-line. The impedance of each of the three lines connecting the load to a bus at a substation is $Z_L = 1.4 \angle 75^\circ$. Find the line-to-line voltage at the substation bus.



The magnitude of voltage-to-neutral at the load is:

$$V_{a'n'} = \frac{4400}{\sqrt{3}} = 2540\,\text{V}\ \angle 0^\circ \quad \left(\text{as reference}\right)$$

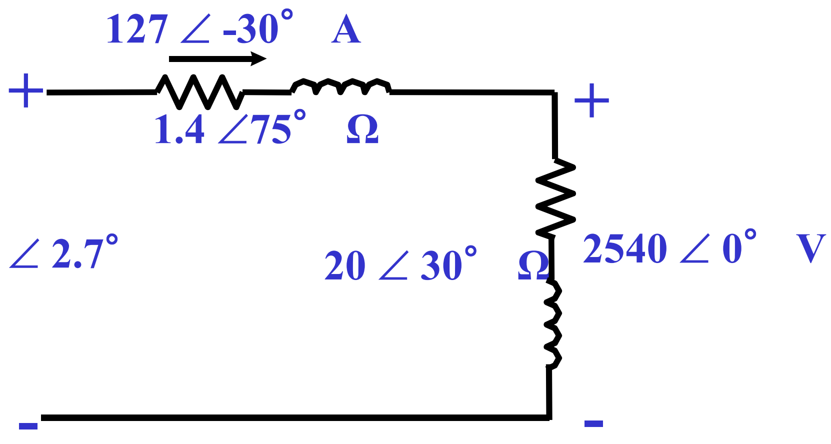
$$I_{a'n'} = \frac{2540\ \angle 0^\circ}{20\,\angle 30^\circ} = 127\,\text{A}\,\angle -30^\circ$$

The line-to-neutral voltage at the substation is:

$$V_{a'n'} + I_{a'n'} \cdot Z_L = 2540\,\angle 0^\circ + 127\,\angle -30^\circ\left(1.4\,\angle 75^\circ\right)$$

$$= 2666 + j125.7$$

$$= 2670\,\angle 2.70^\circ\,\text{V}$$



The voltage magnitude at the substation is

$$4.62\,\text{kV} \cong 2.67\sqrt{3}$$

Homework

Balanced 3-phase system with $V_{ab} = 173.2 \angle 0^\circ$ and wye-connected load with $Z_{LOAD} = 10 \angle 20^\circ \ \Omega$. Assume a-b-c phase sequence.

1. Draw the described balanced three phase system. Don't forget to properly label the voltages and currents
2. Determine **all** phasor voltages and currents.