EEE 51: Second Semester 2017 - 2018

Lecture 15

Frequency Response

Frequency Characteristics of Transistor Circuits

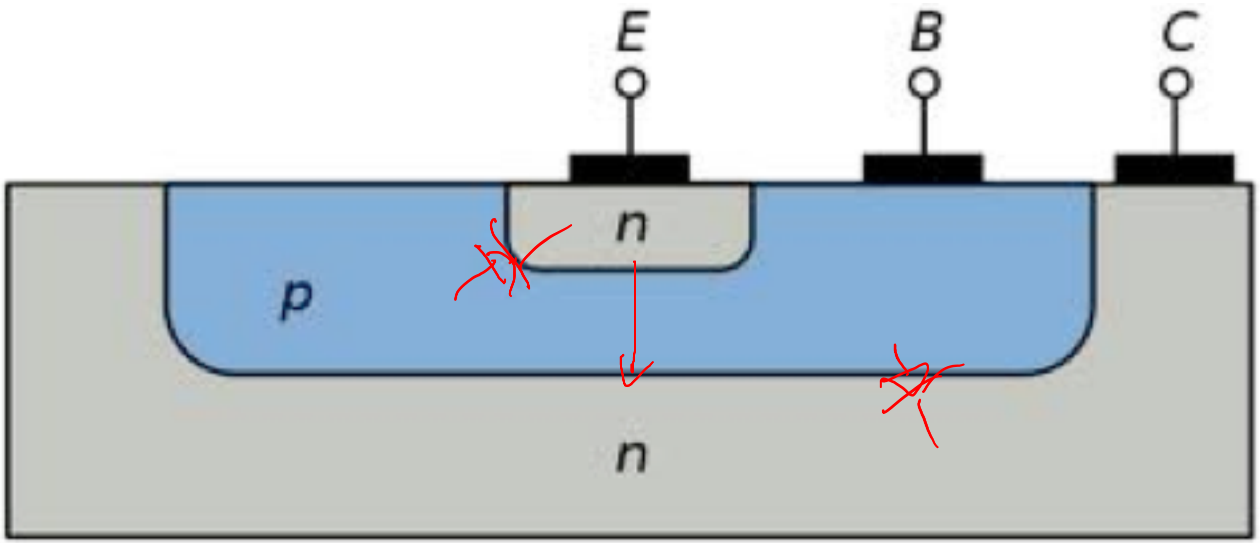
- Due to frequency-dependent impedances
  - Capacitors, inductors

- BJT Parasitic Capacitances
  - Junction capacitances
  - Nonlinear (voltage dependent)

  - Base-Charging capacitance (C_b)
  - Base-Emitter junction capacitance (C_{je})
  - Base-Collector junction capacitance (C_μ)

NPN

BJT Capacitances



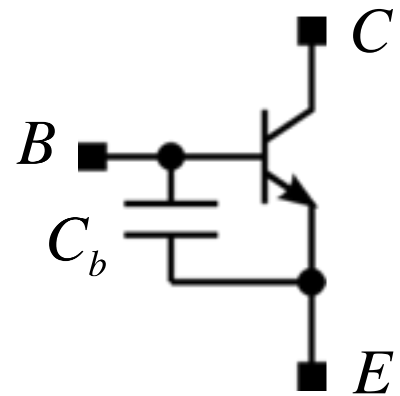
[http://commons.wikimedia.org/wiki/File:Npn_bjt_cross_section.svg]

BJT Base-Charging Capacitance

- Capacitance due to the change in majority carrier charge inside the base of the BJT
  - Cancels out the change in minority carriers in the base due to v_{BE}
  - ~ 100's of fF

$\uparrow 10^{-15}$

$$C_b = \tau_F g_m = \tau_F \frac{I_{C,Q}}{V_T}$$



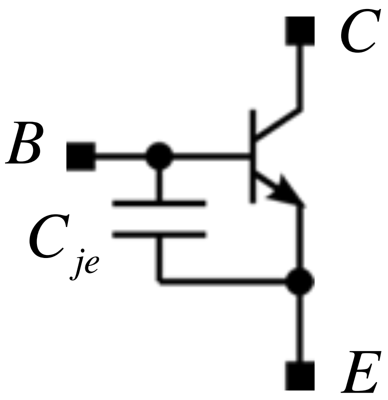
- τ_F – forward base transit time
  - Represents the average time (per carrier) spent crossing the base

BJT Base-Emitter Junction Capacitance

- PN junction capacitance ~ 10's of fF
- In a BJT in the forward active region, BE junction is forward-biased

$$C_{je} = \frac{C_{je0}}{\sqrt{1 - \frac{V_{BE}}{V_{j,BE}}}}$$

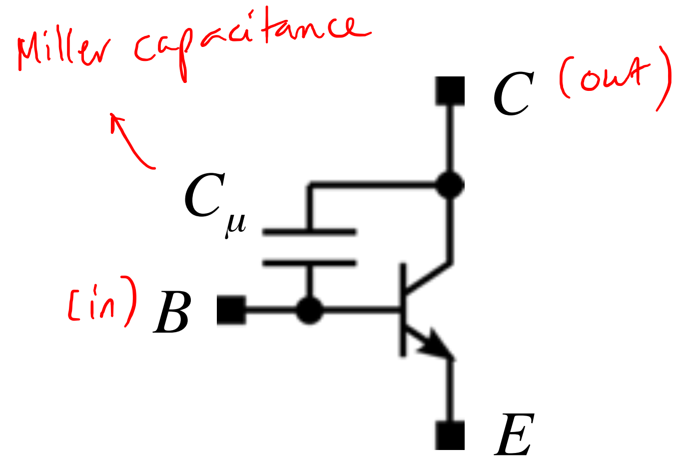
$$C_{je} \approx 2C_{je0}$$



BJT Base-Collector Junction Capacitance

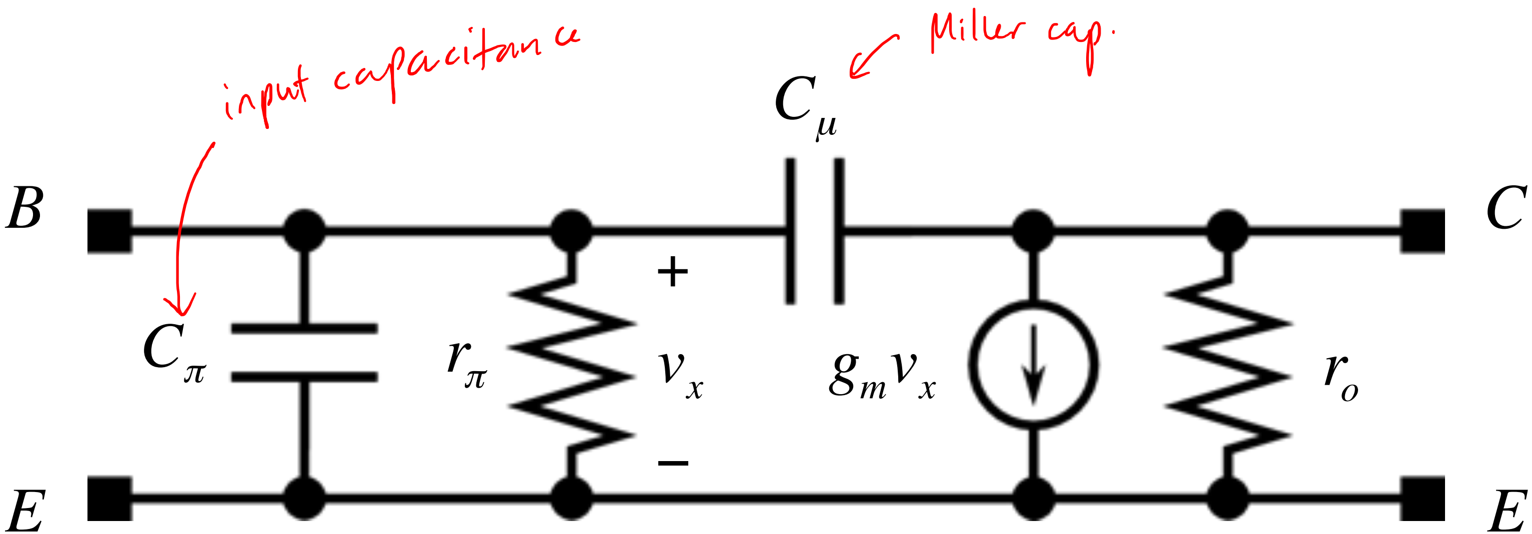
- PN junction capacitance 5 to 10 fF
- Reversed-biased in forward active BJTs

$$C_\mu = \frac{C_{\mu 0}}{\sqrt{1 + \frac{V_{CB}}{V_{j,CB}}}}$$



Can be amplified by the Miller effect

BJT Small Signal Model (with Capacitances)



$$C_\pi = \underline{C_b + C_{je}}$$

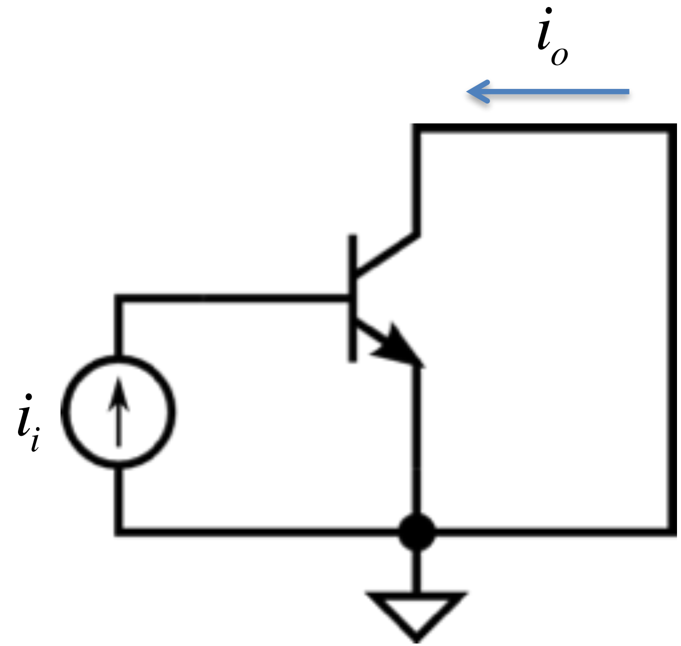
$$C_b = \tau_F g_m$$

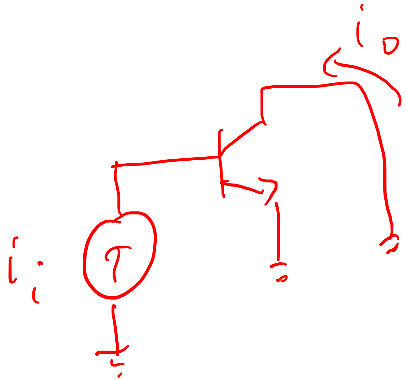
$$C_{je} = \frac{C_{je0}}{\sqrt{1 - \frac{V_{BE}}{V_{j,BE}}}}$$

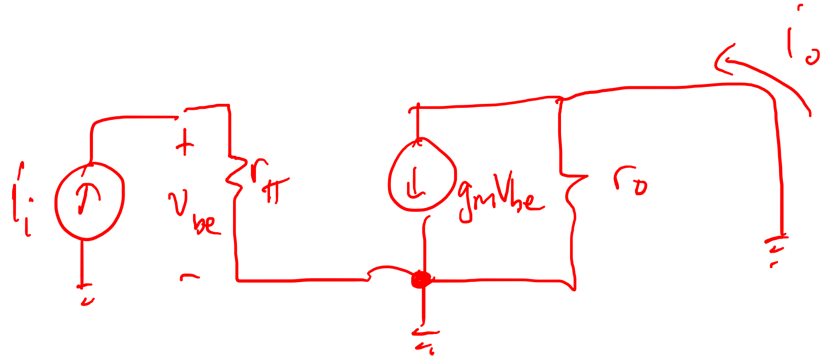
$$C_\mu = \frac{C_{\mu 0}}{\sqrt{1 + \frac{V_{CB}}{V_{j,CB}}}}$$

BJT Transition Frequency (f_T)

- Frequency at which the short-circuit common emitter current gain falls to unity







$$i_o = g_m v_{be} = g_m (i_i \cdot r_\pi)$$

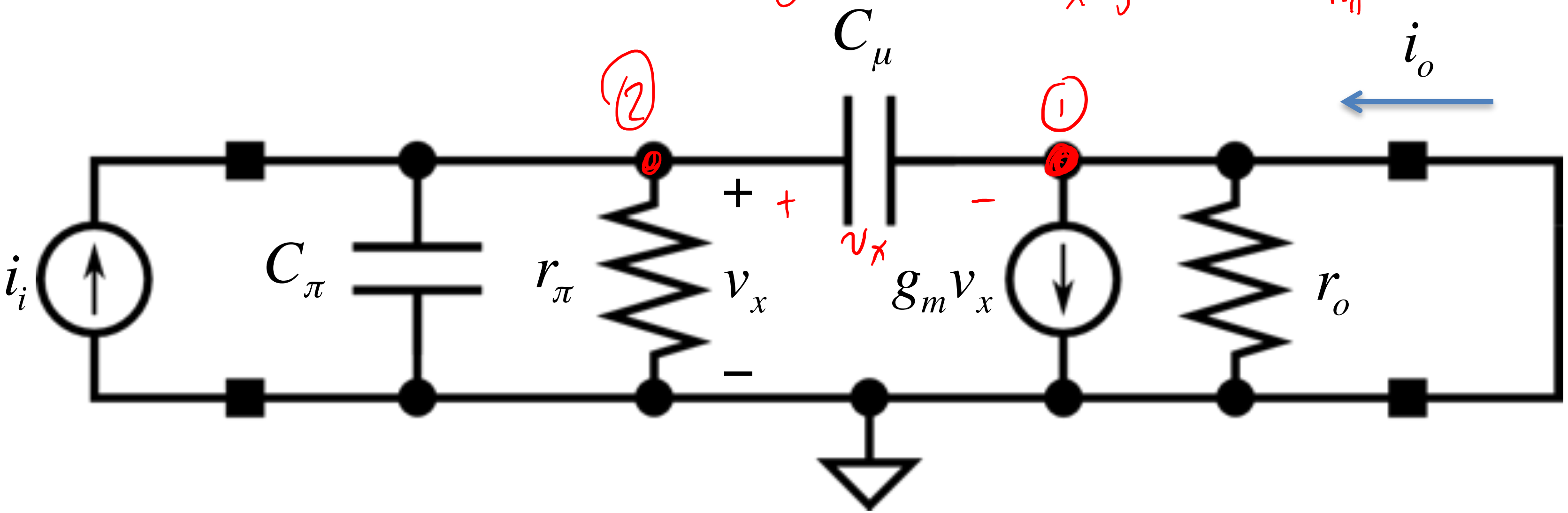
$$\frac{i_o}{i_i} = g_m r_\pi = g_m \cdot \frac{\beta}{g_m} = \beta > 1$$

BJT f_T

- Small signal model

① : $i_o = g_m v_x + (-v_x \cdot j\omega C_\mu)$

② : $i_i = v_x \cdot j\omega C_\pi + \frac{v_x}{r_\pi} + v_x \cdot j\omega C_\mu$



$$\frac{i_o}{i_i} = 1 \quad \Rightarrow \quad \omega_T \approx \frac{g_m}{C_\mu + C_\pi} = 2\pi f_T$$

$$\frac{i_o}{i_i} = \frac{g_m - j\omega C_\mu}{\frac{1}{r_\pi} + j\omega(C_\mu + C_\pi)} \cdot \frac{r_\pi}{r_\pi} = \frac{g_m r_\pi - j\omega C_\mu \cdot r_\pi}{1 + j\omega(C_\mu + C_\pi) r_\pi}$$

at what $\omega = \omega_T$ does $\left|\frac{i_o}{i_i}\right| = 1$

$$\frac{\sqrt{(g_m r_\pi)^2 + (\omega_T \cancel{C_\mu} r_\pi)^2}}{\sqrt{1 + \omega_T^2 (C_\mu + C_\pi)^2 r_\pi^2}} = 1$$

$$(g_m r_\pi)^2 - 1 = \omega_T^2 r_\pi^2 \left[(C_\mu + C_\pi)^2 - (C_\mu)^2\right]$$

approx :
$$(g_m \cancel{r_\pi})^{\cancel{2}} = \omega_T^2 \cancel{r_\pi^2} [C_\mu + C_\pi]^2$$

$$\omega_T^2 \approx \frac{g_m^2}{(C_\mu + C_\pi)^2}$$

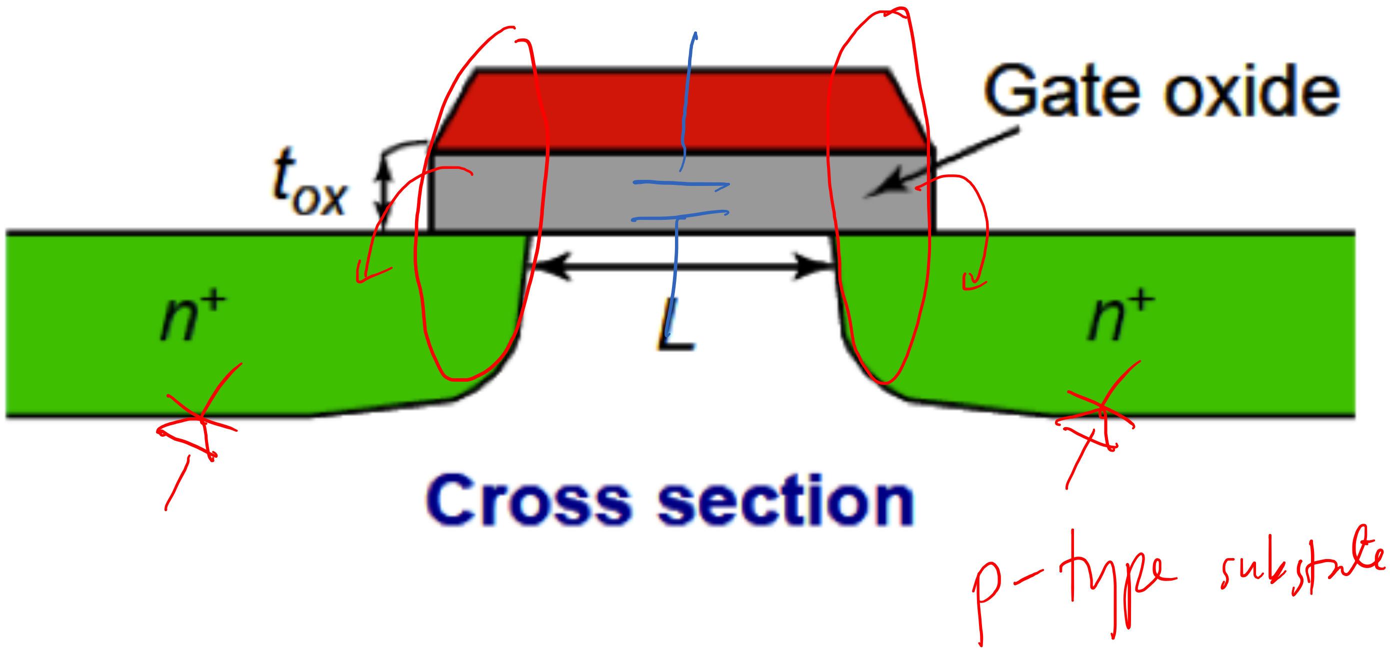
$$\omega_T \approx \frac{g_m}{C_\mu + C_\pi}$$

MOS Capacitances

- MOS parasitic capacitances
  - Nonlinear

- Gate oxide capacitance (“parallel plate”)
- Gate overlap capacitance (fringe)
- Drain/Source-Bulk junction capacitance (PN junction)

MOS Capacitances



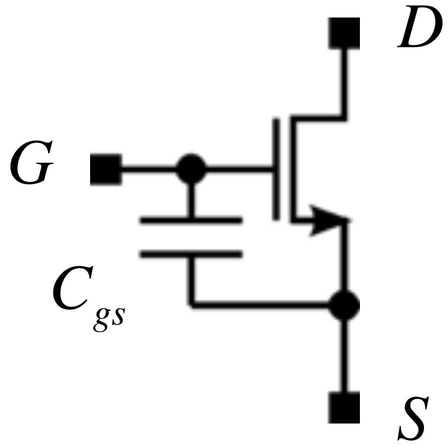
MOS Capacitances

- Gate Capacitance
  - Dependent on the thickness of the gate oxide and area of the gate
  - Nonlinear
    - Dependent on the gate and source/drain voltages
- Gate Overlap Capacitance
  - "parallel-plate" and fringing fields

MOS Gate-Source Capacitance

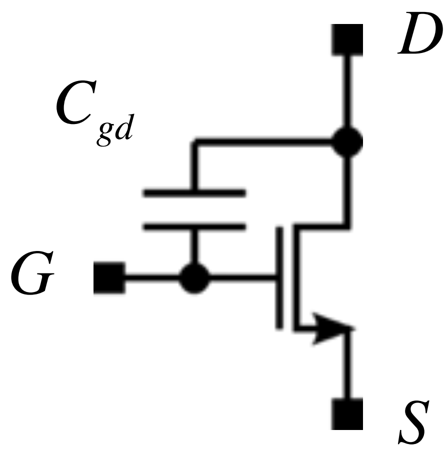
- Components:
  - gate "parallel plate" capacitance
  - Gate-source overlap capacitance

→ dominant value



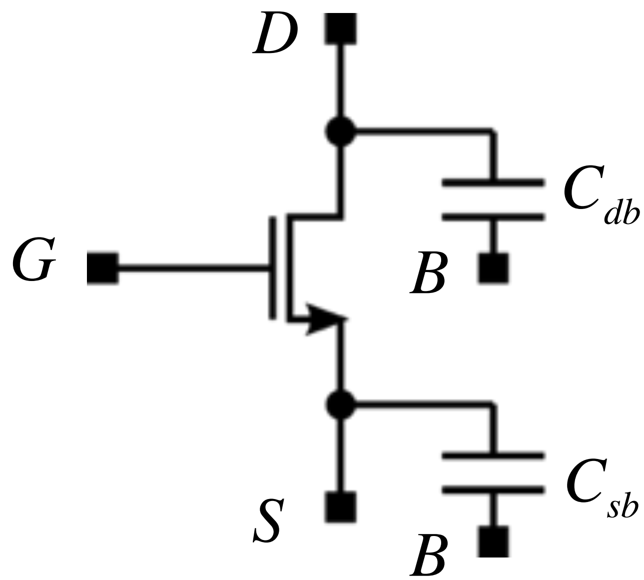
MOS Gate-Drain Capacitance

- Composed of the overlap capacitance between the gate and drain



MOS Source/Drain Junction Capacitance

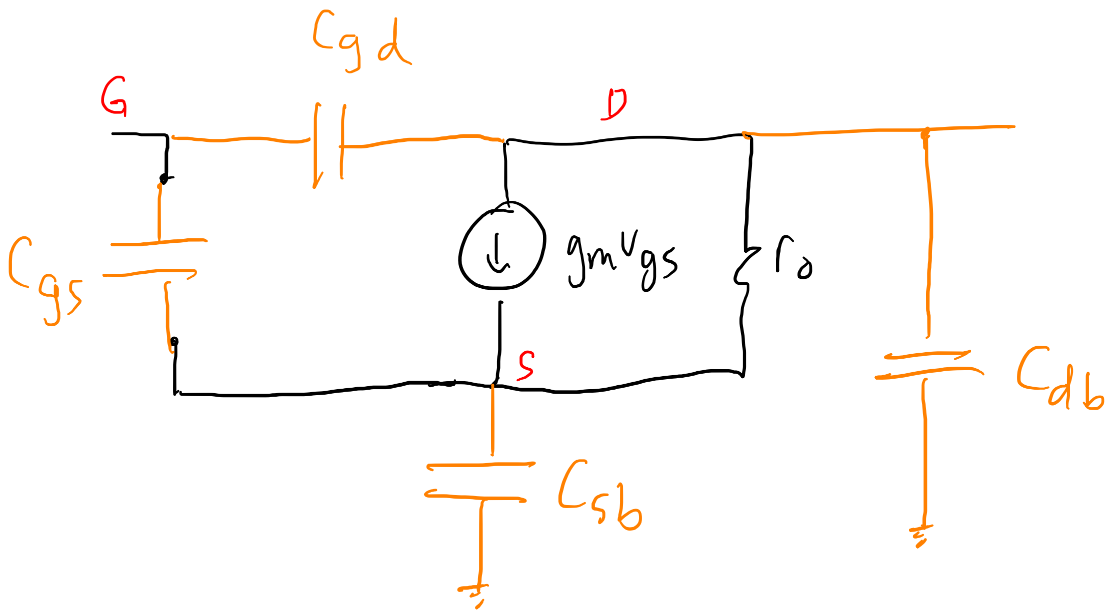
- Formed by the drain/source PN junction (to the substrate)
- Normally reversed biased



$\rightarrow f(V_{DB})$

$\rightarrow f(V_{SB})$

MOSFET small signal (w/ caps)

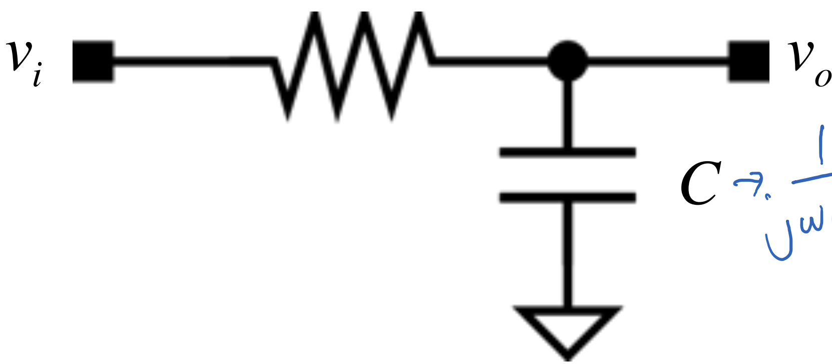


in saturation: $C_{gs} > C_{gd} > C_{sb} / C_{db}$

Capacitance Effects

- A Simple RC Circuit

$s = \sigma + j\omega$

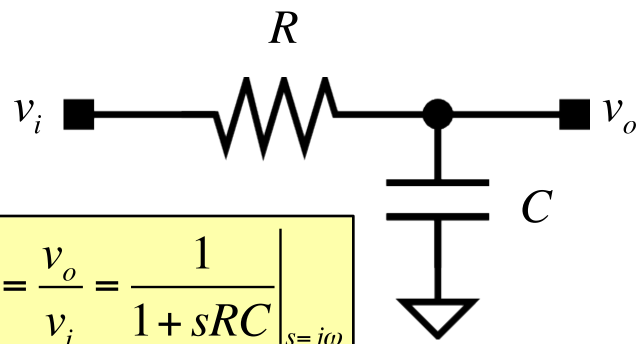


applying voltage division:

$$v_o = \frac{\frac{1}{sC}}{\frac{1}{sC} + R} v_i = \frac{1}{1 + sRC} v_i$$

$$\boxed{A_v(\omega) = \frac{v_o}{v_i} = \left.\frac{1}{1 + sRC}\right|_{s=j\omega}}$$

Frequency Response



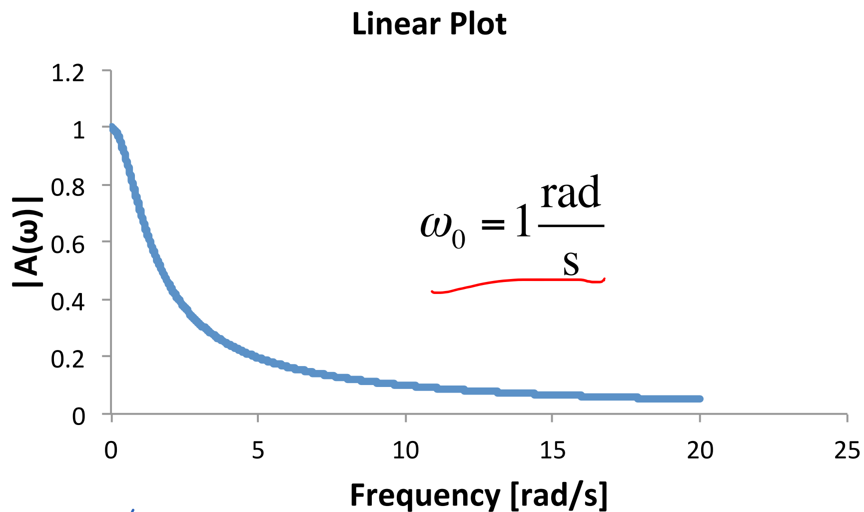
$$\boxed{A_v(\omega) = \frac{v_o}{v_i} = \left.\frac{1}{1+sRC}\right|_{s=j\omega}}$$

$$= \frac{1}{1+j\omega RC}$$

$$|A_v(\omega)| = \frac{1}{\sqrt{1+\underbrace{\omega^2R^2C^2}}}$$

$$= \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^2}}$$

where $\omega_0 = \frac{1}{RC}$

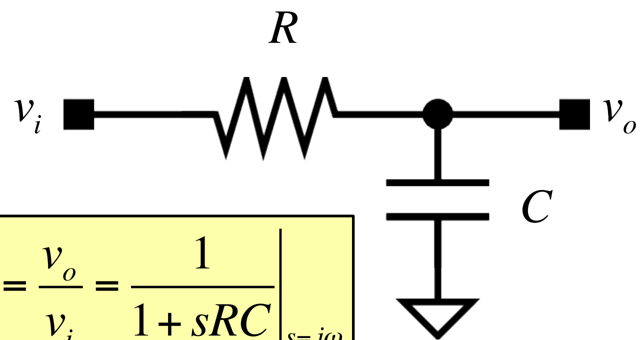


① : $\omega < \omega_0$, $|A_v(\omega)| \to 1$

② : $\omega = \omega_0$, $|A_v(\omega)| = \frac{1}{\sqrt{2}}$

③ : $\omega > \omega_0$, $|A_v(\omega)| \to \frac{\omega_0}{\omega} \to \emptyset$

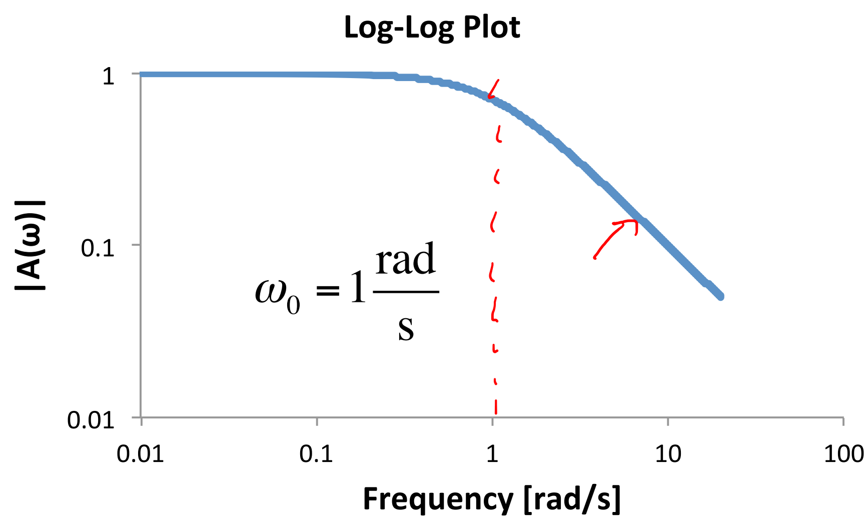
Frequency Response



$$\boxed{A_v(\omega) = \frac{v_o}{v_i} = \left.\frac{1}{1+sRC}\right|_{s=j\omega}}$$

$$\left|A_v(\omega)\right| = \frac{1}{\sqrt{1+\omega^2 R^2 C^2}}$$

$$= \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^2}}$$

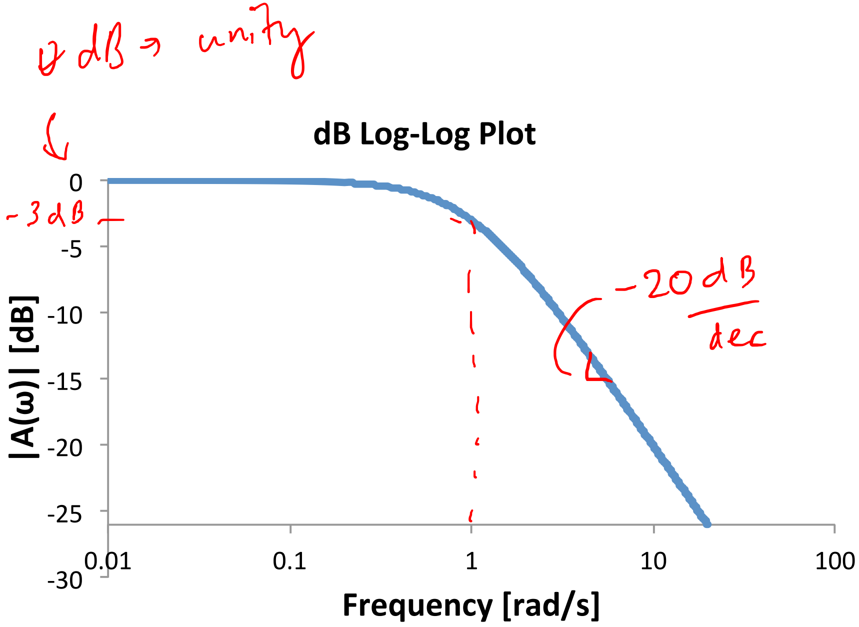


Decibels (dB)

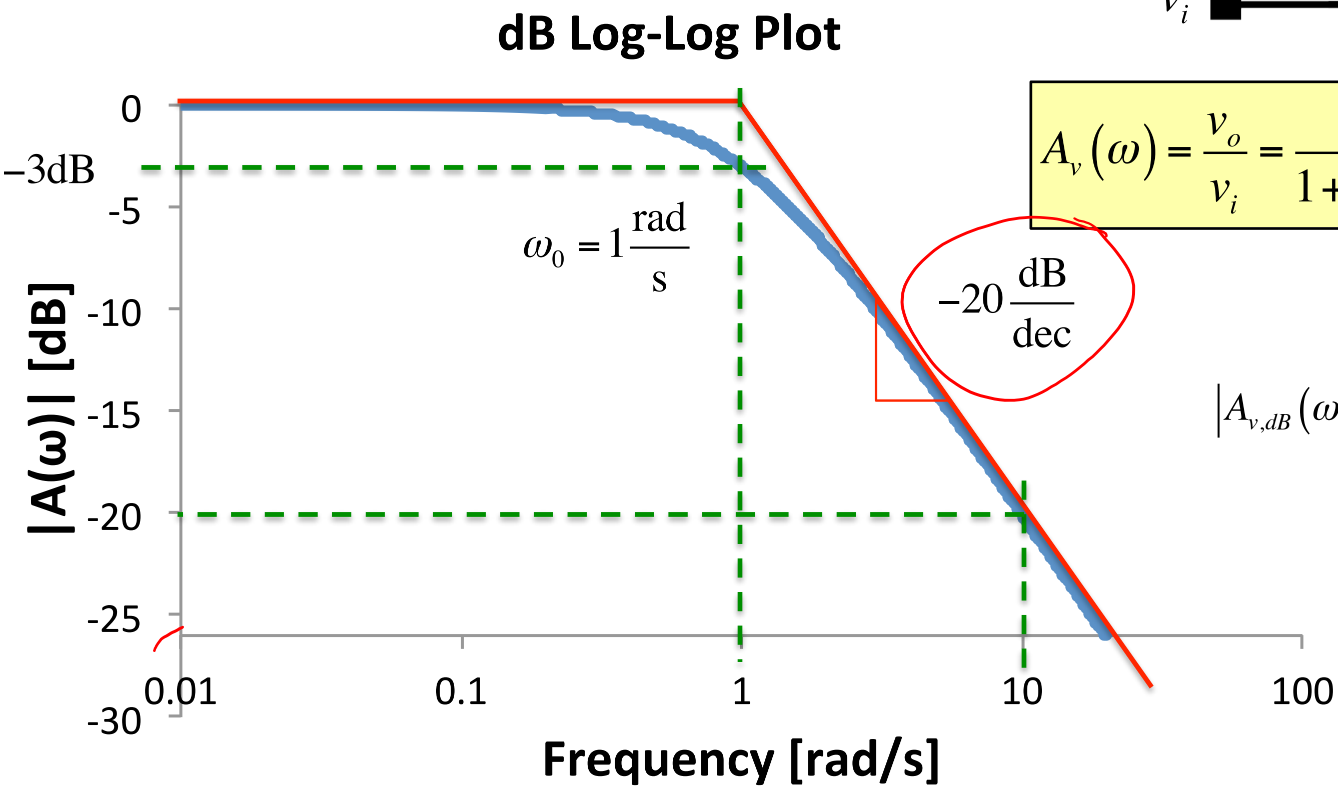
- Voltage gain:

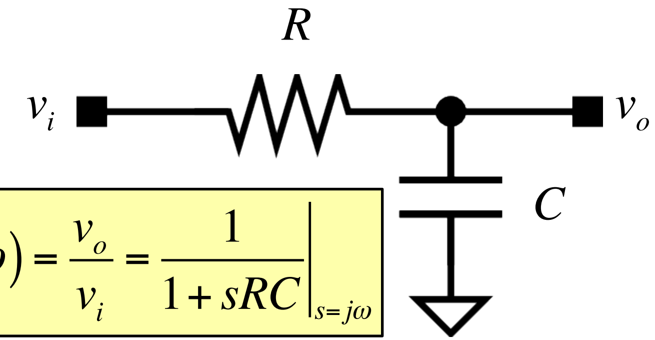
$$\left|A_{v,dB}(\omega)\right| = 20\log\left(\left|\frac{v_o}{v_i}\right|\right)$$

$$= 20\log\left(\frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^2}}\right)$$



Frequency Response: Magnitude

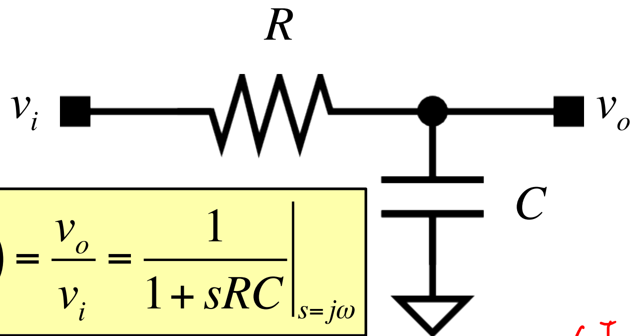




$$A_v(\omega) = \frac{v_o}{v_i} = \left.\frac{1}{1+sRC}\right|_{s=j\omega}$$

$$\left|A_{v,dB}(\omega)\right| = 20\log\left(\frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^2}}\right)$$

Phase Response



$$A_v(\omega) = \frac{v_o}{v_i} = \left.\frac{1}{1+sRC}\right|_{s=j\omega}$$

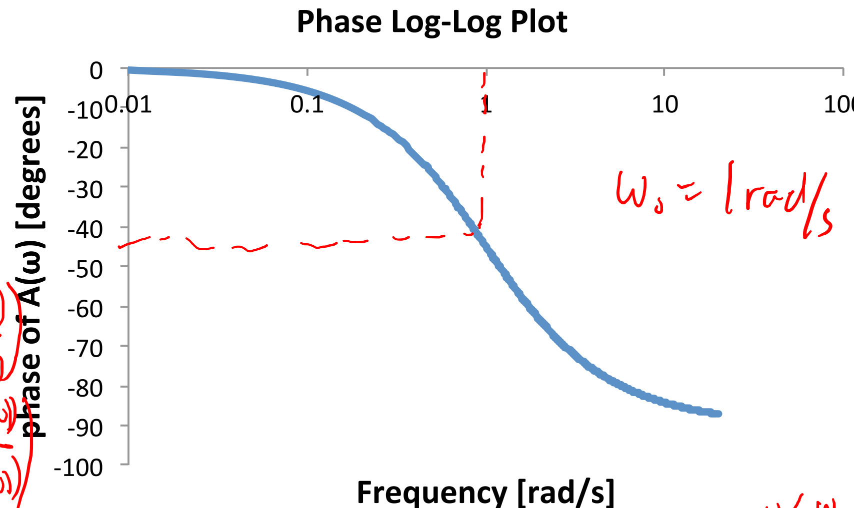
$A_v(\omega) = \dfrac{N(\omega)}{D(\omega)}$

$\angle A_v(\omega) = \tan^{-1}\left(\dfrac{Im(N(\omega))}{Re(N(\omega))}\right) - \tan^{-1}\left(\dfrac{Im(D(\omega))}{Re\,D(\omega)}\right)$

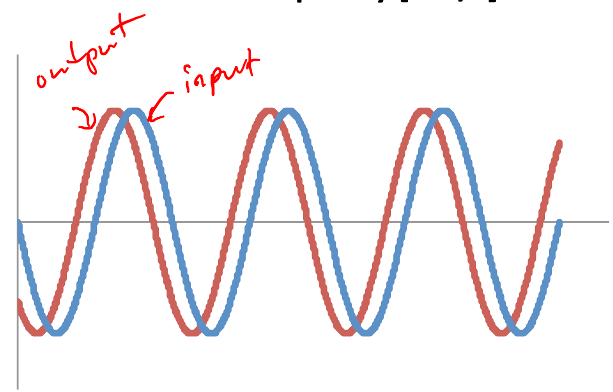
$$\angle A_v(\omega) = \tan^{-1}\left(\frac{\text{Im}[A(\omega)]}{\text{Re}[A(\omega)]}\right)$$

$$= \underbrace{\tan^{-1}\left(\frac{0}{1}\right)}_{0^\circ} - \tan^{-1}\left(\frac{\omega RC}{1}\right)$$

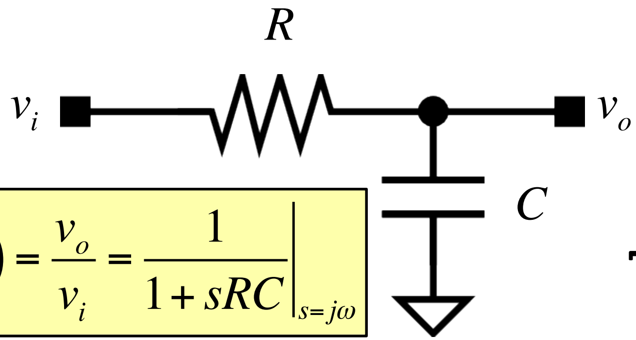
$$= -\tan^{-1}(\omega RC) = -\tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$



- $\omega < \omega_0$: $\angle A_v(\omega) \to 0^\circ$
- $\omega = \omega_0$: $\angle A_v(\omega) = -45^\circ$
- $\omega > \omega_0$: $\angle A_v(\omega) \to -90^\circ$



Phase Response

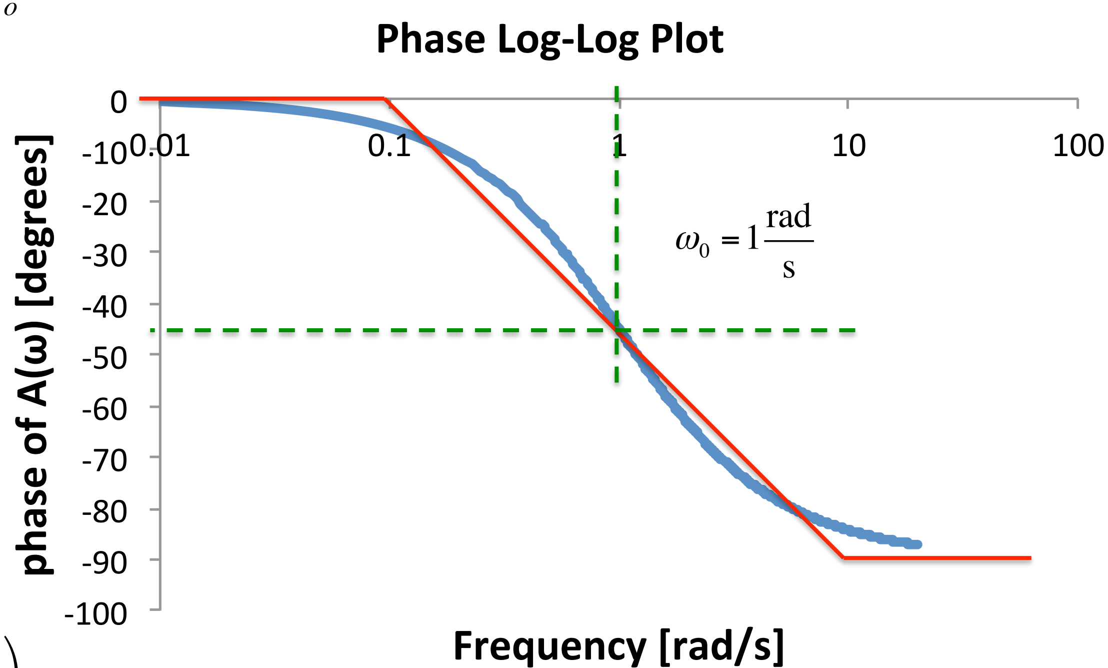


$$A_v(\omega) = \frac{v_o}{v_i} = \left.\frac{1}{1+sRC}\right|_{s=j\omega}$$

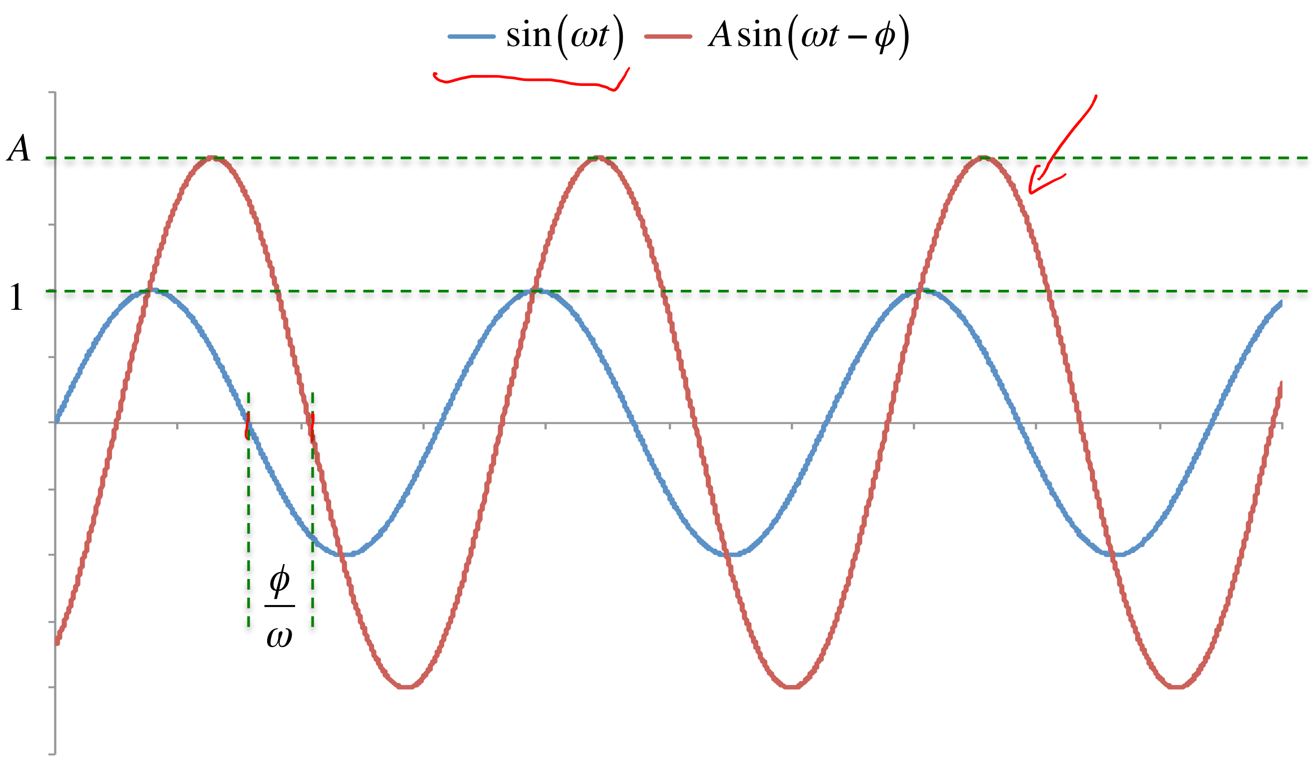
$$\angle A_v(\omega) = \tan^{-1}\left(\frac{\mathrm{Im}\left[A(\omega)\right]}{\mathrm{Re}\left[A(\omega)\right]}\right)$$

$$= \tan^{-1}\left(\frac{0}{1}\right) - \tan^{-1}\left(\frac{\omega RC}{1}\right)$$

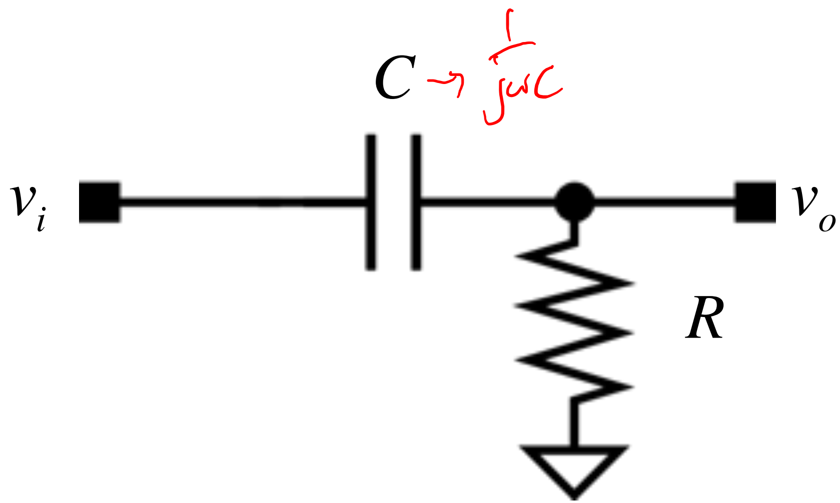
$$= -\tan^{-1}(\omega RC) = -\tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$



Magnitude and Phase



Another RC Example



applying voltage division:

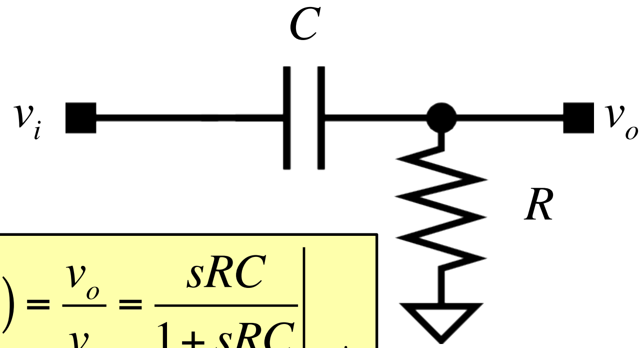
$$v_o = \frac{R}{\frac{1}{sC} + R} v_i = \frac{sRC}{1 + sRC} v_i$$

zero

$$\boxed{A_v(\omega) = \frac{v_o}{v_i} = \left. \frac{sRC}{1 + sRC} \right|_{s = j\omega}}$$

pole

Frequency Response: Magnitude Response

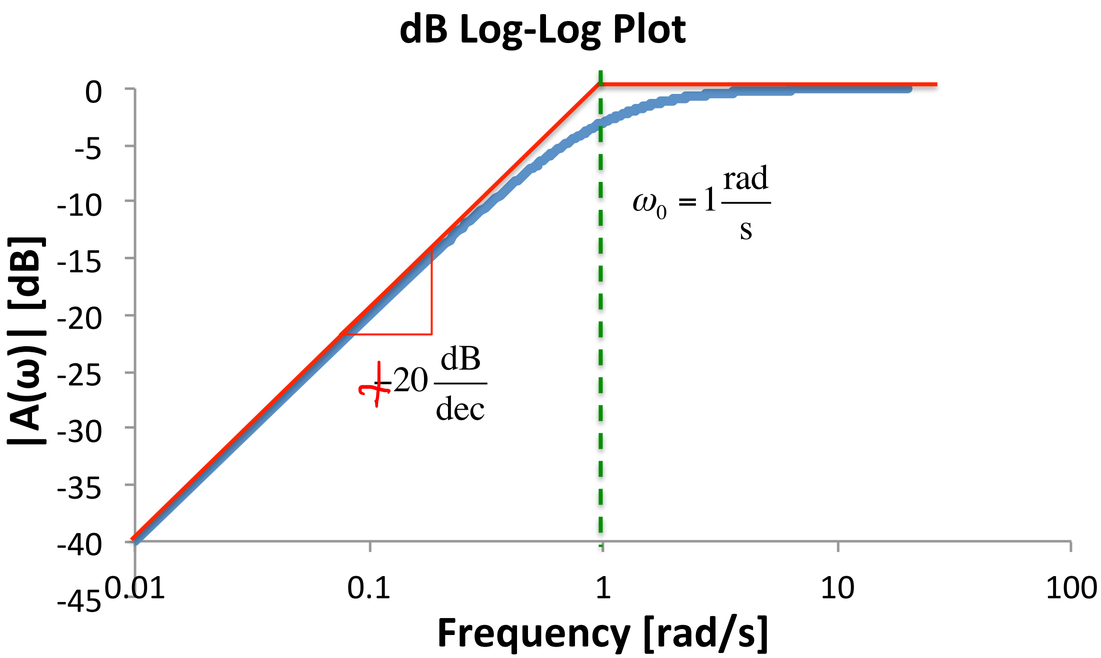


$$\boxed{A_v(\omega) = \frac{v_o}{v_i} = \left.\frac{sRC}{1+sRC}\right|_{s=j\omega}}$$

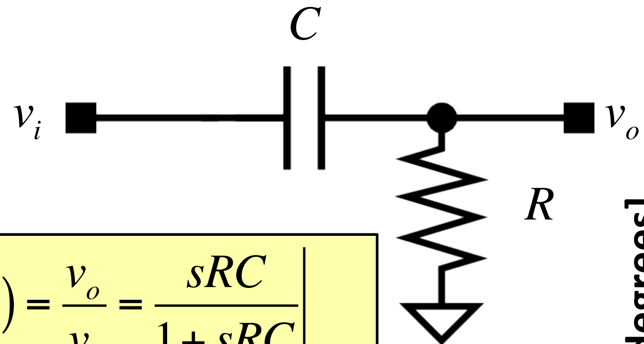
$$|A_v(\omega)| = \frac{\omega RC}{\sqrt{1+\omega^2R^2C^2}}$$

$$= \frac{\dfrac{\omega}{\omega_0}}{\sqrt{1+\left(\dfrac{\omega}{\omega_0}\right)^2}}$$

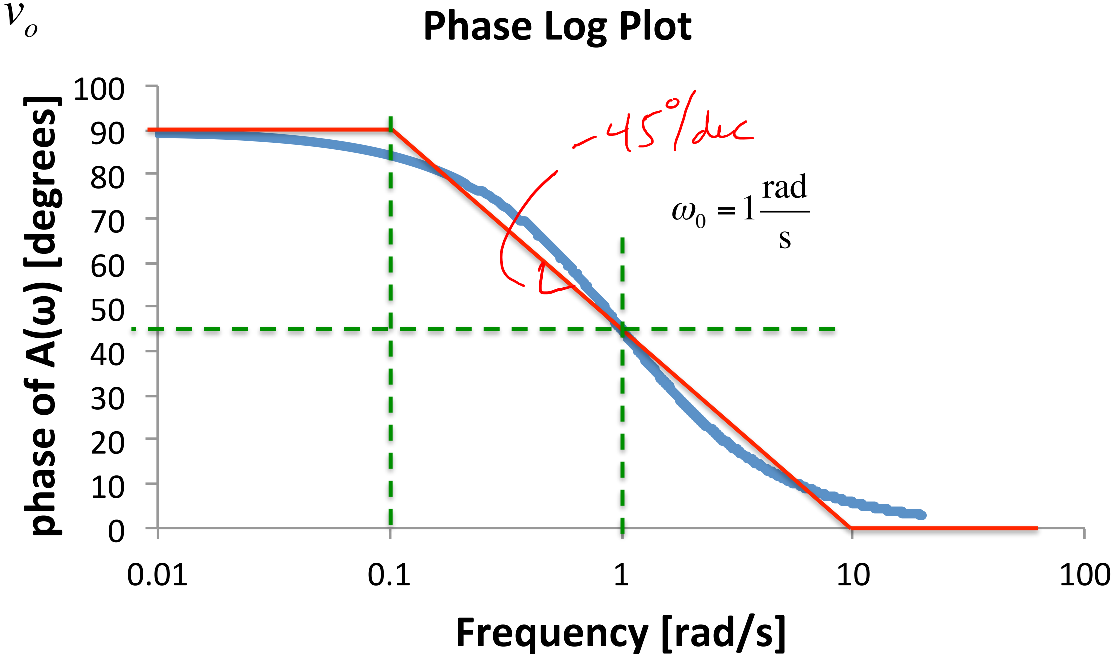
- $\omega < \omega_0$: $|A_v(\omega)| \rightarrow 0$
- $\omega = \omega_0$: $|A_v(\omega)| = \frac{1}{\sqrt{2}}$
- $\omega > \omega_0$: $|A_v(\omega)| \rightarrow 1$



Phase Response



$$A_v(\omega) = \frac{v_o}{v_i} = \left.\frac{sRC}{1+sRC}\right|_{s=j\omega}$$



$$\angle A_v(\omega) = \tan^{-1}\left(\frac{\mathrm{Im}\left[A(\omega)\right]}{\mathrm{Re}\left[A(\omega)\right]}\right)$$

$$= \tan^{-1}\left(\frac{\omega RC}{0}\right) - \tan^{-1}\left(\frac{\omega RC}{1}\right)$$

$$= 90^\circ - \tan^{-1}(\omega RC) = 90^\circ - \tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

- $\omega < \omega_0$: $\angle A_v(\omega) \rightarrow 90^\circ$
- $\omega = \omega_0$: $\angle A_v(\omega) = 45^\circ$
- $\omega > \omega_0$: $\angle A_v(\omega) \rightarrow 0^\circ$

A General Zero Term

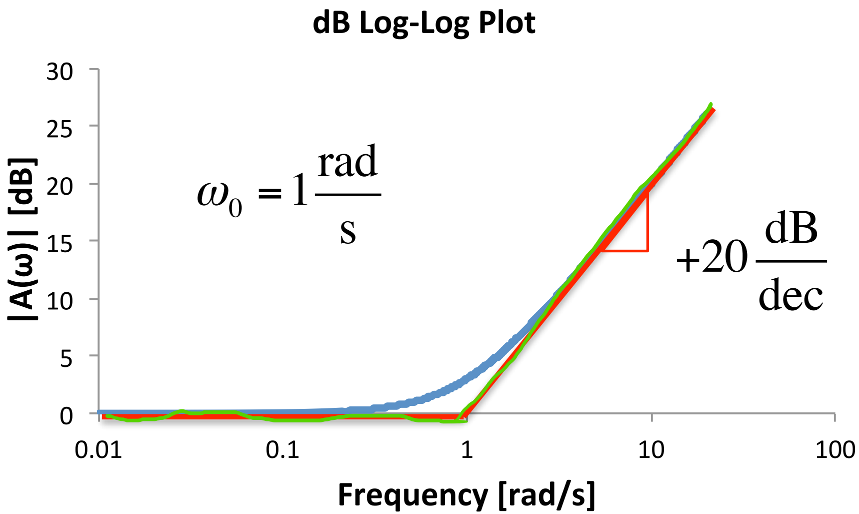
$$\angle B(\omega) = \tan^{-1}\left(\frac{-\omega}{\omega_0}\right) = -\tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

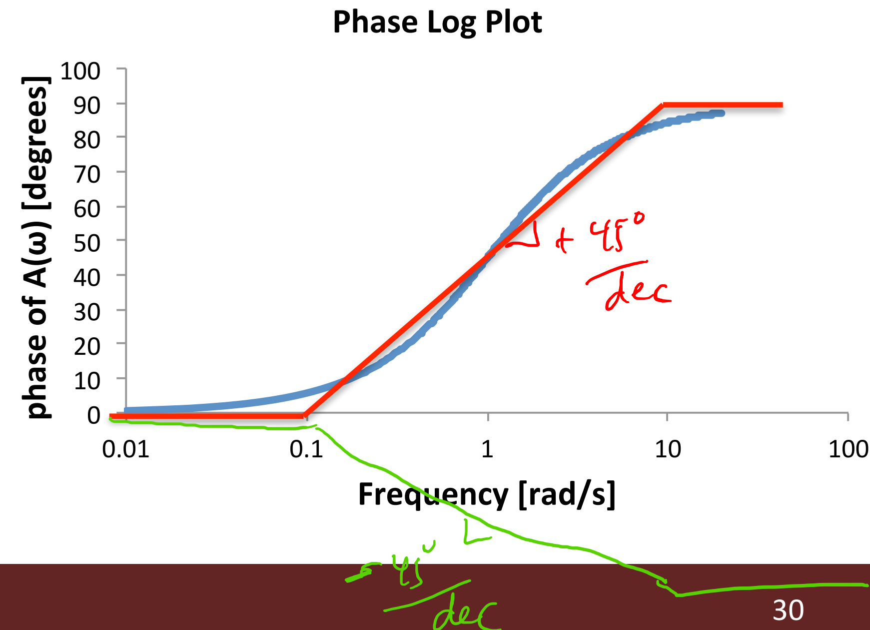
$$\boxed{A(\omega) = 1 + j\frac{\omega}{\omega_0}}$$

$$B(\omega) = 1 - \frac{j\omega}{\omega_0}$$

$$20\log\left[\left|A(\omega)\right|\right] = 20\log\left[\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^2}\right]$$

$$\angle A(\omega) = \tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$





Generalized Transfer Function

- Factor the numerator and denominator to get the poles and zeros of the system

$$A(s) = \overset{\text{DC gain}}{A_0} \frac{\left(1+\frac{s}{z_1}\right)\left(1+\frac{s}{z_2}\right)\cdots\left(1+\frac{s}{z_N}\right)}{\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)\cdots\left(1+\frac{s}{p_D}\right)}$$

$$A(\omega) = A_0 \frac{\left(1+j\frac{\omega}{\omega_{z_1}}\right)\left(1+j\frac{\omega}{\omega_{z_2}}\right)\cdots\left(1+j\frac{\omega}{\omega_{z_N}}\right)}{\left(1+j\frac{\omega}{\omega_{p_1}}\right)\left(1+j\frac{\omega}{\omega_{p_2}}\right)\cdots\left(1+j\frac{\omega}{\omega_{p_D}}\right)}$$

Magnitude Response Using the dB Scale

$$\left|A_{dB}(\omega)\right| = 20\log\left[\left|A_0 \frac{\left(1+j\frac{\omega}{\omega_{z_1}}\right)\left(1+j\frac{\omega}{\omega_{z_2}}\right)\cdots\left(1+j\frac{\omega}{\omega_{z_N}}\right)}{\left(1+j\frac{\omega}{\omega_{p_1}}\right)\left(1+j\frac{\omega}{\omega_{p_2}}\right)\cdots\left(1+j\frac{\omega}{\omega_{p_D}}\right)}\right|\right]$$

$$= 20\log\left[\left|A_0\right|\right] + \sum_N 20\log\left[\left|1+j\frac{\omega}{\omega_{z_i}}\right|\right] + \underbrace{\sum_D 20\log\left[\left|\frac{1}{1+j\frac{\omega}{\omega_{p_i}}}\right|\right]}_{-\sum_D 20\log\left[\left|1+j\frac{\omega}{\omega_{p_i}}\right|\right]}$$

Phase Response

$$A(\omega) = A_0 \frac{\left(1 + j\frac{\omega}{\omega_{z_1}}\right)\left(1 + j\frac{\omega}{\omega_{z_2}}\right)\cdots\left(1 + j\frac{\omega}{\omega_{z_N}}\right)}{\left(1 + j\frac{\omega}{\omega_{p_1}}\right)\left(1 + j\frac{\omega}{\omega_{p_2}}\right)\cdots\left(1 + j\frac{\omega}{\omega_{p_D}}\right)}$$

$A_0 > 0$

$$\angle A(\omega) = \underbrace{\sum_N \angle\left(1 + j\frac{\omega}{\omega_{z_i}}\right) + \sum_D \angle\left(\frac{1}{1 + j\frac{\omega}{\omega_{p_i}}}\right)}$$

$A_0 < 0$

$\angle A(\omega) = 180^\circ +$

Next Meeting

- Frequency Response of Amplifiers