Lecture
Complex Power Equations and Computations

Agenda

- **Lecture**

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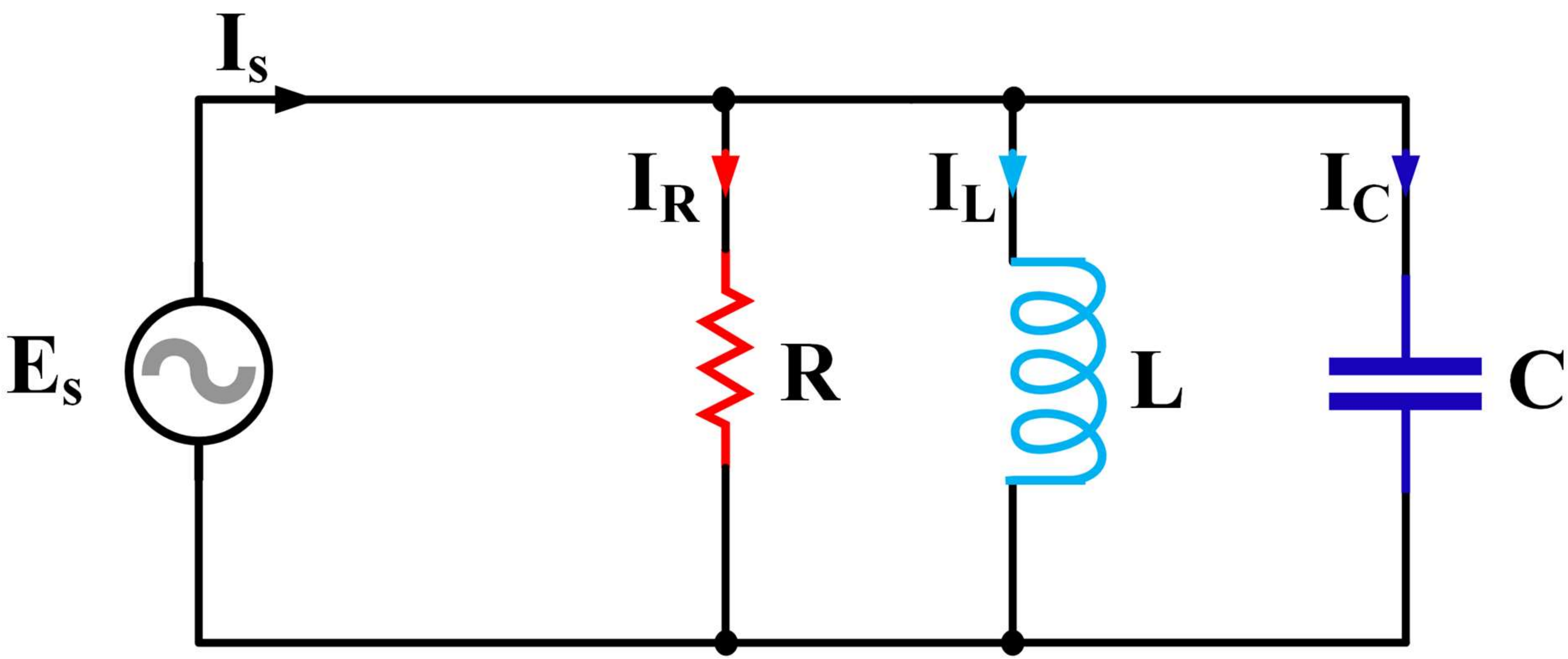
Visualizing Phasors

- http://www.swarthmore.edu/NatSci/echeeve1/Ref/phasors/phasors.html

What is the difference between real and reactive power?



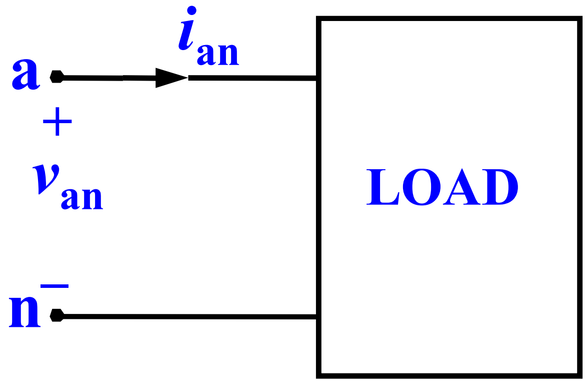
Lecture Outcomes

at the end of the lecture, the student must be able to …

- Formulate Complex Power Equations in AC Circuits
- Solve Problems using Complex Power Equations

POWER IN SINGLE-PHASE AC CIRCUITS

Instantaneous Power



$$v_{an} = V_{max} \cos(\omega t)$$
$$i_{an} = I_{max} \cos(\omega t - \theta)$$

The product of voltage and current at any instant of time is called *instantaneous power*, and is given by,

$$p = v_{an} i_{an} = V_{max} I_{max} \cos(\omega t) \cos(\omega t - \theta)$$

A positive *p* indicates a transfer of energy from the source to the network, while a negative *p* corresponds to a transfer of energy from the network to the source.

Average Power

Consider the ideal case where the passive network consists only of an inductive element, and apply to the network a sinusoidal voltage of the form,

$$v = V_m \sin \omega t$$

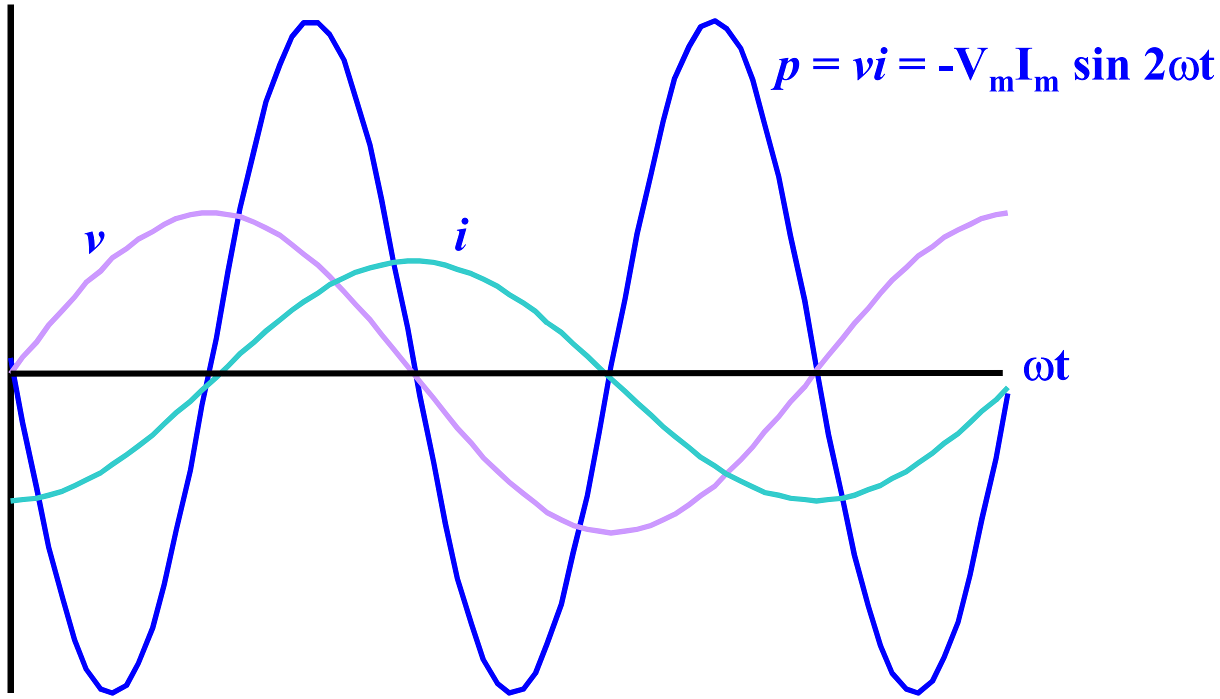
The resulting current will have the form,

$$i = I_m \sin (\omega t - \pi/2)$$

Then the power at any instant of time is

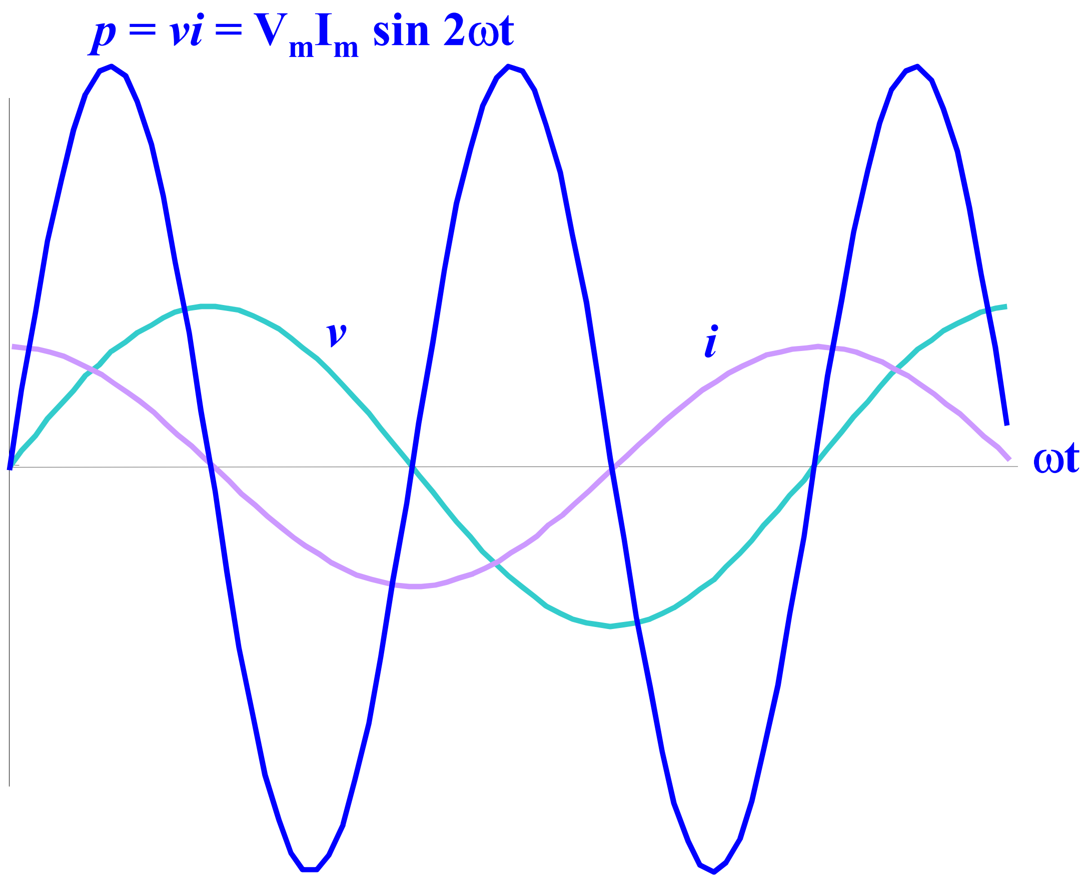
$$p = vi = V_m I_m [\sin (\omega t)][\sin (\omega t - \pi/2)]$$

Since sin (ωt - $\pi/2$) = -cos ωt and sin x cos x = sin 2x, we have,



When v and i are both positive, the power p is positive and energy is delivered from the source to the inductance. When v and i have opposite sign, the power p is negative and energy is returning from the inductance to the source.

In the ideal case of a pure capacitive network, analogous results can also be obtained.

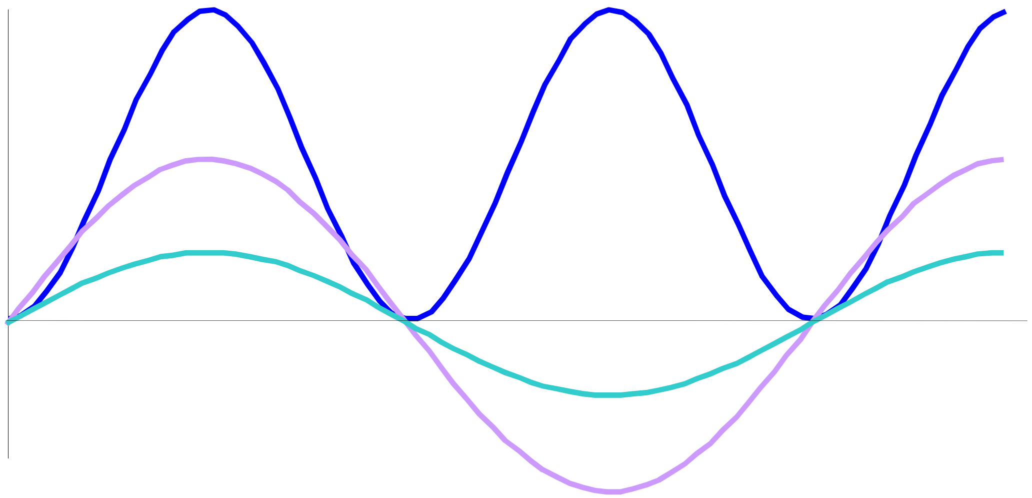


Apply now a voltage $v = V_m \sin \omega t$ to a network containing only resistance. The resulting current is $i = I_m \sin \omega t$, and the corresponding power is,

$$p = vi = V_m I_m \sin^2 \omega t$$

Since $\sin^2 x = ½ (1 - \cos 2x)$, we have,

$$p = ½\, V_m I_m (1 - \cos 2\omega t)$$



Finally, consider the case of a general passive network. For an applied sinusoidal voltage $v = V_m \sin \omega t$, we have a resulting current $i = I_m \sin (\omega t + \theta)$. The phase angle will be positive or negative depending on the capacitive or inductive character of the network. Then,

$$p = vi = V_m I_m \sin \omega t \sin (\omega t + \theta)$$

Note:

$$\sin \alpha \sin \beta = ½ [\cos (\alpha - \beta) - \cos (\alpha + \beta)]$$
$$\cos (-\alpha) = \cos \alpha$$

Therefore,

$$p = ½ V_m I_m [\cos \theta - \cos (2\omega t + \theta)]$$

Average Power

The instantaneous power p consists of a sinusoidal term $-\frac{1}{2} V_m I_m \cos(2\omega t + \theta)$ which has an average value of zero and a constant term $\frac{1}{2} V_m I_m \cos\theta$.

Then, the average value of p is,

$$\mathbf{P} = \tfrac{1}{2} V_m I_m \cos\theta = VI \cos\theta$$

where $V = V_m/\sqrt{2}$ and $I = I_m/\sqrt{2}$ are the effective values of the phasors V and I respectively.

The term $\cos\theta$ is called the *Power Factor* (pf). The angle θ is the angle between V and I, and its value is between $\pm 90°$.

Review

- For a general passive network, the power factor depends only on the resistance value of that network. True or False?

Transitioning from Time Domain to Frequency

- Phasors makes use of the Power Triangle Concept to simplify power calculations

COMPLEX POWER AND THE POWER TRIANGLE

Complex Power = Voltage x Current*

- Delivering bulk power (large amount of energy in short time) will require large current.
- Delivering the same bulk power in higher voltage will result in lower current.

Complex Power

The *Complex Power (S)* can be obtained from the product *VI**. It's real part equals the average power P and it's imaginary part is equal to the reactive power Q.

Consider, $V = |V|\angle\alpha$ and $I = |I|\angle\beta$

$$
\begin{aligned}
S = VI^* &= |V|\,e^{j\alpha}\,|I|\,e^{-j\beta} \\
&= |V|\,|I|\,e^{j(\alpha-\beta)} \\
&= |V|\,|I|\angle(\alpha-\beta) \\
&= |V|\,|I|\cos(\alpha-\beta) + j|V|\,|I|\sin(\alpha-\beta) \\
&= |V|\,|I|\cos\theta + j|V|\,|I|\sin\theta \\
&= P + jQ
\end{aligned}
$$

Complex Power

The equations associated with the average, apparent and reactive power can be developed geometrically on a right triangle called the *power triangle*.

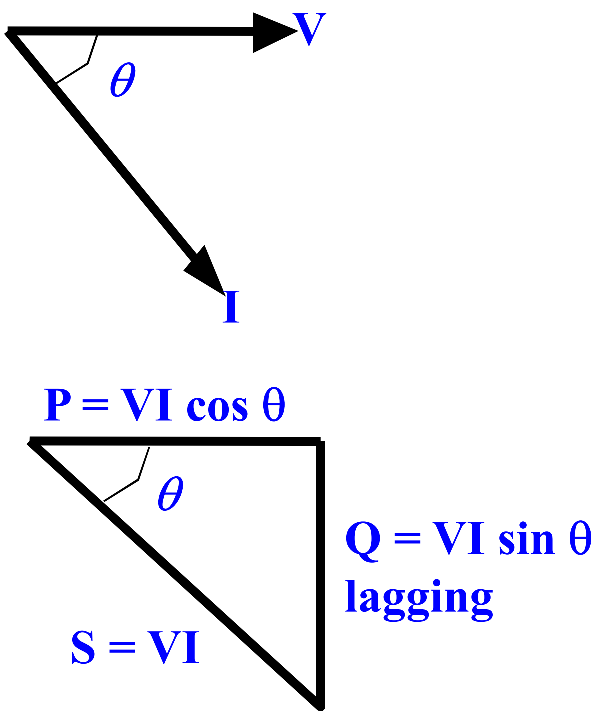
Apparent Power (S) = voltage x current
= VI

Average Power (P) = voltage x in-phase component of current
= VI cos θ

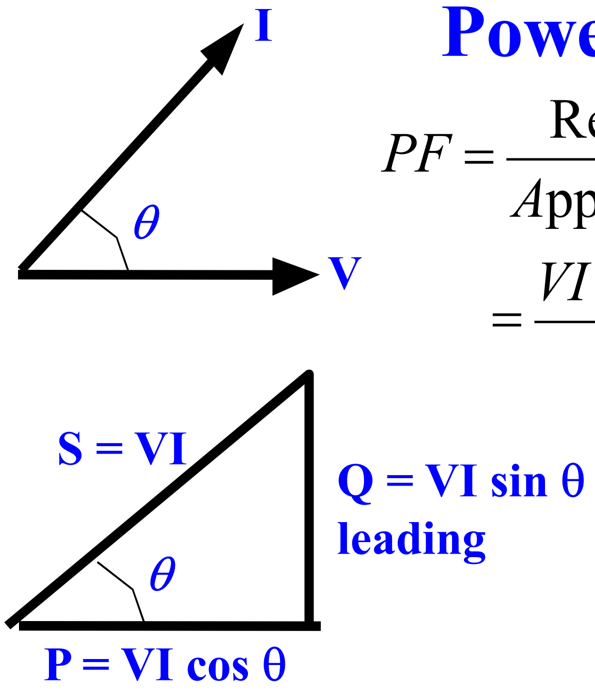
Reactive Power Q = voltage x quadrature comp. of current
= VI sin θ

(note: Average Power is also called Real Power or Active Power)

Power Triangle



Inductive Circuit



Capacitive Circuit

Power Factor

$$PF = \frac{\text{Real Power } (P)}{\text{Apparent Power } (S)}$$

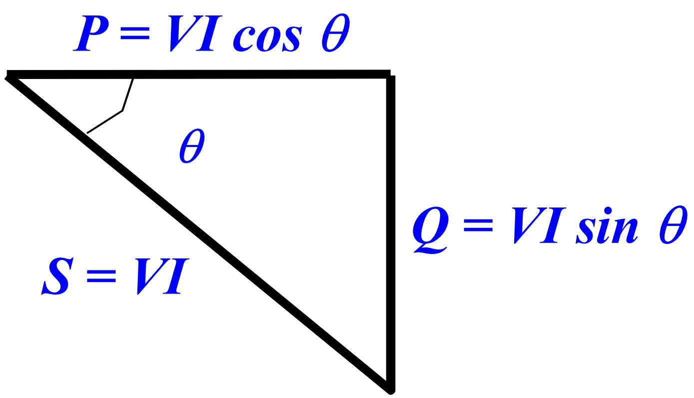
$$= \frac{VI\cos\theta}{VI} = \cos\theta$$

Power Factor

Power Factor

Measure of Efficient Utilization of Power

$$PF = \frac{Real\ Power(P)}{Apparent\ Power(S)} = \frac{VI\cos\theta}{VI}$$



$$\boxed{PF = \cos\theta}$$

Example

Given a circuit with an impedance $\mathbf{Z} = 3 + j4$ and an applied phasor voltage $\mathbf{V} = 100\angle 30^\circ$, draw the power triangle.

Solution:

$I = V/Z = (100\angle 30^\circ) / (5\angle 53.1^\circ) = 20\angle -23.1^\circ$

$|\mathbf{S}| = VI = 100(20) = 2000$ VA

$P = VI \cos\theta = 2000 \cos 53.1^\circ = 1200$ W

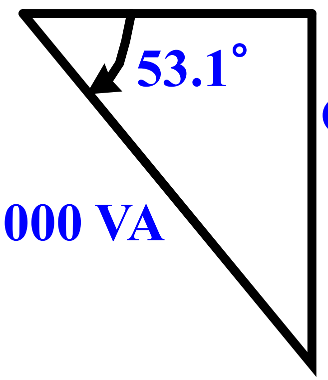
$Q = VI \sin\theta = 2000 \sin 53.1^\circ = 1600$ VARs lagging

$pf = \cos\theta = \cos 53.1^\circ = 0.6$ lagging

Example

Alternative Solution:

$$S = VI^* = (100\angle 30^\circ)(20\angle 23.1^\circ)$$
$$= 2000\angle 53.1^\circ$$
$$= 1200 + j1600$$



We get,

P = 1200 W

Q = 1600 VArs lagging

S = 2000 VA and

$pf = \cos 53.1^\circ = 0.6$ lagging

Concept Test! Again? Yes! Again!

- Work by pairs or individually
- Box your final answer

A small manufacturing plant is located 2 km down a transmission line, which has a series reactance of 0.5 Ω/km. The line resistance is negligible. The line voltage at the plant is $480\angle 0^\circ$ V (rms), and the plant consumes 120 kW at 0.85 power factor lagging. Determine the voltage and power factor at the sending end of the transmission line by using ~~(a) a complex power approach and (b) a circuit analysis approach.~~
any approach.

- Sending end voltage = 682.4 < 21.49° V
- Sending end power factor = 0.6 lagging

Summary

- Average Power
- Power Triangle
- Power Factor