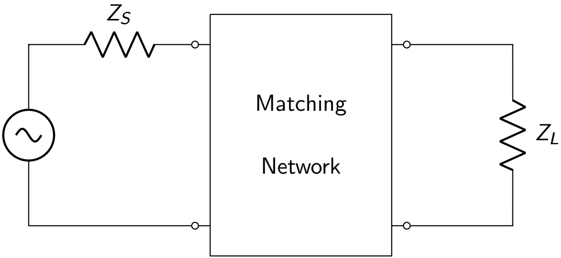
ECE 113: Communication Electronics

Meeting 11: Impedance Matching I

February 27, 2019

RF Impedance Matching

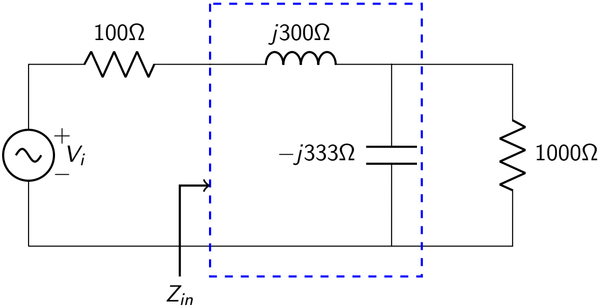


- Maximum power transfer between source and load

$$Z_{L,opt} = Z_S^*$$

- Perfect match can only occur at one frequency (Resonance)
- At resonance, maximum power transfer occurs between R_S and R_L at 50% efficiency

LC Matching

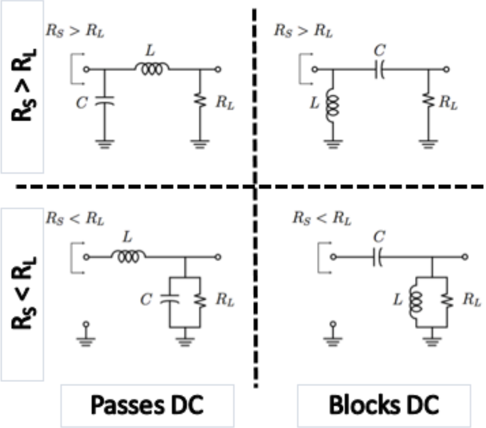


- The input impedance looking into the matching network becomes

$$Z_{in} = j300 + (100 - j300) = 100$$

- System is matched and maximum power transfer occurs
- Without the matching network, $loss = 4.8dB$

The L-Network



Design Equations:

$$R_p = (Q^2 + 1)R_s$$

$$Q = Q_s = Q_p$$

$$Q = \sqrt{\frac{R_p}{R_s} - 1} = (\frac{X_p}{X_s} - 1)^2$$

$$Q_s = \frac{X_s}{R_s} \qquad Q_p = \frac{R_p}{X_p}$$

Example

Design a circuit to match a 100Ω source resistance to a 1000Ω load resistance at 100MHz. Assume also that a DC voltage must also be transferred from source to load and that all elements are ideal.

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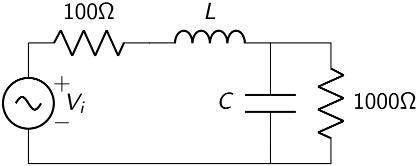
- The matching circuit



$$Q_s = Q_p = \sqrt{\frac{1000}{100} - 1} = 3$$

$$X_s = Q_s R_s = 300\Omega$$

$$X_p = \frac{R_p}{Q_p} = 333\Omega$$

$$X_s = \omega L \rightarrow L = \frac{X_s}{\omega} = 477nH$$

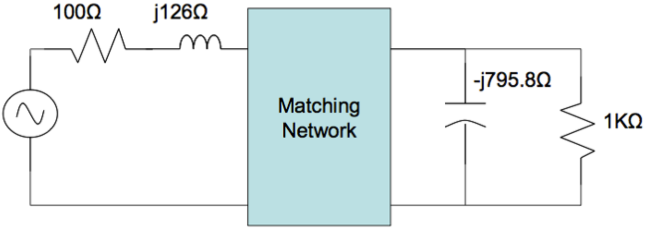
$$X_p = \frac{1}{\omega C} \rightarrow C = \frac{1}{\omega X_p} = 4.8pF$$

Complex Source and Load Impedances

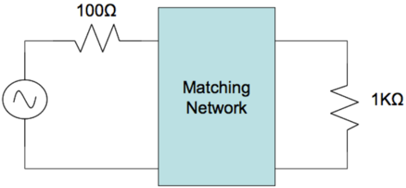
- Previous examples deal with real source and load impedances
- In practice, source and load impedances are complex
- Two basic methods in matching complex loads/sources
  - Absorption Method
    - Stray source/load reactances can be absorbed into the impedance matching network by proper placement of matching elements
  - Resonance Method
    - Stray source/load reactances can be resonated with equal and opposite reactances at the frequency of interest

Absorption Method (Example)

- Use absorption method to match the source and load impedances at 100MHz.



- Ignore stray reactances and solve for the matching network.



- From the previous example

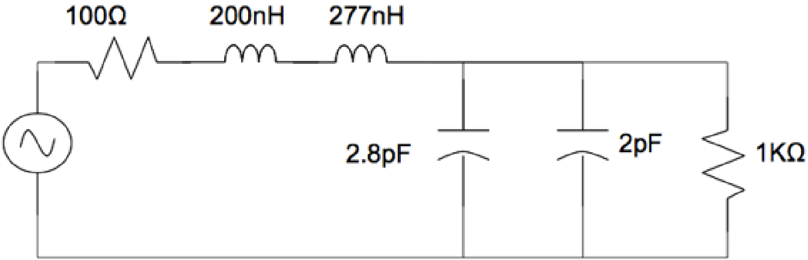
$L = 477nH \qquad\qquad C = 4.8pF$

- The new element values L' and C' are given by:

$$L' = L - L_{stray} = 277nH$$

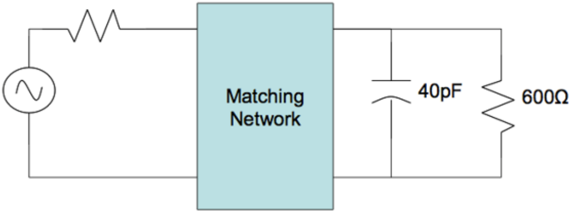
$$C' = C - C_{stray} = 2.8pF$$

- The complete circuit will be



Resonance Method (Example)

- Use resonance method to design a matching network that will match a source resistance of 50Ω to a capacitive load at 75MHz. The matching network should block DC at the output.



Resonate shunt $40pF$ with shunt inductor

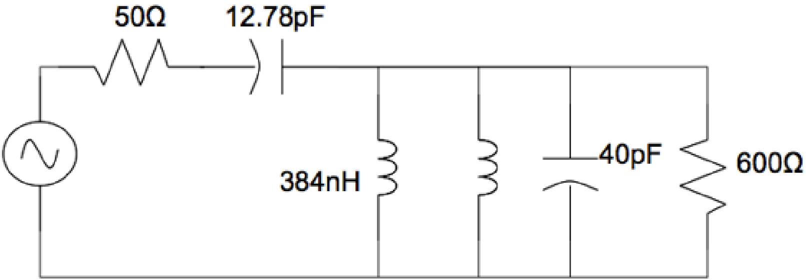
$$\omega L_1 = \frac{1}{\omega C} \rightarrow L_1 = 112.6nH$$

- Match the 50Ω source with the 600Ω load.

$$Q_s = Q_p = \sqrt{\frac{600}{50} - 1} = 3.32$$

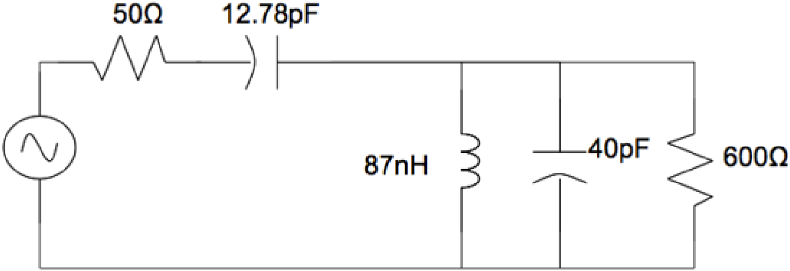
$$X_s = Q_s R_s = 166\Omega \rightarrow C = \frac{1}{\omega X_s}$$

$$X_p = \frac{R_p}{Q_p} = 181\Omega \rightarrow L_2 = \frac{X_p}{\omega} = 384nH$$



- Combining the inductors in parallel

$$L_{tot} = \frac{L_1 L_2}{L_1 + L_2} = 87nH$$



Notes on L-Matching Network

- Major limitation with 2-element matching networks is restriction on circuit Q

$$Q_s = Q_p = \sqrt{\frac{R_p}{R_s} - 1}$$

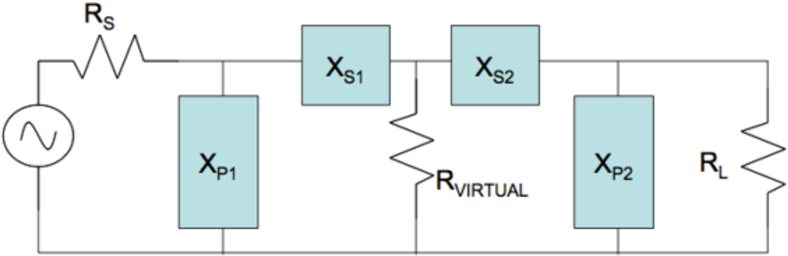
- R_S and R_L determine circuit Q
- Designer does not have freedom to specify circuit Q
- Becomes an issue for high-Q (narrow bandwidth) and even for low-Q applications

3-Element Matching

- Addresses the issue regarding flexibility of Q
  - Designer can specify practical circuit Q required
  - Can achieve larger Q than what can be achieved using L-network
- Two basic types
  - Pi-Network
  - T-Network

Pi-Network

- The two L-networks are designed to match with a **smaller** virtual resistance between source and load resistances
  - Analyze each branch using the usual L-network matching
  - Shunt elements must have opposite types with their corresponding series elements



Pi-Network

- Virtual resistance can have any value as long as it is smaller than both R_S and R_L.
- Start with L-branch with higher terminating resistance (R_S or R_L).
- Approximate expression for the circuit Q of the Pi-Network is given as:

$$Q = \sqrt{\frac{R_{higer}}{R_{virtual}} - 1}$$

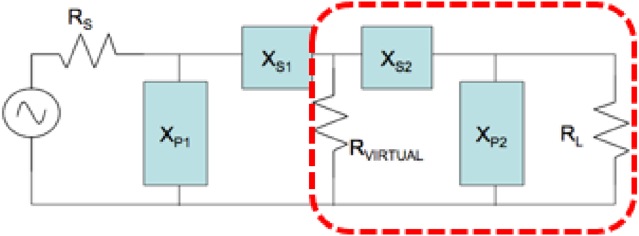
Pi-Network (Example)

Design a Pi-matching network to match a 100Ω source to a 1000Ω load. The loaded Q must be 15.

- Determine value of the virtual resistance. Start with the load since $R_L > R_S$

$$R_{virtual} = \frac{R_L}{Q^2 + 1} = \frac{1000}{226} = 4.42\Omega$$

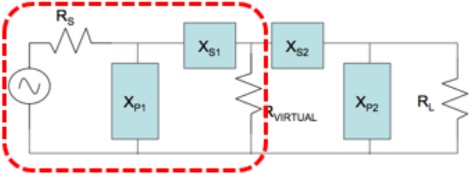
- Solve for the L-networks to match to this virtual resistance



- For the network connected to the load resistance

$$X_{P2} = \frac{R_P}{Q_P} = \frac{R_L}{Q} = \frac{1000}{15} = 66.7\Omega$$

$$X_{S1} = QR_{virtual} = 15(4.42) = 66.3\Omega$$



- For the network connected to the source resistance

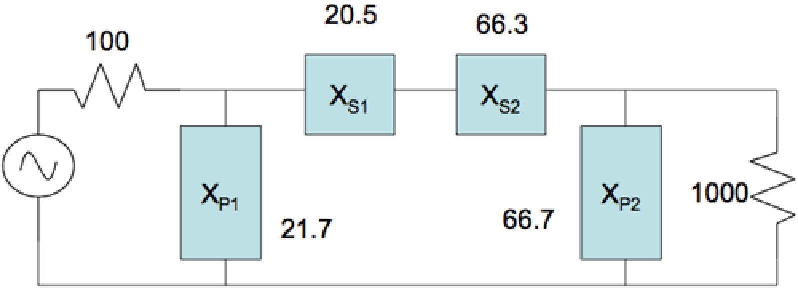
$$Q = \sqrt{\frac{R_S}{R_{virtual}} - 1} = \sqrt{\frac{100}{4.42} - 1} = 4.6$$

  - This Q is different with the specified Q? $\rightarrow$ The circuit Q is defined with the branch with the highest Q

- Determining the reactance values

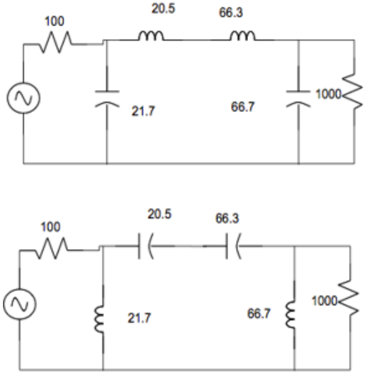
$$X_{P1} = \frac{R_S}{Q} = \frac{100}{4.6} = 21.7\Omega$$

$$X_{S1} = QR_{virtual} = 4.6(4.46) = 20.51\Omega$$



- You are left to specify what types of elements to use.

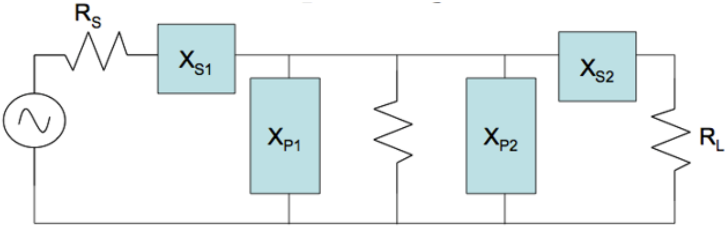
- Different Pi-matching network implementations



T-Network

- Designed to match with a **larger** virtual resistance between source and load resistances.
- Design of T-Network is the same as that of Pi-Network
- Difference is that the virtual resistance must be larger than both R_S and R_L
- Q of network is approximated by:

$$Q = \sqrt{\frac{R_{virtual}}{R_{smaller}} - 1}$$



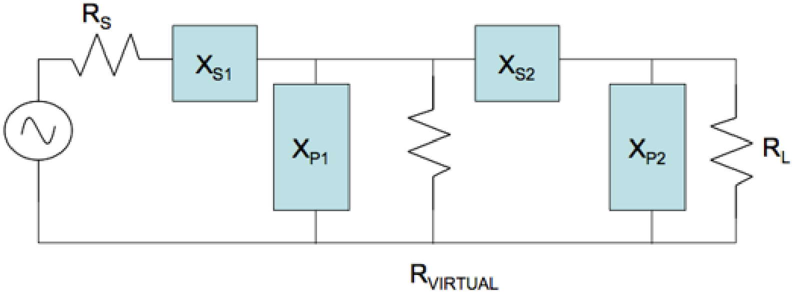
General Steps: 3-element Matching

- R_S and R_L values
  - High-valued $\rightarrow$ use Pi-network
  - Low-valued $\rightarrow$ use T-network
- Determine the value of the virtual resistance $R_{virtual}$
- Proceed with L-network matching starting with branch with larger/smaller terminating resistance for Pi/T respectively.
- Solve for the reactances of the other branch using the new Q defined by $R_{virtual}$
- Decide the types of $X_{S1},\ X_{P1},\ X_{S2},\ and\ X_{P2}$
- Combine elements to reduce number of elements to 3
- Compute for actual capacitance and inductance values.

Low-Q Matching Network

- Pi- and T- matching networks are usually used for narrow bandwidth (high-Q) applications
- For low-Q applications, L-networks must be stacked end-to-end
- Maximum bandwidth can be achieved when

$$R_{virtual} = \sqrt{R_S R_L}$$



Low-Q Matching Network

- The loaded Q of the network

$$Q = \sqrt{\frac{R_{virtual}}{R_{small}} - 1} = \sqrt{\frac{R_{large}}{R_{virtual}} - 1}$$

- For even wider bandwidths, we can cascade more L networks end-to-end

- Optimum bandwidth is obtained when ratios between successive resistances are equal

$$\frac{R_1}{R_{small}} = \frac{R_2}{R_1} = \frac{R_3}{R_2} = \ldots = \frac{R_{large}}{R_N}$$

END