Lecture 17
TRANSMISSION LINE – SHUNT ADMITTANCE

Agenda

- **ANNOUNCEMENTS**
  LECTURE

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Revisiting Our Current Model

R L X

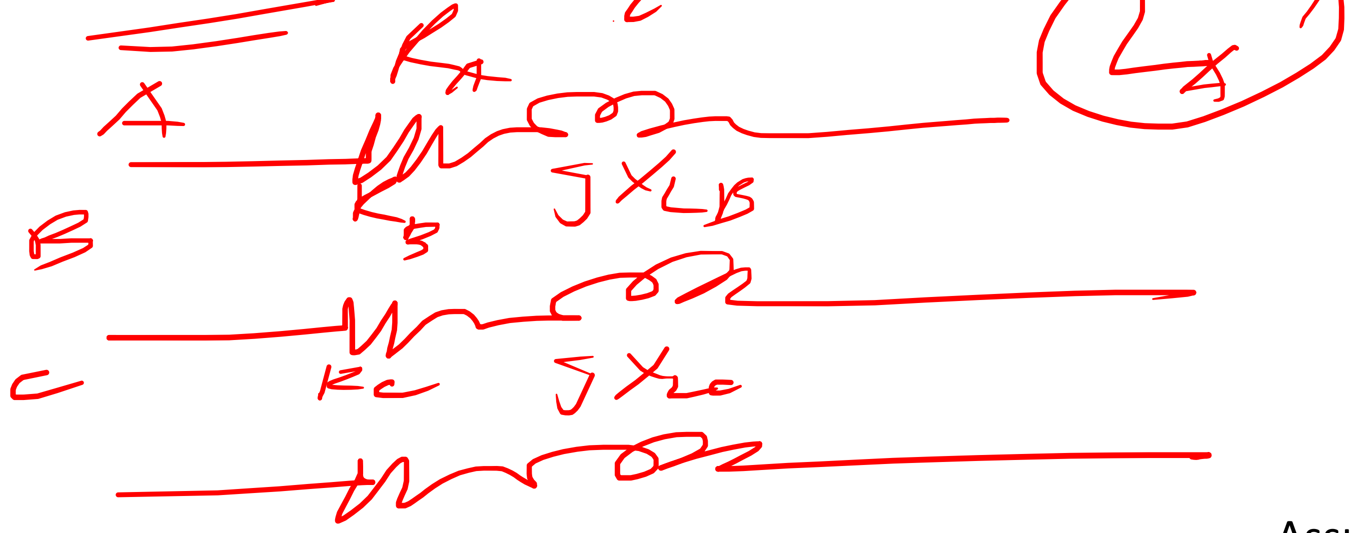
→ series impedance

→ shunt admittance → Y → C

X

feeders

jX_{ca}



Assumptions:

1. Transposition – Makes the system Balanced.

Announcements

- Long Quiz 2 is on March 25, 2019 from 7 to 9AM
  - Early Exam(6 to 8AM) Takers should answer the survey in UVLE.

Why are transmission Lines so high?

Review of Previous Lecture

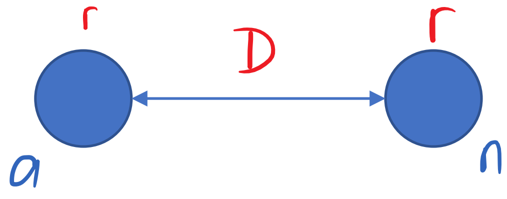
$$L_x = 2 \times 10^{-7} \ln \frac{GMD}{GMR_x} \; H/m$$

$$GMD = \sqrt[mn]{(D_{aa'}D_{ab'} \cdots D_{am})\cdots(D_{na'}D_{nb'} \cdots D_{nm})}$$

$$GMR_x = \sqrt[n^2]{(D_{aa}D_{ab} \cdots D_{an})\cdots(D_{na}D_{nb} \cdots D_{nn})} \quad D_{ii} = r_i'$$

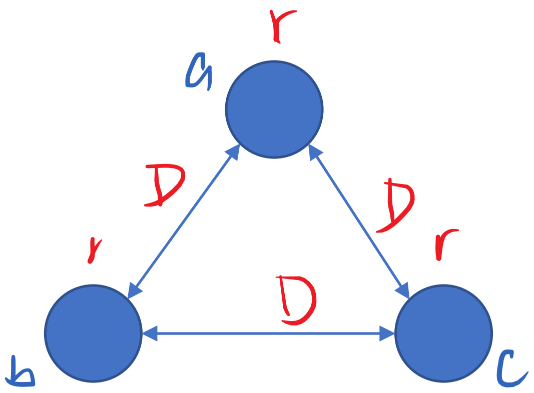
Making Sense of GMD AND GMR

Let $r' = 0.7788\, r$



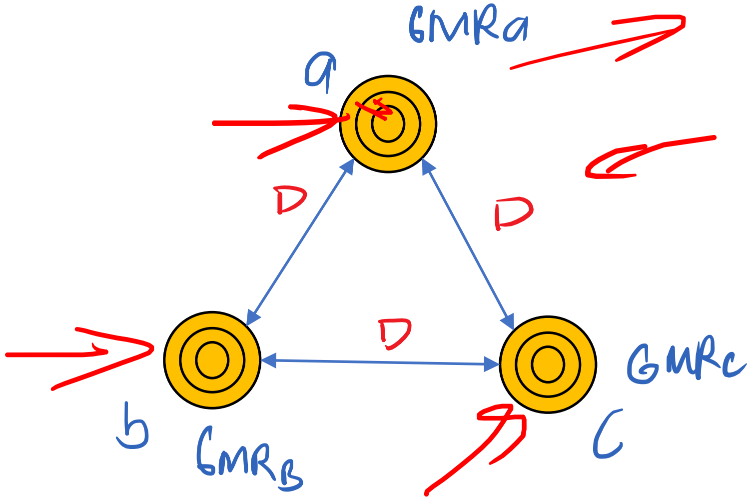
GMD= D

GMR= r'



GMD= D

GMR= r'

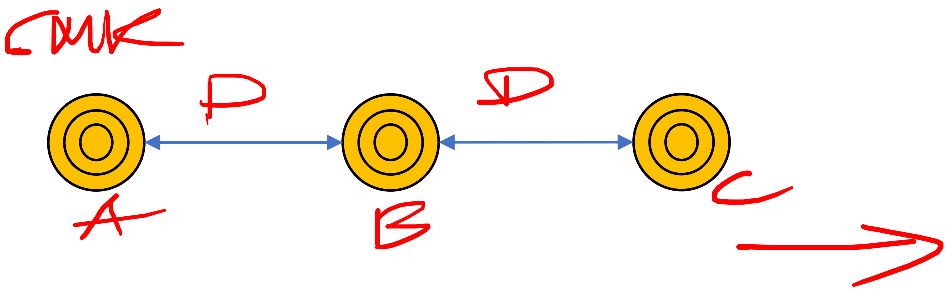


GMD= D

GMR= GMRa = GMRB = GMRc

*Assuming that Distance between conductor sets are larger the distance between subconductors of a conductor set

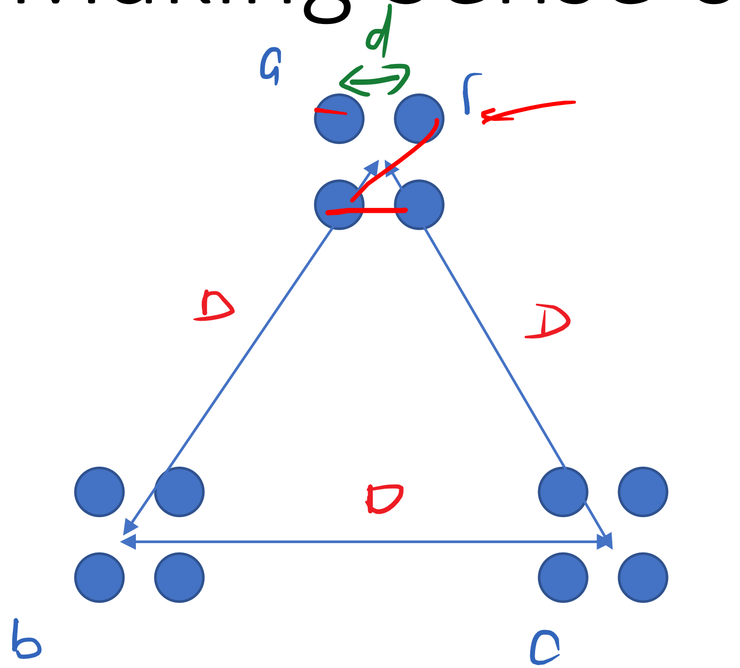
$$\sqrt[3]{D_{AB} D_{BC} D_{CA}}$$



GMD= $\sqrt[3]{D D 2D} = \sqrt[3]{2}D$

GMR= GMR

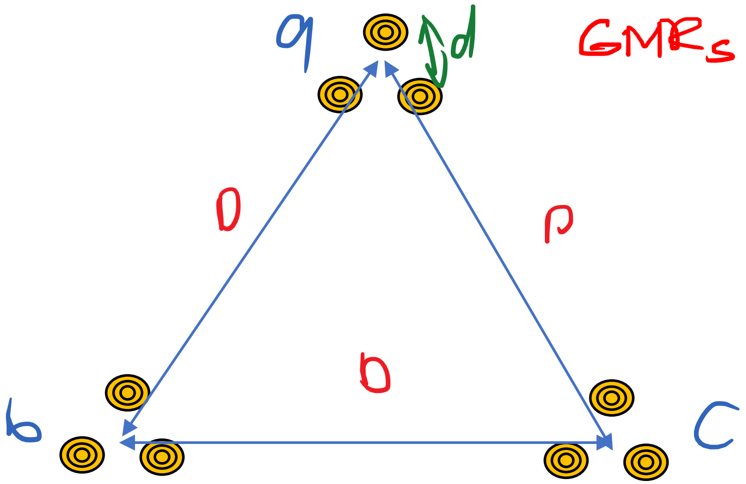
Making Sense of GMD AND GMR



GMD= D

GMR= $\sqrt[16]{\left(r' d d \sqrt{2} d\right)^4}$

*Assuming that Distance between conductor sets are larger the distance between subconductors of a conductor set



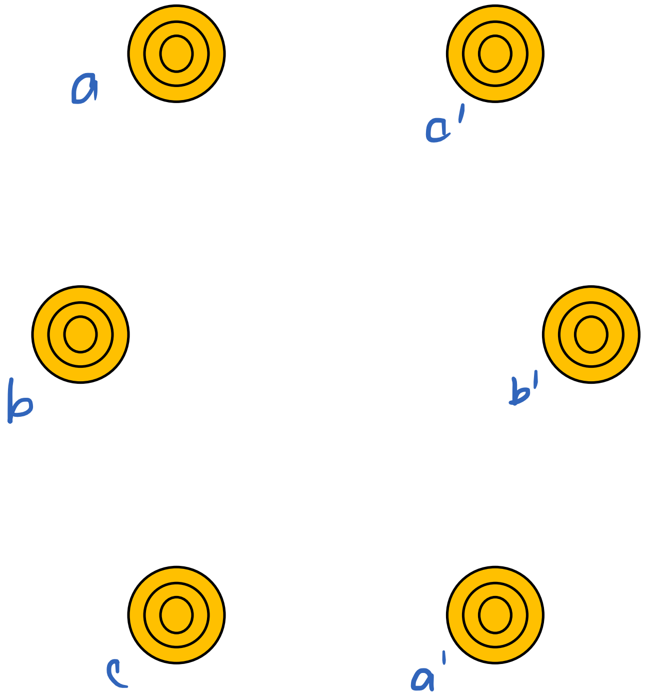
GMD= D

GMR= $\sqrt[9]{\left(GMR_s d d\right)^3}$

Making Sense of GMD AND GMR

Let Distance

D_{ki} *be distance between conductor k and conductor i*



*NOT Assuming that Distance between conductor sets are larger the distance between subconductors of a conductor set

GMD=

GMR=

Lecture Outcomes

at the end of the lecture, the student must be able to …

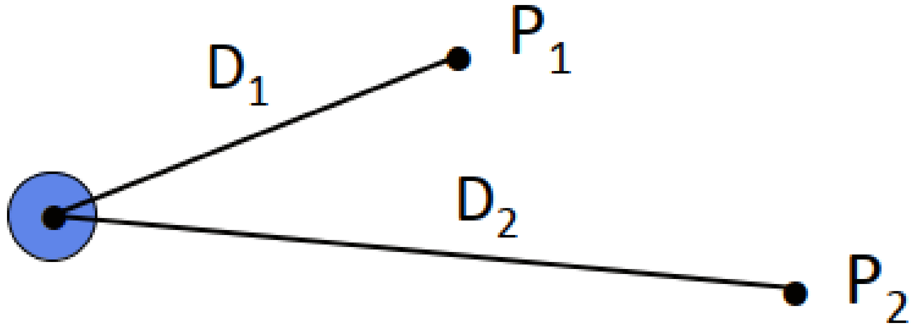
- Compute the Shunt admittance of T&D lines
- Identify the variables that affect the shunt admittance of T&D Lines

THE TREND OF OUR DISCUSSION

- Capacitance of conductors of different configurations
- Incorporating the effect of earth return
- Sequence Capacitance
- Special Case – Parallel Circuit Lines

Line Capacitance

Consider a long cylindrical conductor with a positive charge *q* in C/meter.



The electric field intensity at a point x meters from the charge is

$$E = \frac{q}{2\pi\varepsilon_o x} \quad V/m$$

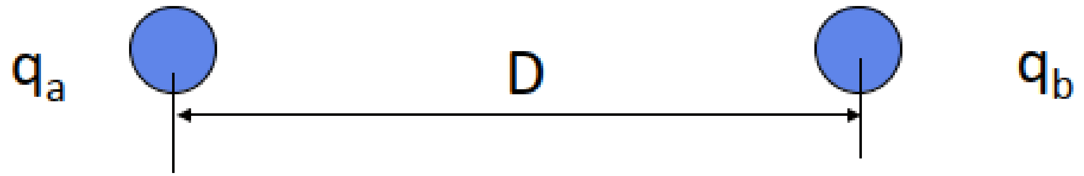
The voltage drop between points P_1 and P_2 is:

$$v_{12} = \int_{D_1}^{D_2} E\,dx = \frac{q}{2\pi\varepsilon_0} \ln\frac{D_2}{D_1}$$

Capacitance of a Two-Wire Line

The capacitance between two conductors of a two-wire line is defined as the charge on the conductors per unit of potential difference between them.

$$C = \frac{q}{v}$$



The voltage drop from a to b, due to charge q_a alone is:

$$v_{ab} = \frac{q_a}{2\pi\varepsilon_o} \ln \frac{D}{r_a}$$

The voltage drop from a to b, due to charge q_b alone is:

$$v_{ba} = \frac{q_b}{2\pi\varepsilon_o} \ln \frac{D}{r_b} \quad \text{or} \quad v_{ab} = \frac{-q_b}{2\pi\varepsilon_o} \ln \frac{D}{r_b}$$

Using the principle of superposition, the total voltage drop from a to b due to charges q_a and q_b taken together is:

$$v_{ab} = \frac{q_a}{2\pi\varepsilon_O}\ln\frac{D}{r_a} - \frac{q_b}{2\pi\varepsilon_O}\ln\frac{D}{r_b}$$

For an isolated system, $q_a + q_b = 0$, or $q_a = -q_b$. Therefore,

$$v_{ab} = \frac{q_a}{2\pi\varepsilon_O}\ln\frac{D^2}{r_a r_b}$$

The capacitance between conductors is the ratio of the conductor charge to the potential difference across the conductors:

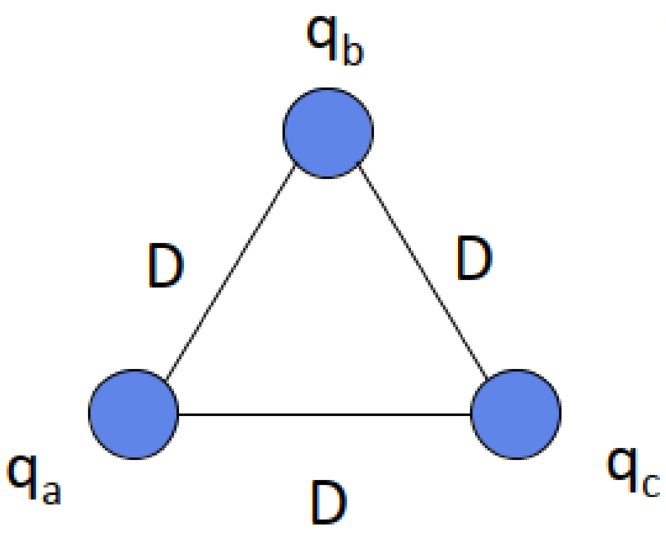
$$C_{ab} = \frac{q_a}{v_{ab}} = \frac{2\pi}{\ln\frac{D^2}{r_a r_b}} \ F/m$$

For identical conductors:

$$C_{ab} = \frac{\pi\varepsilon_o}{\ln\frac{D}{r}} \ F/m$$

Capacitance of a Three-Wire Line with Equilateral Spacing

Consider the three-phase line shown.



The voltage drop from a to b:

$$v_{ab} = \frac{1}{2\pi\varepsilon_O}\left[q_a \ln\frac{D}{r} + q_b \ln\frac{r}{D} + q_c \ln\frac{D}{D}\right]$$

Recall:

$$v_{12} = \frac{q}{2\pi\varepsilon_0}\ln\frac{D_2}{D_1}$$

Similarly, the voltage drop from a to c:

$$V_{ac} = \frac{1}{2\pi\varepsilon_O}\left[q_a \ln\frac{D}{r} + q_b \ln\frac{D}{D} + q_c \ln\frac{r}{D}\right]$$

Adding the two voltage equations:

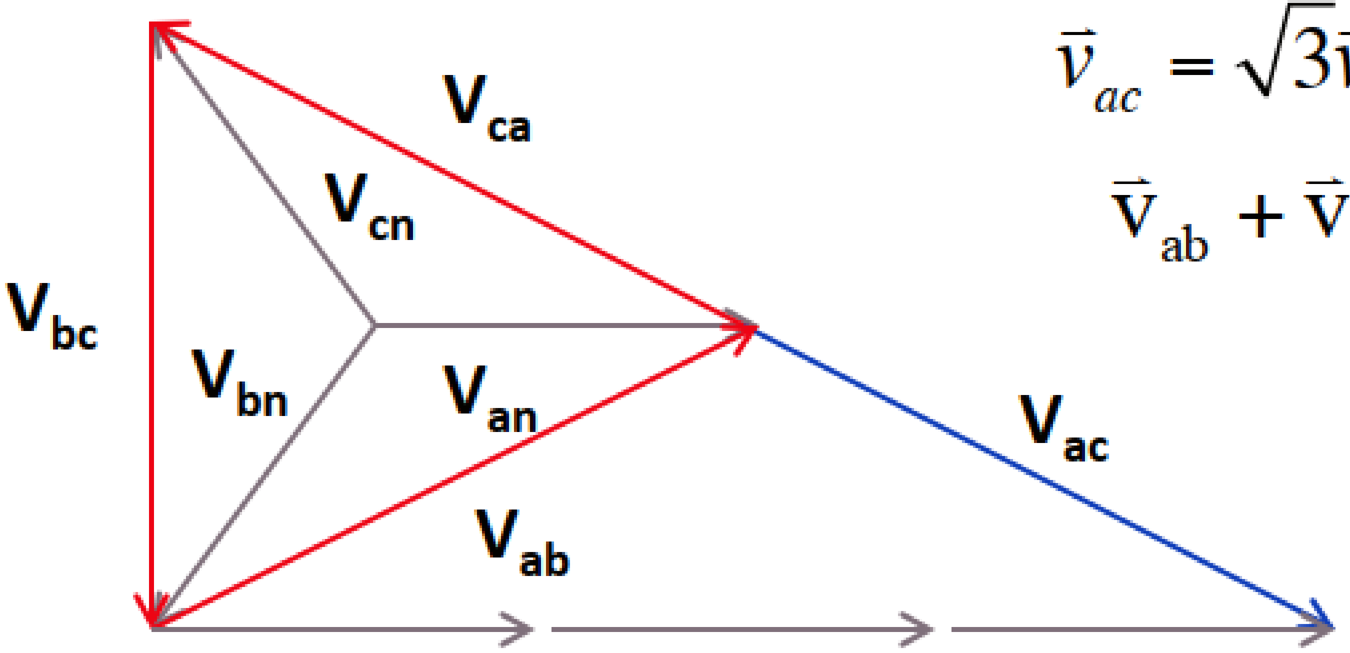
$$V_{ab} + V_{ac} = \frac{1}{2\pi\varepsilon_O}\left[2q_a \ln\frac{D}{r} + (q_b + q_c)\ln\frac{r}{D}\right]$$

For an isolated system: $q_a + q_b + q_c = 0$

$$V_{ab} + V_{ac} = \frac{3q_a}{2\pi\varepsilon_O}\ln\frac{D}{r} \quad V$$

What is (v_{ab} + v_{ac})?

Using the phasor diagram for a balanced 3-phase system:



$$\vec{v}_{ab} = \sqrt{3}\vec{v}_{an}\angle 30°$$

$$\vec{v}_{ac} = \sqrt{3}\vec{v}_{an}\angle -30°$$

$$\vec{\mathrm{V}}_{\mathrm{ab}} + \vec{\mathrm{V}}_{\mathrm{ac}} = 3\vec{\mathrm{V}}_{\mathrm{an}}$$

Therefore,

$$\vec{v}_{ab} + \vec{v}_{ac} = 3\vec{v}_{an} = \frac{3q_a}{2\pi\varepsilon_o} \ln \frac{D}{r} V$$

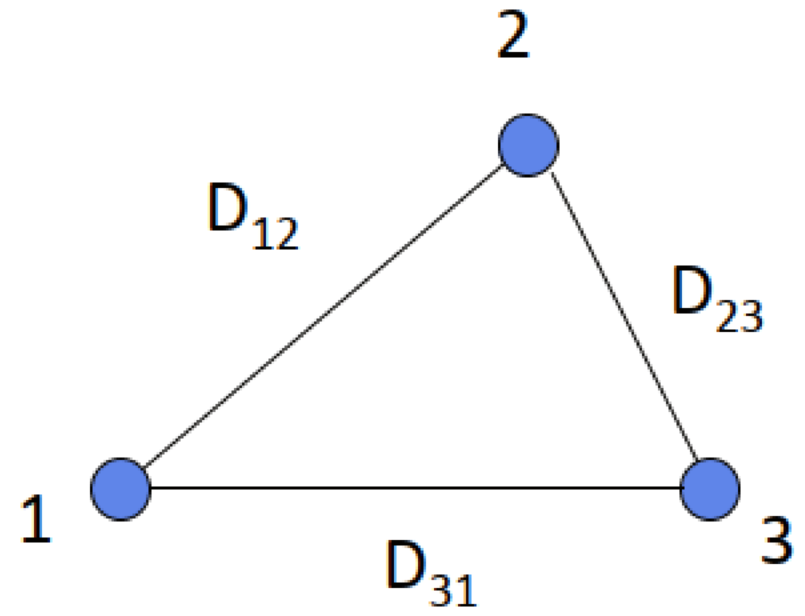
$$v_{an} = \frac{q_a}{2\pi\varepsilon_o} \ln \frac{D}{r}$$

Obtaining the capacitance-to-neutral:

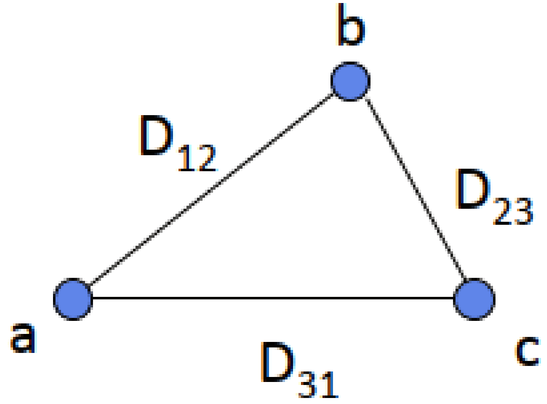
$$\boxed{C_{an} = \frac{q_a}{v_{an}} = \frac{2\pi\varepsilon_o}{\ln \frac{D}{r}}}$$

Capacitance of a Three-Wire Line with Unsymmetrical Spacing

Consider each section of the transposition cycle:



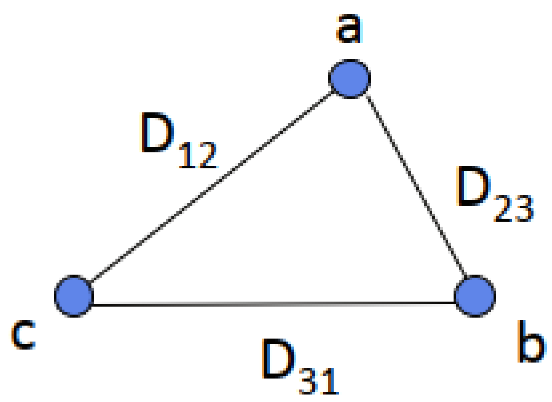
Phase a in position 1
Phase b in position 2
Phase c in position 3



$$v_{ab1} = \frac{1}{2\pi\varepsilon_O}\left[q_a \ln\frac{D_{12}}{r} + q_b \ln\frac{r}{D_{12}} + q_c \ln\frac{D_{23}}{D_{31}}\right]$$

$$v_{ac1} = \frac{1}{2\pi\varepsilon_O}\left[q_a \ln\frac{D_{31}}{r} + q_b \ln\frac{D_{23}}{D_{12}} + q_c \ln\frac{r}{D_{31}}\right]$$

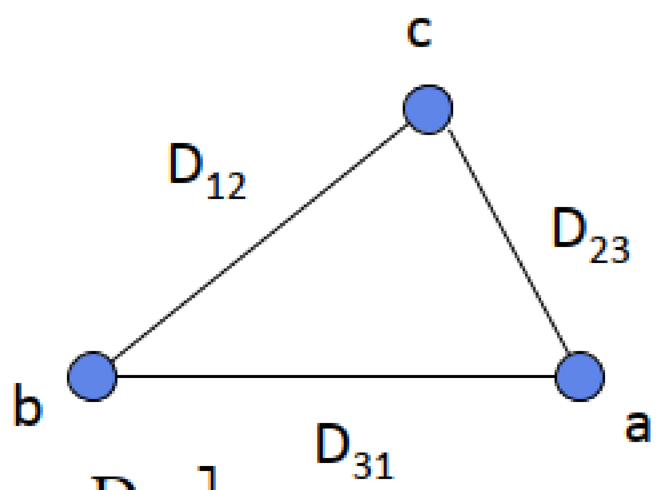
Phase a in position 2
Phase b in position 3
Phase c in position 1



$$v_{ab2} = \frac{1}{2\pi\varepsilon_O}\left[q_a \ln\frac{D_{23}}{r} + q_b \ln\frac{r}{D_{23}} + q_c \ln\frac{D_{31}}{D_{12}}\right]$$

$$v_{ac2} = \frac{1}{2\pi\varepsilon_O}\left[q_a \ln\frac{D_{12}}{r} + q_b \ln\frac{D_{31}}{D_{23}} + q_c \ln\frac{r}{D_{12}}\right]$$

Phase a in position 3
Phase b in position 1
Phase c in position 2



$$v_{ab3} = \frac{1}{2\pi\varepsilon_O}\left[q_a \ln\frac{D_{31}}{r} + q_b \ln\frac{r}{D_{31}} + q_c \ln\frac{D_{12}}{D_{23}}\right]$$

$$v_{ac3} = \frac{1}{2\pi\varepsilon_O}\left[q_a \ln\frac{D_{23}}{r} + q_b \ln\frac{D_{12}}{D_{31}} + q_c \ln\frac{r}{D_{23}}\right]$$

For a completely transposed line, vab is equal to the average of the voltage drops between a and b when the two phase occupy all possible positions:

$$V_{ab} = \frac{V_{ab1} + V_{ab2} + V_{ab3}}{3}$$

$$v_{ab1} = \frac{1}{2\pi\varepsilon_O}\left[q_a \ln\frac{D_{12}}{r} + q_b \ln\frac{r}{D_{12}} + q_c \ln\frac{D_{23}}{D_{31}}\right]$$

$$v_{ab2} = \frac{1}{2\pi\varepsilon_O}\left[q_a \ln\frac{D_{23}}{r} + q_b \ln\frac{r}{D_{23}} + q_c \ln\frac{D_{31}}{D_{12}}\right]$$

$$v_{ab3} = \frac{1}{2\pi\varepsilon_O}\left[q_a \ln\frac{D_{31}}{r} + q_b \ln\frac{r}{D_{31}} + q_c \ln\frac{D_{12}}{D_{23}}\right]$$

$$v_{ab} = \frac{1}{6\pi\varepsilon_O}\left[q_a \ln\frac{D_{12}D_{23}D_{31}}{r^3} + q_b \ln\frac{r^3}{D_{12}D_{23}D_{31}} + q_c \ln\frac{D_{12}D_{23}D_{31}}{D_{12}D_{23}D_{31}}\right]$$

$$v_{ab} = \frac{1}{6\pi\varepsilon_O}\left[q_a \ln\frac{D_{12}D_{23}D_{31}}{r^3} + q_b \ln\frac{r^3}{D_{12}D_{23}D_{31}} + q_c \ln\frac{D_{12}D_{23}D_{31}}{D_{12}D_{23}D_{31}}\right]$$

$$= \frac{1}{2\pi\varepsilon_O}\left[q_a \ln\frac{GMD}{r} + q_b \ln\frac{r}{GMD}\right]$$

Similarly

$$v_{ac} = \frac{1}{2\pi\varepsilon_O}\left[q_a \ln\frac{GMD}{r} + q_c \ln\frac{r}{GMD}\right]$$

$$v_{ab} + v_{ac} = 3v_{an} = \frac{1}{2\pi\varepsilon_O}\left[2q_a \ln\frac{GMD}{r} + (q_b + q_c)\ln\frac{r}{GMD}\right]$$

$$= \frac{1}{2\pi\varepsilon_O}\left[3q_a \ln\frac{GMD}{r}\right]$$

Since $q_a = -(q_b + q_c)$ in an isolated system

Therefore,

$$v_{an} = \frac{1}{2\pi\varepsilon_o} q_a \ln\frac{GMD}{r}$$

The capacitance of phase a to neutral is $$C_{an} = \frac{2\pi\varepsilon_o}{\ln\frac{GMD}{r}}$$

Due to symmetry (from the transposition of the lines):

$$C_n = C_{an} = C_{bn} = C_{cn}$$

*C_n is the positive sequence capacitance of the line (no ground wire)

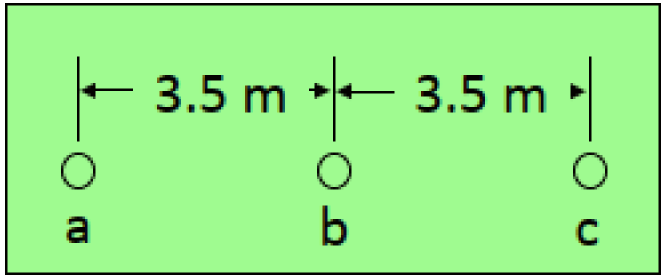
The capacitive reactance of the line is:

$$X_C = \frac{1}{2\pi fC}$$

$$= \frac{2.862\times 10^6}{f} \ln\frac{GMD}{r} \ \Omega\cdot\text{km (to neutral)}$$

Shunt Capacitance of Lines

Example: Find the capacitance to neutral per km of the 69-kV line shown. Also find the capacitive reactance and charging current per km.



Conductor diameter = 0.0143 m

$$GMD = \sqrt[3]{3.5 \times 3.5 \times 7} = 4.41 \text{ m}$$

$$C_{an} = \frac{2\pi\varepsilon_O}{\ln \dfrac{GMD}{r}} = \frac{2\pi \times 8.854 \times 10^{-12}}{\ln \dfrac{4.41}{0.0072}} = 8.6594 \times 10 \text{ pF/m}$$

$$X_C = \frac{1}{2\pi \times 60 \times 8.6594 \times 10^{-12}} = 306.3 \times 10^6 \ \Omega \cdot \text{m}$$

$$= 306.3 \times 10^3 \ \Omega \cdot \text{km}$$

$$I_{chg} = \frac{69\times10^3/\sqrt{3}}{306.3\times10^3} = 130\,\frac{mA}{km}$$

If the total line length is 200 km, the total charging current and charging MVAR are

$$I_{chg} = 130\,\frac{mA}{km}\times 200\text{ km} = 26\text{ A}$$

$$Q_{chg} = \sqrt{3}\times 69kV\times 26A = 3.108\text{ MVAR}$$

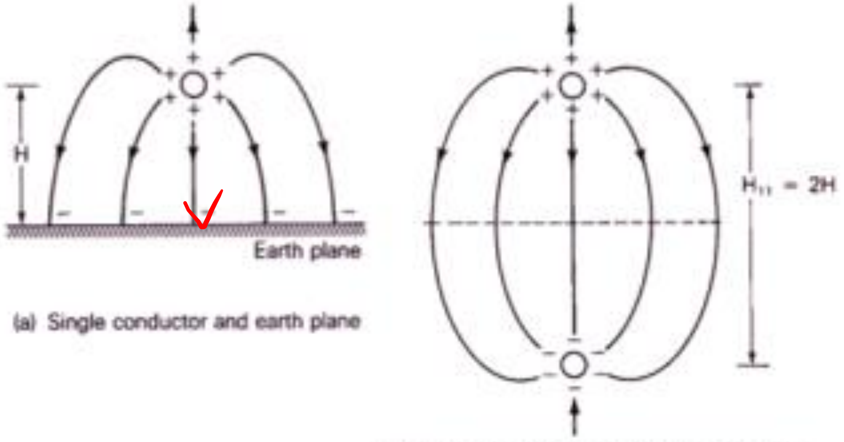
What about the effect of earth?

- We have discussed lines neglecting earth return and neutral conductors?
- They have an effect-> How do we incorporate them?
- Derivation -> Method of Images
- Result is incorporating earth return **increases** the **capacitance.**
- **Method of solution is using matrix – typically we use computers to model the problem.**

Shunt Capacitance of Lines:

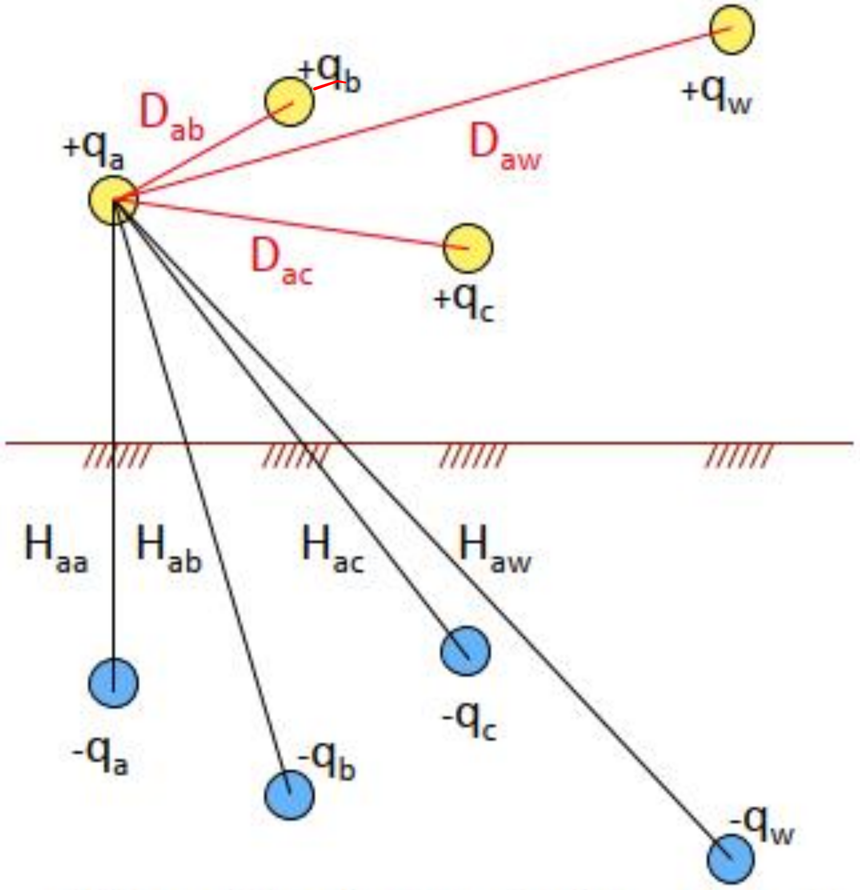
Conductors with Neutral Conductors and Earth Return

In capacitance calculations, the earth is assumed as a perfectly conducting plane. The electric field that results is the same if an image conductor is used for every conductor above ground.



(a) Single conductor and earth plane

(b) Earth plane replaced by image conductor



Mirror Conductors below ground

Shunt Capacitance of Lines:

Conductors with Neutral Conductors and Earth Return

The voltage drop from conductor a to ground is

$$v_a = \tfrac{1}{2} v_{aa'} = \frac{1}{4\pi\varepsilon}\left(q_a \ln\frac{H_{aa}}{r_a} + q_b \ln\frac{H_{ab}}{D_{ab}} + \dots + q_n \ln\frac{H_{an}}{D_{an}} \right.$$

$$\left. - q_a \ln\frac{r_a}{H_{aa}} - q_b \ln\frac{D_{ab}}{H_{ab}} - \dots - q_n \ln\frac{D_{an}}{H_{an}} \right)$$

$$v_a = \frac{1}{2\pi\varepsilon}\left(q_a \ln\frac{H_{aa}}{r_a} + q_b \ln\frac{H_{ab}}{D_{ab}} + \dots + q_n \ln\frac{H_{an}}{D_{an}} \right)$$

Recall:

$$v_{12} = \frac{q}{2\pi\varepsilon_0} \ln\frac{D_2}{D_1}$$

Shunt Capacitance of Lines:

Conductors with Neutral Conductors and Earth Return

$$v_k = \frac{1}{2\pi\varepsilon}\left(q_a \ln\frac{H_{ak}}{D_{ak}} + q_b \ln\frac{H_{bk}}{D_{bk}} + \ldots + q_k \ln\frac{H_{kk}}{r_k} + \ldots + q_n \ln\frac{H_{nk}}{D_{nk}}\right)$$

Involving all voltages and charges:

$$\begin{bmatrix} v_a \\ v_b \\ \vdots \\ v_n \end{bmatrix} = \begin{bmatrix} P_{aa} & P_{ab} & P_{ac} & \ldots & P_{an} \\ P_{ba} & P_{bb} & P_{bc} & \ldots & P_{bn} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ P_{na} & P_{nb} & P_{nc} & \ldots & P_{nn} \end{bmatrix} \begin{bmatrix} q_a \\ q_b \\ \vdots \\ q_n \end{bmatrix}$$

$$P_{kk} = \frac{1}{2\pi\varepsilon} \ln\frac{H_{kk}}{r_k}$$

$$P_{kj} = \frac{1}{2\pi\varepsilon} \ln\frac{H_{kj}}{D_{kj}}$$

$$[v] = [P][q]$$

$$[v] = [P][q]$$

Since Q = CV

$$C = P^{-1}$$

Inversion of matrix P gives

$$C = \begin{bmatrix} +C_{aa} & -C_{ab} & -C_{ac} & \cdots & -C_{an} \\ -C_{ba} & +C_{bb} & -C_{bc} & \cdots & -C_{bn} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -C_{na} & -C_{nb} & -C_{nc} & \cdots & +C_{nn} \end{bmatrix}$$

Shunt Capacitance of Lines

The Shunt Admittance is

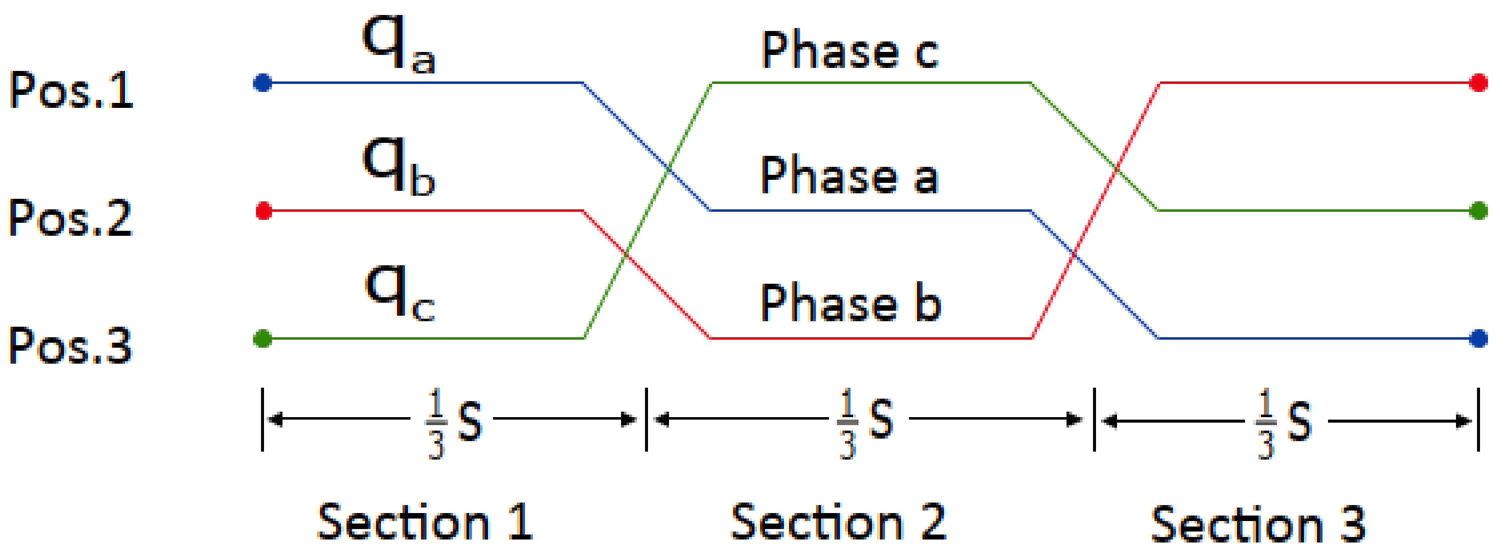
$$Y_{bus} = \begin{bmatrix} +j\omega C_{aa} & -j\omega C_{ab} & -j\omega C_{ac} & \dots & -j\omega C_{an} \\ -j\omega C_{ba} & +j\omega C_{bb} & -j\omega C_{bc} & \dots & -j\omega C_{bn} \\ & & & & \\ -j\omega C_{na} & -j\omega C_{nb} & -j\omega C_{nc} & \dots & +j\omega C_{nn} \end{bmatrix}$$

The difference between the magnitude of a diagonal element and its associated off-diagonal elements is the capacitance to ground. For example, the capacitance of line a to ground is

$$C_{ag} = C_{aa} - C_{ab} - C_{ac} - \dots - C_{an}$$

Shunt Capacitance of Lines

Capacitance of a Transposed Line (3 PHASE ONLY)



For the untransposed line, let

$$C_P = \begin{bmatrix} C_{aa} & -C_{ab} & -C_{ac} \\ -C_{ba} & C_{bb} & -C_{bc} \\ -C_{ca} & -C_{cb} & C_{cc} \end{bmatrix}$$

Shunt Capacitance of Lines

For a completely transposed line,

$$C_{P,T} = \begin{bmatrix} C_{s0} & -C_{m0} & -C_{m0} \\ -C_{m0} & C_{s0} & -C_{m0} \\ -C_{m0} & -C_{m0} & C_{s0} \end{bmatrix}$$

$$C_{s0} = \frac{1}{3}(C_{aa} + C_{bb} + C_{cc}) \qquad C_{m0} = \frac{1}{3}(C_{ab} + C_{bc} + C_{ac})$$

Shunt Capacitance of Lines

Sequence Capacitance

$Let\ Y_{abc} = Y_P$ $\qquad \vec{I}_{abc} = Y_{abc}\vec{V}_{abc} \qquad \vec{I}_{abc} = j\omega C_{abc}\vec{V}_{abc}$

Since $\vec{V}_{abc} = A\vec{V}_{012} \qquad \vec{I}_{abc} = Y_{abc}\vec{V}_{abc}$

$$A\vec{I}_{012} = j\omega C_{abc} A\vec{V}_{012}$$

or

$$\vec{I}_{012} = j\omega A^{-1} C_{abc} A\vec{V}_{012}$$

Thus, $C_{012} = A^{-1} C_{abc} A$

Shunt Capacitance of Lines

For a completely transposed line,

$$C_{s0} = C_{aa} = C_{bb} = C_{cc}$$

$$C_{m0} = C_{ab} = C_{bc} = C_{ac}$$

Substitution gives

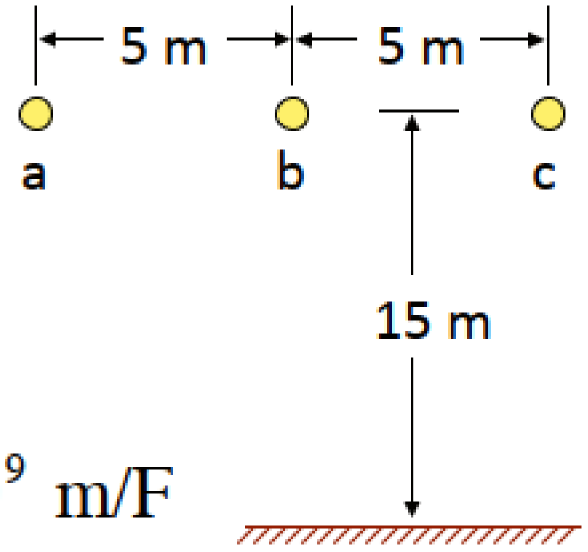
$$C_{012} = \begin{bmatrix} (C_{s0} - 2C_{m0}) & 0 & 0 \\ 0 & (C_{s0} + C_{m0}) & 0 \\ 0 & 0 & (C_{s0} + C_{m0}) \end{bmatrix}$$

or

$$C_0 = C_{s0} - 2C_{m0} \qquad C_1 = C_2 = C_{s0} + C_{m0}$$

Shunt Capacitance of Lines

Example: Determine the phase and sequence capacitances of the transmission line shown. The phase conductors are 477 MCM ACSR 26/7 with radius of 0.0357 ft. The line is 60 km long and is completely transposed.



Radius = r = 0.0109 m

$H_{aa} = H_{bb} = H_{cc} = 30$ m

$H_{ab} = H_{bc} = (5^2 + 30^2)^{1/2} = 30.414$ m

$H_{ac} = (10^2 + 30^2)^{1/2} = 31.623$ m

$$P_{aa} = P_{bb} = P_{cc} = \frac{1}{2\pi\varepsilon_0} \ln\frac{\mathrm{H}_{\mathrm{aa}}}{r_a} = 142.37 \times 10^9 \ \mathrm{m/F}$$

$$P_{ab} = P_{bc} = \frac{1}{2\pi\varepsilon_0} \ln\frac{H_{ab}}{D_{ab}} = 32.454 \times 10^9 \ \mathrm{m/F}$$

$$P_{ac} = \frac{1}{2\pi\varepsilon_0} \ln\frac{H_{ac}}{D_{ac}} = 20.695\times10^9 \text{ m/F}$$

Therefore,

$$P = \begin{bmatrix} 142.37 & 32.45 & 20.70 \\ 32.45 & 142.37 & 32.45 \\ 20.70 & 32.45 & 142.37 \end{bmatrix}\times10^9$$

$$C = \begin{bmatrix} 7.482 & -1.537 & 0.737 \\ -1.537 & 7.725 & -1.537 \\ 0.737 & -1.537 & 7.482 \end{bmatrix}\times10^{-12} \text{ F/m}$$

$$C = \begin{bmatrix} 7.482 & -1.537 & -0.737 \\ -1.537 & 7.725 & -1.537 \\ -0.737 & -1.537 & 7.482 \end{bmatrix} \times 10^{-12} \text{ F/m}$$

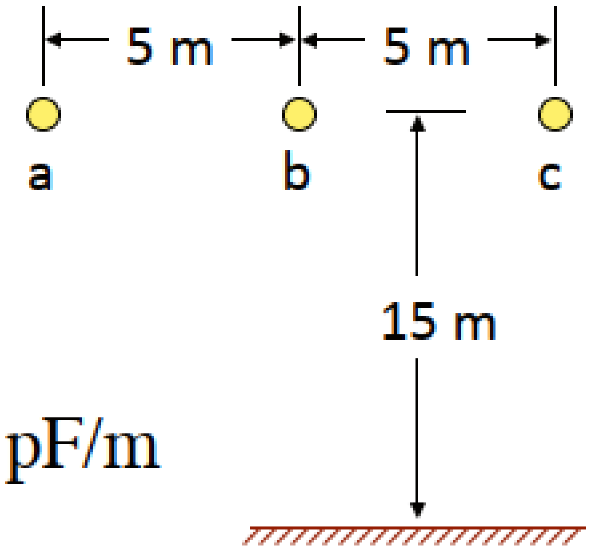
If there is complete transposition:

$$C = \begin{bmatrix} 7.562 & -1.271 & -1.271 \\ -1.271 & 7.562 & -1.271 \\ -1.271 & -1.271 & 7.562 \end{bmatrix} \times 10^{-12} \text{ F/m}$$

The sequence capacitance are:

$C_1 = C_2 = (7.562 + 1.271) \times 10^{-12} = 8.833$ pF/m
$C_0 = (7.562 - 2(1.271)) \times 10^{-12} = 5.020$ pF/m

If the effect of earth is not considered:



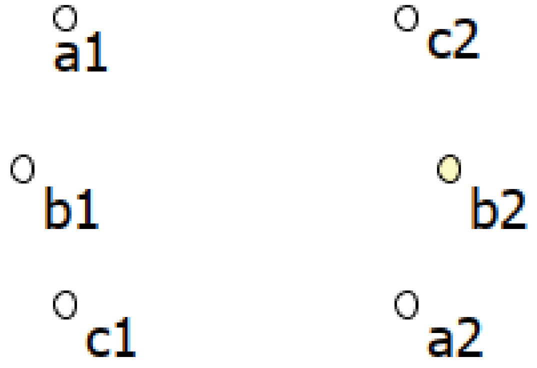
$$GMD = \sqrt[3]{(5)(5)(10)} = 6.3 \text{ m}$$

$$C_{an} = \frac{2\pi\varepsilon_0}{\ln\frac{GMD}{r}} = \frac{2\pi(8.854\times10^{-12})}{\ln\frac{6.3}{0.0109}} = 8.748 \text{ pF/m}$$

Parallel-Circuit Lines

Let

$$V_P = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} V_{a'} \\ V_{b'} \\ V_{c'} \end{bmatrix} \quad \mathrm{q}_{P1} = \begin{bmatrix} q_{a1} \\ q_{b1} \\ q_{c1} \end{bmatrix} \quad \mathrm{q}_{P2} = \begin{bmatrix} q_{a2} \\ q_{b2} \\ q_{c2} \end{bmatrix}$$



$$\begin{bmatrix} V_P \\ V_P \end{bmatrix} = \mathbf{P_P} \begin{bmatrix} q_{P1} \\ q_{P2} \end{bmatrix}$$

where

$$P_{kk} = \frac{1}{2\pi\varepsilon} ln \frac{H_{kk}}{r_k} \qquad P_{kj} = \frac{1}{2\pi\varepsilon} ln \frac{H_{kj}}{D_{kj}}$$

$$\begin{bmatrix} q_{P1} \\ q_{P2} \end{bmatrix} = \mathbf{P_P^{-1}} \begin{bmatrix} V_P \\ V_P \end{bmatrix} = \mathbf{C_P} \begin{bmatrix} V_P \\ V_P \end{bmatrix}$$

$$\begin{bmatrix} q_{P1} \\ q_{P2} \end{bmatrix} = \mathbf{C_P} \begin{bmatrix} V_P \\ V_P \end{bmatrix} = \begin{bmatrix} C_A & C_B \\ C_C & C_D \end{bmatrix} \begin{bmatrix} V_P \\ V_P \end{bmatrix} = \begin{bmatrix} C_A + C_B \\ C_C + C_D \end{bmatrix} V_P$$

Since $q_{P1} + q_{P2} = q_P$,

$$\mathbf{q_P} = (\mathbf{C_A} + \mathbf{C_B} + \mathbf{C_C} + \mathbf{C_D})\mathbf{V_P} = \mathbf{C_{Peq}}\mathbf{V_P}$$

$$\mathbf{Y_{Peq}} = j\omega\mathbf{C_{Peq}}$$

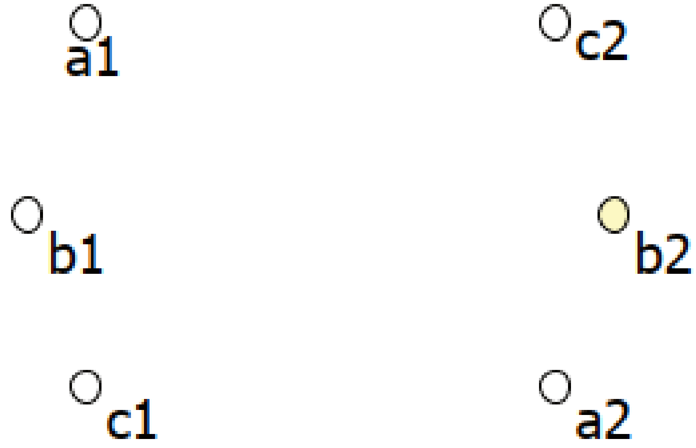
If the line has ground wires:

$$\begin{bmatrix} V_P \\ V_P \\ 0 \end{bmatrix} = \mathbf{P_P} \begin{bmatrix} q_{P1} \\ q_{P2} \\ q_G \end{bmatrix} \rightarrow \begin{bmatrix} V_P \\ V_P \end{bmatrix} = \mathbf{P}_{Peq} \begin{bmatrix} q_{P1} \\ q_{P2} \end{bmatrix} \rightarrow \begin{bmatrix} q_{P1} \\ q_{P2} \end{bmatrix} = \mathbf{C}_{Peq} \begin{bmatrix} V_P \\ V_P \end{bmatrix}$$

Parallel Circuit Lines

Alternate Computation

- Transposition may be assumed
- The distance D^p_{xy} and H^p_{xy} between phases is assumed to be the GMD between pairs of conductors of both phases
- The 'radius' of a phase is computed by treating the parallel conductors as bundled conductors



$$D^p_{ab} = \sqrt[4]{D_{a1,b1}D_{a1,b2}D_{a2,b1}D_{a2,b2}}$$

$$GMD = \sqrt[3]{D^p_{ab}D^p_{bc}D^p_{ac}}$$

$$r_a = \sqrt{D_a r}$$

$$r_{eq} = \sqrt[3]{r_a r_b r_c}$$

$$C_{an} = \frac{2\pi\varepsilon_O}{\ln\dfrac{GMD}{r_{eq}}}$$

Summary of Reactances for Three-Phase Systems

$$L_a = 2x10^{-7}\ ln\frac{GMD}{GMR} \quad H/m$$

$$C_{an} = \frac{2\pi x 8.85\, x10^{-12}}{ln\dfrac{GMD}{r}} \quad F/m$$

$$GMD = \sqrt[3]{D_{ab}D_{bc}D_{ca}}$$

$$GMR = \sqrt[n^2]{(D_{aa}D_{ab}\cdots D_{an})\cdots(D_{na}D_{nb}\cdots D_{nn})}$$

Where, $D_{aa} = D_{bb} = D_{nn} = r' = r\varepsilon^{-1/4}$