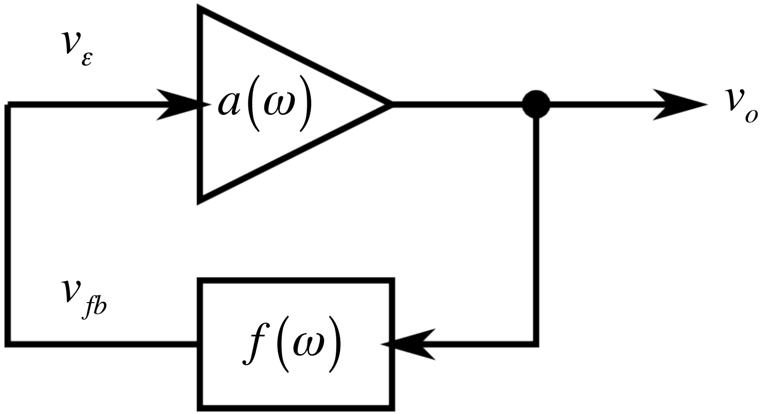
EEE 51: Second Semester 2017 - 2018

Lecture 23

Oscillators

General Form of an Oscillator



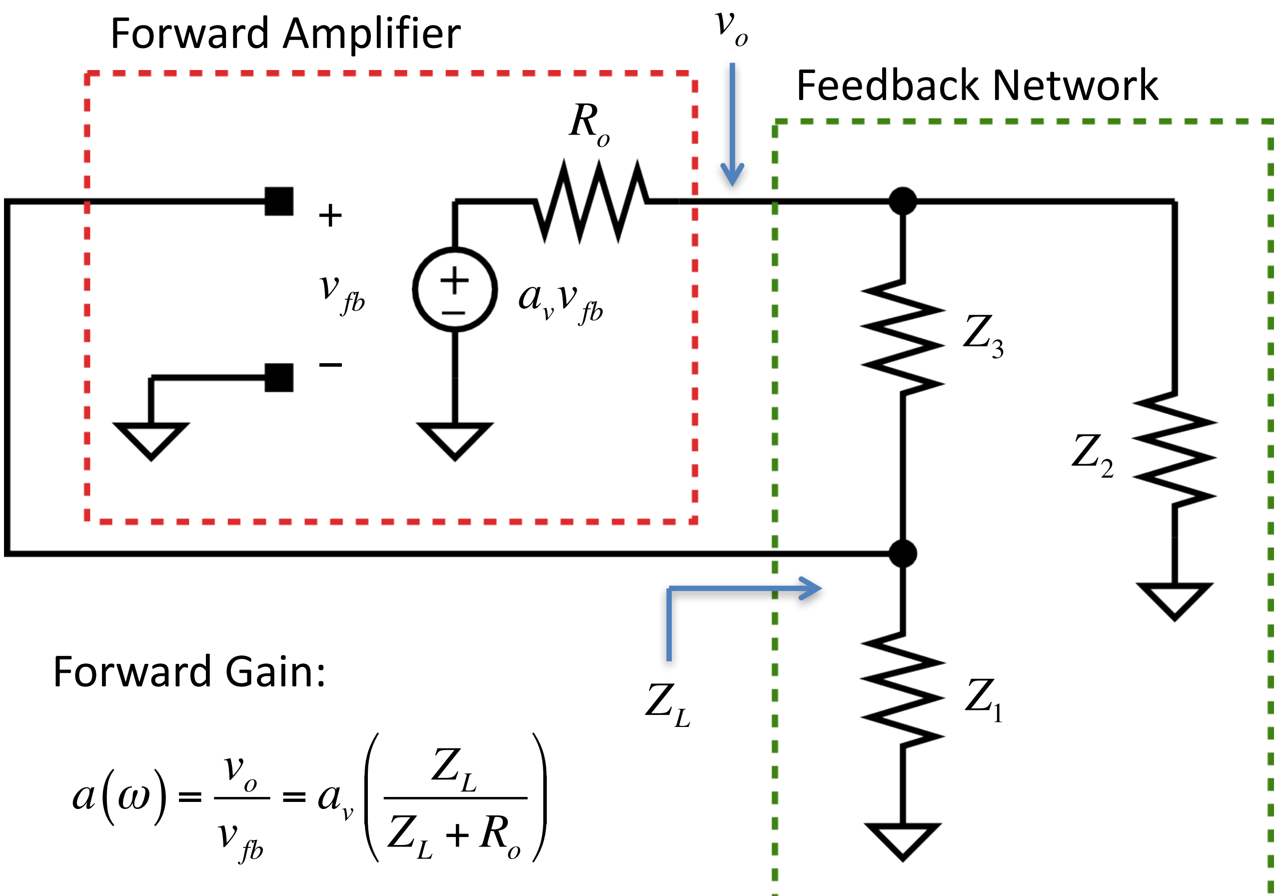
$$a(\omega) = \frac{v_o}{v_\varepsilon} = \frac{v_o}{v_{fb}}$$

Barkhausen Criteria:

$$|T(\omega)| = |a(\omega) \cdot f(\omega)| = 1$$

$$\angle T(\omega) = 360^\circ \cdot n \quad n \in 0,1,2,3\ldots$$

General Form of an Oscillator



Barkhausen Criteria:

$$T(\omega) = a(\omega) \cdot f(\omega) = 1\angle 0^\circ$$

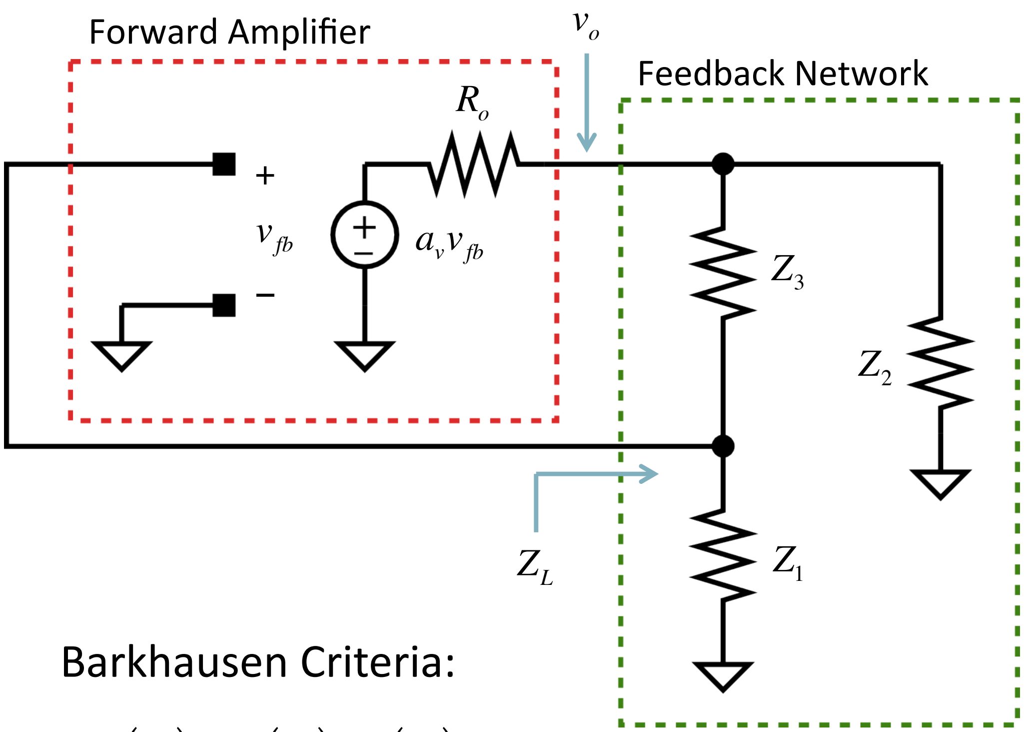
Feedback Factor:

$$f(\omega) = \frac{v_{fb}}{v_o} = \frac{Z_1}{Z_1 + Z_3}$$

Forward Gain:

$$a(\omega) = \frac{v_o}{v_{fb}} = a_v\left(\frac{Z_L}{Z_L + R_o}\right)$$

General Form of an Oscillator



$$a(\omega) = a_v\left(\frac{Z_L}{Z_L + R_o}\right)$$

$$f(\omega) = \frac{Z_1}{Z_1 + Z_3}$$

Loop Gain:

$$T(\omega) = a(\omega)\cdot f(\omega)$$

$$= a_v \frac{Z_L}{Z_L + R_o}\frac{Z_1}{Z_1 + Z_3}$$

$$Z_L = Z_2 \,\|\, (Z_1 + Z_3)$$

$$= \frac{Z_2(Z_1 + Z_3)}{Z_1 + Z_2 + Z_3}$$

Barkhausen Criteria:

$$T(\omega) = a(\omega)\cdot f(\omega)$$

$$= 1\angle 0^\circ$$

General Form of an Oscillator

Loop Gain: $T(\omega) = a(\omega)\cdot f(\omega)$

$$= a_v \cdot \frac{Z_L}{Z_L + R_o} \cdot \frac{Z_1}{Z_1 + Z_3}$$

$$= a_v \cdot \frac{\dfrac{Z_2(Z_1+Z_3)}{Z_1+Z_2+Z_3}}{\dfrac{Z_2(Z_1+Z_3)}{Z_1+Z_2+Z_3} + R_o} \cdot \frac{Z_1}{Z_1+Z_3} = a_v \cdot \frac{Z_2 Z_1}{Z_2(Z_1+Z_3) + R_o(Z_1+Z_2+Z_3)}$$

Barkhausen Criteria:

$$T(\omega) = a(\omega)\cdot f(\omega)$$

$$= a_v \cdot \frac{Z_2 Z_1}{Z_2(Z_1+Z_3) + R_o(Z_1+Z_2+Z_3)} = 1\angle 0^\circ$$

General Form of an Oscillator

Barkhausen Criteria: $T(\omega) = a(\omega) \cdot f(\omega)$

$$= a_v \cdot \frac{Z_2 Z_1}{Z_2(Z_1 + Z_3) + R_o(Z_1 + Z_2 + Z_3)} = 1\angle 0°$$

If we assume that Z_1, Z_2 and Z_3 are purely reactive elements: $\begin{cases} Z_1 = jX_1 \\ Z_2 = jX_2 \\ Z_3 = jX_3 \end{cases}$

$$T(\omega) = a_v \cdot \frac{Z_2 Z_1}{Z_2(Z_1 + Z_3) + R_o(Z_1 + Z_2 + Z_3)} = a_v \cdot \frac{jX_2 \cdot jX_1}{jX_2(jX_1 + jX_3) + R_o(jX_1 + jX_2 + jX_3)}$$

$$= a_v \cdot \frac{-X_2 \cdot X_1}{-X_2(X_1 + X_3) + jR_o(X_1 + X_2 + X_3)}$$

General Form of an Oscillator

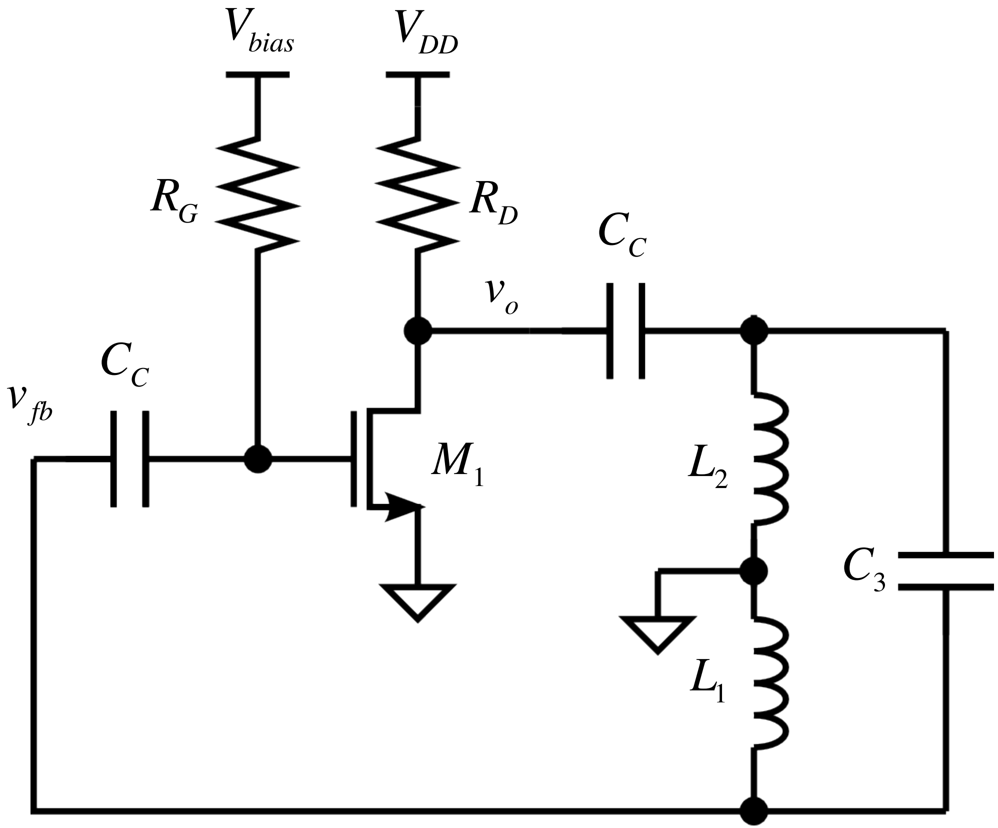
Loop Gain: $$T(\omega) = a_v \cdot \frac{-X_2 \cdot X_1}{-X_2(X_1 + X_3) + jR_o(X_1 + X_2 + X_3)}$$

For oscillations to occur: $$\mathrm{Im}\{T(\omega)\} = 0 \quad \Rightarrow \quad X_1 + X_2 + X_3 = 0 \quad \Rightarrow \quad X_1 + X_3 = -X_2$$

Thus: $$T(\omega_0) = a_v \cdot \frac{-X_2 \cdot X_1}{-X_2(X_1 + X_3)} = a_v \cdot \frac{X_1}{(X_1 + X_3)} = -a_v \cdot \frac{X_1}{X_2} = 1\angle 0^\circ$$

Case 1: $a_v > 0$ → X_1 and X_2 must have different signs
Case 2: $a_v < 0$ → X_1 and X_2 must have the same sign

Example: The Hartley Oscillator



MOSFET bias

$$I_D = k\left(V_{GS} - V_{TH}\right)^2$$

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = 2k\left(V_{GS} - V_{TH}\right)$$

$$r_o \rightarrow \infty$$

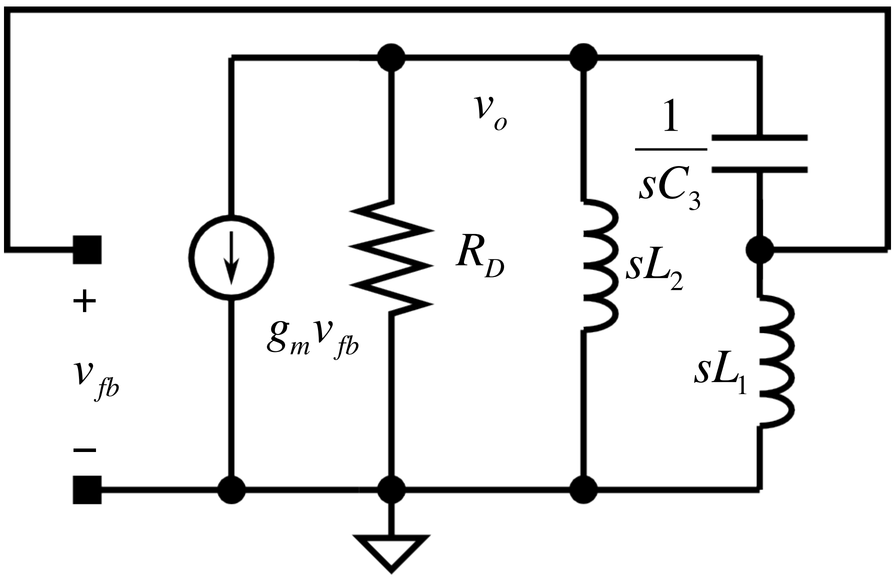
Assume

- R_G is large
- C_C is large

Named after Ralph Vinton Lyon Hartley (November 30, 1888 – May 1, 1970)

Example: The Hartley Oscillator

Small-signal model:



$$X_1 = \omega L_1 \qquad X_2 = \omega L_2 \qquad X_3 = -\frac{1}{\omega C_3}$$

Forward unloaded amplifier:

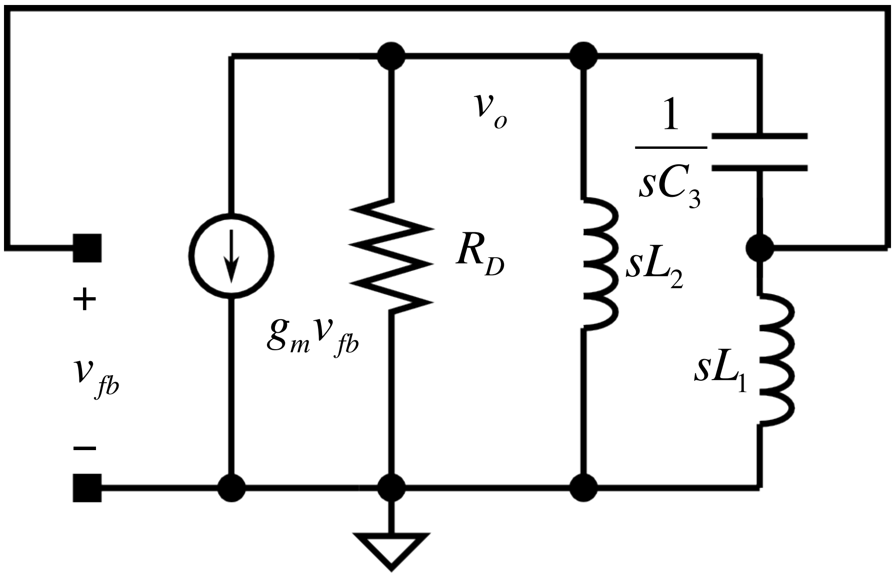
$$a_v = -g_m R_D$$

$$R_o = R_D$$

Note: X_1 and X_2 must have the same sign

Example: The Hartley Oscillator

Small-signal model:



$$a_v = -g_m R_D \qquad X_2 = \omega L_2$$

$$R_o = R_D \qquad X_1 = \omega L_1$$

$$X_3 = -\frac{1}{\omega C_3}$$

Recall:

$$T(\omega) = a_v \cdot \frac{-X_2 \cdot X_1}{-X_2(X_1 + X_3) + jR_o(X_1 + X_2 + X_3)}$$

To oscillate:

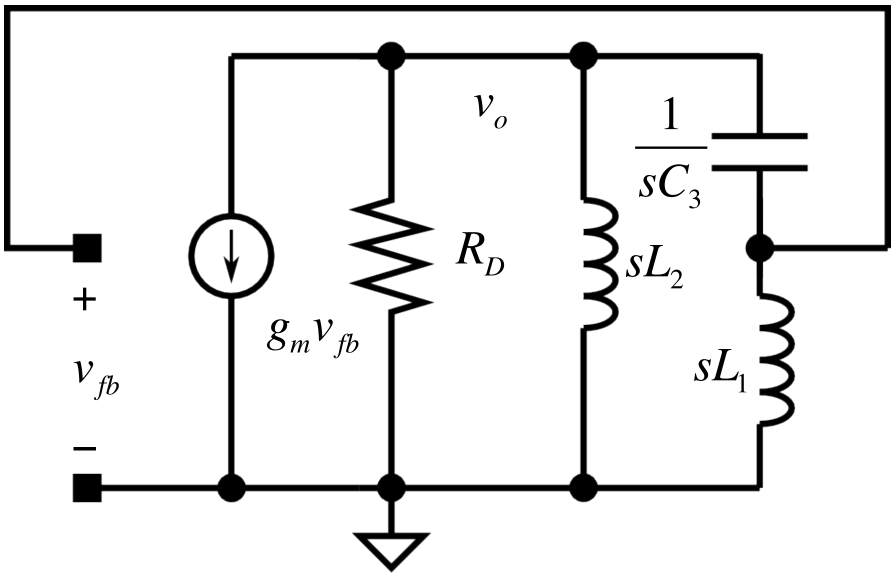
$$X_1 + X_2 + X_3 = 0 \quad \Rightarrow \quad \omega_0(L_1 + L_2) = \frac{1}{\omega_0 C_3}$$

Thus,

$$\omega_0 = \frac{1}{\sqrt{(L_1 + L_2)C_3}}$$

Example: The Hartley Oscillator

Frequency of oscillation: $\omega_0 = \dfrac{1}{\sqrt{(L_1 + L_2)C_3}}$



$a_v = -g_m R_D$

$R_o = R_D$

$X_2 = \omega L_2$

$X_1 = \omega L_1$

$X_3 = -\dfrac{1}{\omega C_3}$

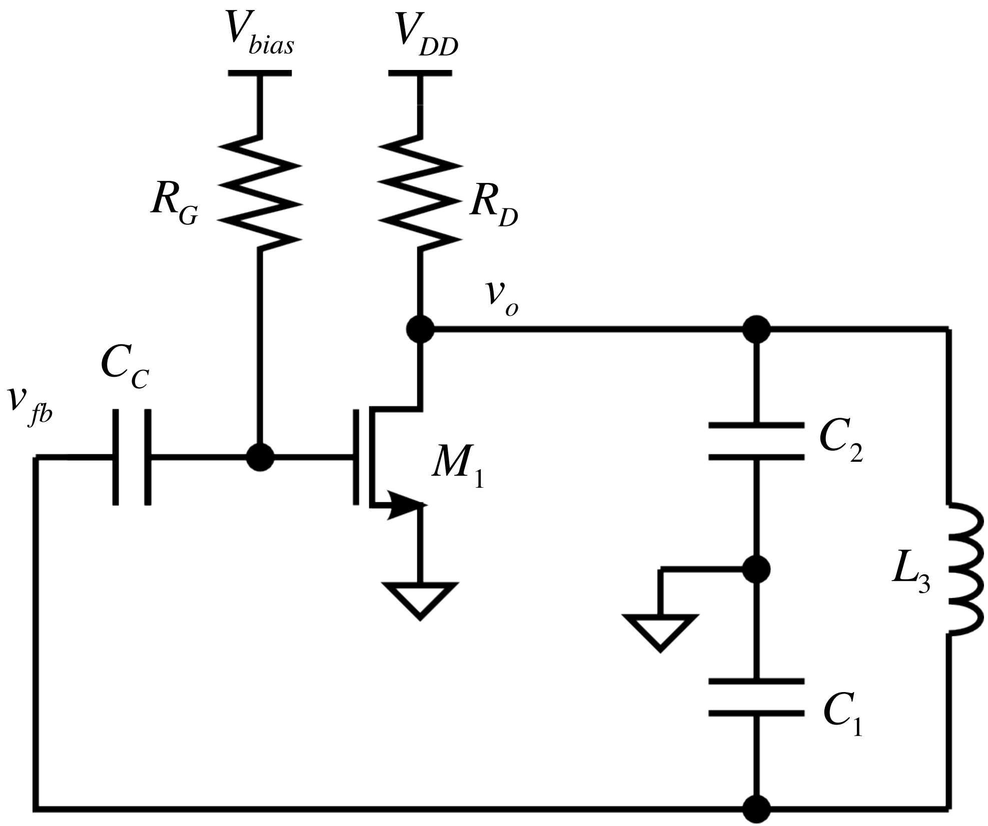
Loop Gain:

$$T(\omega_0) = -a_v \cdot \frac{X_1}{X_2} = 1\angle 0^\circ$$

$$= g_m R_D \frac{L_1}{L_2} = 1\angle 0^\circ$$

For $L_1 = L_2$: $g_m R_D = 1$

The Colpitts Oscillator



MOSFET bias

$$I_D = k\left(V_{GS} - V_{TH}\right)^2$$

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = 2k\left(V_{GS} - V_{TH}\right)$$

$$r_o \rightarrow \infty$$

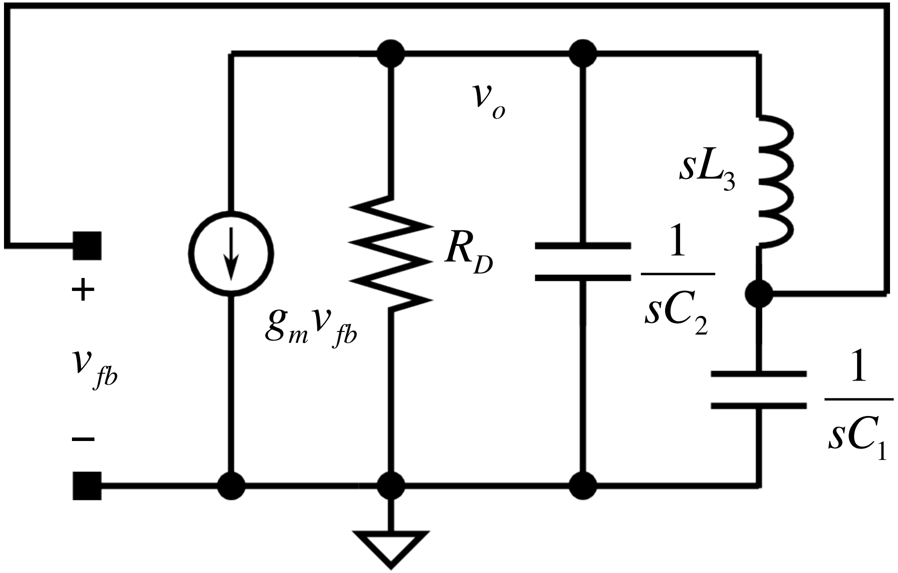
Assume

- R_G is large
- C_C is large

Named after Edwin Henry Colpitts (January 19, 1872 - March 6, 1949)

The Colpitts Oscillator

Small-signal model:



$$X_1 = -\frac{1}{\omega C_1} \quad X_2 = -\frac{1}{\omega C_2} \quad X_3 = \omega L_3$$

Forward unloaded amplifier:

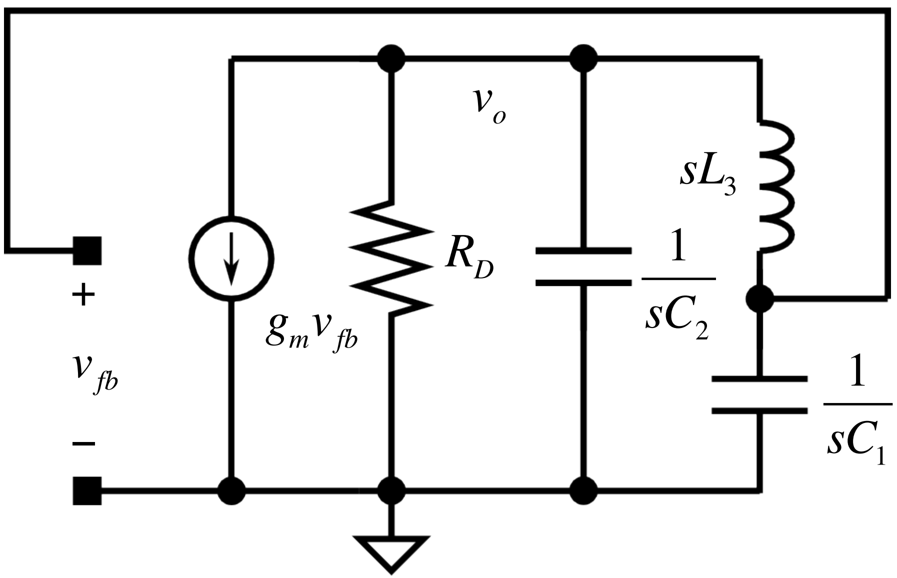
$$a_v = -g_m R_D$$

$$R_o = R_D$$

Note: X_1 and X_2 must have the same sign

The Colpitts Oscillator

Small-signal model:



$$a_v = -g_m R_D \qquad X_1 = -\frac{1}{\omega C_1} \qquad X_2 = -\frac{1}{\omega C_2}$$

$$R_o = R_D \qquad X_3 = \omega L_3$$

Recall:

$$T(\omega) = a_v \cdot \frac{-X_2 \cdot X_1}{-X_2(X_1 + X_3) + jR_o(X_1 + X_2 + X_3)}$$

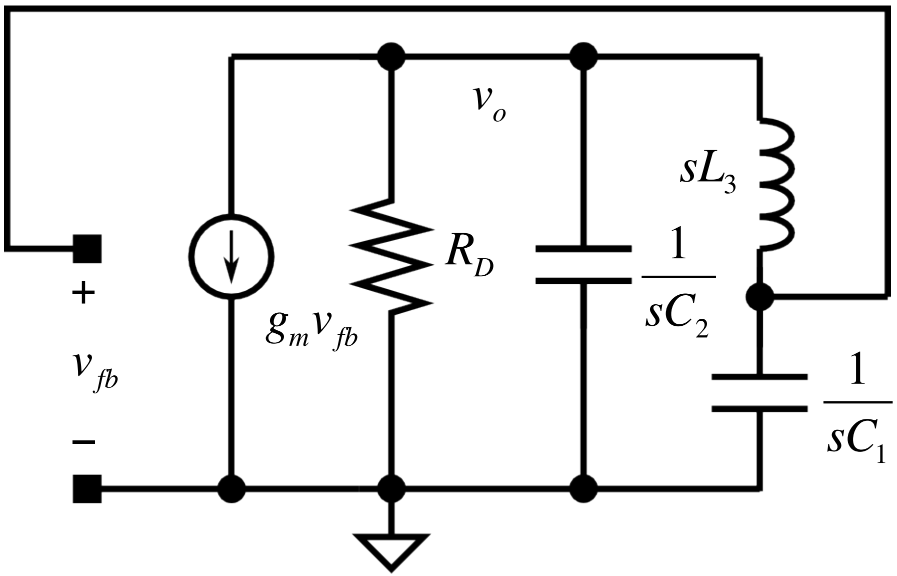
To oscillate:

$$X_1 + X_2 + X_3 = 0 \quad \Rightarrow \quad \omega_0 L_3 = \frac{1}{\omega_0}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

Thus, $$\omega_0 = \frac{1}{\sqrt{L_3 \cdot \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}}}}$$

The Colpitts Oscillator

Frequency of oscillation: $\omega_0 = \dfrac{1}{\sqrt{L_3 \cdot \dfrac{C_1 C_2}{C_1 + C_2}}}$



Loop Gain:

$$T(\omega_0) = -a_v \cdot \frac{X_1}{X_2} = 1\angle 0^\circ$$

$$= g_m R_D \frac{C_2}{C_1} = 1\angle 0^\circ$$

For $C_1 = C_2$: $g_m R_D = 1$

$$a_v = -g_m R_D \qquad X_1 = -\frac{1}{\omega C_1} \qquad X_2 = -\frac{1}{\omega C_2}$$

$$R_o = R_D$$

$$X_3 = \omega L_3$$

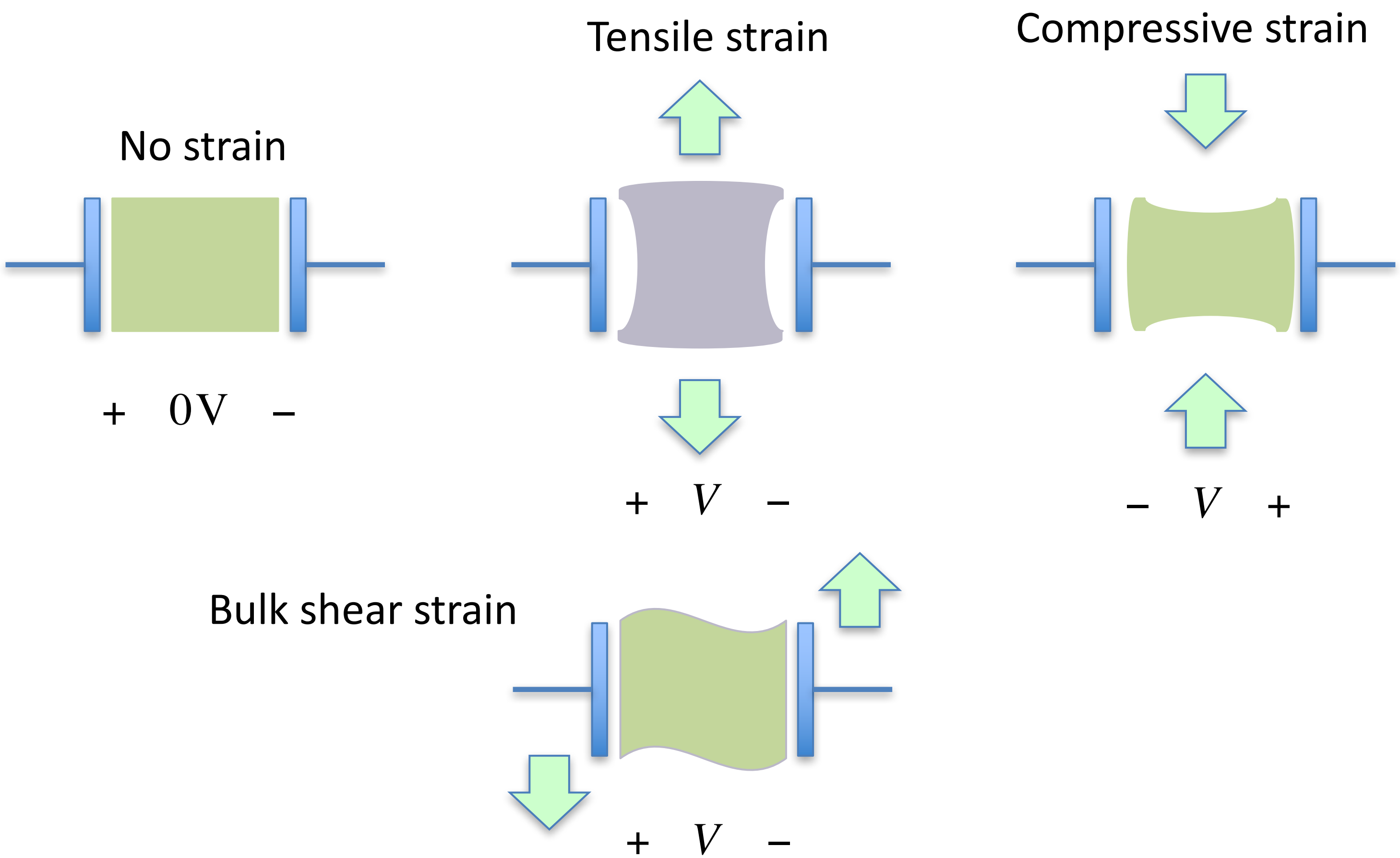
Crystal Oscillators

- Crystals are materials that exhibits the piezoelectric effect
  - When stress is applied, voltage is generated between opposite faces of the crystal
  - When a voltage is applied, the crystal deforms at the frequency of the applied voltage

- Mechanical to electrical energy conversion

- Provides stable frequency of oscillation
  - Resonant frequency dependent on physical crystal size
  - 1 ppm/°C or 0.0001%/°C
  - Compare with LC oscillator: ~1% drift

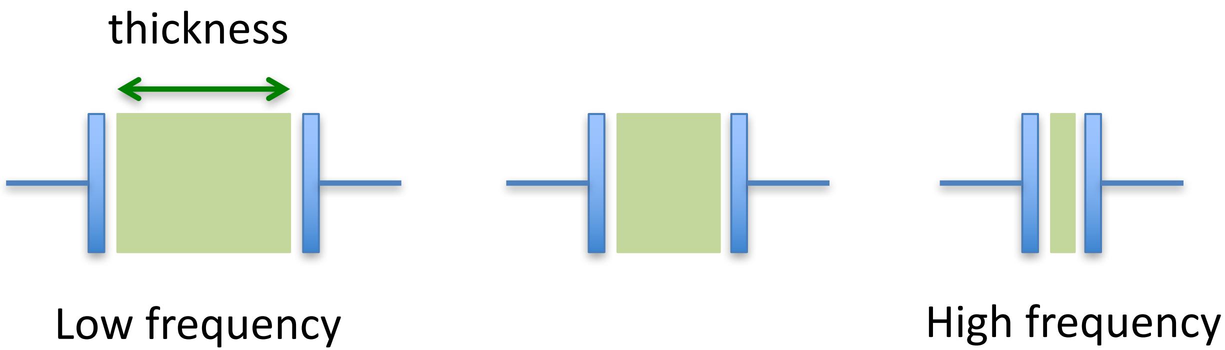
Crystal Strain



Natural Crystal Frequency

- Proportional to crystal thickness

$$f \propto \frac{1}{\text{thickness}}$$

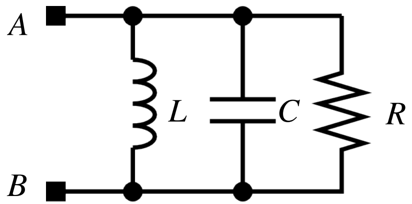


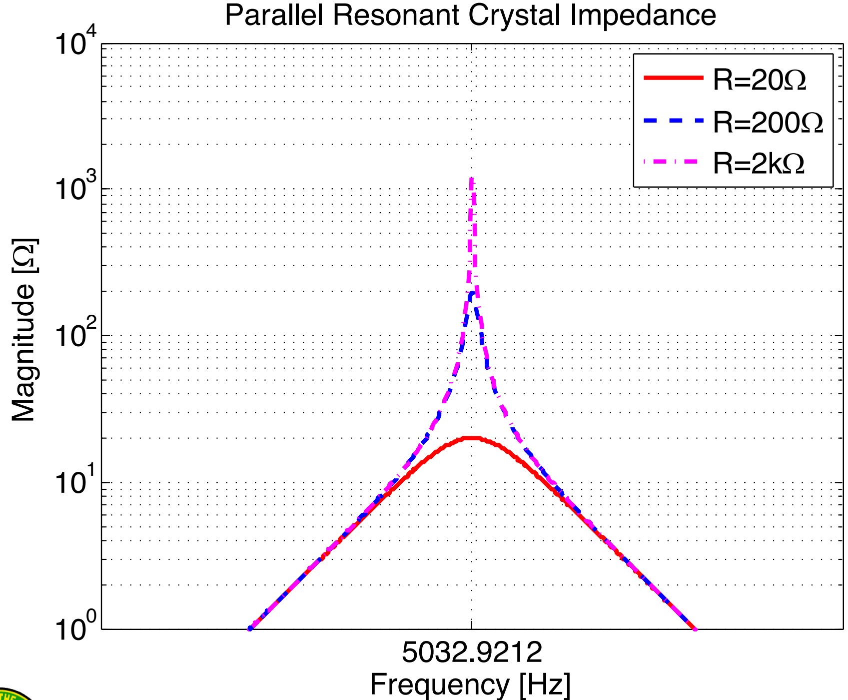
Typical natural frequencies below 20-30 MHz

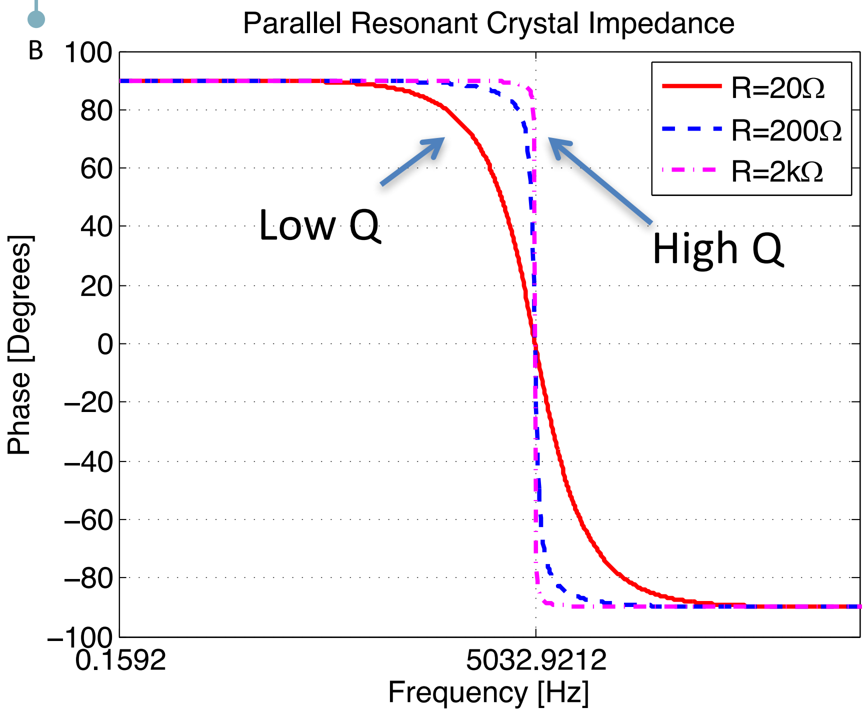
- For 100 MHz, thickness ~ 17µm thick

The Parallel Resonant Mode

$$L = 1\text{mH} \qquad C = 1\mu\text{F}$$

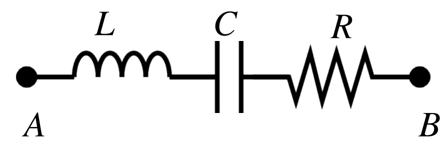


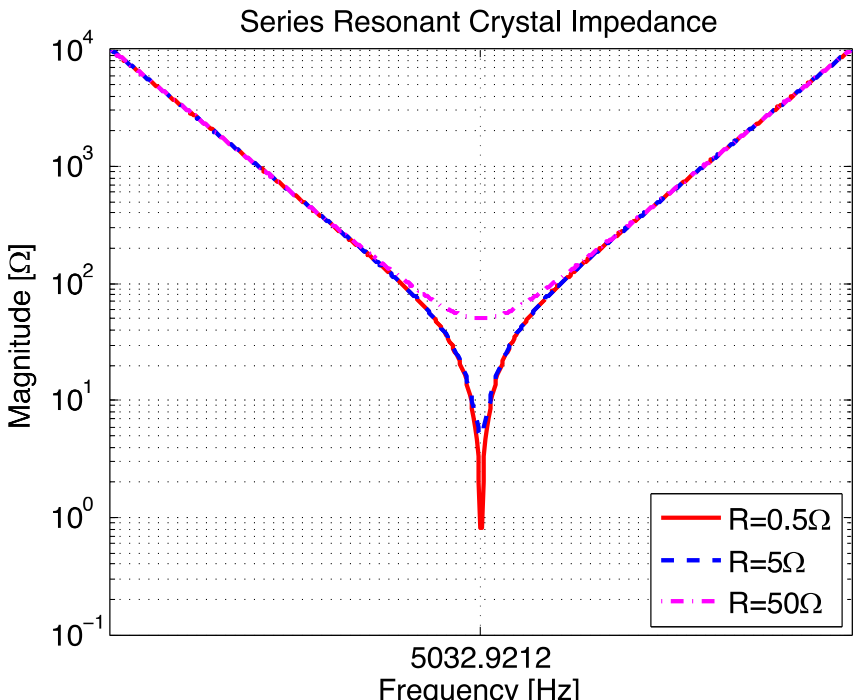


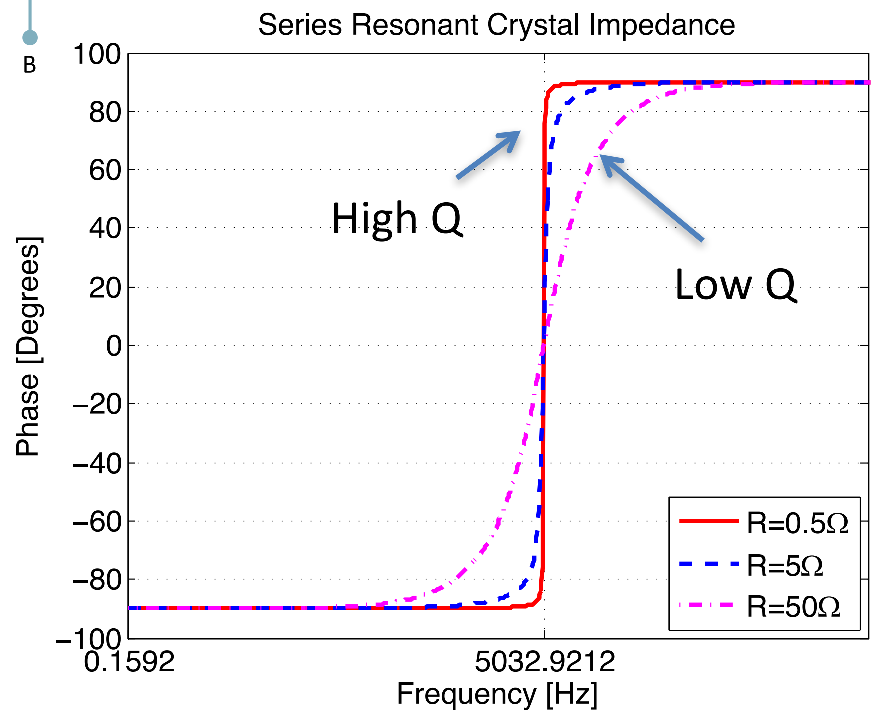


The Series Resonant Mode

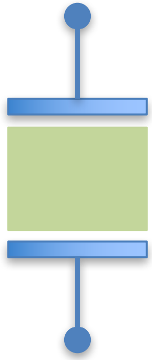
$$L = 1\text{mH} \qquad C = 1\mu\text{F}$$

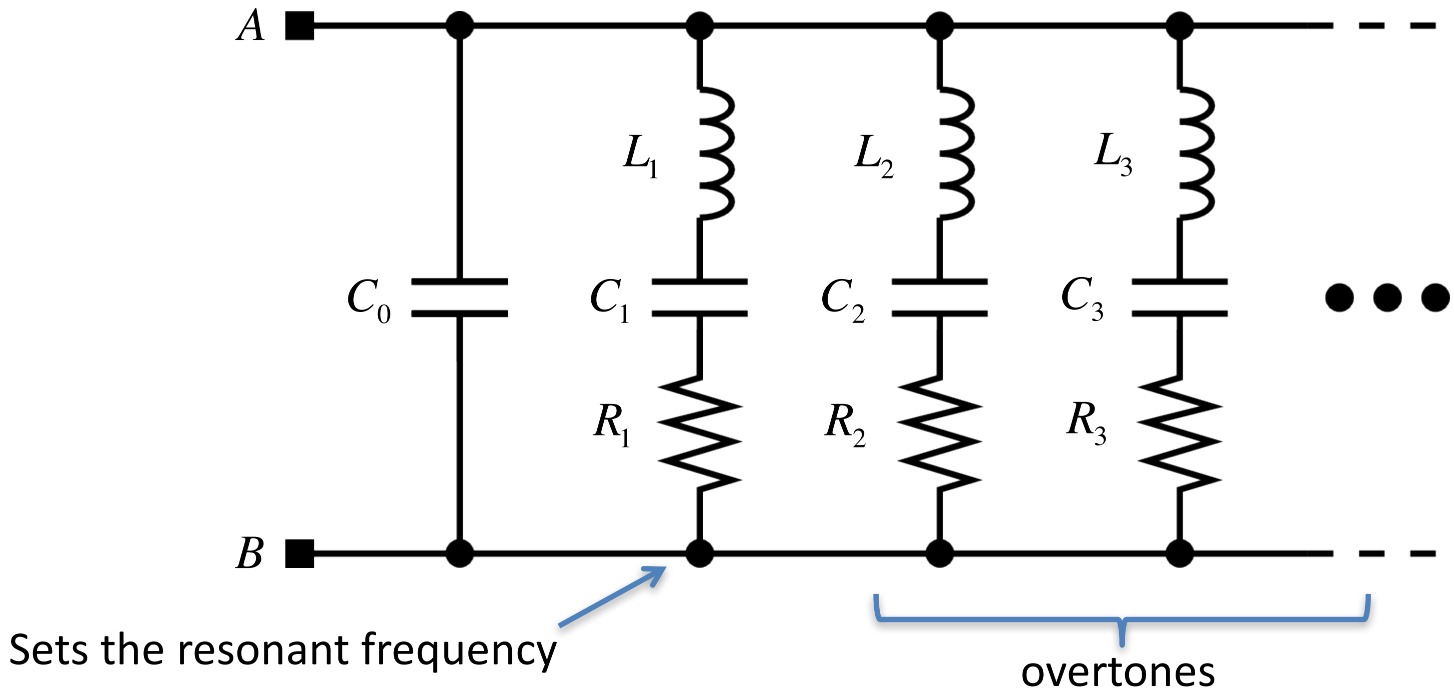






Electrical Equivalent Circuit



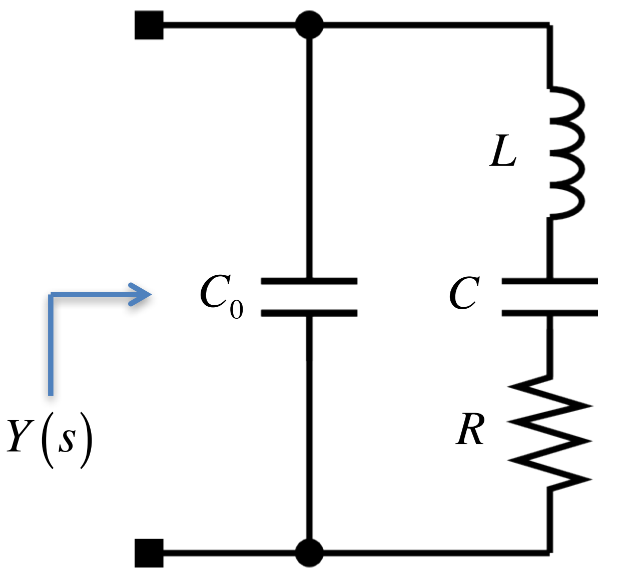


$C_0 \equiv$ parallel capacitances due to contacts and wires

$L_i, C_i \equiv$ mechanical energy storage (mass & spring effects)

$R_i \equiv$ electrical losses due to mechanical effects (e.g. friction)

Crystal Equivalent Circuit

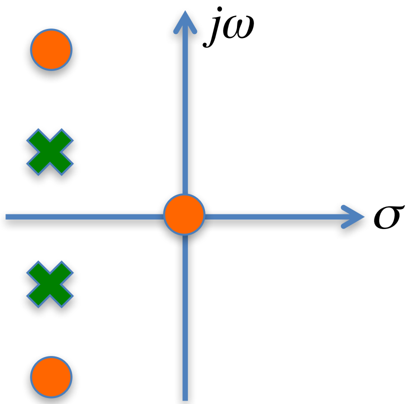


$$Y(s) = sC_0 + \frac{1}{sL + \frac{1}{sC} + R} = sC_0 + \frac{sC}{s^2LC + sRC + 1}$$

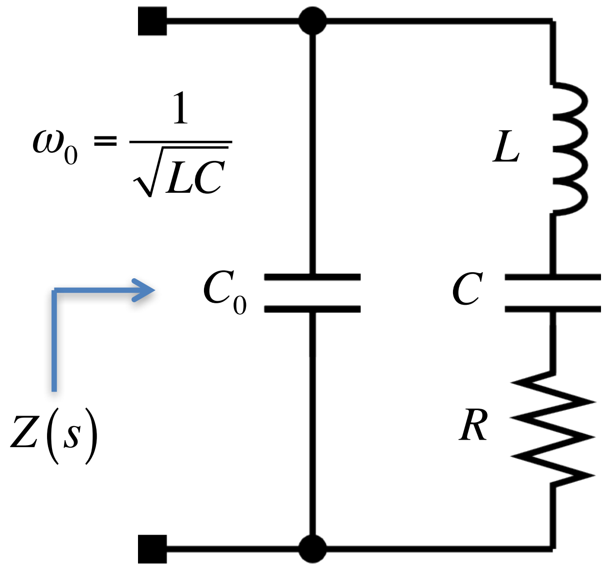
$$= \frac{sC_0\left[s^2 + s\frac{R}{L} + \left(\frac{C}{C_0} + 1\right)\omega_0^2\right]}{s^2 + s\frac{R}{L} + \omega_0^2}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad Z(s) = \frac{1}{Y(s)}$$

p-z plot:



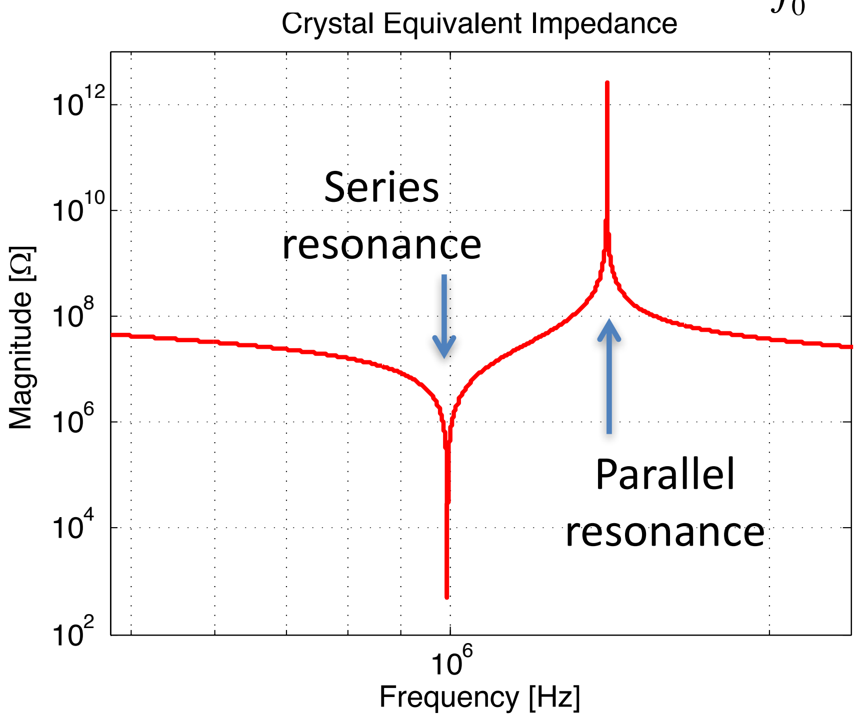
Crystal Equivalent Circuit



$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$Z(s) = \frac{s^2 + s\frac{R}{L} + \omega_0^2}{sC_0\left[s^2 + s\frac{R}{L} + \left(\frac{C}{C_0} + 1\right)\omega_0^2\right]}$$

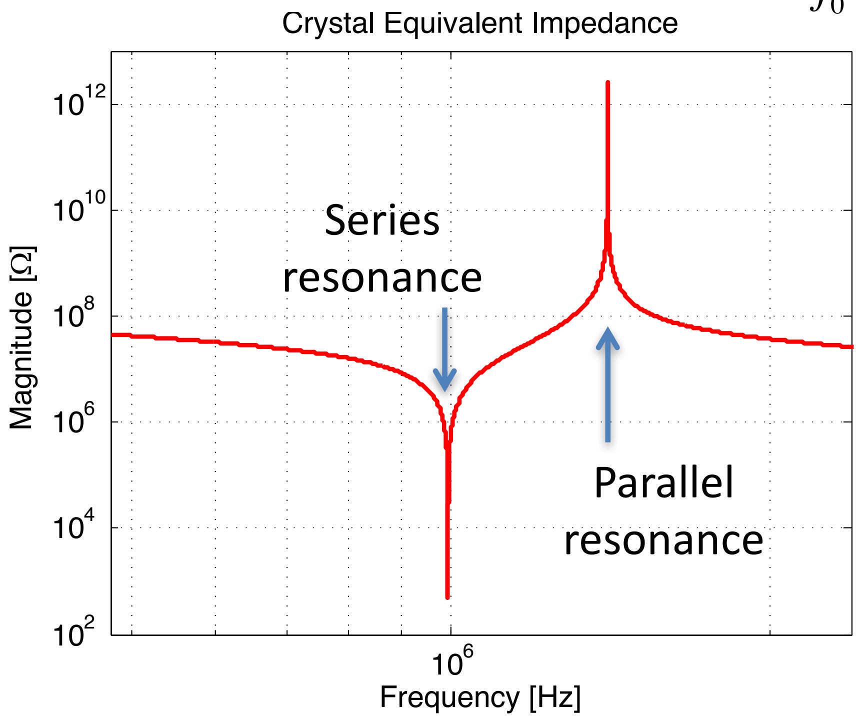
$$L = 10\text{H} \qquad C = C_0 = 3.2\text{fF} \qquad R \approx \frac{5\times10^8}{f_0}$$

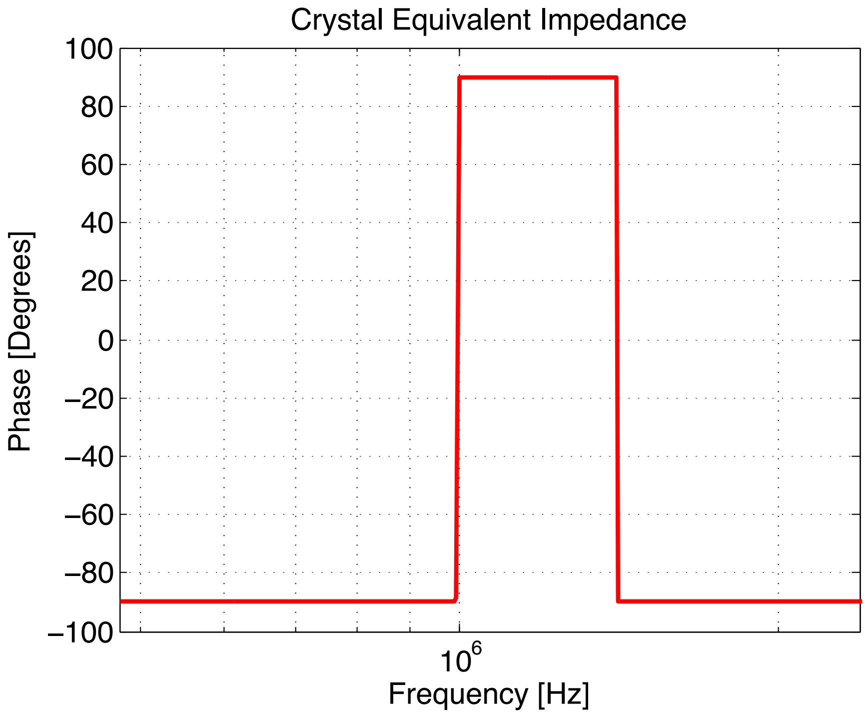


How can we use this to create an oscillator?

Crystal Equivalent Circuit

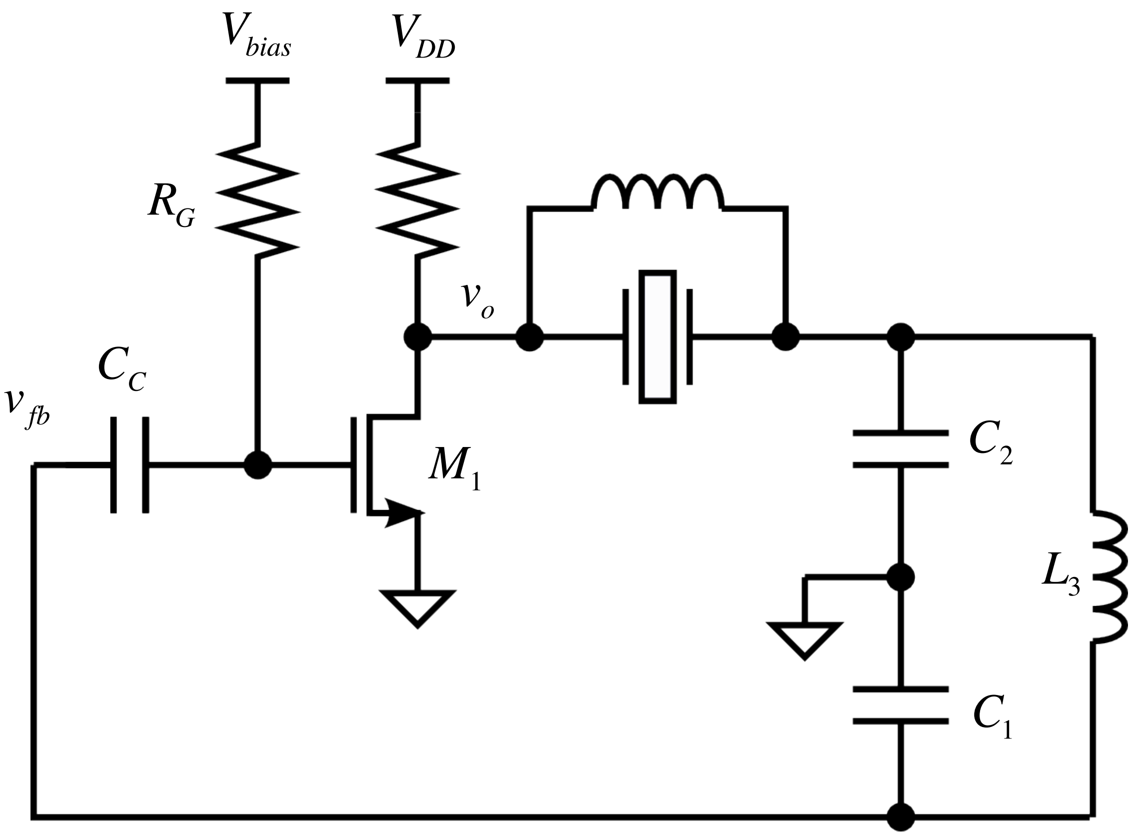
$$L = 10\text{H} \qquad C = C_0 = 3.2\text{fF} \qquad R \approx \frac{5\times10^8}{f_0}$$





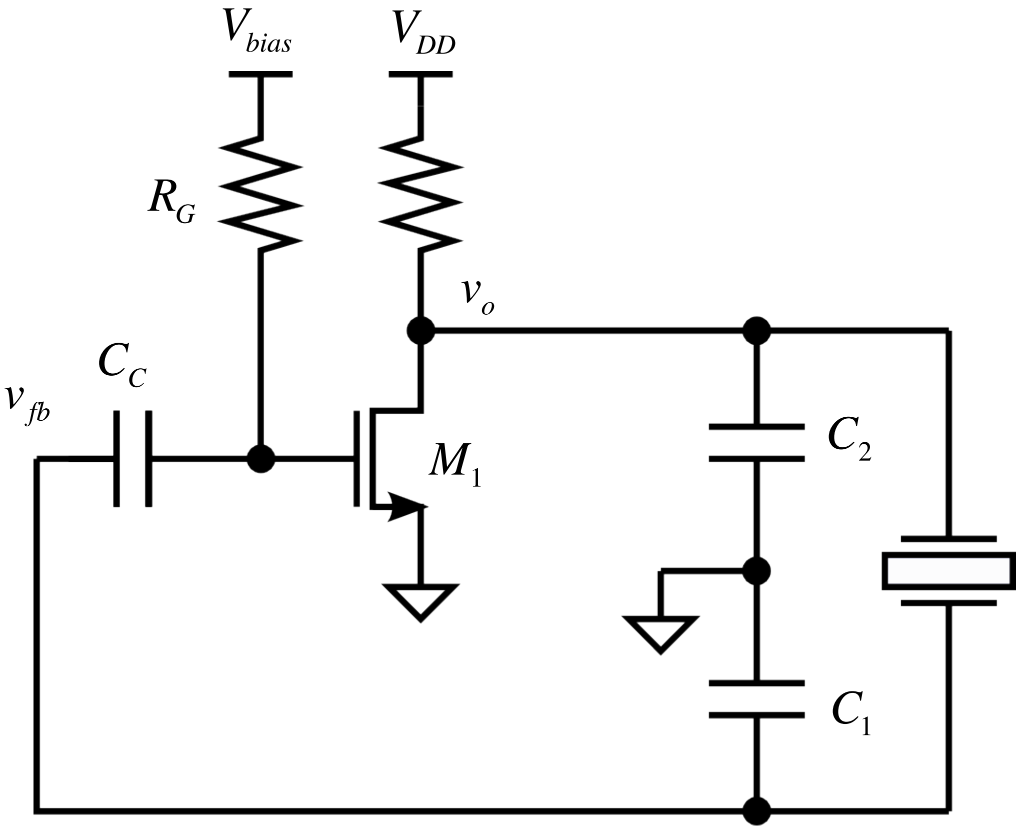
How can we use this to create an oscillator?

Direct Application: A Colpitts Crystal Oscillator



- The crystal is used to close the feedback loop only at the desired frequency
- The parallel inductance is used to cancel out the crystal parallel capacitance C_0
    - Only the series RLC branch controls the feedback path

Another Colpitts Crystal Oscillator



- The crystal is used as an inductance
- The oscillation frequency is set to be a little above series resonance
  - The crystal impedance is inductive
- Note that the crystal series resonant frequency is not the same as the output oscillation frequency
  - Crystal is cut to oscillate at a specified load capacitance

Frequency Bands

Frequency Range	Designation	Wavelength	
3 kHz – 30 kHz	VLF	100 km – 10 km	Phase shift
30 kHz – 300 kHz	LF	10 km – 1 km	
300 kHz – 3 MHz	MF	1 km – 100 m	LC
3 MHz – 30 MHz	HF	100 m – 10 m	Crystal
30 MHz – 300 MHz	VHF	10 m – 1 m	LC, ring oscillators, SAW, MEMS
300 MHz – 3 GHz	UHF	1 m – 0.1 m	
3 GHz – 30 GHz	SHF	0.1 m – 1 cm	
30 GHz – 300 GHz	EHF	1 cm – 1 mm	LC, distributed, MEMS

Next Meeting

- Negative Resistance Oscillators