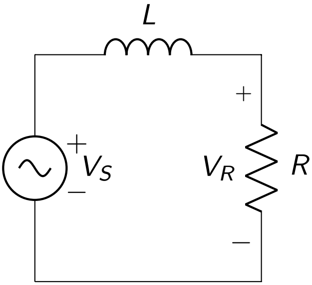
ECE 113: Communication Electronics

Meeting 7: Resonant Circuits II

February 12, 2019

Series RL Circuit

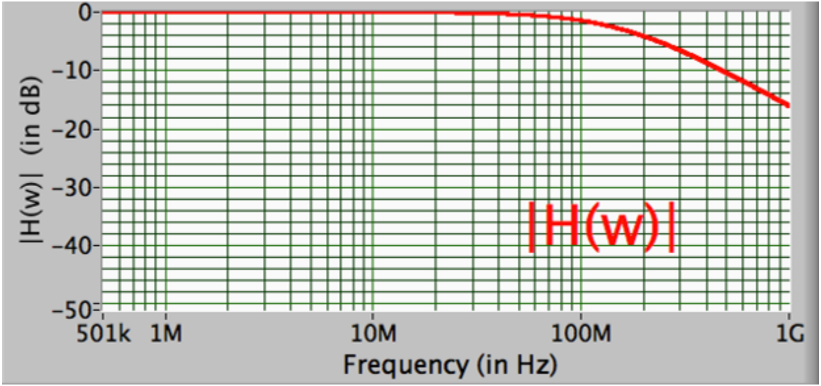


- Example of low pass filter
- Determine the cutoff frequency
  - $|H(\omega)|_{dB} = -3dB$

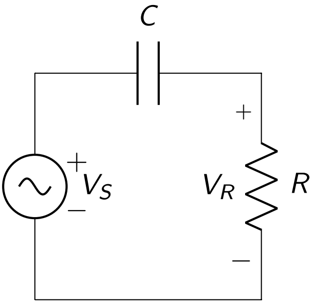
$$\frac{R}{\sqrt{R^2 + \omega^2 L^2}} = \frac{1}{\sqrt{2}}$$

  - $\omega_{cutoff} = \dfrac{R}{L}$

Low Pass Series RL Response



Series RC Circuit

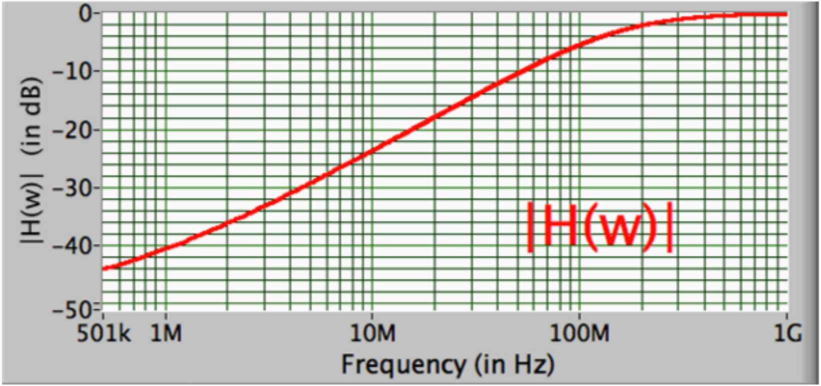


- Example of high pass filter
- Determine the cutoff frequency
    - $|H(\omega)|_{dB} = -3dB$

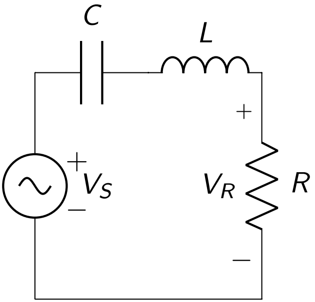
$$\frac{\omega RC}{\sqrt{1+\omega^2R^2C^2}} = \frac{1}{\sqrt{2}}$$

    - $\omega_{cutoff} = \dfrac{1}{RC}$

High Pass Series RC Response



Series RLC Circuit



- Resonant frequency

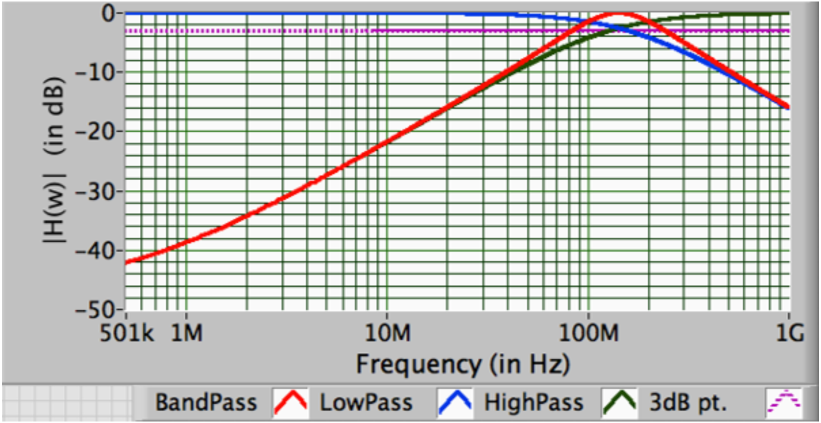
$$\omega_o = \frac{1}{\sqrt{LC}}$$

- Example of band pass filter
- Determine the cutoff frequency
  - $|H(\omega)|_{dB} = -3dB$

$$\frac{R}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} = \frac{1}{\sqrt{2}}$$

  - $\omega_{Hcutoff} = \frac{R}{2L} + \sqrt{(\frac{R}{2L})^2 + \frac{1}{LC}}$
  - $\omega_{Lcutoff} = -\frac{R}{2L} + \sqrt{(\frac{R}{2L})^2 + \frac{1}{LC}}$

Band Pass Series RLC Response

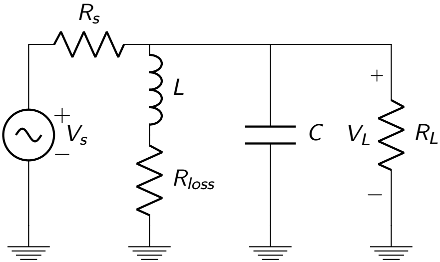


Loaded Q

- Assumptions in RLC resonant circuits
  - Ideal reactive elements (Lossless L & C)
  - Ideal resistive elements (R without parasitics)
  - Voltage source with zero source resistance
  - No load impedance
- Loaded Q takes into account actual "in-circuit" conditions

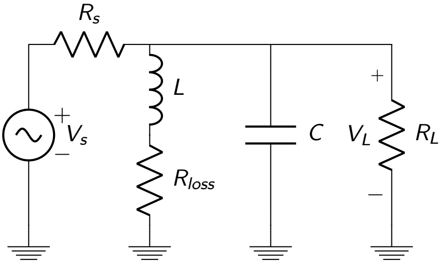
Loaded Q in Parallel LC Circuit

- C - ideal (lossless) capacitor
- L - lossy inductor
- R_{loss} - DC resistance of L
- R_s - source resistance of V_s
- R_L - load resistance



Effects of Source Impedance on Circuit Q

- C = 25pF
- L = 50nH
- $R_{loss} = 0\Omega$ (lossless L)
- $R_s = 50\Omega$
- $R_L = \infty$ (no load)

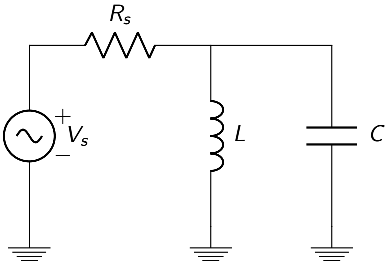


- Resonant Frequency

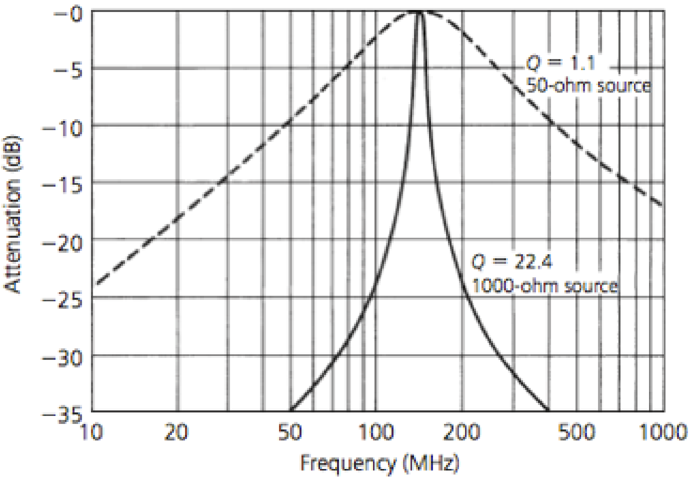
$$f_o = \frac{1}{2\pi\sqrt{LC}} = 142.35MHz$$

- Circuit Q

$$Q = R_s\sqrt{\frac{C}{L}}$$

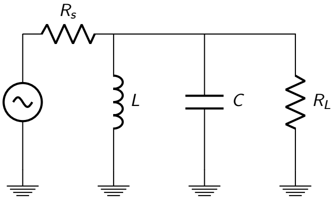


Effects of R_s on Circuit Q

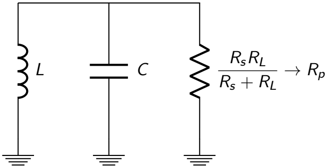


Equivalent Circuit Approach

1. Resonant Circuit with an external node



2. Equivalent circuit for Q calculations



- At resonance

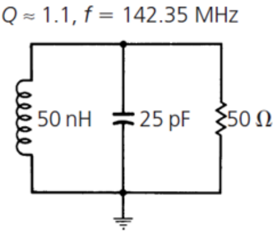
$$Q = \frac{R_p}{X_p}$$

R_p - equivalent load (parallel resistance of R_s and R_L)

X_p - inductive/capacitive reactance (they are equal at resonance)

Effects of R_s and R_L on Circuit Q

- In general, low values of source and load impedances load down ("de-Q") resonant circuits



(A) *Large inductor, small capacitor*

(B) *Small inductor, large capacitor*

- Becomes difficult to design high Q circuits with low source/load impedances
  - Theoretical design can be made, but component values might be impractical

Example

- Design a parallel LC circuit with the following specifications
    - Resonant frequency: $f_o = 1GHz$
    - Source Resistance: $R_s = 50\Omega$
    - Load Resistance: $R_L = 10\text{k}\Omega$

- List L and C for various Q

$$f_o = \frac{1}{2\pi\sqrt{LC}},\ Q_p = (R_s \parallel R_L)\sqrt{\frac{C}{L}}$$

Example

- Design a parallel LC circuit with the following specifications
  - Resonant frequency: $f_o = 1GHz$
  - Source Resistance: $R_s = 50\Omega$
  - Load Resistance: $R_L = 10\text{k}\Omega$

- List L and C for various Q

$$f_o = \frac{1}{2\pi\sqrt{LC}},\ Q_p = (R_s \parallel R_L)\sqrt{\frac{C}{L}}$$

Q	**C**	**L**
0.5	0.66pF	6.59nH
1	1.33pF	3.30nH
10	13.30pF	329.90pH

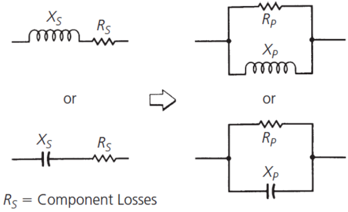
  - Low R_s or R_L and High-Q requirements $\rightarrow$ impractical values for L and/or C

Component Q Transformation

- To convert series equivalent component to its parallel equivalent and vice versa

$$Q_s = \frac{X_s}{R_s},\ Q_p = \frac{R_p}{X_p}$$

$$R_p = (Q^2 + 1)R_s$$



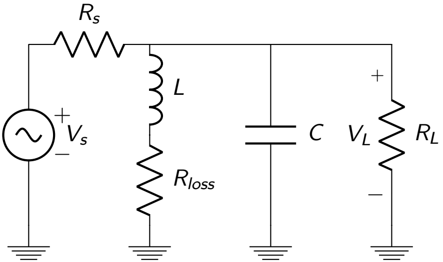
where: $Q_p = Q_s = Q(component\ Q)$

These transformations are valid at only one frequency

Example

Suppose lossy $L = 50nH$ with $R_{loss} = 10\Omega$ at $100MHz$

- Solve for component Q
- Transform series R_{loss} into equivalent parallel RL circuit.



- The component Q of the inductor

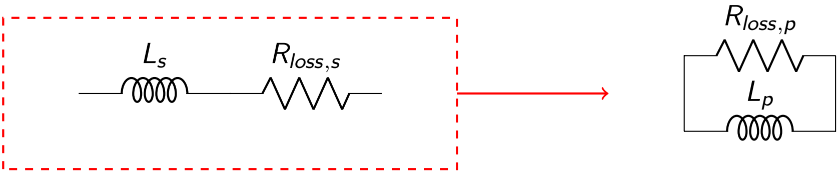
$$Q = \frac{X_{L,s}}{R_{loss,s}} = \frac{2\pi(100 \times 10^6)(50 \times 10^{-9})}{10} = 3.14$$

- Equivalent parallel resistance

$$R_{loss,p} = (Q^2 + 1)R_{loss,s} = 108.7\Omega$$

- Equivalent parallel reactance

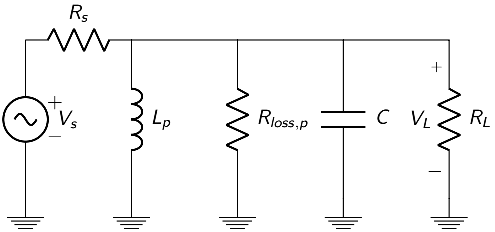
$$X_{L,p} = \frac{R_{loss,p}}{Q_p} = \frac{108.7}{3.14} = 34.62\Omega$$



- The parallel inductance
$$L_p = \frac{X_{L,p}}{\omega} = 55.1nH$$
$$R_{loss,p} = 108.7\Omega$$

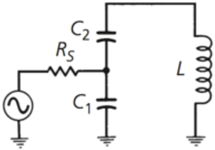
- Equivalent circuit can be used to find loaded Q of circuit

- In general, lossy components tend to reduce loaded Q



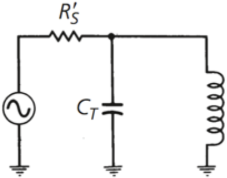
Impedance Transformation

- We may use a tapped-C or tapped-L impedance transformer to make the small source or load impedance appear larger to the resonant circuit
    - For a tapped-C circuit



$$R_s' = R_s\left(1 + \frac{C_1}{C_2}\right)^2$$

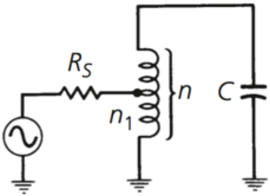
$$C_T = \frac{C_1 C_2}{C_1 + C_2}$$



    - We always set $R_s' = R_L$ for maximum power transfer

Impedance Transformation

- For a tapped-L circuit



From C. Bowick, RF Circuit Design, Newnes 2008

$$R_S' = R_S\left(\frac{n}{n_1}\right)^2$$

$$Q = \omega_0 R_S' C$$

$$Q_1 = \frac{R_S}{\omega_0 L_1}$$

$$R_S' = R_S\left(\frac{L}{L_1}\right)^2$$

$$L - L_1 = \frac{QQ_1 - Q_1^2}{1 + Q_2^2} L_1$$

- Similarly, we set $R_s' = R_L$ for maximum power transfer

Example

Design a resonant circuit with a loaded Q of 20 at a center frequency of 100MHz that will operate between a source resistance of 50Ω and a load resistance of 2000Ω. The inductor has a Q of 100 at 100MHz.

- Loaded Q = 20
- $f_o = 100\text{MHz}$
- $R_s = 50\Omega$
- $R_L = 2000\Omega$

- Used a tapped-C transformer to match R_s with R_L

  - Set $R_s' = R_L = 2000\Omega$

$$\frac{C_1}{C_2} = \sqrt{\frac{R_s'}{R_s}} - 1 = \sqrt{\frac{2000}{50}} - 1 = 5.3 \rightarrow C_1 = 5.3C_2$$

  - Using the component Q of the inductor

$$R_{loss,p} = QX_p \rightarrow R_{loss,p} = 100X_p$$

  - Using the loaded Q of the circuit

$$Q_{loaded} = 20 = \frac{R_{total}}{X_p} \qquad R_{total} = \frac{1000R_{loss,p}}{1000 + R_{loss,p}}$$

  - Solving the equations simultaneously

$$X_p = 40\Omega \qquad R_{loss,p} = 4000\Omega$$

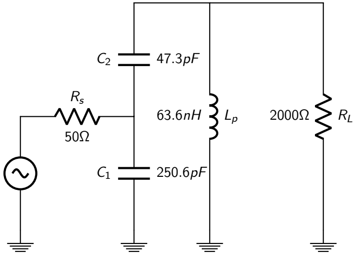
- Solving for the values of the components

$$L_p = \frac{X_p}{\omega} = 63.6nH \qquad C_T = \frac{1}{\omega X_p} = 39.78pF$$

- We can now solve for C_1 and C_2

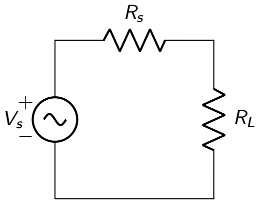
$$C_T = \frac{C_1 C_2}{C_1 + C_2} = 39.78pF \qquad C_1 = 5.3C_2$$

$$C_1 = 250.6pF \qquad C_2 = 47.3pF$$



Insertion Loss

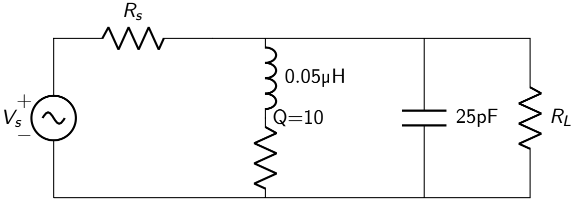
- Part of the power delivered by the source is dissipated by lossy resonant circuit placed in between source and load
- Consider the following circuit where $R_s = R_L$



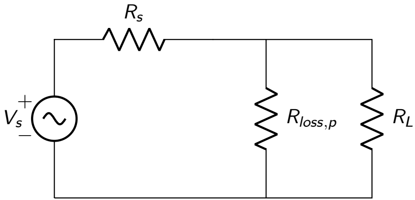
  - By voltage division,

$$\frac{V_L}{V_s} = \frac{R_L}{R_s + R_L} = \frac{1}{2}$$

- Inserting a parallel resonant circuit between source and load resistances



- At resonance, the circuit is reduced to

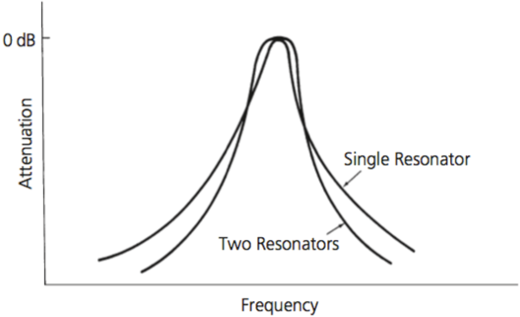


$$\frac{V_L}{V_s} = \frac{(R_{loss,p} \parallel R_L)}{R_s + (R_{loss,p} \parallel R_L)} < \frac{1}{2}$$

$$IL_{dB} = 20log(\frac{V_L/V_s}{0.5})$$

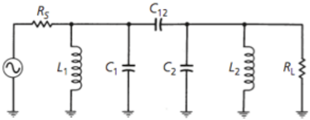
Coupling of Resonant Circuits

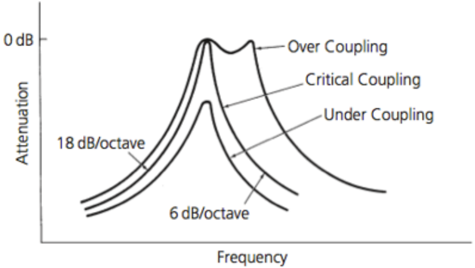
- In some applications, one resonant circuit may not be enough to meet necessary requirements
  - "steep" passband skirts
  - Small shape factors
- Resonant circuits may be coupled together to produce more attenuation at certain frequencies



Capacitive Coupling

- Most common due to simplicity and low cost



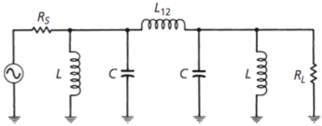


- Critical coupling
  - Reasonable bandwidth
  - Lowest insertion loss
  - Maximum power transfer

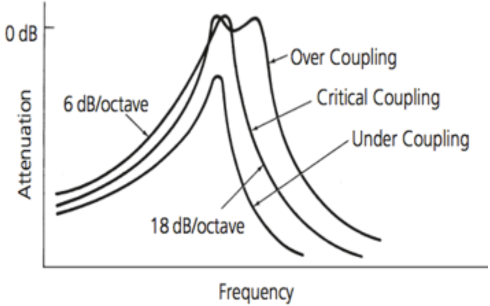
$$C_{12} = \frac{C}{Q}$$

  - $Q = 0.707 Q_{circuit}$

Inductive Coupling



(A) Series inductor



- For Critical Coupling

$$L_{12} = QL$$

- For same operating Q, $X_{L_12} = X_{C_12}$ at resonant frequency

$$\omega_o L_{12} = \frac{1}{\omega_o C_{12}}$$

Example

- Design a parallel resonant circuit to provide a 3dB bandwidth of 10 MHz at a center frequency of 100 MHz. The source and load impedances are 100Ω each. Assume the capacitor to be lossless, and the inductor have a Q of 85. What is the insertion loss of the network?

END