Lecture 16
TRANSMISSION LINE AND DISTRIBUTION LINE MODELS

Agenda

- **ANNOUNCEMENTS**
  LECTURE

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Announcements

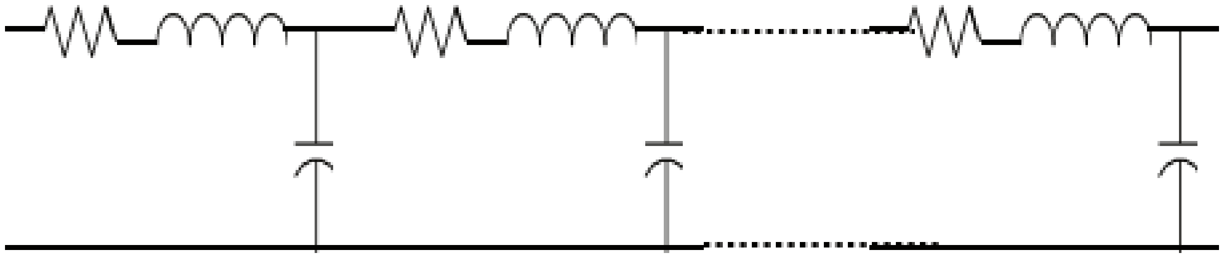
- Long Quiz 2 is on March 25, 2019 from 7 to 9AM
  - Early Exam(6 to 8AM) Takers should answer the survey in UVLE.

Lecture Outcomes

at the end of the lecture, the student must be able to ...

- Specify the equivalent circuit of short, medium, and long transmission lines.
- Identify the differences in performance of the transmission lines models

Equivalent Circuit of Transmission & Distribution Lines



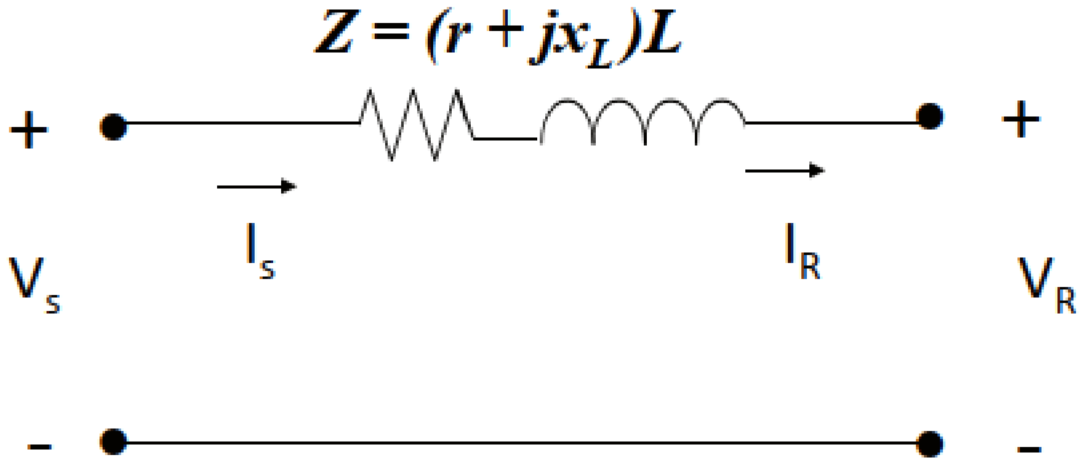
T&D Lines can be represented by a series of resistance and inductive reactance and shunt capacitance.

Equivalent circuit depends on the length of the Lines

a) Short Line………….ℓ < 80km (50 mi)
b) Medium Line……...80km (50mi) < ℓ < 240km (150mi)
c) Long Line………….ℓ > 240 km (150mi)

Short Line

Line length up to 80km (50mi)



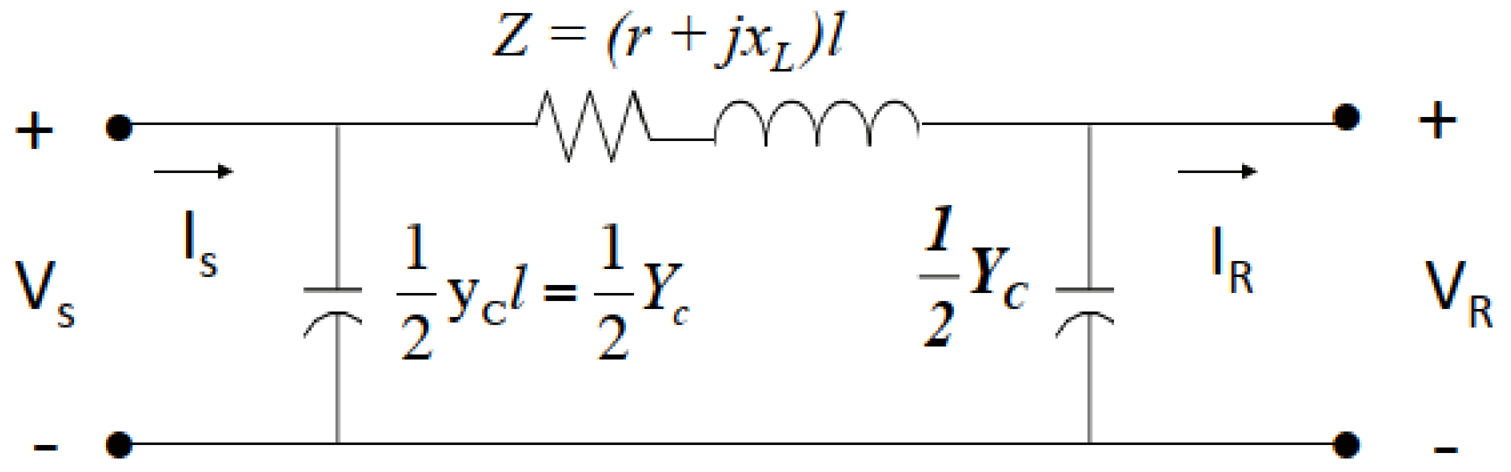
Lumped parameters; Capacitance neglected

$$I_S = I_R$$

$$V_S = ZI_S + V_R$$

Medium-Length Line

Line length from 80km (50mi) to 240km (150mi)



Lumped parameters Capacitance at both ends

$$V_S = (\, 1/2\, Z\, Y_C + 1\,)\, V_R + Z\, I_R$$

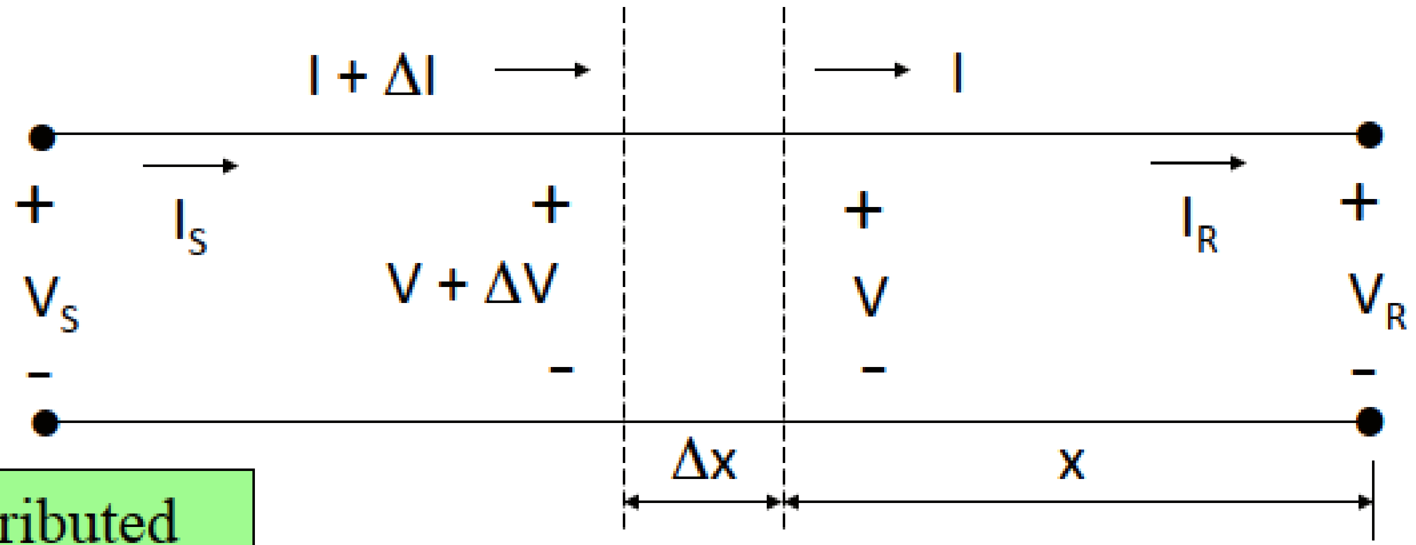
$$I_S = Y_C(\, 1 + 1/4\, ZY_C\,)V_R + (1/2\, ZY_C + 1\,)\, I_R$$

Long Line

Line length longer than 240km (150mi)

z = series impedance per unit length/phase

y = shunt admittance per unit length/phase, to neutral



Distributed parameters

$$\Delta V = Iz\Delta x \qquad \Delta I = Vy\Delta x$$

Long Line

$$\Delta V = Iz\Delta x \qquad \Delta I = Vy\Delta x$$

These equations may be re-written as

$$\frac{\Delta V}{\Delta x} = Iz \quad \frac{\Delta I}{\Delta x} = Vy \longrightarrow \frac{dV}{dx} = Iz \quad \frac{dI}{dx} = Vy$$

Another differentiation with respect to x yields

$$\frac{d^2V}{dx^2} = z\frac{dI}{dx} \qquad \frac{d^2I}{dx^2} = y\frac{dV}{dx}$$

$$\frac{d^2V}{dx^2} = Vyz \qquad \frac{d^2I}{dx^2} = Iyz$$

The solution to the simultaneous differential equations are:

$$V(x) = \frac{V_R + I_R Z_C}{2} e^{\gamma x} + \frac{V_R - I_R Z_c}{2} e^{-\gamma x}$$

$$I(x) = \frac{\frac{V_R}{Z_C} + I_R}{2} e^{\gamma x} - \frac{\frac{V_R}{Z_C} - I_R}{2} e^{-\gamma x}$$

$Z_C = \sqrt{\frac{z}{y}}$ = characteristic impedance

$\gamma = \sqrt{zy}$ = propagation constant = $\alpha + j\beta$

α = attenuation constant, nepers/length

β = phase constant, rad/length

Since $$\sinh\theta = \frac{e^{\theta} - e^{-\theta}}{2} \qquad \cosh\theta = \frac{e^{\theta} + e^{-\theta}}{2}$$

For Long Transmission Line of length ($x = l$)

$$V = V_R \cosh\gamma l + I_R Z_C \sinh\gamma l$$
$$I = I_R \cosh\gamma l + \frac{V_R}{Z_C}\sinh\gamma l$$

Note that $\gamma = \alpha + j\beta$

$$\cosh\gamma l = \cosh(\alpha l + j\beta l) = \cosh\alpha l\cos\beta l + j\sinh\alpha l\sin\beta l$$
$$\sinh\gamma l = \sinh(\alpha l + j\beta l) = \sinh\alpha l\cos\beta l + j\cosh\alpha l\sin\beta l$$

Example

A single-circuit 60-Hz 3-phase line is 160 km long. The phase conductors are Drake with flat horizontal configuration and 7.5 m between conductors. The load at the receiving end is 50 MW at 0.85 pf and 220 kV.

a. Determine the equivalent circuit of the line.
b. Find the sending-end voltage, current, and power

Drake: 795,000 CM ACSR 26/7

outside diameter = 1.108 in. = 0.02814 m

resistance = 0.1284 Ω/mi. = 0.0803 Ω/km @ 50 ^{O}C

GMR = 0.0373 ft. = 0.01137 m

$$GMD = \sqrt[3]{7.5 \times 7.5 \times 15} = 9.45 \text{ m}$$

$$L = 2 \times 10^{-7} \ln \frac{GMD}{GMR} \frac{\text{H}}{\text{m}} = 2 \times 10^{-7} \ln \frac{9.45}{0.01137} = 1.345 \ \mu\text{H/m}$$

$$X_L = 2\pi \times 60 \times L = 506.9 \times 10^{-6} \ \Omega/\text{m}$$

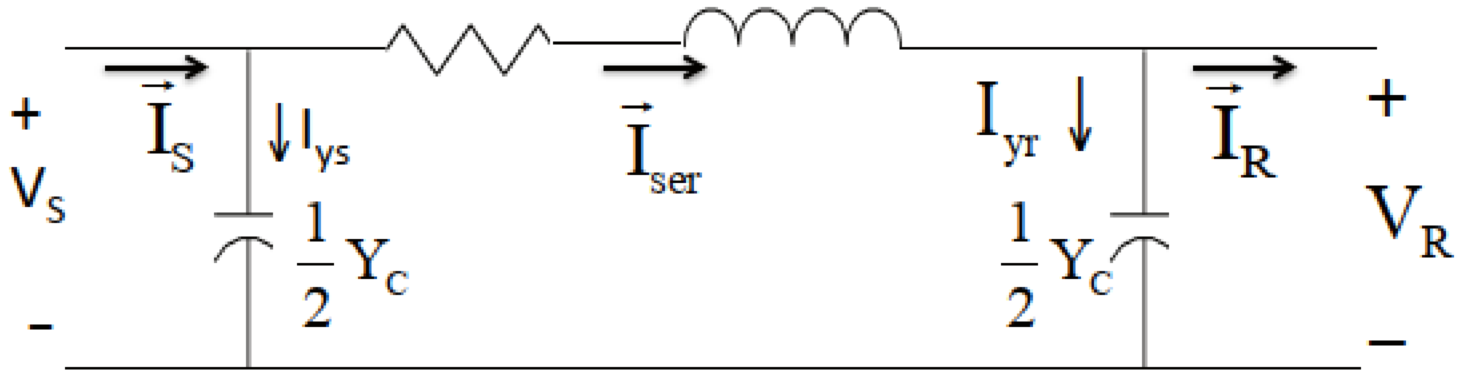
$$z = 0.0803 + j0.5069 \ \Omega/\text{km}$$

$$Z = 12.85 + j81.10 \ \Omega$$

$$C_{an} = \frac{2\pi\varepsilon_0}{\ln GMD/r} = \frac{2\pi\varepsilon_0}{\ln 9.45/0.01407} = 8.546 \text{ pF/m}$$

$$X_C = \frac{1}{2\pi 60 \cdot C_{an} \cdot s} = 1{,}939.9 \ \Omega \rightarrow Y_C = 0.515 \text{ mS}$$

$$Z = 12.85 + j81.10\ \Omega$$



$$Y_C = 0.515\ \text{mS}$$

$$V_R = \frac{220k}{\sqrt{3}} = 127.02\ \text{kV} \rightarrow \vec{V}_R = 127.02\angle 0^\circ\ \text{kV}$$

$$I_R = \frac{50,000}{\sqrt{3}(220k)(0.85)} = 154.37\ \text{A} \rightarrow \vec{I}_R = 154.37\angle -31.79^\circ\ \text{A}$$

$$\vec{I}_{YR} = \frac{1}{2}\vec{Y}_C\vec{V}_R = \frac{1}{2}\left(j515\times 10^{-6}\right)\left(127.02\times 10^{3}\right) = j32.71\ \text{A}$$

$$\vec{I}_{ser} = \vec{I}_{YR} + \vec{I}_R = 154.37\angle -31.79^\circ + j32.71 = 139.93\angle -20.33^\circ$$

$$\vec{V}_S = \vec{V}_R + \vec{I}_{ser}\vec{Z} = 127.02k\angle 0° + (139.93\angle -20.33°)(12.85 + j81.10)$$
$$= 133.03k\angle 4.32°$$

$$\vec{I}_{ys} = \frac{1}{2}\vec{Y}_C\vec{V}_S = \frac{1}{2}\left(j515\times 10^{-6}\right)\left(133.03\times 10^{3}\angle 4.32\right)$$
$$= 34.25\angle 94.32°$$

$$\vec{I}_S = \vec{I}_{ys} + \vec{I}_{ser} = 34.25\angle 94.32° + 139.93\angle -20.33°$$
$$= 129.44\angle -6.41° \text{ A}$$

At the sending end,

$$V_S = \sqrt{3}\left(133.03k\right) = 230.4 \text{ kV}$$
$$I_S = 129.44 \text{ A}$$
$$P_S = \sqrt{3}\left(230.4\right)\left(129.44\right)\cos\left(4.32 + 6.41\right) = 50.76 \text{ MW}$$

Example

A single-circuit, 60-Hz, 3-phase line is 370 km long. The phase conductors are *Rook* with flat horizontal configuration and 7.25 m between conductors. The load at the receiving end is 125 MW at 1.0 pf and 215 kV. Find the voltage, current, and power at the sending end. Also, find the wavelength and velocity of propagation of the line.

Rook: 636,000 CM ACSR 24/7

outside diameter = 0.977 in = 0.0248 m

resistance = 0.1603 Ω/mi = 0.1002 Ω/km @ 50 ^{O}C

GMR = 0.0327 ft. = 0.0100 m

$$GMD = \sqrt[3]{7.25 \times 7.25 \times 14.5} = 9.134 \text{ m}$$

$$x_L = 2\pi fL = 2\pi \cdot 60 \cdot 2 \cdot 10^{-7} \ln \frac{GMD}{GMR} = 514 \times 10^{-6} \frac{\Omega}{\text{m}} = 0.514 \frac{\Omega}{\text{km}}$$

$$z = 0.1002 + j0.514 \ \frac{\Omega}{\text{km}}$$

$$y_C = 2\pi fC_{an} = 2\pi \cdot 60 \cdot \left(\frac{2\pi\varepsilon_0}{\ln GMD/r} \right) = 3.177 \times 10^{-6} \text{ S/km}$$

$$Z_C = \sqrt{\frac{z}{y_C}} = \sqrt{\frac{0.1002 + j0.514}{j3.177 \times 10^{-6}}}$$

$$= \sqrt{164.83 \times 10^{3} \angle -11.03^\circ} = 406 \angle -5.52^\circ \ \Omega$$

$$\gamma = \sqrt{zy_C} = \sqrt{1.664 \times 10^{-6} \angle 168.97^\circ} = 1.29 \times 10^{-3} \angle 84.49^\circ \text{ /km}$$

$$= 0.00012 + j0.00128 \text{ / km}$$

$$\gamma l = 0.0458 + j0.4751$$

$$\vec{V}_R = \frac{215 \text{ kV}}{\sqrt{3}} = 124.13\angle 0° \text{ kV}$$

$$\vec{I}_R = \frac{125{,}000}{\sqrt{3}(215)} = 335.7\angle 0° \text{ A}$$

$$\begin{aligned}\cosh\gamma l &= \cosh 0.0458 \cos 0.4751 + j\sinh 0.0458 \sin 0.4751 \\ &= 0.8902 + j0.0210\end{aligned}$$

$$\begin{aligned}\sinh\gamma l &= \sinh 0.0458 \cos 0.4751 + j\cosh 0.0458 \sin 0.4751 \\ &= 0.0407 + j0.4579\end{aligned}$$

$$\begin{aligned}\vec{V}_S &= \vec{V}_R \cosh\gamma l + \vec{I}_R \vec{Z}_C \sinh\gamma l \\ &= \left(124.13\times 10^3\right)\left(0.8902 + j0.0210\right) \\ &\quad + \left(335.7\right)\left(406\angle -5.52\right)\left(0.0407 + j0.4579\right) \\ &= 137.88\angle 27.75° \text{ kV}\end{aligned}$$

$$\vec{I}_S = \vec{I}_R \cosh \gamma l + \frac{\vec{V}_R}{\vec{Z}_C} \sinh \gamma l$$

$$= (335.7)(0.8902 + j0.0210) + \left(\frac{124.13 \times 10^3}{406\angle -5.52} \right)(0.0407 + j0.4579)$$

$$= 332.33\angle 26.36° \text{ A}$$

At the sending end,

$$V_S = \sqrt{3}(137.88k) = 238.82 \text{ kV}$$

$$I_S = 332.33 \text{ A}$$

$$P_S = \sqrt{3}(238.82)(332.33)\cos(27.75 - 26.36) = 137.04 \text{ MW}$$

END