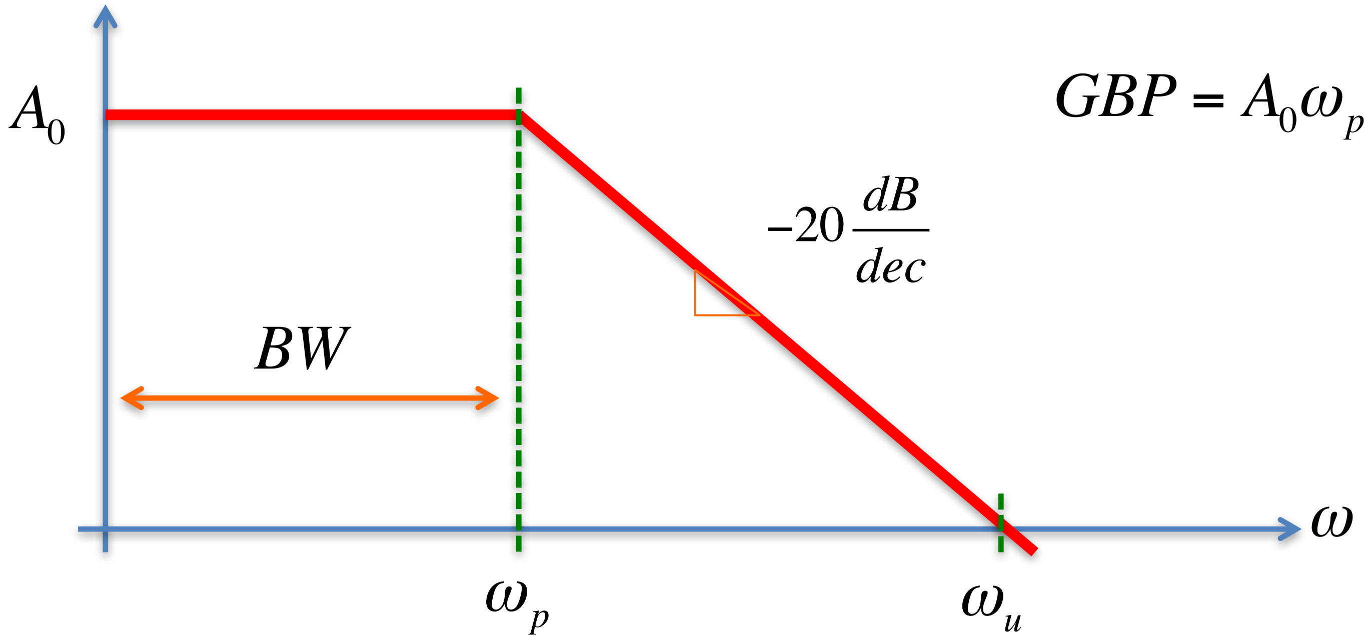
# EEE 51: Second Semester 2017 - 2018

## Lecture 17

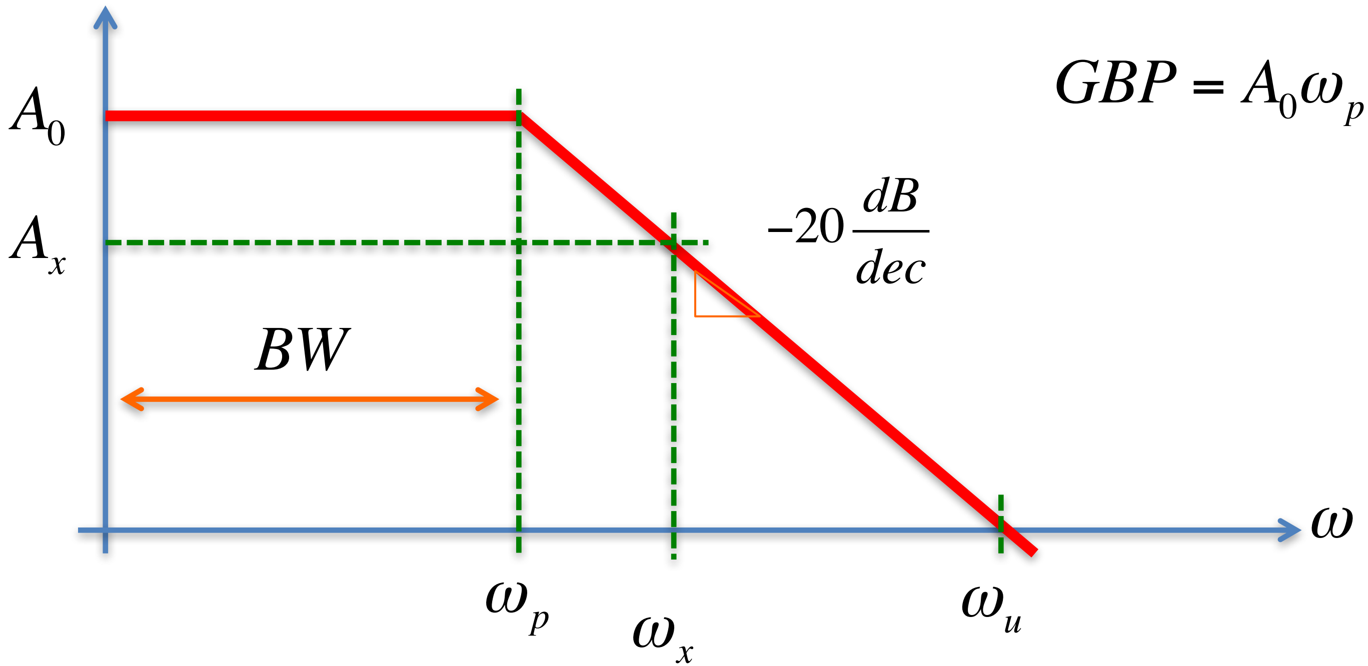
# Frequency Response

# Gain-Bandwidth Product (GBP)




$$|A| = 1 = \frac{A_0}{\sqrt{1+\left(\frac{\omega_u}{\omega_p}\right)^2}} = \frac{A_0^2}{1+\left(\frac{\omega_u}{\omega_p}\right)^2} \approx \frac{A_0^2}{\left(\frac{\omega_u}{\omega_p}\right)^2} \Rightarrow \omega_u = A_0\omega_p = GBP$$

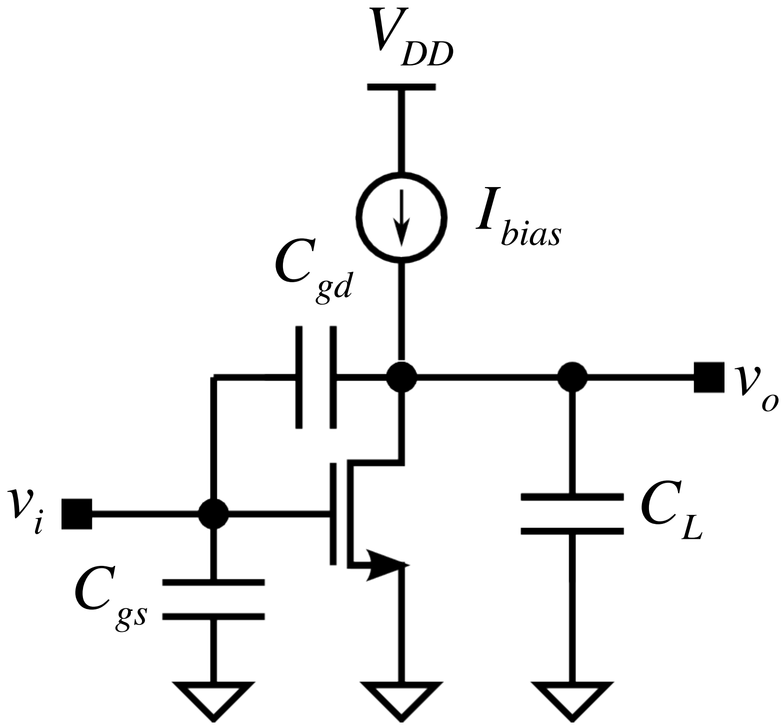
# Gain-Bandwidth Product (GBP)



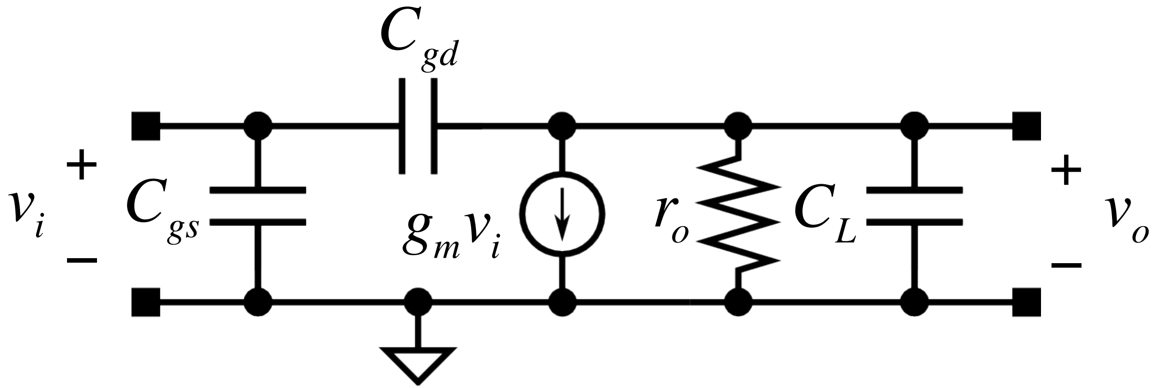

$$GBP = A_0\omega_p$$

$$|A|^2 = A_x^2 = \frac{A_0^2}{1+\left(\frac{\omega_x}{\omega_p}\right)^2} \approx \frac{A_0^2}{\left(\frac{\omega_x}{\omega_p}\right)^2} \Rightarrow A_x\omega_x = A_0\omega_p = GBP$$

# Common Source Amplifier



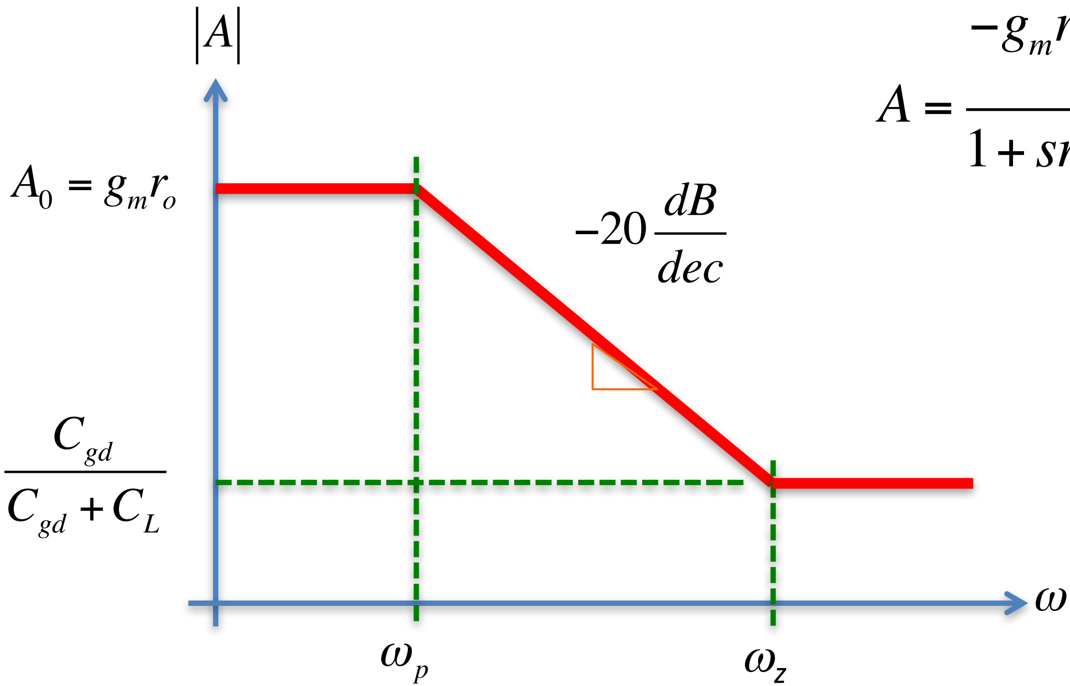
# Common Source Amplifier




$$v_o\left(\frac{1}{r_o}+sC_L+sC_{gd}\right)-v_i sC_{gd}+g_m v_i=0$$

$$\frac{v_o}{v_i}=\frac{-g_m+sC_{gd}}{\frac{1}{r_o}+s\left(C_L+C_{gd}\right)}=\frac{-g_m r_o\left(1-s\frac{C_{gd}}{g_m}\right)}{1+sr_o\left(C_L+C_{gd}\right)}=\frac{-A_0\left(1-\frac{s}{z}\right)}{\left(1+\frac{s}{p}\right)}$$

# Common Source Amplifier



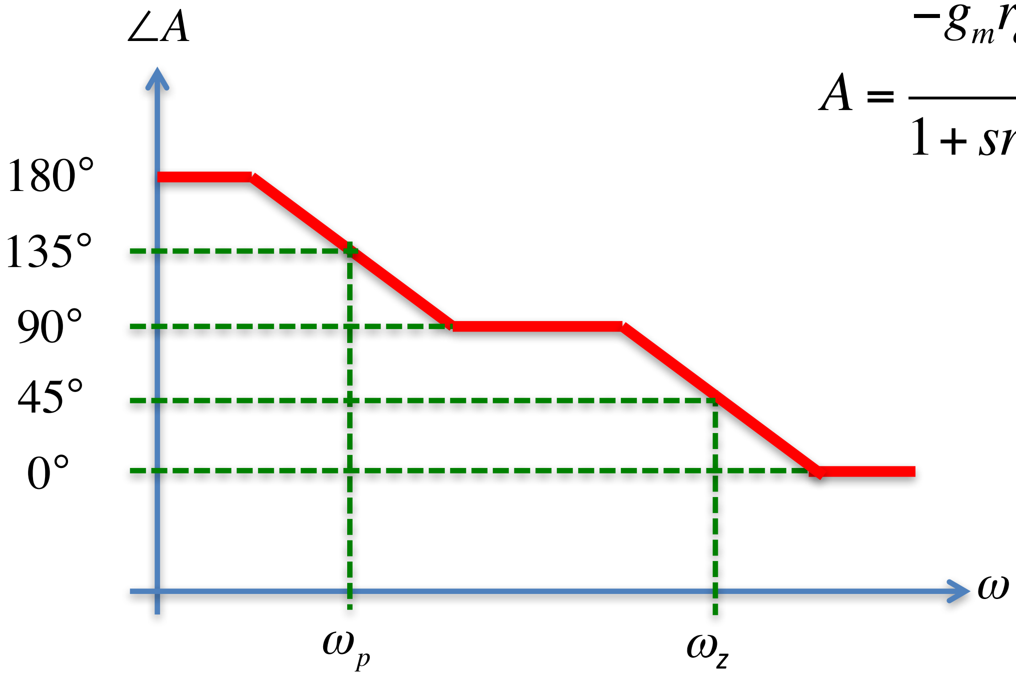

$$A = \frac{-g_m r_o\left(1 - s\frac{C_{gd}}{g_m}\right)}{1 + s r_o\left(C_L + C_{gd}\right)} = \frac{-A_0\left(1 - \frac{s}{z}\right)}{\left(1 + \frac{s}{p}\right)}$$

$$A_0 = g_m r_o$$

$$\omega_p = \frac{1}{r_o\left(C_L + C_{gd}\right)}$$

$$\omega_z = \frac{g_m}{C_{gd}}$$

# Common Source Amplifier



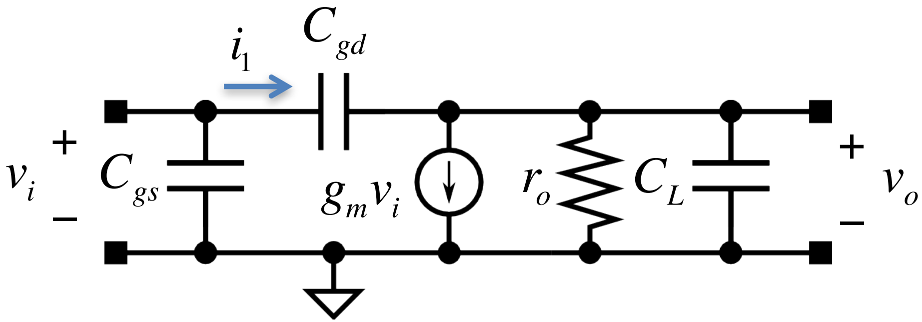

$$A = \frac{-g_m r_o\left(1 - s\frac{C_{gd}}{g_m}\right)}{1 + s r_o\left(C_L + C_{gd}\right)} = \frac{-A_0\left(1 - \frac{s}{z}\right)}{\left(1 + \frac{s}{p}\right)}$$

$$A_0 = g_m r_o$$

$$\omega_p = \frac{1}{r_o\left(C_L + C_{gd}\right)}$$

$$\omega_z = \frac{g_m}{C_{gd}}$$

# Common Source Amplifier Input Impedance

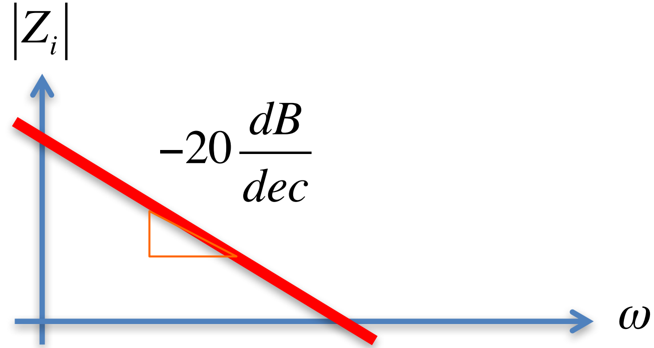



$$A = \frac{-g_m r_o\left(1 - s\frac{C_{gd}}{g_m}\right)}{1 + s r_o\left(C_L + C_{gd}\right)}$$

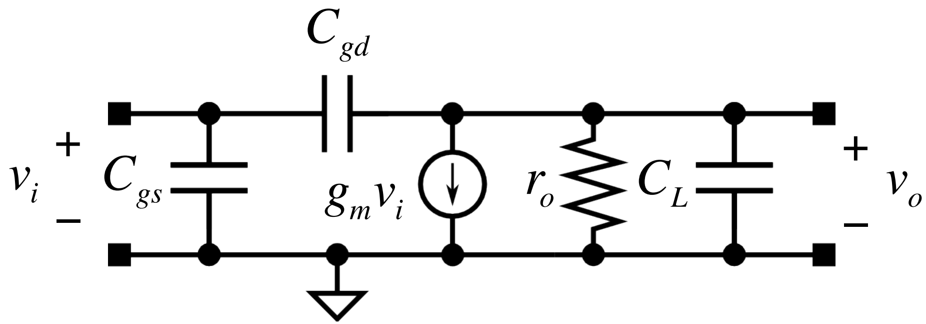
$$i_1 = \left(v_i - v_o\right) sC_{gd} = v_i\left(1 - \frac{v_o}{v_i}\right) sC_{gd} = v_i\left(1 - A\right) sC_{gd}$$

$$\frac{v_i}{i_1} = \frac{1}{s\left(1 - A\right) C_{gd}}$$

$$Z_i = \frac{1}{s\left[C_{gs} + \left(1 - A\right) C_{gd}\right]}$$

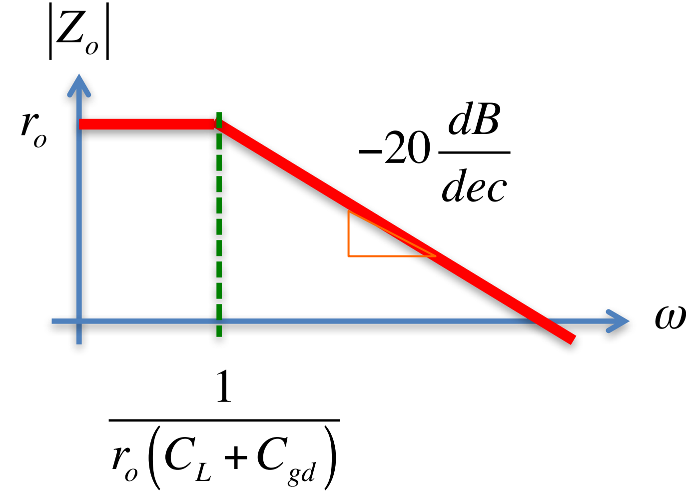


# Common Source Amplifier Output Impedance

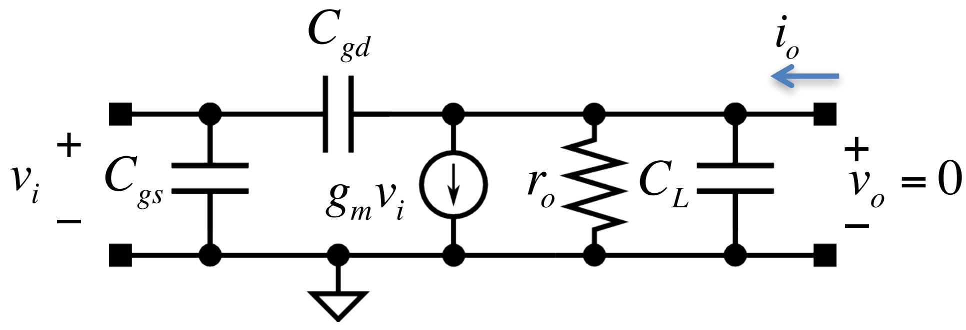



$$A = \frac{-g_m r_o \left(1 - s\frac{C_{gd}}{g_m}\right)}{1 + s r_o \left(C_L + C_{gd}\right)}$$

$$Z_o = r_o \parallel \frac{1}{s\left(C_L + C_{gd}\right)} = \frac{r_o}{1 + s r_o \left(C_L + C_{gd}\right)}$$



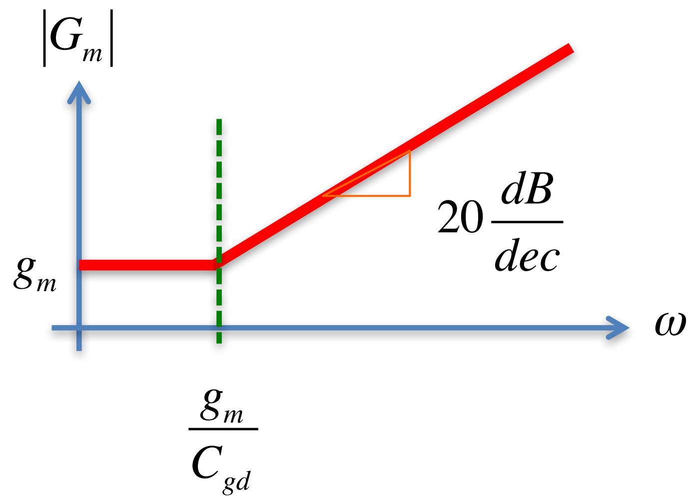
# CS Amplifier Effective Transconductance



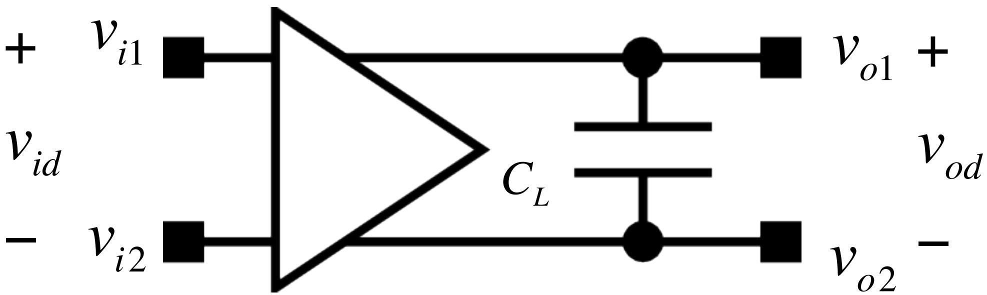

$$A = \frac{-g_m r_o \left(1 - s\frac{C_{gd}}{g_m}\right)}{1 + s r_o \left(C_L + C_{gd}\right)}$$

$$i_o = g_m v_i - v_i s C_{gd}$$

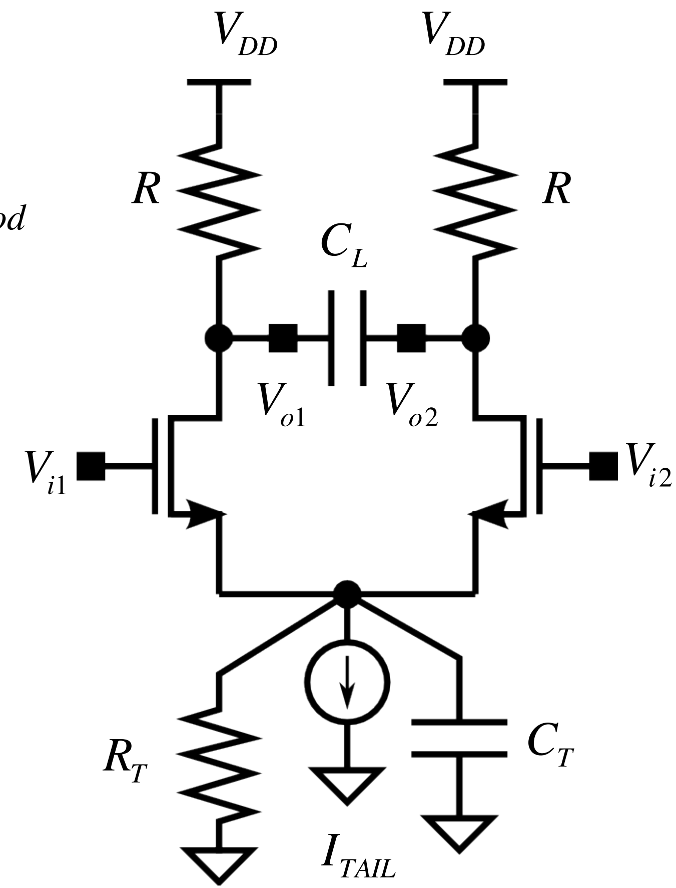
$$G_m = \frac{i_o}{v_i} = g_m - sC_{gd} = g_m\left(1 - s\frac{C_{gd}}{g_m}\right)$$



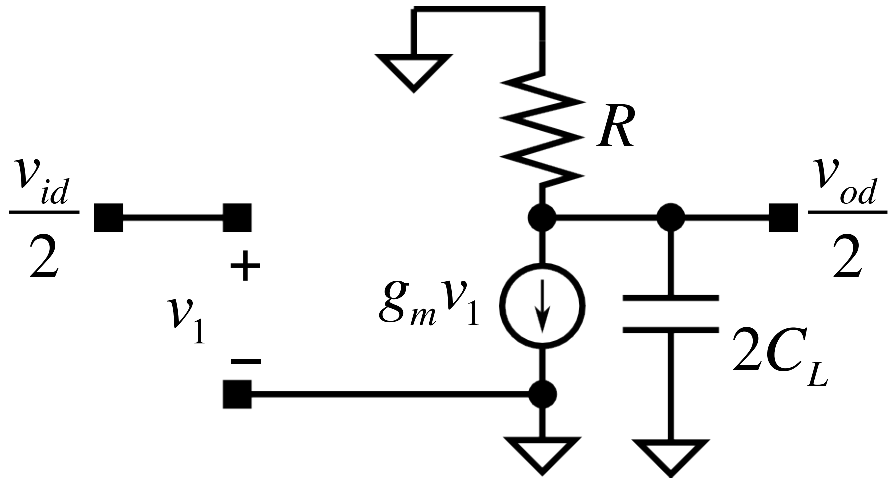
# Differential Amplifier Frequency Response




Differential-Mode Gain?
Common-Mode Gain?

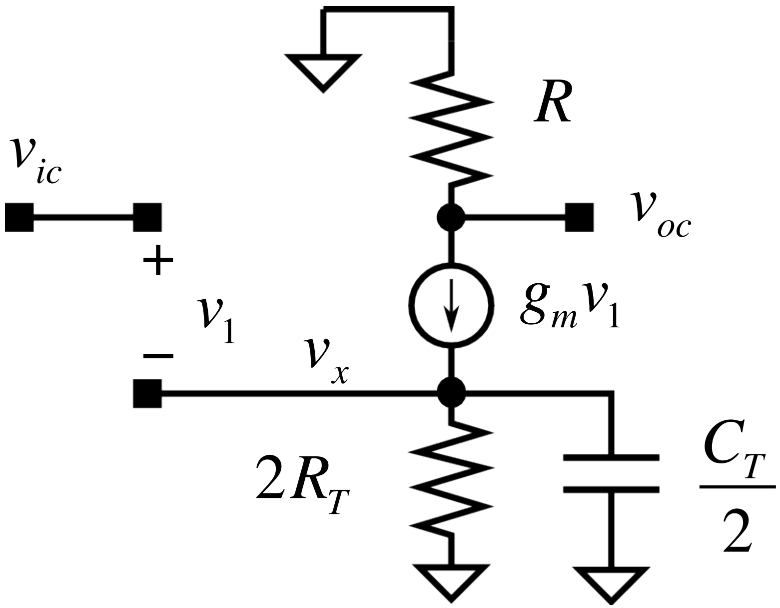


# Differential Mode Half Circuit




$$A_{dm} = \frac{-g_m R}{1 + s\left(2RC_L\right)}$$

# Common Mode Half Circuit

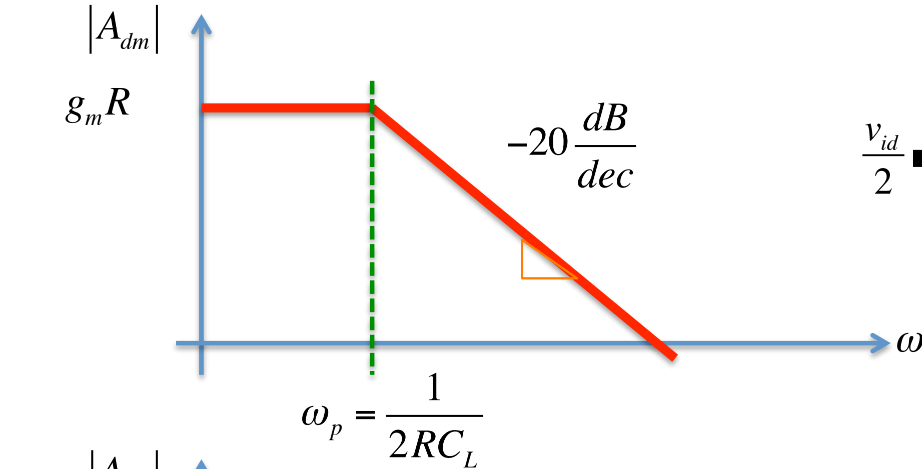


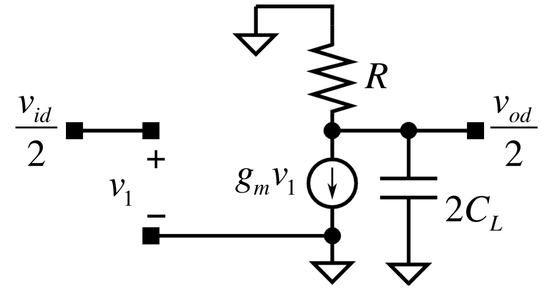

$$v_x\left(\frac{1}{2R_T}+s\frac{C_T}{2}\right)-g_m\left(v_{ic}-v_x\right)=0$$

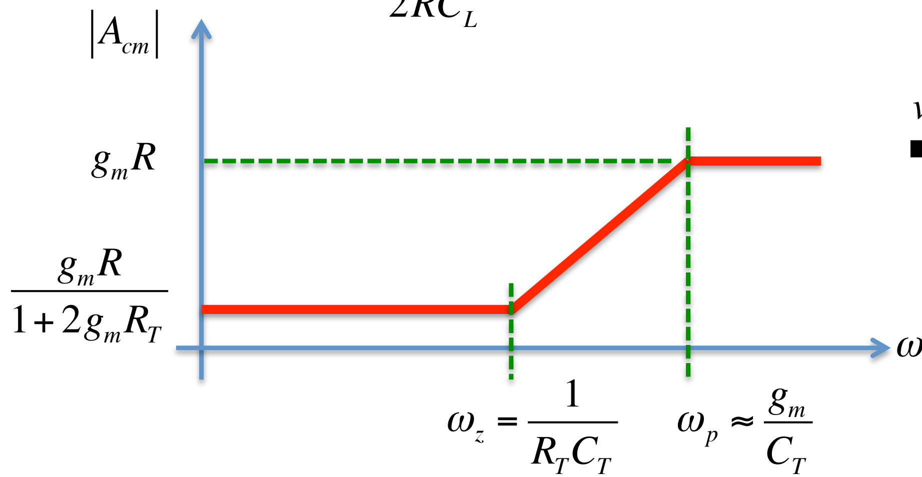
$$v_o\left(\frac{1}{R}\right)+g_m\left(v_{ic}-v_x\right)=0$$

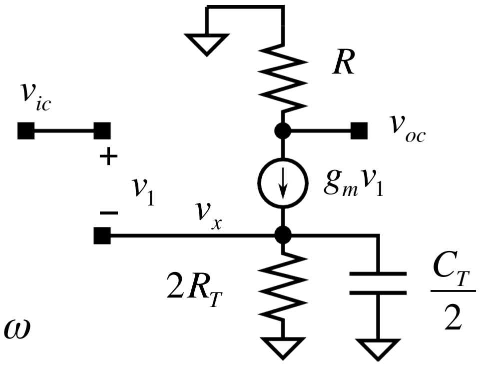
$$A_{cm}=\frac{-g_m R}{1+2g_m R_T}\left(\frac{1+sR_T C_T}{1+s\frac{R_T C_T}{1+2g_m R_T}}\right)$$

# Differential Amplifier Frequency Response

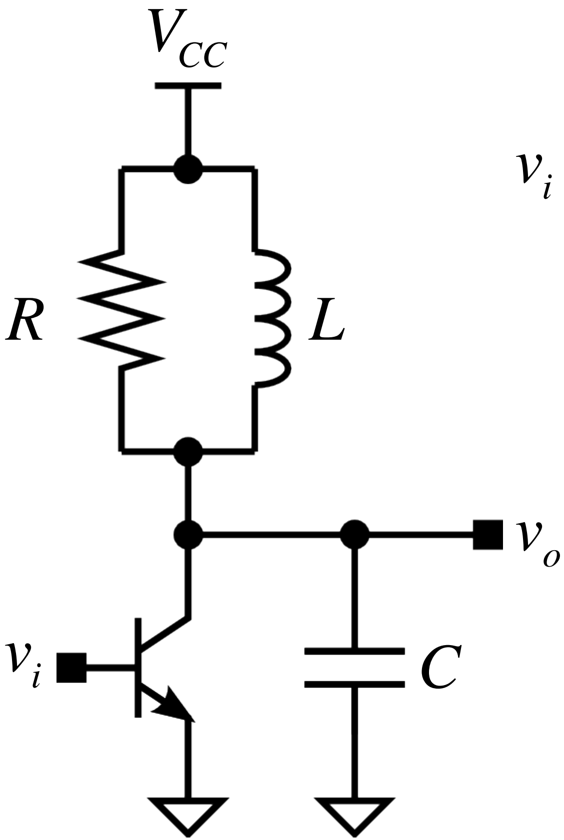


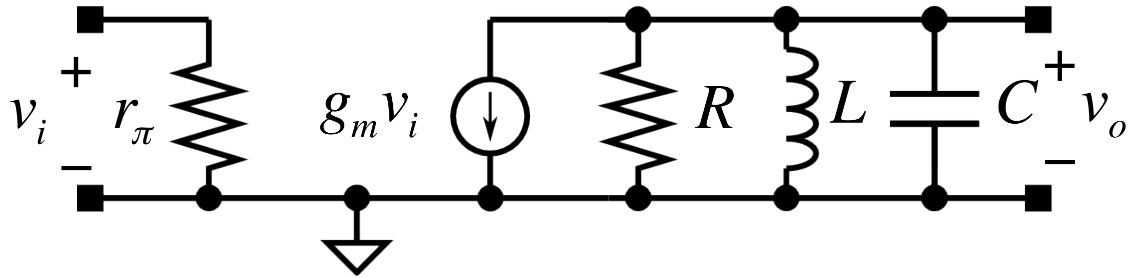



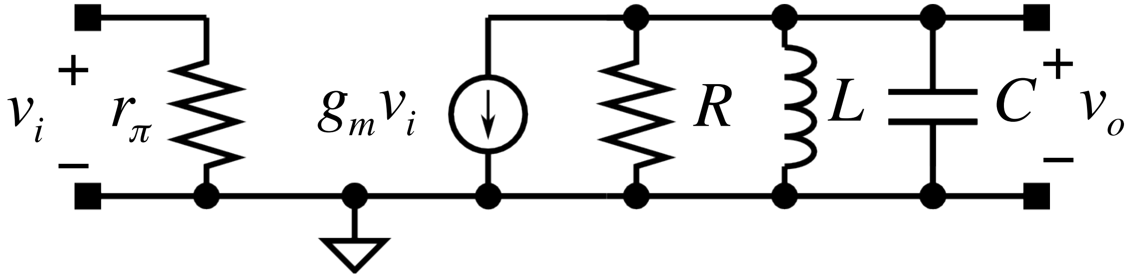
# Tuned CE Amplifier




$$A_v = \frac{v_o}{v_i} = -g_m \left( R \parallel sL \parallel \frac{1}{sC} \right)$$

# Tuned CE Amplifier




$$\omega_0 = \frac{1}{\sqrt{LC}}$$

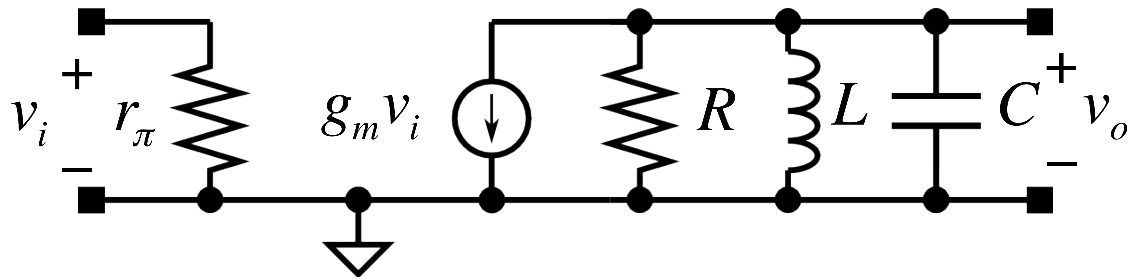
$$a = \frac{1}{2RC}$$

$$A_v = -g_m \left( R \parallel sL \parallel \frac{1}{sC} \right)$$

$$= -g_m R \frac{sL}{s^2 RLC + sL + R} = -g_m R \frac{s\frac{1}{RC}}{s^2 + s\frac{1}{RC} + \frac{1}{LC}}$$

$$= -g_m R \frac{2as}{s^2 + 2as + \omega_0^2}$$

# Tuned CE Amplifier



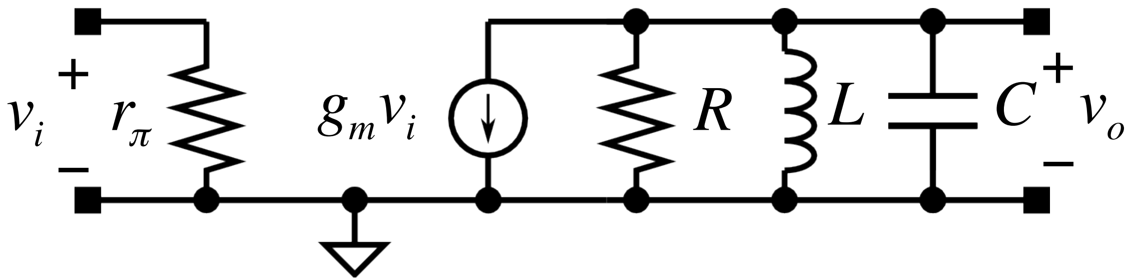

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$a = \frac{1}{2RC}$$

$$|A_v| = \left| -g_m R \frac{2as}{s^2 + 2as + \omega_0^2} \right| = g_m R \left| \frac{j2a\omega}{j2a\omega + \left(\omega_0^2 - \omega^2\right)} \right|$$

$$= g_m R \frac{2a\omega}{\sqrt{\left(2a\omega\right)^2 + \left(\omega_0^2 - \omega^2\right)^2}}$$

# Tuned CE Amplifier Bandwidth



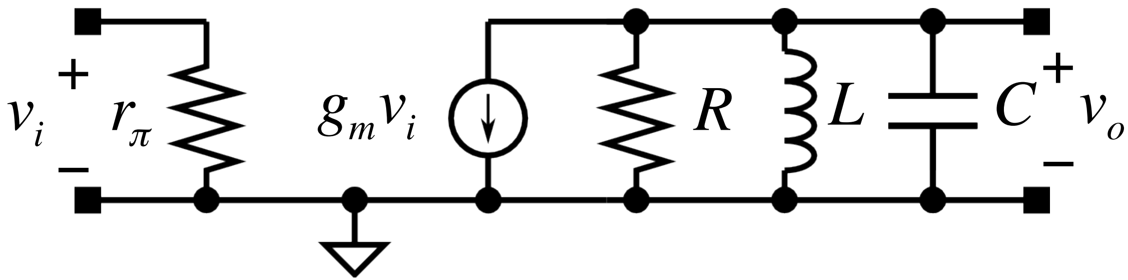

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$a = \frac{1}{2RC}$$

$$\left|A_v\right| = g_m R \frac{2a\omega}{\sqrt{\left(2a\omega\right)^2 + \left(\omega_0^2 - \omega^2\right)^2}}$$

$$\left|A_{v,3dB}\right| = \frac{g_m R}{\sqrt{2}} \quad \Rightarrow \quad \left(\omega_0^2 - \omega^2\right)^2 = \left(2a\omega\right)^2$$

# Tuned CE Amplifier Bandwidth




$$\omega_0 = \frac{1}{\sqrt{LC}}$$

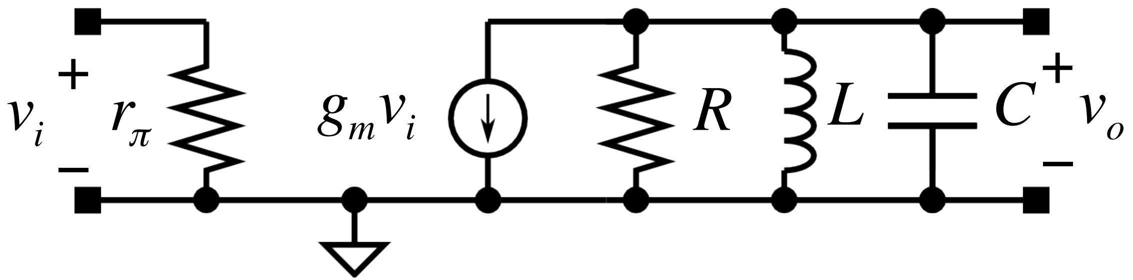
$$a = \frac{1}{2RC}$$

$$\left(\omega_0^2 - \omega^2\right)^2 = \left(2a\omega\right)^2$$

$$\pm\left(\omega_0^2 - \omega^2\right) = \pm 2a\omega$$

$$\omega^2 + 2a\omega - \omega_0^2 = 0 \qquad\qquad \omega^2 - 2a\omega - \omega_0^2 = 0$$

# Tuned CE Amplifier Bandwidth




$$\omega_0 = \frac{1}{\sqrt{LC}}$$

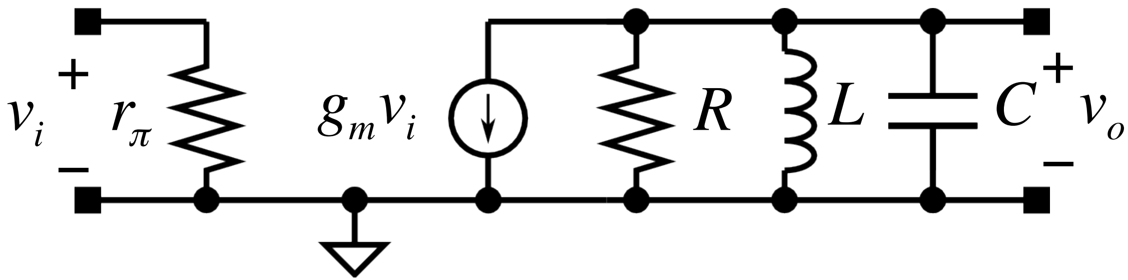
$$a = \frac{1}{2RC}$$

$$\omega^2 + 2a\omega - \omega_0^2 = 0 \qquad \omega^2 - 2a\omega - \omega_0^2 = 0$$

$$\omega = \frac{-2a \pm \sqrt{(2a)^2 + 4\omega_0^2}}{2} = \begin{cases} -a + \sqrt{a^2 + \omega_0^2} \\ -a - \sqrt{a^2 + \omega_0^2} \end{cases}$$

$$\omega = \frac{2a \pm \sqrt{(2a)^2 + 4\omega_0^2}}{2} = \begin{cases} a + \sqrt{a^2 + \omega_0^2} \\ a - \sqrt{a^2 + \omega_0^2} \end{cases}$$

# Tuned CE Amplifier Bandwidth




$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$a = \frac{1}{2RC}$$

$$\omega_1 = -a + \sqrt{a^2 + \omega_0^2}$$

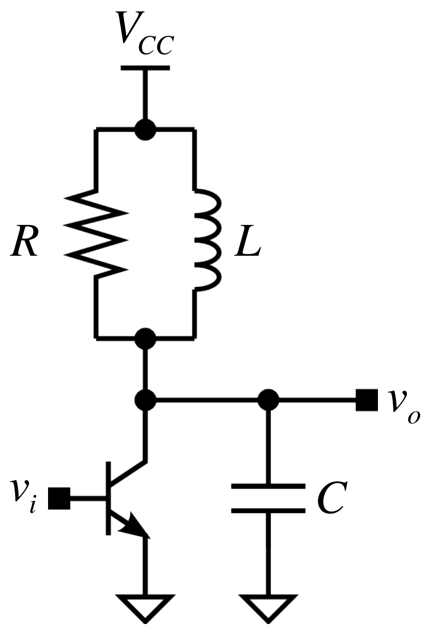
$$\omega_2 = a + \sqrt{a^2 + \omega_0^2}$$

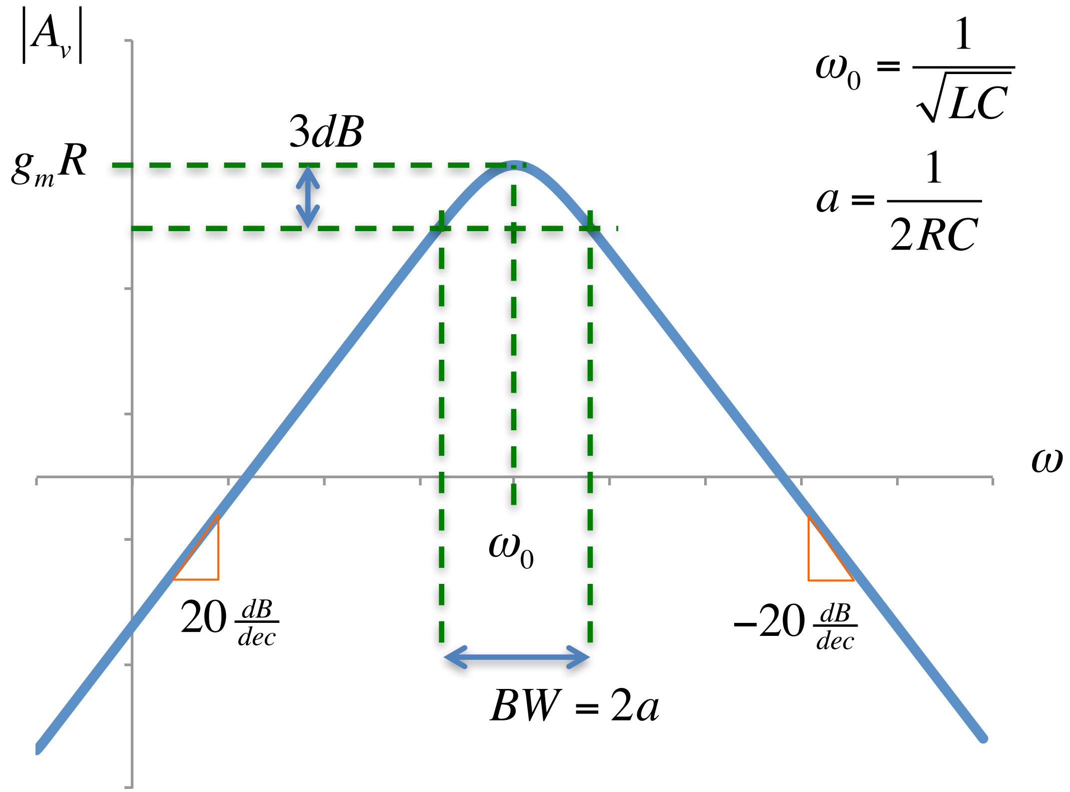
$$BW = \omega_2 - \omega_1 = 2a$$

$$\begin{aligned}\omega_1\omega_2 &= \left(-a + \sqrt{a^2+\omega_0^2}\right)\left(a + \sqrt{a^2+\omega_0^2}\right)\\ &= -a^2 + a\sqrt{a^2+\omega_0^2} - a\sqrt{a^2+\omega_0^2} + a^2 + \omega_0^2\\ &= \omega_0^2\end{aligned}$$

$$\omega_0 = \sqrt{\omega_2\omega_1}$$

# Magnitude Response




$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$a = \frac{1}{2RC}$$

$$Q = \frac{\omega_0}{BW} = \frac{1}{\sqrt{LC}} \cdot 2\left(\frac{RC}{2}\right) = R\sqrt{\frac{C}{L}}$$

# Next Meeting

- Feedback Amplifiers