ECE 113: Communication Electronics

Meeting 4: Network Analysis I

February 4, 2019

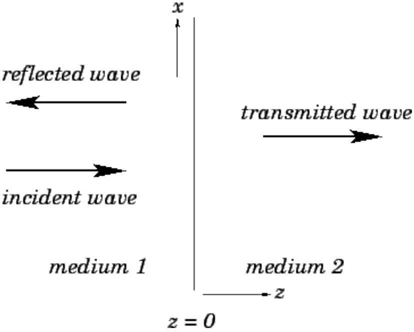
Circuits at Low Frequencies

- Circuit dimensions are relatively small relative to its wavelength ($\lambda = \frac{c}{f}$)
  - e.g. $f = 100\text{kHz} \rightarrow \lambda = 3\text{km}$
- Voltages and Currents are defined at any point in the circuit
  - Use KVL, KCL and Ohm's Law to analyze these circuits
- Treat as interconnection of lumped passive/active elements

Circuits at High Frequencies

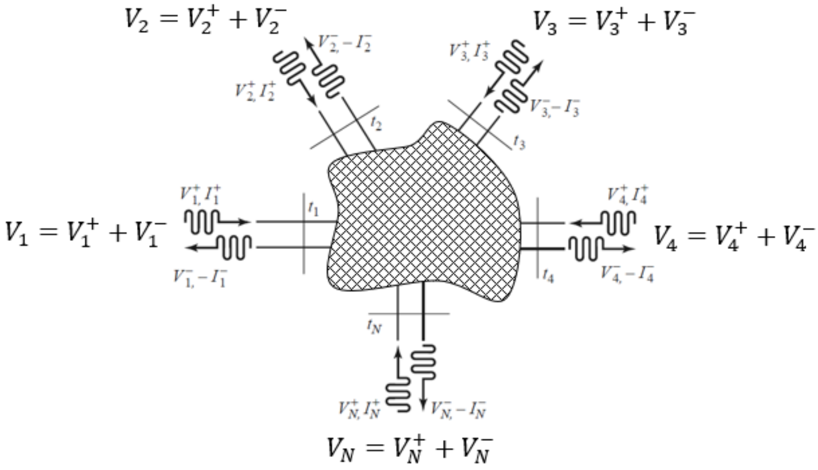
- Circuit dimension becomes comparable with its wavelength
  - e.g. $f = 1\text{GHz} \rightarrow \lambda = 30\text{cm}$
- Measurement of voltages and currents becomes difficult (or almost impossible)
  - Complete circuit analysis requires solving Maxwell's Equations
- It is more convenient to express our circuits (networks) in terms of traveling waves (incident and reflected waves)

Traveling Waves



- Voltages and Currents at a specified terminal (port) can be identified as its incident/reflected wave
- At any given terminal/port n, the voltage and current is given by:
  - $V_n = V_n^+ + V_n^-$
  - $I_n = I_n^+ + I_n^-$

N-Port Network



Impedance Parameters

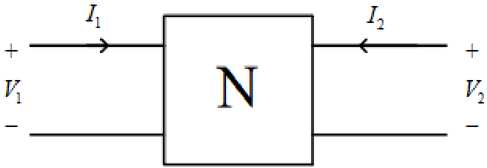
- The impedance matrix [Z] of a network relates the total voltages and currents:

$$\begin{bmatrix} V_1 \\ V_2 \\ \cdot \\ \cdot \\ V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdot & \cdot & Z_{1N} \\ Z_{21} & Z_{22} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ Z_{N1} & \cdot & \cdot & \cdot & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \cdot \\ \cdot \\ I_N \end{bmatrix}$$

- In matrix notation

$$[V]_{N\times 1} = [Z]_{N\times N}[I]_{N\times 1}$$

Z-matrix Elements



- The elements of the Z-matrix Z_{ij} can be evaluated:

$$Z_{ij} = \frac{V_i}{I_j}|_{I_k=0} \text{ for } k \neq j$$

- Drive port **j** with current I_j, open circuit all other ports, and measure the open circuit voltage at port **i**.

Admittance Parameters

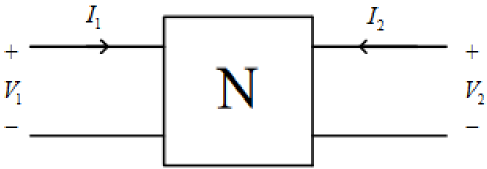
- The admittance matrix [Y] of a network relates the total voltages and currents

$$\begin{bmatrix} I_1 \\ I_2 \\ \cdot \\ \cdot \\ I_N \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdot & \cdot & Y_{1N} \\ Y_{21} & Y_{22} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ Y_{N1} & \cdot & \cdot & \cdot & Y_{NN} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \cdot \\ \cdot \\ V_N \end{bmatrix}$$

- In matrix notation

$$[I]_{N\times 1} = [Y]_{N\times N}[V]_{N\times 1}$$

Y-matrix Elements



- The elements of the Y-matrix Y_{ij} can be evaluated:

$$Y_{ij} = \frac{I_i}{V_j}|_{V_k=0} \text{ for } k \neq j$$

- Drive port **j** with voltage V_j, short circuit all other ports, and measure the short circuit current at port **i**.

Z and Y Parameters

- Elements of the Z and Y matrices are complex
- Z and Y matrices are inverse of each other: $[Y] = [Z]^{-1}$
- For an N-port network, Y and Z matrices are $N \times N$ in size
  - N^2 independent quantities that characterize an arbitrary N-port network
- $\#$ of independent parameters are reduced for special cases
  - Reciprocal and Lossless Networks

Reciprocal Network

- A reciprocal network is a network that does not contain any non-reciprocal (or non-linear) media
  - passive and contains only isotropic materials
  - Results in a **symmetric** matrix

$$Z_{ij} = Z_{ji}$$

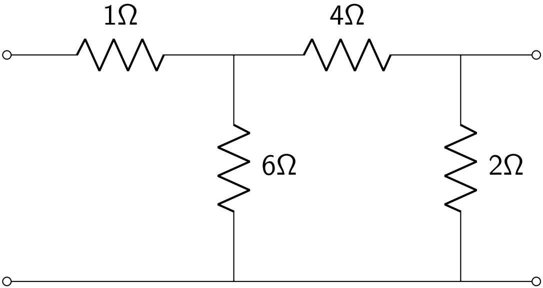
$$Y_{ij} = Y_{ji}$$

Lossless Network

- A lossless network is a network with no elements that introduce loss
  - No power is dissipated (i.e. converted to heat or radiation)
  - Results in a **purely imaginary** matrix

Example

- Determine the impedance parameter of the following 2-port network



$$Z = \begin{bmatrix} 4 & 1 \\ 1 & 1.667 \end{bmatrix}$$

Two-Port Networks

- Specialized case of N-port Networks
- Parameter matrices formed are 2×2
- Most RF/Microwave networks are cascaded two-port networks

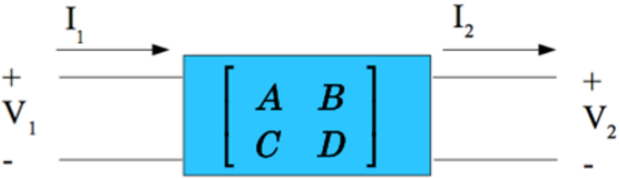
Transmission Parameters

- Only applicable to two-port networks

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

- Note the direction of I_2
  - opposite direction of current with respect to conventional two-port representations
- relates Port 1 to Port 2

ABCD Matrix Elements

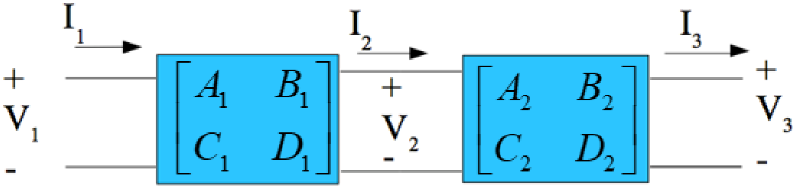


- The elements of the ABCD matrix can be evaluated:

$$A = \frac{V_1}{V_2}\Big|_{I_2=0} \qquad B = \frac{V_1}{I_2}\Big|_{V_2=0}$$

$$C = \frac{I_1}{V_2}\Big|_{I_2=0} \qquad D = \frac{I_1}{I_2}\Big|_{V_2=0}$$

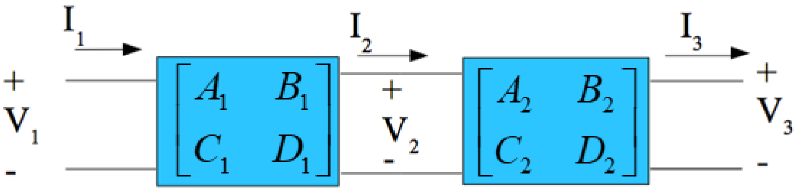
Cascaded Networks



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \qquad \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$

Cascaded Networks



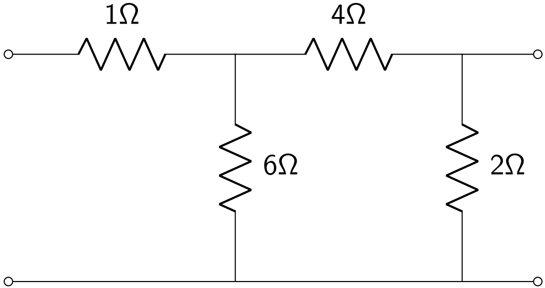
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \qquad \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 A_2 + B_1 C_2 & A_1 B_2 + B_1 D_2 \\ C_1 A_2 + D_1 C_2 & C_1 B_1 + D_1 D_2 \end{bmatrix} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$

Example

- Determine the ABCD parameter of the following 2-port network



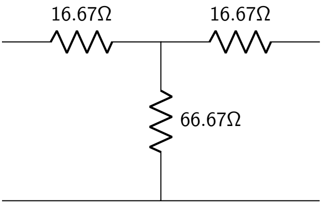
END

Consider 2 networks that are cascaded together.

Network A

$$S = \begin{bmatrix} 0 & 0.89 \\ 0.89 & 0 \end{bmatrix}$$

Network B



1. Obtain the S-parameters for Network B.
2. Derive the cascaded S-parameters.
3. Describe the function of each network and how it relates to the cascaded network.