EEE 51: Second Semester 2017 - 2018

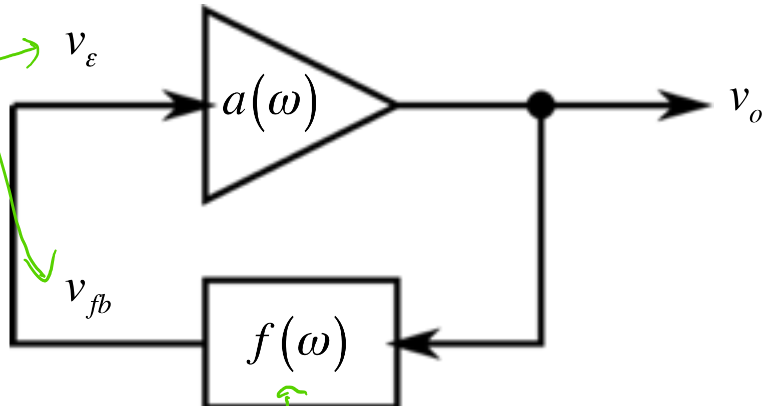
Lecture 23

Oscillators

General Form of an Oscillator

(Positive Feedback)

$v_\varepsilon = v_{fb}$



freq. dependent

$$A_{CL}(s) = \frac{a(s)}{1 - T(s)}$$

$$a(\omega) = \frac{v_o}{v_\varepsilon} = \frac{v_o}{v_{fb}}$$

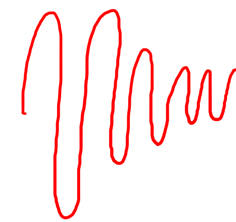
$$f(\omega) = \frac{v_{fb}}{v_o} \rightarrow a(\omega) = \frac{1}{f(\omega)}$$

Barkhausen Criteria:

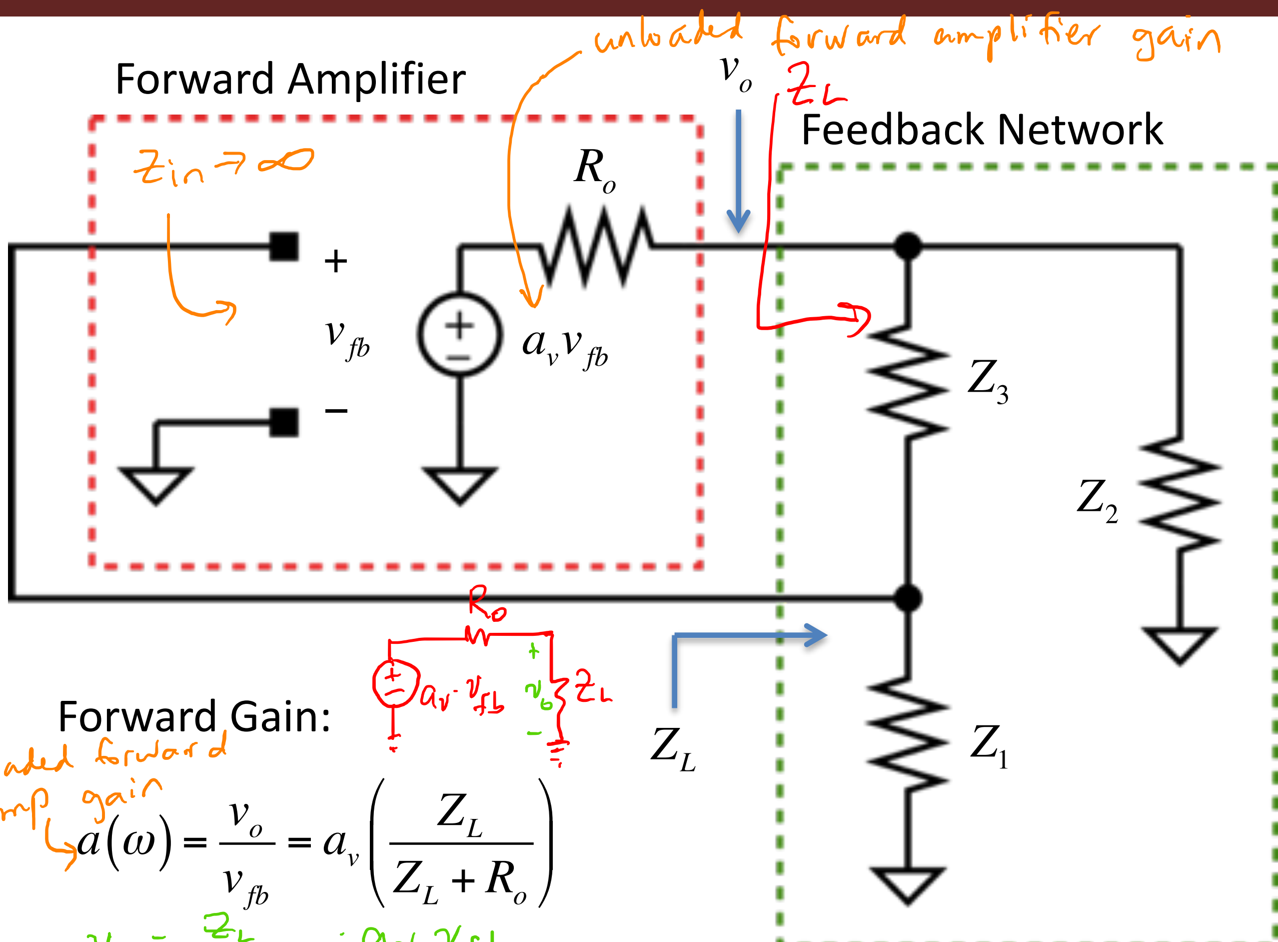
$$|T(\omega)| = |a(\omega) \cdot f(\omega)| = 1 \quad \geq 1$$

and

$$\angle T(\omega) = 360° \cdot n \quad n \in 0,1,2,3\ldots$$



General Form of an Oscillator



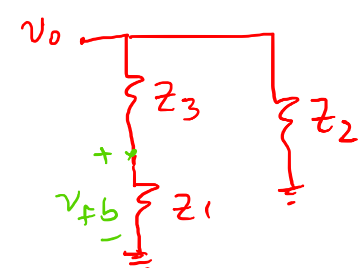
Barkhausen Criteria:

$$T(\omega) = a(\omega) \cdot f(\omega)$$
$$= 1\angle 0^\circ$$

$$v_{fb} = \frac{Z_1}{Z_1 + Z_3} \cdot v_o$$

Feedback Factor:

$$f(\omega) = \frac{v_{fb}}{v_o} = \frac{Z_1}{Z_1 + Z_3}$$



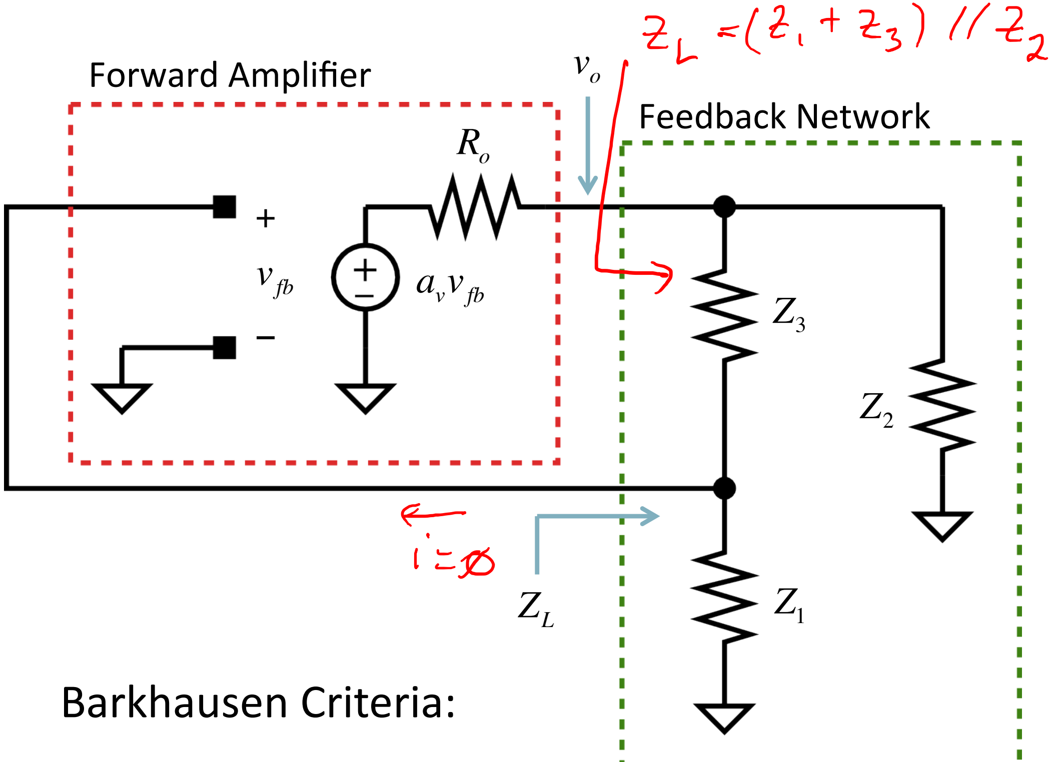
Forward Gain:

loaded forward amp gain

$$a(\omega) = \frac{v_o}{v_{fb}} = a_v\left(\frac{Z_L}{Z_L + R_o}\right)$$

$$v_o = \frac{Z_L}{Z_L + R_o} \cdot a_v v_{fb}$$

General Form of an Oscillator



$$a(\omega) = a_v \left(\frac{Z_L}{Z_L + R_o} \right)$$

$$f(\omega) = \frac{Z_1}{Z_1 + Z_3}$$

Loop Gain:

$$T(\omega) = a(\omega) \cdot f(\omega)$$

$$= a_v \frac{Z_L}{Z_L + R_o} \frac{Z_1}{Z_1 + Z_3}$$

$$Z_L = Z_2 \parallel (Z_1 + Z_3)$$

$$= \frac{Z_2 (Z_1 + Z_3)}{Z_1 + Z_2 + Z_3}$$

Barkhausen Criteria:

$$T(\omega) = a(\omega) \cdot f(\omega)$$

$$= 1\angle 0° = 1 + j\emptyset$$

General Form of an Oscillator

Loop Gain: $T(\omega) = a(\omega) \cdot f(\omega)$

$$= a_v \cdot \frac{Z_L}{Z_L + R_o} \cdot \frac{Z_1}{Z_1 + Z_3}$$

$$= a_v \cdot \frac{\dfrac{Z_2 \cancel{\left(Z_1 + Z_3\right)}}{Z_1 + Z_2 + Z_3}}{\dfrac{Z_2\left(Z_1 + Z_3\right)}{Z_1 + Z_2 + Z_3} + R_o} \cdot \frac{Z_1}{\cancel{Z_1 + Z_3}} = a_v \cdot \frac{Z_2 Z_1}{Z_2\left(Z_1 + Z_3\right) + R_o\left(Z_1 + Z_2 + Z_3\right)}$$

Barkhausen Criteria:

$$T(\omega) = a(\omega) \cdot f(\omega)$$

$$= a_v \cdot \frac{Z_2 Z_1}{Z_2\left(Z_1 + Z_3\right) + R_o\left(Z_1 + Z_2 + Z_3\right)} = 1\angle 0^\circ$$

General Form of an Oscillator

Barkhausen Criteria: $T(\omega) = a(\omega) \cdot f(\omega)$

$$= a_v \cdot \frac{Z_2 Z_1}{Z_2(Z_1 + Z_3) + R_o(Z_1 + Z_2 + Z_3)} = 1\angle 0°$$

If we assume that Z_1, Z_2 and Z_3 are purely reactive elements: $\begin{cases} Z_1 = jX_1 \\ Z_2 = jX_2 \\ Z_3 = jX_3 \end{cases}$

if not:

$Z_1 = R_1 + jX_1$

$Z_2 = R_2 + jX_2$

$Z_3 = R_3 + jX_3$

note: $j^2 = -1$

$$T(\omega) = a_v \cdot \frac{Z_2 Z_1}{Z_2(Z_1 + Z_3) + R_o(Z_1 + Z_2 + Z_3)} = a_v \cdot \frac{jX_2 \cdot jX_1}{jX_2(jX_1 + jX_3) + R_o(jX_1 + jX_2 + jX_3)}$$

$$= a_v \cdot \frac{-X_2 \cdot X_1}{-X_2(X_1 + X_3) + jR_o(X_1 + X_2 + X_3)}$$

General Form of an Oscillator

Loop Gain: $T(\omega) = a_v \cdot \dfrac{-X_2 \cdot X_1}{-X_2(X_1 + X_3) + \underbrace{jR_o(X_1 + X_2 + X_3)}}$

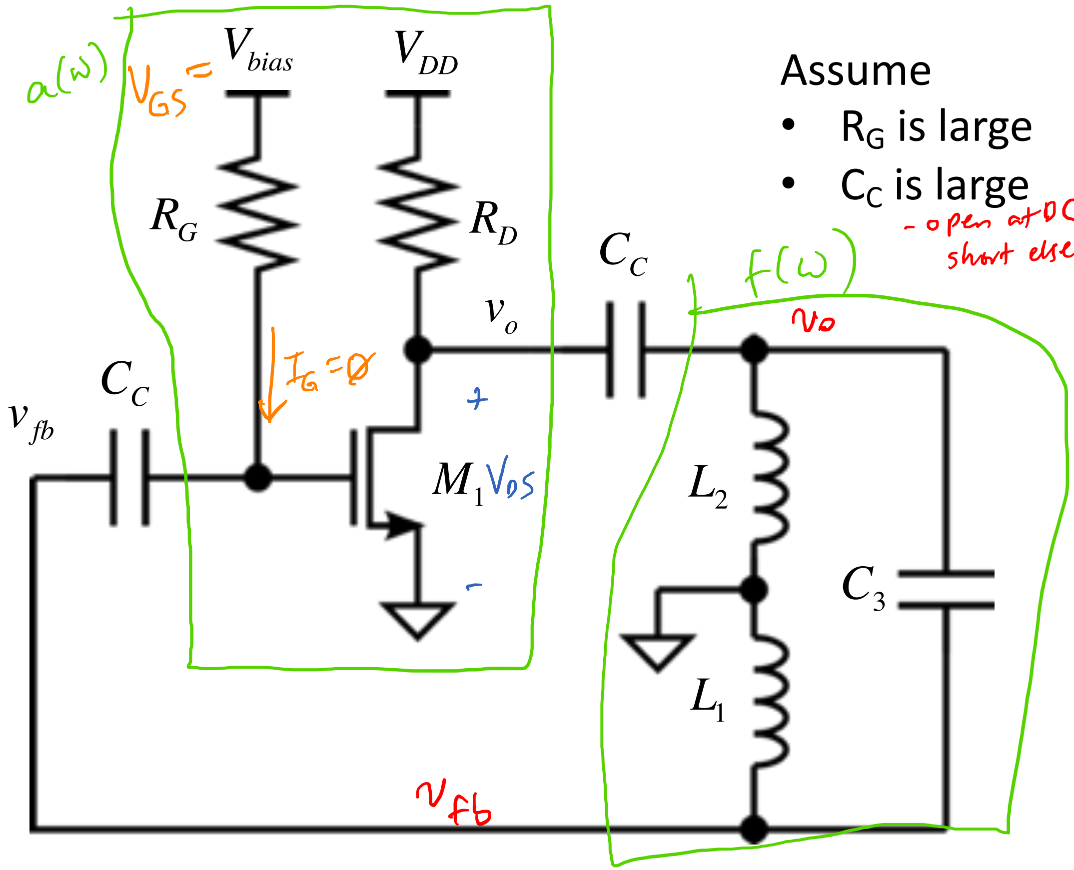
For oscillations to occur: $\mathrm{Im}\{T(\omega)\} = 0 \Rightarrow X_1 + X_2 + X_3 = 0$ ① $\Rightarrow X_1 + X_3 = -X_2$

Thus: $T(\omega_0) = a_v \cdot \dfrac{\cancel{+X_2} \cdot X_1}{\cancel{+X_2}(X_1 + X_3)} = a_v \cdot \dfrac{X_1}{(X_1 + X_3)} = \boxed{-a_v \cdot \dfrac{X_1}{X_2} = 1\angle 0°}$ ②

Case 1: $a_v > 0$ → X_1 and X_2 must have different signs

Case 2: $a_v < 0$ → X_1 and X_2 must have the same sign

Example: The Hartley Oscillator



Assume

- R_G is large
- C_C is large

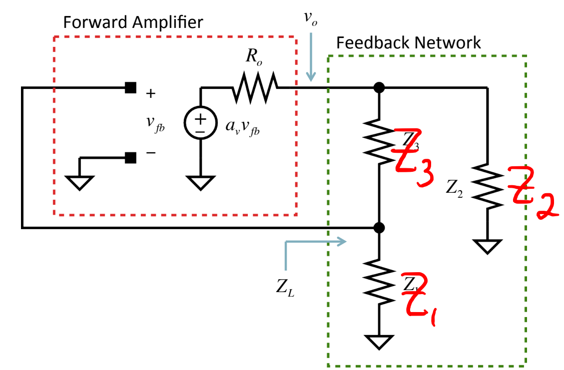
- open at DC, short elsewhere

MOSFET bias

$$I_D = k\left(V_{GS} - V_{TH}\right)^2 = k\left(V_{bias} - V_{TH}\right)^2$$

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = 2k\left(V_{GS} - V_{TH}\right)$$

$$r_o \to \infty$$

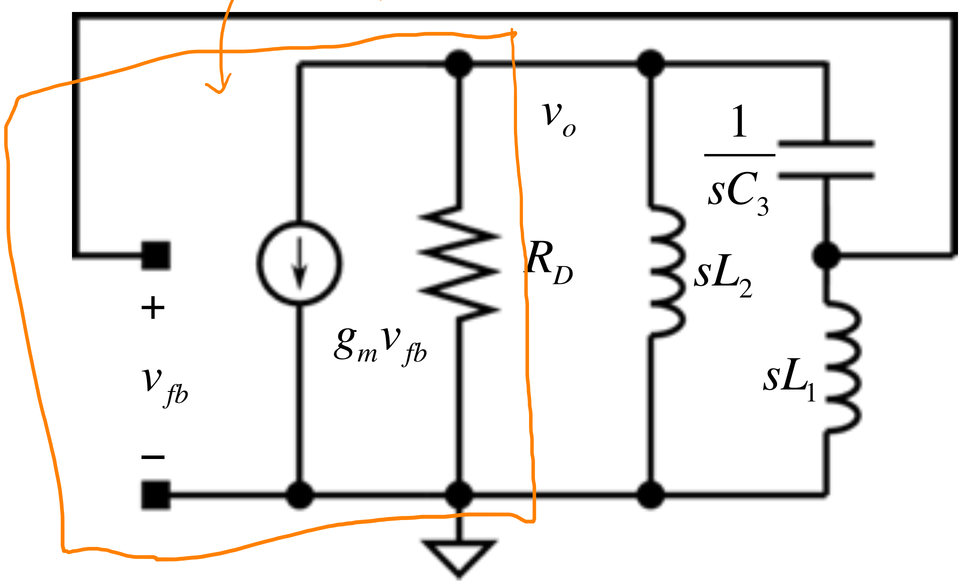


Named after Ralph Vinton Lyon Hartley (November 30, 1888 – May 1, 1970)

Example: The Hartley Oscillator

Small-signal model:

$a(\omega)$



$X_1 = \omega L_1$ $\quad X_2 = \omega L_2$ $\quad X_3 = -\frac{1}{\omega C_3}$

Forward unloaded amplifier:

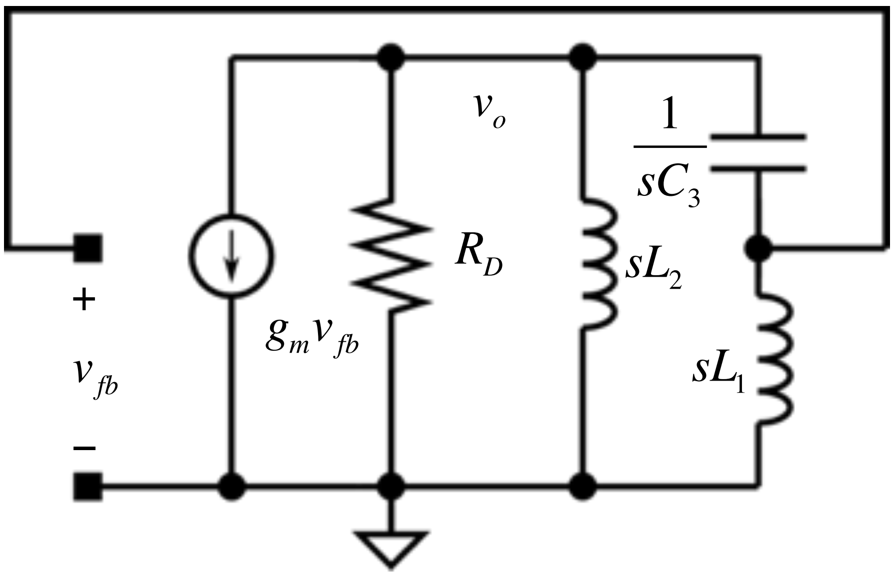
$$a_v = -g_m R_D$$

$$R_o = R_D = R_D // r_o \text{ if } r_o \text{ is finite.}$$

Note: X_1 and X_2 must have the same sign since $a_v < 0$

Example: The Hartley Oscillator

Small-signal model:



$$a_v = -g_m R_D \qquad X_2 = \omega L_2$$

$$R_o = R_D \qquad X_1 = \omega L_1$$

$$X_3 = -\frac{1}{\omega C_3}$$

Recall:

$$T(\omega) = a_v \cdot \frac{-X_2 \cdot X_1}{-X_2(X_1 + X_3) + jR_o(X_1 + X_2 + X_3)}$$

To oscillate:

1 $\underline{X_1 + X_2 + X_3 = 0} \Rightarrow \omega_0(L_1 + L_2) = \frac{1}{\omega_0 C_3}$

$\omega L_1 + \omega L_2 - \frac{1}{\omega C_3} = \emptyset$

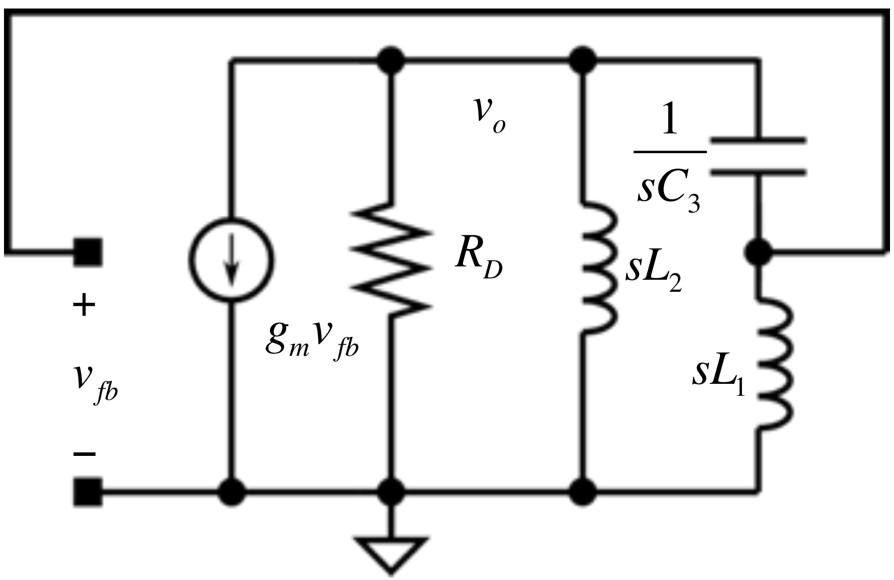
Thus,

$\omega_o^2 = \frac{1}{(L_1 + L_2) C_3}$

$$\omega_0 = \frac{1}{\sqrt{(L_1 + L_2) C_3}}$$

Example: The Hartley Oscillator

Frequency of oscillation: $\omega_0 = \dfrac{1}{\sqrt{(L_1 + L_2)C_3}}$



$a_v = -g_m R_D$ $\quad X_2 = \omega L_2$ $\quad X_3 = -\dfrac{1}{\omega C_3}$

$R_o = R_D$ $\quad X_1 = \omega L_1$

$a_v = -g_m \cdot R_D$

Loop Gain:

2

$$T(\omega_0) = \boxed{-a_v \cdot \frac{X_1}{X_2} = 1\angle 0^\circ}$$

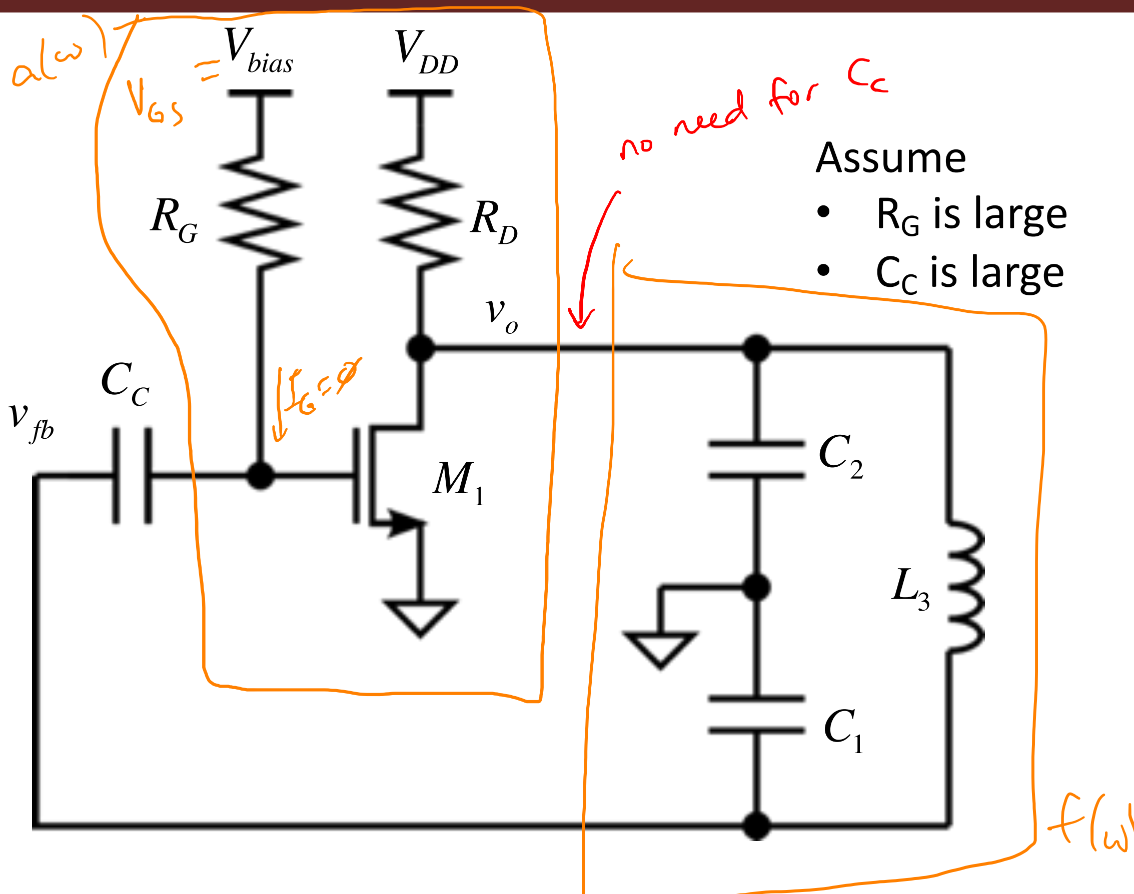
$$= g_m R_D \frac{L_1}{L_2} = 1\angle 0^\circ$$

For $L_1 = L_2$: $\quad g_m R_D = 1$

if $g_m R_D = 10$, $\dfrac{L_1}{L_2} = \dfrac{1}{10}$

$L_2 = 10 \cdot L_1$

The Colpitts Oscillator



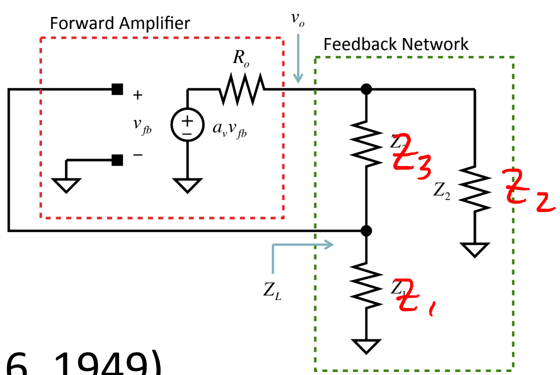
Assume
- R_G is large
- C_C is large

MOSFET bias

$$I_D = k\left(V_{GS} - V_{TH}\right)^2 = k\left(V_{bias} - V_{TH}\right)^2$$

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = 2k\left(V_{GS} - V_{TH}\right)$$

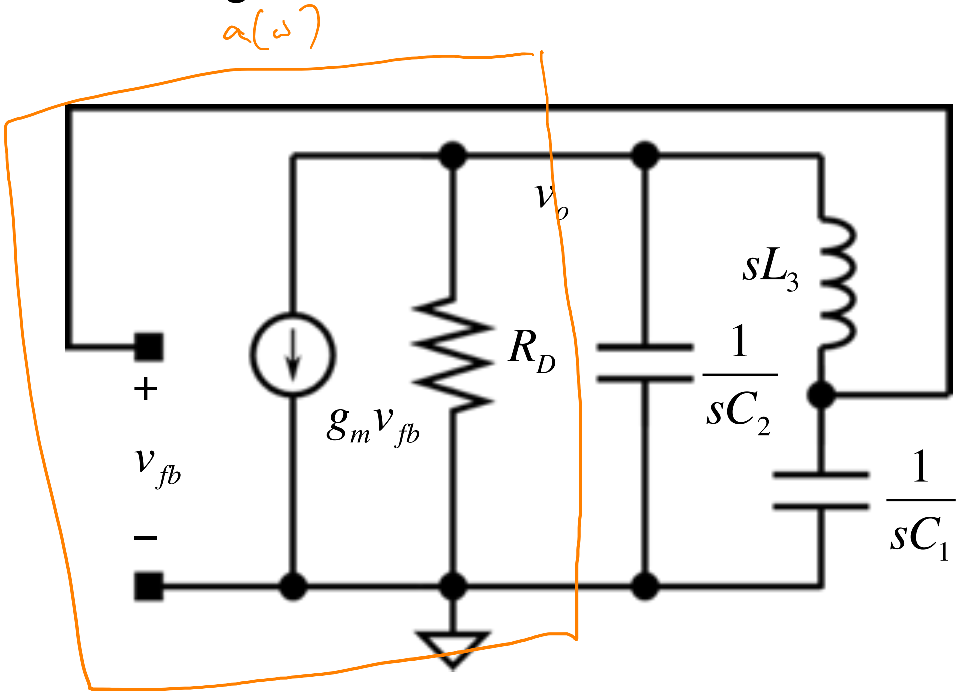
$$r_o \rightarrow \infty$$



Named after Edwin Henry Colpitts (January 19, 1872 - March 6, 1949)

The Colpitts Oscillator

Small-signal model:



$$X_1 = -\frac{1}{\omega C_1} \quad X_2 = -\frac{1}{\omega C_2} \quad X_3 = \omega L_3$$

Forward unloaded amplifier:

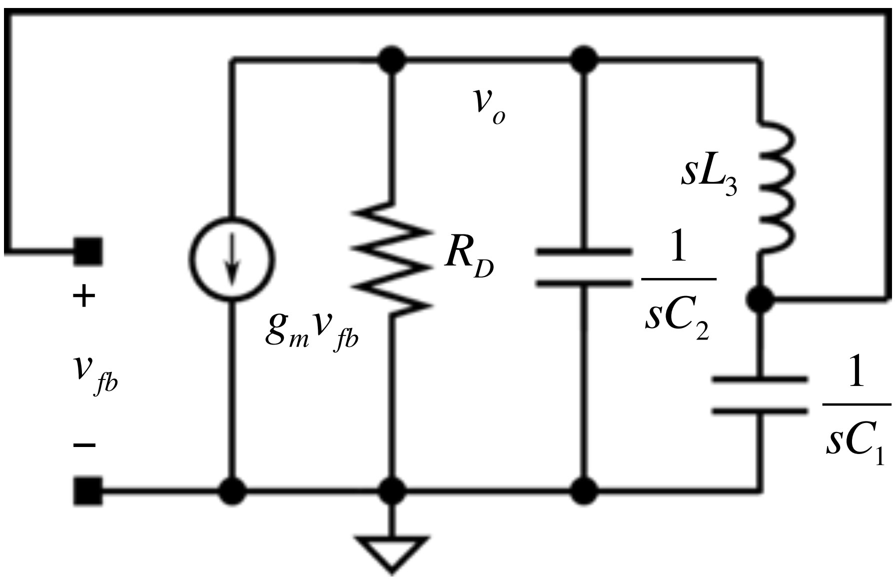
$$a_v = -g_m R_D$$

$$R_o = R_D$$

Note: X_1 and X_2 must have the same sign since $a_v < 0$.

The Colpitts Oscillator

Small-signal model:



$$a_v = -g_m R_D \qquad X_1 = -\frac{1}{\omega C_1} \qquad X_2 = -\frac{1}{\omega C_2}$$

$$R_o = R_D \qquad X_3 = \omega L_3$$

Recall:

$$T(\omega) = a_v \cdot \frac{-X_2 \cdot X_1}{-X_2(X_1 + X_3) + jR_o(X_1 + X_2 + X_3)}$$

To oscillate:

1

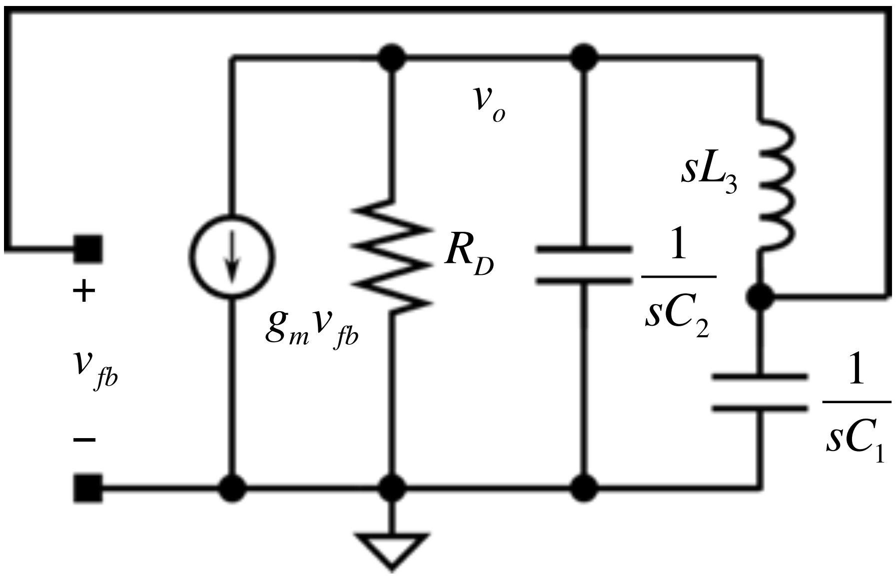
$$X_1 + X_2 + X_3 = 0 \quad \Rightarrow \quad \omega_0 L_3 = \frac{1}{\omega_0}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

Thus,

$$\omega_0 = \frac{1}{\sqrt{L_3 \cdot \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}}}}$$

The Colpitts Oscillator

Frequency of oscillation: $\omega_0 = \dfrac{1}{\sqrt{L_3 \cdot \dfrac{C_1 C_2}{C_1 + C_2}}}$



$a_v = -g_m R_D$

$R_o = R_D$

$X_1 = -\dfrac{1}{\omega C_1}$ $\quad X_2 = -\dfrac{1}{\omega C_2}$

$X_3 = \omega L_3$

Loop Gain: 2

$$\boxed{T(\omega_0) = -a_v \cdot \frac{X_1}{X_2} = 1\angle 0^\circ}$$

$$= g_m R_D \frac{C_2}{C_1} = 1\angle 0^\circ$$

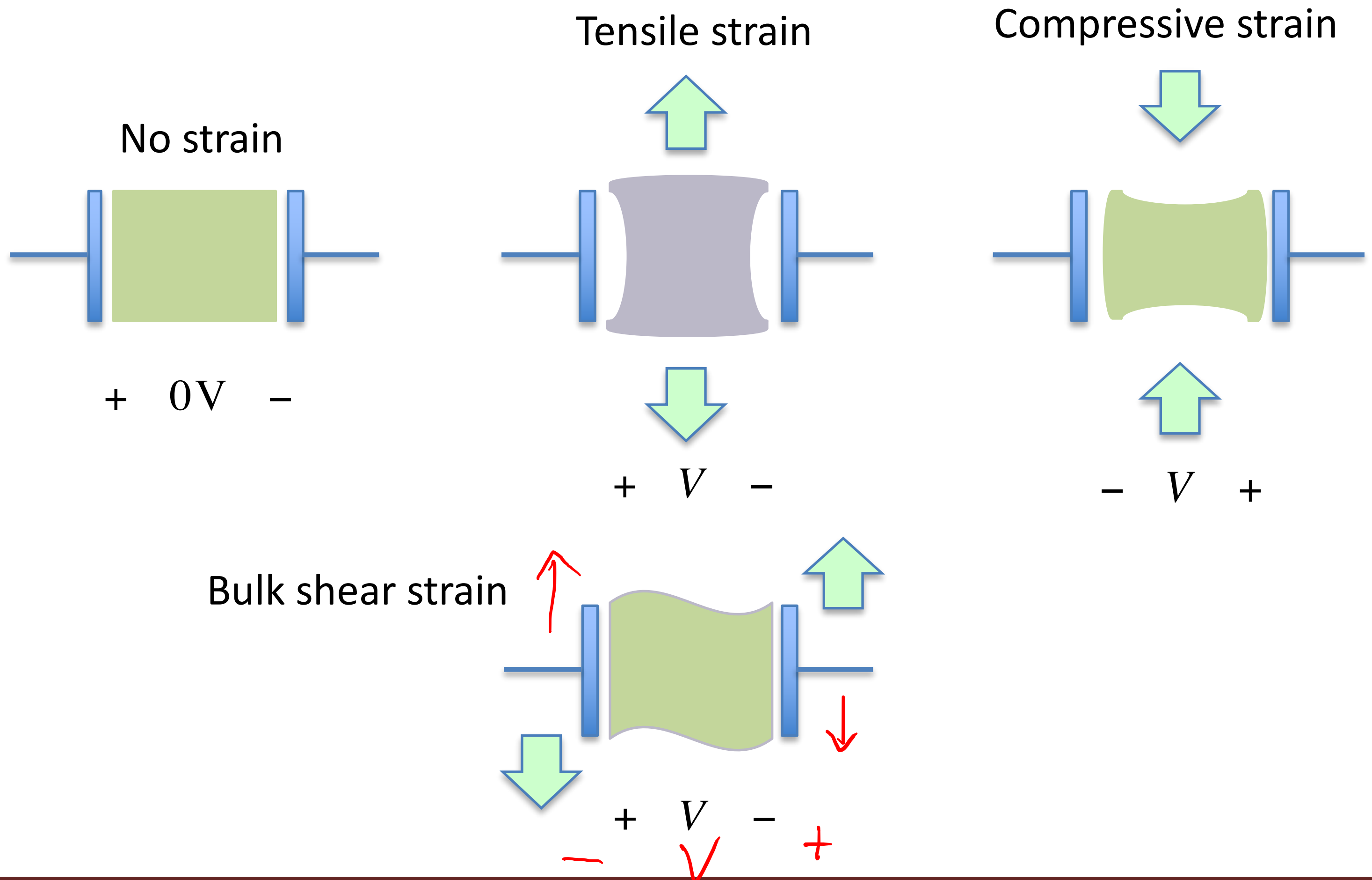
For $C_1 = C_2$: $\quad g_m R_D = 1$

if $g_m R_D = 10$; $\quad \frac{C_2}{C_1} = \frac{1}{10}$ or $C_1 = 10 \cdot C_2$

Crystal Oscillators

- Crystals are materials that exhibits the piezoelectric effect
  – When stress is applied, voltage is generated between opposite faces of the crystal
  – When a voltage is applied, the crystal deforms at the frequency of the applied voltage
- Mechanical to electrical energy conversion
- Provides stable frequency of oscillation
  – Resonant frequency dependent on physical crystal size
  – 1 ppm/°C or 0.0001%/°C
  – Compare with LC oscillator: ~1% drift

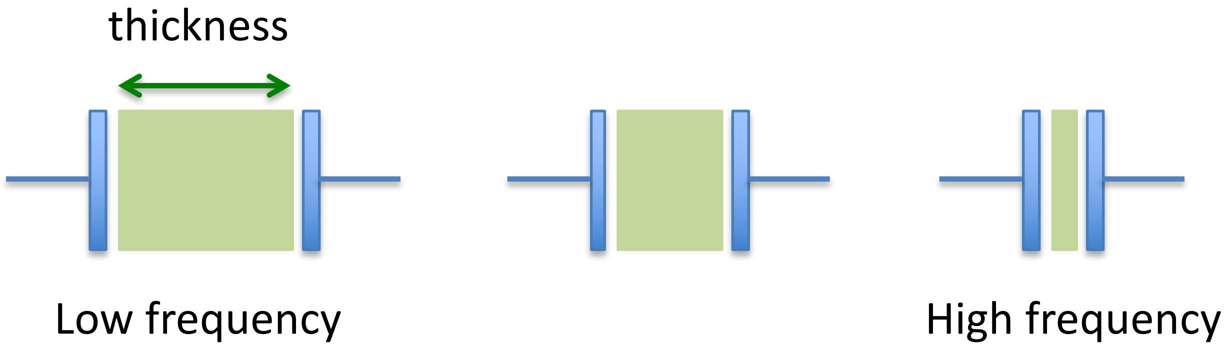
Crystal Strain



Natural Crystal Frequency

- Proportional to crystal thickness

$$f \propto \frac{1}{\text{thickness}}$$

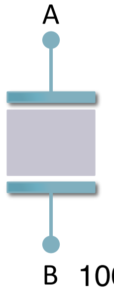


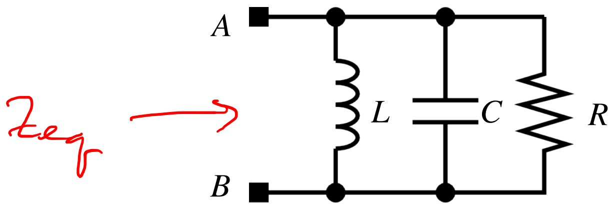
Typical natural frequencies below 20-30 MHz

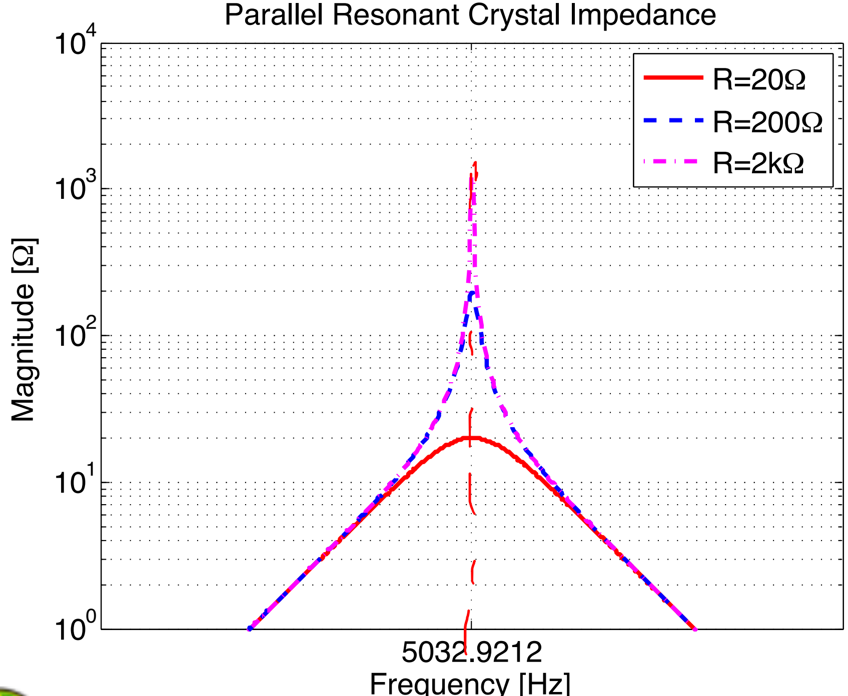
- For 100 MHz, thickness ~ 17μm thick

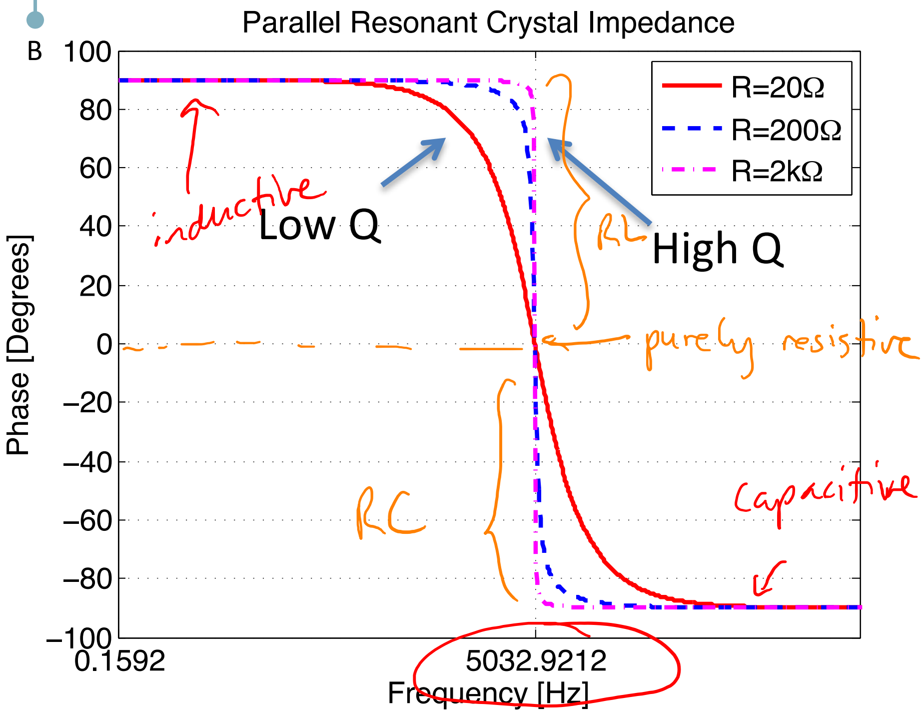
The Parallel Resonant Mode

$$L = 1\text{mH} \quad C = 1\mu\text{F}$$



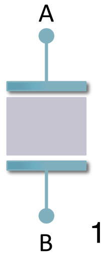


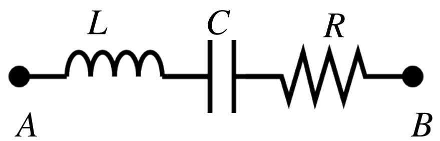


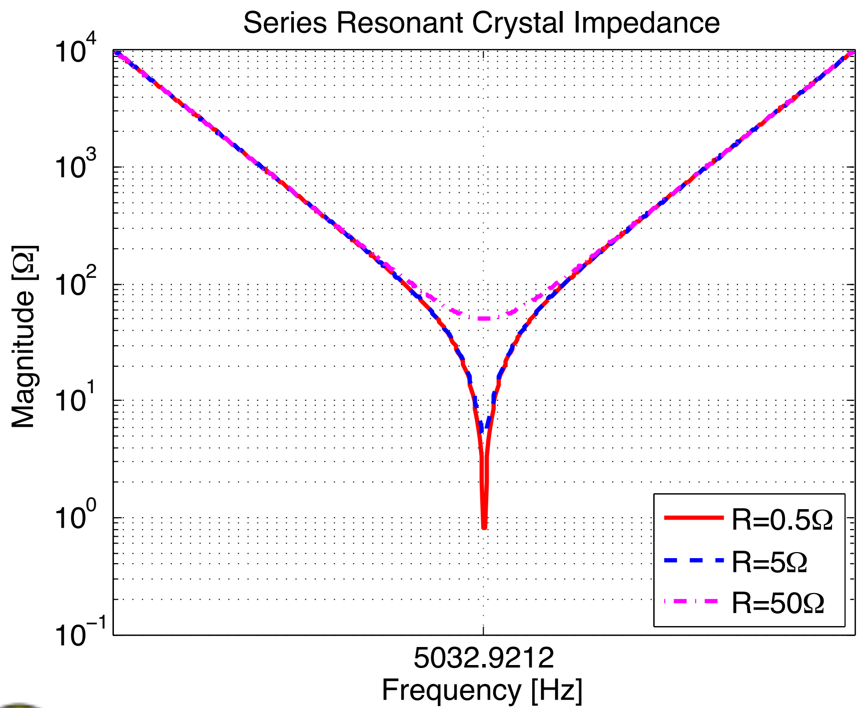


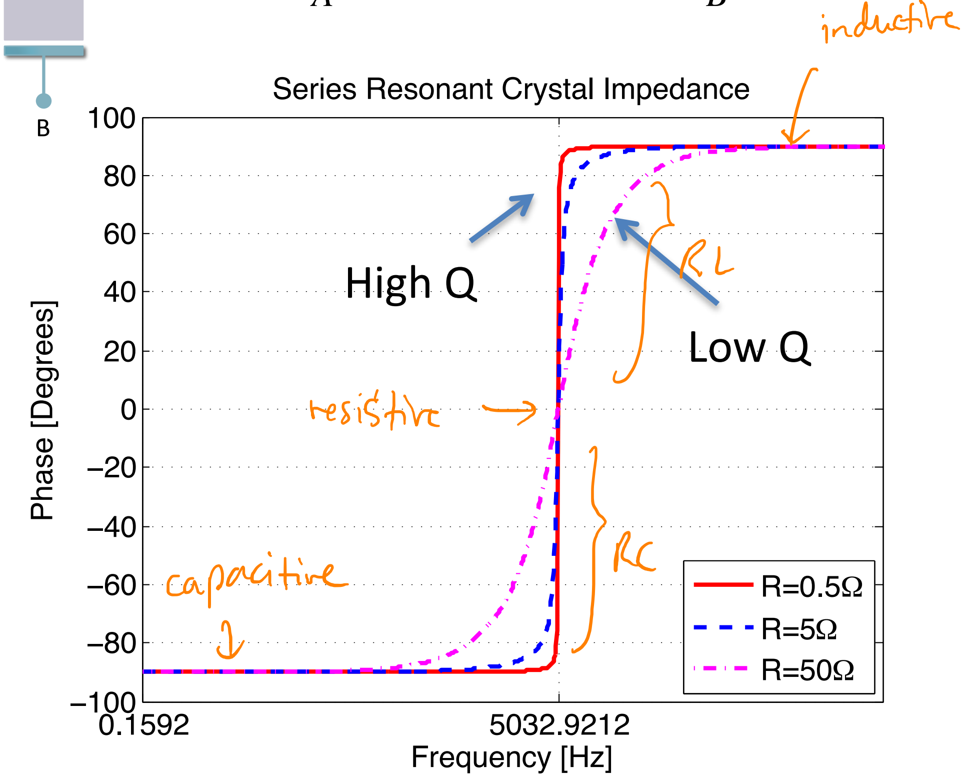
The Series Resonant Mode

$$L = 1\text{mH} \quad C = 1\mu\text{F}$$

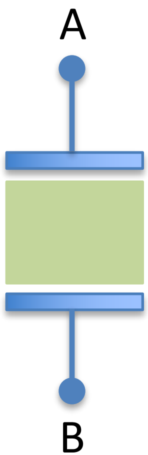


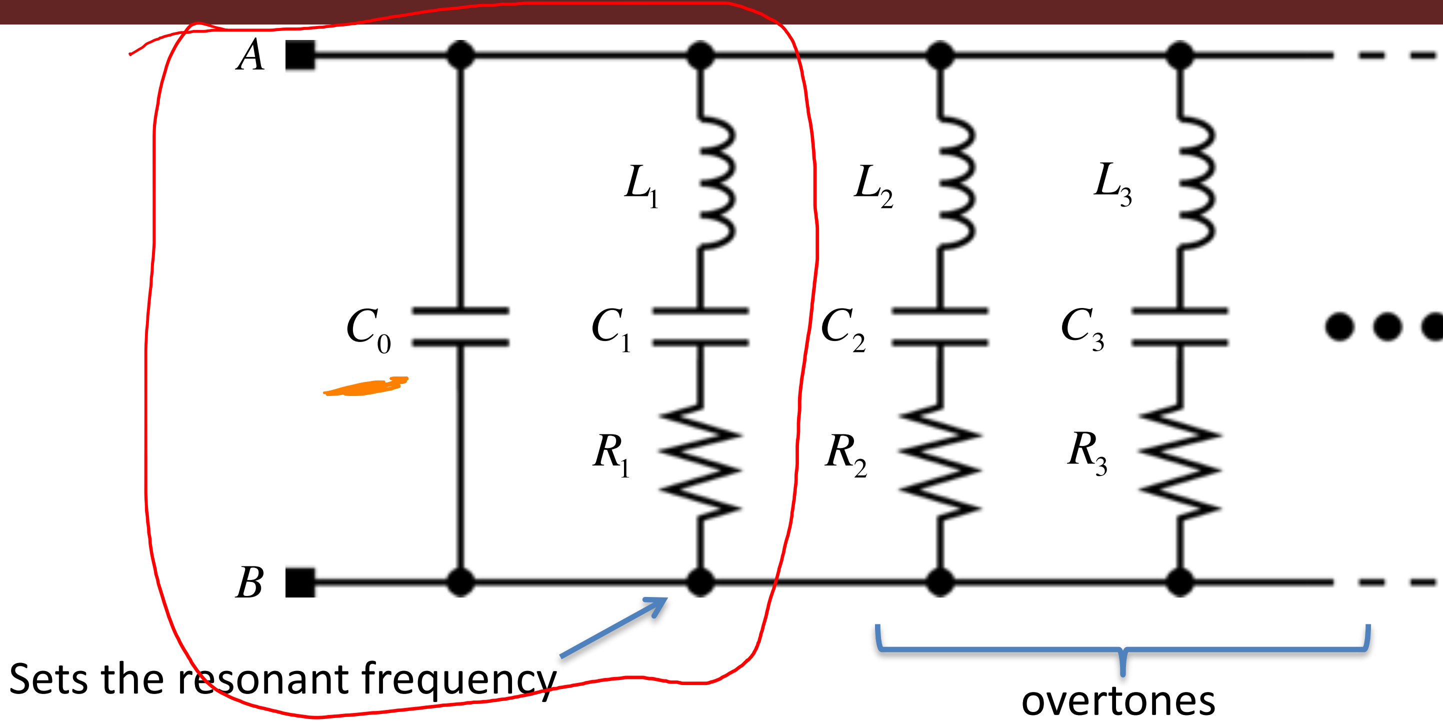






Electrical Equivalent Circuit



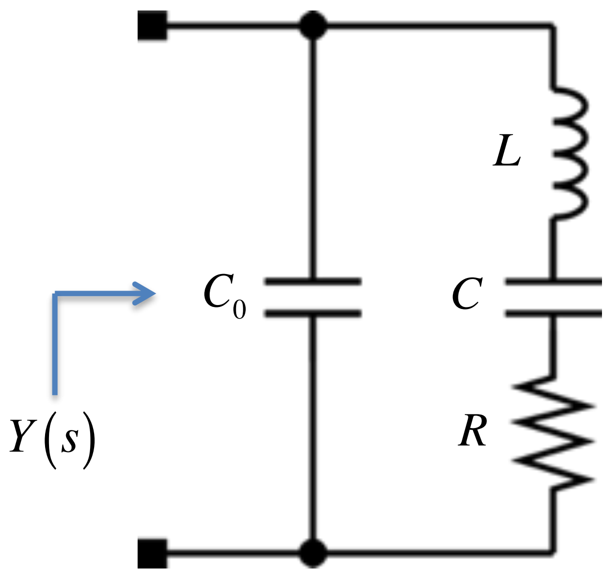


$C_0 \equiv$ parallel capacitances due to contacts and wires

$L_i, C_i \equiv$ mechanical energy storage (mass & spring effects)

$R_i \equiv$ electrical losses due to mechanical effects (e.g. friction)

Crystal Equivalent Circuit



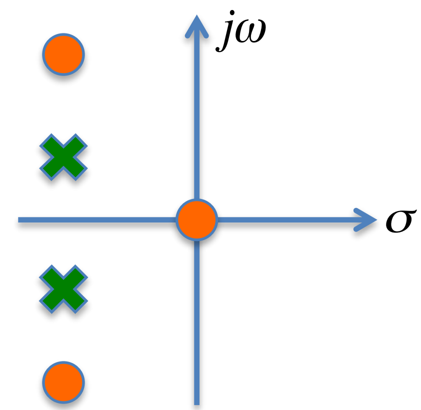
admittance

$$Y(s) = sC_0 + \frac{1}{sL + \frac{1}{sC} + R} = sC_0 + \frac{sC}{s^2LC + sRC + 1}$$

$$= \frac{sC_0\left[s^2 + s\frac{R}{L} + \left(\frac{C}{C_0} + 1\right)\omega_0^2\right]}{s^2 + s\frac{R}{L} + \omega_0^2}$$

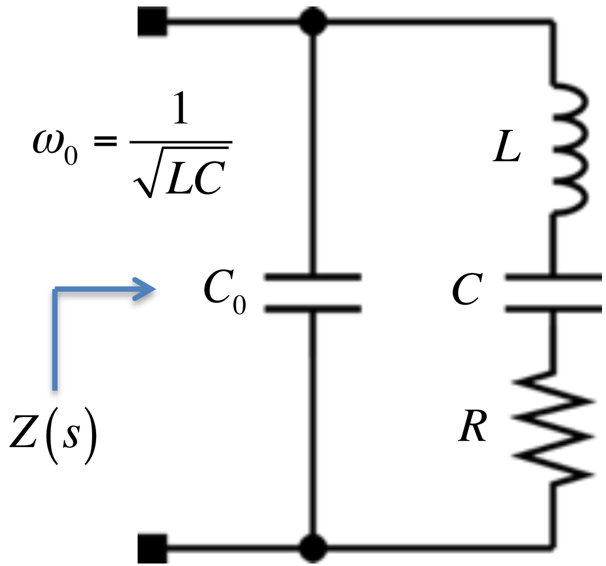
$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad \boxed{Z(s) = \frac{1}{Y(s)}}$$

p-z plot:



$Z(s) = \frac{1}{Y(s)}$

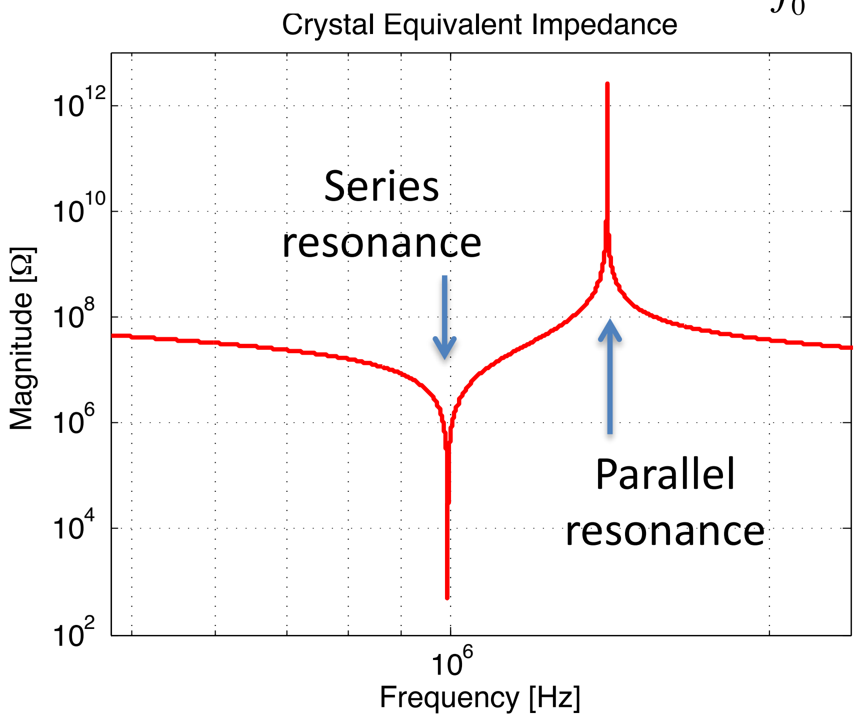
Crystal Equivalent Circuit



$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$Z(s) = \frac{s^2 + s\frac{R}{L} + \omega_0^2}{sC_0\left[s^2 + s\frac{R}{L} + \left(\frac{C}{C_0} + 1\right)\omega_0^2\right]}$$

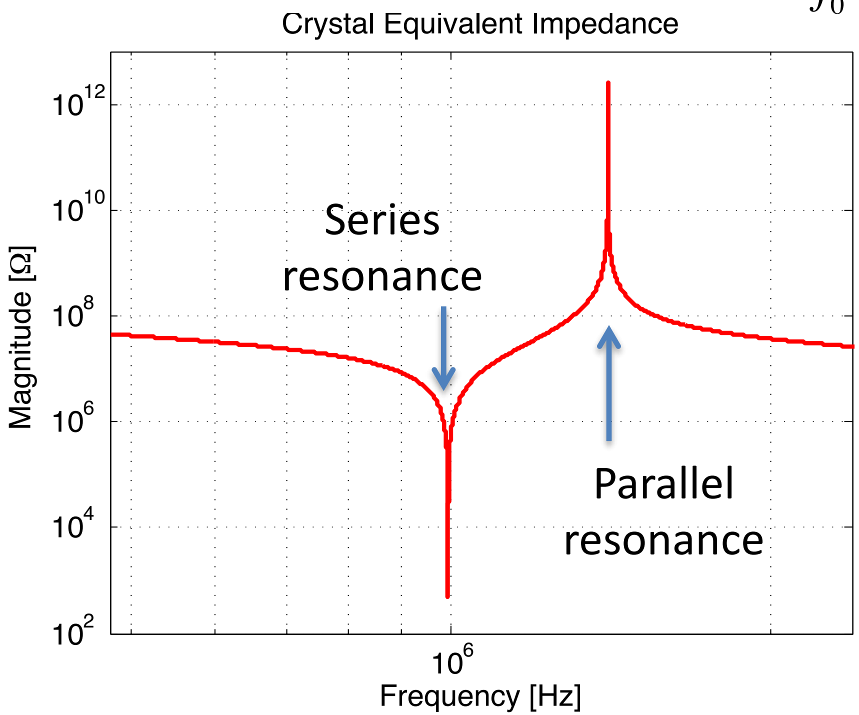
$$L = 10\text{H} \qquad C = C_0 = 3.2\text{fF} \qquad R \approx \frac{5\times 10^8}{f_0}$$

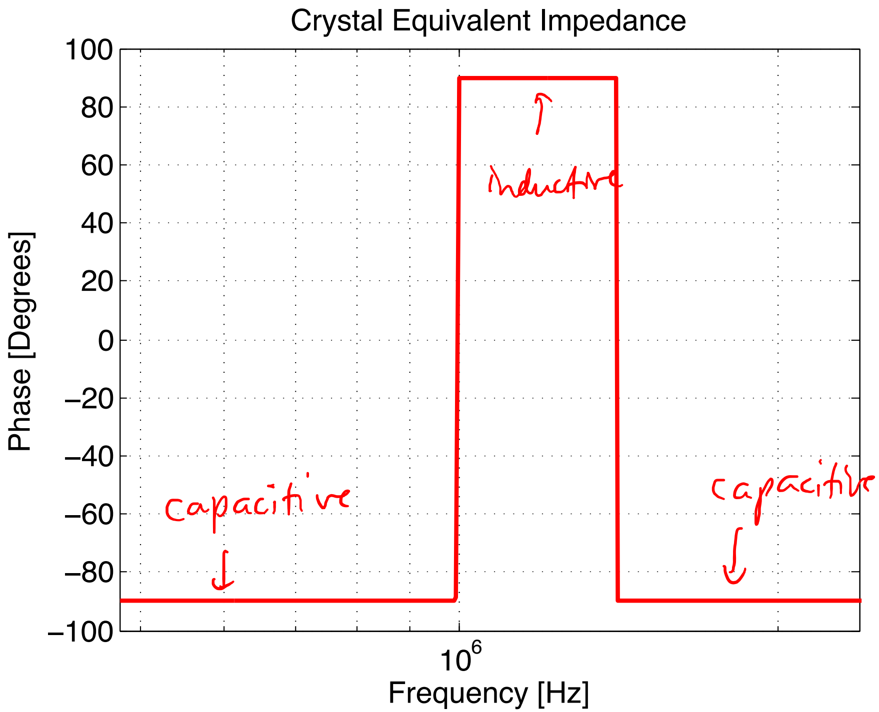


How can we use this to create an oscillator?

Crystal Equivalent Circuit

$$L = 10\text{H} \qquad C = C_0 = 3.2\text{fF} \qquad R \approx \frac{5\times 10^8}{f_0}$$

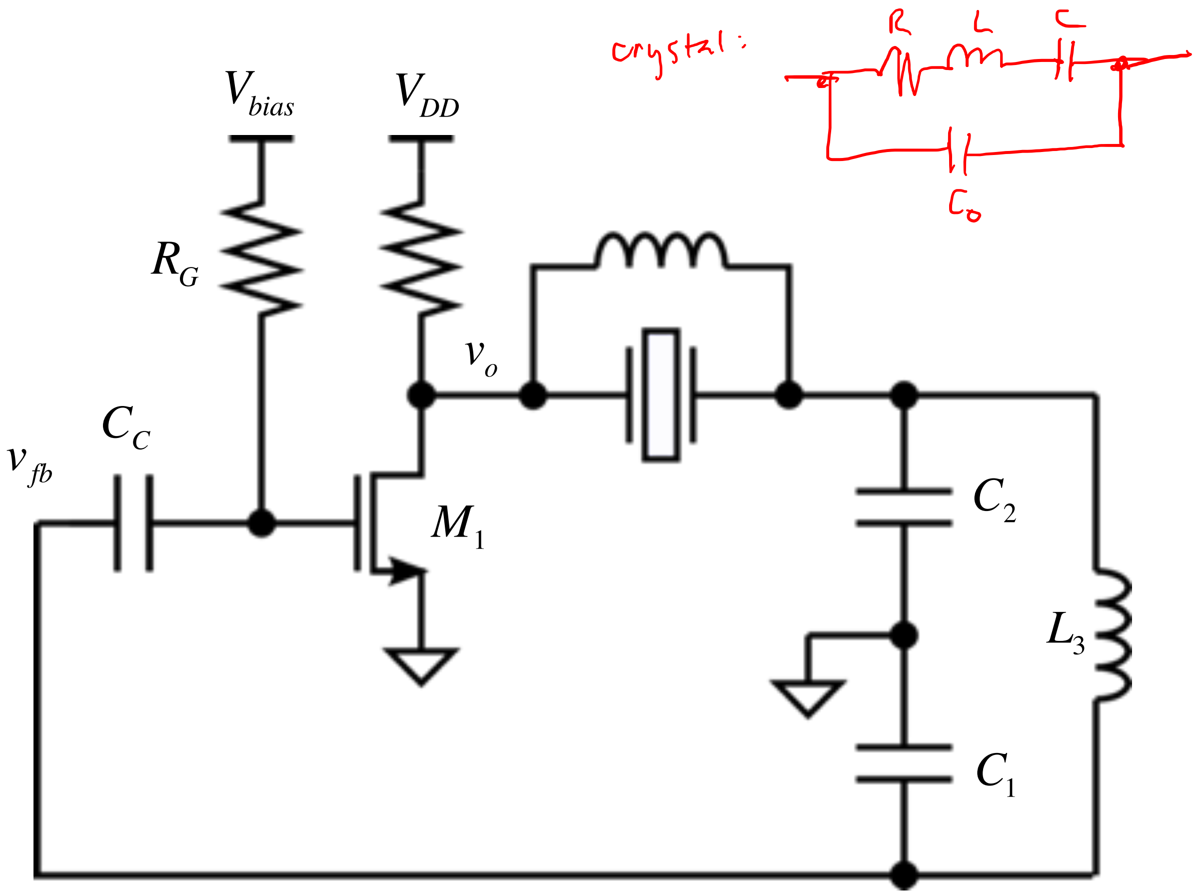




How can we use this to create an oscillator?

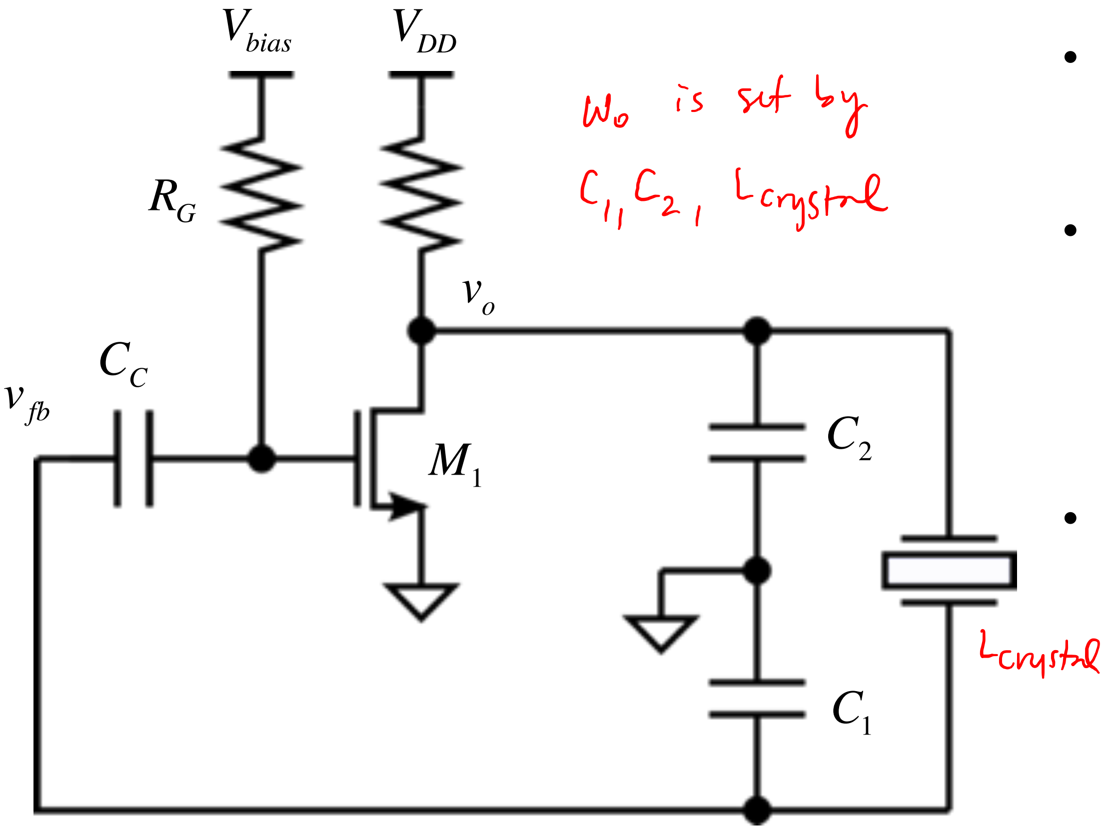
Direct Application: A Colpitts Crystal Oscillator

crystal:



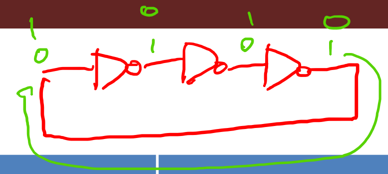
- The crystal is used to close the feedback loop only at the desired frequency
- The parallel inductance is used to cancel out the crystal parallel capacitance C_0
  - Only the series RLC branch controls the feedback path

Another Colpitts Crystal Oscillator



- The crystal is used as an inductance
- The oscillation frequency is set to be a little above series resonance
    - The crystal impedance is inductive
- Note that the crystal series resonant frequency is not the same as the output oscillation frequency
    - Crystal is cut to oscillate at a specified load capacitance

Frequency Bands



Frequency Range	Designation	Wavelength	
3 kHz – 30 kHz	VLF	100 km – 10 km	Phase shift
30 kHz – 300 kHz	LF	10 km – 1 km	
300 kHz – 3 MHz	MF	1 km – 100 m	LC
3 MHz – 30 MHz	HF	100 m – 10 m	Crystal
30 MHz – 300 MHz	VHF	10 m – 1 m	LC, ring oscillators, SAW, MEMS
300 MHz – 3 GHz	UHF	1 m – 0.1 m	
3 GHz – 30 GHz	SHF	0.1 m – 1 cm	
30 GHz – 300 GHz	EHF	1 cm – 1 mm	LC, distributed, MEMS

Next Meeting

- Negative Resistance Oscillators