Lecture 19
BUS ADMITTANCE MATRIX AND ELECTRIC LOAD MODELS

Agenda

- **ANNOUNCEMENTS**
- **LECTURE**

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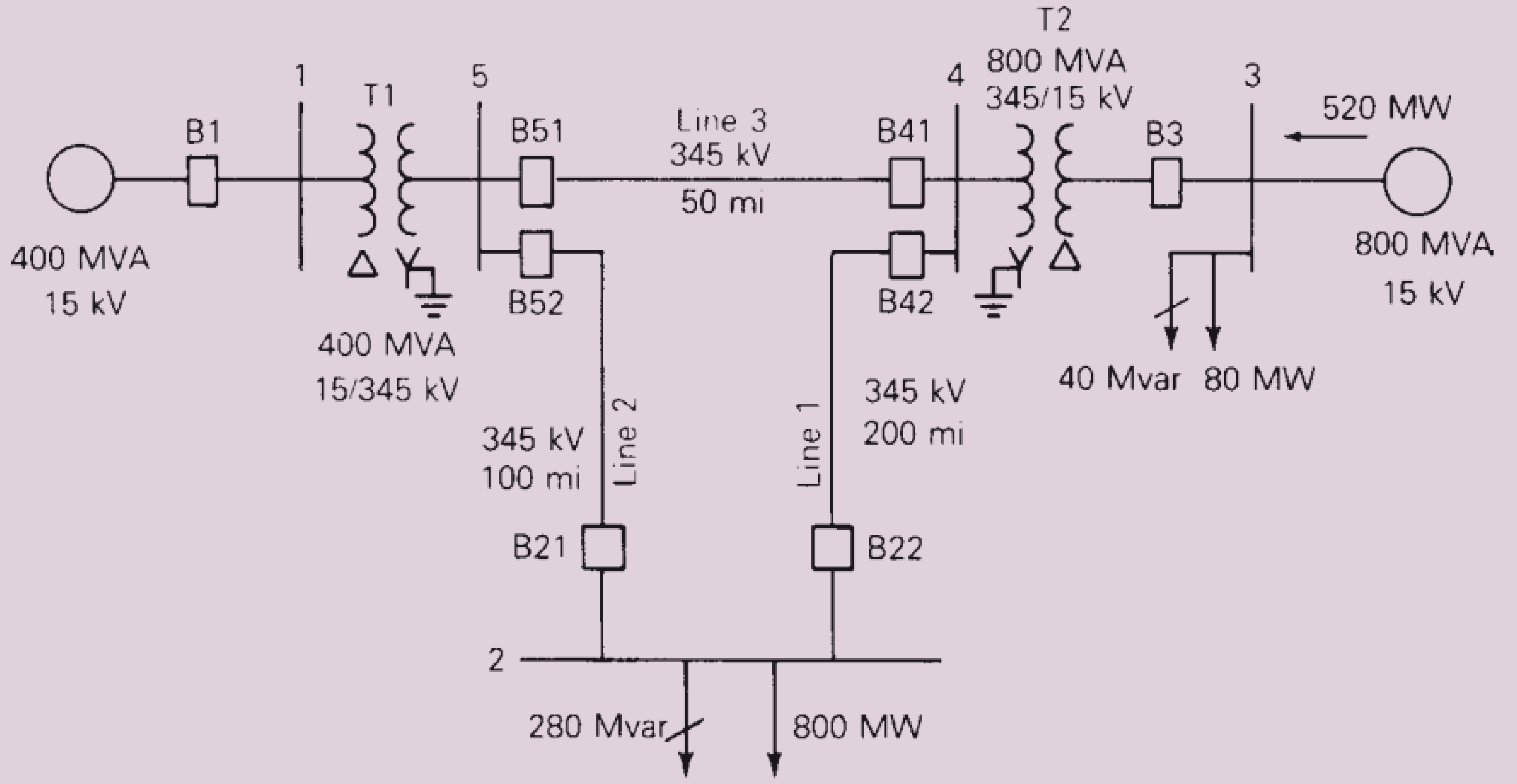
Announcements

- Long Quiz 2 is on March 25, 2019 from 7 to 9AM
  - Early Exam(6 to 8AM) Takers should answer the survey in UVLE.

POWER SYSTEM MODELLING FOR ANALYSIS
RECALL NODAL ANALYSIS

FIGURE 6.2

Single-line diagram for Example 6.9

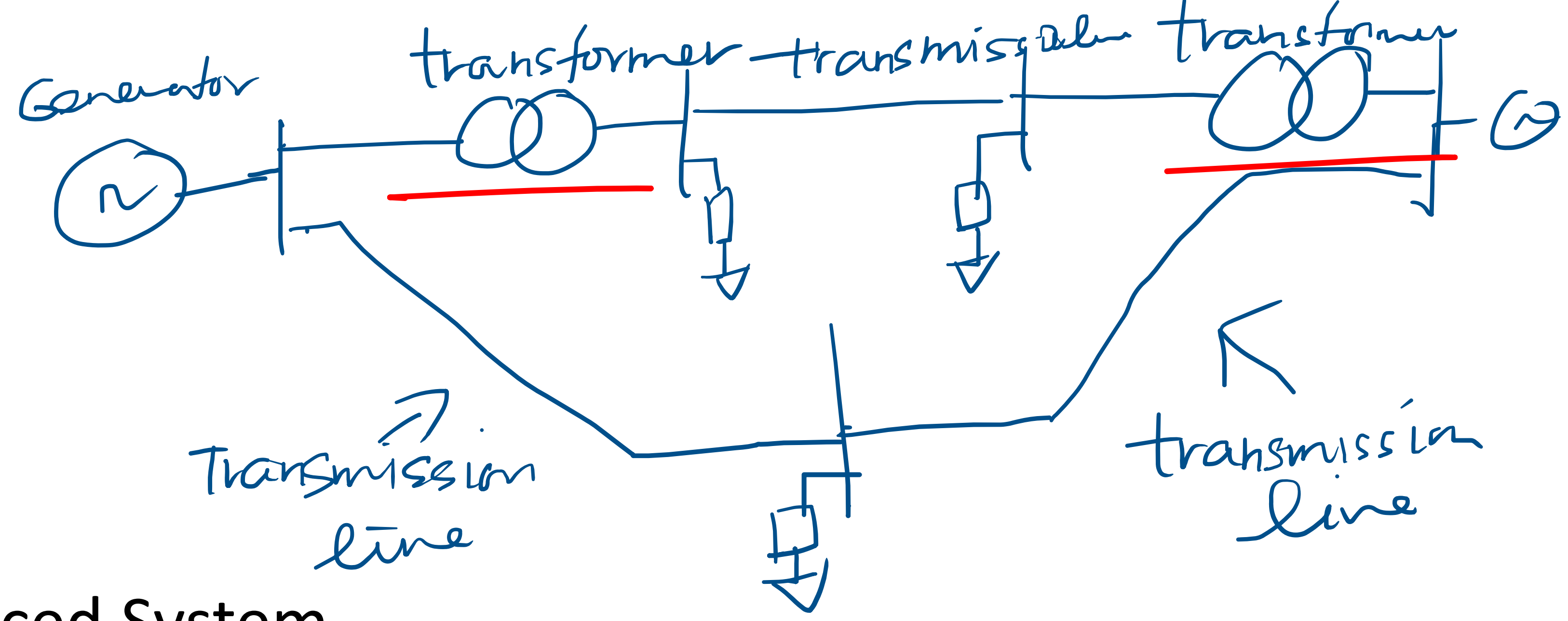


Lecture Outcomes

at the end of the lecture, the student must be able to …

- Derive the bus admittance matrix of an electric power network
- Observe the effect of using one of Z-I-P load models in the analysis

Perspective

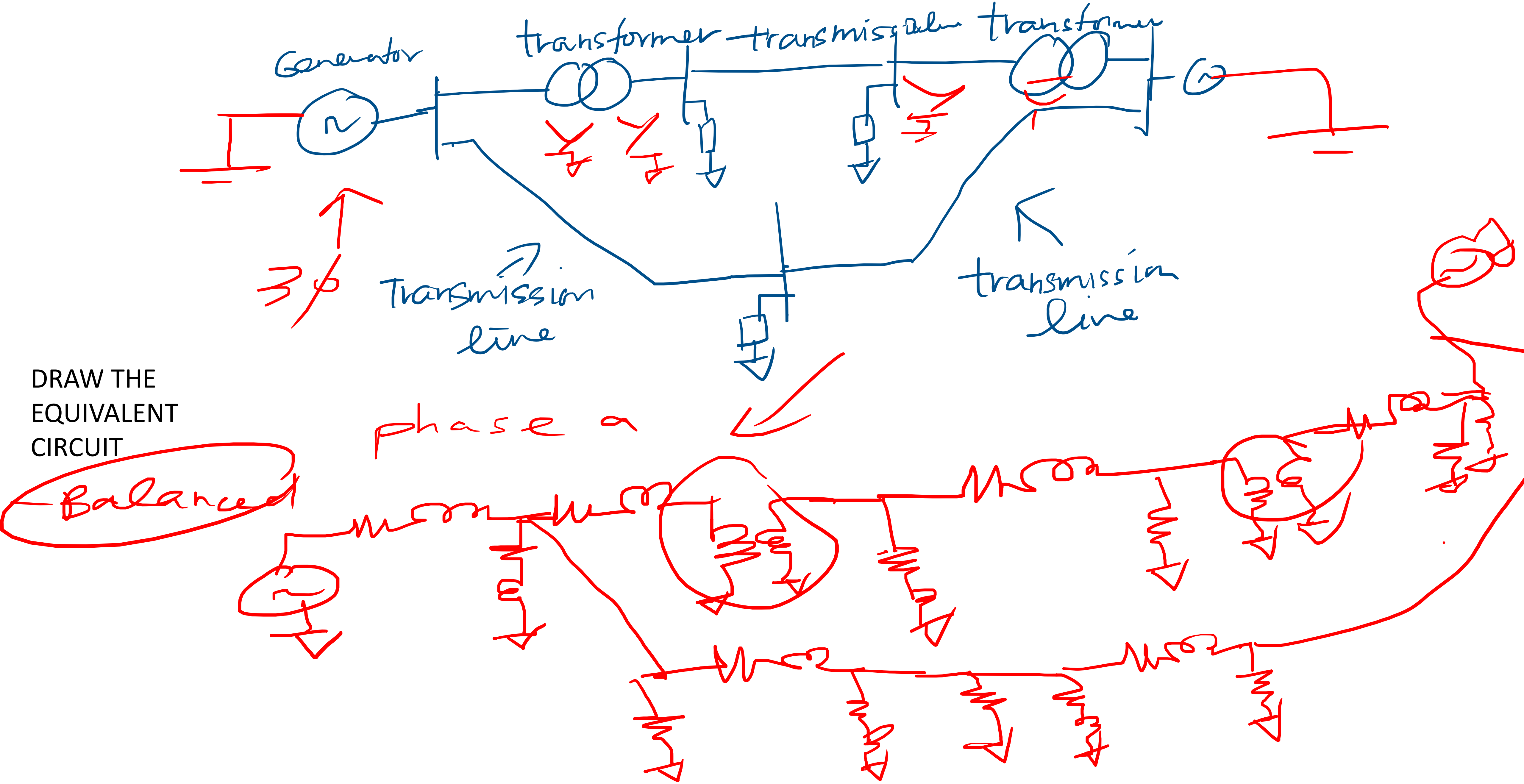


- Circuit Theory
- Per Unit
- Balanced and Unbalanced System
- Sequence Networks

Different voltage levels

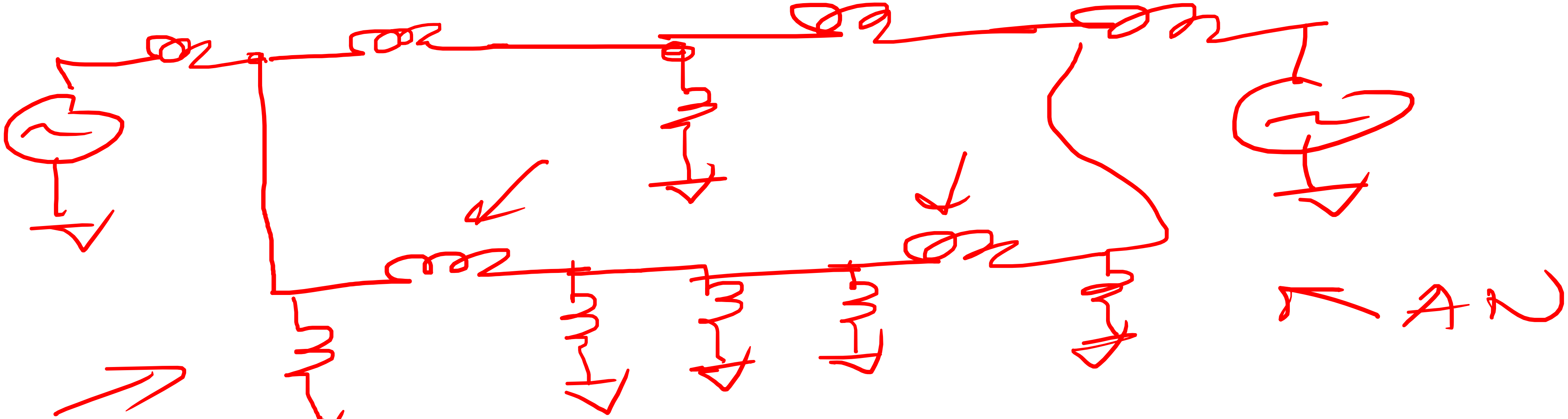
Transmission

Distribution



DRAW THE EQUIVALENT CIRCUIT

DRAW THE PER UNIT CIRCUIT



per unit

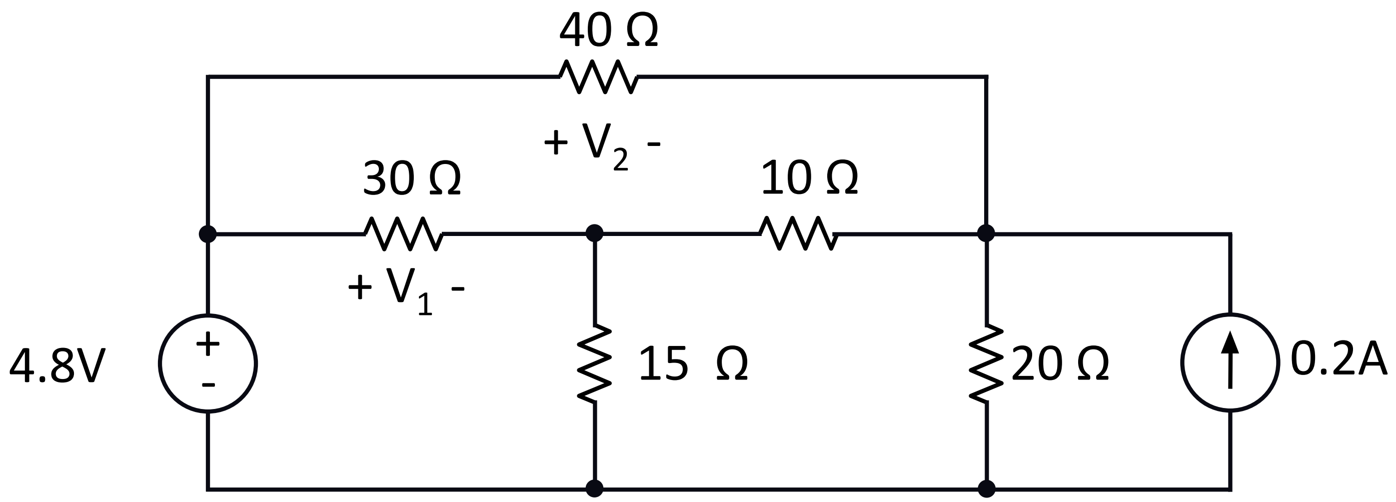
DRAW THE SEQUENCE NETWORKS

positive

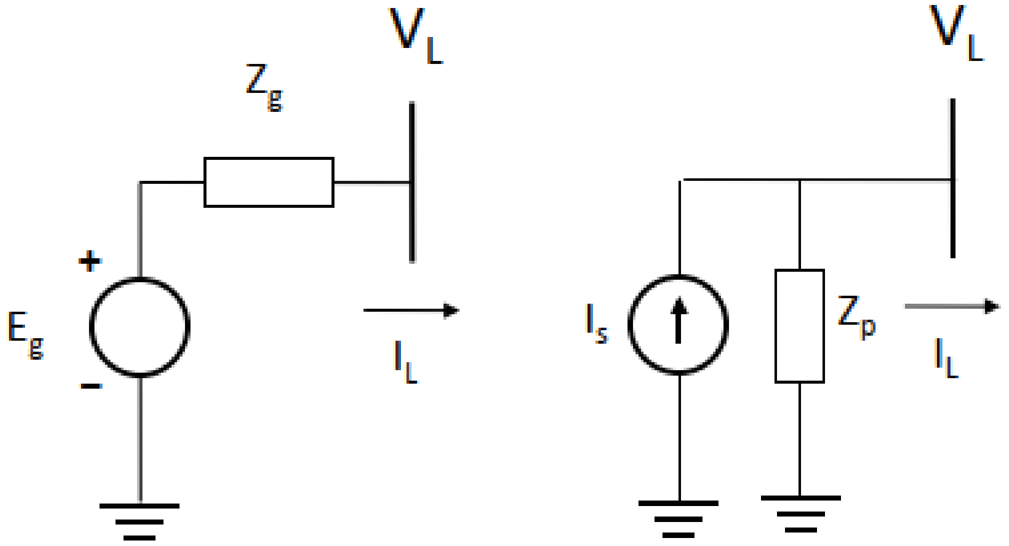
nega

Bus Admittance Matrix

Recall Node Voltage Analysis



Equivalence of Sources

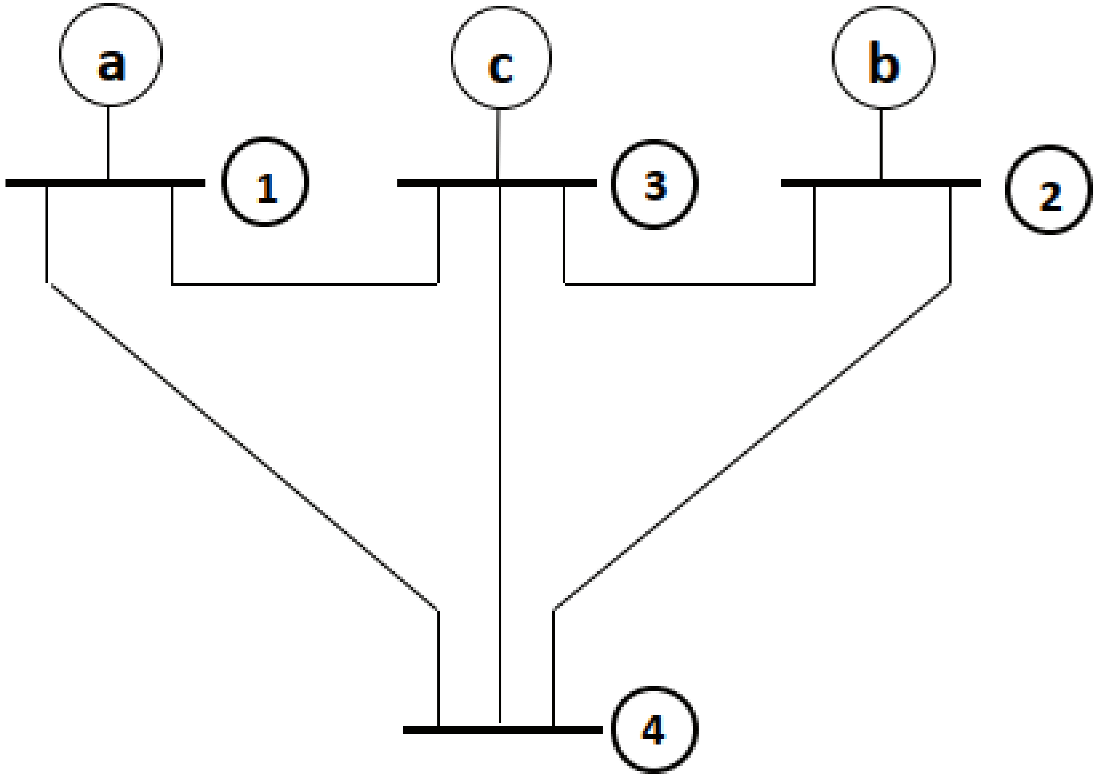


$$\mathbf{E_g = I_S Z_P}$$
$$\mathbf{Z_g = Z_p}$$

The two sources will be equivalent if V_L and I_L are the same for both circuits.

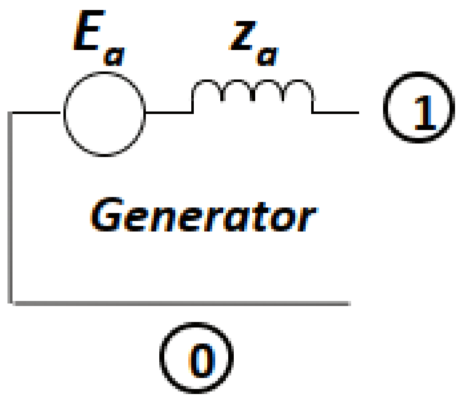
One-Line Diagram

Bus	Gen	Line
1	a	1 - 3
2	b	2 - 3
3	c	1 - 4
4		2 - 4
		3 - 4

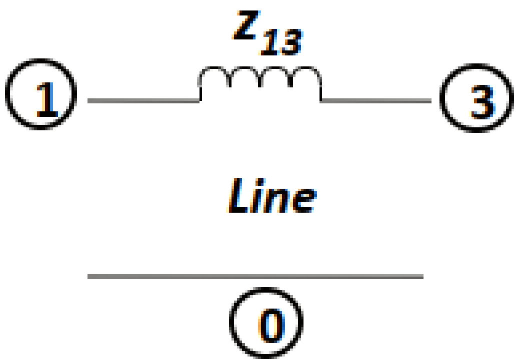


One-Line Diagram

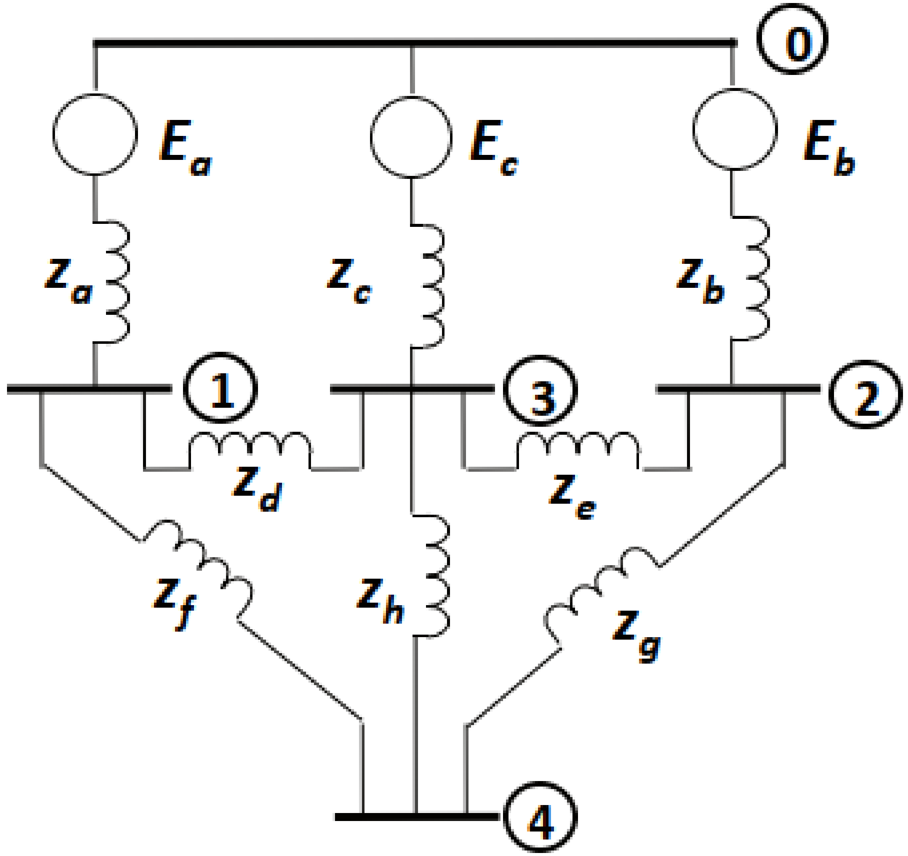
Impedance Diagram



Generator



Line

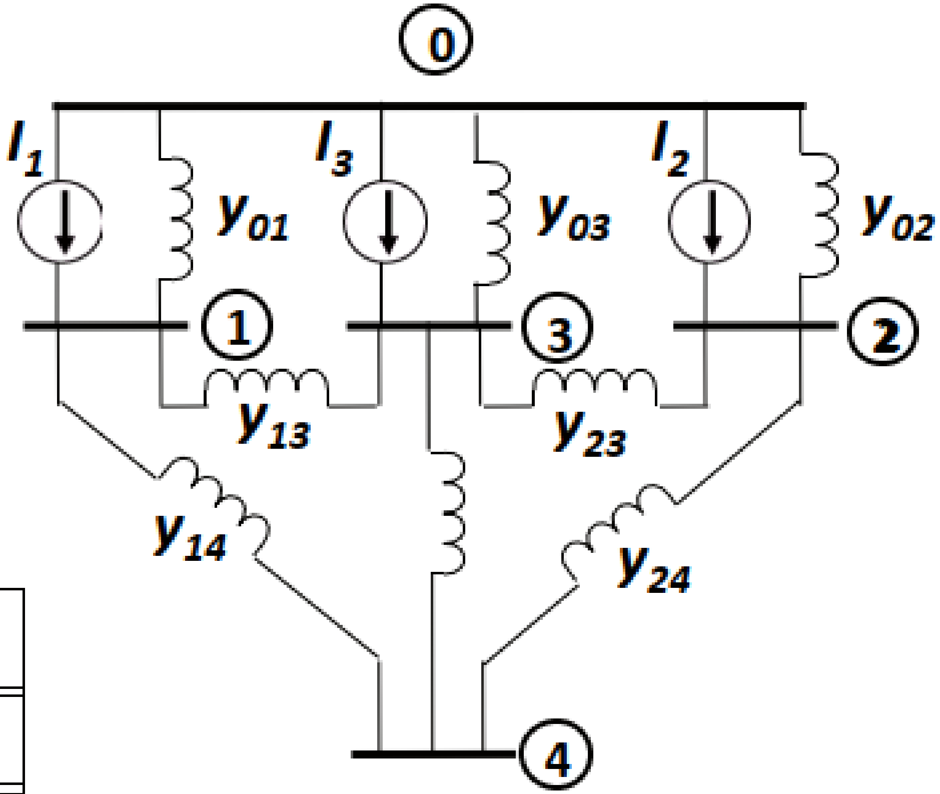


Impedance Diagram

Admittance Diagram

$I_1 = E_a / z_a$ $y_{01} = 1 / z_a$
$I_2 = E_b / z_b$ $y_{02} = 1 / z_b$
$I_3 = E_c / z_c$ $y_{03} = 1 / z_c$

$y_{13} = 1 / z_d$	$y_{14} = 1 / z_f$
$y_{23} = 1 / z_e$	$y_{24} = 1 / z_g$
	$y_{34} = 1 / z_h$



Admittance Diagram

Nodal Analysis

Applying Kirchhoff's Current Law (KCL):

at node 1:

$$I_1 = V_1 y_{01} + (V_1 - V_3) y_{13} + (V_1 - V_4) y_{14}$$

at node 2:

$$I_2 = V_2 y_{02} + (V_2 - V_3) y_{23} + (V_2 - V_4) y_{24}$$

at node 3:

$$I_3 = V_3 y_{03} + (V_3 - V_2) y_{23} + (V_3 - V_4) y_{34} + (V_3 - V_1) y_{13}$$

at node 4:

$$0 = (V_4 - V_1) y_{14} + (V_4 - V_2) y_{24} + (V_4 - V_3) y_{34}$$

Bus Admittance Matrix

Rearranging the equations,

$$I_1 = V_1(y_{01} + y_{13} + y_{14}) - V_3 y_{13} - V_4 y_{14}$$

$$I_2 = V_2(y_{02} + y_{23} + y_{24}) - V_3 y_{23} - V_4 y_{24}$$

$$I_3 = V_3(y_{03} + y_{23} + y_{34} + y_{13}) - V_1 y_{13} - V_2 y_{23} - V_4 y_{34}$$

$$0 = V_4(y_{14} + y_{24} + y_{34}) - V_1 y_{14} - V_2 y_{24} - V_3 y_{34}$$

In matrix form,

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ 0 \end{bmatrix} = \begin{bmatrix} y_{01}+y_{13}+y_{14} & 0 & -y_{13} & -y_{14} \\ 0 & y_{02}+y_{23}+y_{24} & -y_{23} & -y_{24} \\ -y_{13} & -y_{23} & y_{03}+y_{23}+y_{34}+y_{13} & -y_{34} \\ -y_{14} & -y_{24} & -y_{34} & y_{14}+y_{24}+y_{34} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

Bus Admittance Matrix

The standard form of n independent equations:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & \cdots Y_{1n} \\ Y_{21} & Y_{22} & Y_{23} & \cdots Y_{2n} \\ Y_{31} & Y_{32} & Y_{33} & \cdots Y_{3n} \\ \vdots & \vdots & \vdots & \vdots \\ Y_{n1} & Y_{n2} & Y_{n3} & \cdots Y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ \vdots \\ V_n \end{bmatrix}$$

$[\mathbf{I}] = [Y_{bus}][\mathbf{V}]$ $\qquad$ $[\mathbf{y}] = [\mathbf{A}][\mathbf{x}]$

Y_{bus} is also called Bus Admittance Matrix

Bus Admittance Matrix

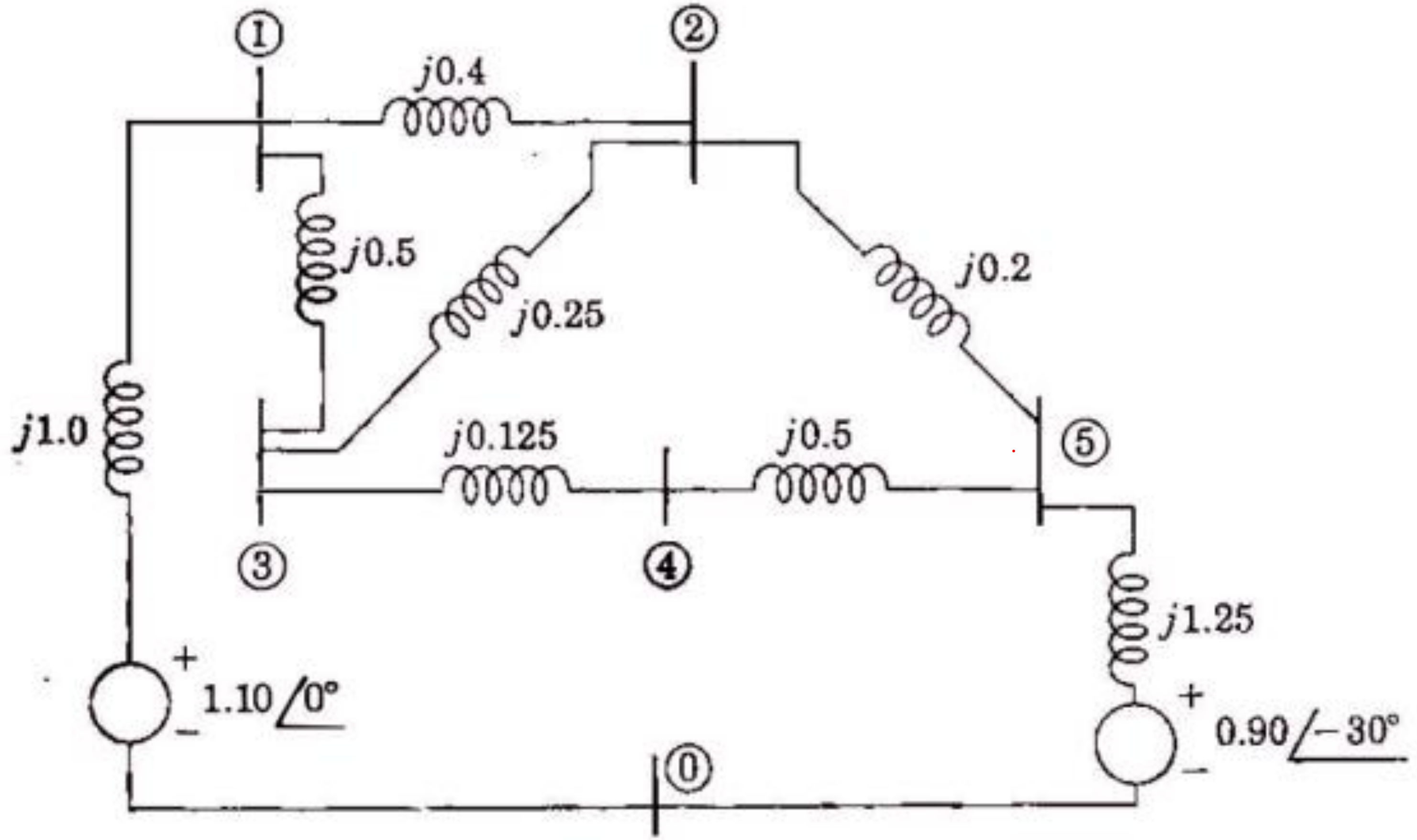
$$[Y_{BUS}] = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & \cdots Y_{1n} \\ Y_{21} & Y_{22} & Y_{23} & \cdots Y_{2n} \\ Y_{31} & Y_{32} & Y_{33} & \cdots Y_{3n} \\ \vdots & \vdots & \vdots & \vdots \\ Y_{n1} & Y_{n2} & Y_{n3} & \cdots Y_{nn} \end{bmatrix}$$

Y_{ii} = self-admittance, the sum of all admittances terminating on the node (diagonal elements)

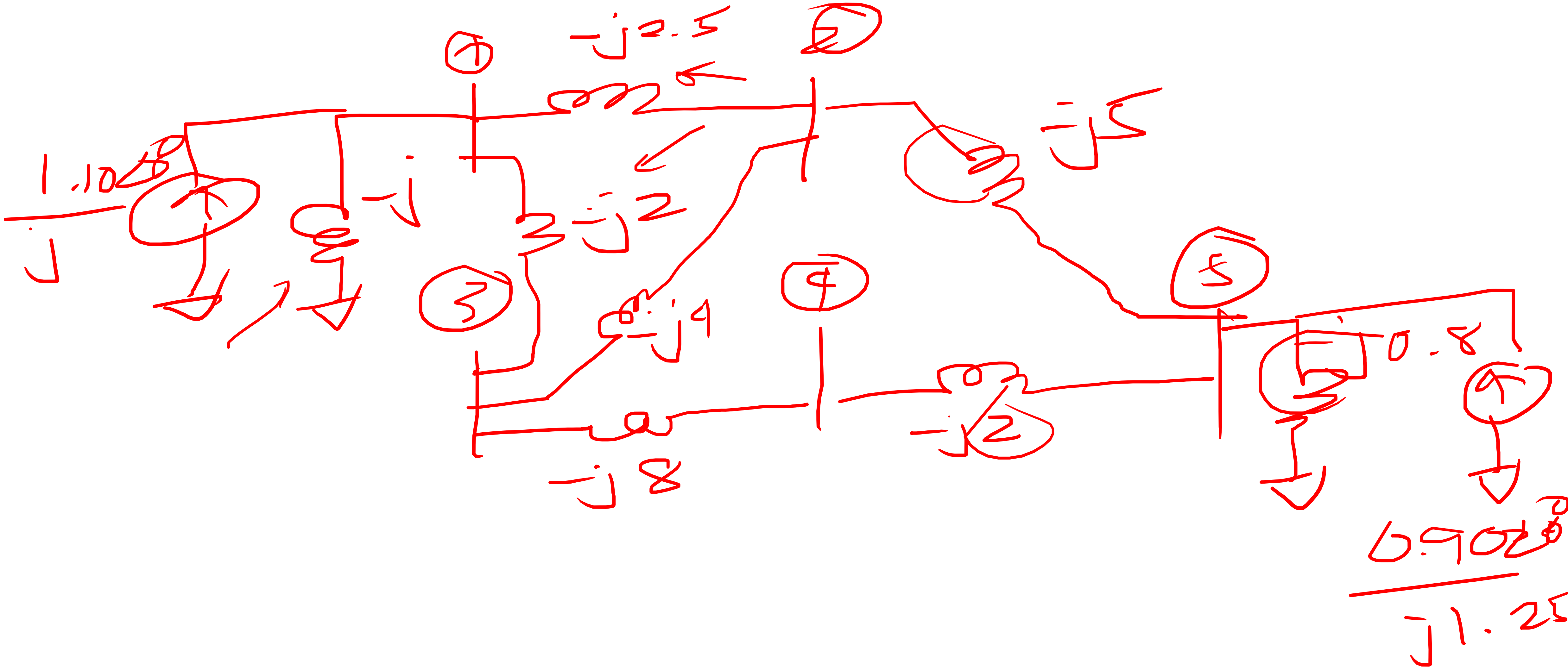
Y_{ij} = mutual admittance, the negative of the admittances connected directly between the nodes identified by the double subscripts

Example

- Find the Y-bus for the following power system:



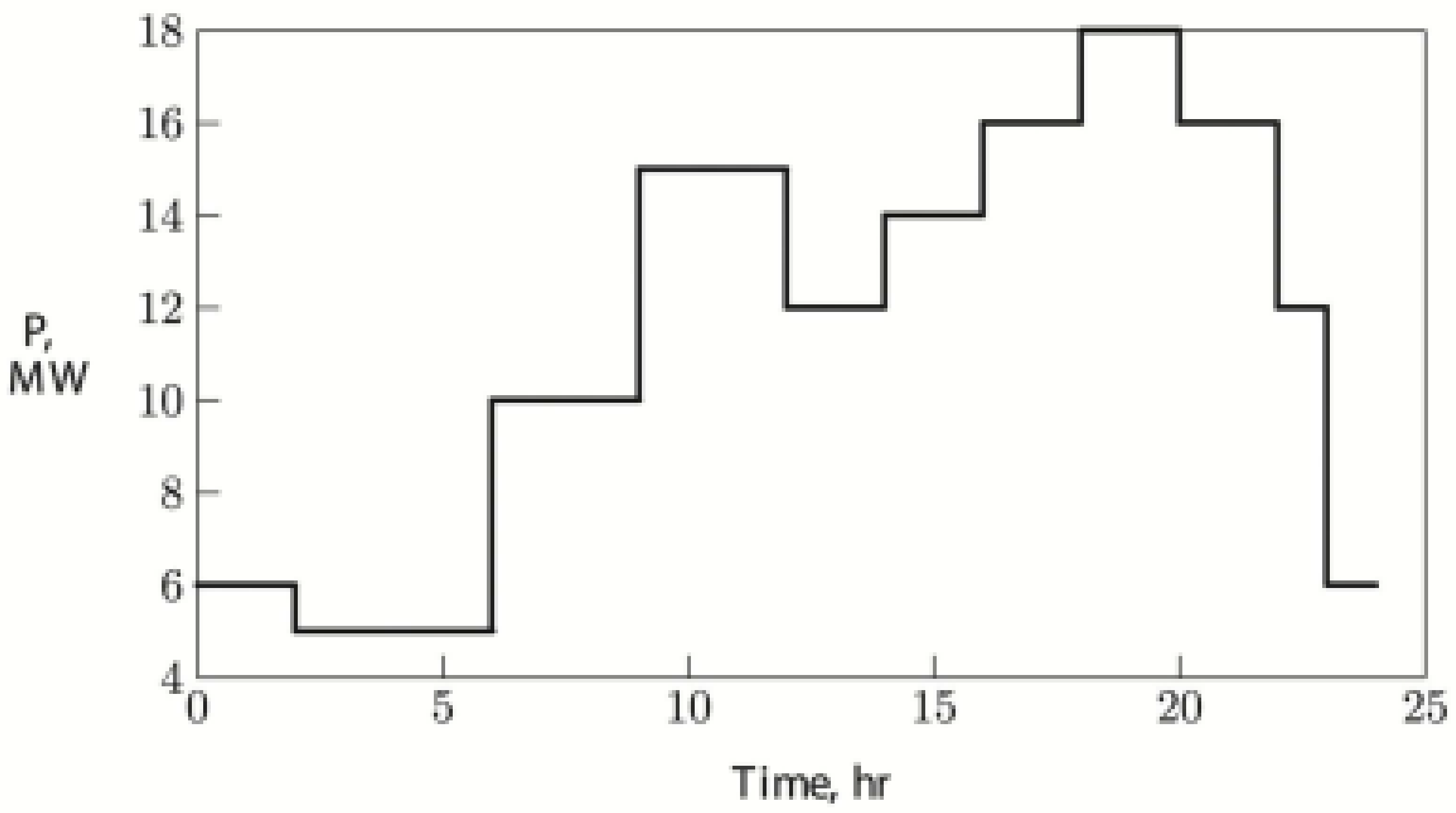
ADMITTANCE DIAGRAM



$\rightarrow$ $-j(1+2.5+2) = -j5.5$	$-(-j2.5)$ $j2.5$	$-(-j2)$ $j2$	0	0
$j2.5$	$-j(2.5+4+5)$	$j4$	0	$j5$
$j2$	$j4$	$-j(2+4+8)$	$j8$	0
0	0	$j8$	$-j(2+8)$	$j2$
0	$j5$	0	$j2$	$-j(5+2+0.8)$

Load Models

LOAD PROFILE



Load Models

$$Load = \%Z + \%I + \%P$$

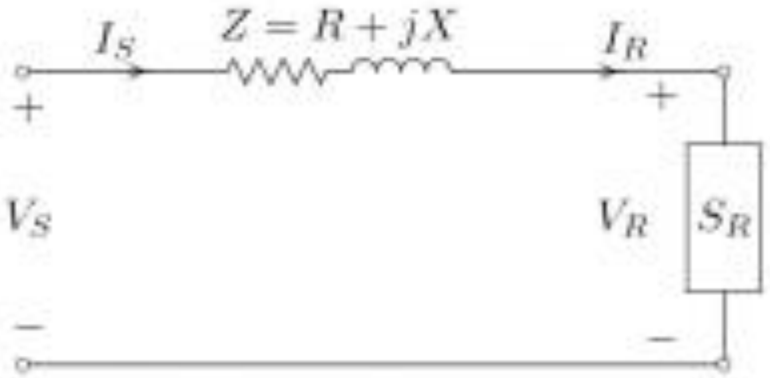
$$P_i = \frac{|V_a^2|}{|V_n^2|} * |S_n| * Z_{\%} * cos(Z_\theta) + \frac{|V_a|}{|V_n|} * |S_n| * I_{\%} * cos(I_\theta) + |S_n| * P_{\%} * cos(P_\theta)$$

$$Q_i = \frac{|V_a^2|}{|V_n^2|} * |S_n| * Z_{\%} * sin(Z_\theta) + \frac{|V_a|}{|V_n|} * |S_n| * I_{\%} * sin(I_\theta) + |S_n| * P_{\%} * sin(P_\theta)$$

where:

- P_i: Real power consumption of the ith load
- Q_i: Reactive power consumption of the ith load
- V_a: Actual terminal voltage
- V_n: Nominal terminal voltage
- S_n: Apparent Power consumption at nominal voltage
- $Z_{\%}$: Percent of load that is constant impedance
- $I_{\%}$: Percent of load that is constant current
- $P_{\%}$: Percent of load that is constant power
- Z_θ: Phase angle of constant impedance fraction
- I_θ: Phase angle of constant current fraction
- P_θ: Phase angle of constant power fraction

Load Models



If V_R, I_R are known, calculate V_S, I_S if load is defined as:

a. P + jQ (constant power)

b. Z (constant impedance)

c. I (constant current)

a.) constant Power

$$V_R I_R^* = P + jQ = \text{constant}$$

$$(V_S = Z I_R + V_R) I_R^*$$

$$V_S I_R^* = Z I_R^2 + P + jQ$$

$$V_S = \frac{Z I_R^2 + P + jQ}{I_R^*}$$

$$I_S = I_R$$

b.) Constant Impedance

$$\frac{V_R}{I_R} = Z_L = \text{constant}$$

$$I_S = I_R$$

$$\frac{V_S}{I_S} = \frac{Z I_R + V_R}{I_R}$$

$$\frac{V_S}{I_S} = Z + Z_L$$

$$\frac{V_S}{I_R} = Z + Z_L$$

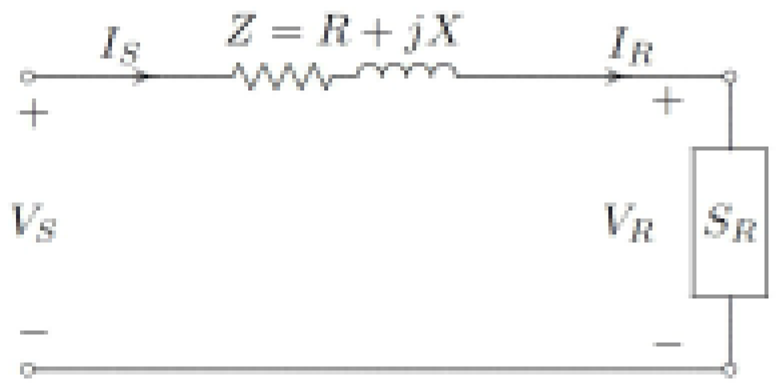
$$V_S = (Z + Z_L) I_R$$

c.) constant current

$$I_s = I_R = \text{constant}$$

$$V_s = Z I_R + V_R$$

Load Models



If V_S, I_S are known, calculate V_R, I_R if load is defined as:

a. P + jQ (constant power)

b. Z (constant impedance)

c. I (constant current)

a) constant Power

$$V_R = V_S - I_S$$

$$I_R = I_S$$

$$V_S = \frac{Z I_R^2 + P + jQ}{I_R^*}$$

$$V_S = \frac{Z I_S^2}{I_S^*} + \frac{P + jQ}{I_S^*}$$

b) Constant impedance

$$\frac{V_R}{I_R} = Z_L = \frac{V_R}{I_S}$$

$$\frac{V_S}{I_S} = \frac{V_S}{I_R} = \underbrace{Z + Z_L}_{\text{constant}}$$

$$V_R = \underline{V_S} - Z\underline{I_S}$$

$$I_S = I_R$$

a) constant current

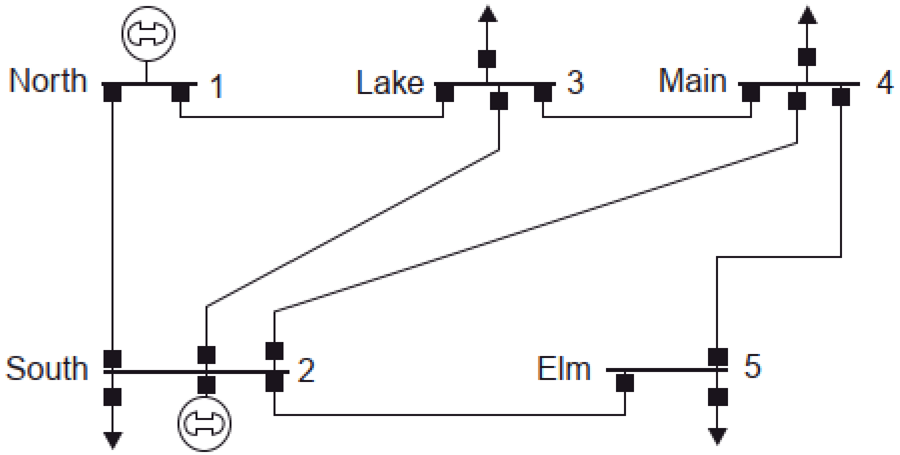
I_R = constant

$V_R = V_S - Z I_S$

$I_R = I_S$

FIGURE 6.18

Sample System Diagram



Homework 5

Determine the bus admittance matrix for the following power three phase system) Assume a three phase 100 MVA per unit base. You are given a partial bus admittance matrix. Ignore generator reactance and loads. Numbers are exact.

TABLE 6.9

Bus input data for Problem 6.30

Bus-to-Bus	R per unit	X per unit	B per unit
1-2	0.02	0.06	0.06
1-3	0.08	0.24	0.05
2-3	0.06	0.18	0.04
2-4	0.08	0.24	0.05
2-5	0.02	0.06	0.02
3-4	0.01	0.04	0.01
4-5	0.03	0.10	0.04

Assume Medium Length Lines.

TABLE 6.10

Partially Completed Bus Admittance Matrix (Y_{bus})

$6.25 - j18.695$	$-5.00 + j15.00$	$-1.25 + j3.75$	0	0
$-5.00 + j15.00$	$12.9167 - j38.665$			

Complete Bus matrix is 5x5