EEE 51: Second Semester 2017 - 2018

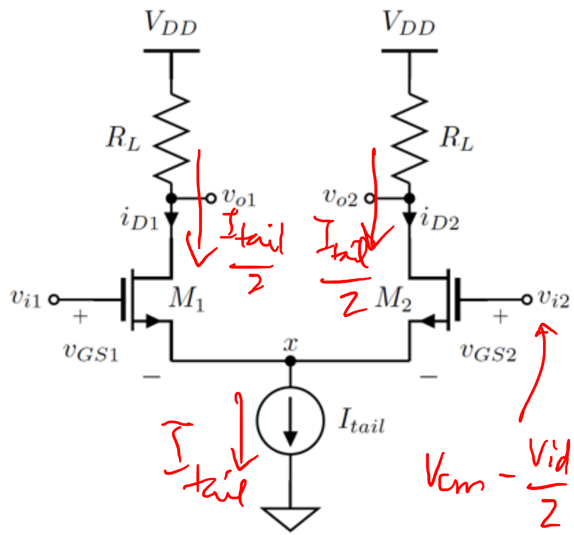
Lecture 10

Differential Circuits

Today

- MOSFET Differential Circuits

The MOSFET Differential Amplifier



* M_1 & M_2 must be matched
  - $K_1 = K_2$, $\lambda_1 = \lambda_2$, $V_{TH1} = V_{TH2}$

* we want to look at how V_{od} changes with V_{id}

case 1: $V_{id} = \varnothing$ $[V_{i1} = V_{i2}]$

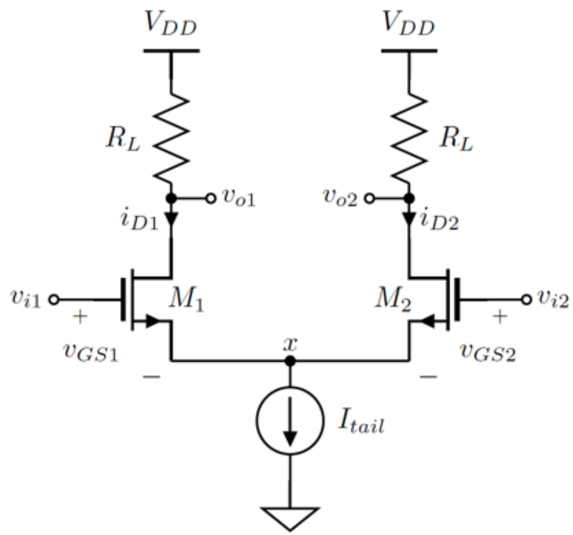
$$I_{D1} = I_{D2} = \frac{I_{tail}}{2}$$

$$V_{o1} = V_{DD} - I_{D1} R_L = V_{DD} - \frac{I_{tail}}{2} R_L$$

$$V_{o2} = V_{DD} - I_{D2} R_L = V_{DD} - \frac{I_{tail}}{2} R_L$$

$$V_{od} = \varnothing$$

The MOSFET Differential Amplifier



case 2: $V_{id} > \emptyset$ [$V_{i1} > V_{i2}$]

$I_{D1} > I_{D2} \Rightarrow V_{o1} < V_{o2}$

$V_{od} < \emptyset$ max $I_{D1} = I_{tail}$

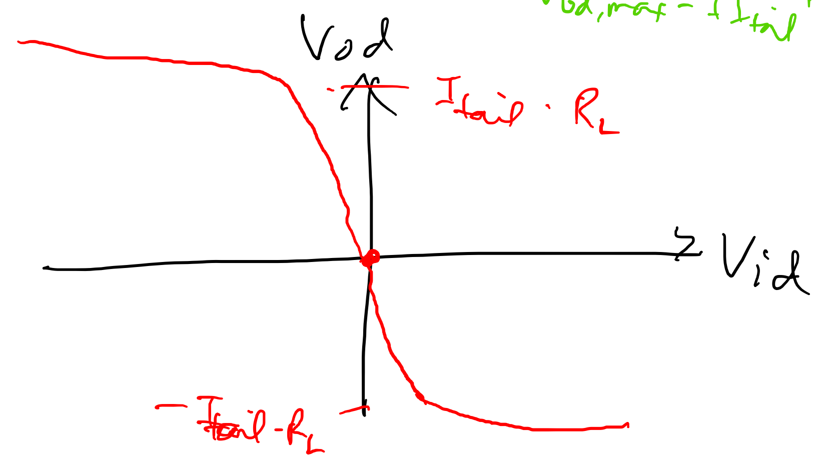
$\left.\begin{array}{l} V_{o1} = V_{DD} - I_{tail} \cdot R_L \\ V_{o2} = V_{DD} \end{array}\right\} |V_{od,max}| = |- I_{tail} \cdot R_L|$

case 3: $V_{id} < \emptyset$ [$V_{i1} < V_{i2}$]

$I_{D1} < I_{D2} \Rightarrow V_{o1} > V_{o2}$

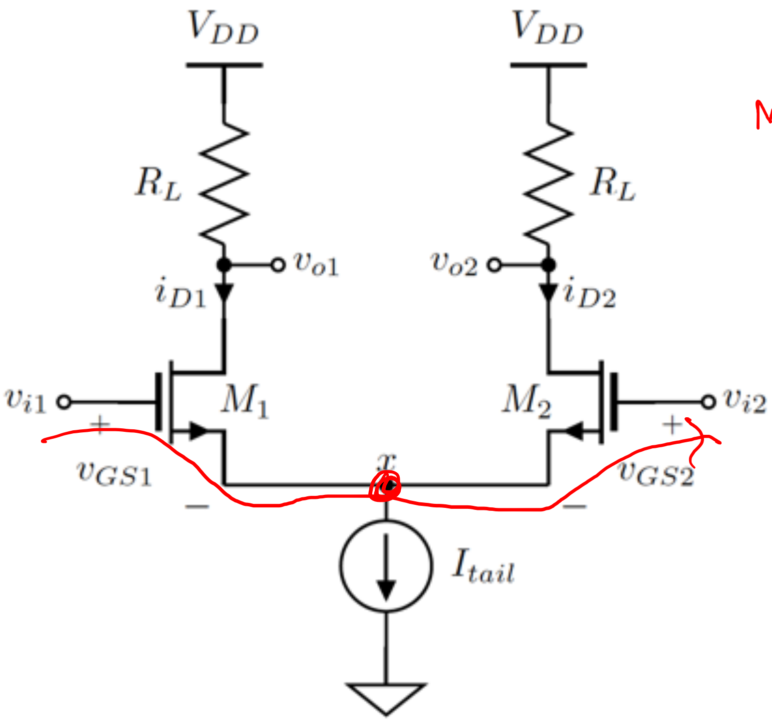
$V_{od} > \emptyset$ max $I_{D2} = I_{tail}$

$V_{od,max} = +I_{tail} \cdot R_L$



The MOSFET Differential Amplifier (1)

- Source-coupled pair: DC Analysis



KVL at the input loop: $V_{i1} - V_{GS1} + V_{GS2} - V_{i2} = 0$

MOSFET in sat: $I_D \approx k(V_{GS} - V_{TH})^2$

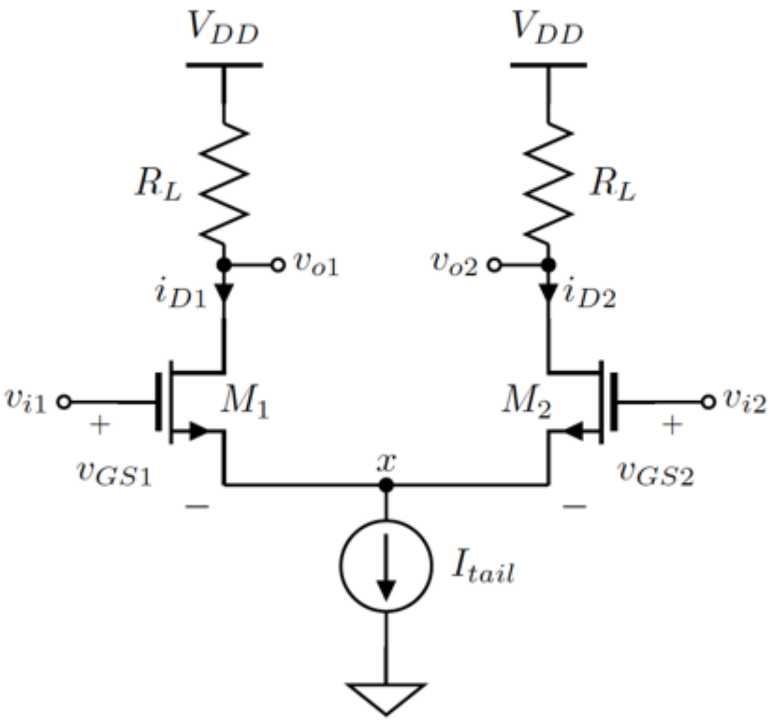
$$\underbrace{V_{i1} - V_{i2}}_{V_{id}} = V_{GS1} - V_{GS2}$$

$$V_{id} = \left(\cancel{V_{TH}} + \sqrt{\frac{I_{D1}}{k}}\right) - \left(\cancel{V_{TH}} + \sqrt{\frac{I_{D2}}{k}}\right)$$

$$\boxed{V_{id} = \sqrt{\frac{I_{D1}}{k}} - \sqrt{\frac{I_{D2}}{k}}}$$

The MOSFET Differential Amplifier (2)

- Drain currents



KCL at node x: $I_{tail} = I_{D1} + I_{D2}$ $\leftarrow$ $I_{D1} = I_{tail} - I_{D2}$

Recall: $\boxed{V_{id} = \sqrt{\frac{I_{D1}}{k}} - \sqrt{\frac{I_{D2}}{k}}}$

$$\boxed{I_{D1} = \frac{I_{tail}}{2} + V_{id}\frac{k}{2}\sqrt{\frac{2 \cdot I_{tail}}{k} - V_{id}^2}} = \frac{I_{tail}}{2} + \Delta I$$

$$\boxed{I_{D2} = \frac{I_{tail}}{2} - V_{id}\frac{k}{2}\sqrt{\frac{2 \cdot I_{tail}}{k} - V_{id}^2}} = \frac{I_{tail}}{2} - \Delta I$$

$$V_{id} = \sqrt{\frac{I_{D1}}{K}} - \sqrt{\frac{I_{D2}}{K}} \quad (1) \qquad I_{tail} = I_{D1} + I_{D2} \quad (2)$$

$$\left(V_{id}\right)^2 = \left(\sqrt{\frac{I_{D1}}{K}} - \sqrt{\frac{I_{tail} - I_{D1}}{K}}\right)^2$$

$$V_{id}^2 = \cancel{\frac{I_{D1}}{K}} - 2\sqrt{\frac{I_{D1}\left(I_{tail} - I_{D1}\right)}{K^2}} + \frac{I_{tail} - \cancel{I_{D1}}}{K}$$

$$\left(\sqrt{I_{D1}\left(I_{tail} - I_{D1}\right)}\right)^2 = \left(\frac{K}{2}\left(\frac{I_{tail}}{K} - V_{id}^2\right)\right)^2$$

$$I_{D1}\left(I_{tail} - I_{D1}\right) = \frac{K^2}{4}\left(\frac{I_{tail}}{K} - V_{id}^2\right)^2$$

Rearranging the previous equation:

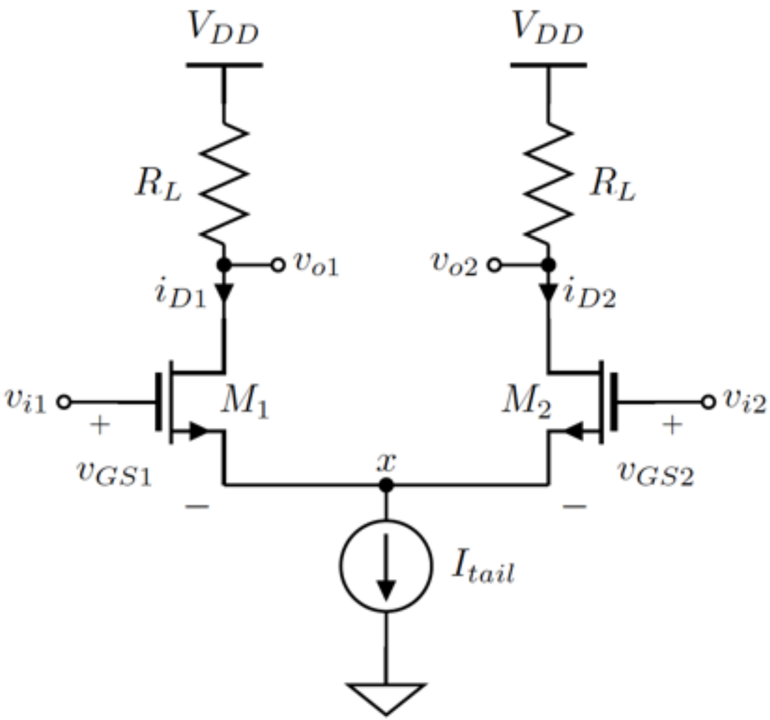
$$I_{D1}^2 - I_{tail} \cdot I_{D1} + \frac{K^2}{4}\left(\frac{I_{tail}}{K} - V_{id}^2\right)^2 = 0$$

Applying the quadratic formula:

$$I_{D1} = \frac{I_{tail}}{2} \pm \frac{V_{id} \cdot K}{2} \sqrt{\frac{2 I_{tail}}{K} - V_{id}^2}$$

The MOSFET Differential Amplifier (3)

- Drain currents



$$I_{D1} = \frac{I_{tail}}{2} + V_{id}\frac{k}{2}\sqrt{\frac{2 \cdot I_{tail}}{k} - V_{id}^2} = \frac{I_{tail}}{2} + \Delta I$$

$$I_{D2} = \frac{I_{tail}}{2} - \underbrace{V_{id}\frac{k}{2}\sqrt{\frac{2 \cdot I_{tail}}{k} - V_{id}^2}}_{=\frac{I_{tail}}{2}} = \frac{I_{tail}}{2} - \Delta I$$

$I_{D1,max} = I_{tail}$

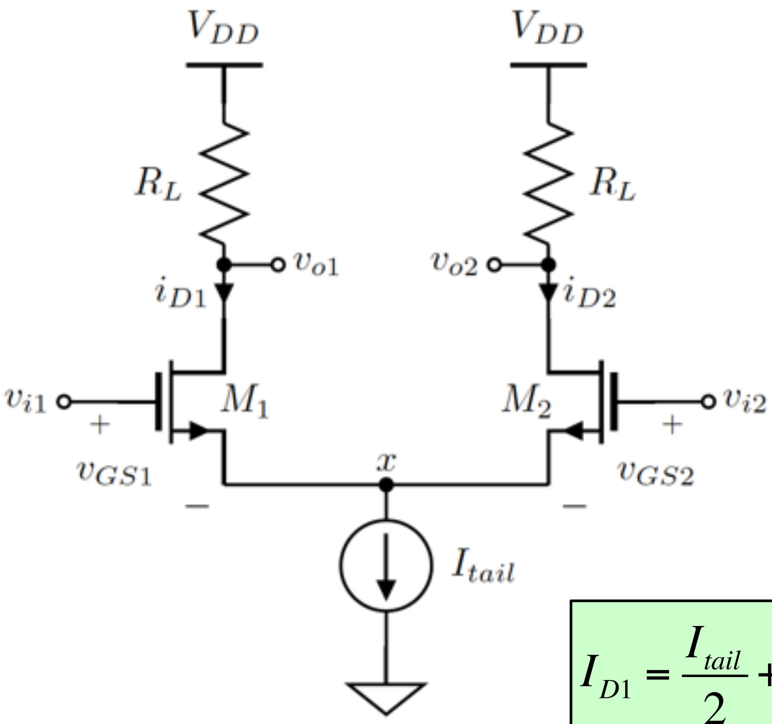
Note: $\boxed{\Delta I_{\max} = \frac{I_{tail}}{2}}$

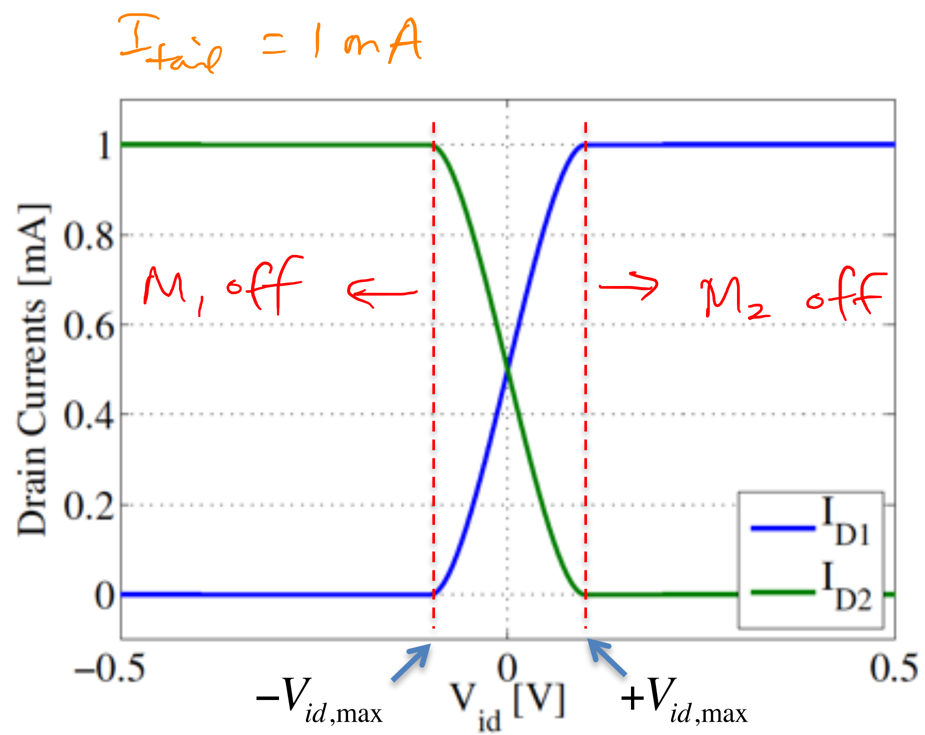
$$\boxed{V_{id,\max} = \sqrt{\frac{I_{tail}}{k}}}$$

Beyond this, all the tail current flows in one branch

The MOSFET Differential Amplifier (4)

- The source-couple pair



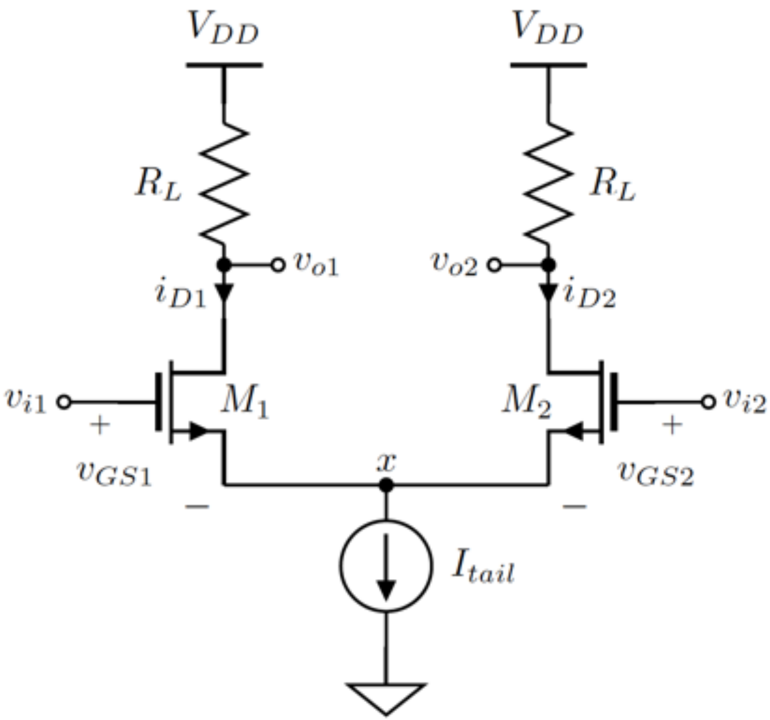


$$I_{D1} = \frac{I_{tail}}{2} + V_{id}\frac{k}{2}\sqrt{\frac{2 \cdot I_{tail}}{k} - V_{id}^2}$$

$$I_{D2} = \frac{I_{tail}}{2} - V_{id}\frac{k}{2}\sqrt{\frac{2 \cdot I_{tail}}{k} - V_{id}^2}$$

The MOSFET Differential Amplifier (5)

- Output voltage



$V_{o1} = V_{DD} - I_{D1} \cdot R_L$

$V_{o2} = V_{DD} - I_{D2} R_L$

KVL at the output loop: $V_{od} = V_{o1} - V_{o2} = R_L\left(I_{D2} - I_{D1}\right)$

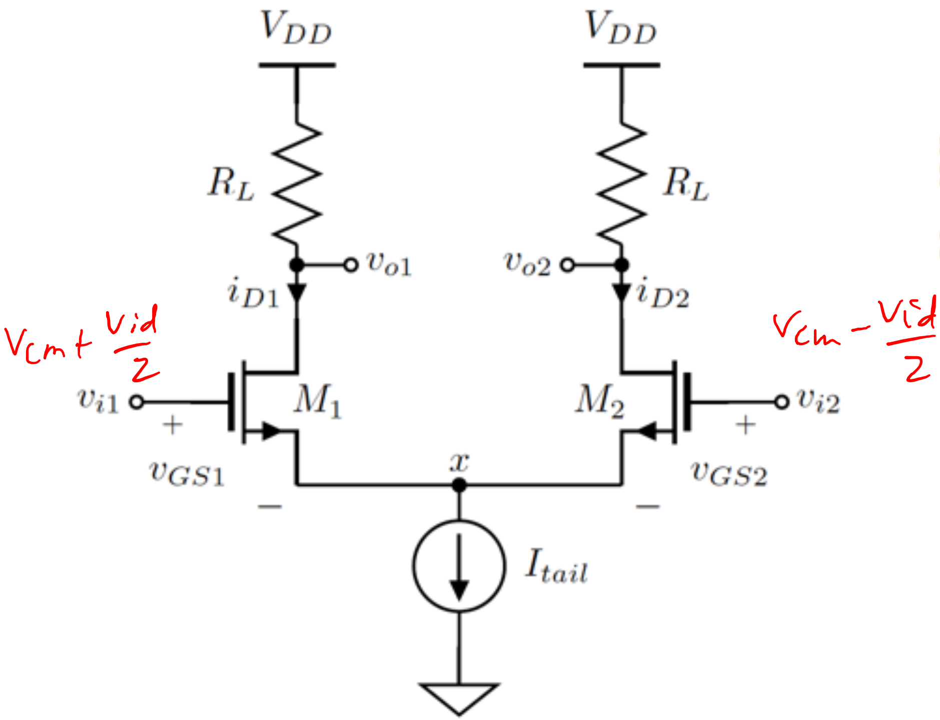
$$I_{D2} = \frac{I_{tail}}{2} - V_{id}\frac{k}{2}\sqrt{\frac{2 \cdot I_{tail}}{k} - V_{id}^2}$$

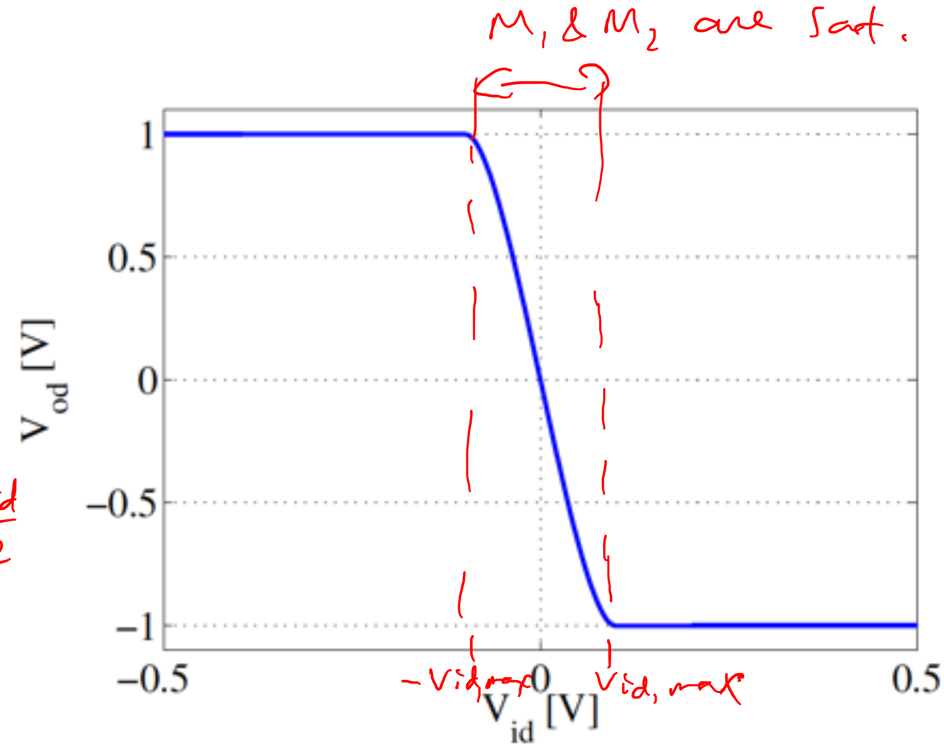
$$I_{D1} = \frac{I_{tail}}{2} + V_{id}\frac{k}{2}\sqrt{\frac{2 \cdot I_{tail}}{k} - V_{id}^2}$$

$$V_{od} = -R_L \cdot V_{id} \cdot k\sqrt{\frac{2 \cdot I_{tail}}{k} - V_{id}^2}$$

The MOSFET Differential Amplifier (6)

- Transfer characteristic



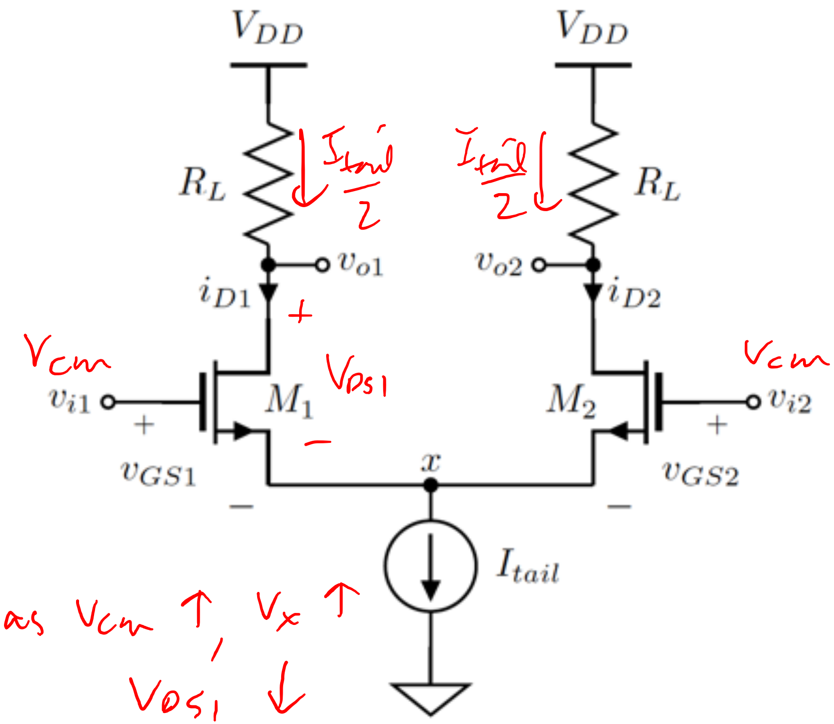


$$V_{od} = -R_L \cdot V_{id} \cdot k \sqrt{\frac{2 \cdot I_{tail}}{k} - V_{id}^2}$$

Common-Mode Input Range (1)

- MOSFET operating region?

to keep M_1 sat, $V_{DS1} > V_{GS1} - V_{TH}$



as $V_{cm} \uparrow$, $V_x \uparrow$, $V_{DS1} \downarrow$

Assume zero differential input

$V_{i1} = V_{i2} = V_{cm}$

KVL: $V_{DS1} = V_{DD} - I_{D1}R_L - V_X > V_{GS1} - V_{TH}$

$\frac{I_{tail}}{2}$

Note: $V_X = V_{I1} - V_{GS1}$

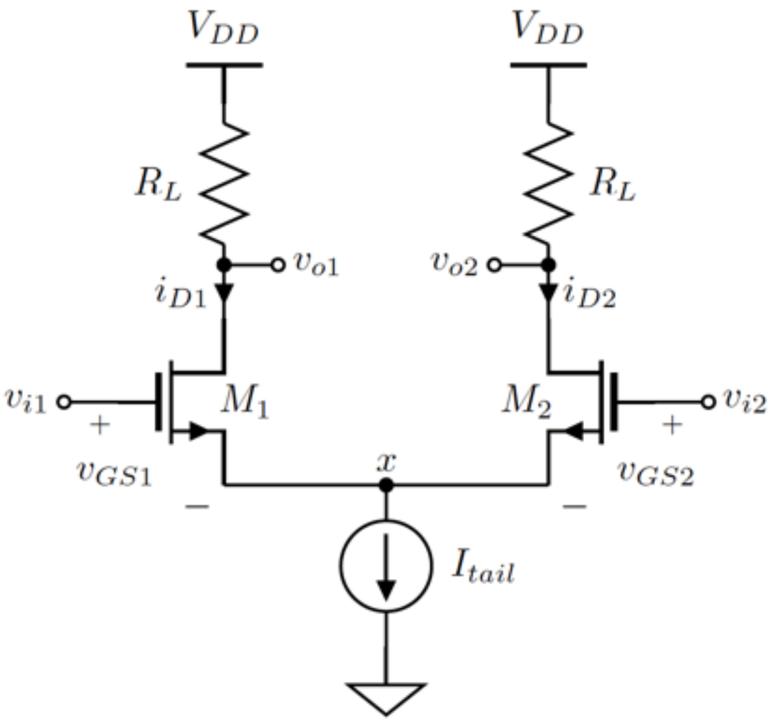
V_{cm}

V_X is controlled by the input common-mode (DC input)!

$$V_{DS1} = V_{DD} - I_{D1}R_L - V_{I1} + V_{GS1}$$

Common-Mode Input Range (2)

- MOSFET operating region?



For zero differential input: $v_{ic} = V_{I1} = V_{I2} = V_{cm}$

$$\boxed{I_{D1} = I_{D2} = \frac{I_{tail}}{2}}$$

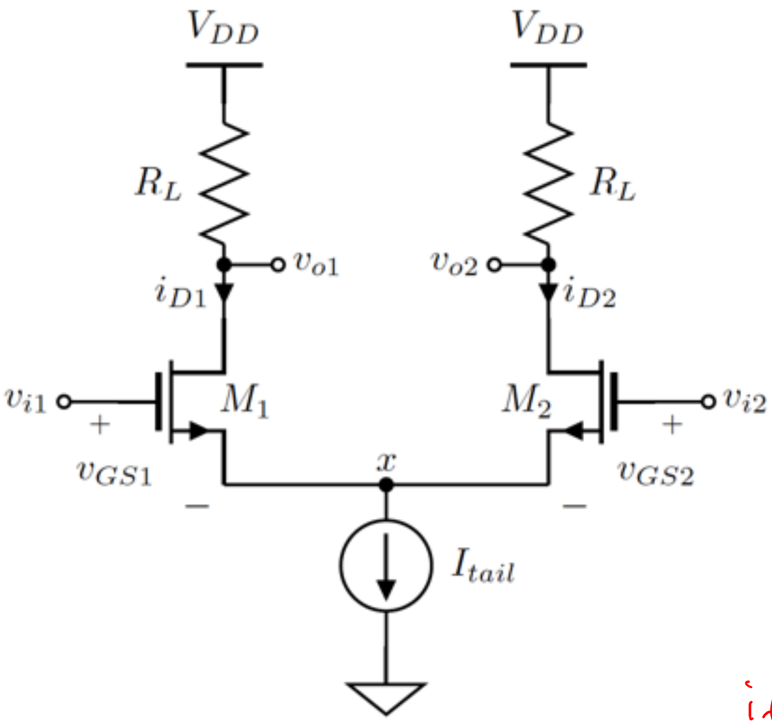
For a range of common-mode inputs!

$$\boxed{V_{DS1} = V_{DD} - \frac{I_{tail}R_L}{2} - V_{cm} + V_{GS1}}$$

$$\boxed{V_{DS1} = V_{DD} - \frac{I_{tail}R_L}{2} - V_{cm} + \cancel{V_{GS1}} > \cancel{V_{GS1}} - V_{TH}}$$

Common-Mode Input Range (3)

- MOSFET operating region?



$$V_{DS1} = V_{DD} - \frac{I_{tail}R_L}{2} - V_{cm} + V_{GS1} > V_{GS1} - V_{TH}$$

$$V_{DD} - \frac{I_{tail}R_L}{2} - V_{cm} > -V_{TH}$$

$V_{cm} < V_{DD} - \frac{I_{tail}}{2} R_L + V_{TH}$

Maximum common-mode input voltage:

$$\boxed{V_{cm,\max} = V_{DD} - \frac{I_{tail}R_L}{2} + V_{TH}}$$

if $V_{cm} > V_{cm,max}$, M_1 & M_2 goes to linear region.

Common-Mode Input Range (4)

- MOSFET operating region?

If the tail current is not ideal → V_{min}

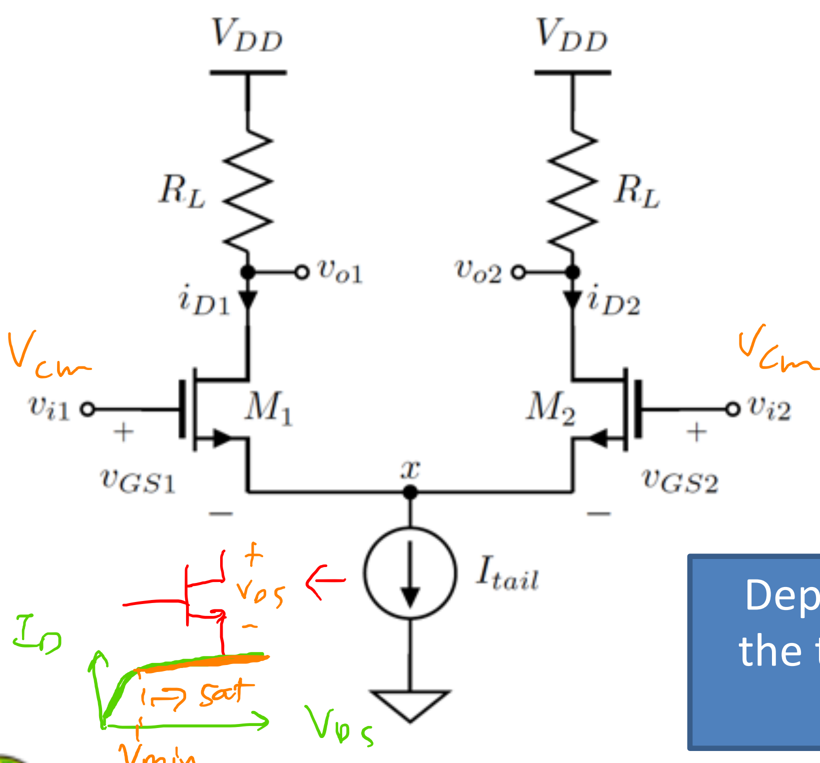
for a simple C.M

$V_{min} = V_{DSAT} = V_{GS} - V_{TH}$

$$V_X = V_{I1} - V_{GS1}$$

$$= \cancel{V_{I1}} - V_{TH} - \sqrt{\frac{I_{D1}}{k}} = V_{cm} - V_{TH} - \sqrt{\frac{I_{tail}}{2 \cdot k}} > V_{min}$$

V_{cm}



$$V_{cm} > V_{min} + V_{TH} + \sqrt{\frac{I_{tail}}{2 \cdot k}}$$

Minimum common-mode input voltage:

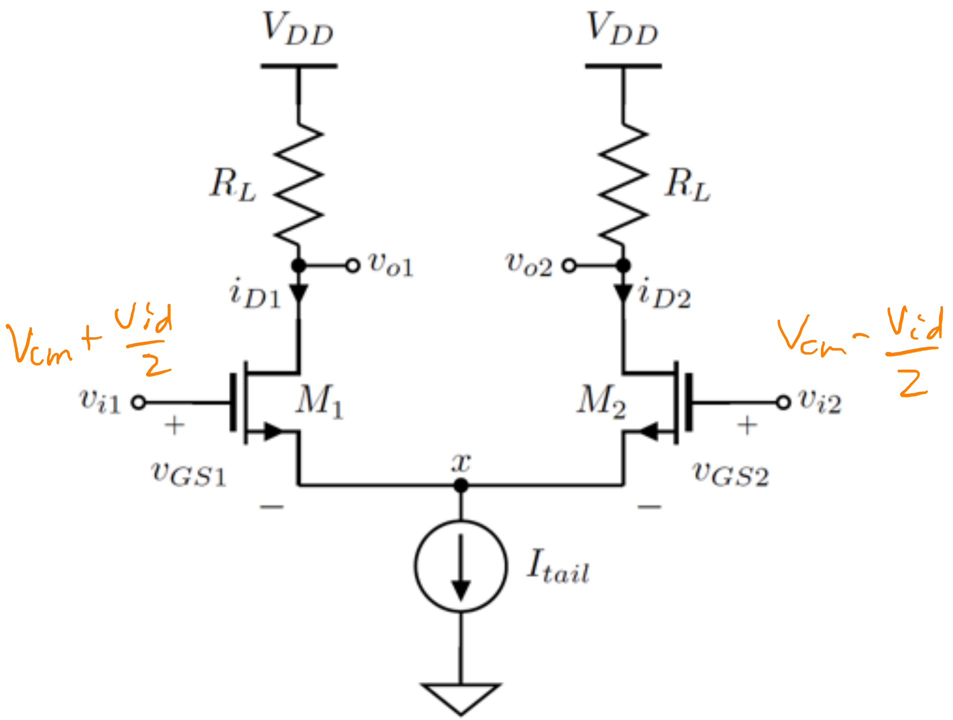
Dependent on the tail current source

$$\boxed{V_{cm,\min} = V_{\min} + V_{TH} + \sqrt{\frac{I_{tail}}{2 \cdot k}}}$$

if $V_{cm} < V_{cm,min}$, no tail current source

Common-Mode Input Range (5)

- MOSFET operating region: also set by the input common-mode



$$V_{cm,\max} = V_{DD} - \frac{I_{tail} R_L}{2} + V_{TH}$$

Common-mode input range

$$V_{cm,\min} = V_{\min} + V_{TH} + \sqrt{\frac{I_{tail}}{2 \cdot k}}$$

Next Meeting

- Differential Circuit Small Signal Analysis
- Compound Amplifiers