EEE 51 Assignment 9 Solution

2nd Semester SY 2017-2018
Due: 5pm Tuesday, May 8, 2018 (Rm. 220)

Instructions: Write legibly. Show all solutions and state all assumptions. Write your full name, student number, and section at the upper-right corner of each page. Start each problem on a new sheet of paper. Box or encircle your final answer.

Answer sheets should be colored according to your lecture section. The color scheme is as follows:

THQ – yellow
THR – blue
THU – white
THX – green
WFX – pink

1. **What is Feedback?** Chaeyoung is an engineer who is building a communication system. She wants to reduce the effect of noise from unwanted nearby signals by using a negative feedback circuit, which is shown in Figure 1. Assume that the op amp is ideal, and $\beta \to \infty$ for the transistor.

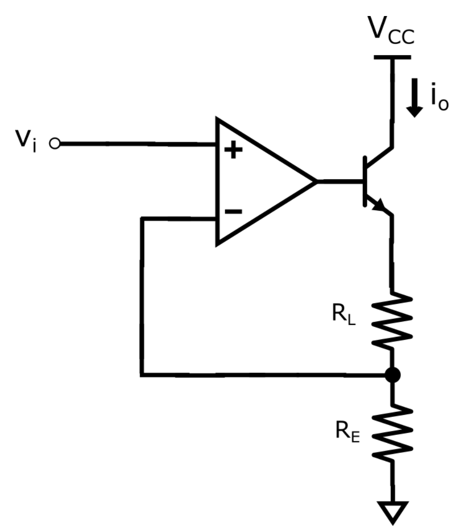

Figure 1: Tzuyu's negative feedback circuit

(a) What is the quantity being sampled (i.e. what is the output quantity): voltage or current? [0.5 pt]

current [0.5 pt]

(b) What is the quantity being mixed with the input: voltage or current? [0.5 pt]

voltage [0.5 pt]

(c) From your answers in (a) and (b), what type of feedback topology is being used? [1 pt]

series-series [1 pt]

(d) Recall that the forward gain, A, is the ratio of the output signal (s_o) to the error signal (s_e). Express A in terms of R_L and R_E. [2 pts]

Remember that the quantity being sampled is current, and the quantity being mixed is voltage. Therefore:

$$A = \frac{s_o}{s_e} = \frac{i_o}{v_e} \tag{1}$$

The feedback voltage, v_{fb}, is the voltage at the inverting input of the op amp. So the error voltage is:

$$v_e = v_i - v_{fb} \tag{2}$$

We can then express Eq. (1) as:

$$A = \frac{i_o}{v_i - v_{fb}} \tag{3}$$

But since the op amp is ideal,

$$v_i = v_{fb} \tag{4}$$

therefore the open loop gain is:

$$\boxed{A \rightarrow \infty \text{ [2 pts]}}$$

(e) On the other hand, the feedback factor F is the ratio of the signal being fed back to the input (s_{fb}) to the output signal (s_o). Express T in terms of R_L and R_E. [2 pts]

$$F = \frac{s_{fb}}{s_o} = \frac{v_{fb}}{i_o} \tag{5}$$

v_{fb} can be expressed as the voltage across R_E:

$$v_{fb} = R_E i_o \tag{6}$$

F then becomes:

$$F = \frac{R_E i_o}{i_o} \tag{7}$$

$$\boxed{F = R_E \text{ [2 pts]}}$$

(f) Given your answers in (d) and (e), what is the closed loop gain, A_{CL}? [2 pts]

The general expression for A_{CL} is:

$$A_{CL} = \frac{A}{1 + AF} \tag{8}$$

But since $A \rightarrow \infty >> 1$, A_{CL} simplifies to:

$$A_{CL} = \frac{1}{F} \tag{9}$$

$$\boxed{A_{CL} = \frac{1}{R_E} \text{ [2 pts]}}$$

2. **Frequency response of a common emitter amplifier with feedback.** A common emitter amplifier is shown in **??**. Given that $V_{CC} = 12\,\text{V}$, $V_{IN} = 5\,\text{V}$, $V_A = 100\,\text{V}$, $I_S = 1\,\text{fA}$, $R_S = 10\,\text{k}\Omega$, $R_C = 500\,\Omega$, $R_f = 1\,\text{k}\Omega$, and $\beta = 50$, answer the following questions.

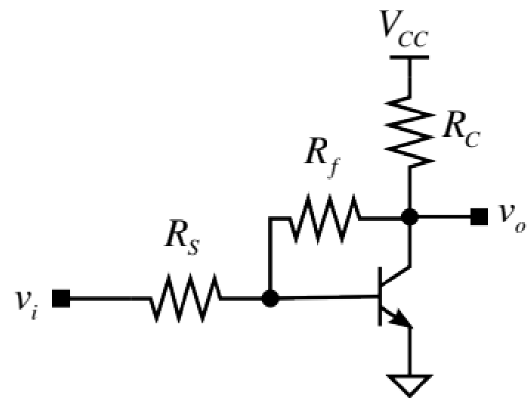

Figure 2: Common Emitter Amplifier with Resistive Feedback Network

(a) Draw the equivalent small-signal circuit for both open-loop and closed-loop circuits. (Ignore parasitic capacitances.)

In order to find the bias current, the equations that we need are the following,

$$\beta I_B = I_C \tag{10}$$

$$I_B = \frac{1}{\beta+1} I_S(e^{\frac{V_{BE}}{V_T}} - 1) \tag{11}$$

$$I_B = \frac{\beta}{\beta+1} I_S(e^{\frac{V_{BE}}{V_T}} - 1) \tag{12}$$

$$\frac{V_o - V_{BE}}{R_f} = I_{R_f} \tag{13}$$

$$\frac{V_{CC} - V_o}{R_C} = I_{R_C} \tag{14}$$

KCL at the output node:

$$I_{R_C} = I_C + I_{R_f} \tag{15}$$

KCL at the base terminal node:

$$I_B = I_{R_f} + I_{R_s} \tag{16}$$

Combining equations, we get two simultaneous equation:

$$\frac{V_{CC} - V_o}{R_C} = \frac{\beta}{\beta+1} I_S(e^{\frac{V_{BE}}{V_T}} - 1) + \frac{V_o - V_{BE}}{R_f} \tag{17}$$

$$\frac{1}{\beta+1} I_S(e^{\frac{V_{BE}}{V_T}} - 1) = \frac{V_o - V_{BE}}{R_f} + \frac{V_{IN} - V_{BE}}{R_S} \tag{18}$$

Substituting actual values and the isolating V_o:

$$V_o = \frac{V_{BE}}{3} + \frac{24}{3} - \frac{50000}{153}(1\,\text{fA})(e^{\frac{V_{BE}}{0.026}} - 1) \tag{19}$$

$$V_o = \frac{1000}{51}(1\,\text{fA})(e^{\frac{V_{BE}}{0.026}} - 1) + \frac{11V_{BE}}{10} - \frac{1}{2} \tag{20}$$

Using these simultaneous equations, we can find V_{BE}:

$$V_{BE} = 0.827\,815\,\mathrm{V} \tag{21}$$

Solving for I_C:

$$I_C = \frac{\beta}{\beta+1}(1\,\mathrm{fA})(e^{\frac{0.827815}{0.026}} - 1) \tag{22}$$

$$\boxed{\begin{array}{c} g_m = \frac{I_C}{V_T} = 2.53\,\mathrm{S} \\ r_o = \frac{V_A}{I_C} = 1517.3\,\Omega \\ r_\pi = \frac{\beta V_T}{I_C} = 19.7253\,\Omega \end{array}}$$

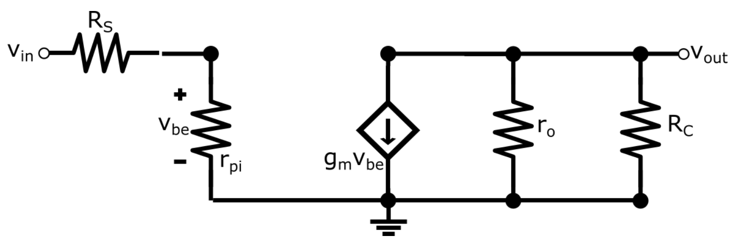

Figure 3: Common Emitter Amplifier with Resistive Feedback Network Open-Loop Small-Signal Circuit [1 pts]

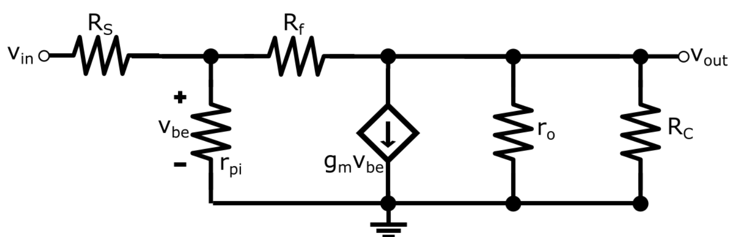

Figure 4: Common Emitter Amplifier with Resistive Feedback Network Closed-Loop Small-Signal Circuit [1 pts]

(b) What is the small-signal open-loop gain of the amplifier?
The gain of a simple common-emitter is given by:

$$A_v = -g_m R_o = -g_m(r_o||R_L) \text{ [2 pts]} \tag{23}$$

$$\boxed{A_v = -g_m(R_C||r_o) = 951.46 \text{ [1 pt]}}$$

(c) What is the small-signal closed-loop gain of the amplifier?
Solution 1: Assuming direct solution, the gain of the amplifier is given by(from the slides):

$$R_1 = R_S||R_f||r_\pi = 19.2068\,\Omega \tag{24}$$

$$R_2 = R_C||R_f = 333.33\,\Omega \tag{25}$$

$$\frac{v_o}{v_i} = -\frac{R_f}{R_s}\frac{\frac{R_1R_2}{R_f}\left(g_m - \frac{1}{R_f}\right)}{1 + \frac{R_1R_2}{R_f}\left(g_m - \frac{1}{R_f}\right)} \tag{26}$$

$$\boxed{\frac{v_o}{v_i} = -0.0942 \text{ [1 pt]}}$$

Solution 2: Assuming feedback analysis, the gain of the amplifier is given by(again, from the slides):

$$\frac{v_o}{v_i} = -\frac{R_f}{R_s}\frac{\frac{g_m R_1 R_2}{R_f}}{1+\frac{g_m R_1 R_2}{R_f}} \tag{27}$$

$$\boxed{\frac{v_o}{v_i} = -0.0942 \text{ [1 pt]}}$$

3. **Analysis of a feedback amplifier.** Consider the feedback amplifier shown in Fig. 5. Provided that all transistors have $\beta \to \infty$, $V_{be,on} = 0.7V$, $V_A \to \infty$, an input DC voltage of 1V and an output DC voltage of 1.5V,

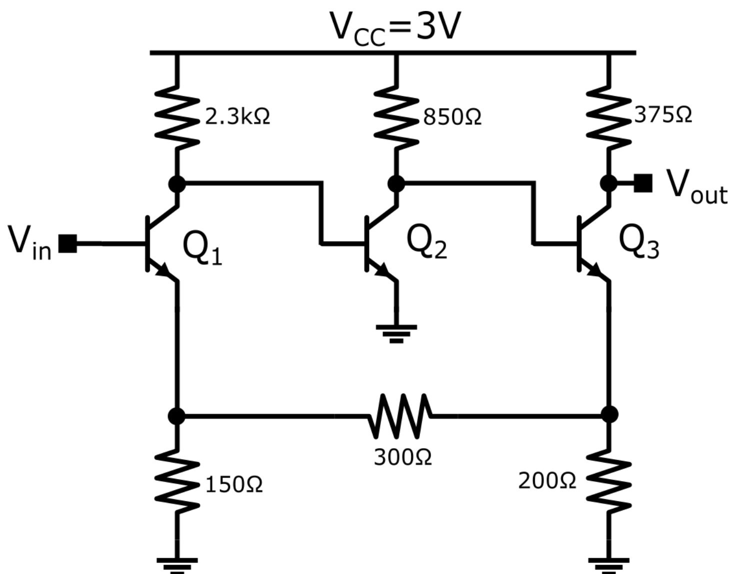

Figure 5: Basic feedback amplifier

(a) Determine the collector currents and transconductances of Q_1, Q_2 and Q_3. [1 pt]

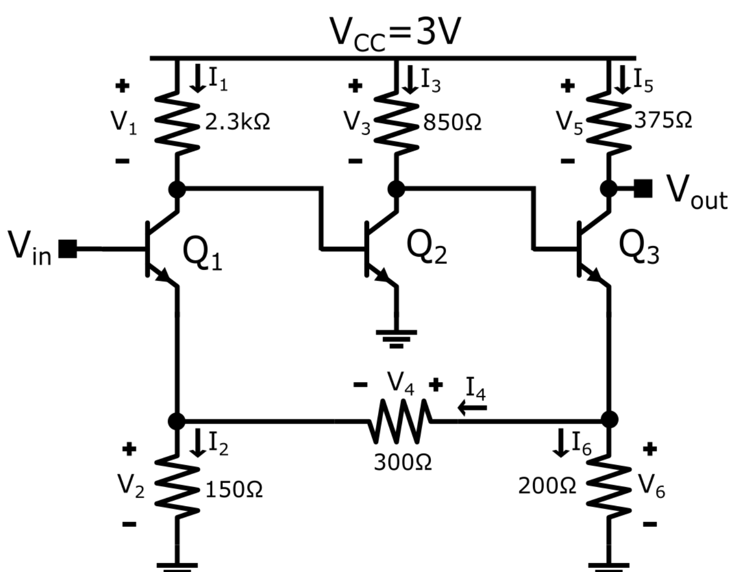

Figure 6: Basic feedback amplifier with voltage and current notations

Refer to Fig. 6 above. First, with $\beta \to \infty$, all of the base currents of Q_1, Q_2 and Q_3 are zero ($I_{b1} = I_{b2} = I_{b3} = 0A$). Also from Fig. 6, we solve the indicated voltages and currents as follows:

$$V_1 = V_{CC} - V_{be,on\_Q2} = 3 - 0.7 = 2.3V \quad \rightarrow I_1 = I_{C\_Q1} = \frac{2.3}{2.3k} = 1mA \tag{28}$$

$$V_2 = V_{in} - V_{be,on\_Q1} = 1 - 0.7 = 0.3V \quad \rightarrow I_2 = \frac{0.3}{150} = 2mA \tag{29}$$

$$I_4 = I_2 - I_1 = 2mA - 1mA = 1mA \quad \rightarrow V_4 = 300 * I_4 = 0.3V \tag{30}$$

$$V_6 = V_4 + V_2 = 0.3 + 0.3 = 0.6V \quad \rightarrow I_6 = \frac{0.6}{200} = 3mA \tag{31}$$

$$V_5 = V_{CC} - V_{out} = 3 - 1.5 = 1.5V \quad \rightarrow I_5 = I_{C\_Q3} = \frac{1.5}{375} = 4mA \tag{32}$$

$$V_3 = V_{CC} - V_{be,on\_Q3} - V_6 = 3 - 0.7 - 0.6 = 1.7V \quad \rightarrow I_3 = I_{C\_Q2} = \frac{1.7}{850} = 2mA \tag{33}$$

$$I_{C\_Q1} = 1mA$$
$$I_{C\_Q2} = 2mA$$
$$I_{C\_Q3} = 4mA$$
[0.5pts all or nothing]

For the transconductances we use the following equation and considering room temperature ($T = 300K$),

$$g_m = \frac{I_C}{V_T} \tag{34}$$

$$g_{m\_Q1} = \frac{I_{C1}}{V_T} = \frac{1mA}{26mV} = 38.5mS$$
$$g_{m\_Q2} = \frac{I_{C2}}{V_T} = \frac{2mA}{26mV} = 76.9mS$$
$$g_{m\_Q3} = \frac{I_{C3}}{V_T} = \frac{4mA}{26mV} = 153.8mS$$
[0.5pts all or nothing]

(b) What type of feedback topology is used in this circuit? Draw the feedback network. [2 pts]
The feedback network is shown below in Fig. 7

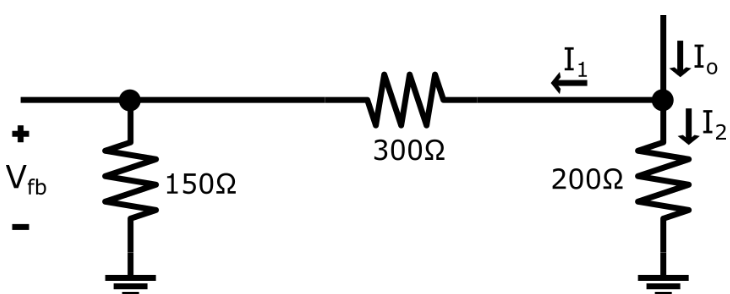

Figure 7: Feedback network

The sampled signal is a current signal and the feedback signal is a voltage signal, hence the feedback topology of this amplifier is **series-series**.

[1 pt] will be given for the drawing of the feedback network and [1 pt] will be given for the correct feedback topology.

(c) Find the feedback factor, F, the input and output resistances of the feedback network. [3 pts]
From Fig. 7,

$$V_{fb} = 150 * I_1 \tag{35}$$

$$(300 + 150)I_1 = 200I_2 = 200(I_o - I_1) \tag{36}$$

$$I_1 = \frac{200}{200+300+150} I_0 \tag{37}$$

Substitute Eq. 37 to Eq. 35,

$$V_{fb} = 150 * \frac{200}{200+300+150} I_0 \quad \rightarrow F = \frac{V_{fb}}{I_0} = \frac{600}{13} V/A = 46.15 V/A \tag{38}$$

For the input resistance,

$$R_{i,fb} = 200//450 = 138.46\Omega \tag{39}$$

For the output resistance,

$$R_{o,fb} = 150//500 = 115.38\Omega \tag{40}$$

$$\boxed{\begin{gathered} F = 46.15V/A \text{ [1pt]} \\ R_{i,fb} = 138.46\Omega \text{ [1 pt]} \\ R_{o,fb} = 115.38\Omega \text{ [1 pt]} \end{gathered}}$$

(d) Draw the small signal model of the amplifier. [2 pts]
Since $\beta \rightarrow \infty$ and $V_A \rightarrow \infty$, $r_\pi = r_o = \infty$,

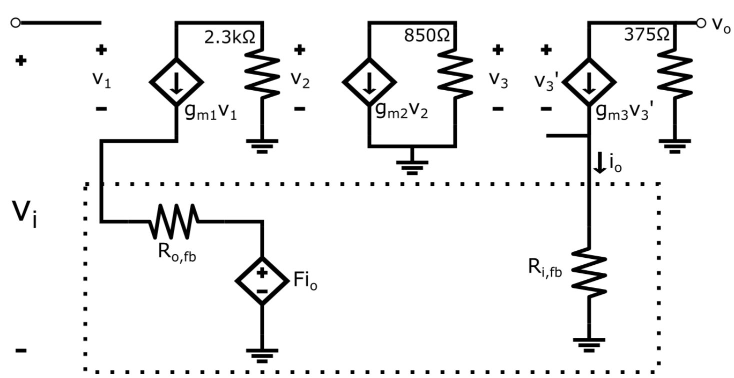

Figure 8: Small signal model

(e) Determine the open-loop gain of the amplifier. [2 pts]
To determine the open-loop gain of the amplifier, we analyze the circuit shown in Fig. 9 below and get the expression for $\frac{i_o}{v_i}$,

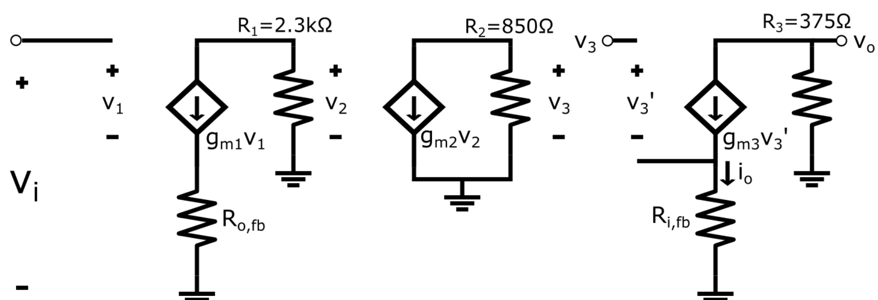

Figure 9: Open loop circuit

Thru basic circuit analysis techniques (KCL, KVL) we get the following expressions:

$$v_i = v_1 + g_{m1} v_1 R_{o,fb} = v_1(1 + g_{m1} R_{o,fb}) \tag{41}$$

$$v_2 = -g_{m1} v_1 R_1 \tag{42}$$

$$v_3 = -g_{m2} v_2 R_2 \tag{43}$$

$$v_3 = v_{3'} + g_{m3} v_{3'} R_{i,fb} = v_{3'}(1 + g_{m3} R_{i,fb}) \tag{44}$$

$$i_o = g_{m3} v_{3'} \tag{45}$$

Combining Eqs. 41-45 together, we get the expression for the open-loop gain as,

$$A = \frac{i_o}{v_i} = \frac{g_{m1} R_1}{1 + g_{m1} R_{o,fb}} * g_{m2} R_2 * \frac{g_{m3}}{1 + g_{m3} R_{i,fb}} \tag{46}$$

Substitute the values for the resistances and transconductances, we get the following open-loop gain:

$A = \frac{i_o}{v_i} = 7.337 A/V$ [0.5 pts]
[1.5 pts] will be given for the solution

(f) What is the loop gain of the feedback network? [1 pt]
The loop gain of the amplifier is,

$T = AF = 7.337 A/V \; * \; 46.15 V/A = 338.6$ [1 pt]

(g) What is the closed-loop gain of the amplifier? [1 pt]
The closed-loop **transconductance** gain is,

$$G_{m,cl} = \frac{A}{1 + AF} = 0.0216 A/V \tag{47}$$

However, this is not the closed-loop gain of the whole amplifier. Since

$$i_o = G_{m,cl} v_i \tag{48}$$

and

$$v_o = -i_o * 375 \tag{49}$$

$$\frac{v_o}{v_i} = -G_{m,cl} * 375 \tag{50}$$

$A_v = \frac{v_o}{v_i} = -8.1$ [1 pt]

TOTAL: 30 points.