Lecture
Unbalanced Three Phase Systems and Three Phase Power Measurement

Agenda

Lecture Agenda

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How do we measure power?

→ Real

→ Reactive power

Limitation

Devices can only

Real power

Lecture Outcomes

at the end of the lecture, the student must be able to …

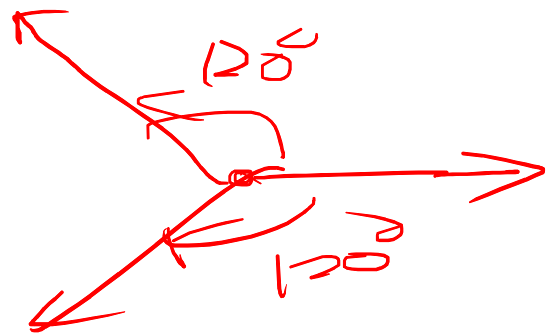
- Define what makes a Three Phase system unbalanced.
- Outline how a single-phase wattmeter can be used for single-phase and three phase power measurements.

UNBALANCED THREE-PHASE SYSTEMS

Unbalanced Three-Phase Systems

An unbalanced system contains at least one of the following:

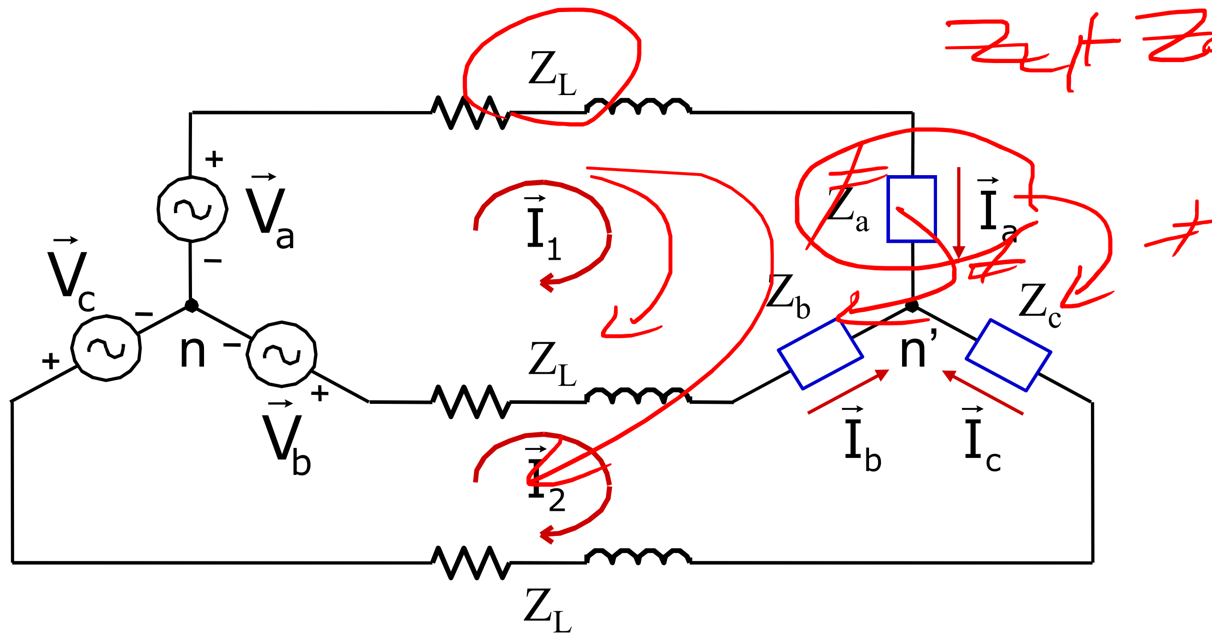
- Unbalanced three-phase source(s);
- Unbalanced loads; or
- Lines have unequal impedances.



Unbalance can be due to:

- ☞Difference in magnitudes; and/or
- ☞Phase angle displacements $\neq 120°$.

$Z_{L1} + Z_a = Z_{L2} + Z_b$



- Source phase voltages are not equal
- Load impedances are not equal
- Line impedances are not equal

Methods of Solution

1. Simultaneous Equations → mesh → nodal

   Network analysis on three phases.

   – Simplifying assumption – source is a balanced three-phase source, either
     - Balanced three-phase voltage source; or
     - Balanced three-phase current source.

2. Symmetrical Components

Review Questions

- An Unbalanced System contains at least one of the following:
  - Unbalanced three-phase source(s);
  - Unbalanced loads; or
  - Lines have unequal impedances.

POWER MEASUREMENTS

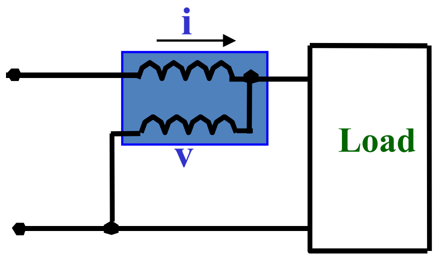
The Wattmeter

An instrument that has a potential coil and a current coil so arranged that its deflection is proportional to

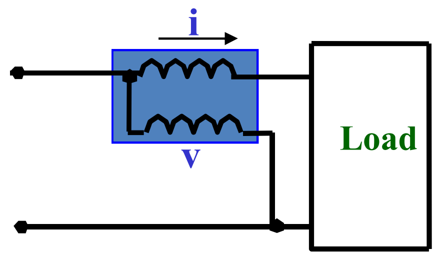
$$VI \cos\theta$$

Single-Phase Power Measurement

Power Meter



Power Meter



$$v(t) = V_m \cos \omega t$$

$$i(t) = I_m \cos(\omega t - \theta)$$

$$p(t) = v(t) \cdot i(t) = V_m I_m \cos \omega t \cos(\omega t - \theta)$$

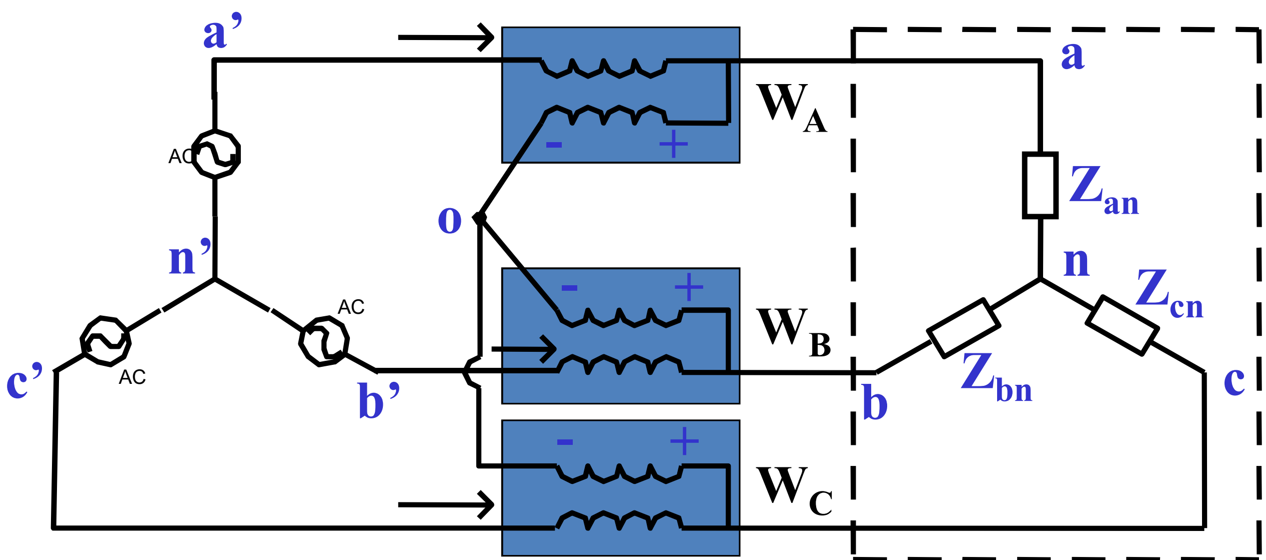
$$= \tfrac{1}{2} V_m I_m \cos\theta + \tfrac{1}{2} V_m I_m \cos(2\omega t) + \tfrac{1}{2} V_m I_m \sin(2\omega t) \sin\theta$$

$$P_{ave} = \frac{1}{T}\int_0^T p(t)\,dt = \operatorname{Re}\left\{V \cdot I^*\right\} = |V||I|\cos\theta$$

$$\cos^2 \omega t = \frac{1}{2}(1 + \cos 2\omega t)$$

$$\cos \omega t \sin \omega t = \frac{1}{2} \sin 2\omega t$$

Three-Phase Power Measurement



Three-wattmeter Method for 3-wire 3-phase Systems.

We know that total average power to 3 φ load over T:

$$P_{abc} = \frac{1}{T}\int_0^T (v_{an} i_{a'a} + v_{bn} i_{b'b} + v_{cn} i_{c'c})dt$$

Total average power measured by the 3 wattmeters:

$$P_{meters} = \frac{1}{T}\int_0^T \left(v_{ao} i_{a'a} + v_{bo} i_{b'b} + v_{co} i_{c'c} \right) dt$$

From KVL:

$$v_{ao} = v_{an} - v_{on}$$

$$v_{bo} = v_{bn} - v_{on}$$

$$v_{co} = v_{cn} - v_{on}$$

For a 3-wire 3-phase system:

$$i_{a'a} + i_{b'b} + i_{c'c} = 0$$

$$P_{meters} = \frac{1}{T}\int_0^T \left(v_{an} i_{a'a} + v_{bn} i_{b'b} + v_{cn} i_{c'c} \right) dt$$

$$\cancel{- \frac{1}{T}\int_0^T v_{on} \left(i_{a'a} + i_{b'b} + i_{c'c} \right) dt} \rightarrow 0$$

For a 3-wire 3-phase system:

$$P_{3\phi} = \mathrm{Re}\left\{V_{ao}I^{*}_{a'a}\right\} + \mathrm{Re}\left\{V_{bo}I^{*}_{b'b}\right\} + \mathrm{Re}\left\{V_{co}I^{*}_{c'c}\right\}$$

$$= \left|V_{ao}\right|\left|I_{a'a}\right|\cos\theta_a + \left|V_{bo}\right|\left|I_{b'b}\right|\cos\theta_b + \left|V_{co}\right|\left|I_{c'c}\right|\cos\theta_c$$

which we have shown to be equal to

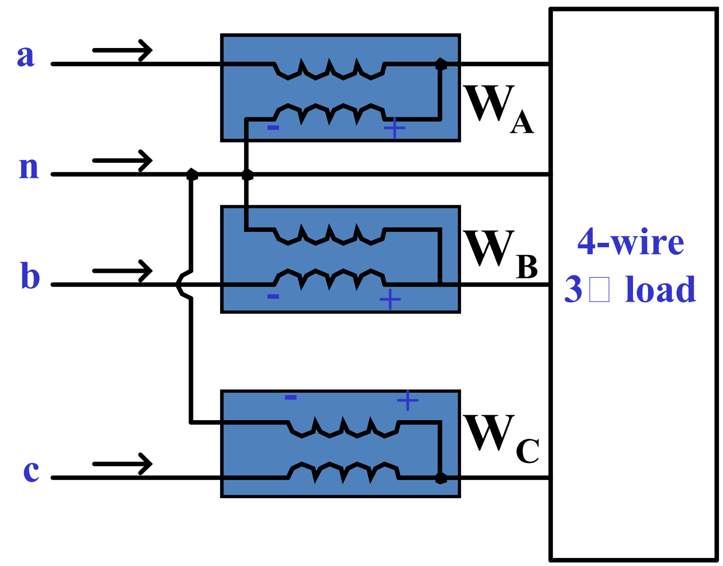
$$= \left|V_{an}\right|\left|I_{a'a}\right|\cos\theta_a + \left|V_{bn}\right|\left|I_{b'b}\right|\cos\theta_b + \left|V_{cn}\right|\left|I_{c'c}\right|\cos\theta_c$$

$$P_{3\phi} = \underline{W_A} + \underline{W_B} + \underline{W_C}$$

For balanced loads:

$$\mathbf{W_A = W_B = W_C}$$

Three-Wattmeter Method for 4-wire 3-phase systems

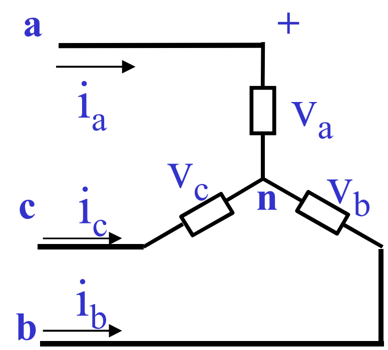


$$P_{3\phi} = \mathrm{Re}\left\{V_{an} I_a^*\right\} + \mathrm{Re}\left\{V_{bn} I_b^*\right\} + \mathrm{Re}\left\{V_{cn} I_c^*\right\}$$

$$= \left|V_{an}\right| \left|I_a\right| \cos\theta_a + \left|V_{bn}\right| \left|I_b\right| \cos\theta_b + \left|V_{cn}\right| \left|I_c\right| \cos\theta_c$$

$$P_{3\phi} = W_A + W_B + W_C$$

Two Wattmeter Method for 3-wire 3-phase Wye-Connected Systems



Recall: $$p_{3\phi} = v_a i_a + v_b i_b + v_c i_c$$

If the system is 3-wire wye-connected (KCL at the neutral point *n*):

$$i_a + i_b + i_c = 0$$

$$p_{3\phi} = v_a i_a + v_b i_b + v_c\left(-i_a - i_b\right)$$

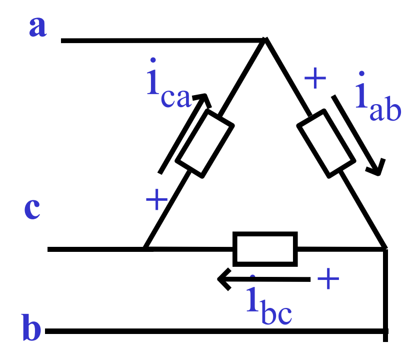
$$p_{3\phi} = v_a i_a + v_b i_b - v_c i_a - v_c i_b$$

$$p_{3\phi} = \left(v_a i_a - v_c i_a\right) + \left(v_b i_b - v_c i_b\right)$$

$$p_{3\phi} = \left(v_a - v_c\right) i_a + \left(v_b - v_c\right) i_b$$

$$P_{3\phi} = \left|V_{ac}\right|\left|I_a\right|\cos\theta_{Ia}^{Vac} + \left|V_{bc}\right|\left|I_b\right|\cos\theta_{Ib}^{Vbc}$$

Two Wattmeter Method for 3-phase Delta-Connected Systems



Recall: $$p_{3\phi} = v_{ab}i_{ab} + v_{bc}i_{bc} + v_{ca}i_{ca}$$

If the system is 3-wire delta-connected (KVL along the delta legs):

$$v_{ab} + v_{bc} + v_{ca} = 0$$

$$p_{3\phi} = \left(-v_{bc} - v_{ca}\right)i_{ab} + v_{bc}i_{bc} + v_{ca}i_{ca}$$

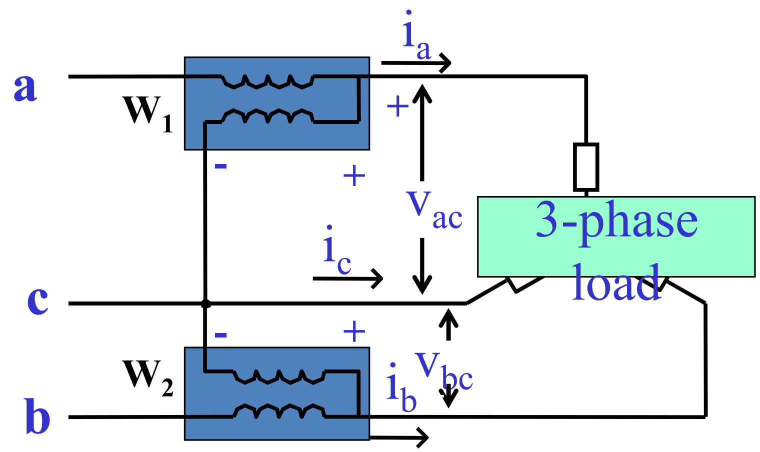
$$p_{3\phi} = \left(v_{bc}i_{bc} - v_{bc}i_{ab}\right) + \left(v_{ca}i_{ca} - v_{ca}i_{ab}\right)$$

$$p_{3\phi} = v_{bc}\left(i_{bc} - i_{ab}\right) + v_{ca}\left(i_{ca} - i_{ab}\right)$$

$$p_{3\phi} = v_{bc}\left(i_{b}\right) + v_{ca}\left(-i_{a}\right) = v_{ac}i_{a} + v_{bc}i_{b}$$

$$\boxed{P_{3\phi} = \left|V_{ac}\right|\left|I_{a}\right|\cos\theta_{Ia}^{Vac} + \left|V_{bc}\right|\left|I_{b}\right|\cos\theta_{Ib}^{Vbc}}$$

Two Wattmeter Method for 3-wire 3-phase Systems

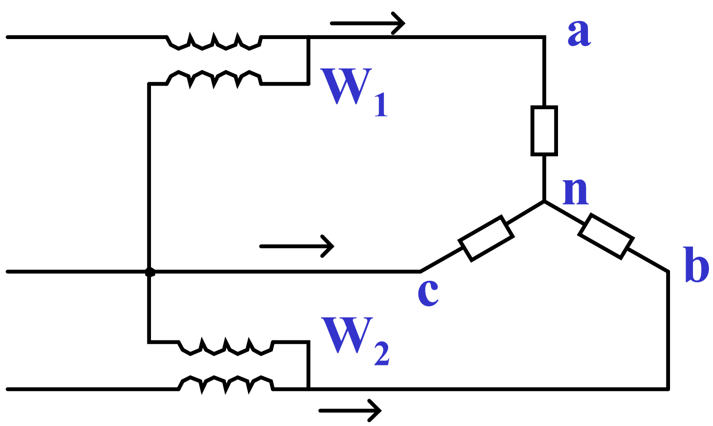


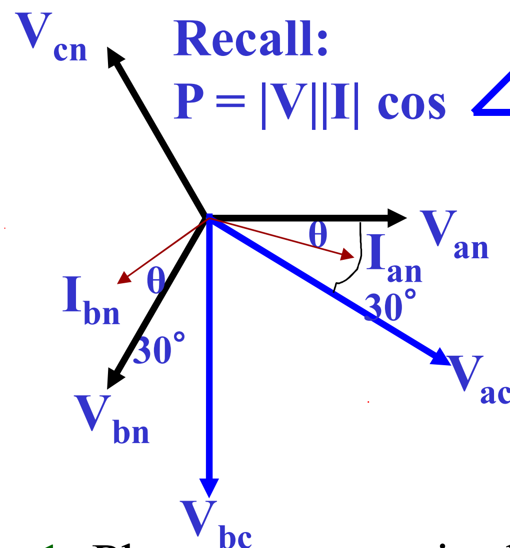
$$W_1 = |V_a - V_c||I_a|\cos\theta_{Ia}^{(Va-Vc)} = |V_{ac}||I_a|\cos\theta_{Ia}^{Vac}$$

$$W_2 = |V_b - V_c||I_b|\cos\theta_{Ib}^{(Vb-Vc)} = |V_{bc}||I_b|\cos\theta_{Ib}^{Vbc}$$

$$P_{3\phi} = W_1 + W_2$$

For a balanced 3-phase Y-connected load:





Recall:

P = |V||I| cos ∠V I

$W_1 = |V_{ac}|\ |I_{an}| \cos(30° - \theta)$

$W_2 = |V_{bc}|\ |I_{bn}| \cos(30° + \theta)$

$W_1 = |V_{LL}|\ |I_L| \cos(30° - \theta)$

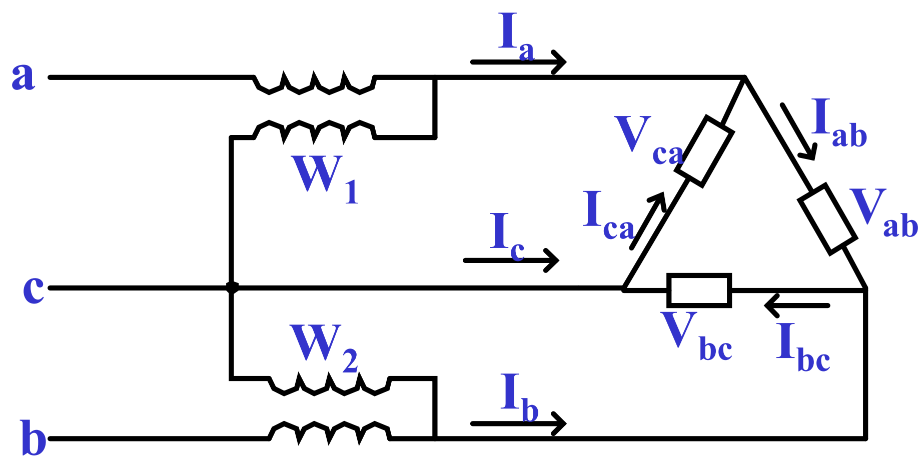
$W_2 = |V_{LL}|\ |I_L| \cos(30° + \theta)$

$$\boxed{P_{3\phi} = W_1 + W_2}$$

Note 1 Phase sequence is abc. What would happen if phase sequence is acb?

Note 2 PF is assumed lagging. What would happen if PF is leading?

For a balanced Delta-connected load:



$$W_1 = |V_{ac}|\,|I_a| \cos(30^\circ - \theta)$$
$$W_2 = |V_{bc}|\,|I_b| \cos(30^\circ + \theta)$$

$$W_1 = |V_{LL}|\,|I_L| \cos(30^\circ - \theta)$$
$$W_2 = |V_{LL}|\,|I_L| \cos(30^\circ + \theta)$$

$$P_{3p} = W_1 + W_2$$

$$W_1 = |V_{LL}|\,|I_L| \cos(30^\circ - \theta)$$
$$W_2 = |V_{LL}|\,|I_L| \cos(30^\circ + \theta)$$

$$P_{3p} = W_1 + W_2$$

θ	$\cos(30 - \theta)$	$\cos(30 + \theta)$	W1 + W2	sqrt(3) cos θ
-90	-0.500	0.500	0.000	0.000
-60	0.000	0.866	0.866	0.866
-45	0.259	0.966	1.225	1.225
-30	0.500	1.000	1.500	1.500
0	0.866	0.866	1.732	1.732
30	1.000	0.500	1.500	1.500
45	0.966	0.259	1.225	1.225
60	0.866	0.000	0.866	0.866
90	0.500	-0.500	0.000	0.000

Consider $W_1 - W_2$:

$$W_1 - W_2 = |V||I| \cos(30 - \theta) - |V||I| \cos(30 + \theta)$$

$$= |V||I|[\cos 30 \cos\theta + \sin 30 \sin\theta$$
$$- \cos 30 \cos\theta + \sin 30 \sin\theta]$$

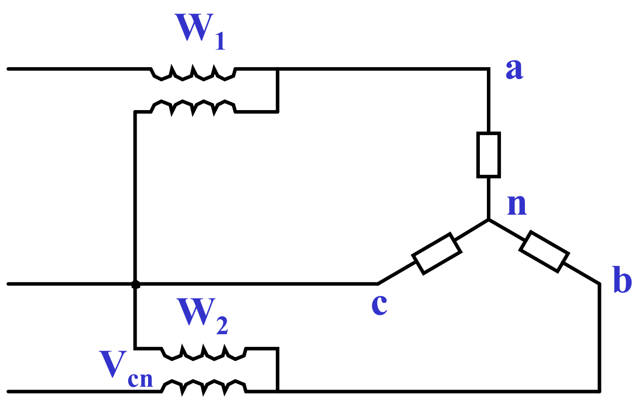
$$= |V||I| \sin\theta$$

For balanced 3φ: $Q_{3p} = \text{Sqrt}(3)(W_1 - W_2)$

Thus, the for a balanced 3p

$$\tan\theta = Q_{3p} / P_{3p} = (\text{Sqrt}(3)(W_a - W_b)) / (W_a + W_b)$$

EXAMPLE: $W_1 = 800, W_2 = -400$



$$P = W_a + W_b = 800 + (-400) = 400 \text{ W}$$

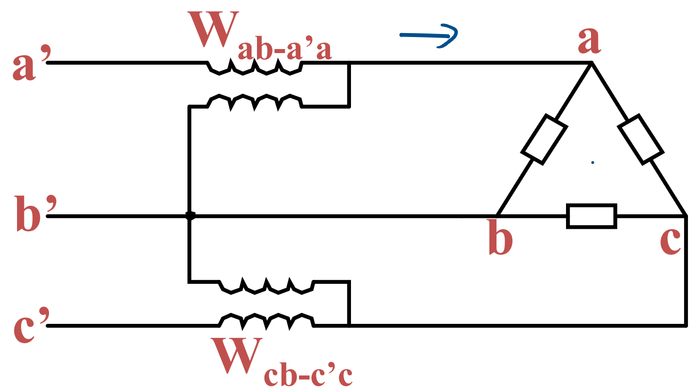
$$Q = \text{Sqrt}(3)\,(W_a - W_b) = \text{Sqrt}(3)\,[800 - (-400)] = 2078 \text{ Vars}$$

CONCEPT TEST

Solve for the power readings in each wattmeter:

Sequence is negative

$V_{ab} = 200\angle 0^\circ$ $V_{bc} = 200\angle -240^\circ$ $V_{ca} = 200\angle -120^\circ$



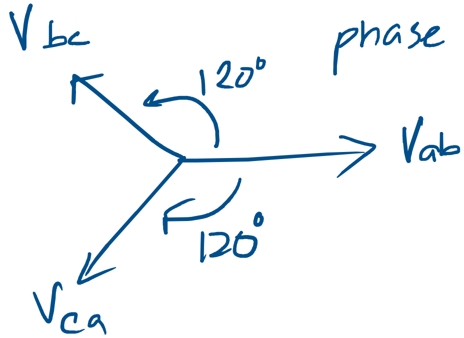
6 kW, 0.8 pf Delta-connected load

SOLUTION:

1. Calculate phase voltages (phasor).
2. Calculate phase currents (phasor).
3. Calculate line currents (phasor).
4. Calculate power in each wattmeter.
5. Check with the given total complex power.

Phase Voltages

1) $V_{ab} = 200\angle 0^\circ$ V
$V_{bc} = 200\angle -240^\circ$ V
$V_{ca} = 200\angle -120^\circ$ V



phase sequence is a-c-b

2.) Phase Currents

since System is balanced,

$P_{3\phi} = 3 V_p I_p \cos\theta_{pf} = 6\text{kW}$

$\theta_{pf} = 36.87^\circ$

$I_p = 12.5$

$I_{ab} = 12.5\angle -36.87^\circ$ A
$I_{bc} = 12.5\angle 83.13^\circ$ A
$I_{ca} = 12.5\angle -156.87^\circ$ A

OR

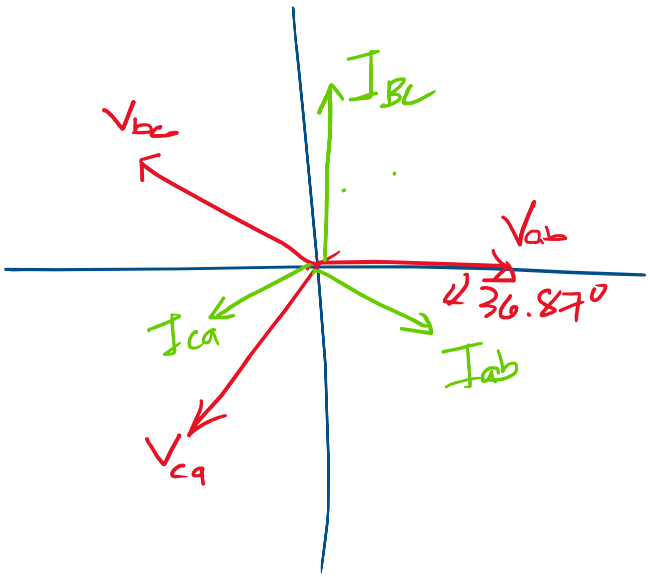
$$I_{ab} = \left(\frac{S_{ab}}{V_{ab}}\right)^* = \frac{\frac{1}{3}(7.5\angle 36.87)}{200\angle 0^\circ}$$

$$I_{bc} = \left(\frac{S_{bc}}{V_{bc}}\right)^* = \frac{\frac{1}{3}(7.5\angle 36.87)}{200\angle 240^\circ}$$

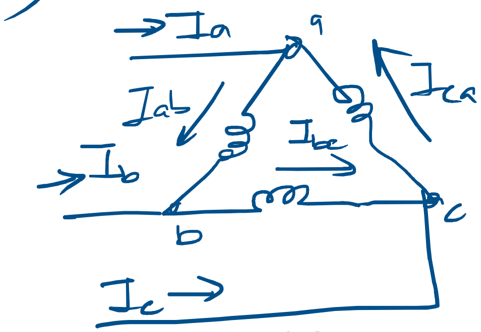
$$I_{ca} = \left(\frac{S_{ca}}{V_{ca}}\right)^* = \frac{\frac{1}{3}(7.5\angle 36.87)}{200\angle 120^\circ}$$

$I_{ab} = 12.5\angle -36.87^\circ$ A
$I_{bc} = 12.5\angle 83.13^\circ$ A
$I_{ca} = 12.5\angle -156.87^\circ$

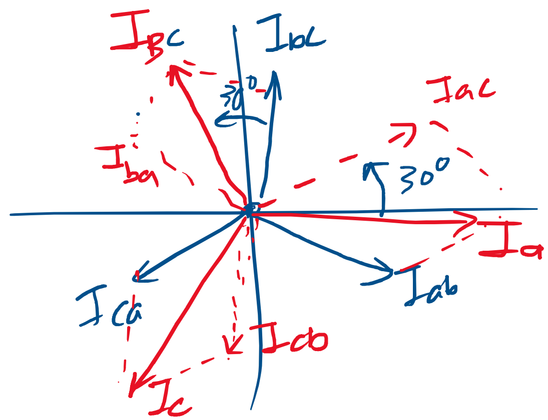
Visualizing Phasors



3.) Line currents



Phasor Diagram



$$I_a = I_{ab} - I_{ca}$$
$$I_b = I_{bc} - I_{ab}$$
$$I_c = I_{ca} - I_{bc}$$

Let $|I_{ab}| = |I_{bc}| = |I_{ca}| = I_p$

If I_{ab} is reference, for any I_L:

$$I_L = I_p\angle 0^0 - I_p\angle -120^0$$
$$I_L = \sqrt{3} I_p \angle 30^0$$

therefore for a-c-b (negative) sequence

$$I_L = \sqrt{3} I_p \angle \theta + 30^0$$

Solving for line current

Method 1

for a-c-b (negative)

$$I_L = \sqrt{3}\, I_p \angle \theta + 30^\circ$$

$(-36.87 + 30)$

$$\boxed{\begin{aligned} I_a &= 21.65\angle{-6.87^\circ}\ A \\ I_b &= 21.65\angle 113.13^\circ\ A \\ I_c &= 21.65\angle{-126.87} \end{aligned}}$$

Method 2

$$I_a = I_{ab} - I_{ca} = 21.65\angle{-6.87^\circ}\ A$$

$$I_b = I_{bc} - I_{ab} = 21.65\angle 113.13^\circ\ A$$

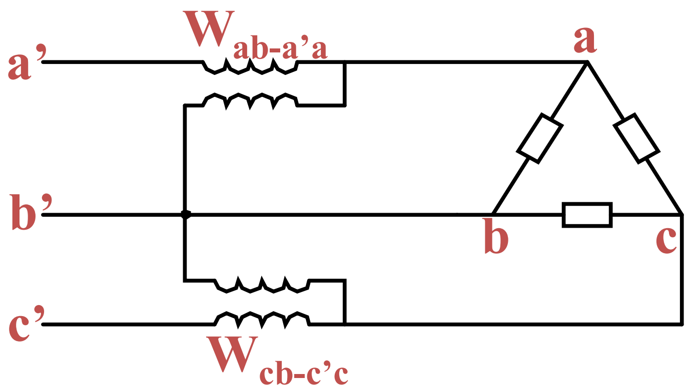
$$I_c = I_{ca} - I_{bc} = 21.65\angle{-126.87^\circ}\ A$$

4.) Solving for W_1 & W_2

$$W_1 + W_2 = P_{3\phi} = 6\text{kW}$$

$$\sqrt{3}(W_1 - W_2) = Q_{3\phi} = 4.5\text{ kvars}$$

$$\boxed{W_1 = 4.3\text{kW} \quad W_2 = 1.7\text{kW}}$$



6 kW, 0.8 pf
Delta-connected
load

confirmed

5.)

$$W_{ab-a'a} = V_{ab} I_a \cos\theta^{V_{ab}}_{I_a} = 200(21.65)\cos(0^\circ - (-6.87^\circ)) = \boxed{4.3k}$$

$$W_{cb-c'c} = V_{cb} I_c \cos\theta^{V_{cb}}_{I_c} = 200(21.65)\cos(-60^\circ - (-126.87^\circ)) = \boxed{1.7k}$$

OR

$$W_1 = V_{LL} I_L \cos(\theta - 30^\circ) = 200(21.65)(\overset{\theta v - \theta i}{36.87^\circ} - 30^\circ) \boxed{= 4.3k}$$

$$W_2 = V_{LL} I_L \cos(\theta + 30^\circ) = 200(21.65)(36.87^\circ + 30^\circ) \boxed{= 1.7k}$$

End of Presentation