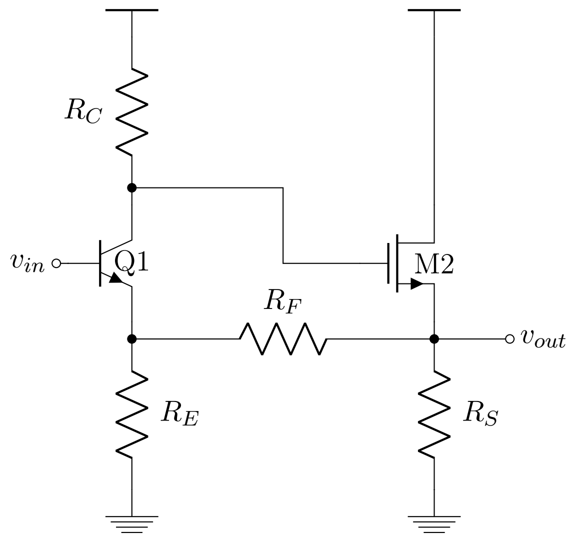
EEE 51 Assignment 9 Answer Key

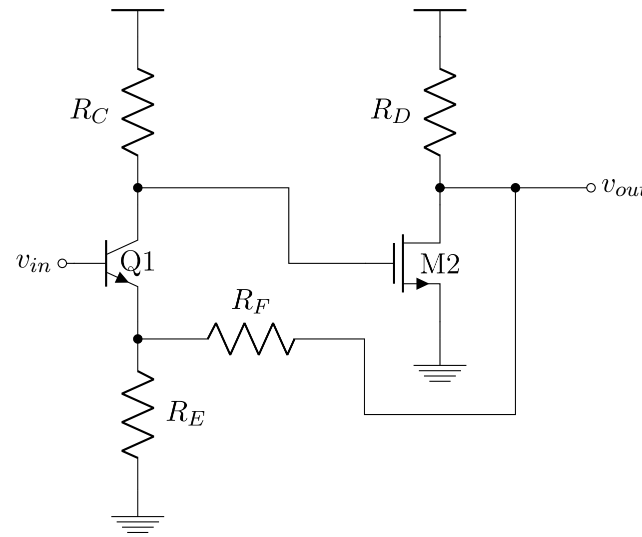
2nd Semester SY 2018-2019

1. **Positive-Negative.**

You are to implement an amplifier with feedback. Given in Figure 1 are two possible designs.



(a) Amplifier A



(b) Amplifier B

Figure 1: Feedback Amplifiers.

(a) Both of the designs use the same type of feedback. Identify the type of feedback being used. (1 pt)

- Looking at the input side, the input to the amplifiers is both v_{be} which is the difference between the input voltage and the voltage across R_E. We can say that a voltage is sampled at the input, and the input side is therefore in **series**. (0.5 pt for this answer)
- At the output, the feedback path is directly connected at the output node. We can say that the voltage is being sampled at the output and thus the output side is **shunt**. (0.5 pt for this answer)
- The type of feedback used is **SERIES-SHUNT**.

(b) Which of the two designs is valid (employs negative feedback and will not blow-up)? Be careful since your answers in the next item will be dependent here. Committing a mistake in this item will automatically void the answers for the next item. (2 pts)

- The output voltage at amplifier A is negative of the input. Feeding it back to the input, the error voltage will be $v_{be} = v_{in} - (-\left|a_v f\right| v_{in})$. A positive feedback. This might overload the transistors and blow them up at worse.
- At amplifier B, the output is a positive voltage. Feeding the output back yields $v_{be} = vin - (\left|a_v f\right| v_{in})$. A negative feedback.
- **AMPLIFIER B** is the valid design.
- 1 point will be given if you answered incorrectly in this item but still answered the rest of the problem.

(c) Using your answer in (b), use the parameters β, g_{mx}, and r_π; and the resistors R_C, R_D, R_E, R_F, and R_S to express your answers. Assume that $r_o \to \infty$ for both transistors. For parallel resistors, just use $(R_1||R_2||...||R_n)$ notation instead. Do not expand the expressions.

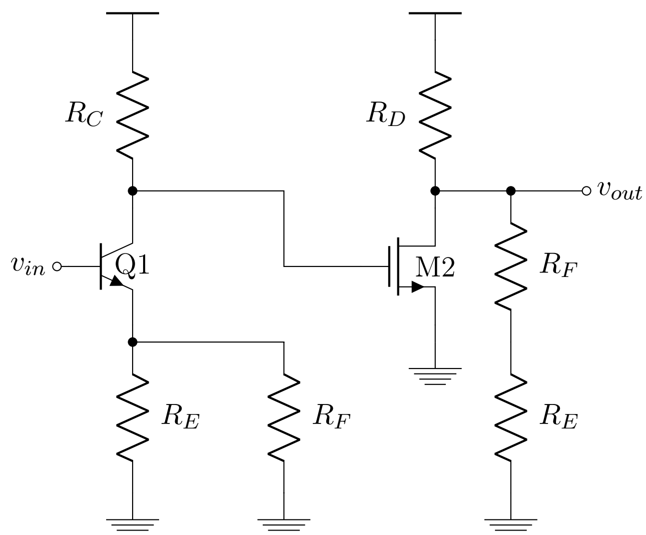
- We need to identify the open-loop circuit first.

  i. For easier analysis, we assume that the feedback network is ideal such that is does not load the amplifier. The supposed load will be "moved" to the open-loop amplifier instead.

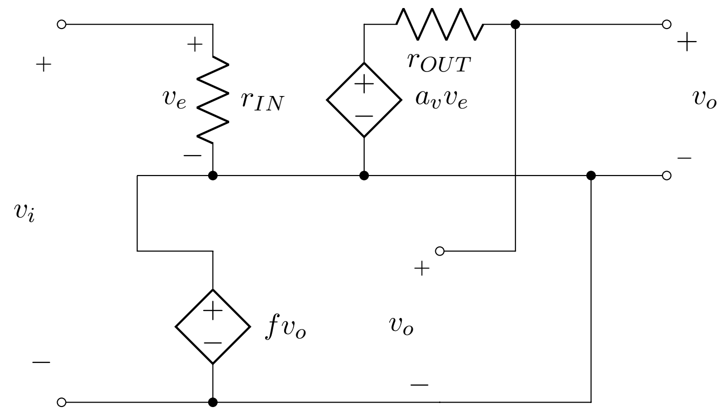
  ii. To "move" the load at the INPUT side, we look first at the output. We will break the loop at the output side (cut the connection of R_F to the v_{out} node). Since we know that the output network is shunt, the loose end of R_F will be grounded (we leave it open/hanging for the case of a series network). We do this so the output has zero direct influence on the resistance that can be "seen" at the input.

  iii. To "move" the load at the OUTPUT side, we look first at the input. Break the loop at the input side as well (separate emitter of Q_1 to the node where R_E and R_F meet). Since we know that the input network is series, we leave the node of R_F-R_E connection hanging (we short that node to ground for the case of a shunt network). We do this so the input has zero direct influence on the resistance that can be "seen" at the output.

  iv. The open-loop circuit for amplifier B is:



- The closed-loop small signal is:



- Note that the open-loop circuit is just for simpler analysis. Any circuit connected to amplifier B will still "see" it in its closed-loop form.

i. Feedback gain $f = \frac{v_{fb}}{v_o}$. (1 pt)

   - The feedback voltage is just the voltage across R_E in series with R_F (output side of the open-loop circuit). Note that we need to sample v_{out} so we solve for f with respect to v_{out}.

$$v_{fb} = v_{out} \frac{R_E}{R_E + R_F}$$
$$f = \frac{v_{fb}}{v_{out}} = \boxed{\frac{R_E}{R_E + R_F}}$$

ii. Open-loop gain (with feedback loading) $a_v = \frac{v_{out}}{v_{in}}$. (1 pt)

   - Gain at first stage (common-emitter with degeneration resistance) (0.5 pt. for correct expression):

$$a_{v1} = \frac{-g_{m1} R_C}{1 + g_{m1} (R_E || R_F)}$$

   - Gain at second stage (common source) (0.5 pt. for correct expression):

$$a_{v2} = -g_{m2} \left[R_D || \left(R_E + R_F \right) \right]$$

   - Open-loop gain:

$$\boxed{a_v = \frac{g_{m1} g_{m2} R_C \left[R_D || \left(R_E + R_F \right) \right]}{1 + g_{m1} (R_E || R_F)}}$$

iii. Closed-loop gain $A_V = \frac{v_{out}}{v_{in}}$. (1 pt)

$$\begin{aligned} A_V &= \frac{a_v}{1 + a_v f} \\ &= \boxed{\frac{\frac{g_{m1} g_{m2} R_C [R_D || (R_E + R_F)]}{1 + g_{m1} (R_E || R_F)}}{1 + \frac{g_{m1} g_{m2} R_C [R_D || (R_E + R_F)]}{1 + g_{m1} (R_E || R_F)} \frac{R_E}{R_E + R_F}}} \text{ or} \\ &= \boxed{\frac{g_{m1} g_{m2} R_C \left[R_D || \left(R_E + R_F \right) \right] \left(R_E + R_F \right)}{R_E + R_F + g_{m1} (R_E || R_F) \left(R_E + R_F \right) + g_{m1} g_{m2} R_C R_E \left[R_D || \left(R_E + R_F \right) \right]}} \end{aligned}$$

iv. Closed-loop input resistance $R_{IN} = \frac{v_{in}}{i_{in}}$. (2 pts)

- Get the open-loop open resistance first. This input resistance is just the resistance seen at the base of Q_1 (1 pt. for having this correct):

$$r_{IN} = r_\pi + (\beta + 1)(R_E || R_F)$$

- The closed-loop input resistance is:

$$\begin{aligned} R_{IN} &= r_{IN}(1 + a_v f) \\ &= \boxed{[r_\pi + (\beta+1)(R_E||R_F)]\left(1 + \frac{g_{m1}g_{m2}R_C[R_D||(R_E+R_F)]}{1+g_{m1}(R_E||R_F)}\frac{R_E}{R_E+R_F}\right)} \end{aligned}$$

- Since we sample a voltage at the input, it is reasonable that we have a large input resistance.
- Moreover, Q_1 is in CE configuration which is a voltage amplifier. A large input resistance will "absorb" most of the input voltage.

v. Closed-loop output resistance $R_{OUT} = \frac{v_{out}}{i_{out}}$. (2 pts)

- Get the open-loop output resistance, which is the resistance at the output node (1 pt. for having this correct):

$$r_{OUT} = R_D || (R_E + R_F)$$

- The closed-loop output resistance is:

$$\begin{aligned} R_{OUT} &= \frac{r_{OUT}}{1 + a_v f} \\ &= \boxed{\frac{R_D||(R_E+R_F)}{1 + \frac{g_{m1}g_{m2}R_C[R_D||(R_E+R_F)]}{1+g_{m1}(R_E||R_F)}\frac{R_E}{R_E+R_F}}} \text{ or} \\ &= \boxed{\frac{[R_D||(R_E+R_F)][1+g_{m1}(R_E||R_F)](R_E+R_F)}{R_E+R_F+g_{m1}(R_E||R_F)(R_E+R_F)+g_{m1}g_{m2}R_CR_E[R_D||(R_E+R_F)]}} \end{aligned}$$

- For an amplifier that has voltage as an output (M_2 is a buffer or a voltage follower), we want a small output resistance so we can deliver the output to any load of greater resistance.

- Sbeve

2. **Feed me, feed me, feed me back!**
The parameters for all transistors are the following: $V_{be,on} = 0.7$V, β and V_A approaches infinity. $v_{in,DC} = 0.9$V, $v_{out,DC} = 2.5$V, $V_{DD} = 5$V. All transistors operate in the forward active region at T = 300 K.

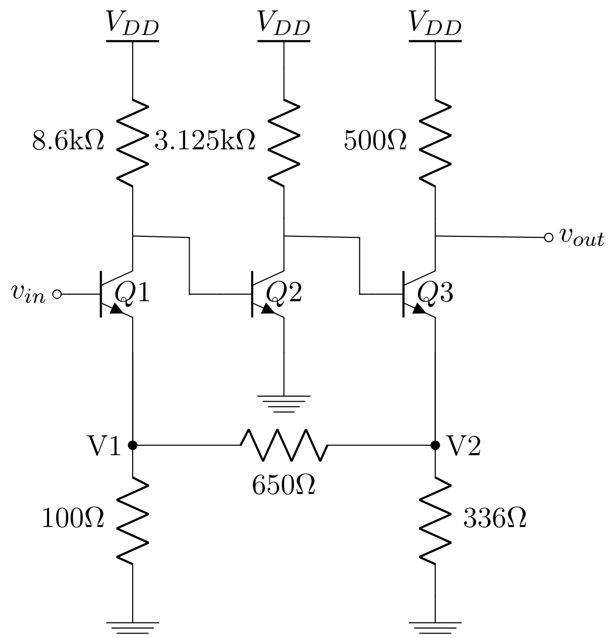


Figure 2: BJT Circuit

(a) Solve for the collector currents, I_{C1}, I_{C2}, I_{C3}, and small signal g_{m1}, g_{m2}, g_{m3} for transistors Q1, Q2, and Q3 (3 pts)

- I_{C1} and I_{C3} can be solved readily by nodal equations in the collector terminals of Q1 and Q3

$$I_{C1} = \frac{V_{DD} - Vbe2}{8.6k\Omega} = \boxed{0.5mA}$$
$$I_{C3} = \frac{V_{DD} - VOUT,DC}{500\Omega} = \boxed{5mA}$$

For I_{C2}, the value of V2 must be solved first. Note that $V1 = v_{in,DC} - 0.7V = 0.2$V. The nodal equation at node V2 is

$$I_{C3} = \frac{V2 - V1}{650\Omega} + \frac{V2}{336\Omega}$$
$$V2 = 1.175V$$
$$I_{C2} = \frac{V_{DD} - Vbe3 - V2}{3.125k\Omega} = \boxed{1mA}$$

The small signal gms are

$$g_{m1} = \frac{I_{C1}}{26mV} = \boxed{19.23mS}$$
$$g_{m2} = \frac{I_{C2}}{26mV} = \boxed{38.46mS}$$
$$g_{m3} = \frac{I_{C3}}{26mV} = \boxed{192.3mS}$$

(b) Identify the feedback configuration (1 pt)
The feedback configuration is series-series

(c) Draw the two-port network of the feedback circuit then solve for the feedback parameters $R_{i,fb}$, $R_{o,fb}$, and feedback factor F (4 pts)

The two-port network of the feedback circuit is shown in Figure 3. Since this is a series-series feedback, the input is a current i_{in} and supplies a voltage V_{fb} to some load.

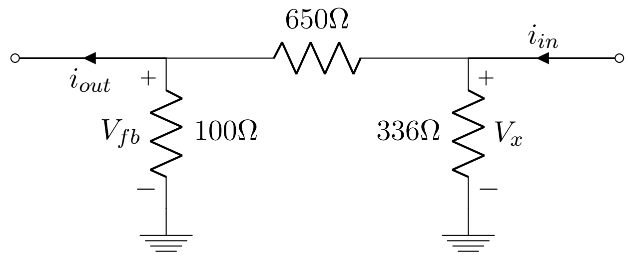


Figure 3: Feedback circuit

To get $R_{i,fb}$, set i_{out} to zero to satisfy a no-load condition then get equivalent resistance seen at i_{in}.

$$R_{i,fb} = 336||(650 + 100) = \boxed{232\Omega}$$

To get $R_{o,fb}$, set i_{in} to zero to satisfy a zero-input condition then get equivalent resistance seen at i_{out}.

$$R_{o,fb} = 100||(650 + 336) = \boxed{90.79\Omega}$$

The feedback factor F, is

$$\begin{aligned} F &= \frac{V_{fb}}{i_{in}} \\ &= \frac{336}{336 + 650 + 100} 100 \\ &= \boxed{30.94 \frac{V}{A}} \end{aligned}$$

(d) Solve for the loaded open-loop "gain" (depends on your answer in (b)) (2 pts)

Since the feedback configuration is series-series, an open-loop transconductance gain is expected. Moreover, the load resistance R_L is not part of the forward path for the open-loop transconductance. Figure 4 shows the small-signal equivalent circuit.

To solve for the open-loop forward transconductance gain, set the feedback factor F to zero and absorb the feedback resistances to the forward path. Moreover, since the β's of the transistors approach infinity, the circuit can be divided into three stages with corresponding gains such that

$$\frac{i_{out}}{v_e} = \frac{v_{o1}}{v_e} \frac{v_{o2}}{v_{o1}} \frac{i_{out}}{v_{o2}}$$

where V_e is the voltage across V_{in} to node 1.

For the first stage, it can be noted that it is a degenerated common-emitter amplifier. Therefore the gain $\frac{v_{o2}}{v_{o1}}$ can be written as

$$\begin{aligned} \frac{v_{o1}}{v_{in}} &= -\frac{g_{m1} R_{C1}}{1 + g_{m1} R_{o,fb}} \\ &= \boxed{-60.23 V/V} \end{aligned}$$

For the second stage, it is a simple common-emitter amplifier. Therefore its voltage gain can be solved using

$$\begin{aligned}\frac{v_{o2}}{v_{o1}} &= -g_{m2}R_{C2}\\ &= \boxed{-120.23V/V}\end{aligned}$$

For the third stage, it is again an emitter-degenerated common-emitter amplifier. Therefore its transconductance can be solved using

$$\begin{aligned}\frac{i_{out}}{v_{o2}} &= \frac{g_{m3}}{1+g_{m3}R_{i,fb}}\\ &= \boxed{4.216mS}\end{aligned}$$

Therefore, the open-loop transconductance gain is $A_{f,OL}$ = (-60.23)(-120.23)(4.216mS) = $\boxed{\textbf{30.53 A/V}}$

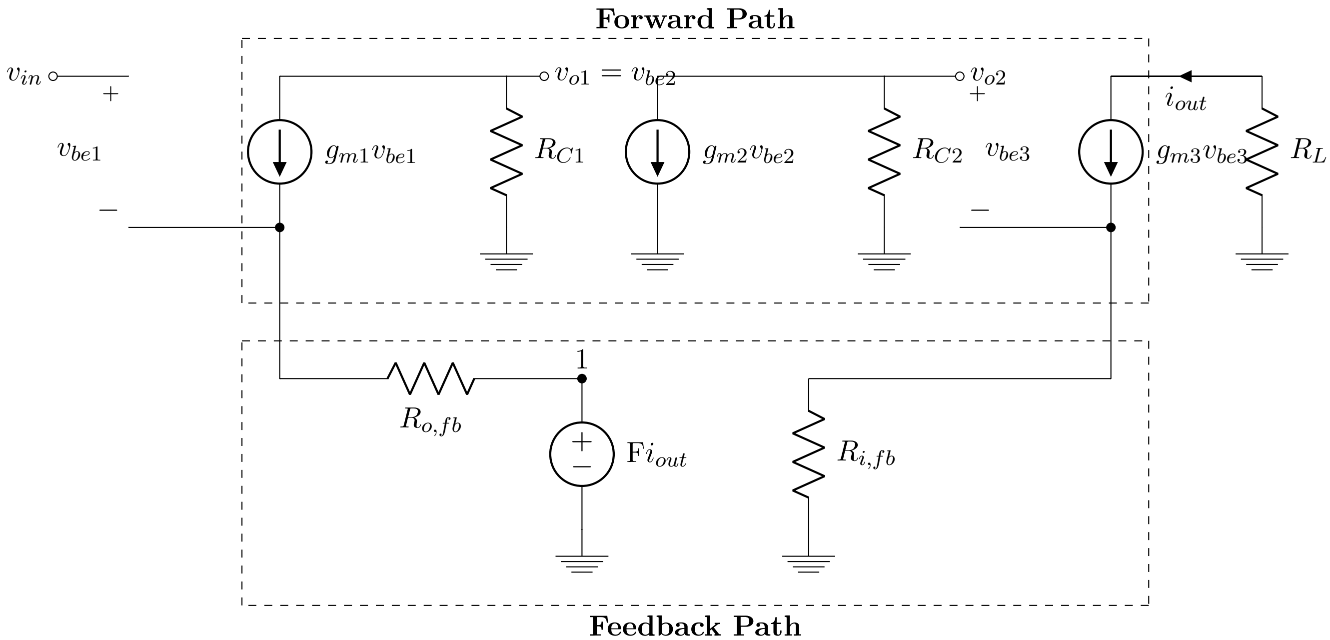


Figure 4: small signal circuit

(e) Solve for the closed-loop voltage gain, $\frac{v_{out}}{v_{in}}$ of the whole circuit (3 pts) First, solve for the closed loop transconductance gain of the whole circuit. From feedback analysis, the equation below can be written

$$\begin{aligned}A_{f,CL} &= \frac{i_{out}}{v_{in}}\\ &= \frac{A_{f,OL}}{1+FA_{f,OL}}\\ &= \boxed{32.29mS}\end{aligned}$$

Note that v_{out} = -$i_{out}R_L$. Dividing both sides by v_{in}, the expression below can be obtained.

$$\begin{aligned}\frac{v_{out,CL}}{v_{in}} &= \frac{-i_{out}R_L}{v_{in}}\\ A_{v,CL} &= -A_{f,CL}R_L\\ &= \boxed{-16.14V/V}\end{aligned}$$

(f) Suppose the open-loop "gain" increased by 10%, by how much is the approximate increase, in percentage, of the closed-loop voltage gain? (2 pts)

It has been established in (e) that $A_{v,CL} = -A_{f,CL}R_L$. Express $A_{f,CL}$ in the form where the loaded open-loop gain ang feedback factor F is involved such that

$$A_{v,CL} = -\frac{A_{f,OL}R_L}{1 + FA_{f,OL}}$$

Get the derivative of both side to get the change of the closed loop gain with respect to a change in the open loop gain.

$$dA_{v,CL} = d(-\frac{A_{f,OL}R_L}{1 + FA_{f,OL}})$$
$$dA_{v,CL} = \frac{R_L}{(1 + FA_{f,OL})^2}dA_{f,OL}$$

To get the percentage change, divide it by the original expression. Also note that 10% change in open loop gain means $\frac{dA_{f,OL}}{A_{f,OL}} = 0.1$.

$$\begin{aligned}
\frac{dA_{v,CL}}{A_{v,CL}} &= \frac{R_L}{(1 + FA_{f,OL})^2}dA_{f,OL} \cdot \frac{1}{A_{v,CL}} \\
&= \frac{R_L}{(1 + FA_{f,OL})^2}dA_{f,OL} \cdot \frac{1 + FA_{f,OL}}{A_{f,OL}R_L} \\
&= \frac{dA_{f,OL}}{A_{f,OL}} \cdot \frac{1}{1 + FA_{f,OL}} \\
&= 0.1 \cdot \frac{1}{1 + 30.94(30.53)} \\
&= .000105753 = \boxed{.0105753\%}
\end{aligned}$$

3. **Give Me Some More Feedback**

Referring to Figure 3a:

(a) The sampled feedback signal going to the input is voltage. Also, the signal sampled at the output is voltage. Therefore the feedback configuration is **series-shunt**.

(b) The feedback circuit is shown:

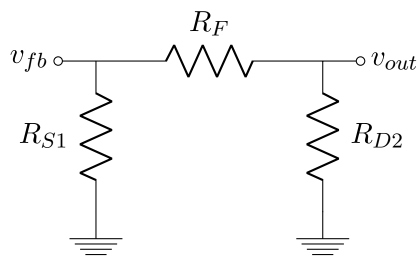


Figure 5: Feedback Circuit

From the feedback circuit above, $R_{I,fb}$, $R_{O,fb}$, and F expressions can be extracted. To get $R_{I,fb}$, look at the resistance seen at v_{out} terminal while remaining v_{fb} terminal open(**series**).

$$\boxed{R_{I,fb} = (R_{S1} + R_F)||R_{D2}}$$

To get $R_{O,fb}$, look at the resistance seen at v_{fb} terminal while shorting(**shunt**) the v_{out} terminal.

$$\boxed{R_{O,fb} = R_{S1}||R_F}$$

The feedback factor F is:

$$F = \frac{v_{fb}}{v_{out}}$$

$$\boxed{F = \frac{R_{S1}}{R_{S1} + R_F}}$$

Next, the unilateral two-port equivalent circuit model of the feedback is shown in Figure 6.

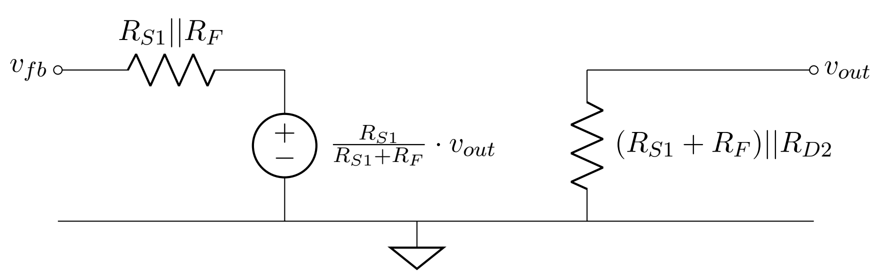


Figure 6: Unilateral Two-Port Model for Feedback Circuit

(c) The small signal equivalent model for the amplifier+feedback is shown in Figure 7. Simply move the resistances ($R_{I,fb}$, $R_{O,fb}$) to the forward gain path to get the ideal feedback path.

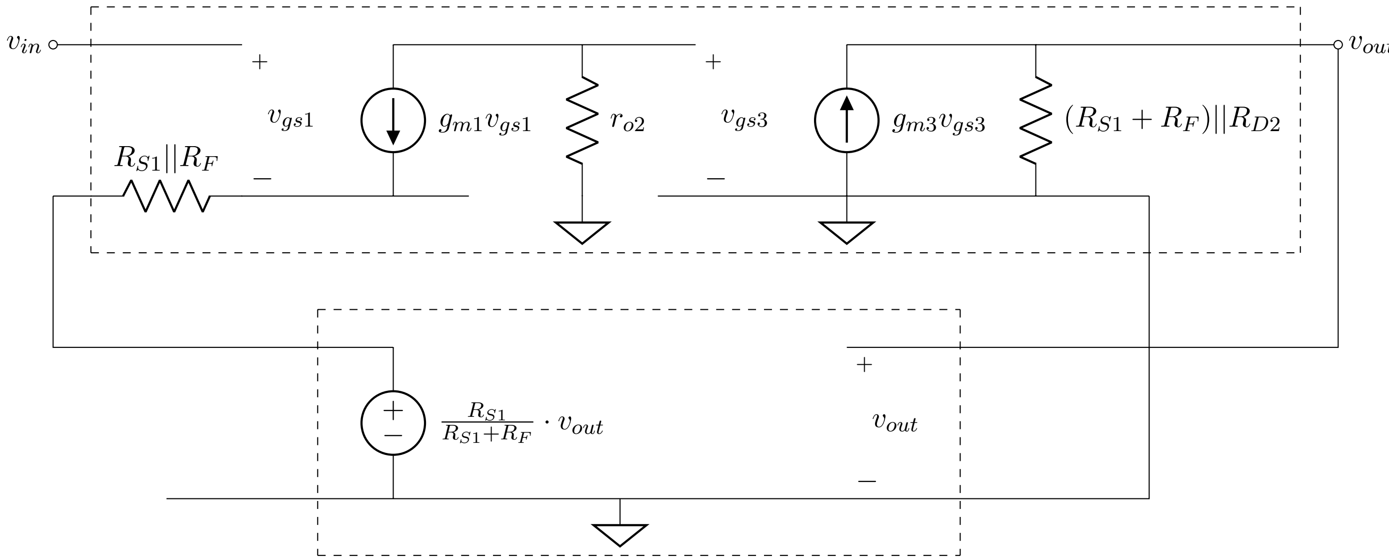


Figure 7: Small signal equivalent model for Amplifier+Feedback emphasizing **Forward Gain Path** and **Ideal Feedback Path**

(d) The Amplifier is two stage both are Common Source configuration. To obtain the open loop gain/forward gain ($A_{v,ol}$), set $F = 0$ (shorting the dependent voltage source in the ideal feedback path). Now, the first stage is a common source with degenerated source resistance and the second stage is a simple common source.

$$\begin{aligned} A_{v,ol} &= A_{v1}\cdot A_{v2} \\ &= -\frac{g_{m1}r_{o2}}{1+g_{m1}(R_{S1}||R_F)}\cdot -g_{m3}[(R_{S1}+R_F)||R_{D2}] \end{aligned}$$

$$\boxed{A_{v,ol} = \frac{g_{m1}g_{m3}r_{o2}[(R_{S1}+R_F)||R_{D2}]}{1+g_{m1}(R_{S1}||R_F)}}$$

$$\boxed{A_{v,ol} = 32.695\, V/V}$$

(e) For Loop Gain (T):

$$T = A_{v,ol}\cdot F$$

$$\boxed{T = \frac{g_{m1}g_{m3}r_{o2}[(R_{S1}+R_F)||R_{D2}]}{1+g_{m1}(R_{S1}||R_F)}\cdot\frac{R_{S1}}{R_{S1}+R_F}}$$

$$\boxed{T = 16.348}$$

(f) For closed-loop gain ($A_{v,cl}$):

$$A_{v,cl} = \frac{A_{v,ol}}{1+T}$$

$$\boxed{A_{v,cl} = \frac{\frac{g_{m1}g_{m3}r_{o2}[(R_{S1}+R_F)||R_{D2}]}{1+g_{m1}(R_{S1}||R_F)}}{1+\left(\frac{g_{m1}g_{m3}r_{o2}[(R_{S1}+R_F)||R_{D2}]}{1+g_{m1}(R_{S1}||R_F)}\cdot\frac{R_{S1}}{R_{S1}+R_F}\right)}}$$

$$\boxed{A_{v,cl} = 1.885\, V/V}$$

(g) To compute for the closed loop output resistance, must first find the expression for the open loop resistance $R_{O,ol}$.

$$R_{O,ol} = (R_{S1}+R_F)||R_{D2}$$

For closed loop output resistance:

$$R_{O,cl} = \frac{R_{O,ol}}{1+T}$$

$$\boxed{R_{O,cl} = \frac{(R_{S1}+R_F)||R_{D2}}{1+\left(\frac{g_{m1}g_{m3}r_{o2}[(R_{S1}+R_F)||R_{D2}]}{1+g_{m1}(R_{S1}||R_F)} \cdot \frac{R_{S1}}{R_{S1}+R_F}\right)}}$$

$$\boxed{R_{O,cl} = 115.29\Omega}$$

Referring to Figure 3b:

(h) No, Feedback configuration (**series-shunt**) is still the same. M_4 accepts the sampled voltage from the feedback and outputs a voltage that goes to the input of the main amplifier depending what type of amplifier configuration M_4 is.

(i) The feedback circuit is shown:

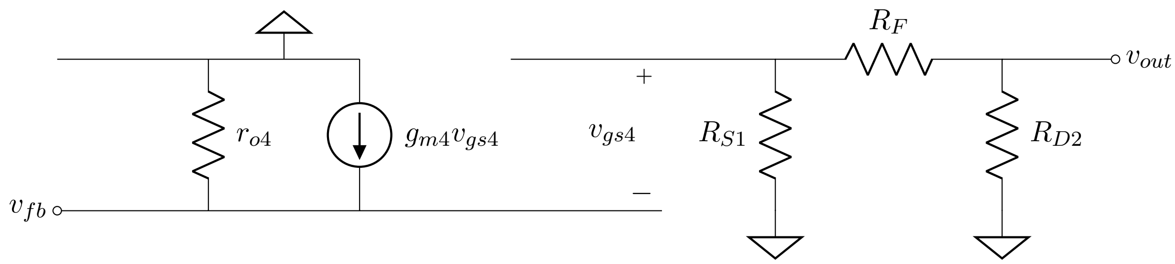


Figure 8: Feedback Circuit with M_4

From the feedback circuit, $R_{I,fb}$, $R_{O,fb}$, and F can be obtained:

$$\boxed{R_{I,fb} = (R_{S1}+R_F)||R_{D2}}$$

$$\boxed{R_{O,fb} = r_{o4}||\frac{1}{g_{m4}}}$$

$$\boxed{F = \frac{g_{m4}r_{o4}}{1+g_{m4}r_{o4}} \cdot \frac{R_{S1}}{R_{S1}+R_F}}$$

Next, the unilateral two-port model of the feedback is shown in Figure 9.

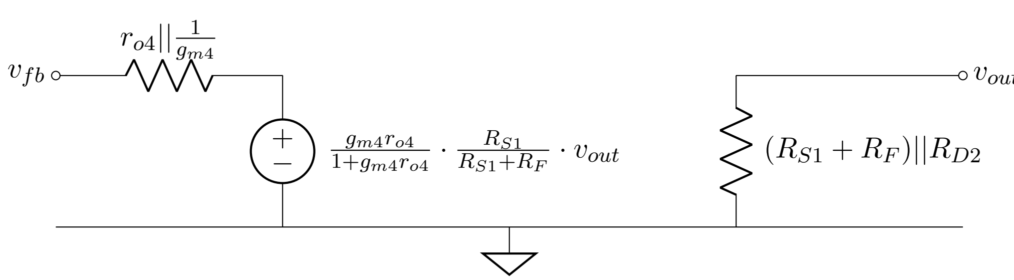


Figure 9: Unilateral Two-Port Model for Feedback Circuit with M_4

The small signal equivalent for the amplifier+feedback is shown in Figure 10.

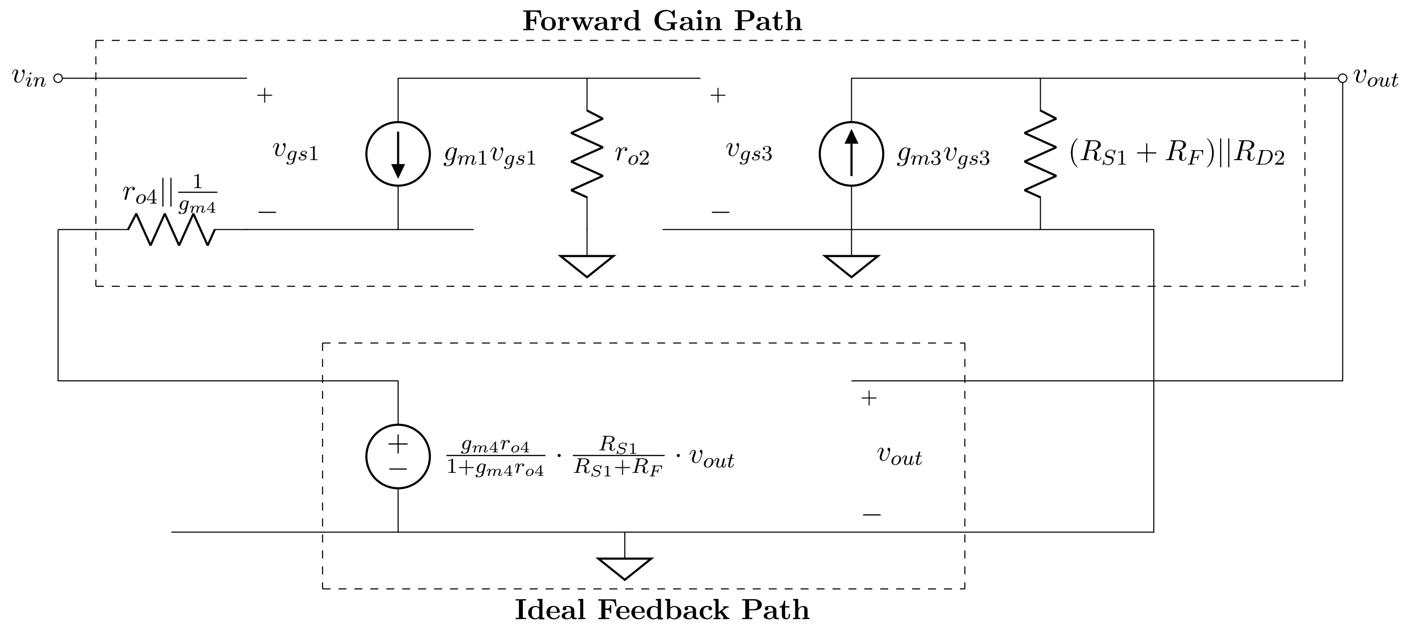


Figure 10: Small signal equivalent model for Amplifier+Feedback with M_4 in Feedback emphasizing **Forward Gain Path** and **Ideal Feedback Path**

The open-loop gain/forward gain ($A_{v,ol}$) is:

$$A_{v,ol} = -\frac{g_{m1}r_{o2}}{1+g_{m1}\left(r_{o4}||\frac{1}{g_{m4}}\right)} \cdot -g_{m3}[(R_{S1}+R_F)||R_{D2}]$$

$$\boxed{A_{v,ol} = \frac{g_{m1}g_{m3}r_{o2}[(R_{S1}+R_F)||R_{D2}]}{1+g_{m1}\left(r_{o4}||\frac{1}{g_{m4}}\right)}}$$

$$\boxed{A_{v,ol} = 144.07\,V/V}$$

The Loop Gain (T) is:

$$\boxed{T = \frac{g_{m1}g_{m3}r_{o2}[(R_{S1}+R_F)||R_{D2}]}{1+g_{m1}\left(r_{o4}||\frac{1}{g_{m4}}\right)} \cdot \frac{g_{m4}r_{o4}R_{S1}}{(1+g_{m4}r_{o4})(R_{S1}+R_F)}}$$

$$\boxed{T = 71.349}$$

The closed-loop gain ($A_{v,cl}$) is:

$$\boxed{A_{v,cl} = \frac{\frac{g_{m1}g_{m3}r_{o2}[(R_{S1}+R_F)||R_{D2}]}{1+g_{m1}\left(r_{o4}||\frac{1}{g_{m4}}\right)}}{1+\left(\frac{g_{m1}g_{m3}r_{o2}[(R_{S1}+R_F)||R_{D2}]}{1+g_{m1}\left(r_{o4}||\frac{1}{g_{m4}}\right)} \cdot \frac{g_{m4}r_{o4}R_{S1}}{(1+g_{m4}r_{o4})(R_{S1}+R_F)}\right)}}$$

$$\boxed{A_{v,cl} = 1.991\,V/V}$$

The closed loop output resistance ($R_{O,cl}$) is:

$$R_{O,cl} = \frac{R_{O,ol}}{1+T}$$

$$\boxed{R_{O,cl} = \frac{(R_{S1}+R_F)||R_{D2}}{1+\left(\frac{g_{m1}g_{m3}r_{o2}[(R_{S1}+R_F)||R_{D2}]}{1+g_{m1}\left(r_{o4}||\frac{1}{g_{m4}}\right)} \cdot \frac{g_{m4}r_{o4}R_{S1}}{(1+g_{m4}r_{o4})(R_{S1}+R_F)}\right)}}$$

$$\boxed{R_{O,cl} = 27.644\Omega}$$

(j) The closed-loop gain and output resistance for the resistive only feedback are $A_{v,cl} = 1.885$ and $R_{O,cl} = 115.29\Omega$ while the resistive feedback with transistor M_4 are $A_{v,cl} = 1.991$ and $R_{O,cl} = 27.644\Omega$. It shows that inserting the transistor M_4 shows an improvement on both the $A_{v,cl}$ and $R_{O,cl}$. This is because M_4 is a source follower/common drain configuration which has the advantage of a very low output impedance that does not load or serves as a buffer of the sampled voltage from the resistive feedback going to the input of the main amplifier.

TOTAL: 45 points.