Lecture 4
Basic Principles and Concepts in Power System Analysis

Agenda

- Reminders – UVLE Quiz!
- Lecture

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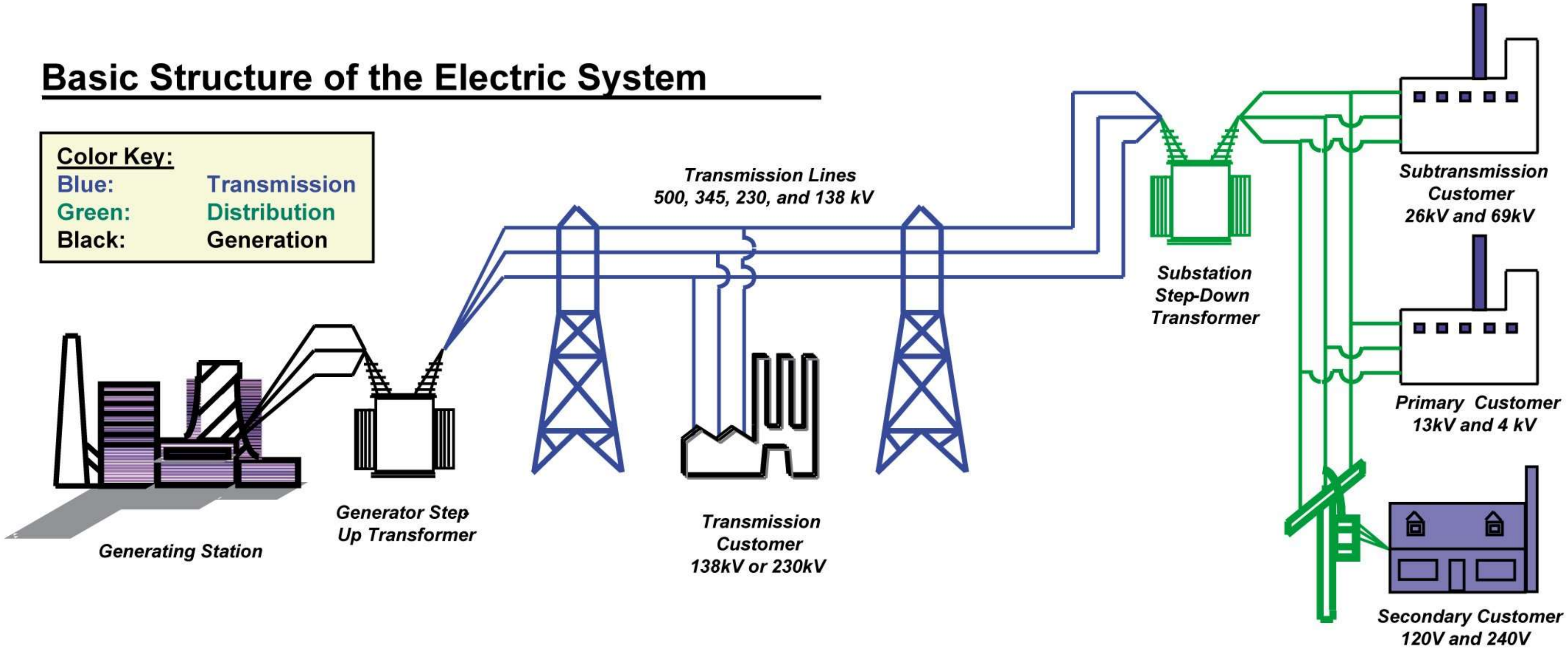
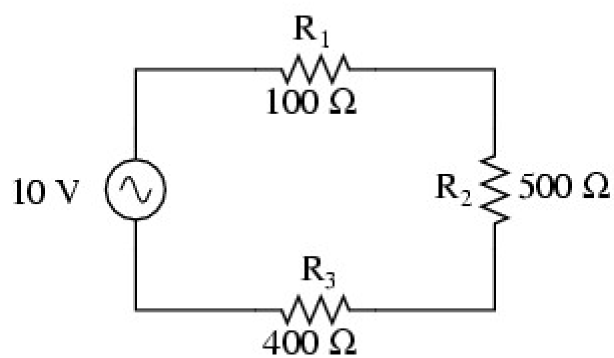
The power system is just a huge circuit.





2. Basic Principles & Concepts

2.1 Circuit Conventions & Notations

2.2 Complex Notation & Phasors

2.3 Complex Impedance

2.4 Network Reduction and Transformations

2.5 Complex Power Equations

2.6 Three-Phase Systems

2.7 Power Measurements

Lecture Outcomes

at the end of the lecture, the student must be able to …

- Apply techniques of steady-state AC circuit analysis
- Identify the AC circuit conventions and notations used for power system analysis.

CIRCUIT CONVENTIONS AND NOTATIONS

Voltage and Current Directions

Polarity Marking of Voltage Source Terminals:

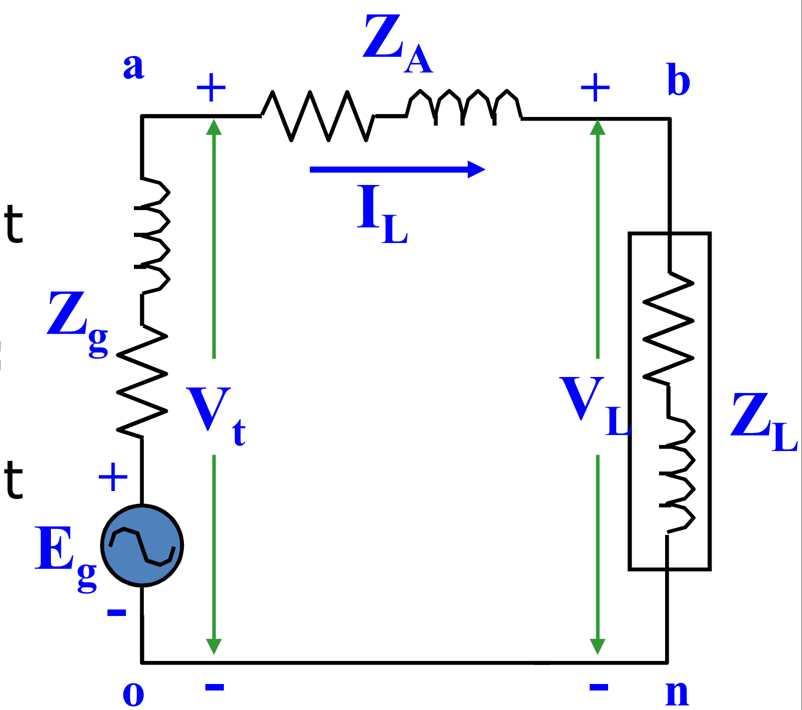
Plus sign (+) for the terminal where positive current comes out

Specification of Load Terminals:

Minus sign (-) for the terminal where positive current comes out

Specification of Current Direction:

Arrows for the positive current (i.e., from the source towards the load)



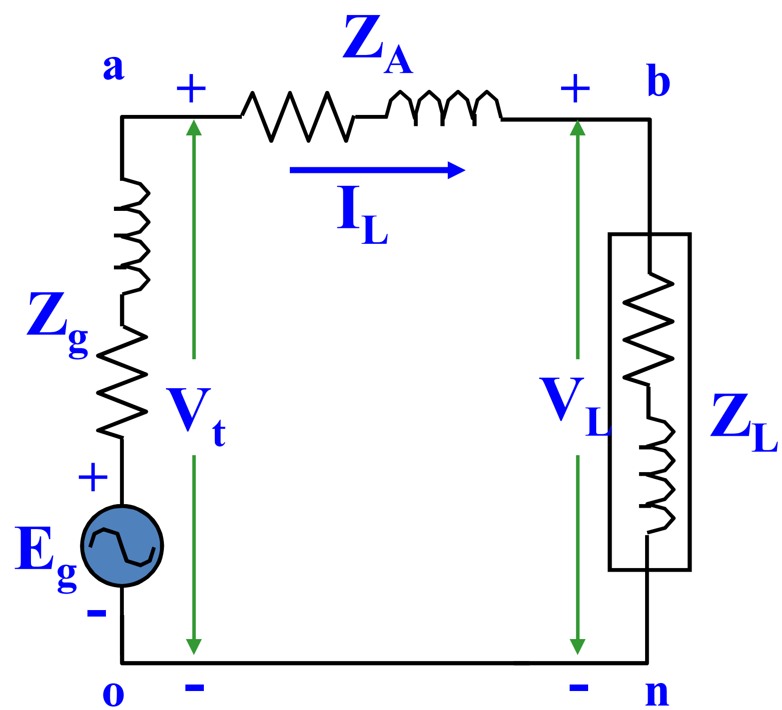
Single Subscript Notation

From the Figure,

$$I_L = \frac{V_T - V_L}{Z_A}$$

$$V_T = E_G - I_L Z_G$$

$$V_a = V_t \quad ; \quad V_b = V_L$$

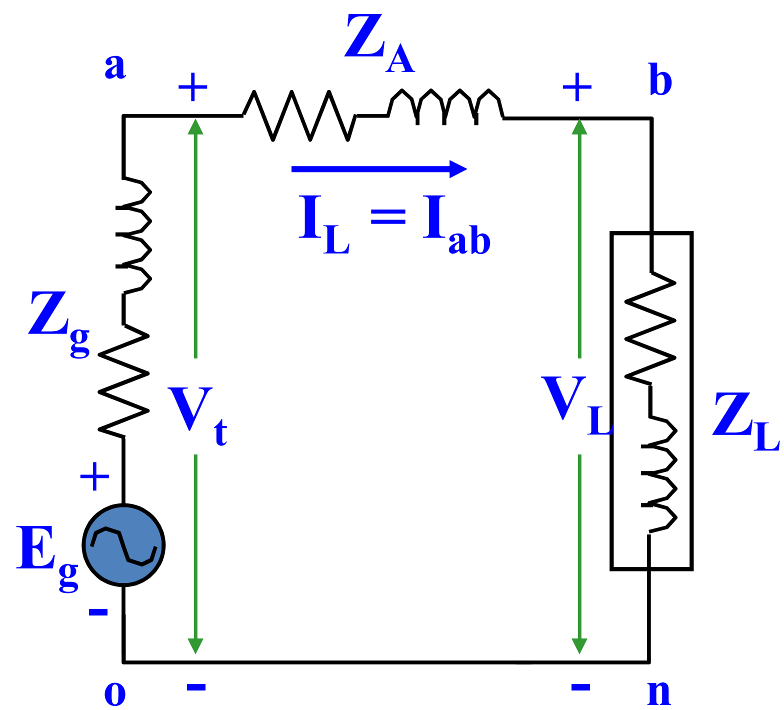


Double Subscript Notation

The use of polarity marks for voltages and direction arrows for currents can be avoided by double subscript notation.

Current $\mathbf{I_L}$ is positive when the current is actually in the direction from a to b.

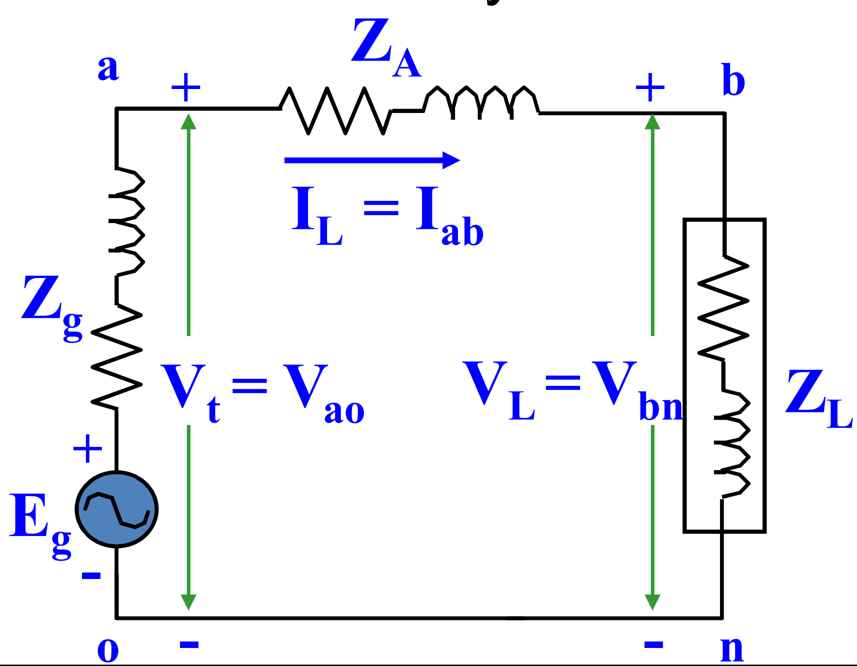
In double subscript notation this current is $\mathbf{I_{ab}}$.



Double Subscript Notation

The letter subscripts on a voltage indicate the nodes of the circuit between which the voltage exists.

The first subscript denotes the voltage of that node with respect to the node identified by the second subscript.



Double Subscript Notation

Reversing the order of the subscripts of either a current or voltage gives a current or voltage 180° out of phase with the original

$$V_{ab} = V_{ba} \angle 180^\circ = -V_{ba}$$

$$I_{ab} = -I_{ba}$$

In writing the Kirchoff's voltage law, the order of the subscripts is the order of tracing the closed path around the circuit. For the figure, with points *n* and *o* at the same potential:

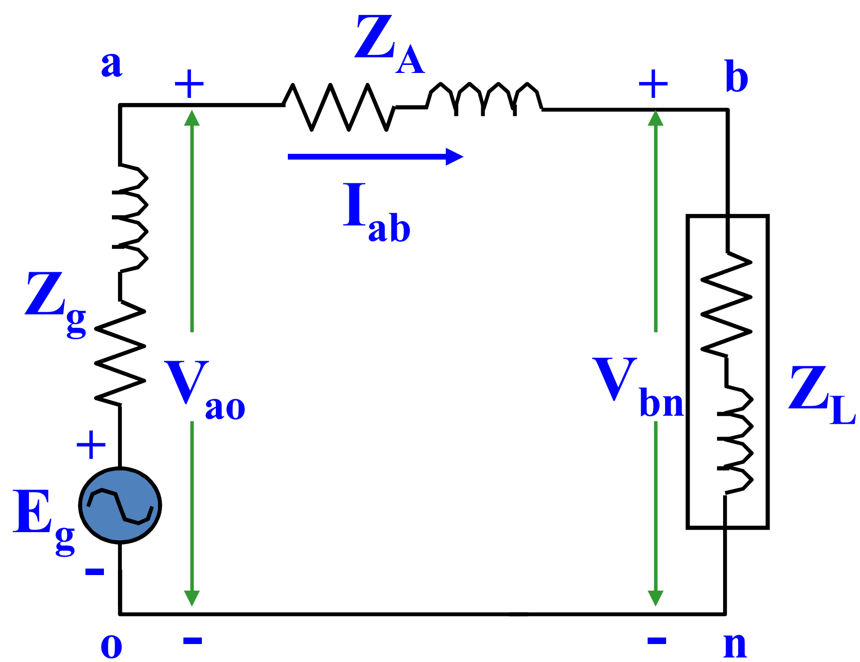
$$V_{oa} + V_{ab} + V_{bn} = 0$$

Double Subscript Notation

Replacing V_{oa} by $-V_{ao}$ and noting that $V_{ab} = I_{ab}Z_A$ yields

$$-\mathbf{V}_{ao} + \mathbf{I}_{ab}\mathbf{Z}_a + \mathbf{V}_{bn} = 0$$

$$\mathbf{I}_{ab} = \frac{\mathbf{V}_{ao} - \mathbf{V}_{bn}}{\mathbf{Z}_a}$$



Time Domain to Frequency Domain

- We can do analysis in Time domain.
  - Model Capacitors and Inductors with Differential Equations
  - OR
- Frequency Domain : Use Phasors (Complex Calculations) and solve the AC Circuit like a DC Circuit.

Phasors

Consider the general sinusoidal function

$$y(t) = Y_{max} \cos(\omega t + \phi)$$

where Y_{max} = amplitude, or max. value
ω = radian frequency
ϕ = phase angle

Recall Euler's identity: $e^{j\theta} = \cos\theta + j\sin\theta$

It can be proven that,

$$y(t) = Y_{max} \cos(\omega t + \phi) = \mathrm{Re}\{Y_{max}\, e^{j(\omega t + \phi)}\}$$

Phasors

$$y(t) = \mathrm{Re}\left\{Y_{max}\, e^{j\phi} e^{j\omega t}\right\}$$

$$= \sqrt{2}\,\mathrm{Re}\left\{(Y_{max}/\sqrt{2})\, e^{j\phi}\, e^{j\omega t}\right\}$$

Let $\mathbf{Y} = (Y_{max}/\sqrt{2}\,)\, e^{j\phi}$ and

$|\mathbf{Y}| = (Y_{max}/\sqrt{2}\,)$ = r.m.s. value of $y(t)$

Then, the sinusoidal function can be written as

$$y(t) = \sqrt{2}\,\mathrm{Re}\left\{\mathbf{Y}\, e^{j\omega t}\right\}$$

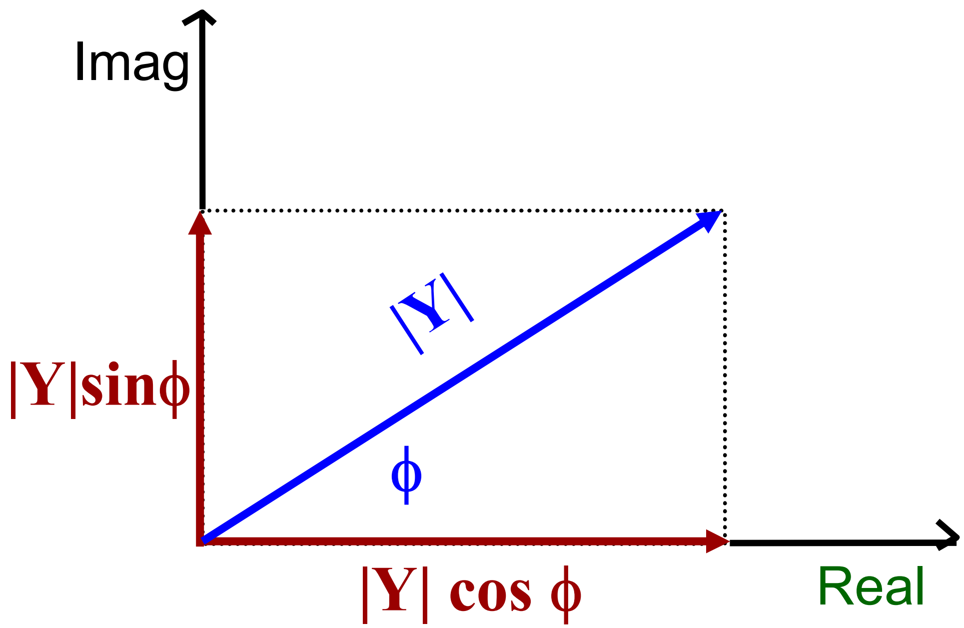
and can be represented by a phasor that is operated by $e^{j\phi}$ or $\angle\phi$.

Phasors

Thus, $\mathbf{Y} = |Y|e^{j\phi}$ or $\mathbf{Y} = |\mathbf{Y}|\angle\phi$

Applying Euler's identity,

$$\mathbf{Y} = |Y|\angle\phi = |Y| \cos \phi + j\,|Y| \sin \phi$$



Phasors

EXAMPLE convert to phasors:

$v = 141.4\cos(\omega t + 30^\circ)$ volts
$i = 7.07\cos(\omega t)$ amperes

$V_{max} = 141.4\ V \rightarrow |\mathbf{V}| = 100\ V_{RMS}$

$v = \mathrm{Re}\{\sqrt{2} \times 100e^{j(\omega t + 30^\circ)}\} = \mathrm{Re}\{100e^{j30}\sqrt{2}e^{j\omega t}\}$

$\mathbf{V} = 100e^{j30^\circ} = 100\angle 30^\circ = 86.6 + j50$ volts

$I_{max} = 7.07\ A \qquad |\mathbf{I}| = 5\ A_{RMS}$

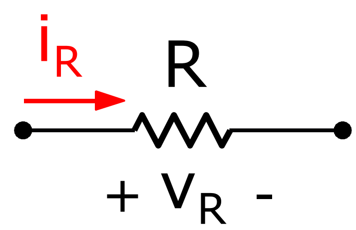
$i = \mathrm{Re}\{\sqrt{2} \times 5e^{j(\omega t + 0^\circ)}\} = \mathrm{Re}\{5e^{j0}\sqrt{2}e^{j\omega t}\}$

$\mathbf{I} = 5e^{j0^\circ} = 5\angle 0^\circ = 5 + j0$ amps

COMPLEX IMPEDANCE

Resistance

In a purely resistive element:

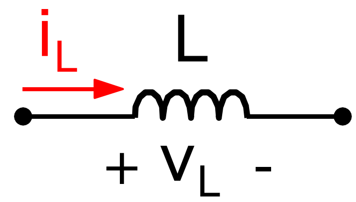


- $\mathbf{V_R}$ and $\mathbf{I_R}$ are in-phase.
- The impedance of the element is equal to the resistance:

$$Z_R = \frac{V_R}{I_R} = R$$

Inductance

In a purely inductive element:



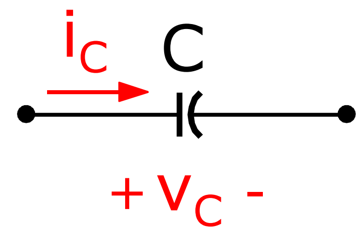
- $\mathbf{I_L}$ lags $\mathbf{V_L}$ by 90° .
- The impedance of the element, or inductive reactance, is:

$$Z_L = \frac{V_L}{I_L} = j\omega L$$

$$\omega = 2\pi f$$

Capacitance

In a purely capacitive element:



- $\mathbf{I_C}$ leads $\mathbf{V_C}$ by 90° .
- The impedance of the element, or capacitive reactance, is:

$$Z_C = \frac{V_C}{I_C} = \frac{1}{j\omega C} = \frac{-j}{\omega C}$$

$$\omega = 2\pi f$$

Impedance

Definition: The ratio of transformed voltage to transformed current is defined as impedance.

$$Z = \frac{V(j\omega)}{I(j\omega)}$$

Note:

(1) For a resistor, $Z_R = R$ in Ω

(2) For an inductor, $Z_L = j\omega L = jX_L$ in Ω

(3) For a capacitor, $Z_C = 1/j\omega C = -jX_C$ in Ω

(4) X_L is the inductive reactance of L

(5) X_C is the capacitive reactance of C

Admittance

Definition: The ratio of transformed current to transformed voltage is defined as admittance.

$$Y = \frac{1}{Z} = \frac{I(j\omega)}{V(j\omega)}$$

Note:

(1) For a resistor, $Y_R = 1/R$ in Ω^{-1}

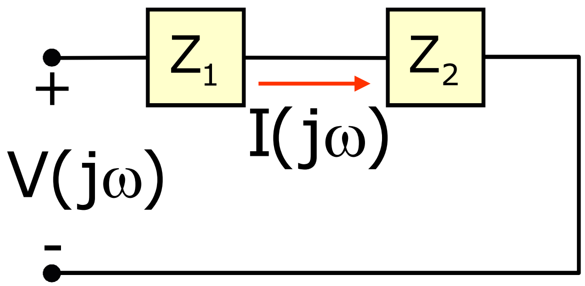
(2) For an inductor, $Y_L = 1/j\omega L = -jB_L$ in Ω^{-1}

(3) For a capacitor, $Y_C = j\omega C = jB_C$ in Ω^{-1}

(4) B_L and B_C are the susceptance of L and C, respectively.

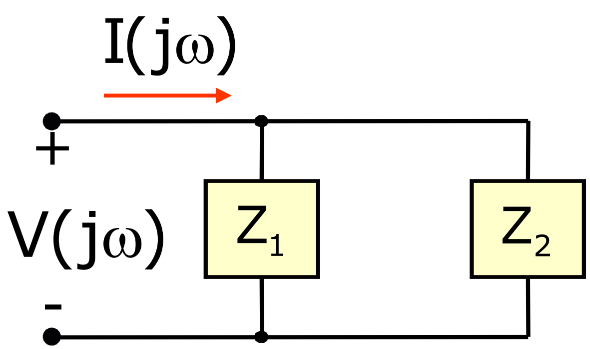
Network Reduction

Impedances in Series:



$$Z_{eq} = \frac{V(j\omega)}{I(j\omega)} = Z_1 + Z_2$$

Impedances in Parallel:



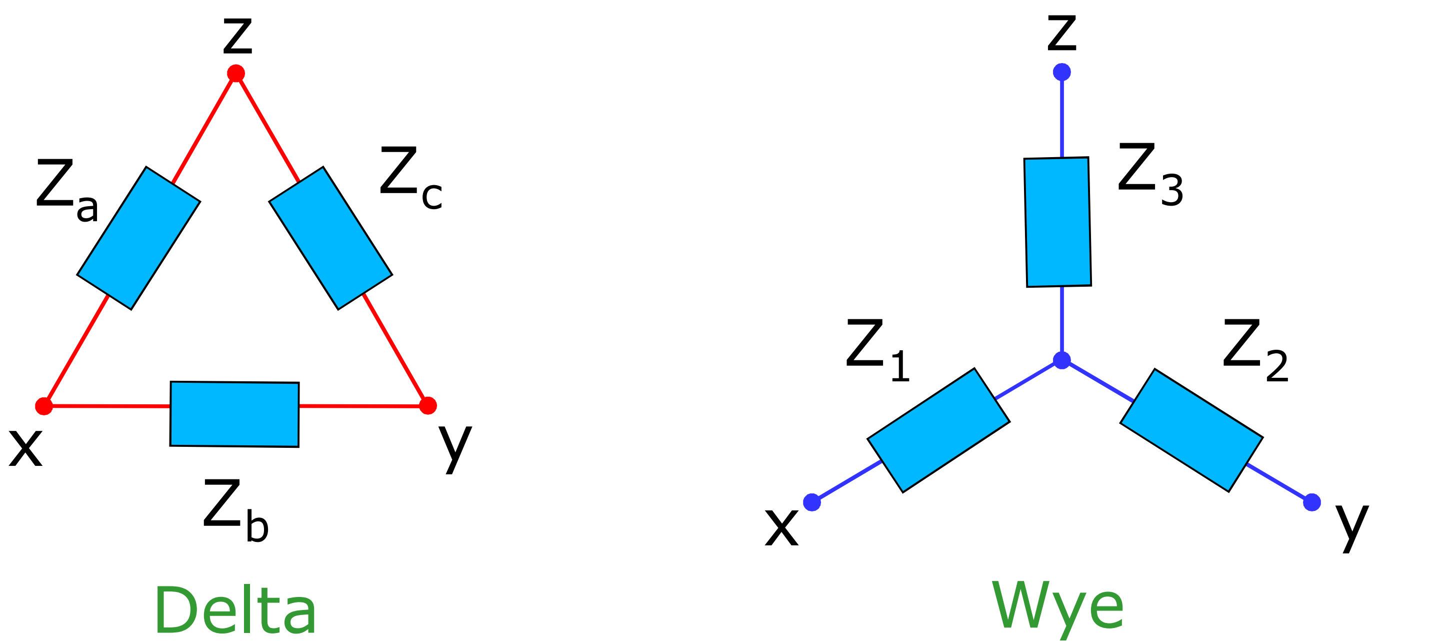
$$Z_{eq} = \frac{V(j\omega)}{I(j\omega)} = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$Y_{eq} = \frac{I(j\omega)}{V(j\omega)} = Y_1 + Y_2$$

DELTA-WYE AND WYE-DELTA TRANSFORMATIONS

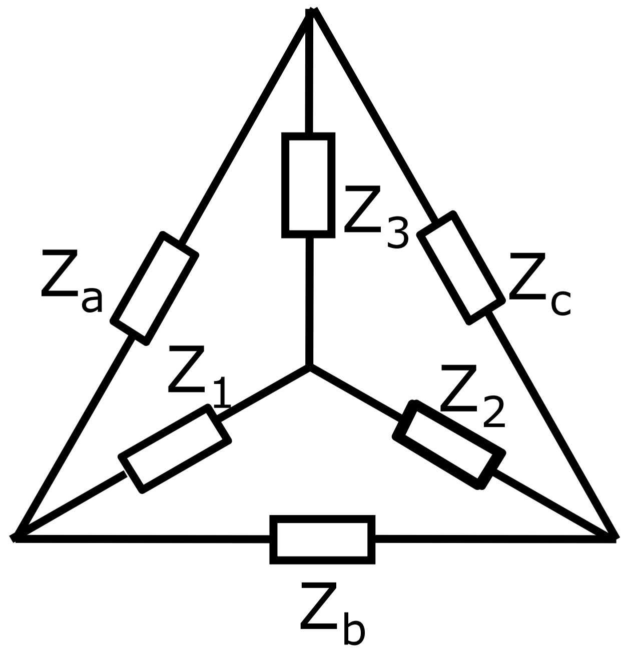
Delta-Wye Transformation

The transformation is used to establish equivalence for networks with 3 terminals.



For equivalence, the resistance between any pair of terminals must be the same for both networks.

Delta-Wye Transformation



$$Z_Y = \frac{\text{product of adjacent } Z_\Delta\text{'s}}{\text{sum of } Z_\Delta\text{'s}}$$

$$Z_\Delta = \frac{\text{sum of pairwise products of } Z_Y\text{'s}}{\text{the opposite } Z_Y}$$

Delta-to-Wye Transformation Equations

$$Z_1 = \frac{Z_a Z_b}{Z_a + Z_b + Z_c} \qquad Z_2 = \frac{Z_b Z_c}{Z_a + Z_b + Z_c}$$

$$Z_3 = \frac{Z_c Z_a}{Z_a + Z_b + Z_c}$$

Wye-to-Delta Transformation Equations

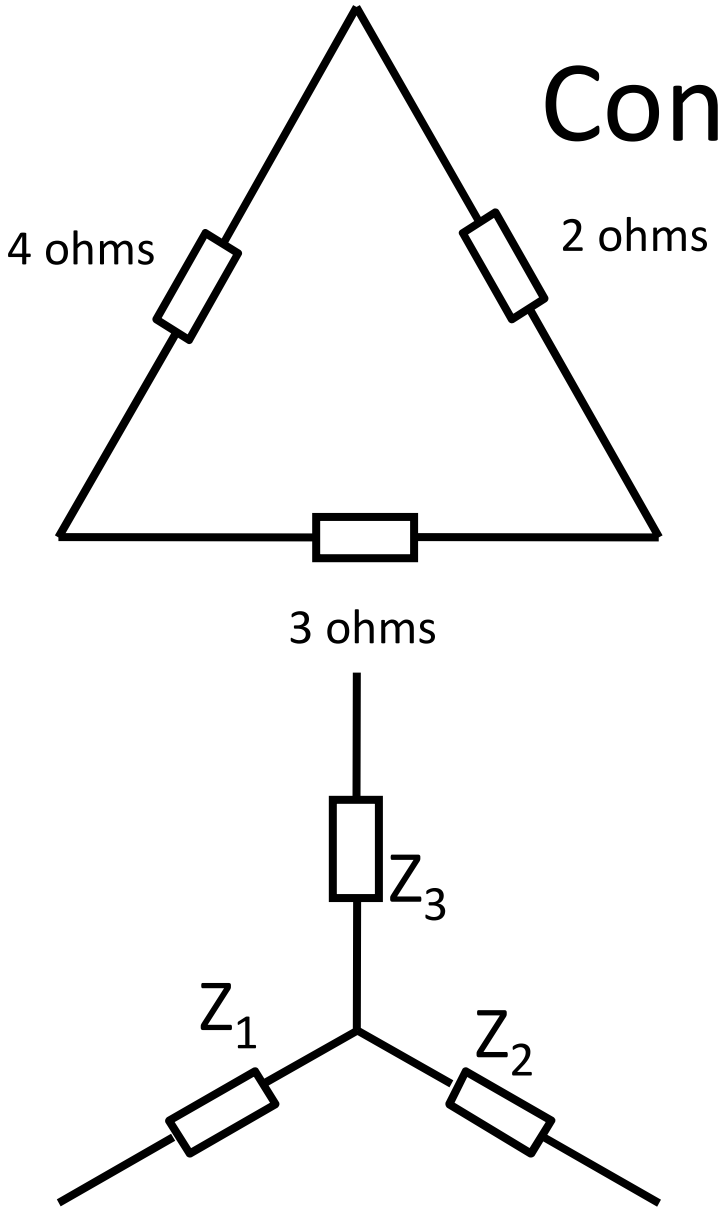
$$Z_a = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_2} \qquad Z_b = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_3}$$

$$Z_c = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_1}$$

Balanced Loads

- If all impedances of the three phase loads are equivalent
- $Z_Y = \frac{Z_\Delta}{3}$
- Proof :

Concept Test!



Solve for Z1, Z2, and Z3

$$Z_1 = \frac{4(3)}{4+3+2} = 1.333$$

$$Z_2 = \frac{2(3)}{4+3+2} = 0.667$$

$$Z_3 = \frac{4(2)}{4+3+2} = 0.889$$

Summary

- Single Subscript and Double Subscript Notation
- Phasor Notation
- Complex Impedance
- Delta and Wye