EEE 51 Assignment 4 Solution

2nd Semester SY 2017-2018

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1. **MOSFET-BJT Amplifier Design.** Figure 1 shows an amplifier configuration that uses current mirrors to generate the DC voltages needed by M_1 and Q_2 with only one DC voltage supply. The output DC voltage requirement is 2.5 V at room temperature, while using a single 5 V supply voltage and M_1 having a quiescent DC drain current of 1 mA. Assume that the given capacitor and inductor elements are ideal and have infinitely large values.

   Given $\beta = 300$, $|I_S|$=$1fA$, and $|V_A|$=$100V$ for the PNP transistors and $k = 2\frac{mA}{V^2}$, $\lambda = 0.1V^{-1}$, and $V_{TH} = 1V$ for the NMOS transistors:

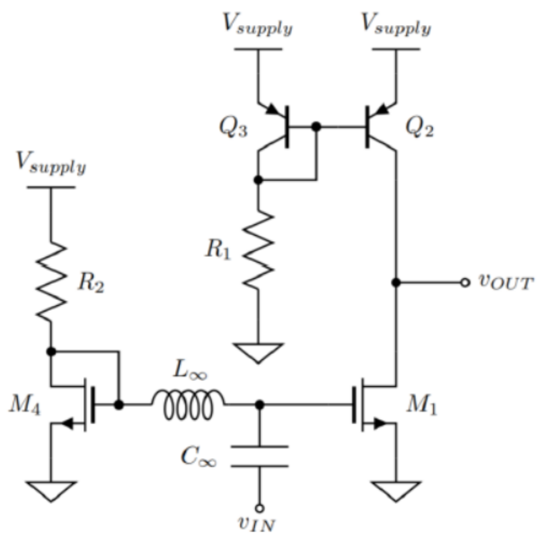

Figure 1: MOSFET-BJT Amplifier

(a) Determine the required R_1 and R_2. [4 pts]
For R_1:

$$R_1 = \frac{V_{R1}}{I_{R1}} \tag{1}$$

Wherein I_{R1} is

$$I_{R1} = I_{C3} + I_{B3} + I_{B2} = I_{C3} + \frac{I_{C3}}{\beta} + \frac{I_{C2}}{\beta} \tag{2}$$

and $I_{C2} = I_{D1} = 1mA$

$$|V_{eb2}| = Vt * ln[\frac{I_{C2}}{(1 + \frac{|V_{ec2}|}{V_a})|I_S|}] = 0.718220V \tag{3}$$

$V_{eb2} = V_{eb3}$ therefore:

$$I_{C3} = |I_S|(e^{\frac{|V_{eb3}|}{Vt}} - 1)(1 + \frac{|V_{eb3}|}{V_a}) = 999.982uA \tag{4}$$

$$I_{R1} = 999.982uA + \frac{999.982uA}{300} + \frac{1mA}{300} = 1.007mA \tag{5}$$

$$V_{R1} = V_{supply} - |V_{eb3}| = 4.282V \tag{6}$$

$$R_1 = \frac{V_{R1}}{I_{R1}} = 4253.5\Omega \tag{7}$$

For R_2:

$$R_2 = \frac{V_{supply} - V_{GS4}}{I_{D4}} \tag{8}$$

Considering that $V_{GS1} = V_{GS4} = V_{DS4}$ and that $V_{DS1} = 2.5V$

$$V_{GS1} = V_{TH} + \sqrt[2]{\frac{I_{D1}}{k(1 + \lambda * V_{ds1})}} = 1.632V \tag{9}$$

and I_{d4} is

$$I_{d4} = \frac{1 + \lambda * V_{ds4}}{1 + \lambda * V_{ds1}} = 930.596uA \tag{10}$$

therefore

$$R_2 = \frac{V_{supply} - V_{GS4}}{I_{D4}} = 3618.69\Omega \tag{11}$$

(b) Determine the small signal gain, $A_V = \frac{v_{out}}{v_{in}}$, of the amplifier. [3 pts]

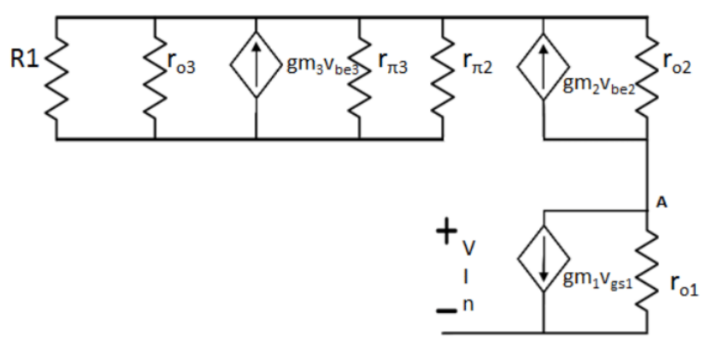

Figure 2: small signal

Since Q2 is a current mirror we assume that $v_{be2} \& v_{be3} = 0$. KCL at node A:

$$V_{out}(\frac{1}{ro_1} + \frac{1}{ro_2}) = -V_{in} * gm_1 \tag{12}$$

$$A_v = \frac{V_{out}}{V_{in}} = -gm_1 * (ro_1 // ro_2) \tag{13}$$

$$ro_1 = \frac{1}{\lambda * I_d} = 10k\Omega \tag{14}$$

$$ro_2 = \frac{V_a}{I_{c2}} = 100k\Omega \tag{15}$$

$$gm_1 = 2k(V_{gs1} - V_{TH}) = 2.53mS \tag{16}$$

$$A_v = -22.998 \tag{17}$$

2. **BJT Current Mirror with Emitter Resistors**. A current mirror was constructed as shown in Figure 3 with a resistor R_A used to generate a bias current. Q_1 and Q_2 are used to mirror that current and generate I_{OUT}. The current mirror has already been biased such that both transistors are in forward active over some range of output voltage, and so $r_{\pi 1}$, $r_{\pi 2}$, r_{o1}, r_{o2}, g_{m1}, g_{m2} are known to be some set of values, as well as R_A, R_B and R_C.

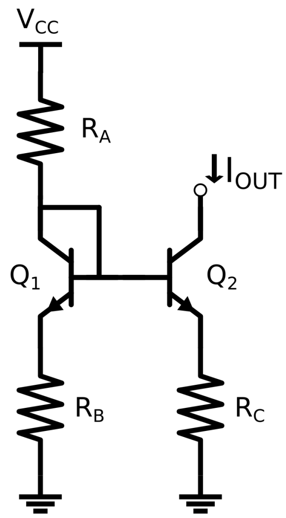

Figure 3: BJT Current Mirror

(a) Draw the small-signal equivalent circuit. Label all parameters, voltages, currents necessary. Label terminal names, and if possible, label the nodes mapping to the pins of the transistors. [3 pts]

In order to simplify and make more visually intuitive the later part of the solution, this method (seen in Figure 4) of representing the small-signal circuit was used.

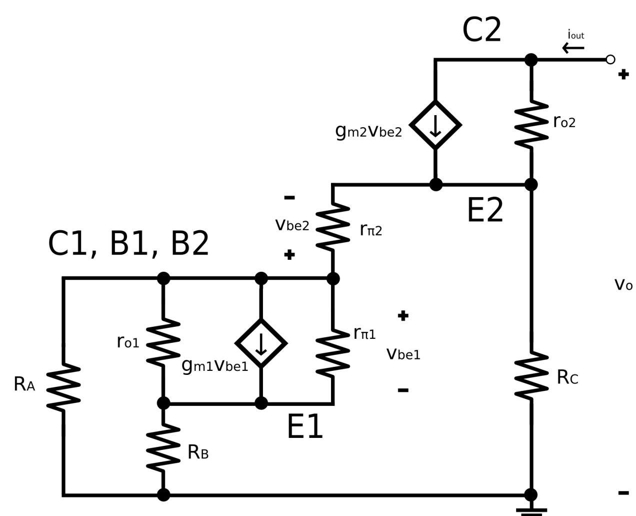

Figure 4: Small-signal equivalent.

[3 pts]

(b) Determine the output resistance R_o in terms of the small-signal parameters and resistor values. Do not omit terms based on common assumptions. If possible, show a detailed step by step solution and simplify or expand terms for clarity. [4 pts]

Before beginning, note that the small-signal circuit can be simplified first. Q_1's $g_{m1}v_{be1}$ term is a current source that refers to its own voltage, as v_{be1} is now applied across it because of collector-base shorting. Therefore it is effectively a resistor with a value of $\frac{1}{g_{m1}}$.
Therefore, the entire left side that refers to parameters from Q_1 can now be simplified as it is essentially a network of resistors, including R_A and R_B. We can refer to the entire network as one effective resistance R_x as seen by the rest of the circuit.

$$R_x = R_A || \left(R_B + \left(r_{o1} || \frac{1}{g_{m1}} || r_{\pi 1} \right) \right) \tag{18}$$

We can redraw the circuit to reflect this change. Figure 5 shows the new equivalent circuit.

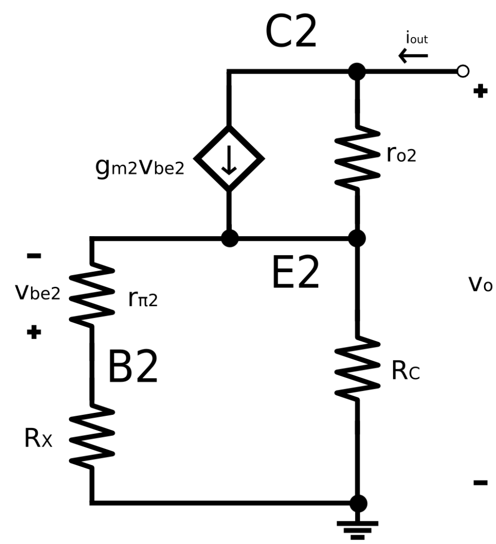

Figure 5: Small-signal equivalent.

In order to find output resistance, start looking for equations for v_o and i_{out}. v_o is the total of the voltage across r_{o2} and the voltage across $R_C || (R_x + r_{\pi 2})$. The latter has i_{out} flowing through it, but the former has a reduced amount of current due to g_{m2}.

$$v_o = i_{out} \left(R_C || \left(R_x + r_{\pi 2} \right) \right) + \left(i_{out} - g_{m2} v_{be2} \right) r_{o2} \tag{19}$$

v_{be2} is itself a voltage division of that voltage across $R_C || (R_x + r_{\pi 2})$.

$$v_{be2} = -i_{out} \left(R_C || \left(R_x + r_{\pi 2} \right) \right) \frac{r_{\pi 2}}{r_{\pi 2} + R_x} \tag{20}$$

Now that everything has an i_{out} factor,

$$v_o = i_{out} \left(\left(R_C || \left(R_x + r_{\pi 2} \right) \right) + r_{o2} + g_{m2} r_{o2} \left(R_C || \left(R_x + r_{\pi 2} \right) \right) \frac{r_{\pi 2}}{r_{\pi 2} + R_x} \right) \tag{21}$$

$$R_o = \frac{v_o}{i_{out}} = \left(R_C || \left(R_x + r_{\pi 2} \right) \right) + r_{o2} + g_{m2} r_{o2} \left(R_C || \left(R_x + r_{\pi 2} \right) \right) \frac{r_{\pi 2}}{r_{\pi 2} + R_x} \tag{22}$$

Rearranging some terms to factor,

$$R_o = \frac{v_o}{i_{out}} = \left(R_C || \left(R_x + r_{\pi 2} \right) \right) \left(1 + \frac{g_{m2} r_{o2} r_{\pi 2}}{r_{\pi 2} + R_x} \right) + r_{o2} \tag{23}$$

Since the problem requires the original small-signal parameters and resistance values,

$$R_o = \left(R_C || \left(\left(R_A || \left(R_B + \left(r_{o1} || \frac{1}{g_{m1}} || r_{\pi 1}\right)\right)\right) + r_{\pi 2}\right)\right) \left(1 + \frac{g_{m2} r_{o2} r_{\pi 2}}{r_{\pi 2} + \left(R_A || \left(R_B + \left(r_{o1} || \frac{1}{g_{m1}} || r_{\pi 1}\right)\right)\right)}\right) + r_{o2}$$

[4 pts]

3. **MOSFET Cascode Current Mirror**. Given the cascode current mirror below, determine the following:

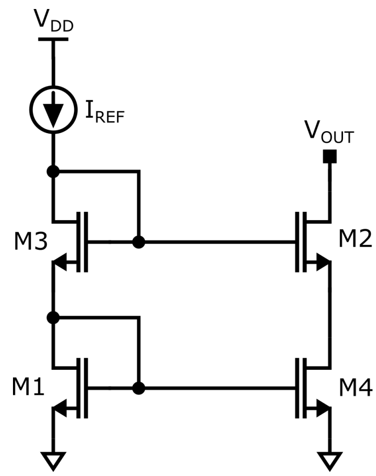

Figure 6: MOSFET Cascode Current Mirror

(a) Draw the *simplified* small-signal equivalent circuit. State all assumptions and properly label all parameters, voltages, and terminal names. [3 pts]

Given a *fixed gate voltage* for M2 and M4 by transistors M1 and M3 respectively, and *no variable voltage* at M1 and M3, the gates of M2 and M4 can be treated as *AC ground*. This also results to $v_{gs4} = 0$. Therefore, the *simplified* small-signal model equivalent of the circuit is as shown in Figure 7.

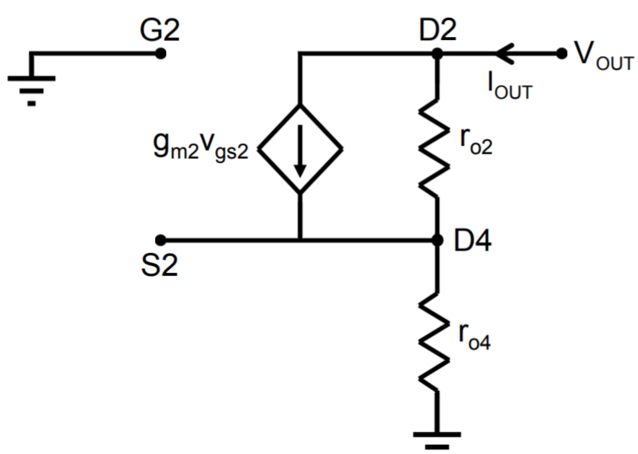

Figure 7: *Simplified* small-signal model of the MOSFET Cascode Current Mirror

(b) Determine the circuit's output resistance R_o in terms of the transistor g_m and r_o. [3 pts]

Using KVL from v_{out} to ground, we get,

$$v_{out} = r_{o2}(i_{out} - g_{m2}V_{GS2}) + r_{o4}i_{out} \tag{24}$$

Since $V_{GS2} = -r_{o4}i_{out}$, we can substitute this to (24), isolate i_{out} from the right-hand side of the resulting expression, and divide the whole expression by i_{out}. This yields,

$$\frac{v_{out}}{i_{out}} = r_{o2} + g_{m2}r_{o2}r_{o4} + r_{o4} \tag{25}$$

Rearranging the terms and rewriting, we get,

$$\boxed{R_o = r_{o2}(1 + g_{m2}r_{o4}) + r_{o4} \text{ or } R_o = r_{o2} + r_{o4}(1 + g_{m2}r_{o2})}$$