EEE 51: Second Semester 2017 - 2018

Lecture 8

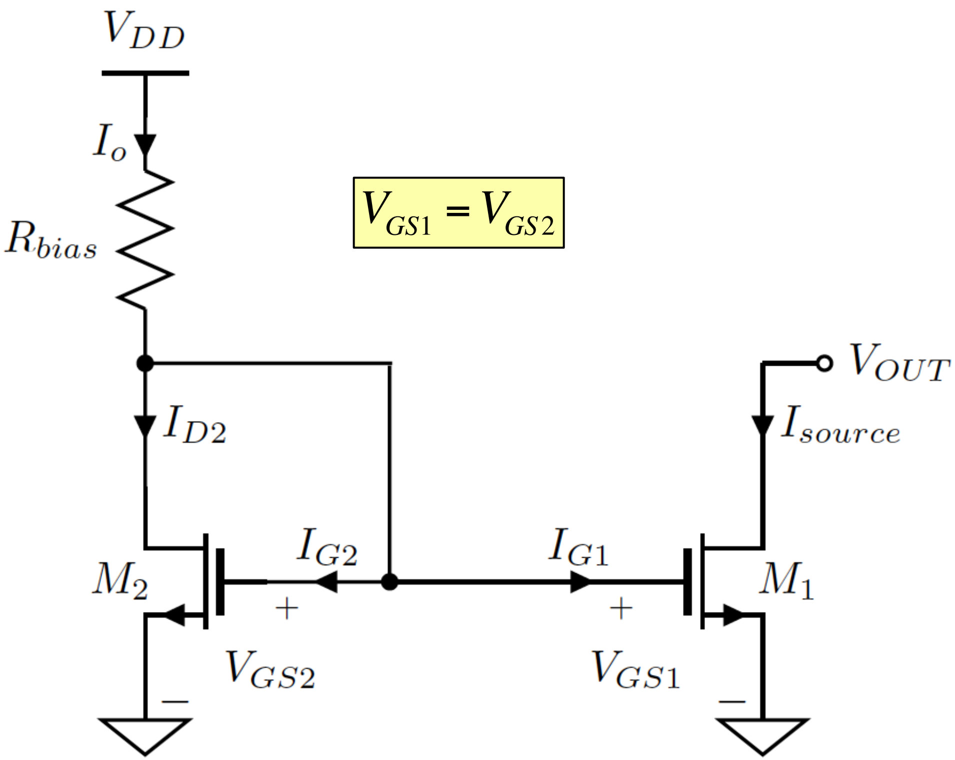
Differential Circuits

Today

- Finish Up Current Mirrors
- Amplifiers Biased Using Current Mirrors
- Differential Circuits

A Simple MOSFET Current Mirror

- No gate currents!



$$V_{GS1} = V_{GS2}$$

Recall: $V_{GS} = V_{TH} + \sqrt{\dfrac{I_D}{k(1+\lambda V_{DS})}}$

$$\Rightarrow \frac{I_{D1}}{(1+\lambda V_{DS1})} = \frac{I_{D2}}{(1+\lambda V_{DS2})}$$

$$I_{source} = I_{D2}\frac{(1+\lambda V_{DS1})}{(1+\lambda V_{DS2})} = I_o\frac{(1+\lambda V_{OUT})}{(1+\lambda V_{GS})}$$

Assume λ=0

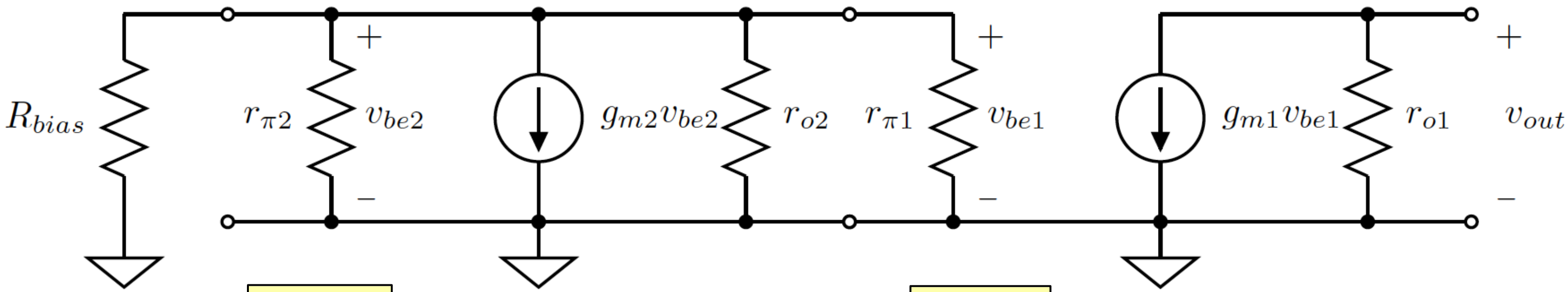
$$I_{source} \approx I_o$$

Mirror!

Mirroring Error:
→ Due to V_{DS} “mismatch” only

Current Mirror Small Signal Model

- Small Signal Output Resistance?



$v_{be2} = ?$

$v_{be1} = ?$

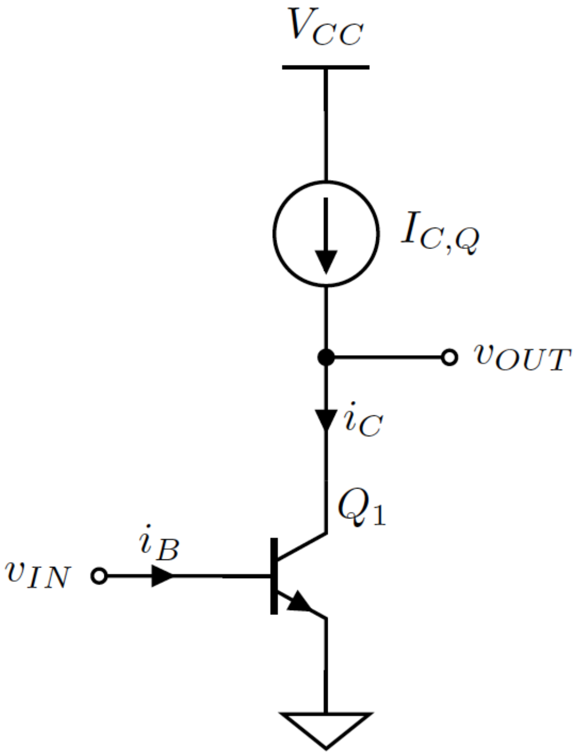
Diode-connected transistor

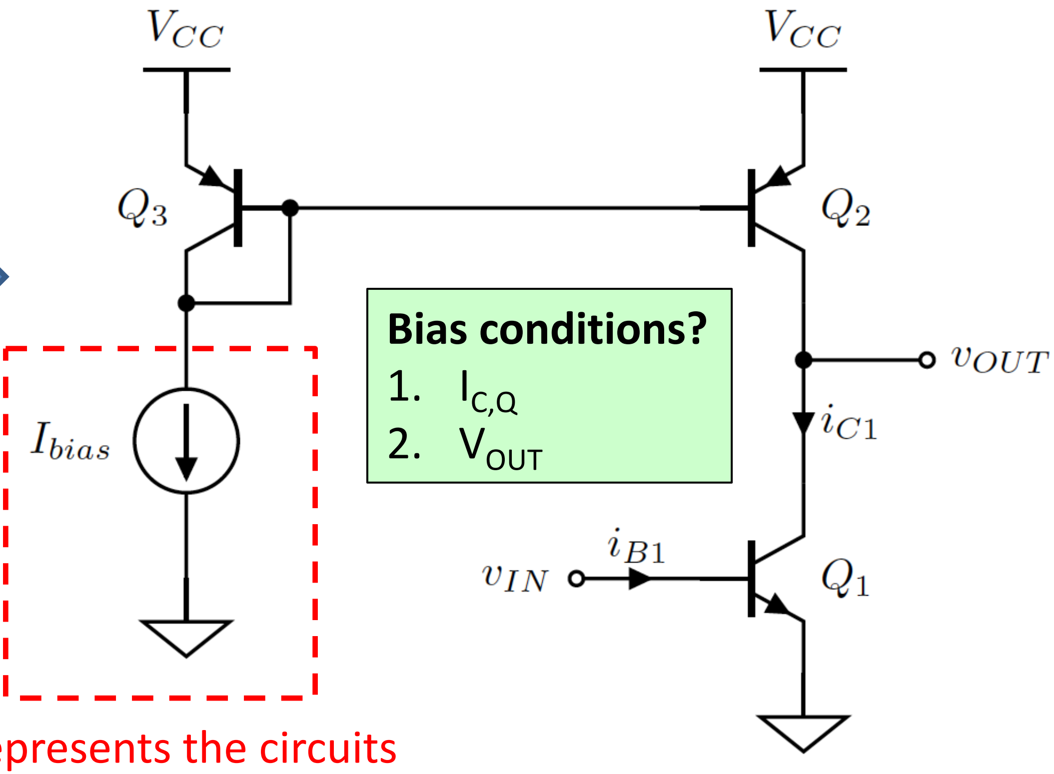
Current source transistor

Why?

$R_o = r_{o1}$

Biasing the Common-Emitter Amplifier (1)





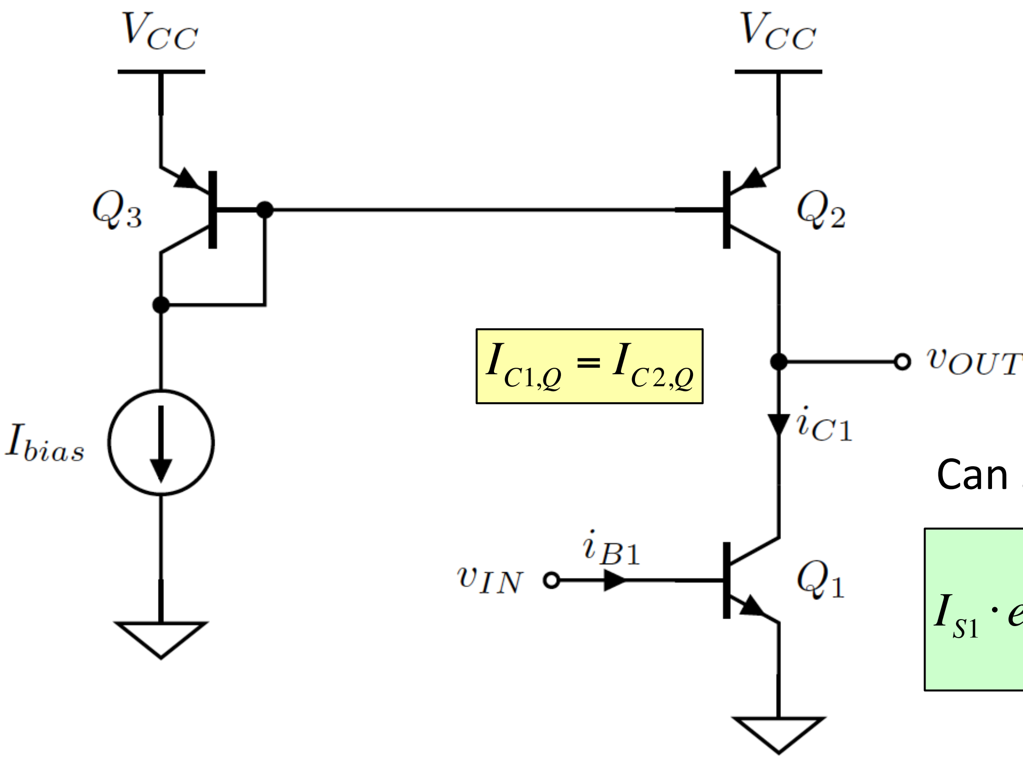
Bias conditions?

1. $I_{C,Q}$
2. V_{OUT}

Represents the circuits used to bias Q_3

Biasing the Common-Emitter Amplifier (2)

- DC Analysis



$$I_{C1,Q} = I_{C2,Q}$$

For $V_A \rightarrow \infty$:

$$I_{C,Q} \approx I_{bias}$$

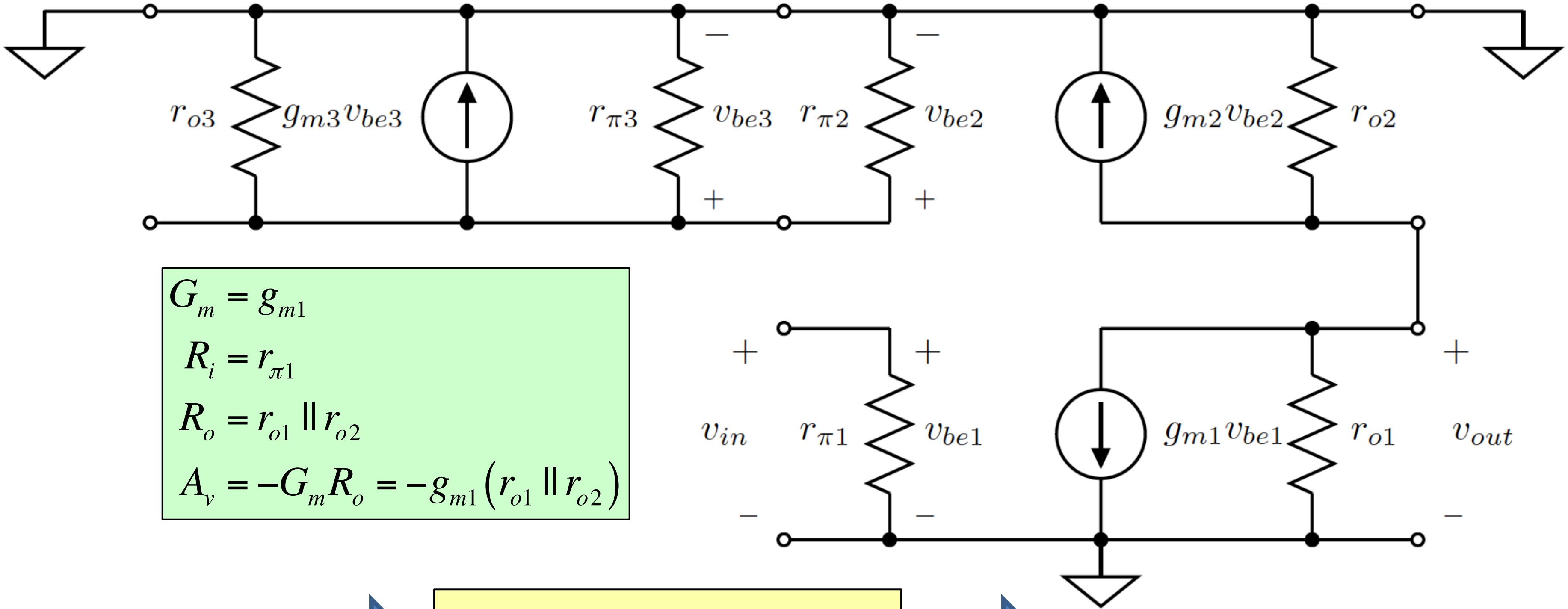
What about V_{OUT}?
→ Need finite V_A!
→ Why?

Can solve for V_{OUT}:

$$I_{S1} \cdot e^{\frac{V_{IN}}{V_T}} \cdot \left(1 + \frac{V_{OUT}}{V_{A1}}\right) = I_{S2} \cdot e^{\frac{|V_{BE2}|}{V_T}} \cdot \left(1 + \frac{|V_{CC} - V_{OUT}|}{V_{A2}}\right)$$

Biasing the Common-Emitter Amplifier (3)

- Small Signal Model



$$G_m = g_{m1}$$
$$R_i = r_{\pi 1}$$
$$R_o = r_{o1} \parallel r_{o2}$$
$$A_v = -G_m R_o = -g_{m1}\left(r_{o1} \parallel r_{o2}\right)$$

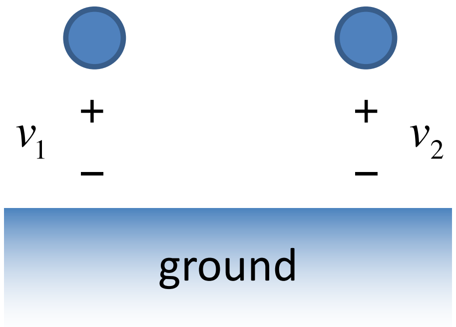
If $r_{o1} \approx r_{o2}$ ➡ $A_v = -g_{m1}\left(r_{o1} \parallel r_{o2}\right) \approx -\frac{g_{m1}r_{o1}}{2}$ ➡ Already half a_o!

Differential Circuits

- Building Blocks (so far…)
  - Single-stage amplifiers
  - Current mirrors
  - **Differential Pair**

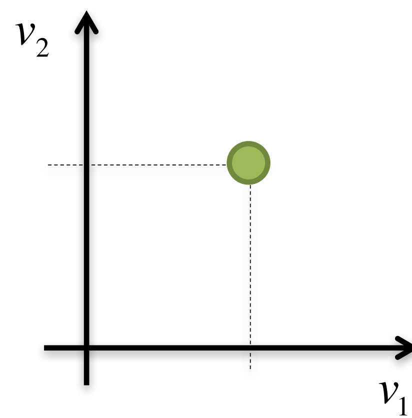
Differential Signals (1)

- Consider a pair of wires



v_1 and v_2 are **single-ended** signals

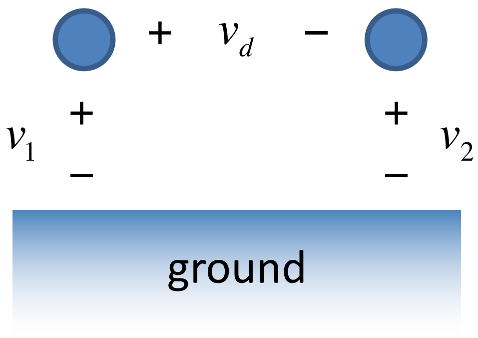
In this case, with two wires, to completely describe the system, we need both v_1 and v_2



How about another set of coordinates?

Differential Signals (2)

- Definitions



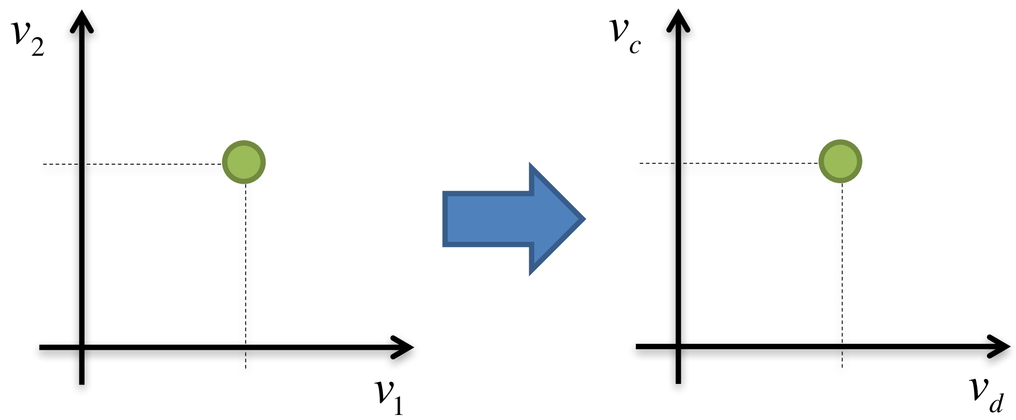
Differential voltage

$$v_d = v_1 - v_2$$

Common-mode voltage

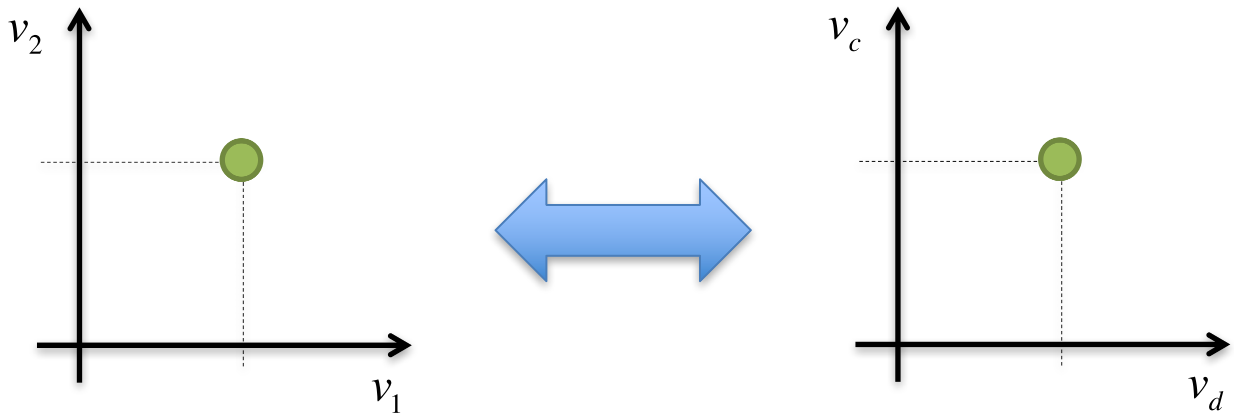
$$v_c = \frac{v_1 + v_2}{2}$$

Same point, different coordinate system



Differential Signals (3)

- Can easily go from single-ended to differential



$$v_1 = v_c + \frac{v_d}{2}$$

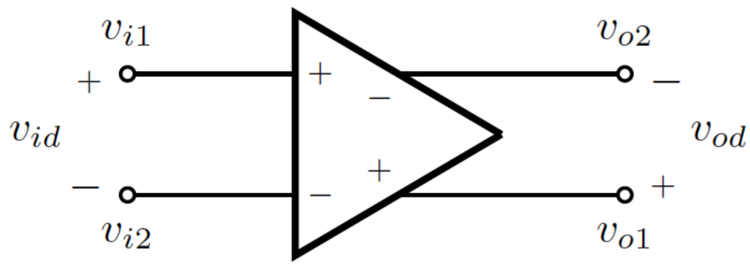
$$v_2 = v_c - \frac{v_d}{2}$$

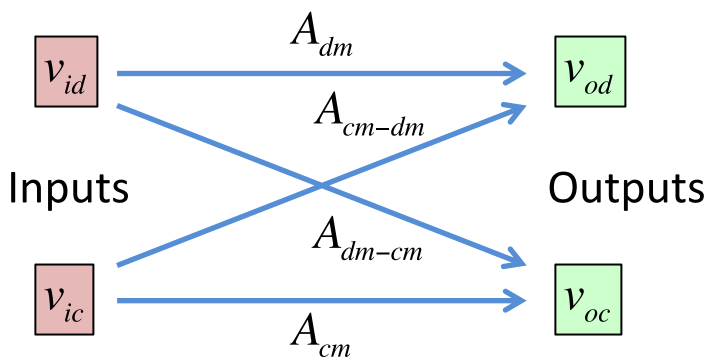
$$v_d = v_1 - v_2$$

$$v_c = \frac{v_1 + v_2}{2}$$

Differential Amplifiers: Gain Definitions

- A Fully-Differential Amplifier





Works with differential signals

$$A_{dm} = \left.\frac{v_{od}}{v_{id}}\right|_{v_{ic}=0} \qquad A_{cm} = \left.\frac{v_{oc}}{v_{ic}}\right|_{v_{id}=0}$$

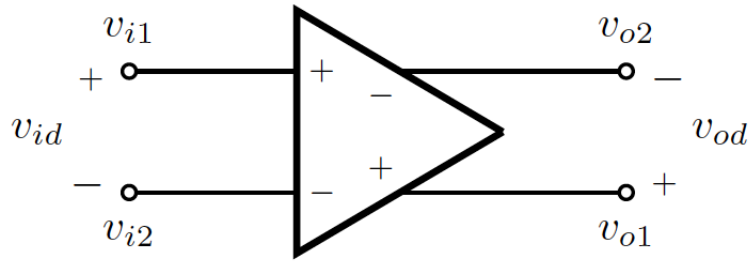
$$A_{dm-cm} = \left.\frac{v_{oc}}{v_{id}}\right|_{v_{ic}=0} \qquad A_{cm-dm} = \left.\frac{v_{od}}{v_{ic}}\right|_{v_{id}=0}$$

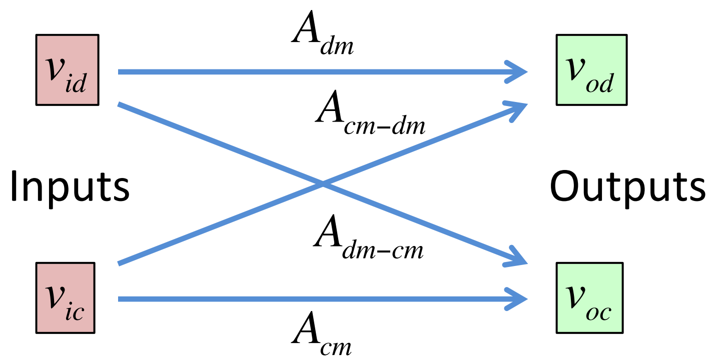
$$v_{od} = A_{dm} v_{id} + A_{cm-cm} v_{ic}$$
$$v_{oc} = A_{dm-cm} v_{id} + A_{cm} v_{ic}$$

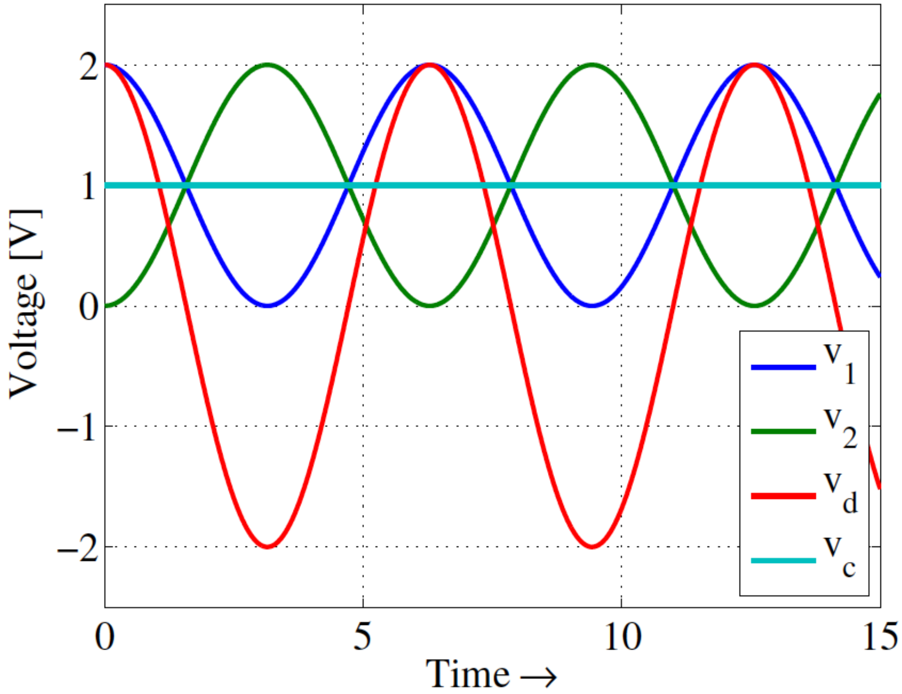
Inversion comes for FREE!

Why Use Differential Signaling? (1)

- Differential vs. single-ended signals

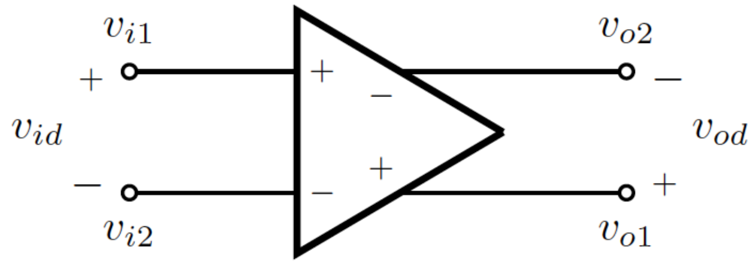


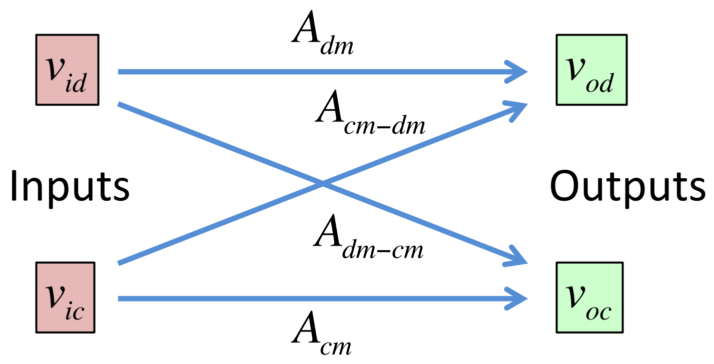


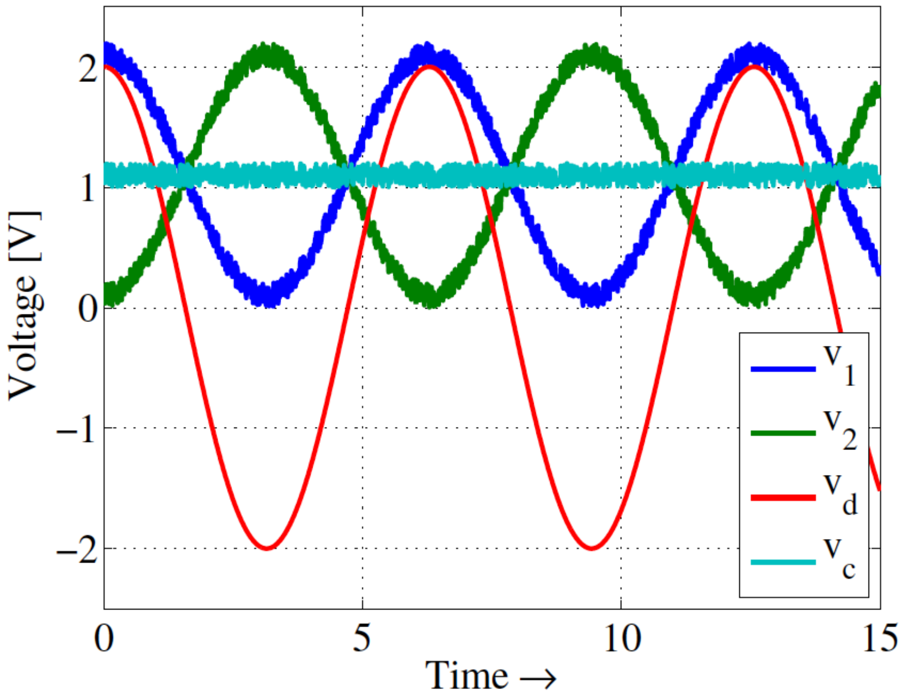


Why Use Differential Signaling? (2)

- Common-mode noise or interference



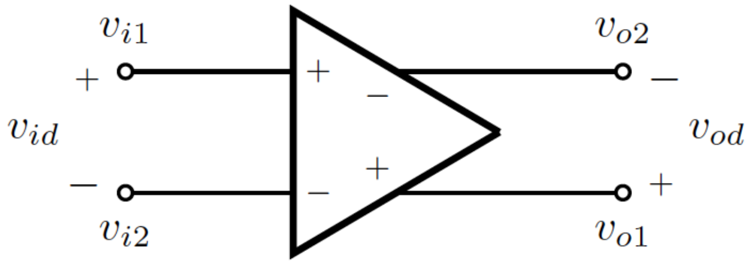


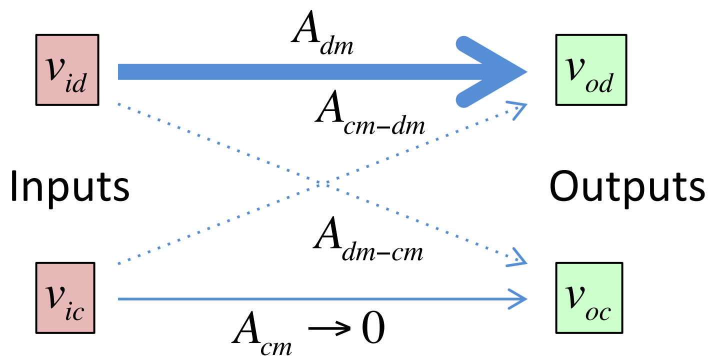


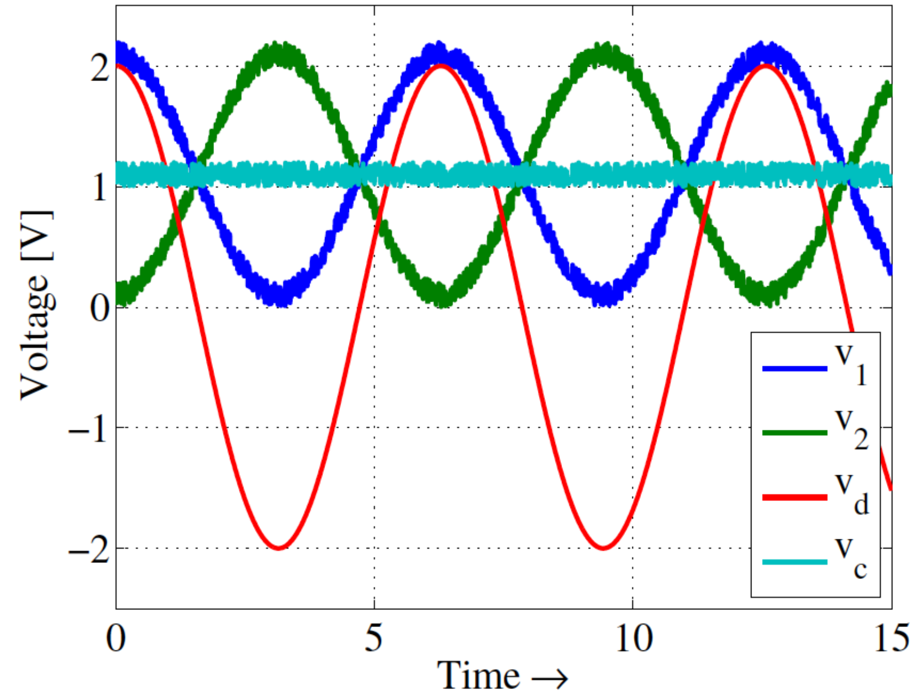
Anything common to both v_1 and v_2 will not be seen in v_{id}!

Why Use Differential Signaling? (3)

- Amplify v_{id}, reject v_{ic}





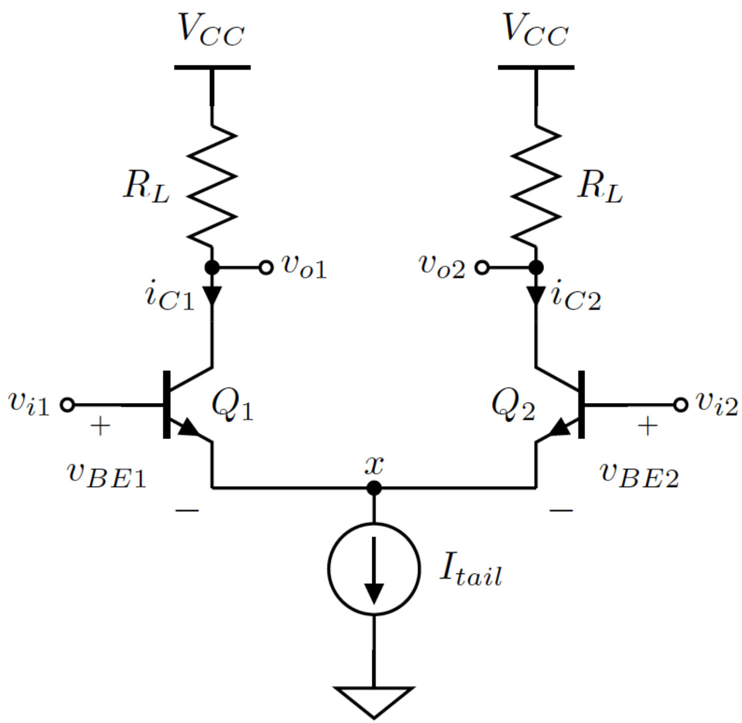


Metric:
Common-Mode Rejection Ratio

$$CMRR = \left|\frac{A_{dm}}{A_{cm}}\right|$$

Basic Building Block: The Differential Pair (1)

- BJT: The emitter-coupled pair DC Analysis



KVL at the input loop: $$V_{i1} - V_{BE1} + V_{BE2} - V_{i2} = 0$$

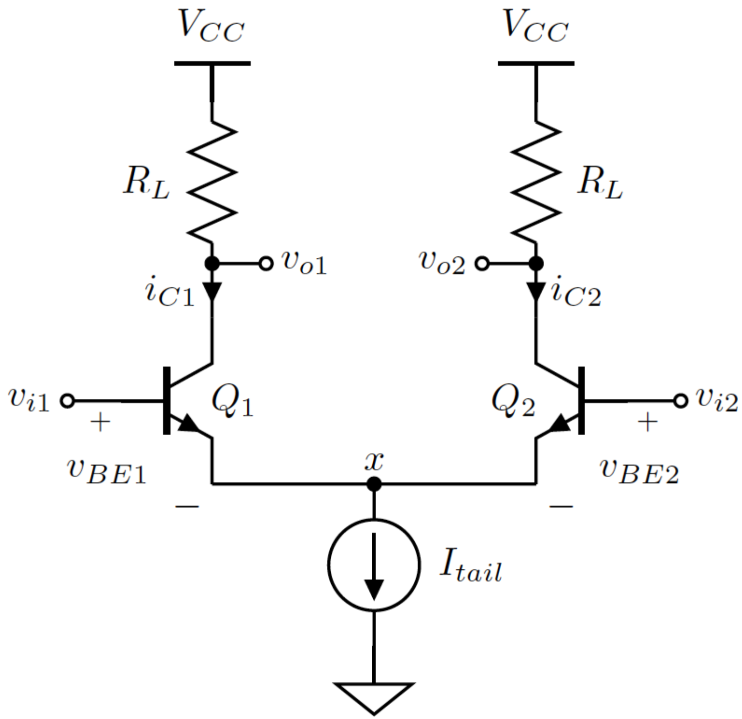
$$V_{i1} - V_{i2} = V_{BE1} - V_{BE2}$$

$$V_{id} = V_T \ln\left(\frac{I_{C1}}{I_S}\right) - V_T \ln\left(\frac{I_{C2}}{I_S}\right) = V_T \ln\left(\frac{I_{C1}}{I_{C2}}\right)$$

$$\boxed{I_{C1} = I_{C2} \cdot e^{\frac{V_{id}}{V_T}}}$$

Basic Building Block: The Differential Pair (2)

- BJT: The emitter-coupled pair DC Analysis



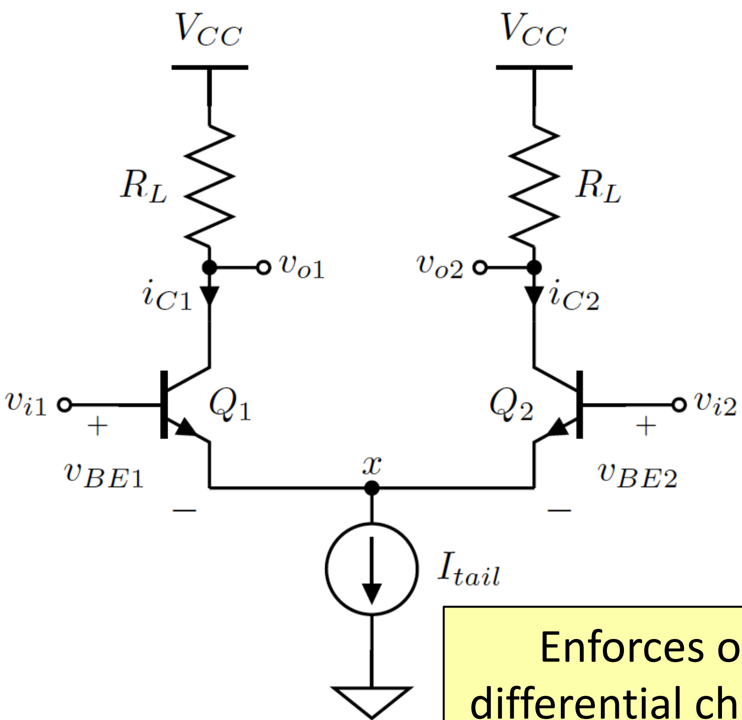
KCL at node x: $I_{tail} = I_{E1} + I_{E2} = \dfrac{I_{C1} + I_{C2}}{\alpha}$

Recall: $\boxed{I_{C1} = I_{C2} \cdot e^{\frac{V_{id}}{V_T}}}$

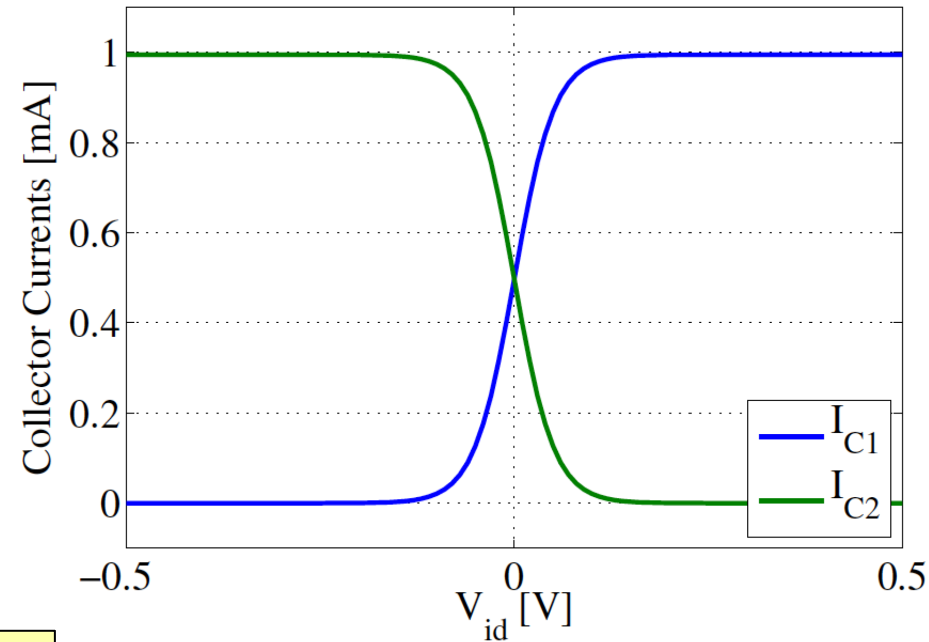
$$\boxed{I_{C1} = \frac{\alpha \cdot I_{tail}}{1 + e^{-\frac{V_{id}}{V_T}}}} \qquad \boxed{I_{C2} = \frac{\alpha \cdot I_{tail}}{1 + e^{+\frac{V_{id}}{V_T}}}}$$

Basic Building Block: The Differential Pair (3)

- BJT: The emitter-coupled pair DC Analysis



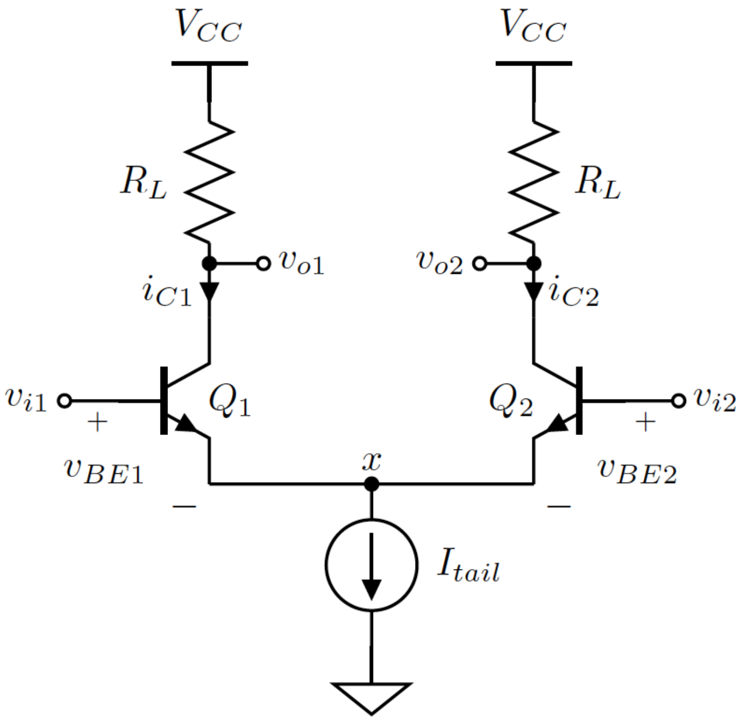
Enforces only differential changes!



$$I_{C1} = \frac{\alpha \cdot I_{tail}}{1 + e^{-\frac{V_{id}}{V_T}}}$$

$$I_{C2} = \frac{\alpha \cdot I_{tail}}{1 + e^{+\frac{V_{id}}{V_T}}}$$

- Output Voltage



$$I_{C1} = \frac{\alpha \cdot I_{tail}}{1 + e^{-\frac{V_{id}}{V_T}}}$$

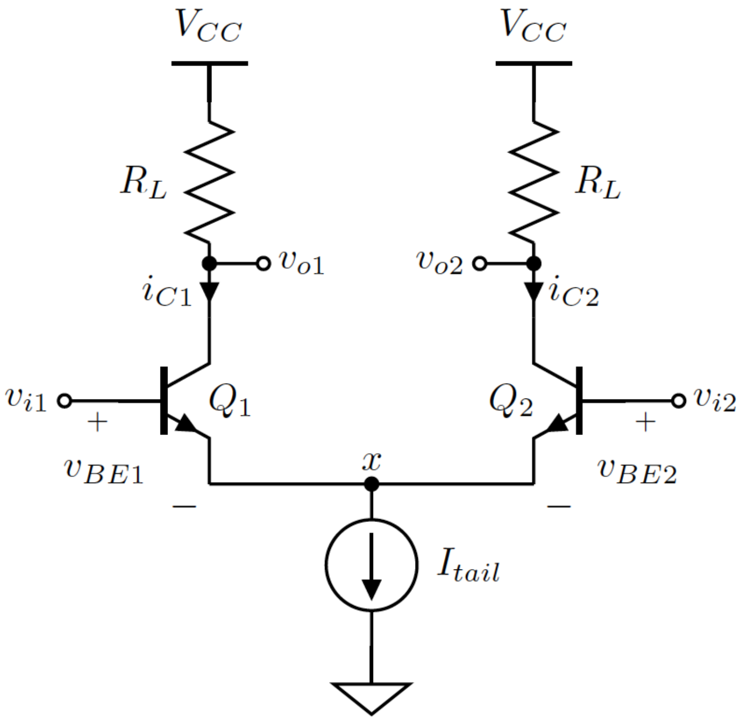
$$I_{C2} = \frac{\alpha \cdot I_{tail}}{1 + e^{+\frac{V_{id}}{V_T}}}$$

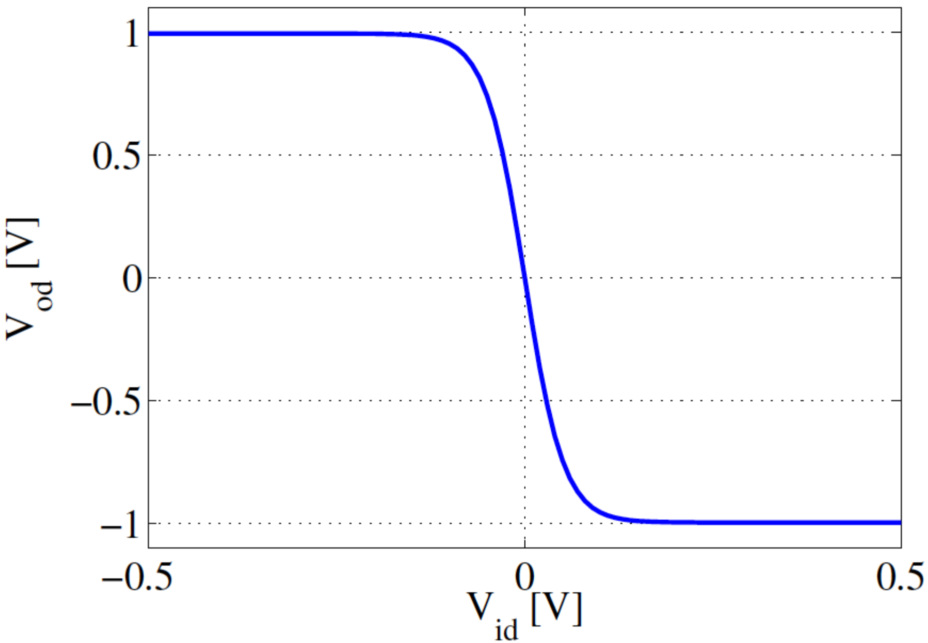
KVL at the output loop: $V_{od} = V_{o1} - V_{o2} = R_L \left(I_{C2} - I_{C1} \right)$

$$V_{od} = \alpha \cdot I_{tail} \cdot R_L \cdot \left(\frac{1}{1 + e^{+\frac{V_{id}}{V_T}}} - \frac{1}{1 + e^{-\frac{V_{id}}{V_T}}} \right)$$

$$= \alpha \cdot I_{tail} \cdot R_L \cdot \tanh\left(-\frac{V_{id}}{2 \cdot V_T} \right)$$

- Transfer Characteristic





$$V_{od} = \alpha \cdot I_{tail} \cdot R_L \cdot \tanh\left(-\frac{V_{id}}{2 \cdot V_T}\right)$$

Next Meeting

- Continue with Differential Circuits