EEE 51: Second Semester 2017 - 2018

Lecture 15

Frequency Response

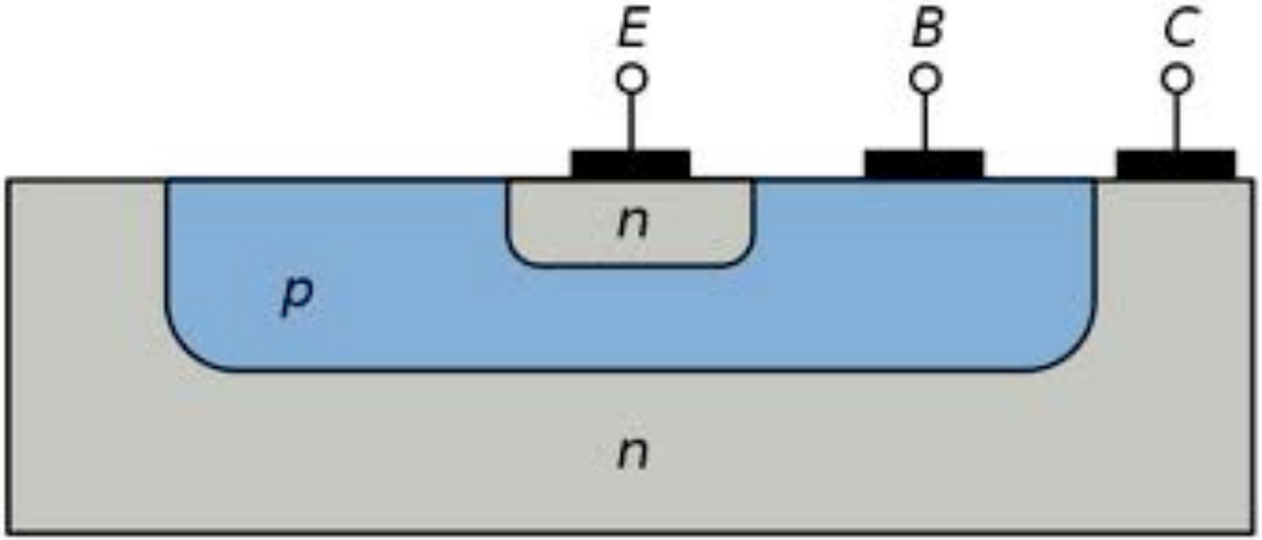
Frequency Characteristics of Transistor Circuits

- Due to frequency-dependent impedances
  - Capacitors, inductors

- BJT Parasitic Capacitances
  - Junction capacitances
  - Nonlinear (voltage dependent)

  - Base-Charging capacitance (C_b)
  - Base-Emitter junction capacitance (C_{je})
  - Base-Collector junction capacitance (C_μ)

BJT Capacitances

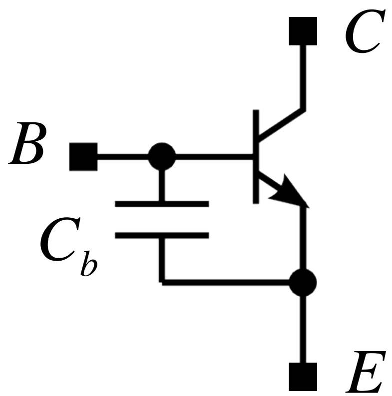


[http://commons.wikimedia.org/wiki/File:Npn_bjt_cross_section.svg]

BJT Base-Charging Capacitance

- Capacitance due to the change in majority carrier charge inside the base of the BJT
  - Cancels out the change in minority carriers in the base due to v_{BE}
  - ~ 100's of fF

$$C_b = \tau_F g_m$$

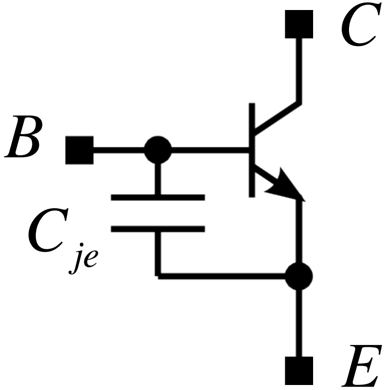


- τ_F – forward base transit time
  - Represents the average time (per carrier) spent crossing the base

BJT Base-Emitter Junction Capacitance

- PN junction capacitance ~ 10's of fF
- In a BJT in the forward active region, BE junction is forward-biased

$$C_{je} = \frac{C_{je0}}{\sqrt{1 - \frac{V_{BE}}{V_{j,BE}}}}$$

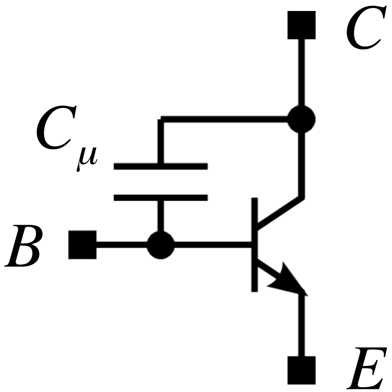


$$C_{je} \approx 2C_{je0}$$

BJT Base-Collector Junction Capacitance

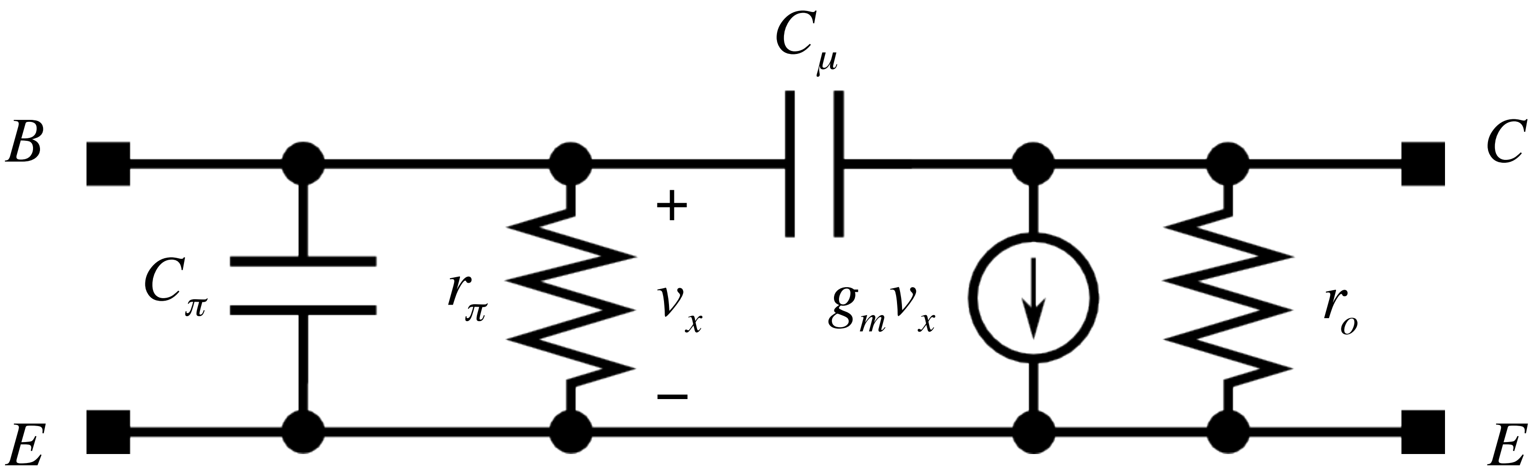
- PN junction capacitance 5 to 10 fF
- Reversed-biased in forward active BJTs

$$C_\mu = \frac{C_{\mu 0}}{\sqrt{1 + \frac{V_{CB}}{V_{j,CB}}}}$$



Can be amplified by the Miller effect

BJT Small Signal Model (with Capacitances)



$$C_\pi = C_b + C_{je}$$

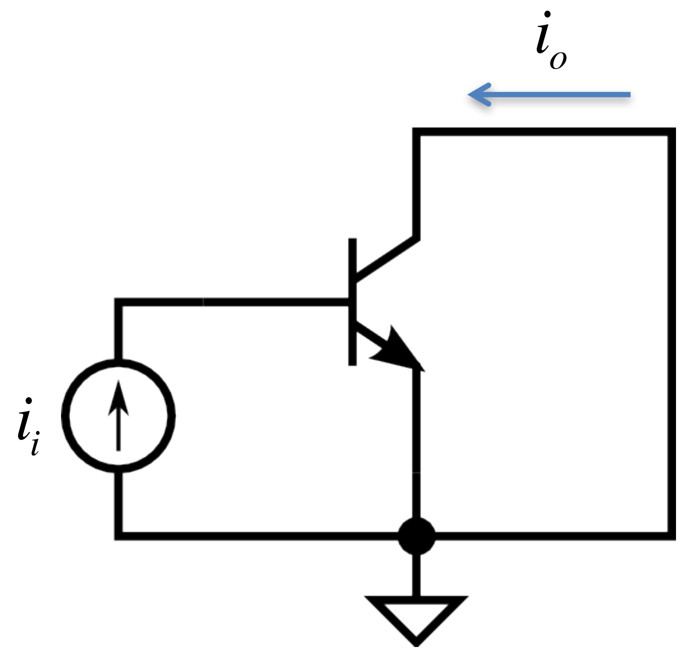
$$C_b = \tau_F g_m$$

$$C_{je} = \frac{C_{je0}}{\sqrt{1 - \frac{V_{BE}}{V_{j,BE}}}}$$

$$C_\mu = \frac{C_{\mu 0}}{\sqrt{1 + \frac{V_{CB}}{V_{j,CB}}}}$$

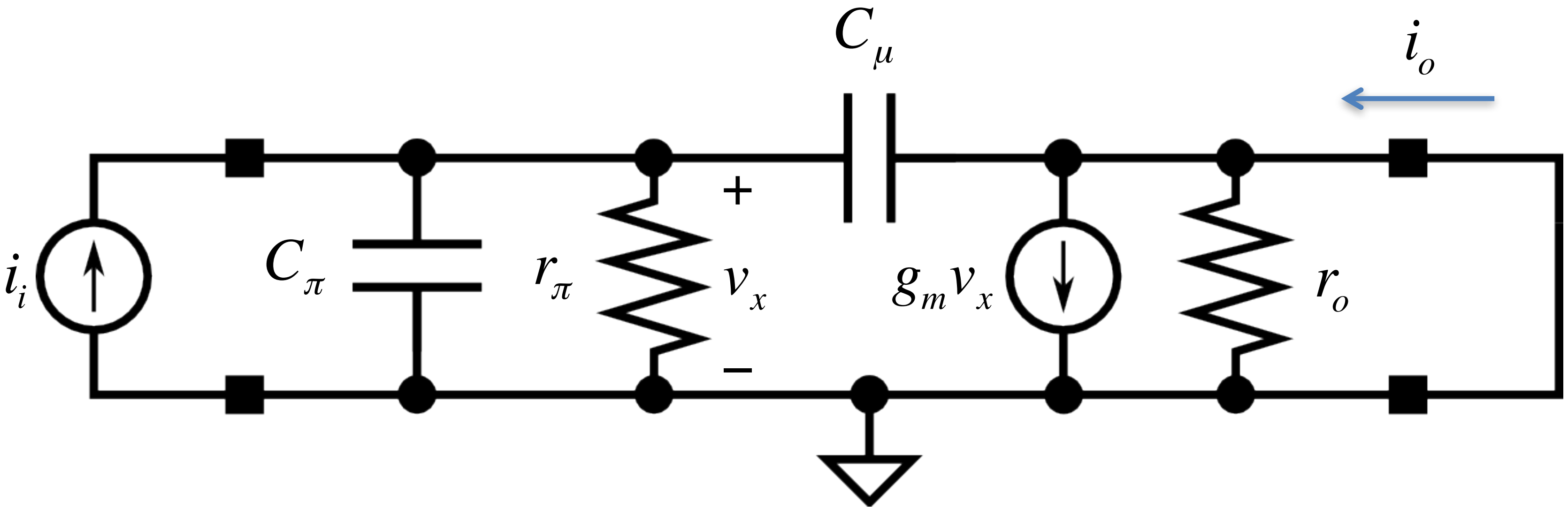
BJT Transition Frequency (f_T)

- Frequency at which the short-circuit common emitter current gain falls to unity



BJT f_T

- Small signal model



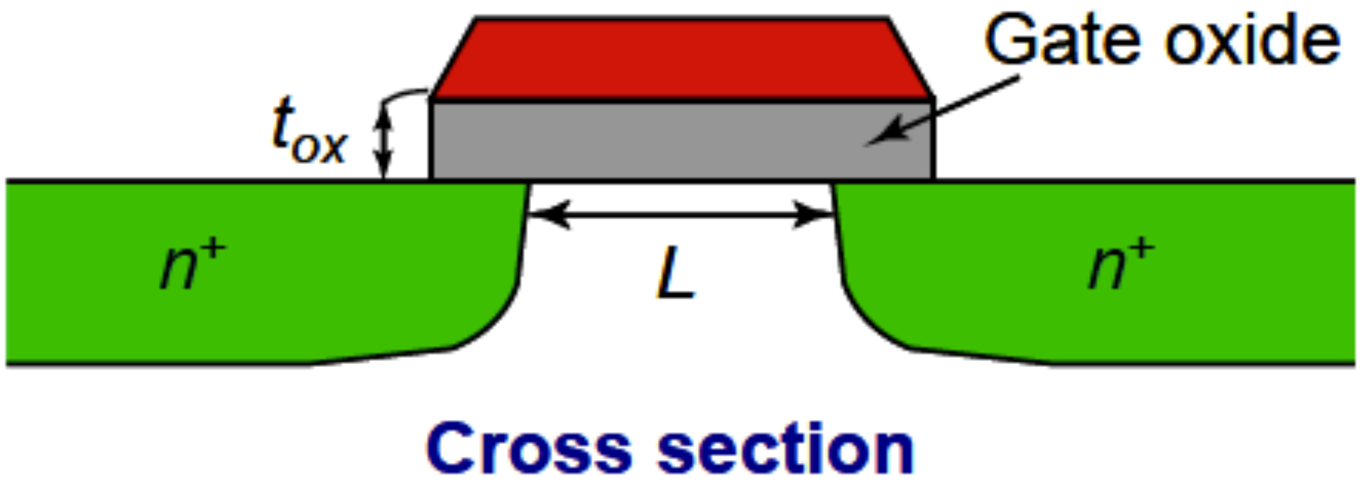
$$\frac{i_o}{i_i} = 1 \quad \Rightarrow \quad \omega_T \approx \frac{g_m}{C_\mu + C_\pi} = 2\pi f_T$$

MOS Capacitances

- MOS parasitic capacitances
  - Nonlinear

- Gate oxide capacitance (“parallel plate”)
- Gate overlap capacitance (fringe)
- Drain/Source-Bulk junction capacitance (PN junction)

MOS Capacitances

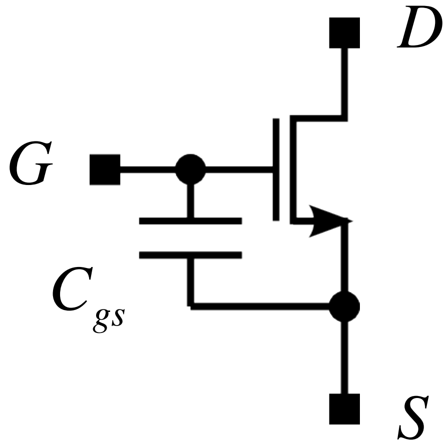


MOS Capacitances

- Gate Capacitance
  - Dependent on the thickness of the gate oxide and area of the gate
  - Nonlinear
    - Dependent on the gate and source/drain voltages
- Gate Overlap Capacitance
  - "parallel-plate" and fringing fields

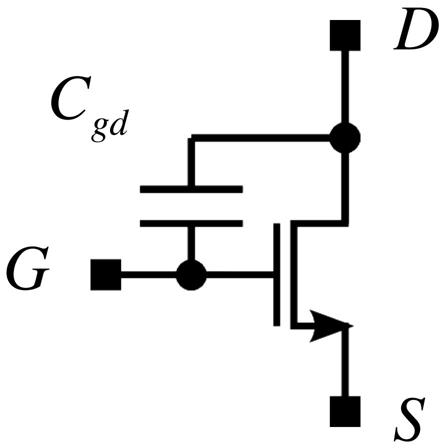
MOS Gate-Source Capacitance

- Components:
  - gate "parallel plate" capacitance
  - Gate-source overlap capacitance



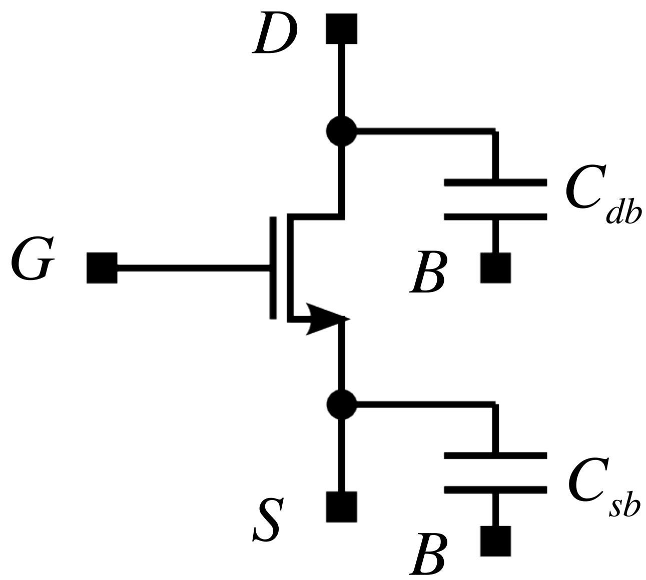
MOS Gate-Drain Capacitance

- Composed of the overlap capacitance between the gate and drain



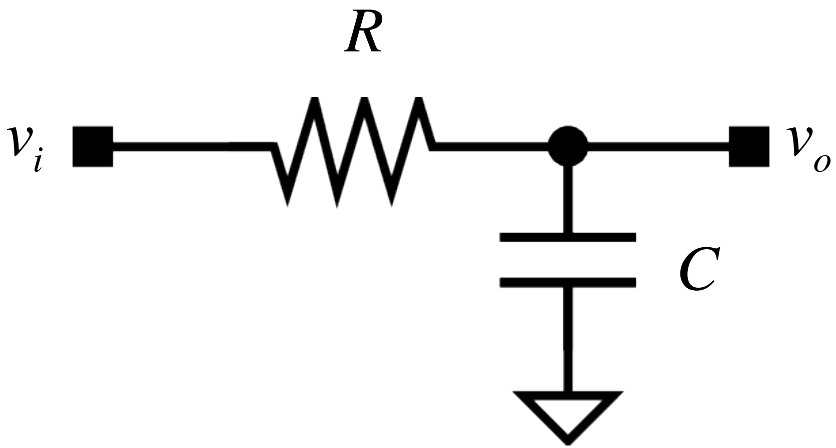
MOS Source/Drain Junction Capacitance

- Formed by the drain/source PN junction (to the substrate)
- Normally reversed biased



Capacitance Effects

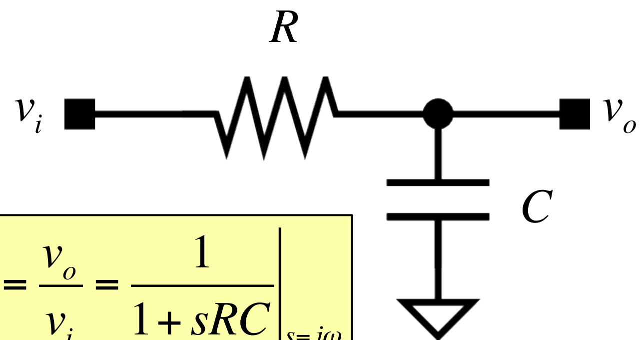
- A Simple RC Circuit



$$v_o = \frac{\frac{1}{sC}}{\frac{1}{sC} + R} v_i = \frac{1}{1 + sRC} v_i$$

$$\boxed{A_v(\omega) = \frac{v_o}{v_i} = \left.\frac{1}{1 + sRC}\right|_{s=j\omega}}$$

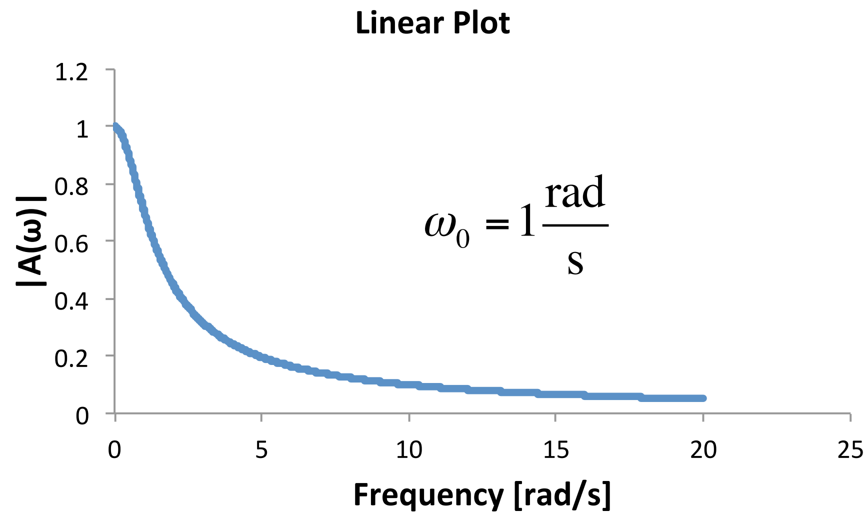
Frequency Response



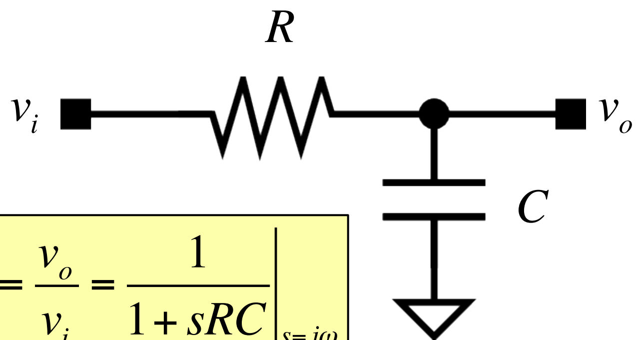
$$\boxed{A_v(\omega) = \frac{v_o}{v_i} = \left.\frac{1}{1+sRC}\right|_{s=j\omega}}$$

$$\left|A_v(\omega)\right| = \frac{1}{\sqrt{1+\omega^2R^2C^2}}$$

$$= \frac{1}{\sqrt{1+\left(\dfrac{\omega}{\omega_0}\right)^2}}$$



Frequency Response



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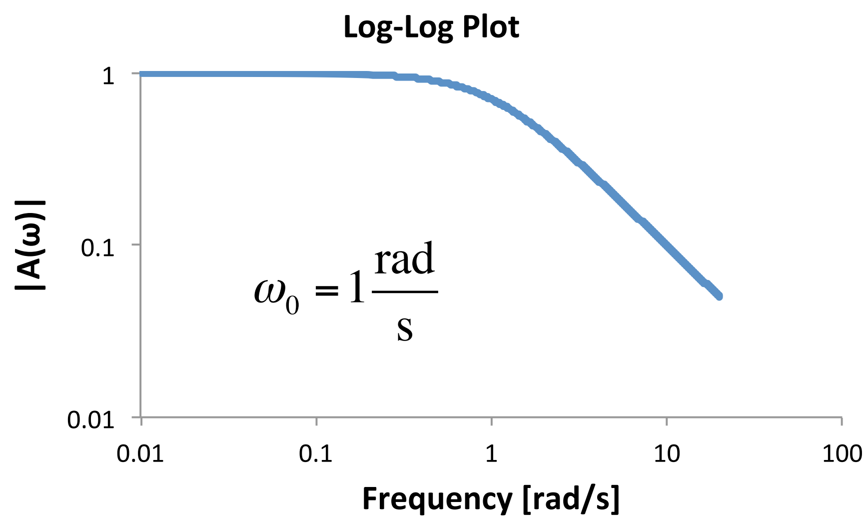
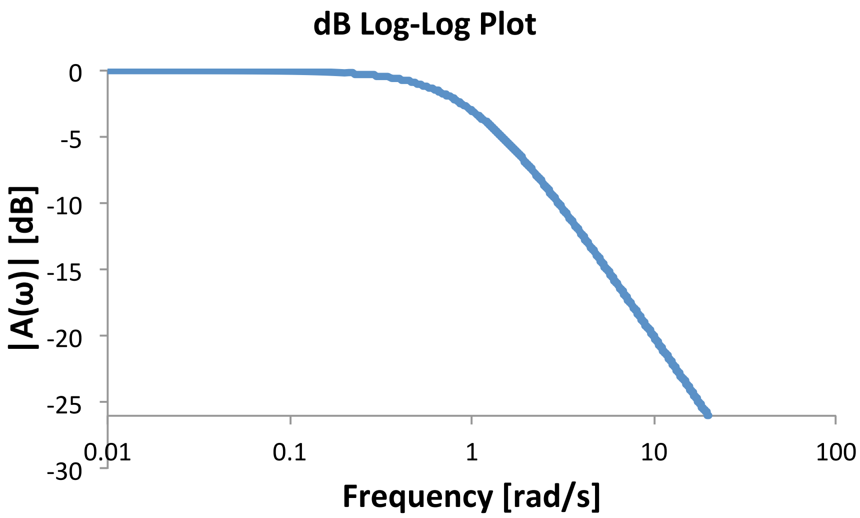


Decibels (dB)

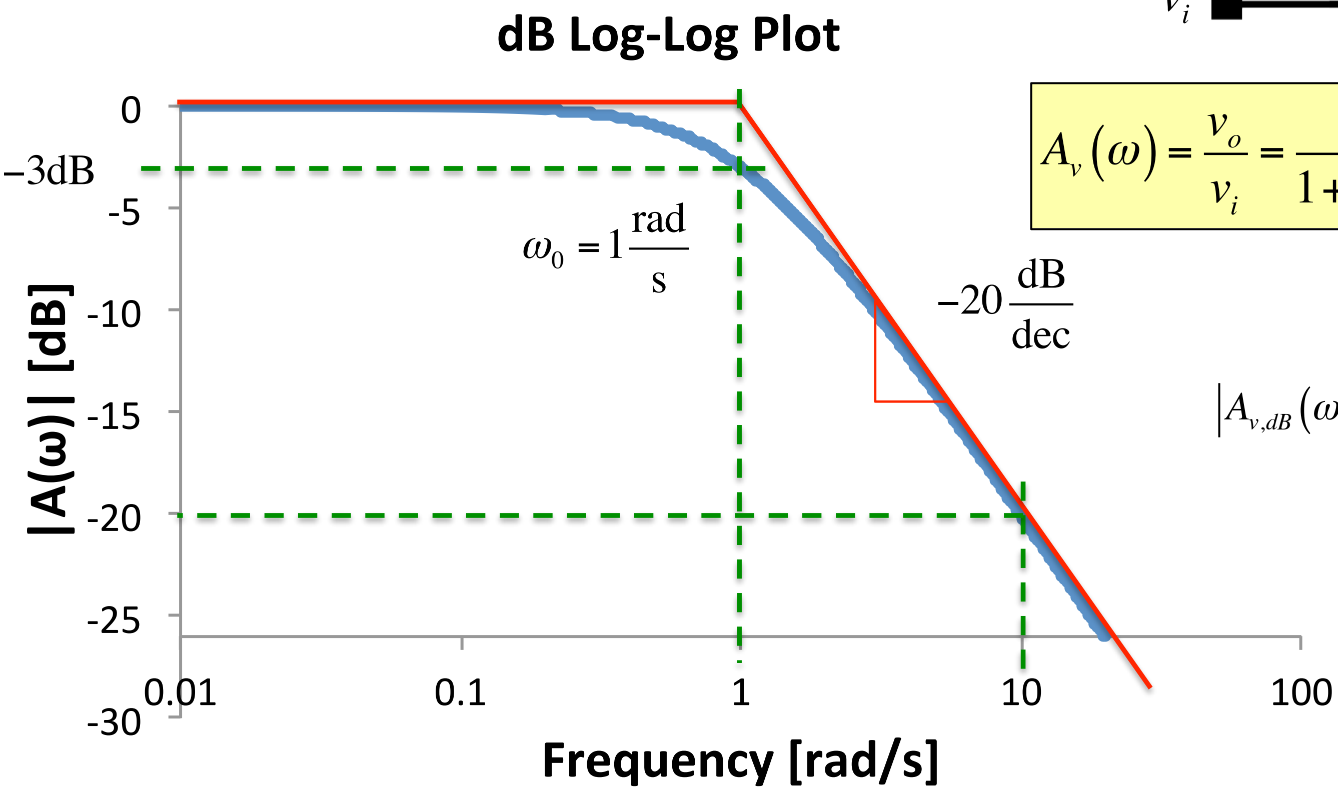
- Voltage gain:

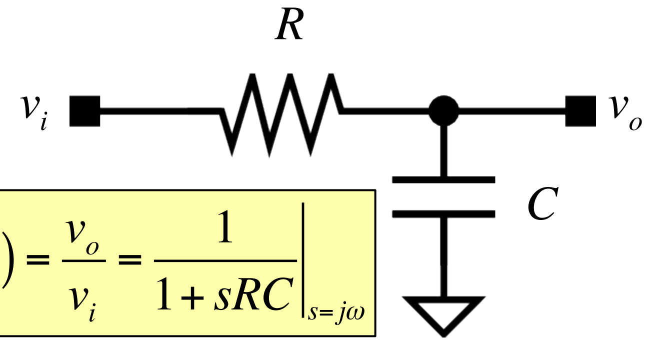
$$\left|A_{v,dB}(\omega)\right| = 20\log\left(\left|\frac{v_o}{v_i}\right|\right)$$

$$= 20\log\left(\frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^2}}\right)$$



Frequency Response: Magnitude

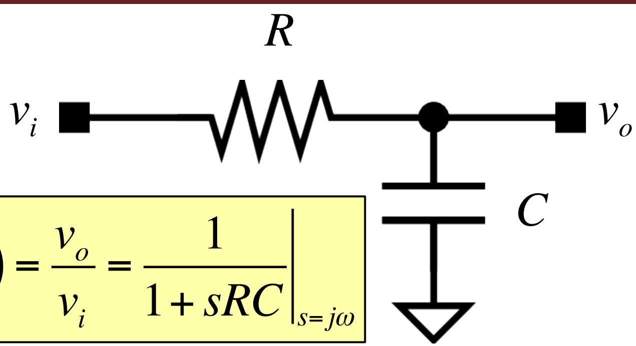




$$A_v(\omega) = \frac{v_o}{v_i} = \left.\frac{1}{1+sRC}\right|_{s=j\omega}$$

$$\left|A_{v,dB}(\omega)\right| = 20\log\left(\frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^2}}\right)$$

Phase Response

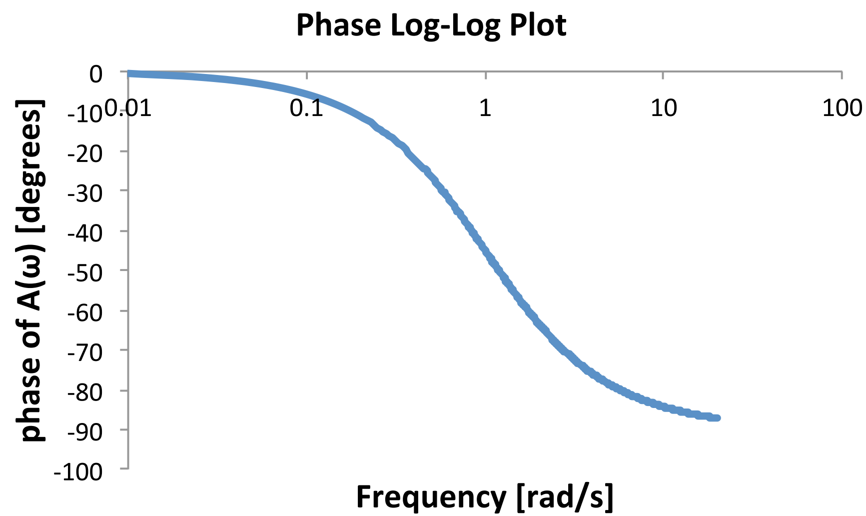


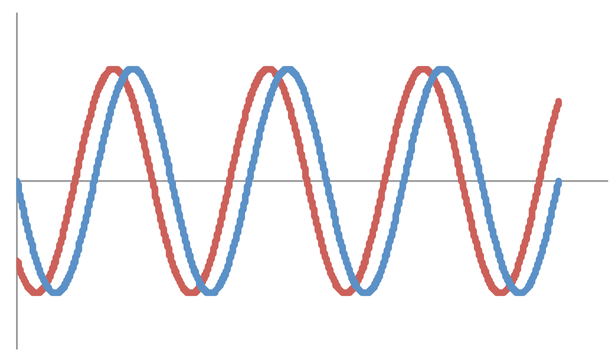
$$A_v(\omega) = \frac{v_o}{v_i} = \left.\frac{1}{1+sRC}\right|_{s=j\omega}$$

$$\angle A_v(\omega) = \tan^{-1}\left(\frac{\mathrm{Im}\left[A(\omega)\right]}{\mathrm{Re}\left[A(\omega)\right]}\right)$$

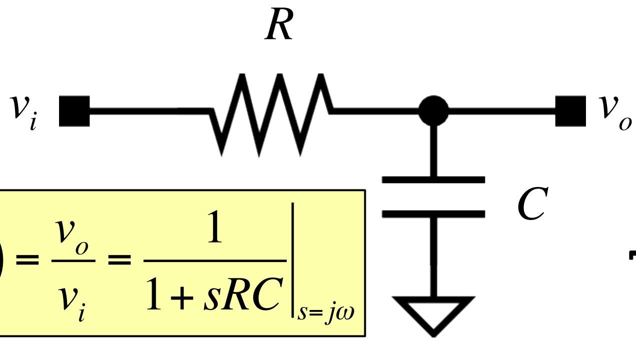
$$= \tan^{-1}\left(\frac{0}{1}\right) - \tan^{-1}\left(\frac{\omega RC}{1}\right)$$

$$= -\tan^{-1}(\omega RC) = -\tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$





Phase Response



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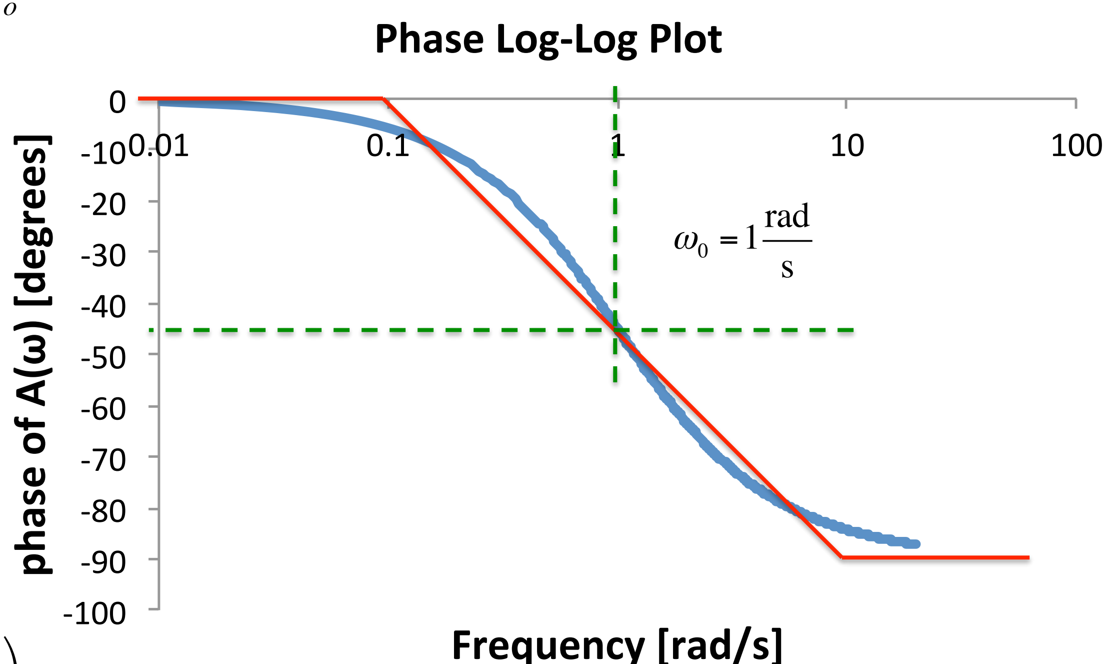
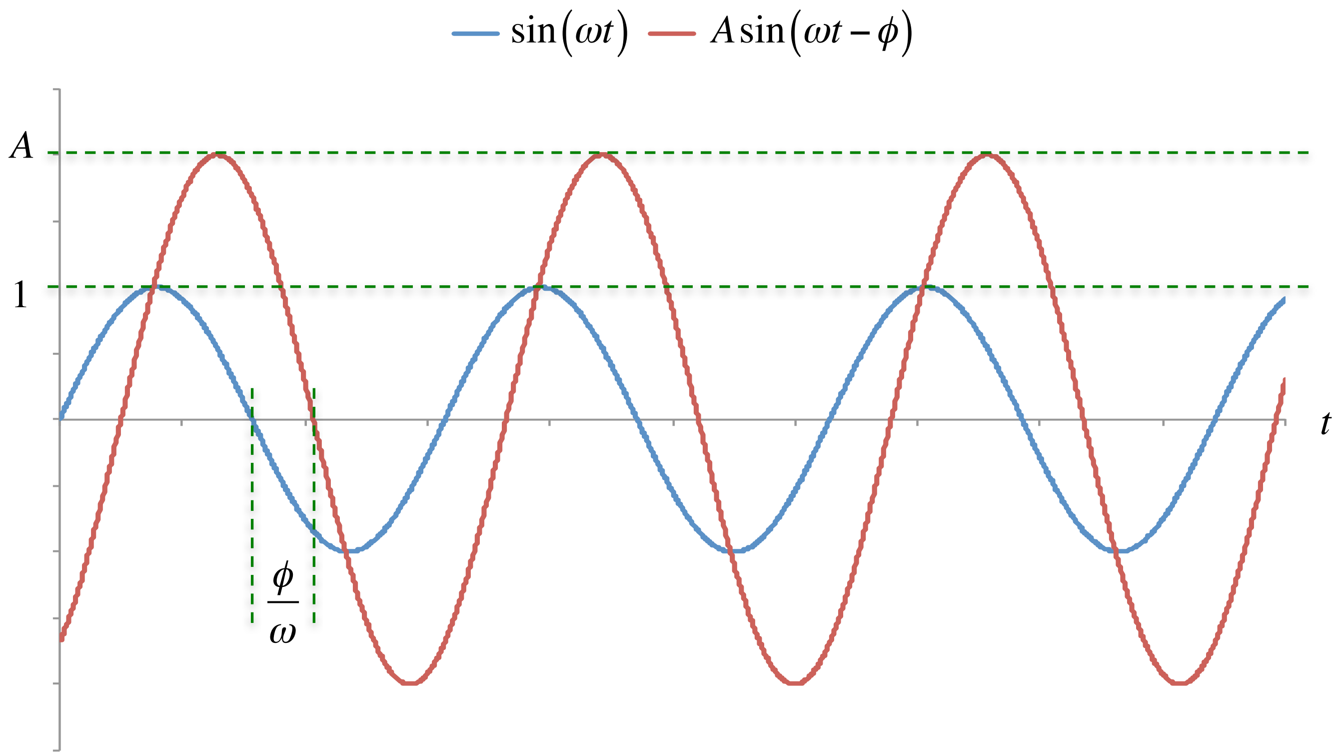
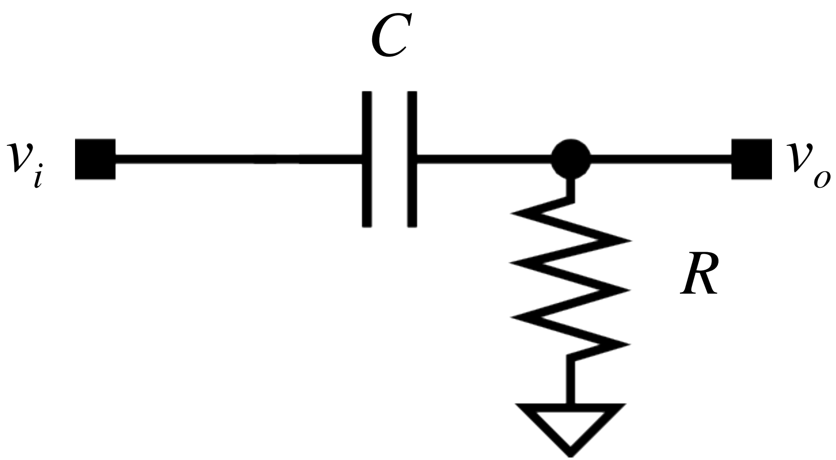


Magnitude and Phase



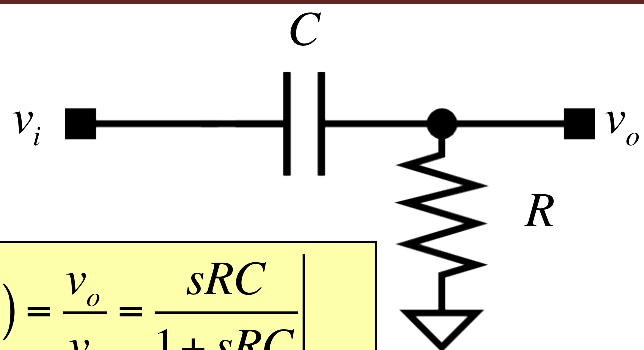
Another RC Example



$$v_o = \frac{R}{\frac{1}{sC} + R} v_i = \frac{sRC}{1 + sRC} v_i$$

$$\boxed{A_v(\omega) = \frac{v_o}{v_i} = \left.\frac{sRC}{1 + sRC}\right|_{s=j\omega}}$$

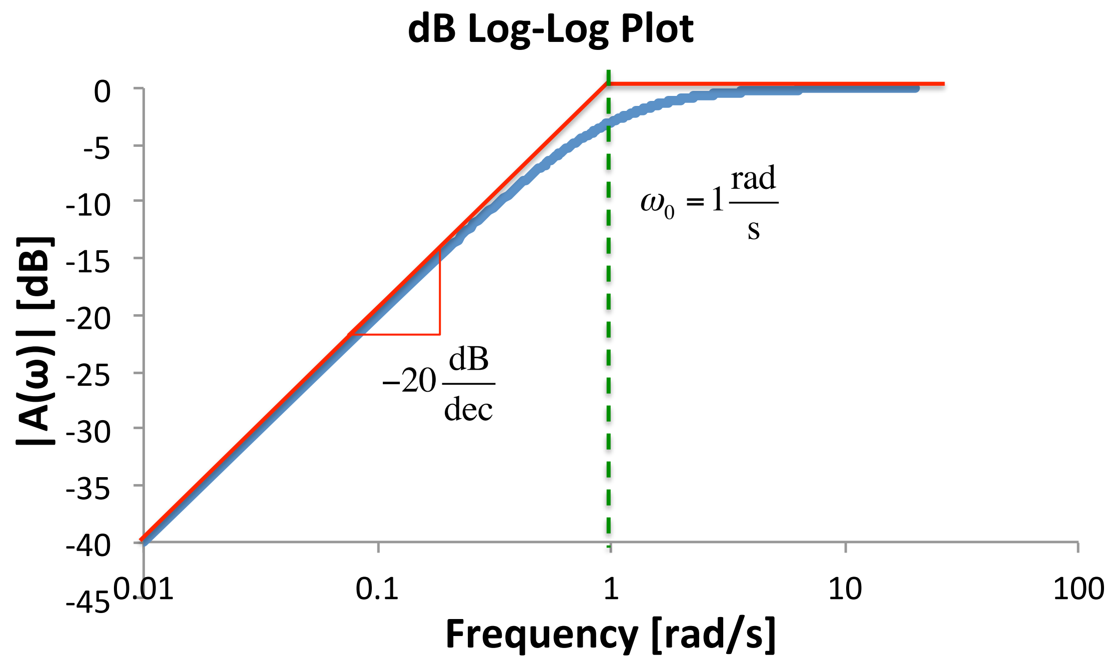
Frequency Response: Magnitude Response



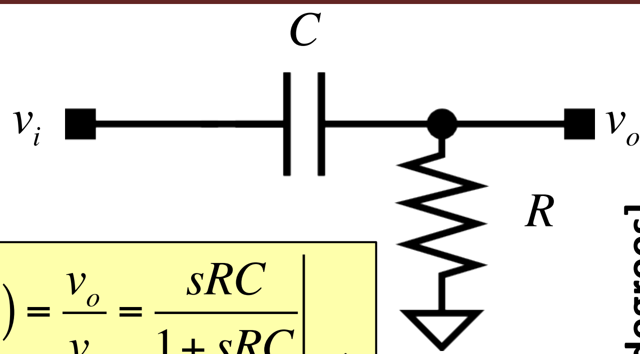
$$A_v(\omega) = \frac{v_o}{v_i} = \left.\frac{sRC}{1+sRC}\right|_{s=j\omega}$$

$$\left|A_v(\omega)\right| = \frac{\omega RC}{\sqrt{1+\omega^2R^2C^2}}$$

$$= \frac{\dfrac{\omega}{\omega_0}}{\sqrt{1+\left(\dfrac{\omega}{\omega_0}\right)^2}}$$



Phase Response

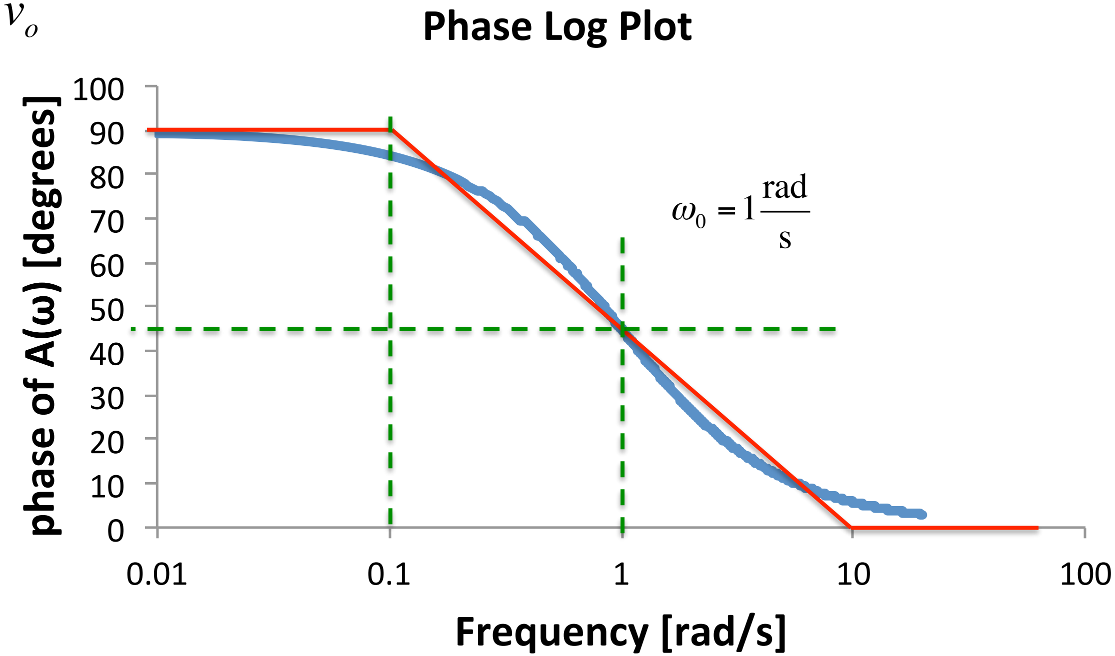


$$A_v(\omega) = \frac{v_o}{v_i} = \left.\frac{sRC}{1+sRC}\right|_{s=j\omega}$$

$$\angle A_v(\omega) = \tan^{-1}\left(\frac{\mathrm{Im}\left[A(\omega)\right]}{\mathrm{Re}\left[A(\omega)\right]}\right)$$

$$= \tan^{-1}\left(\frac{1}{0}\right) - \tan^{-1}\left(\frac{\omega RC}{1}\right)$$

$$= 90° - \tan^{-1}(\omega RC) = 90° - \tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

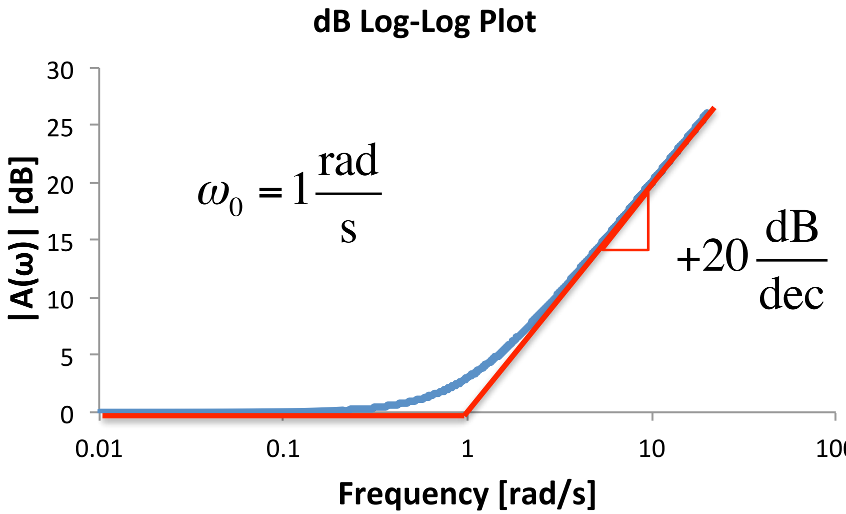


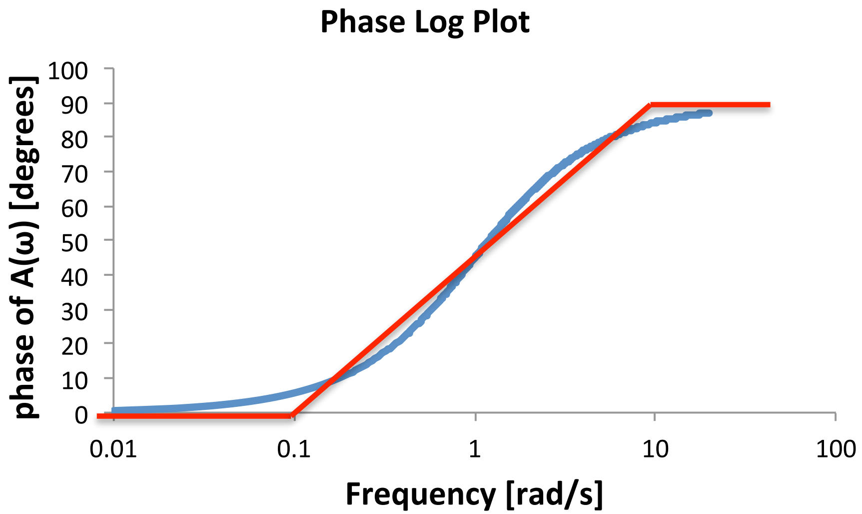
A General Zero Term

$$A(\omega) = 1 + j\frac{\omega}{\omega_0}$$

$$20\log\left[\left|A(\omega)\right|\right] = 20\log\left[\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^2}\right]$$

$$\angle A(\omega) = \tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$





Generalized Transfer Function

- Factor the numerator and denominator to get the poles and zeros of the system

$$A(s) = A_0 \frac{\left(1+\frac{s}{z_1}\right)\left(1+\frac{s}{z_2}\right)\cdots\left(1+\frac{s}{z_N}\right)}{\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)\cdots\left(1+\frac{s}{p_D}\right)}$$

$$A(\omega) = A_0 \frac{\left(1+j\frac{\omega}{\omega_{z_1}}\right)\left(1+j\frac{\omega}{\omega_{z_2}}\right)\cdots\left(1+j\frac{\omega}{\omega_{z_N}}\right)}{\left(1+j\frac{\omega}{\omega_{p_1}}\right)\left(1+j\frac{\omega}{\omega_{p_2}}\right)\cdots\left(1+j\frac{\omega}{\omega_{p_D}}\right)}$$

Magnitude Response Using the dB Scale

$$\left|A_{dB}(\omega)\right| = 20\log\left[\left|A_0 \frac{\left(1+j\frac{\omega}{\omega_{z_1}}\right)\left(1+j\frac{\omega}{\omega_{z_2}}\right)\cdots\left(1+j\frac{\omega}{\omega_{z_N}}\right)}{\left(1+j\frac{\omega}{\omega_{p_1}}\right)\left(1+j\frac{\omega}{\omega_{p_2}}\right)\cdots\left(1+j\frac{\omega}{\omega_{p_D}}\right)}\right|\right]$$

$$= 20\log\left[\left|A_0\right|\right] + \sum_N 20\log\left[\left|1+j\frac{\omega}{\omega_{z_i}}\right|\right] + \sum_D 20\log\left[\left|\frac{1}{1+j\frac{\omega}{\omega_{p_i}}}\right|\right]$$

Phase Response

$$A(\omega) = A_0 \frac{\left(1 + j\frac{\omega}{\omega_{z_1}}\right)\left(1 + j\frac{\omega}{\omega_{z_2}}\right)\cdots\left(1 + j\frac{\omega}{\omega_{z_N}}\right)}{\left(1 + j\frac{\omega}{\omega_{p_1}}\right)\left(1 + j\frac{\omega}{\omega_{p_2}}\right)\cdots\left(1 + j\frac{\omega}{\omega_{p_D}}\right)}$$

$$\angle A(\omega) = \sum_{N} \angle\left(1 + j\frac{\omega}{\omega_{z_i}}\right) + \sum_{D} \angle\left(\frac{1}{1 + j\frac{\omega}{\omega_{p_i}}}\right)$$

Next Meeting

- Frequency Response of Amplifiers