EEE 51: Second Semester 2017 - 2018

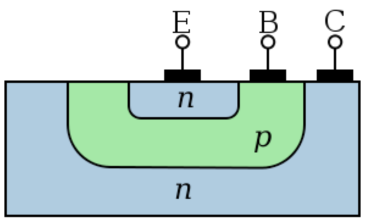
Lecture 2

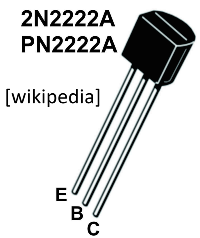
Transistor Models

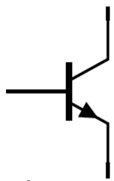
Today

- Transistor Models
  - Large Signal
  - Small Signal

From Transistors to Transistor Circuits

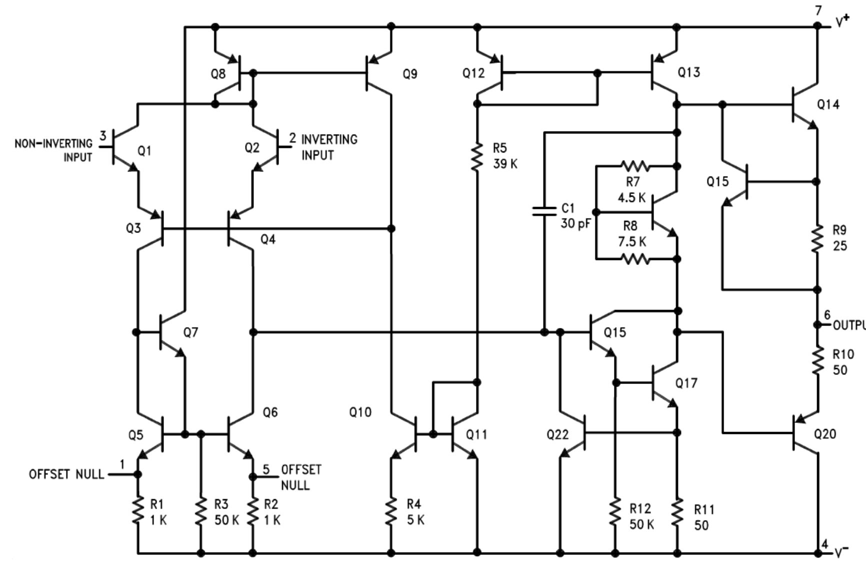






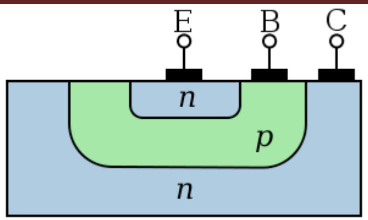
transistors

?



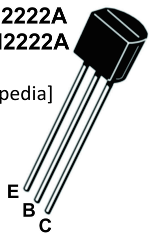
transistor circuits

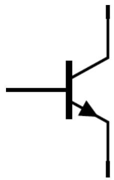
Transistor Models



2N2222A
PN2222A

[wikipedia]

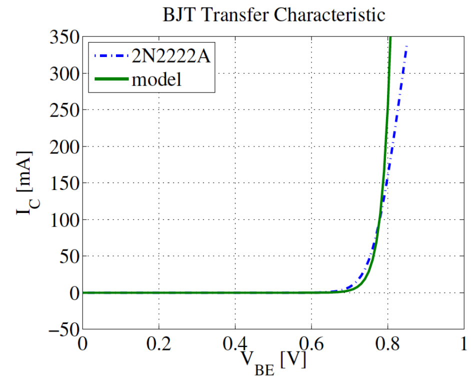


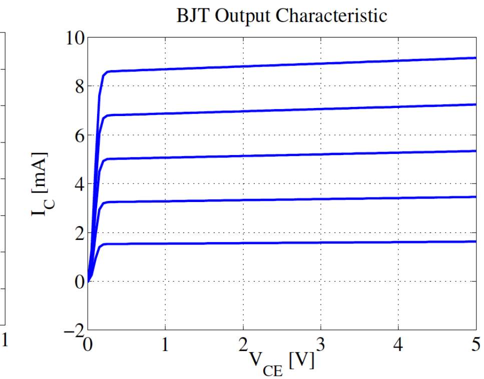


transistor operation

device characterization

EEE 41





large signal terminal behavior (I-V curves)

device models

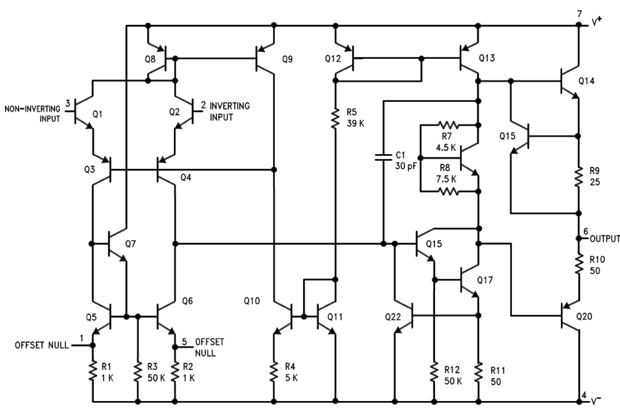
EEE 41

I-V models (**large signal** approximation)

$$I_C = I_S\left(e^{\frac{V_{BE}}{V_T}} - 1\right)\left(1 + \frac{V_{CE}}{V_A}\right) \qquad I_B = \frac{1}{\beta} I_C$$

EEE 51

?

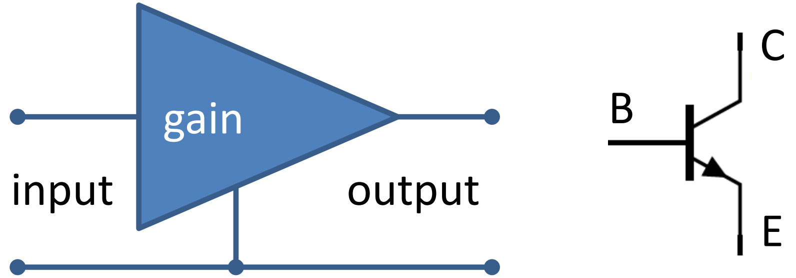


transistor circuits

Large Signal Models

- V and I over **different transistor operating** regions
  - BJT: Forward-active, saturation, cut-off
  - MOSFET: Saturation, linear (ohmic), subthreshold ("cut-off")

- Large signal model <u>reference</u> → largest "gain" topology
  - Transfer Characteristic
  - Output Characteristic
  - Input Characteristic
  - "Unilateral"
  - "Low" frequency

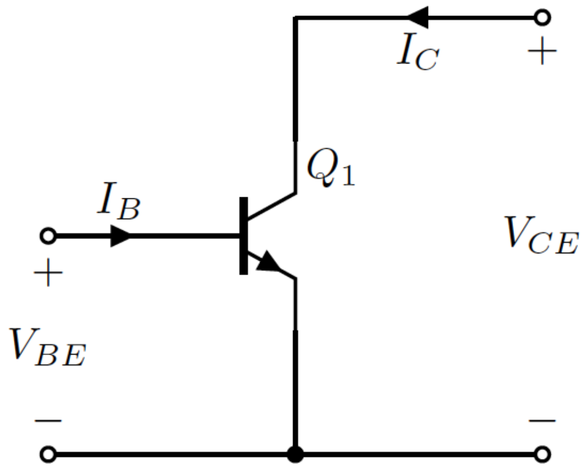


How many ways can you configure a 3-terminal device?

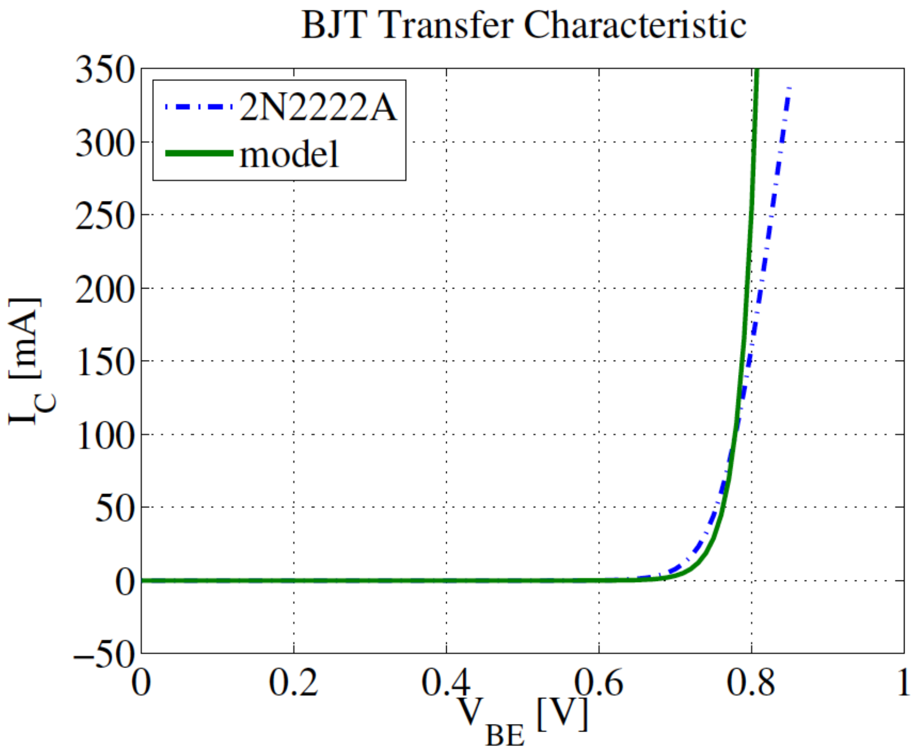
BJT Transfer Characteristics (I_C vs. V_{BE})

$$I_C = I_S\left(e^{\frac{V_{BE}}{V_T}} - 1\right)\left(1 + \frac{V_{CE}}{V_A}\right)$$

$$\approx I_S \cdot e^{\frac{V_{BE}}{V_T}}$$

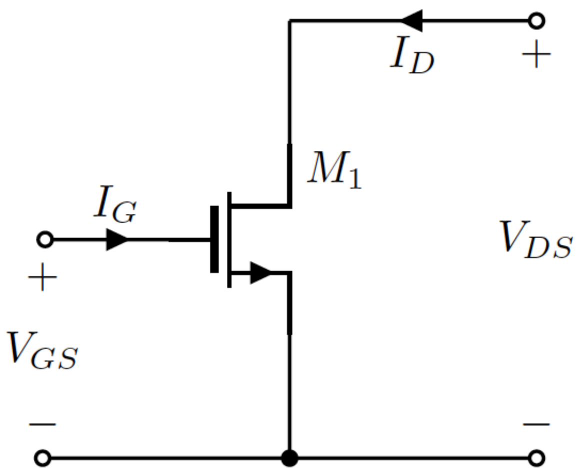


Common-Emitter topology

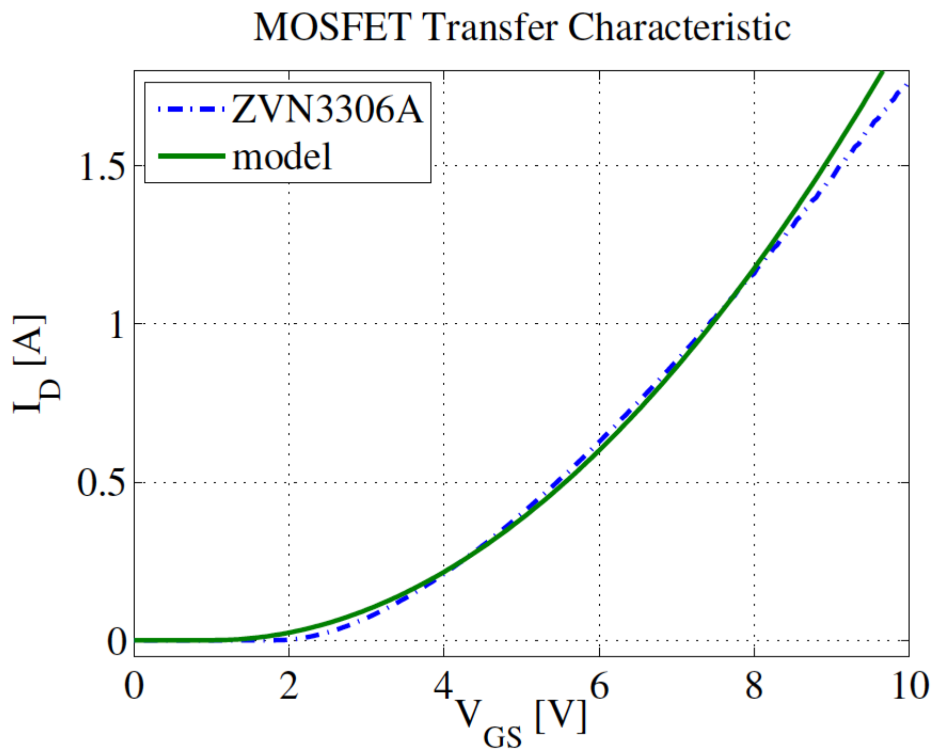


MOSFET Transfer Characteristics (I_D vs. V_{GS})

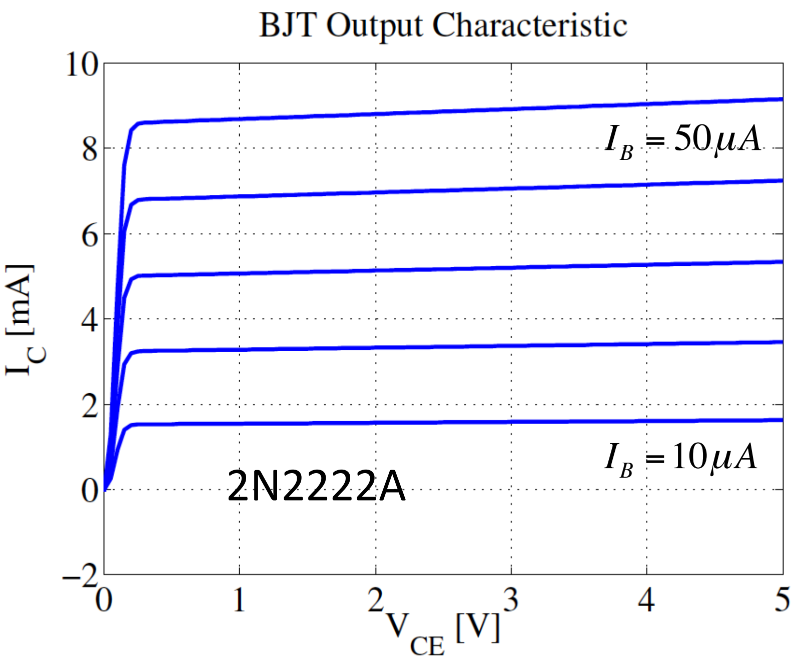
$$I_D = k \cdot (V_{GS} - V_{TH})^2 (1 + \lambda \cdot V_{DS})$$
$$\approx k \cdot (V_{GS} - V_{TH})^2$$

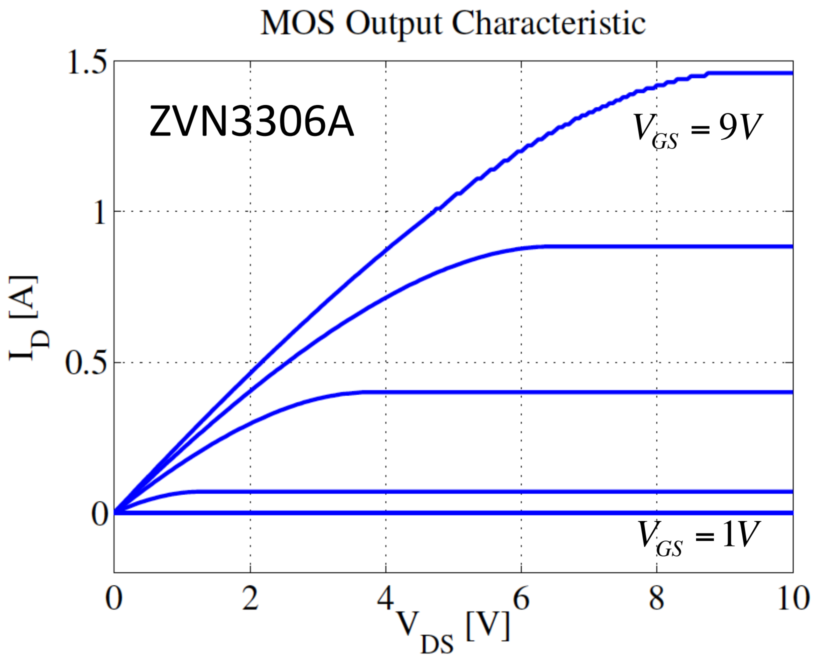


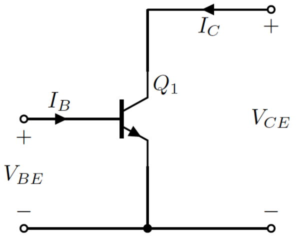
Common-Source topology



Output Characteristics

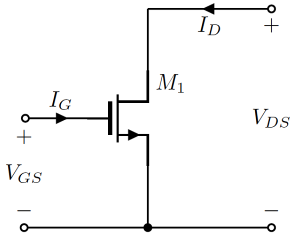






$$I_C = I_S\left(e^{\frac{V_{BE}}{V_T}} - 1\right)\left(1 + \frac{V_{CE}}{V_A}\right)$$

(forward active)



$$I_D = k \cdot \left(V_{GS} - V_{TH}\right)^2 \left(1 + \lambda \cdot V_{DS}\right)$$

(saturation)

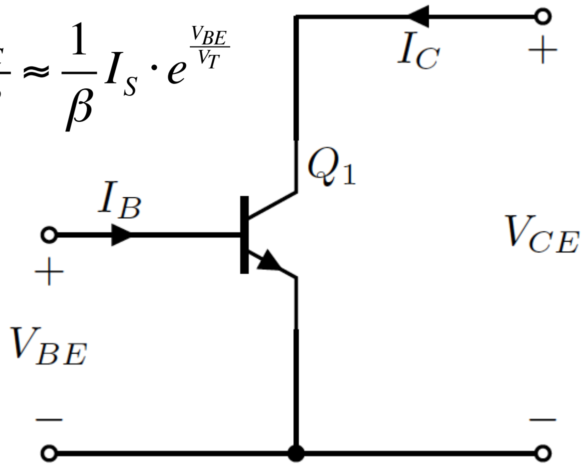
$$I_D = 2k \cdot \left[\left(V_{GS} - V_{TH}\right)V_{DS} - \tfrac{1}{2}V_{DS}^2\right]$$

(linear/ohmic)

Input Characteristics

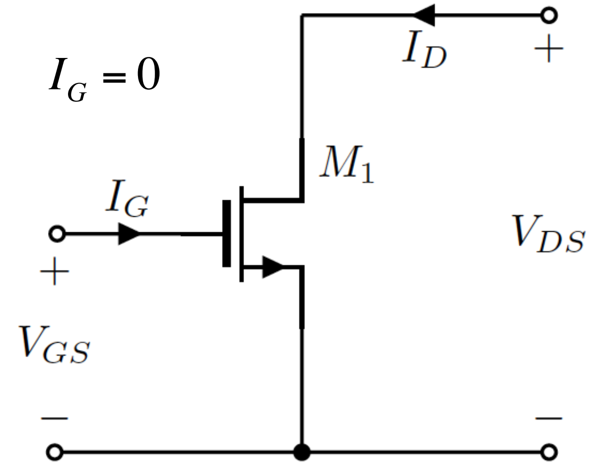
BJT

$$I_B = \frac{I_C}{\beta} \approx \frac{1}{\beta} I_S \cdot e^{\frac{V_{BE}}{V_T}}$$



MOSFET

$$I_G = 0$$



Two Ways to Bridge the Gap

The complex, non-intuitive, non-extendable way…

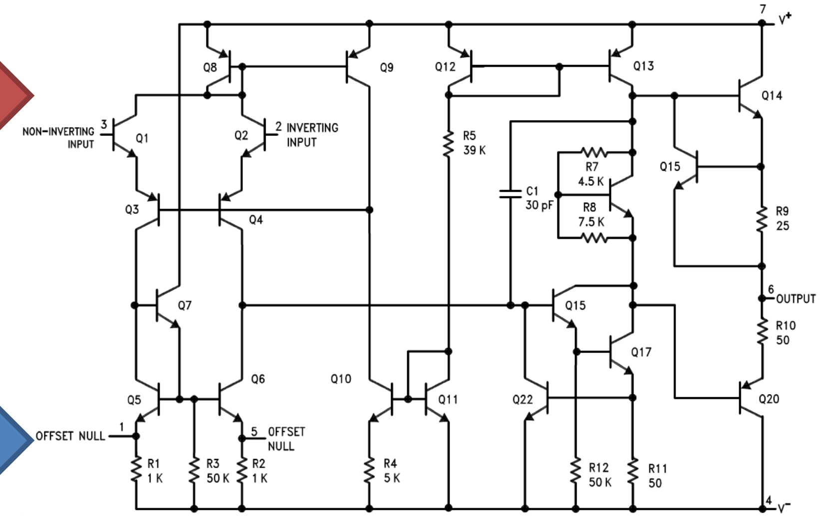
I-V models
(**<u>large signal</u>** transfer,
input and output
characteristics)

$$I_C = I_S\left(e^{\frac{V_{BE}}{V_T}} - 1\right)\left(1 + \frac{V_{CE}}{V_A}\right)$$

$$I_B = \frac{1}{\beta} I_C$$

direct application
of KCL and KVL

linearization +
two-port network
reduction



transistor circuits

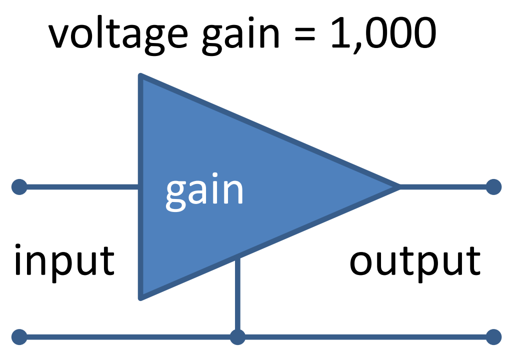
EEE 51 way (aka the fun way ☺)

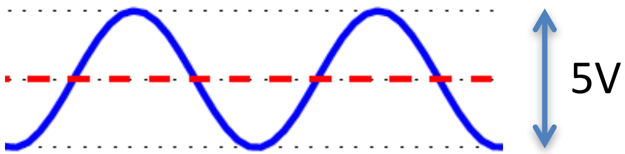
- Allows us to use our EEE 31, 33 skills
- Allows us to break up large circuits into smaller ones
- Gives us more intuition in terms of circuit operation

Linearization (1)

- Consider an amplifier with large gain:

input voltage levels?

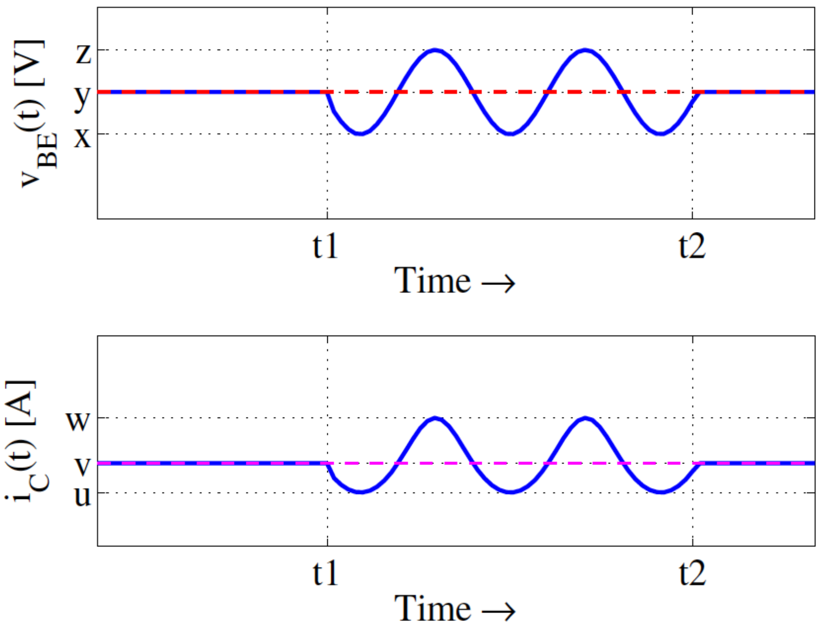


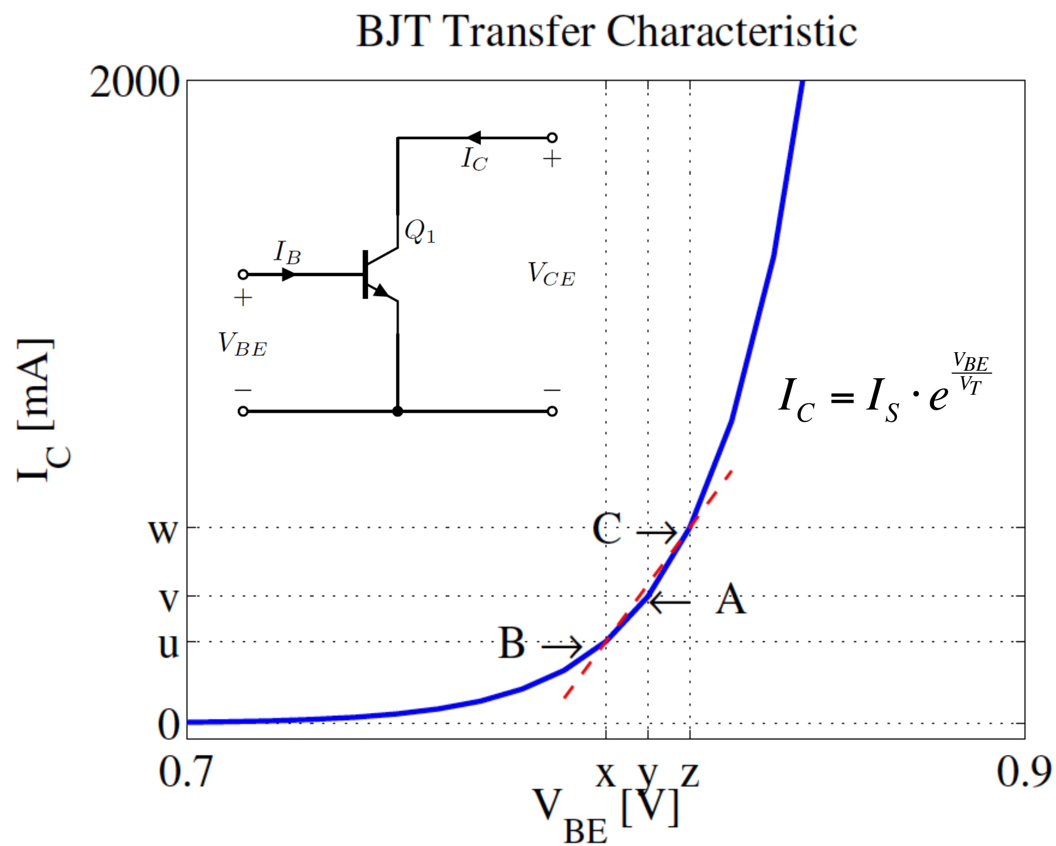


In most amplifier applications, we are interested in the transistor behavior when we apply "small" signals

Linearization (2)

- Consider a BJT in the forward active region:





A is known as the quiescent DC operating point

Linearization (3)

- So what if the signals are "small"?
  - Recall: Taylor Series expansion

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \ldots$$

Example:

$$e^x = e^0 + \frac{e^0}{1!}x + \frac{e^0}{2!}x^2 + \frac{e^0}{3!}x^3 + \ldots$$

$$= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$$

Linearizing the BJT Transfer Characteristic (1)

- Expanding the transfer characteristic

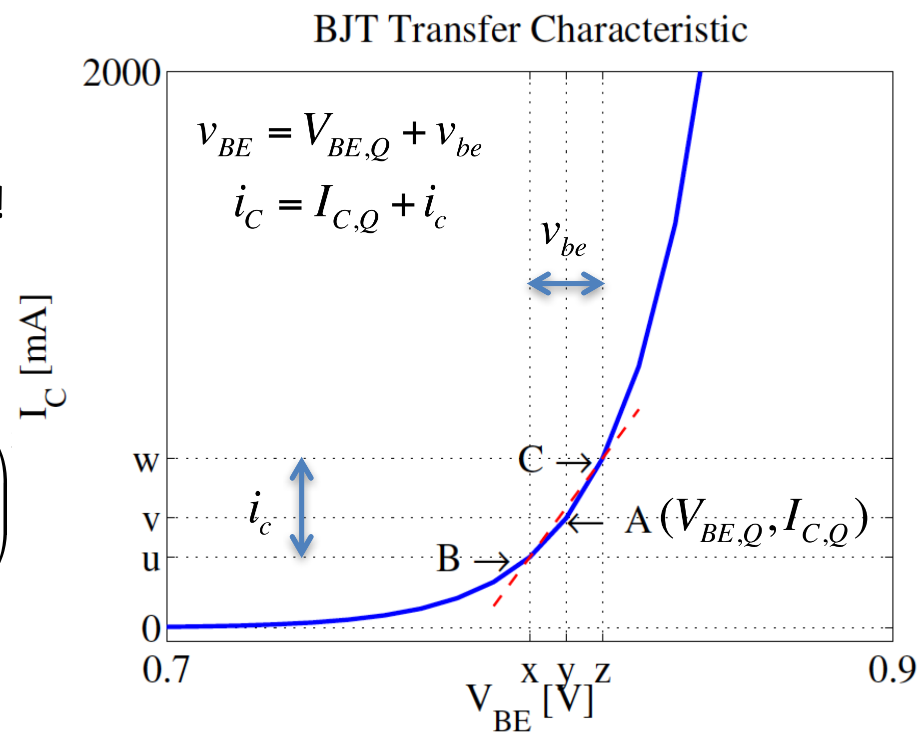
$$i_C = I_{C,Q} + i_c = I_S \cdot e^{\frac{V_{BE,Q}+v_{be}}{V_T}}$$

$$I_{C,Q} + i_c = I_S \cdot e^{\frac{V_{BE,Q}}{V_T}} \cdot e^{\frac{v_{be}}{V_T}} = I_{C,Q} \cdot e^{\frac{v_{be}}{V_T}}$$ ← nonlinear!

$$= I_{C,Q} \cdot \left(1 + \frac{v_{be}}{V_T} + \frac{v_{be}^2}{2V_T^2} + \frac{v_{be}^3}{6V_T^3} + \ldots \right)$$

$$I_{C,Q} + i_c = I_{C,Q} + I_{C,Q}\frac{v_{be}}{V_T} + \frac{I_{C,Q}}{2}\left(\frac{v_{be}}{V_T}\right)^2 + \frac{I_{C,Q}}{6}\left(\frac{v_{be}}{V_T}\right)$$

$$i_c = I_{C,Q}\frac{v_{be}}{V_T} + \frac{I_{C,Q}}{2}\left(\frac{v_{be}}{V_T}\right)^2 + \frac{I_{C,Q}}{6}\left(\frac{v_{be}}{V_T}\right)^3 + \ldots$$



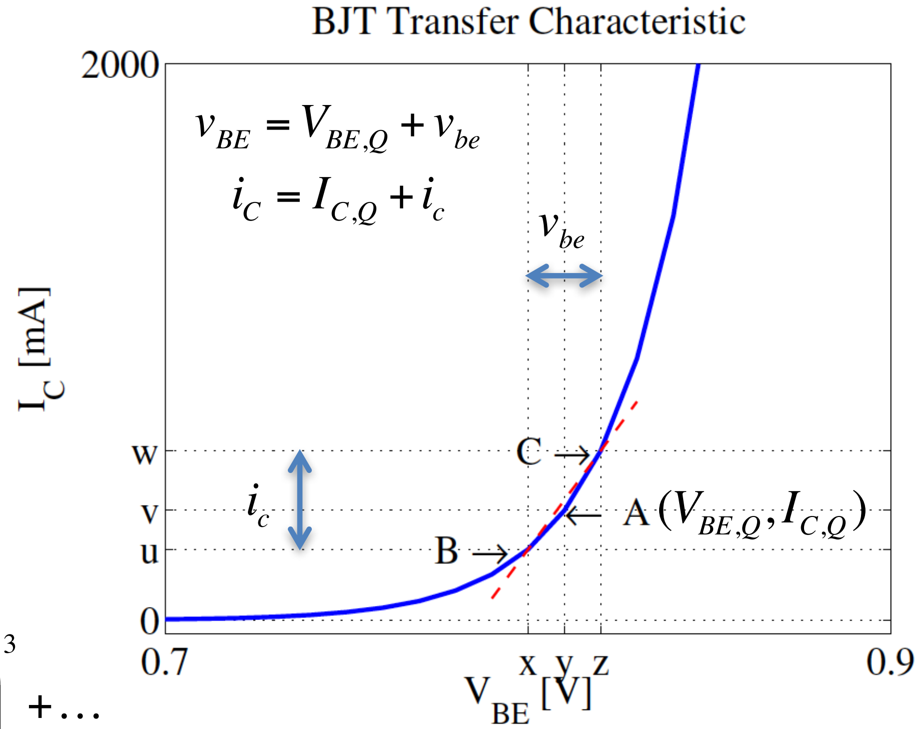
Linearizing the BJT Transfer Characteristic (2)

- If v_{be} is "small"

$$i_c = I_{C,Q}\frac{v_{be}}{V_T} + \frac{I_{C,Q}}{2}\left(\frac{v_{be}}{V_T}\right)^2 + \frac{I_{C,Q}}{6}\left(\frac{v_{be}}{V_T}\right)^3 + \ldots$$

$$\approx I_{C,Q}\frac{v_{be}}{V_T} \quad \leftarrow \text{linear!}$$

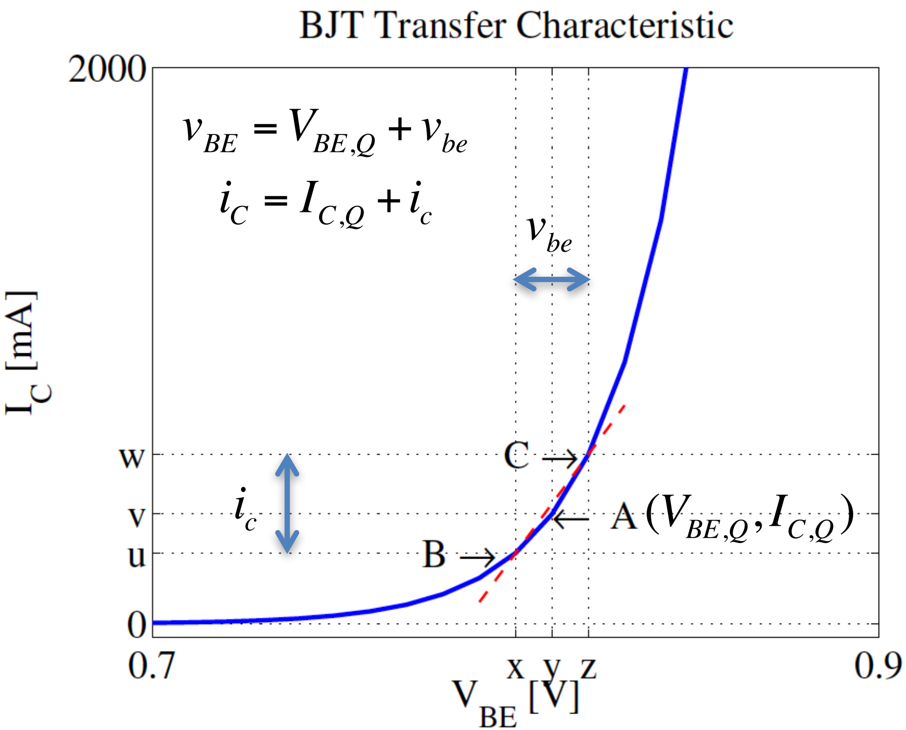
"small" means → $\dfrac{v_{be}}{V_T} << 1$



Approximation error: $\dfrac{I_{C,Q}}{2}\left(\dfrac{v_{be}}{V_T}\right)^2 + \dfrac{I_{C,Q}}{6}\left(\dfrac{v_{be}}{V_T}\right)^3 + \ldots$

Linearizing the BJT Transfer Characteristic (3)

- Another way to think about linearization



If we make $v_{be} \rightarrow 0$

$$m = \lim_{v_{be} \to 0} \frac{i_C\left(V_{BE,Q} + v_{be}\right) - i_C\left(V_{BE,Q}\right)}{V_{BE,Q} + v_{be} - V_{BE,Q}}$$

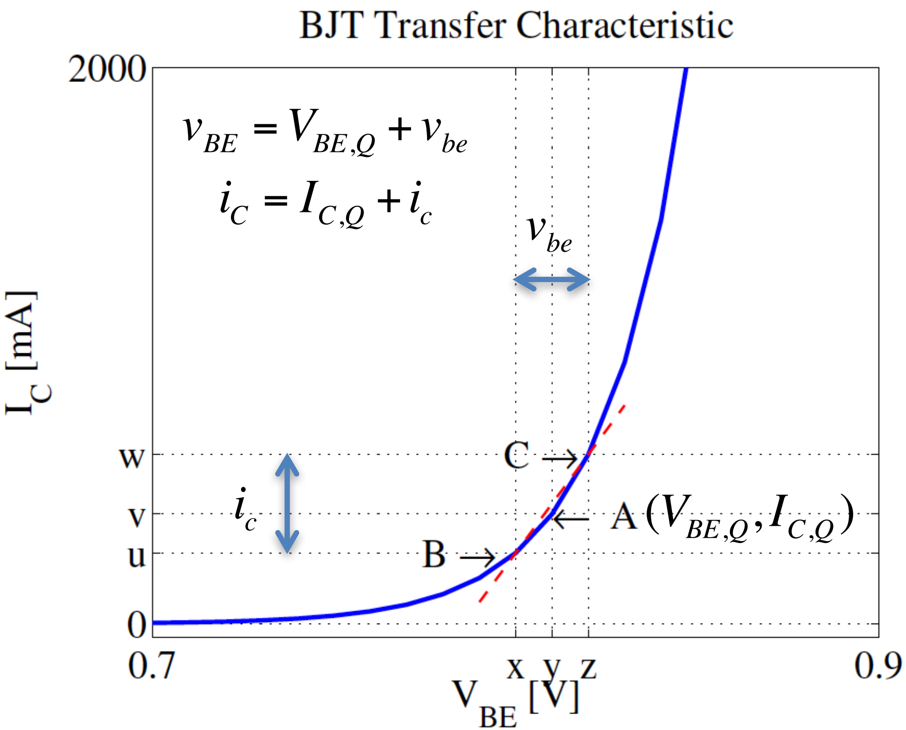
$$= \left.\frac{\partial I_C}{\partial V_{BE}}\right|_{V_{BE}=V_{BE,Q}}$$

We can make the approximation:

$$i_c = m \cdot v_{be} = \left.\frac{\partial I_C}{\partial V_{BE}}\right|_{V_{BE}=V_{BE,Q}} \cdot v_{be}$$

Linearizing the BJT Transfer Characteristic (4)

- Transconductance



Define **transconductance** as

$$g_m = \left. \frac{\partial I_C}{\partial V_{BE}} \right|_{V_{BE} = V_{BE,Q}}$$

For small signals

$$i_c = g_m \cdot v_{be}$$

Linearizing the BJT Transfer Characteristic (5)

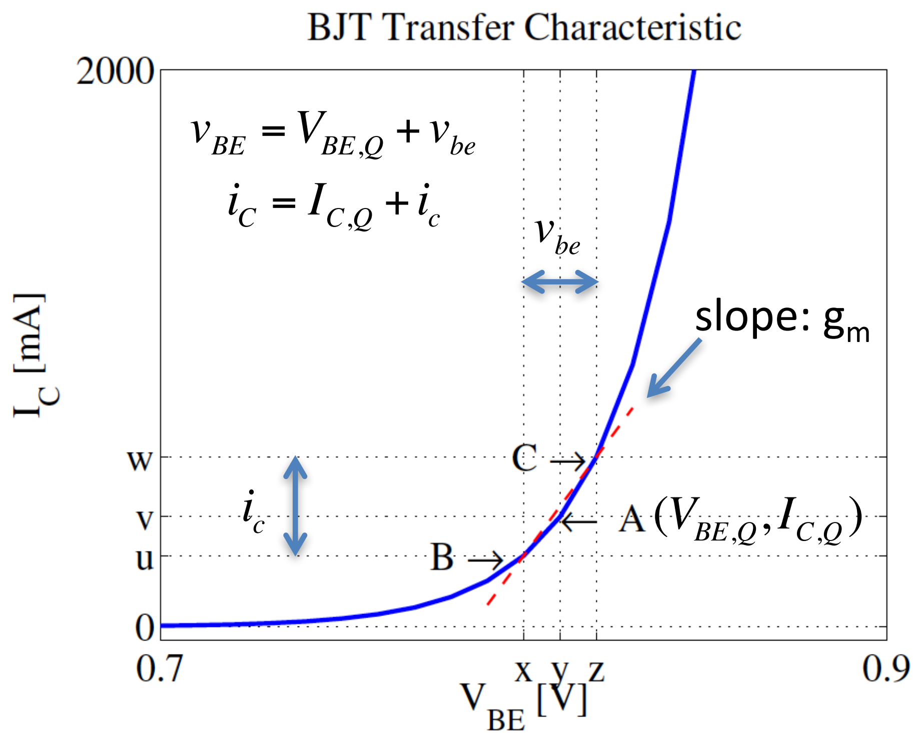
- BJT transconductance

$$g_m = \left.\frac{\partial I_C}{\partial V_{BE}}\right|_{V_{BE}=V_{BE,Q}}$$

$$= \left.\frac{\partial}{\partial V_{BE}}\left(I_S \cdot e^{\frac{V_{BE}}{V_T}}\right)\right|_{V_{BE}=V_{BE,Q}}$$

$$= \frac{I_S \cdot e^{\frac{V_{BE,Q}}{V_T}}}{V_T} = \frac{I_{C,Q}}{V_T}$$

Again, we get: $i_c = g_m \cdot v_{be} = \frac{I_{C,Q}}{V_T} \cdot v_{be}$



Linearizing the BJT Transfer Characteristic

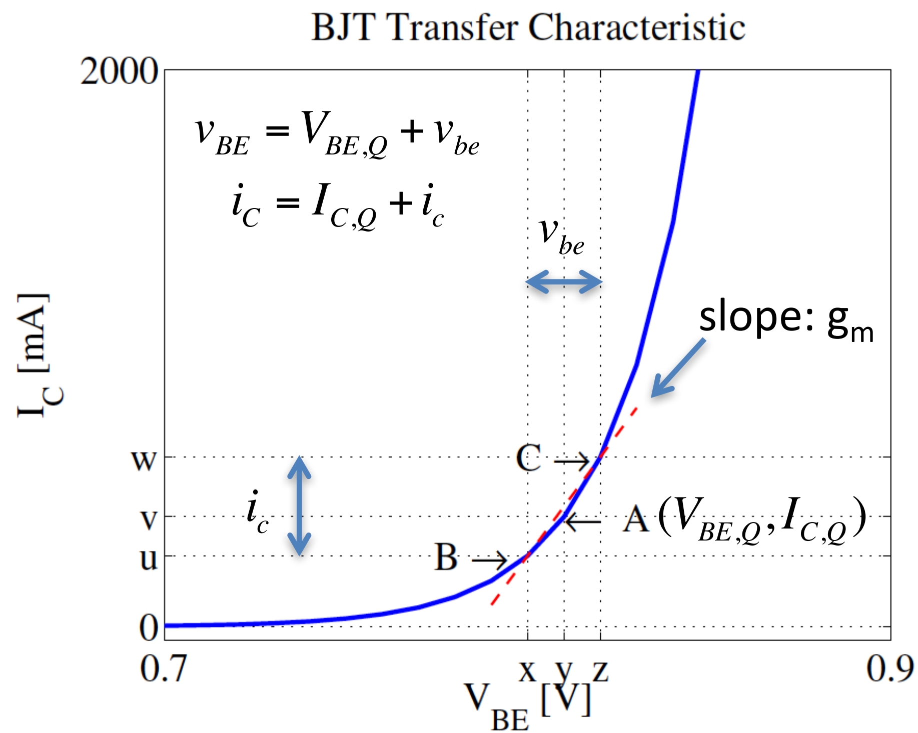
- BJT transconductance

$$g_m = \left.\frac{\partial I_C}{\partial V_{BE}}\right|_{V_{BE}=V_{BE,Q}}$$

$$= \left.\frac{\partial}{\partial V_{BE}}\left(I_S \cdot e^{\frac{V_{BE}}{V_T}}\right)\right|_{V_{BE}=V_{BE,Q}}$$

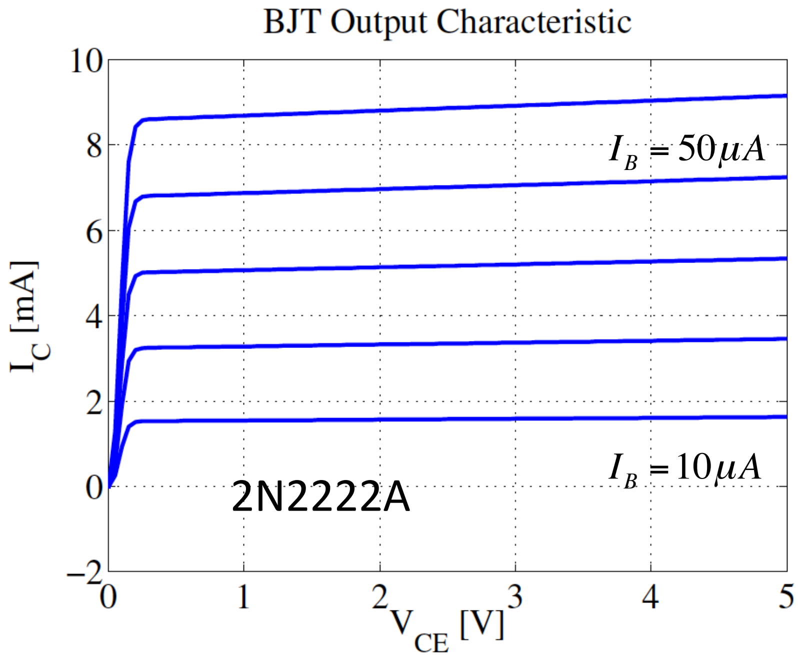
$$= \frac{I_S \cdot e^{\frac{V_{BE,Q}}{V_T}}}{V_T} = \frac{I_{C,Q}}{V_T}$$

Again, we get: $i_c = g_m \cdot v_{be} = \frac{I_{C,Q}}{V_T} \cdot v_{be}$



Does g_m give us the complete picture?

- What else changes i_c?



$$I_C = I_S\left(e^{\frac{V_{BE}}{V_T}} - 1\right)\left(1 + \frac{V_{CE}}{V_A}\right)$$

BJT Transistor Output Resistance

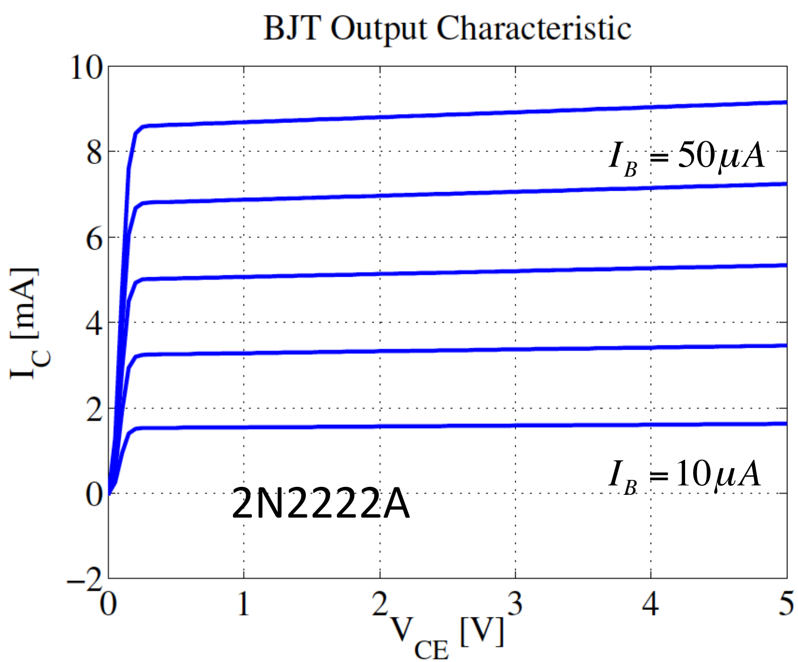
- What happens when there are small changes in V_{CE}?

$$i_c = \left.\frac{\partial I_C}{\partial V_{CE}}\right|_{V_{CE}=V_{CE,Q}} \cdot v_{ce}$$

$$= \left.\frac{\partial}{\partial V_{CE}}\left(I_S\left(e^{\frac{V_{BE}}{V_T}}-1\right)\left(1+\frac{V_{CE}}{V_A}\right)\right)\right|_{V_{CE}=V_{CE,Q}} \cdot v_{ce}$$

$$= \frac{I_S\left(e^{\frac{V_{BE,Q}}{V_T}}-1\right)}{V_A} \cdot v_{ce} = \frac{I_{C,Q}}{V_A} \cdot v_{ce} = g_o \cdot v_{ce} = \frac{v_{ce}}{r_o}$$

Output resistance: $r_o = \frac{V_A}{I_{C,Q}}$



Completing the Picture: Transistor Input Resistance

BJT

- Small signal base current due to v_{be}

$$g_\pi = \left.\frac{\partial I_B}{\partial V_{BE}}\right|_{V_{BE}=V_{BE,Q}}$$

$$= \left.\frac{\partial}{\partial V_{BE}}\left(\frac{I_C}{\beta}\right)\right|_{V_{BE}=V_{BE,Q}} = \frac{1}{\beta}\cdot\left.\frac{\partial I_C}{\partial V_{BE}}\right|_{V_{BE}=V_{BE,Q}}$$

$$= \frac{g_m}{\beta}$$

$$r_\pi = \frac{1}{g_\pi} = \frac{\beta}{g_m} = \frac{\beta \cdot V_T}{I_{C,Q}}$$

Linearization Result: The **Small Signal Model**

BJT

- Total i_c:

$$i_c = \left(\left. \frac{\partial I_C}{\partial V_{BE}} \right|_{V_{BE}=V_{BE,Q}} \cdot v_{be} \right) + \left(\left. \frac{\partial I_C}{\partial V_{CE}} \right|_{V_{CE}=V_{CE,Q}} \cdot v_{ce} \right)$$

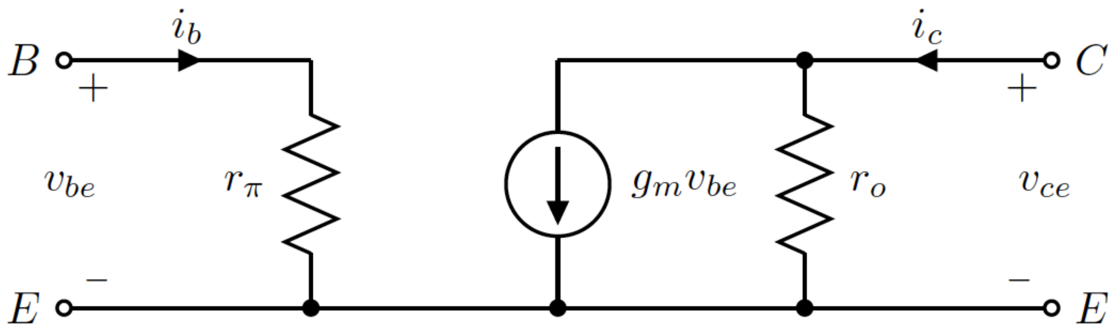
$$= g_m v_{be} + \frac{v_{ce}}{r_o}$$

- Total i_b:

$$i_b = \left. \frac{\partial I_B}{\partial V_{BE}} \right|_{V_{BE}=V_{BE,Q}} \cdot v_{be} = \frac{v_{be}}{r_\pi}$$

The BJT Small Signal Equivalent Circuit

- KCL / KVL results in the small signal model



$$i_b = \frac{v_{be}}{r_\pi} \qquad\qquad i_c = g_m v_{be} + \frac{v_{ce}}{r_o}$$

- Linear!
- Fully describes the response of the BJT to **small signal** disturbances about the quiescent point (no DC information!)
- Dependent on the quiescent DC operating point

Linearizing the MOSFET Transfer Characteristic

- MOSFET transconductance

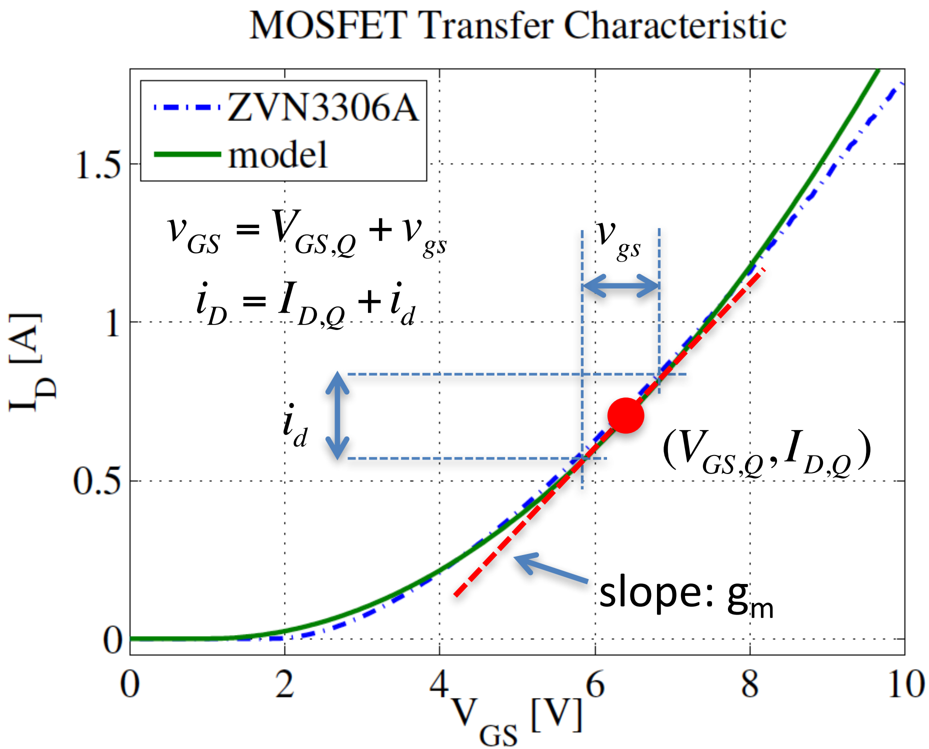
$$g_m = \left.\frac{\partial I_D}{\partial V_{GS}}\right|_{V_{GS}=V_{GS,Q}}$$

$$= \left.\frac{\partial}{\partial V_{GS}}\left(k\cdot\left(V_{GS}-V_{TH}\right)^2\right)\right|_{V_{GS}=V_{GS,Q}}$$

$$= 2k\cdot\left(V_{GS,Q}-V_{TH}\right)$$

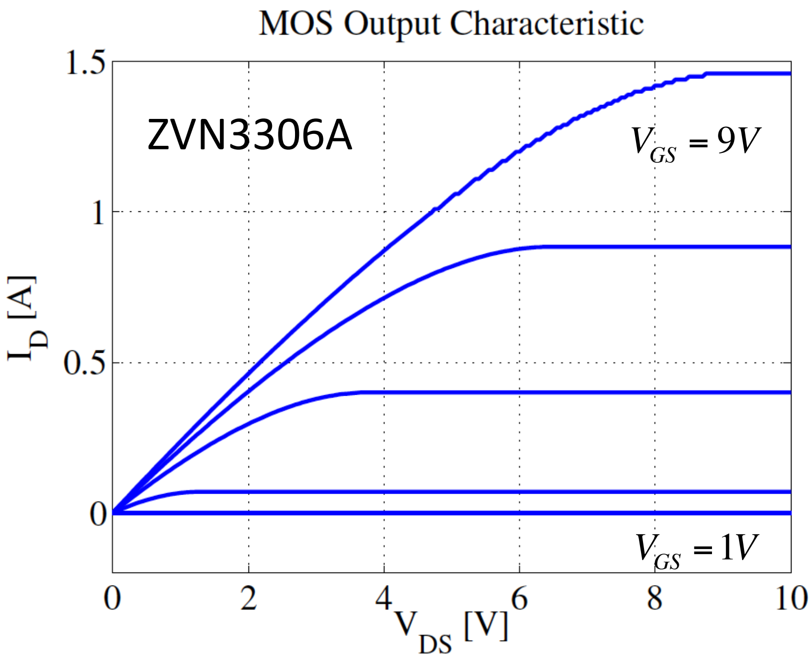
We get the linear relationship:

$$i_d = g_m \cdot v_{gs} = 2k\cdot\left(V_{GS,Q}-V_{TH}\right)\cdot v_{be}$$



Does g_m give us the complete picture?

- What else changes i_d?



$$I_D = k \cdot \left(V_{GS} - V_{TH}\right)^2 \left(1 + \lambda \cdot V_{DS}\right)$$

MOSFET Transistor Output Resistance

- What happens when there are small changes in V_{DS}?

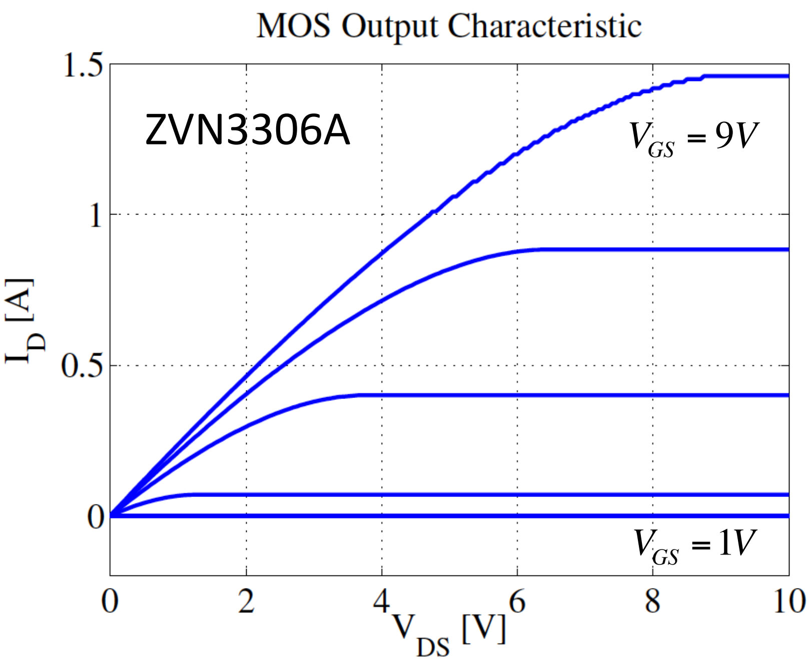
$$i_d = \left.\frac{\partial I_D}{\partial V_{DS}}\right|_{V_{DS}=V_{DS,Q}} \cdot v_{ds}$$

$$= \frac{\partial}{\partial V_{DS}}\left.\left(k\cdot\left(V_{GS}-V_{TH}\right)^2\left(1+\lambda\cdot V_{DS}\right)\right)\right|_{V_{DS}=V_{DS,Q}} \cdot v_{ds}$$

$$= k\cdot\left(V_{GS}-V_{TH}\right)^2\lambda\cdot v_{ds} = \lambda I_{DS,Q}\cdot v_{ds}$$

$$= g_o\cdot v_{ds} = \frac{v_{ds}}{r_o}$$

Output resistance: $r_o = \dfrac{1}{\lambda\cdot I_{C,Q}}$



Completing the Picture: Transistor Input Resistance

MOSFET

- Small signal gate current due to v_{gs}

$$g_\pi = \left.\frac{\partial I_G}{\partial V_{GS}}\right|_{V_{GS}=V_{GS,Q}} = 0$$

$$r_\pi = \frac{1}{g_\pi} \rightarrow \infty$$

Linearization Result: The <u>**Small Signal Model**</u>

BJT

- Total i_c:

$$i_c = \left(\left.\frac{\partial I_C}{\partial V_{BE}}\right|_{V_{BE}=V_{BE,Q}} \cdot v_{be} \right) + \left(\left.\frac{\partial I_C}{\partial V_{CE}}\right|_{V_{CE}=V_{CE,Q}} \cdot v_{ce} \right)$$

$$= g_m v_{be} + \frac{v_{ce}}{r_o}$$

- Total i_b:

$$i_b = \left.\frac{\partial I_B}{\partial V_{BE}}\right|_{V_{BE}=V_{BE,Q}} \cdot v_{be} = \frac{v_{be}}{r_\pi}$$

MOSFET

- Total i_d:

$$i_d = \left(\left.\frac{\partial I_D}{\partial V_{GS}}\right|_{V_{GS}=V_{GS,Q}} \cdot v_{gs} \right) + \left(\left.\frac{\partial I_D}{\partial V_{DS}}\right|_{V_{DS}=V_{DS,Q}} \cdot v_{ds} \right)$$

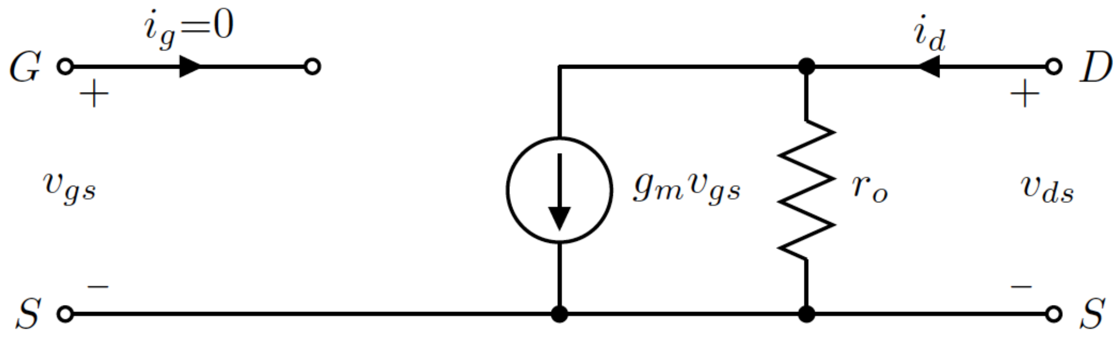
$$= g_m v_{gs} + \frac{v_{ds}}{r_o}$$

- Total i_g:

$$i_g = 0$$

The MOSFET Small Signal Equivalent Circuit

- KCL / KVL results in the small signal model



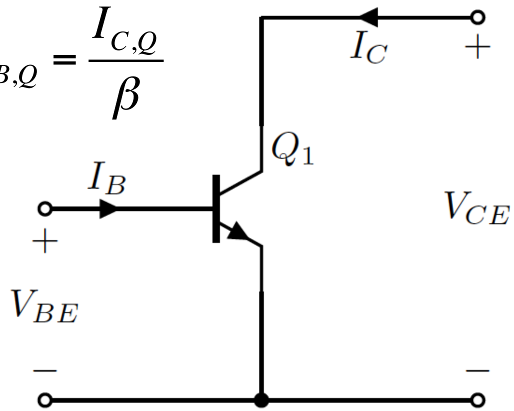
$$i_g = 0$$

$$i_d = g_m v_{gs} + \frac{v_{ds}}{r_o}$$

Large Signal vs. Small Signal

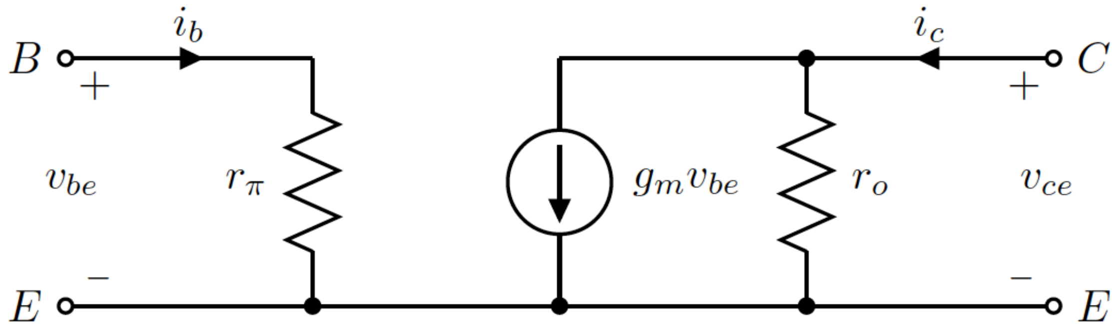
$$I_{C,Q} = I_S\left(e^{\frac{V_{BE,Q}}{V_T}} - 1\right)\left(1 + \frac{V_{CE,Q}}{V_A}\right)$$

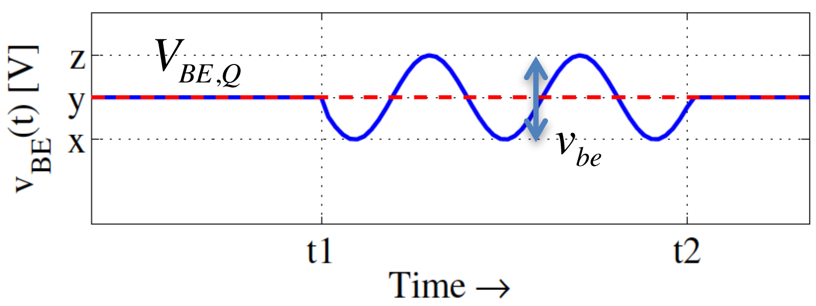
$$I_{B,Q} = \frac{I_{C,Q}}{\beta}$$

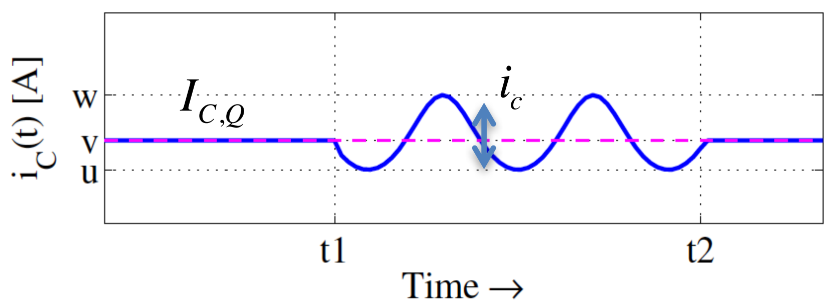


$$i_b = \frac{v_{be}}{r_\pi}$$

$$i_c = g_m v_{be} + \frac{v_{ce}}{r_o}$$

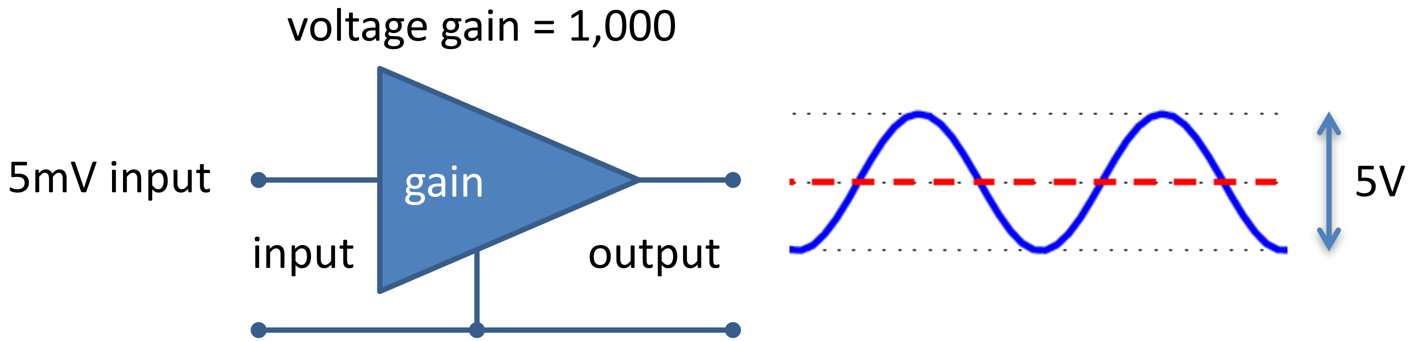






Small Signal Model Implications

- Linear relationships!
  - For small signals
- Linear circuit analysis works!
  - EEE 31 and 33 is useful after all… ☺
  - Can use **two-port network** concepts



Next Meeting

- Review of Two-Port Networks
- Single-Stage Amplifiers