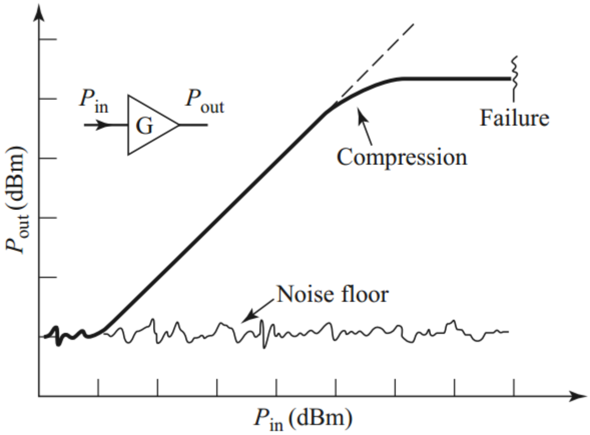
ECE 113: Communication Electronics

Meeting 3: Distortion

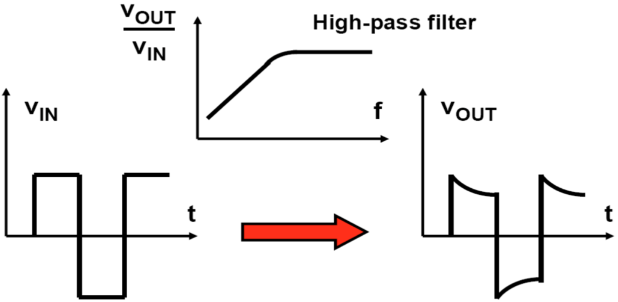
January 23, 2019

Dynamic Range

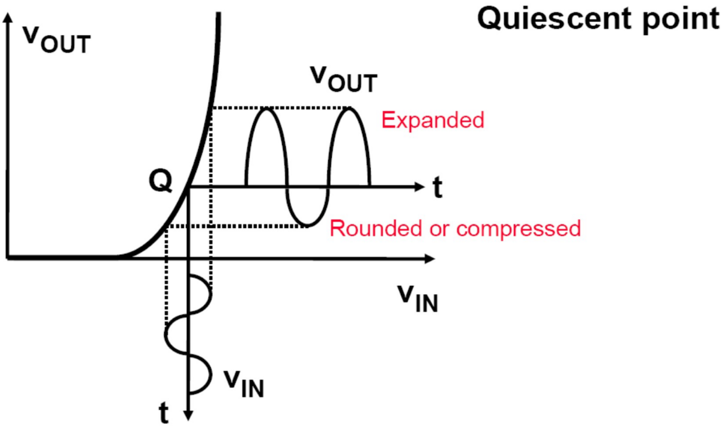
Range of signal Levels over which linear relationship between input and output is valid



Distortion



Distortion


Quiescent point
vOUT
vOUT
Expanded
Q
t
Rounded or compressed
vIN
vIN
t

Nonlinear Device/Network

- In general, the output of a nonlinear network can be modeled as a Taylor series in terms of the input voltage.

$$v_0 = a_0 + a_1 v_i + a_2 v_i^2 + a_3 v_i^3 + \dots$$

Taylor Series Expression

$$\upsilon_0 = a_0 + a_1\upsilon_i + a_2\upsilon_i^2 + a_3\upsilon_i^3 + \ldots$$

$a_0 = \upsilon_0(0)$	DC Output	Rectification (AC to DC)
$a_1 = \frac{d\upsilon_0}{d\upsilon_i}\vert_{v_i=0}$	Linear Output	Attenuator or Amplifier
$a_2 = \frac{d^2\upsilon_0}{d\upsilon_i}\vert_{\upsilon_i=0}$	Squared Output	Mixing or other frequency converting functions

Gain Compression

Consider the case where a single-frequency component is used as the input to the nonlinear network.

$$v_i = V_o cos \omega_o t$$

Applying the tayolor series expansion,

$$\begin{aligned} v_o &= a_0 + a_1 V_0 \cos\omega_0 t + a_2 V_0^2 \cos^2\omega_0 t + a_3 V_0^3 \cos^3\omega_0 t + \cdots \\ &= \left(a_0 + \frac{1}{2}a_2 V_0^2\right) + \left(a_1 V_0 + \frac{3}{4}a_3 V_0^3\right)\cos\omega_0 t + \frac{1}{2}a_2 V_0^2 \cos 2\omega_0 t \\ &\quad + \frac{1}{4}a_3 V_0^3 \cos 3\omega_0 t + \cdots . \end{aligned}$$

Voltage gain of the signal component at frequency ω_o

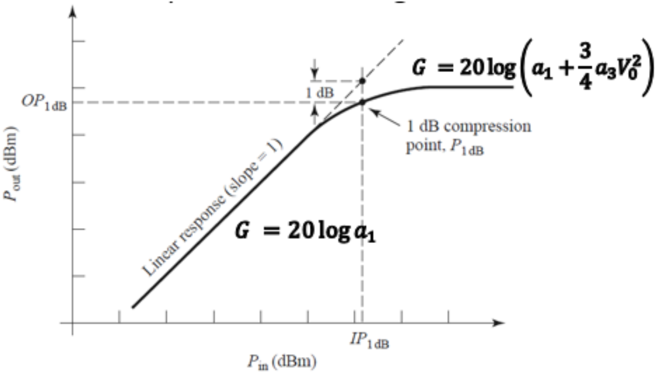
$$G_v = \frac{v_o^{(\omega_0)}}{v_i^{(\omega_0)}} = \frac{a_1 V_0 + \frac{3}{4}a_3 V_0^3}{V_0} = \boxed{a_1 + \frac{3}{4}a_3 V_0^2}$$

As expected

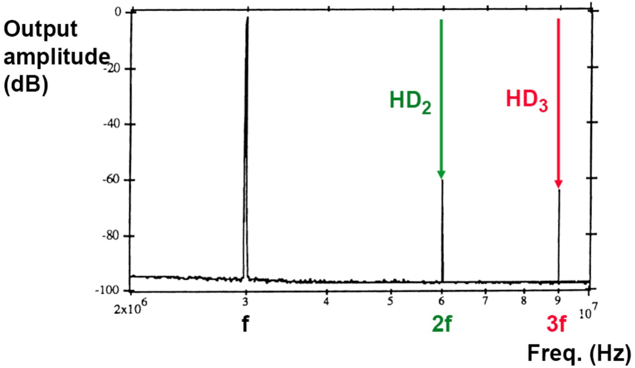
Usually has opposite sign of a1

1dB Compression Point (P_{1dB})

- Output tends to be reduced from the expected linear dependence at large values of V_o
- Physically, the instantaneous output voltage is limited by the power supply voltage used to bias the active circuit.



Harmonic Distortion



- Can be useful in multiplier circuits
- Can lead to signal distortion if harmonics are in the passband of amplifier systems

Harmonic Distortion

- Square power dominates the second harmonic

$$HD_2 = \frac{amplitude\ of\ second\ harmonic}{amplitude\ of\ fundamental} = \frac{a_2}{2a_1}V_o$$

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- Cubic power dominates the third harmonic

$$HD_3 = \frac{amplitude\ of\ third\ harmonic}{amplitude\ of\ fundamental} = \frac{a_3}{4a_1} V_o^2$$

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- Power in distortion relative to the fundamental power

$$\frac{Power\ in\ Distortion}{Power\ in\ Fundamental} = \frac{\upsilon_{o2}^2}{\upsilon_{o1}^2} + \frac{\upsilon_{o3}^2}{\upsilon_{o1}^2} + \ldots$$

$$= HD_2^2 + HD_3^2 + HD_4^2 + \ldots$$

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- Total Harmonic Distortion

$$THD = \sqrt{HD_2^2 + HD_3^2 + HD_4^2 + \dots}$$

Intermodulation Distortion

Consider the case where the input is a two-tone voltage consisting of two closely spaced frequencies.

$$v_i = V_o(cos\omega_1 t + cos\omega_2 t)$$

Applying Taylor series expansion,

$$\begin{aligned}
v_o &= a_0 + a_1 V_0(\cos\omega_1 t + \cos\omega_2 t) + a_2 V_0^2(\cos\omega_1 t + \cos\omega_2 t)^2 \\
&\quad + a_3 V_0^3(\cos\omega_1 t + \cos\omega_2 t)^3 + \cdots \\
&= a_0 + a_1 V_0 \cos\omega_1 t + a_1 V_0 \cos\omega_2 t + \frac{1}{2}a_2 V_0^2(1 + \cos 2\omega_1 t) + \frac{1}{2}a_2 V_0^2(1 + \cos 2\omega_2 t) \\
&\quad + a_2 V_0^2 \cos(\omega_1 - \omega_2)t + a_2 V_0^2 \cos(\omega_1 + \omega_2)t \\
&\quad + a_3 V_0^3\left(\frac{3}{4}\cos\omega_1 t + \frac{1}{4}\cos 3\omega_1 t\right) + a_3 V_0^3\left(\frac{3}{4}\cos\omega_2 t + \frac{1}{4}\cos 3\omega_2 t\right) \\
&\quad + a_3 V_0^3\left[\frac{3}{2}\cos\omega_2 t + \frac{3}{4}\cos(2\omega_1 - \omega_2)t + \frac{3}{4}\cos(2\omega_1 + \omega_2)t\right] \\
&\quad + a_3 V_0^3\left[\frac{3}{2}\cos\omega_1 t + \frac{3}{4}\cos(2\omega_2 - \omega_1)t + \frac{3}{4}\cos(2\omega_2 + \omega_1)t\right] + \cdots.
\end{aligned}$$

Second Order Intermodulation

The second power term gives

DC and HD_2	$a_2 V_o^2 + \frac{a_2 V_o^2}{2}(cos2\omega_1 t + cos2\omega_2 t)$
IM_2	$a_2 V_o^2(cos(\omega_1 + \omega_2)t + cos(\omega_1 - \omega_2)t)$

Definition

$$IM_2 = \frac{Amp\ of\ \ Intermod}{Amp\ of\ \ Fund} = \frac{a_2}{a_1} V_o$$

- Relation between HD_2 and IM_2

$$IM_2 = 2HD_2 = HD_2 + 6dB$$

Third Order Intermodulation

From the cubic term,

$$\frac{3}{4}a_3V_o^3(cos(2\omega_2 \pm \omega_1) + cos(2\omega_1 \pm \omega_2))$$

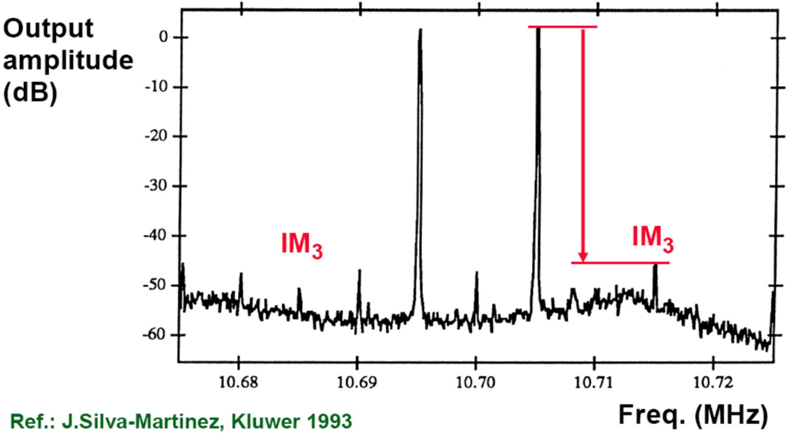
Definition

$$IM_3 = \frac{Amp\ of\ Intermod}{Amp\ of\ Fund} = \frac{3}{4}\frac{a_3}{a_1}V_o^2$$

- Relation between HD_3 and IM_3

$$IM_3 = 3HD_3 = HD_3 + 9.5dB$$

Third Order Intermodulation



Ref.: J.Silva-Martinez, Kluwer 1993

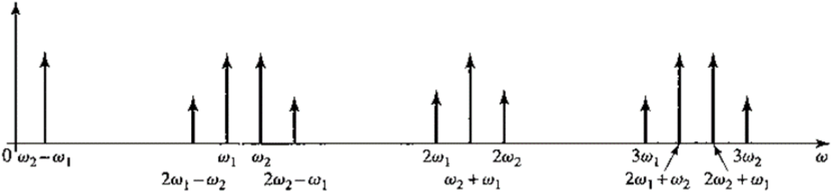
Intermodulation Products

Definition

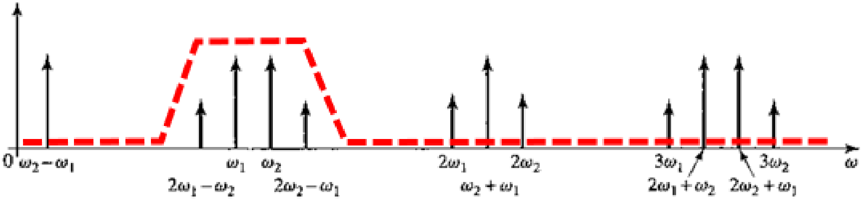
unwanted signals with frequencies

$$m\omega_1 + n\omega_2$$

where $m, n = 0, \pm 1, \pm 2, \pm 3, \ldots$ and the **order** is defined as $|m| + |n|$

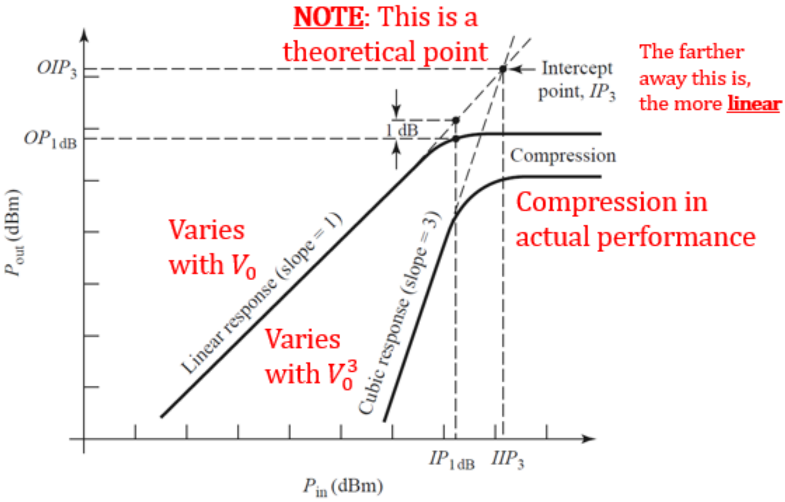


Intermodulation Distortion

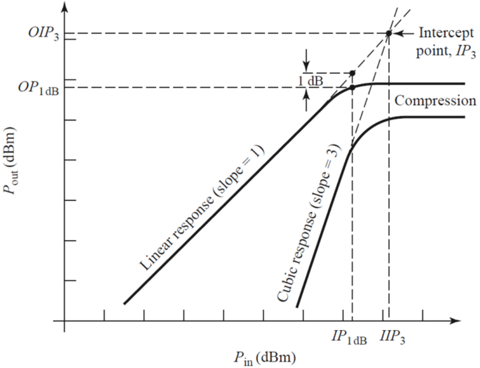


- IM_3 components are very near the fundamental frequency
  - Suppose $\omega_1 \approx \omega_2$, $2\omega_2 - \omega_1 \approx \omega_2$
- Even if the system is narrowband, the output of the amplifier can contain in band intermodulation due to IM_3
  - Filtering might not be easy.

Third Order Intercept Point



Third Order Intercept Point



- Similar with P_1dB, it can be referred either to the input or the output.
- $OIP_{3,dBm} = IIP_{3,dBm} + G_{dB}$

- At the third order intercept point

  Power at Fundamental = Power at Third Order Product

$$P_{\omega_1} = P_{2\omega_1-\omega_2}$$

$$\frac{1}{2}a_1^2 V_o^2 = \frac{1}{2}(\frac{3}{4}a_3 V_o^3)^2$$

$$\frac{1}{2}a_1^2 V_o^2 = \frac{9}{32}a_3^2 V_o^6$$

- Define input signal voltage at intercept point as V_{IP}

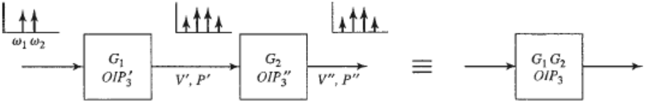
$$\frac{1}{2}a_1^2 V_{IP}^2 = \frac{1}{2}(\frac{3}{4}a_3^2 V_{IP}^6)$$

$$V_{IP} = \sqrt{\frac{4a_1}{3a_3}}$$

- Solving for OIP_3,

$$OIP_3 = P_{\omega_1}|_{V_o=V_{IP}} = \frac{1}{2}a_1^2 V_{IP}^2 = \frac{2a_1^3}{3a_3}$$

Intercept Point of a Cascaded System



- first stage output power of third order intermodulation product

$$P'_{2\omega_1-\omega_2} = \frac{9}{32}a_3^2 V_o^6 = \frac{\frac{1}{8}a_1^6 V_o^6}{\frac{4a_1^6}{9a_3^2}} = \frac{(P'_{\omega_1})^3}{(OIP_3')^2}$$

- Obtaining the voltage output,

$$V'_{2\omega_1-\omega_2} = \frac{\sqrt{(P'_{\omega_1})^3 Z_o}}{OIP_3'},$$ where Z_o is the system impedance

Intercept Point of a Cascaded System

- Worst case at the output of the second stage
  - assumes in-phase addition of distortion components

$$V''_{2\omega_1-\omega_2} = \sqrt{G_2}\frac{\sqrt{(P'_{\omega_1})^3 Z_o}}{OIP'_3} + \frac{\sqrt{(P''_{\omega_1})^3 Z_o}}{OIP''_3}$$

- Using $P''_{\omega_1} = G_2 P'_{\omega_1}$,

$$V''_{2\omega_1-\omega_2} = \left(\frac{1}{G_2(OIP'_3)} + \frac{1}{OIP''_3}\right)\sqrt{(P''_{\omega_1})^3 Z_o}$$

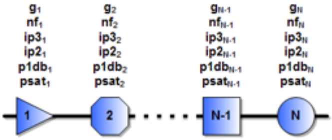
- Total Output Distortion Power

$$OIP_3 = \left(\frac{1}{G_2(OIP'_3)} + \frac{1}{OIP''_3}\right)^{-1}$$

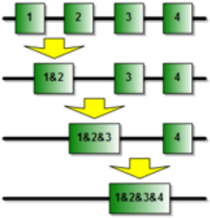
- Note that $OIP_3 = G_2(OIP'_3)$ when $OIP''_3 \rightarrow \infty$ (highly linear components)

Intercept Point of a Cascaded System

- In general,



A Typical Chain of Cascaded Components



Combining 2 Stages at a Time for Calculations

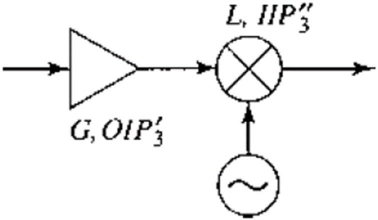
$$ip3_C = \frac{1}{\frac{1}{ip3_{N-1} \cdot g_N} + \frac{1}{ip3_N}} \; [\![mW]\!] \qquad \left| \begin{array}{l} C = \text{cumulative up to and including stage } N \\ N = \text{current stage} \\ N-1 = \text{previous stage} \end{array} \right.$$

$$IP3_C = 10 \cdot \log(ip3_C) \; [\![dBm]\!]$$

Source: www.rfcafe.com

Examples

- The amplifier has a gain of 20 dB and a third-order intercept point of 22 dBm (referenced at output), and the mixer has a conversion loss of 6 dB and a third-order intercept point of 13 dBm (referenced at input).



- Find the intercept points of the cascade network.

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