Lecture 11
SYMMETRICAL COMPONENTS

Agenda

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Why Symmetrical Components? Unbalanced to Balanced Analysis

- https://vignette.wikia.nocookie.net/powerprotection/images/8/88/Sequence_Components_in_the_phase_domain.gif/revision/latest?cb=20151203223611

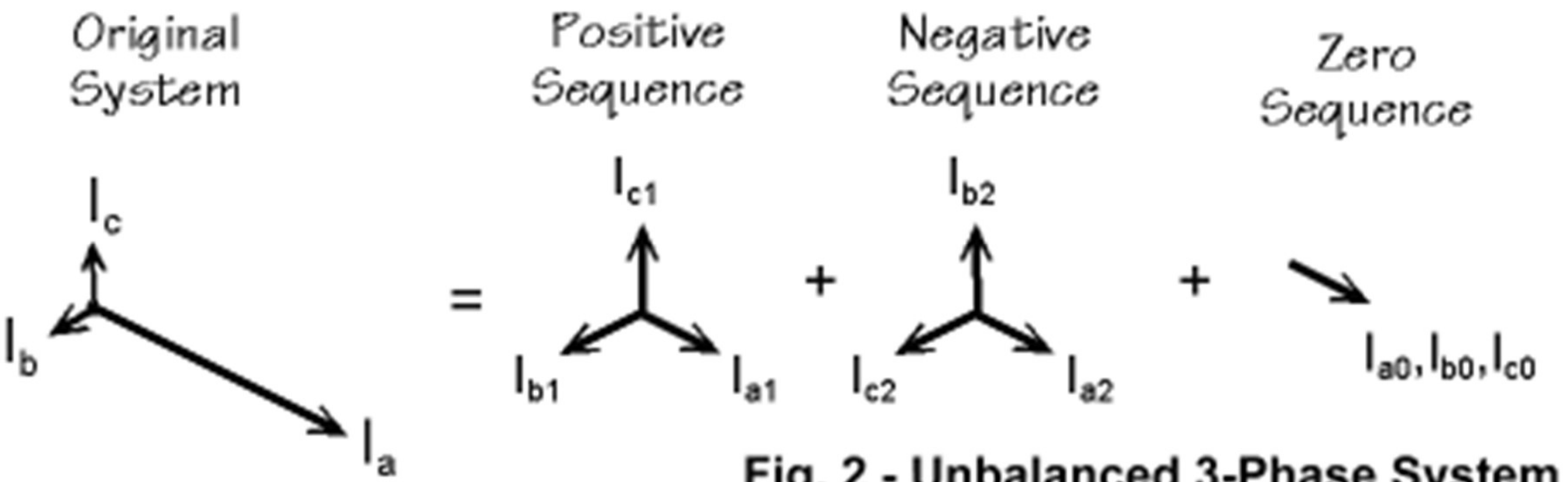


Fig. 2 - Unbalanced 3-Phase System

Lecture Outcomes

at the end of the lecture, the student must be able to …

- Solve for the symmetrical components given a balanced or unbalanced phasor.
- Understand sequence networks and how they simplify analysis.

Symmetrical Components

Symmetrical Components

In Balanced Power Systems,

- Generator Voltages are three-phase balanced
- Line and transformer impedances are balanced
- Loads are three-phased balanced

Single-Phase Representation and Analysis can be used for the Balanced Three-Phase Power System.

Symmetrical Components

In practical Power Systems,

- Lines are not transposed.
- Single-phase transformers used to form three-phase banks are not identical.
- Loads are not balanced.
- Presence of vee-phase and single phase lines.
- Faults

Single-phase Representation <u>CANNOT</u> be used for the analysis of a Three-Phase Power System that cannot be reasonably assumed to be balanced.

Symmetrical Components Method

According to Fortescue, any balanced and unbalanced three-phase system of *phasors* may be resolved into three balanced sets of phasors which are referred to as the symmetrical components of the original phasors, namely:

a) **POSITIVE-SEQUENCE COMPONENT**

b) **NEGATIVE-SEQUENCE COMPONENT**

c) **ZERO-SEQUENCE COMPONENT**

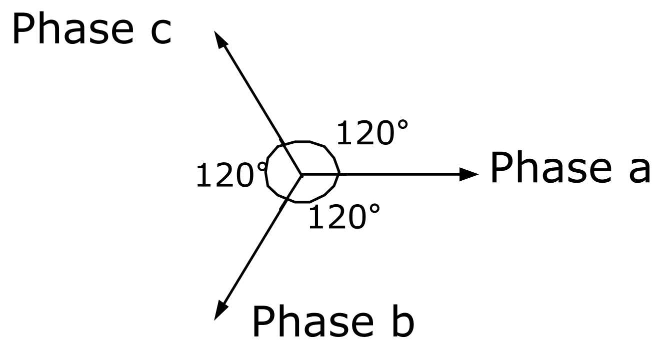
Charles L. Fortescue, "Method of Symmetrical Co-Ordinates Applied to the Solution of Polyphase Networks". Presented at the 34th annual convention of the AIEE (American Institute of Electrical Engineers) in Atlantic City, N.J. on 28 June 1918. Published in: AIEE Transactions, vol. 37, part II, pages 1027–1140 (1918). For a brief history of the early years of symmetrical component theory, see: J. Lewis Blackburn, Symmetrical Components for Power Engineering (Boca Raton, Florida: CRC Press, 1993), pages 3–4.

Sequence Networks

- These Symmetrical components each have their own equivalent circuits, called, sequence networks.
  - Positive Sequence Network
  - Negative Sequence Network
  - Zero- Sequence Network
- For <u>balanced</u> three phase systems, the networks are <u>decoupled</u>
- For <u>unbalanced</u> three phase systems, the networks are <u>connected only</u> on the points of unbalanced.

Symmetrical Components

REFERENCE PHASE SEQUENCE: abc



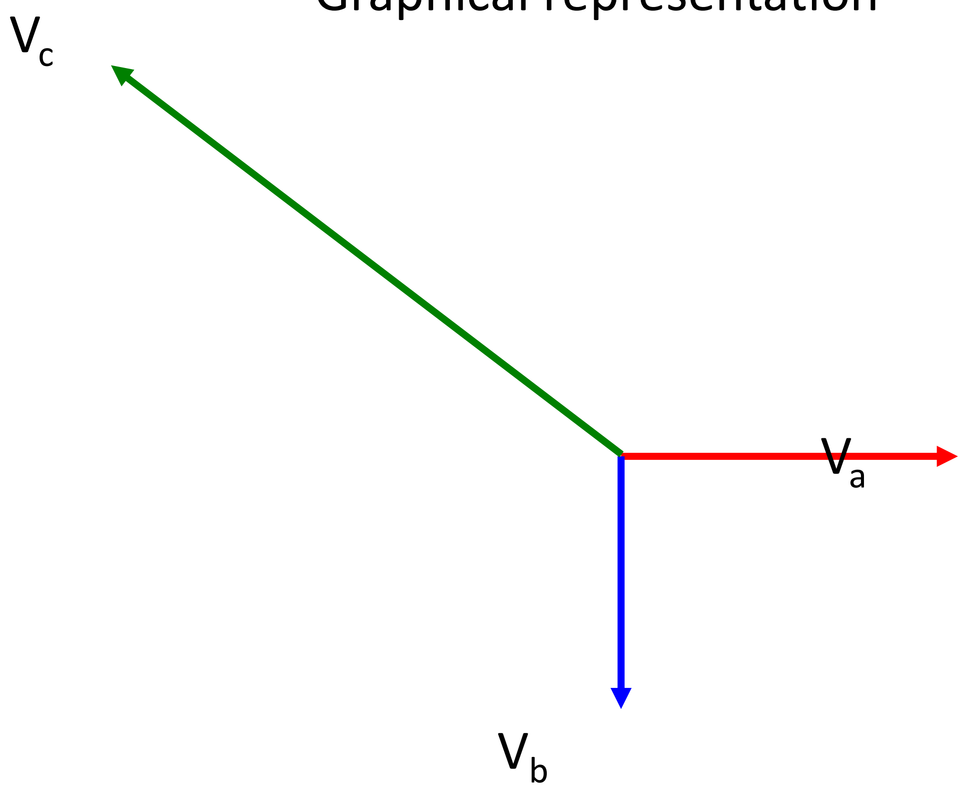
- *Positive Sequence Phasors are three-phase, balanced and have the phase sequence as the original set of unbalanced phasors.*

- Negative Sequence Phasors are three-phase, balanced but with a phase sequence opposite to that original set of unbalnced phasors.

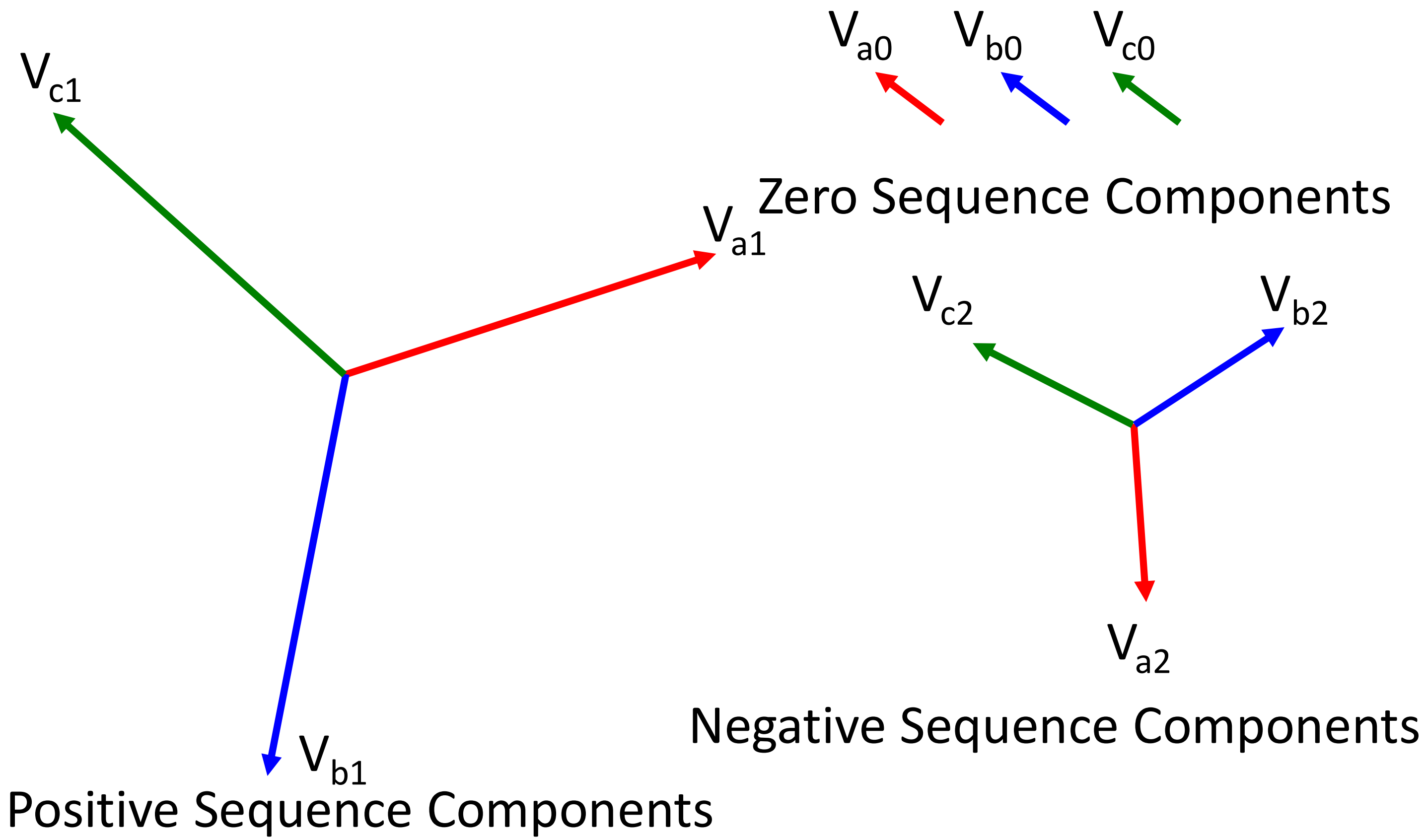
- *Zero Sequence Phasors are single-phase, equal in magnitude and in the same direction.*

Symmetrical Components

Graphical representation

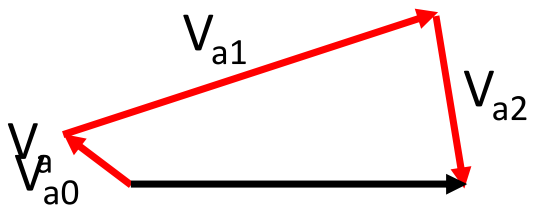


Symmetrical Components

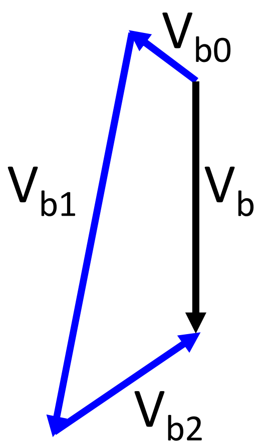


Symmetrical Components

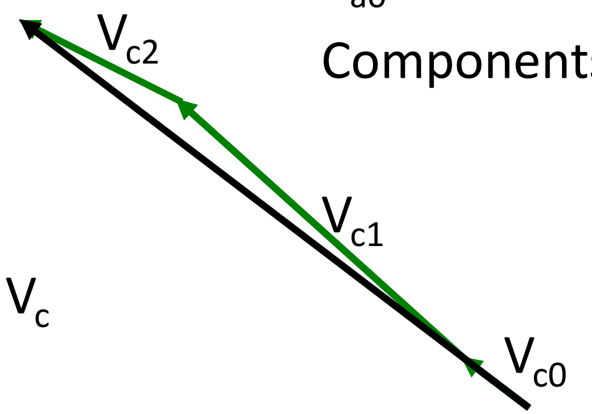
Add Sequence Components Graphically



Components of V_a



Components of V_b



Components of V_c

Symmetrical Components

Each of the original unbalanced phasor is the sum of its sequence components. Thus,

$$V_a = V_{a1} + V_{a2} + V_{a0}$$

$$V_b = V_{b1} + V_{b2} + V_{b0}$$

$$V_c = V_{c1} + V_{c2} + V_{c0}$$

Where,

V_{a1} – Positive Sequence component of Voltage V_a
V_{a2} – Negative Sequence component of Voltage V_a
V_{a0} – Zero Sequence component of Voltage V_a

Symmetrical Components

OPERATOR *"a"*

An operator "a" causes a rotation of 120° in the counter clockwise direction of any phasor.

$a = 1 \angle 120°$ $\quad a^2 = 1 \angle 240°$ $\quad a^3 = 1 \angle 0°$

V_b *in terms of* V_a

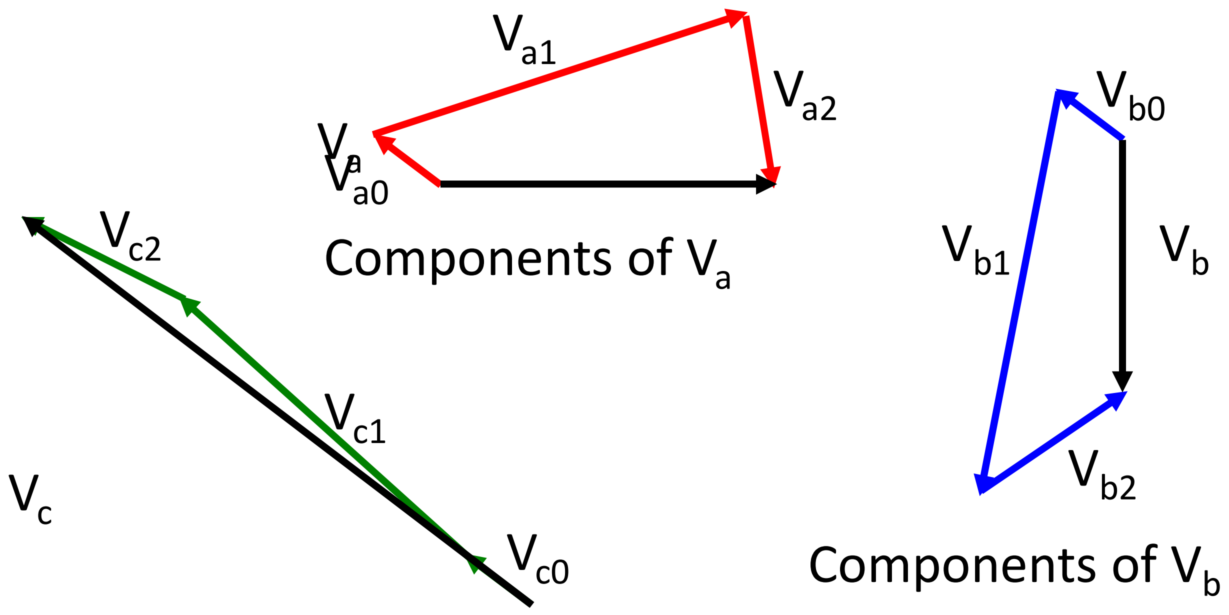
$$V_{b1} = a^2 V_{a1}$$
$$V_{b2} = a V_{a2}$$
$$V_{b0} = V_{a0}$$

V_c *in terms of* V_a

$$V_{c1} = a V_{a1}$$
$$V_{c2} = a^2 V_{a2}$$
$$V_{c0} = V_{a0}$$

Symmetrical Components

Add Sequence Components Graphically



Components of V_a

Components of V_b

Components of V_c

$$V_{b1} = a^2 V_{a1}$$
$$V_{b2} = a\, V_{a2}$$
$$V_{b0} = V_{a0}$$

$$V_{c1} = a\, V_{a1}$$
$$V_{c2} = a^2 V_{a2}$$
$$V_{c0} = V_{a0}$$

Symmetrical Components

Writing again the phasors in terms of phasor V_a and operator “a”,

$$V_a = V_{a0} + V_{a1} + V_{a2}$$
$$V_b = V_{a0} + a^2V_{a1} + aV_{a2}$$
$$V_c = V_{a0} + aV_{a1} + a^2V_{a2}$$

Rearranging and writing in matrix form

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

Symmetrical Components

Let

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

Using numerical techniques, the inverse of A can be obtained as,

$$A^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$$

Symmetrical Components

The symmetrical sequence components can be obtained by pre-multiplying the original phasors (V_a, V_b and V_c) by the inverse of A,

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

Thus,

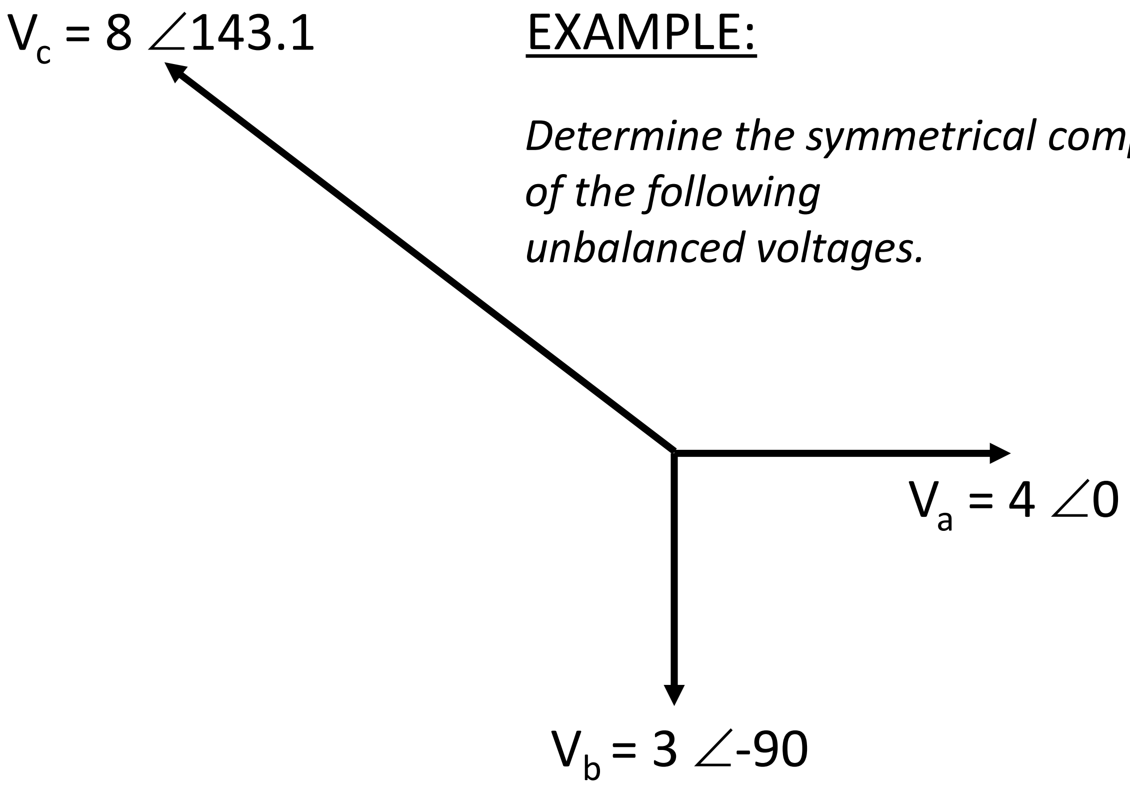
$$V_{a0} = \frac{1}{3}[V_a + \ V_b + \ V_c] \qquad V_{a1} = \frac{1}{3}[V_a + aV_b + \ a^2 V_c]$$

$$V_{a2} = \frac{1}{3}[V_a + a^2 V_b + \ aV_c]$$

Symmetrical Components

<u>EXAMPLE:</u>

Determine the symmetrical components of the following unbalanced voltages.



Symmetrical Components

For Phasor V_a:

$$V_{a0} = \frac{1}{3}(V_a + V_b + V_c)$$

$$= \frac{1}{3}(4\angle 0 + 3\angle -90 + 8\angle 143.1)$$

$$= 1\angle 143.05$$

$$V_{a1} = \frac{1}{3}(V_a + aV_b + a^2 V_c)$$

$$= \frac{1}{3}\left[4\angle 0 + (1\angle 120)(3\angle -90) + (1\angle 240)(8\angle 143.1)\right]$$

$$= 4.9\angle 18.38$$

Symmetrical Components

For Phasor V_a:

$$V_{a2} = \frac{1}{3}(V_a + a^2 V_b + a V_c)$$

$$= \frac{1}{3}\left[4\angle 0 + (1\angle 240)(3\angle -90) + (1\angle 120)(8\angle 143.1)\right]$$

$$= 2.15\angle -86.08$$

Symmetrical Components

Components of V_b can be obtained by operating the sequence components of phasor V_a.

$$V_{b0} = V_{a0}$$

$$= 1\angle 143.05 \qquad = 1\angle 143.05$$

$$V_{b1} = a^2 V_{a1}$$

$$= (1\angle 240)(4.9\angle 18.38)$$

$$= 4.9\angle 258.38$$

$$= 4.9\angle -101.62$$

$$V_{b2} = aV_{a2}$$

$$= (1\angle 120)(2.15\angle -86.08)$$

$$= 2.15\angle 33.92$$

Symmetrical Components

Similarly, components of phasor V_c can be obtained by operating V_a.

$$\mathrm{V_{c0}} = \mathrm{V_{a0}}$$

$$= 1\angle 143.05 \qquad = 1\angle 143.05$$

$$\mathrm{V_{c1}} = \mathrm{a}V_{a1}$$

$$= (1\angle 120)(4.9\angle 18.38)$$

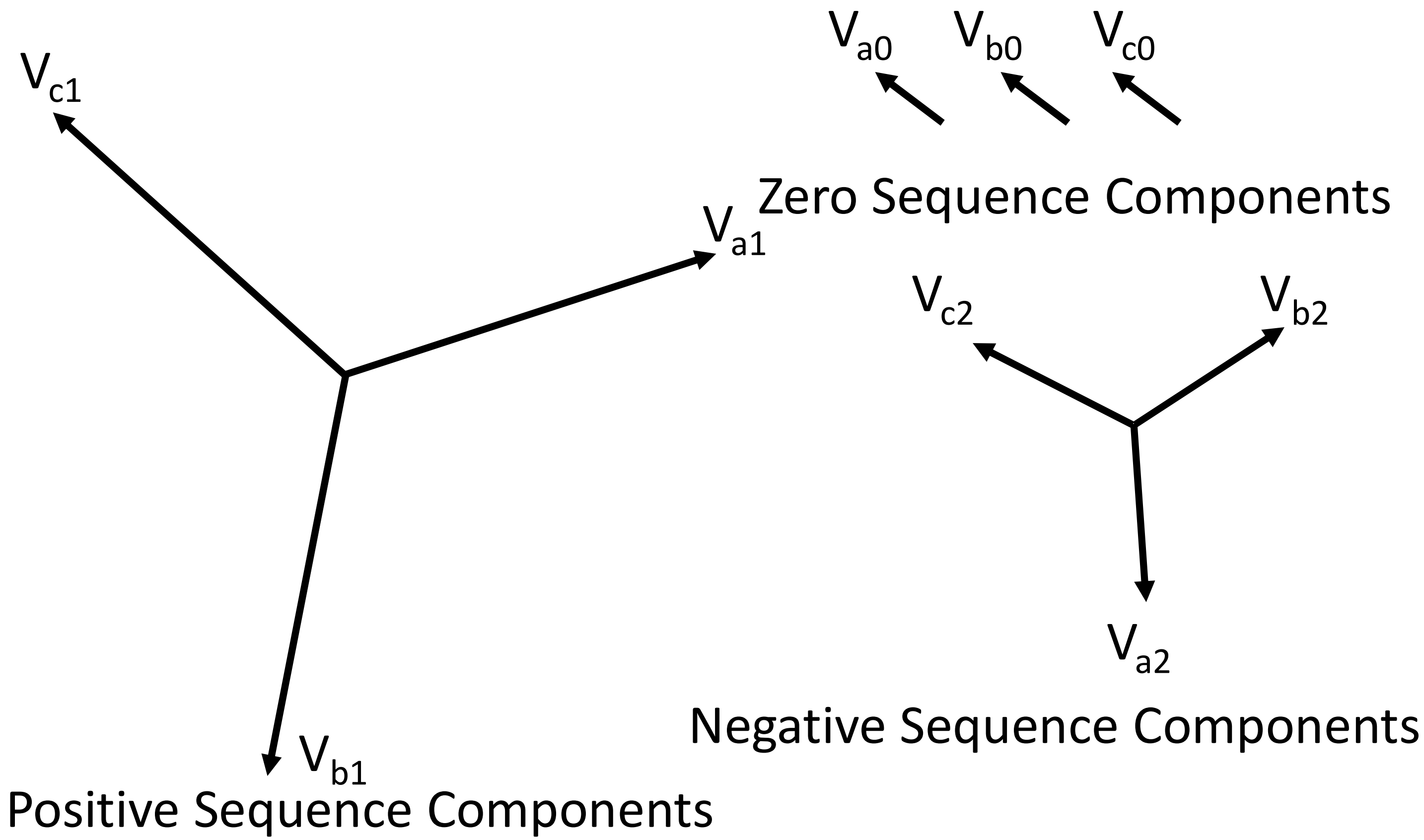
$$= 4.9\angle 138.38$$

$$\mathrm{V_{c2}} = \mathrm{a}^2\mathrm{V_{a2}}$$

$$= (1\angle 240)(2.15\angle -86.08)$$

$$= 2.15\angle 153.92$$

Symmetrical Components



Review Question

- How do we check if the derived symmetrical components are correct?
    - Sum the corresponding sequence components.

Symmetrical Components

The results can be checked either mathematically or graphically.

$$V_a = V_{a0} + V_{a1} + V_{a2}$$

$$= 1 \angle 143.05 + 4.9 \angle 18.38 + 2.15 \angle -86.08$$

$$= 4 \angle 0$$

$$V_b = V_{b0} + V_{b1} + V_{b2}$$

$$= 1 \angle 143.05 + 4.9 \angle -101.62 + 2.15 \angle 33.92$$

$$= 3 \angle -90$$

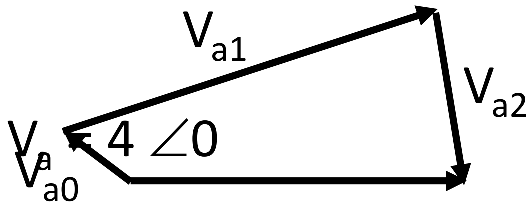
$$V_c = V_{c0} + V_{c1} + V_{c2}$$

$$= 1 \angle 143.05 + 4.9 \angle 138.38 + 2.15 \angle 153.92$$

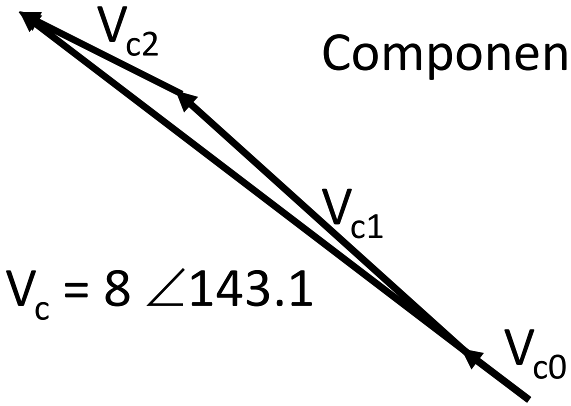
$$= 8 \angle 143.1$$

Symmetrical Components

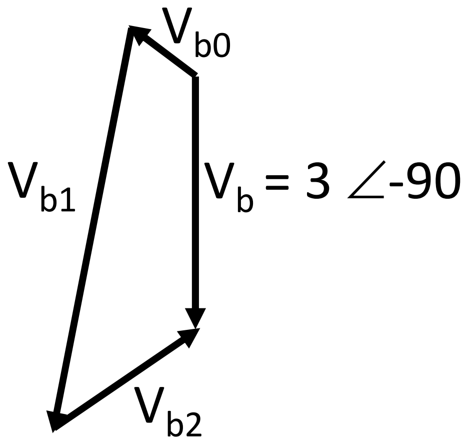
Add Sequence Components Graphically



Components of V_a



Components of V_c



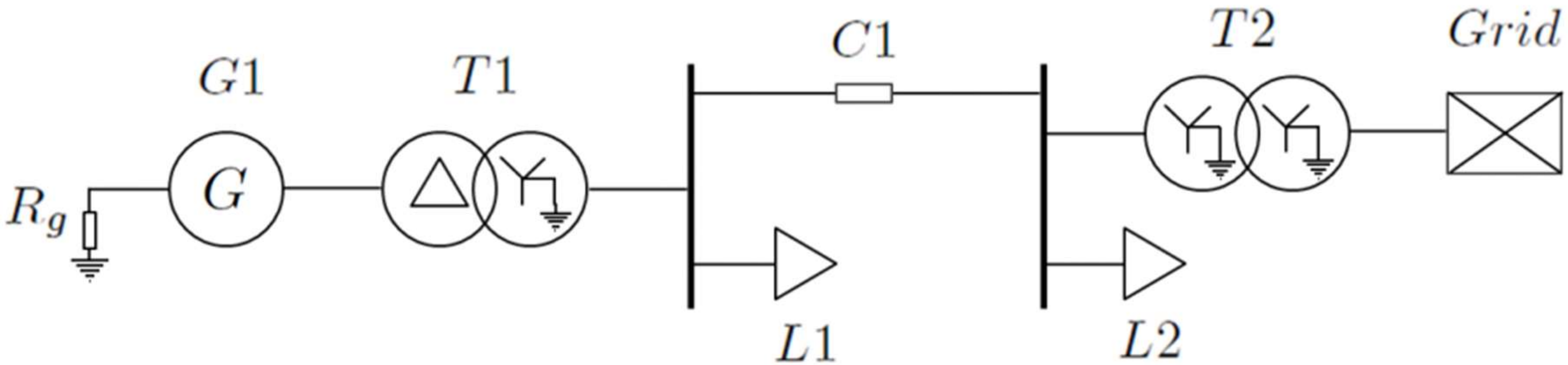
Components of V_b

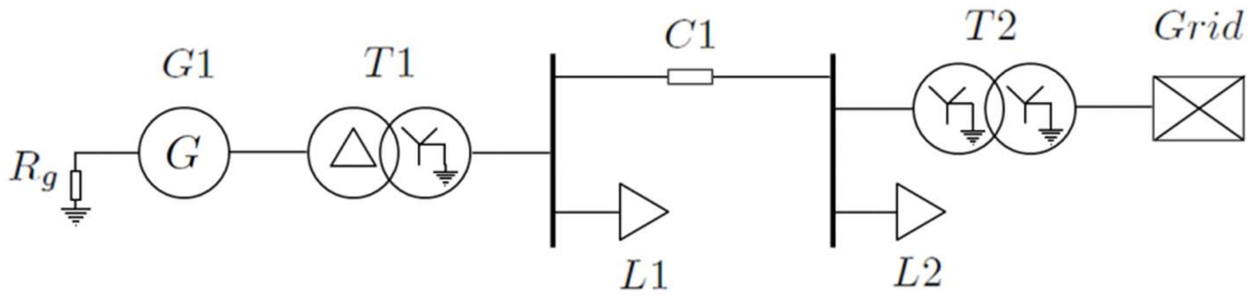
Sequence Networks

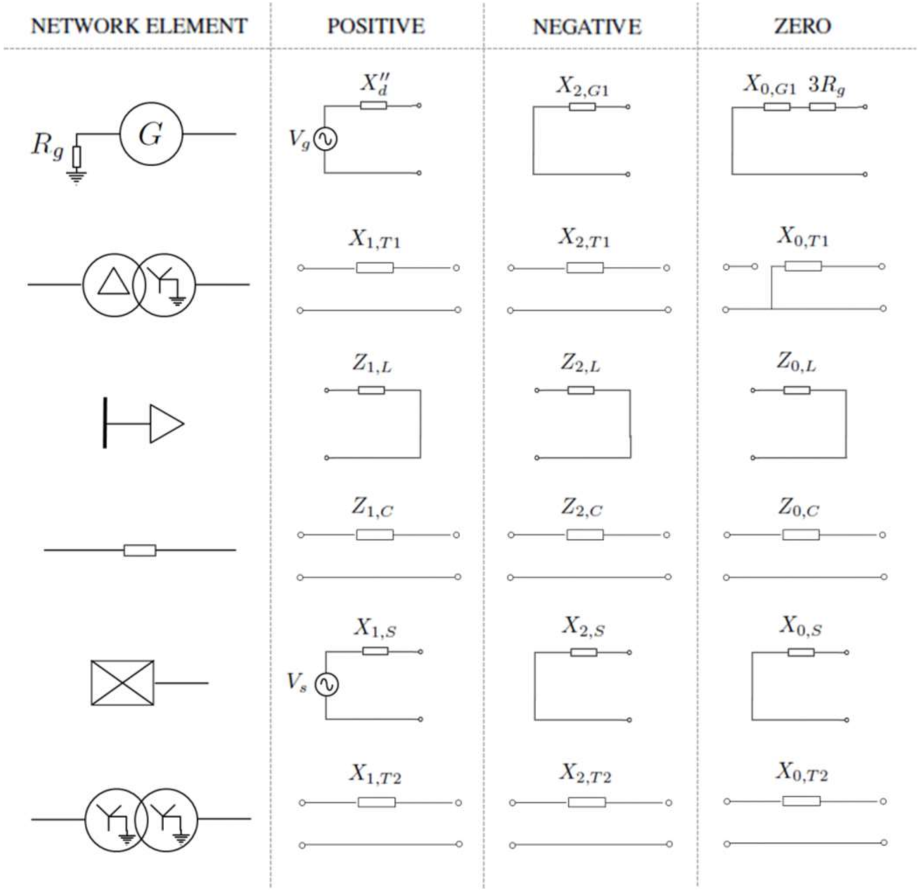
- Recall, in symmetrical component analysis, a balanced three phase electrical network can be broken down into three sequence networks
  - Positive, Negative, and Zero Sequence Networks
- If the three system is balanced, they are decoupled.
- The sequence network circuit depends on the components
  - Loads
  - Transformers
  - Transmission Lines
  - Synchronous Machines

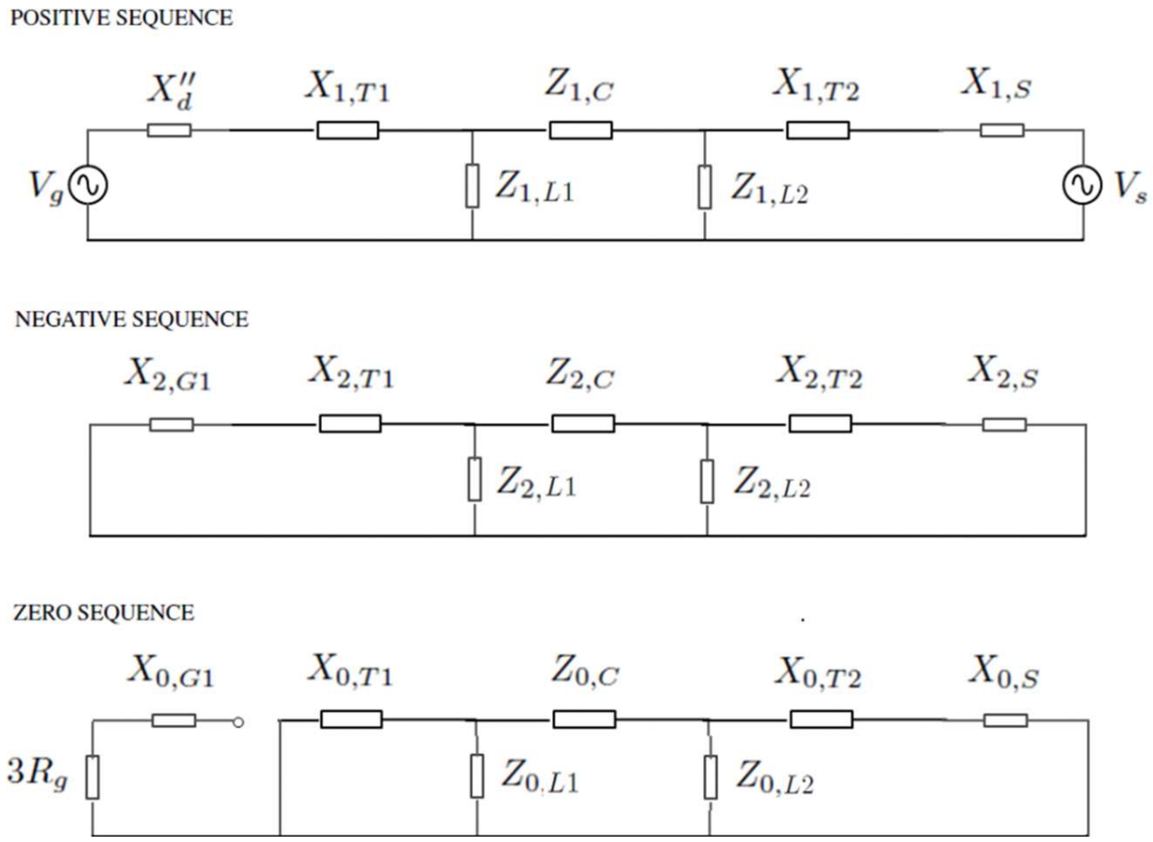
Sequence Networks

- Single-phase equivalent circuits in the form of zero-, positive-, and negative-sequence circuits is available for
  - Load impedances
  - Transformers – depending on the connections
  - Transmission lines
  - Synchronous machines

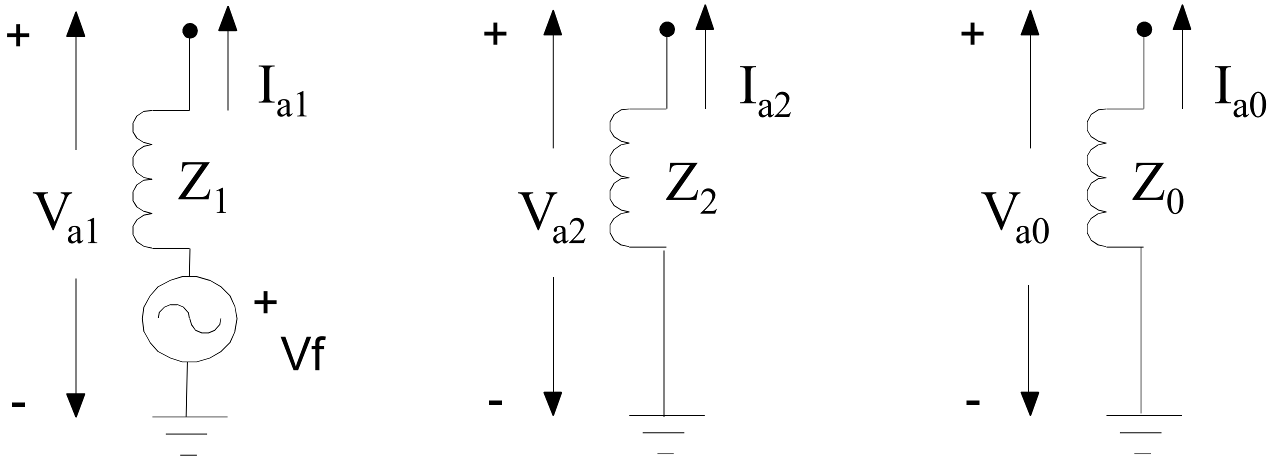



G1
T1
C1
T2
Grid
Rg
G
L1
L2


NETWORK ELEMENT
POSITIVE
NEGATIVE
ZERO
Rg
G
X''d
Vg
X2,G1
X0,G1 3Rg
X1,T1
X2,T1
X0,T1
Z1,L
Z2,L
Z0,L
Z1,C
Z2,C
Z0,C
X1,S
Vs
X2,S
X0,S
X1,T2
X2,T2
X0,T2


POSITIVE SEQUENCE
X''d
X1,T1
Z1,C
X1,T2
X1,S
Vg
Z1,L1
Z1,L2
Vs
NEGATIVE SEQUENCE
X2,G1
X2,T1
Z2,C
X2,T2
X2,S
Z2,L1
Z2,L2
ZERO SEQUENCE
X0,G1
X0,T1
Z0,C
X0,T2
X0,S
3Rg
Z0,L1
Z0,L2

Sequence Networks



$V_{a1} = V_f - I_{a1}Z_1$
Positive Sequence

$V_{a2} = -I_{a2}Z_2$
Negative Sequence

$V_{a0} = -I_{a0}Z_0$
Zero Sequence

Sequence Networks

- The voltage drop caused by a current of a certain sequence depends only on the impedance to the current flow of that sequence.
- The positive- and negative-sequence impedances are equal in any static circuit, and are approximately equal in synchronous machines under sub transient conditions.

Sequence Networks

- Generally, Z_0 is different from Z_1 and Z_2
- Only the positive sequence circuits of rotating machines contain sources that are positive sequence voltages
- The neutral node is the reference of positive- and negative-sequence voltages

Sequence Networks

- No positive- and negative-sequence currents flow between neutral points and ground
- An impedance Z_n between neutral and ground are not included in positive- and negative-sequence circuits, but represented by an impedance equal to $3Z_n$ between neutral and ground for zero-sequence circuits only

Sequence Impedances

- In general,

$$Z_1 \neq Z_2 \neq Z_0 \quad \textit{for generators}$$

$$Z_1 = Z_2 = Z_0 \quad \textit{for transformers}$$

$$Z_1 = Z_2 \neq Z_0 \quad \textit{for lines}$$

HOMEWORK 3

A Y-connected load bank with a three-phase rating of 500 kVA and 2300 V consists of three identical resistors of 10.58 Ω. The load bank has the following applied voltages: $V_{ab} = 1840\underline{/82.8^\circ}$, $V_{bc} = 2760\underline{/-41.4^\circ}$, and $V_{ca} = 2300\underline{/180^\circ}$ V. Determine the symmetrical components of (a) the line-to-line voltages V_{ab0}, V_{ab1}, and V_{ab2}; (b) the line-to-neutral voltages V_{an0}, V_{an1}, and V_{an2}; (c) and the line currents I_{a0}, I_{a1}, and I_{a2}. (Note that the absence of a neutral connection means that zero-sequence currents are not present.)