EEE 51 Assignment 2 Solution

2nd Semester SY 2017-2018

1. **MOSFET Single Stage CS Amplifier with Source Degeneration.** Consider the circuit shown below. Provided that $V_{dd} = 5V$, $V_{out} = 2.5V$, $|V_{TH}| = 0.8V$, $R_L = 50k\Omega$, $R_S = 20k\Omega$, k=200 µA/V^2, $\lambda = 0.001V^{-1}$, determine the following:

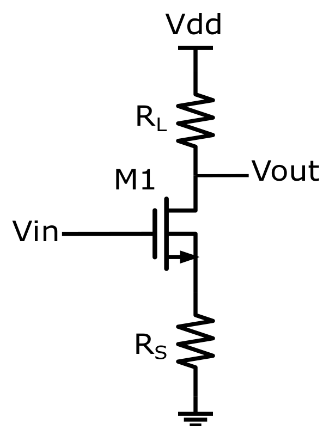

Figure 1: MOSFET Single Stage CS Amplifier

(a) Compute for the I_D, V_{DS}, V_{GS} and V_{in}. State all necessary assumptions. [3 pts] Assume M1 operates at saturation region.

$$V_{dd} - I_D R_L - V_{out} = 0 \tag{1}$$

$$\boxed{\text{Therefore, } I_D = 50\,\mu\text{A [0.5 pt]}}$$

$$V_{out} = V_{DS} + I_D R_S \tag{2}$$

$$\boxed{V_{DS} = 1.5V \text{ [0.5 pt]}}$$

$$I_D = k(V_{GS} - V_{TH})^2(1 + \lambda V_{DS}) \tag{3}$$

$$\boxed{V_{GS} = 1.2962V \text{ [1 pt]}}$$

$$V_{in} = V_{GS} + I_D R_S \tag{4}$$

$$\boxed{V_{in} = 2.2962V \text{ [1 pt]}}$$

(b) Draw the small-signal equivalent circuit with proper labels. [1 pt]

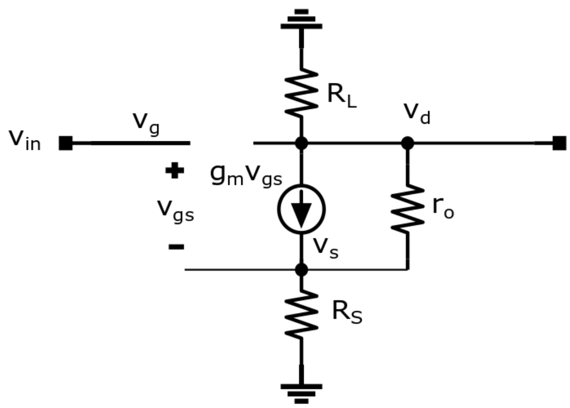

Figure 2: Small Signal Analysis

(c) Compute for the small signal parameters of the MOSFET g_m, r_i and r_o. [1 pts]

$$g_m = \frac{2I_D}{V_{GS} - V_{TH}} \tag{5}$$

$$\boxed{g_m = 201.49\,\frac{\mu\text{A}}{\text{V}}}$$

$$r_o = \frac{1}{\lambda I_D} \tag{6}$$

$$\boxed{r_o = 20\,\text{M}\Omega}$$

$$\boxed{r_i = \infty}$$

(d) Compute for the G_m, R_i, R_o and A_V of the circuit. [3 pts] To Compute for the circuit transconductance we solve for $G_m = \frac{i_o}{v_i}$ at no-load.

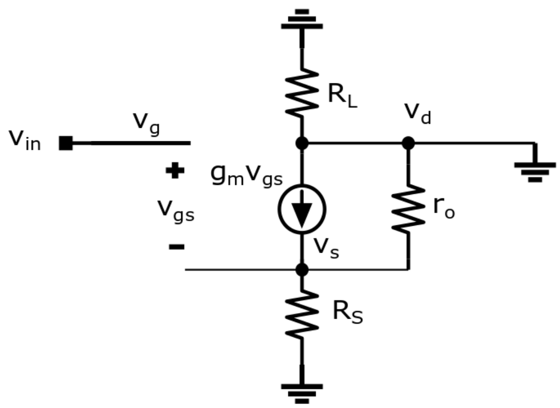

Figure 3: Small Signal Analysis no-load

Since the output current i_o passing to both r_o and the dependent source is the same current passing through R_S.

$$i_o = i_s = \frac{v_s}{R_S} \tag{7}$$

Using KCL at the drain node.

$$i_o = g_m v_{gs} - \frac{v_s}{r_o} \tag{8}$$

Using $v_{gs} = v_{in} - v_s$ to equation 9.

$$i_o = g_m v_{in} - v_s(g_m + \frac{1}{r_o}) \tag{9}$$

Combining equations 10 and 8.

$$i_o = g_m v_{in} - i_o R_S(g_m + \frac{1}{r_o}) \tag{10}$$

We get

$$G_m = \frac{i_o}{v_{in}} = \frac{g_m}{1 + \frac{R_S}{r_o}(g_m r_o + 1} \tag{11}$$

The transconductance $G_m = 40.05\,\frac{\mu A}{V}$ [1 pt]

The input impedance is infinity $R_i = \infty$

To Compute for the circuit output impedance we solve for $R_o = \frac{v_o}{i_o}$ at zero input.

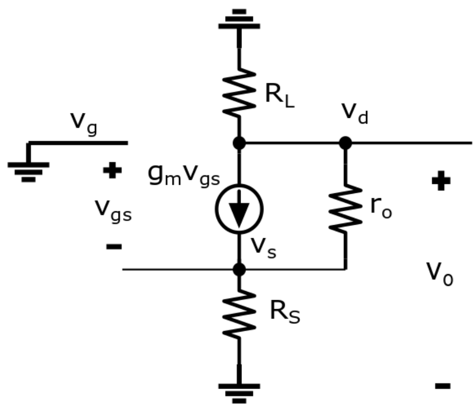

Figure 4: Small Signal Analysis zero input

Since the output impedance will be the parallel impedance of R_L and the whole other side of the small signal circuit, we can first compute the impedance of the other side while ignoring R_L.

$$v_o = i_{ro} r_o + i_s R_S \tag{12}$$

$$i_{ro} = i'_o - gm(-vs) \tag{13}$$

Combining equation 12 and 13.

$$v_o = (i'_o + g_m i_s RS) r_o + i_s R_S \tag{14}$$

Using $i'_o = i_s$ on equation 14 since the current that flows through the r_o and the dependent source is the same as i_s and adding the parallel R_L

$$R_o = \frac{v_o}{i_o} = (r_o + g_m r_o RS + RS) // R_L \tag{15}$$

The output impedance $R_o = 49.975\,\mathrm{k\Omega}$. [1pt]

$$\boxed{\text{Therefore the gain is } A_V = G_m R_o = -2 \text{ [1pt]}}$$

2. **BJT Single Stage Amplifier**. A BJT Q_1 with $\beta = 100$, $I_S = 10\,\text{fA}$, $V_{CE,sat} = 0.2\,\text{V}$ and $V_A = 200\,\text{V}$ is biased with resistors. The resistors used are $R_C = 500\,\Omega$, $R_B = 50\,\text{k}\Omega$, $R_E = 300\,\Omega$. The supply voltage V_{CC} is $5\,\text{V}$. An ideal, DC-blocked input is connected to the base, as shown in Figure 5a.

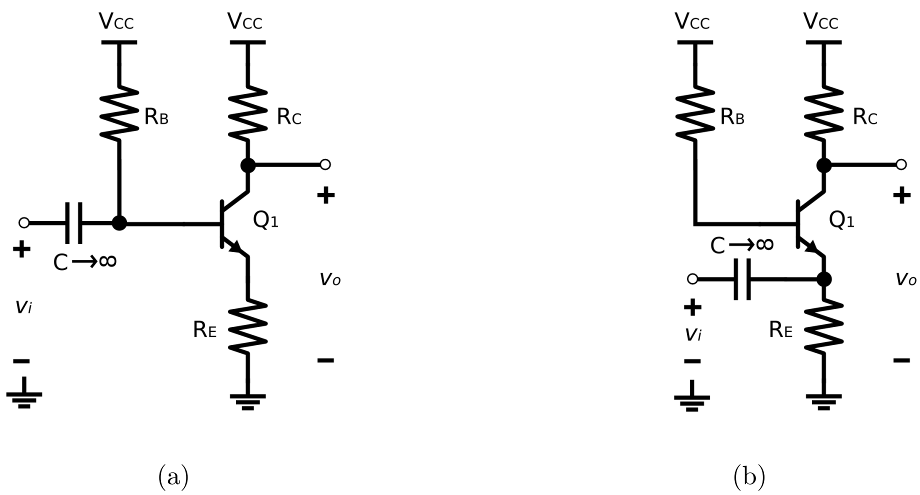

Figure 5: BJT Single-Stage Amplifier

(a) Determine I_C, V_{CE}, and V_O. Confirm that the biasing allows Q_1 to operate in forward active mode. From that, determine Q_1's parameters g_m, r_π, and r_o. State all necessary assumptions. [3 pts]

Start solving for I_C. A KVL equation passing through the base gives

$$V_{CC} = \frac{I_C}{\beta} R_B + V_{BE} + \frac{\beta + 1}{\beta} I_C R_E \tag{16}$$

Assume first that Q_1 is in forward active, and this should be verified later. V_{BE} is then related to I_C with the equation

$$I_C = I_S \left(e^{\frac{V_{BE}}{V_T}} - 1 \right) \left(1 + \frac{V_{CE}}{V_A} \right) \tag{17}$$

Even if V_{CE} is not known, it cannot exceed V_{CC} which is already much smaller than V_A. Therefore $\frac{V_{CE}}{V_A}$ approaches zero and can be assumed to have no significant effect.

Similarly, I_C is expected to be much larger than I_S and therefore $\left(e^{\frac{V_{BE}}{V_T}} - 1 \right)$ can be simplified to $e^{\frac{V_{BE}}{V_T}}$. The resulting equation is now

$$I_C \approx I_S \, e^{\frac{V_{BE}}{V_T}} \tag{18}$$

which is much simpler to manipulate. This can be manipulated to show V_{BE} as a function of I_C as

$$V_{BE} \approx V_T \; ln \left(\frac{I_C}{I_S} \right) \tag{19}$$

Returning to Equation (16), substituting V_{BE} gives

$$V_{CC} = \frac{I_C}{\beta} R_B + V_T \; ln \left(\frac{I_C}{I_S} \right) + \frac{\beta + 1}{\beta} I_C R_E \tag{20}$$

Rearranging and grouping the terms gives

$$\frac{I_C}{\beta}\left(R_B + (\beta+1)\,R_E\right) = V_{CC} - V_T\;ln\left(\frac{I_C}{I_S}\right) \tag{21}$$

Then isolating the I_C term on the left (but not including the I_C term inside the ln() for an iterative solution) gives

$$I_C = \beta\frac{V_{CC} - V_T\;ln\left(\frac{I_C}{I_S}\right)}{R_B + (\beta+1)\,R_E} \tag{22}$$

Note that $(\beta+1)\,R_E$ is **not** much larger than R_B and so R_B cannot be omitted.

Equation (22) can now be used to solve for I_C iteratively. Starting with a value of $I_C = 1\,\mathrm{mA}$, the solution converged to

I_C (inside ln())	I_C (resulting)
1.000 mA	5.407 mA
5.407 mA	5.352 mA
5.352 mA	5.352 mA

And so $I_C = 5.352\,\mathrm{mA}$.

$I_C = 5.352\,\mathrm{mA}$ [1 pt]

With I_C, a KVL equation passing through the collector gives

$$V_{CC} = I_C R_C + V_{CE} + \frac{\beta+1}{\beta} I_C R_E \tag{23}$$

Which results in a V_{CE} of 0.702 V.

Similarly, V_O can be found with

$$V_{CC} - I_C R_C = V_O \tag{24}$$

and so $V_O = 2.324\,\mathrm{V}$.

$V_{CE} = 0.702\,\mathrm{V}$, $V_{OUT} = 2.324\,\mathrm{V}$. Because V_{CE} is well above $V_{CE,sat}$, and I_C is much larger than I_S, which implies V_{BE} is biased properly, it can be confirmed that Q_1 is in forward active. [1 pt]

Also, by knowing I_C, the small-signal parameters intrinsic to Q_1 at this bias point can all be found.

$$r_\pi = \frac{\beta V_T}{I_C} \quad g_m = \frac{I_C}{V_T} \quad r_o = \frac{V_A}{I_C} \tag{25}$$

$r_\pi = 485.8\,\Omega \quad g_m = 205.85\,\mathrm{mA\,V^{-1}} \quad r_o = 37.369\,\mathrm{k\Omega}$ [1 pt]

Note that r_π is rather small.

(b) With the way the input is connected, the amplifier is a common-emitter with emitter degeneration. Determine this amplifier's G_m, R_o, A_v, and R_i. State all necessary assumptions. [3 pts]

In order to account for emitter degeneration, the two-port model first to be analyzed is without the resistors R_C and R_B, as shown with the red box in Figure 6. This is to simplify analysis. It is then understood that for the two-port parameters R_i', G_m', R_o' to be taken, that

$$G_m = G_m' \quad R_o = (R_o'||R_C) \quad R_i = (R_i'||R_B) \tag{26}$$

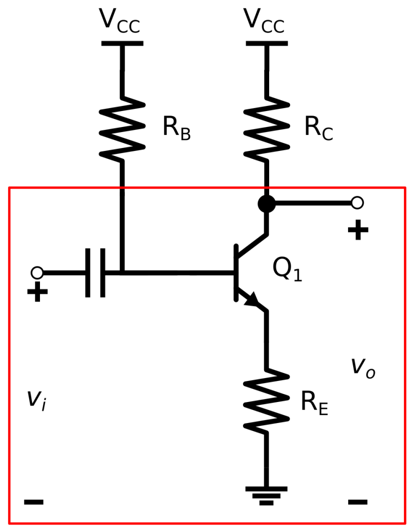

Figure 6: Common-emitter with emitter degeneration

The small signal model is then

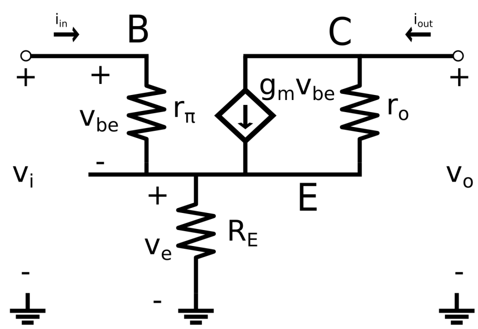

Figure 7: Small-signal model of the CE with degeneration

Begin with solving for G_m' and R_i', which both assume that the output is at no-load condition (v_o is 0), and that there is an input v_i and i_{in}.
Starting by finding an equation for v_e, a KCL at node E gives

$$i_{in} - \frac{v_e}{R_E} - \frac{v_e}{r_o} + g_m v_{be} = 0 \tag{27}$$

By isolating v_e, it is then

$$v_e = (i_{in} + g_m v_{be})\,(r_o||R_E) \tag{28}$$

Since $\frac{v_{be}}{i_{in}} = r_\pi$, it can be substituted and i_{in} be factored out.

$$v_e = i_{in}\,(1 + g_m r_\pi)\,(r_o||R_E) \tag{29}$$

Since i_{in} is in this equation, an equation for v_i will lead to R_i'. A KVL at the input side gives

$$v_i = i_{in} r_\pi + v_e \tag{30}$$

Combining Equation (29) and Equation (30), v_i can be shown as a function of i_{in} such that

$$v_i = i_{in} r_\pi + i_{in}\,(1 + g_m r_\pi)\,(r_o||R_E) \tag{31}$$

Factoring out i_{in} allows it to be brought to the left hand side and so R_i' is

$$R_i' = \frac{v_i}{i_{in}} = r_\pi + (1 + g_m r_\pi)(r_o || R_E) \tag{32}$$

Begin simplifying. Since r_o is much larger than R_E, the parallel resistance changes the equation to

$$R_i' = r_\pi + R_E + g_m r_\pi R_E \tag{33}$$

And then factoring out r_π gives

$$R_i' = r_\pi \left(1 + g_m R_E + \frac{R_E}{r_\pi}\right) \tag{34}$$

Since r_π is **not** much larger than R_E, the fraction cannot be simply canceled out.
For G_m', a KCL at node C gives

$$i_{out} = -\frac{v_e}{r_o} + g_m (v_i - v_e) \tag{35}$$

Grouping together the v_e terms results in

$$i_{out} = g_m v_i - v_e \left(\frac{1}{r_o} + g_m\right) \tag{36}$$

v_e must then be found, now in terms of v_i instead of i_{in}. A KCL at node E gives

$$-\frac{v_i - v_e}{r_\pi} + \frac{v_e}{R_E} + \frac{v_e}{r_o} - g_m (v_i - v_e) = 0 \tag{37}$$

The terms with v_e can be separated from the terms with v_i.

$$\frac{v_e}{r_\pi} + \frac{v_e}{R_E} + \frac{v_e}{r_o} + v_e g_m = v_i g_m + \frac{v_i}{r_\pi} \tag{38}$$

Isolating v_e gives

$$v_e = v_i \frac{g_m + \frac{1}{r_\pi}}{g_m + \frac{1}{r_\pi} + \frac{1}{R_E} + \frac{1}{r_o}} \tag{39}$$

Using this result for Equation (36) results in

$$i_{out} = g_m v_i - v_i \frac{g_m + \frac{1}{r_\pi}}{g_m + \frac{1}{r_\pi} + \frac{1}{R_E} + \frac{1}{r_o}} \left(\frac{1}{r_o} + g_m\right) \tag{40}$$

By factoring out v_i, it can be brought to the left hand side. Also, expanding the large right side term gives

$$G_m' = \frac{i_{out}}{v_i} = g_m - \frac{g_m^2 + \frac{g_m}{r_o} + \frac{g_m}{r_\pi} + \frac{1}{r_o r_\pi}}{g_m + \frac{1}{r_\pi} + \frac{1}{R_E} + \frac{1}{r_o}} \tag{41}$$

Factoring out g_m then including everything in the right hand side into the fraction gives

$$G_m' = g_m \left(\frac{g_m + \frac{1}{r_\pi} + \frac{1}{R_E} + \frac{1}{r_o} - g_m - \frac{1}{r_o} - \frac{1}{r_\pi} - \frac{1}{r_o r_\pi g_m}}{g_m + \frac{1}{r_\pi} + \frac{1}{R_E} + \frac{1}{r_o}}\right) \tag{42}$$

Then canceling the like terms,

$$G_m' = g_m \left(\frac{\frac{1}{R_E} - \frac{1}{r_o r_\pi g_m}}{g_m + \frac{1}{r_\pi} + \frac{1}{R_E} + \frac{1}{r_o}}\right) \tag{43}$$

Multiplying both sides of the fraction by R_E,

$$G'_m = g_m \left(\frac{1 - \frac{R_E}{r_o r_\pi g_m}}{g_m R_E + \frac{R_E}{r_\pi} + 1 + \frac{R_E}{r_o}} \right) \tag{44}$$

The terms divided by r_o can be safely canceled as r_o is very large, but again. r_π is **not** much larger than R_E. Therefore

$$G'_m = g_m \frac{1}{1 + g_m R_E + \frac{R_E}{r_\pi}} \tag{45}$$

Since $G_m = G'_m$, we can solve for G_m.

$$\boxed{G_m = 3.248\,\mathrm{mA\,V^{-1}}\ [1\ \mathrm{pt}]}$$

Finally, for R'_o, the input v_i is set to zero, and so v_e is the same as $-v_{be}$. The voltage v_e is then

$$v_e = i_{out} \left(R_E || r_\pi \right) \tag{46}$$

The output voltage is

$$v_o = \left(i_{out} - g_m v_{be} \right) r_o + v_e \tag{47}$$

Which is then

$$v_o = \left(i_{out} + g_m v_e \right) r_o + v_e \tag{48}$$

$$v_o = i_{out}\ r_o + v_e \left(1 + g_m r_o \right) \tag{49}$$

Since v_e is known,

$$v_o = i_{out}\ r_o + i_{out} \left(R_E || r_\pi \right) \left(1 + g_m r_o \right) \tag{50}$$

Then bringing i_{out} to the left side,

$$R'_o = \frac{v_o}{i_{out}} = r_o + \left(R_E || r_\pi \right) \left(1 + g_m r_o \right) \tag{51}$$

Factoring out r_o,

$$R'_o = r_o \left(1 + \frac{R_E || r_\pi}{r_o} + \left(R_E || r_\pi \right) g_m \right) \tag{52}$$

Since r_o is much larger than the parallel resistance, it can be canceled, but the parallel resistance multiplied to g_m cannot be simplified as the values of R_E and r_π are too close together. Therefore

$$R'_o = r_o \left(1 + g_m \left(R_E || r_\pi \right) \right) \tag{53}$$

Now that these intermediate parameters have been solved,

$$R_i = R'_i || R_B = r_\pi \left(1 + g_m R_E + \frac{R_E}{r_\pi} \right) || R_B \tag{54}$$

$$\boxed{R_i = 19.054\,\mathrm{k\Omega}\ [1\ \mathrm{pt}]}$$

$$R_o = R_o'||R_C = \left(r_o \left(1 + g_m \left(R_E||r_\pi\right)\right)\right)||R_C \tag{55}$$

$$A_v = -G_m R_o \tag{56}$$

$$\boxed{R_o = 499.8\,\Omega \quad A_v = -1.624\tfrac{V}{V}\ \text{[1 pt]}}$$

(c) The ideal, DC-blocked input is then disconnected, and reconnected at the emitter, as shown in Figure 5b. Determine this amplifier's G_m, R_o, A_v, and R_i. State all necessary assumptions. [3 pts]

The input being placed at the emitter changes the topology closer to that of a common-base as commonly known. However, because a resistor was used to bias the base, R_B changes the two-port characteristics in a way somewhat like emitter degeneration for the common-emitter.

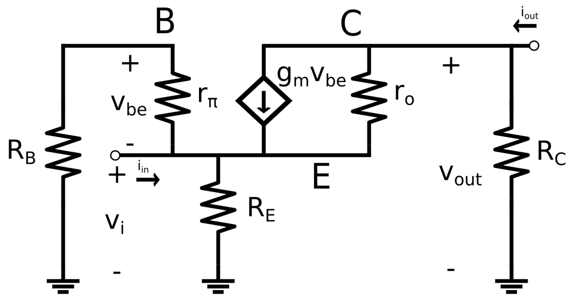

Figure 8: Modified Common-base

R_o is quite simple to solve for. If the input voltage is zero, then the voltages across R_E, r_π and R_B are all zero. This sets v_{be} to zero, which also causes the $g_m v_{be}$ current source to act as an open. Therefore,

$$R_o = r_o||R_C \tag{57}$$

which is then $493.4\,\Omega$.

To start solving for G_m, a KCL at C can be used to have an equation involving i_{out}. Since v_o is zero for no-load conditions, the voltage across r_o is $-v_i$.

$$i_{out} = -\frac{v_i}{r_o} + g_m v_{be} \tag{58}$$

v_{be} is simply a voltage division as v_i drops across the series resistance of R_B and r_π.

$$v_{be} = -v_i \frac{r_\pi}{R_B + r_\pi} \tag{59}$$

Therefore, Equation (58) can be rewritten as

$$i_{out} = -v_i \left(\frac{1}{r_o} + \frac{g_m r_\pi}{R_B + r_\pi}\right) \tag{60}$$

G_m is then

$$G_m = \frac{i_{out}}{v_i} = -\left(\frac{1}{r_o} + \frac{g_m r_\pi}{R_B + r_\pi}\right) \tag{61}$$

Similar to the emitter-degenerated common-emitter, it might be useful to bring out g_m for analysis.

$$G_m = -g_m \left(\frac{1}{g_m r_o} + \frac{r_\pi}{R_B + r_\pi}\right) \tag{62}$$

Since $g_m r_o$ is meant to be large, and in this case it is, the first term can be assumed to be insignificant (changes the result by about 1% in this case).

$$G_m \approx -g_m \left(\frac{r_\pi}{R_B + r_\pi} \right) \tag{63}$$

$$\boxed{G_m = -2.008\,\mathrm{mA\,V^{-1}}\ [1\text{ pt}]}$$

A_v is now known.

$$A_v = -G_m R_o = g_m \left(\frac{1}{g_m r_o} + \frac{r_\pi}{R_B + r_\pi} \right) (r_o || R_C) \tag{64}$$

$$\boxed{R_o = 493.4\,\Omega \quad A_v = 0.9905 \tfrac{V}{V}\ [1\text{ pt}]}$$

To solve for R_i, set v_o to zero again. A KCL at E gives

$$i_{in} + g_m v_{be} - \frac{v_i}{r_o} - \frac{v_i}{R_E} + \frac{v_{be}}{r_\pi} = 0 \tag{65}$$

Equation (59) still holds, so v_{be} can be replaced.

$$i_{in} = g_m v_i \frac{r_\pi}{R_B + r_\pi} + \frac{v_i}{r_o} + \frac{v_i}{R_E} + \frac{v_i}{r_\pi} \frac{r_\pi}{R_B + r_\pi} \tag{66}$$

Factoring out v_i,

$$i_{in} = v_i \left(\frac{g_m r_\pi}{R_B + r_\pi} + \frac{1}{R_B + r_\pi} + \frac{1}{r_o} + \frac{1}{R_E} \right) \tag{67}$$

R_i is then

$$R_i = \frac{v_i}{i_{in}} = \left(\frac{g_m r_\pi}{R_B + r_\pi} + \frac{1}{R_B + r_\pi} + \frac{1}{r_o} + \frac{1}{R_E} \right)^{-1} \tag{68}$$

Or as expressed as parallel resistances,

$$R_i = R_E || r_o || \left(R_B + r_\pi \right) || \frac{R_B + r_\pi}{g_m r_\pi} \tag{69}$$

$$\boxed{R_i = 186.5\,\Omega\ [1\text{ pt}]}$$

3. **MOSFET Single Stage CD Amplifier**. In the circuit shown in Figure 3, the transistor is biased with an ideal current source $I_S = 0.82mA$. The voltage input to the transistor is a purely AC signal. Given that $|V_{TH}| = 3V$, k $= 400\,\mu\mathrm{A}/V^2$ and $\lambda = 0.001V^{-1}$, assuming there is no body effect and ignoring channel length modulation while biasing, determine the following:

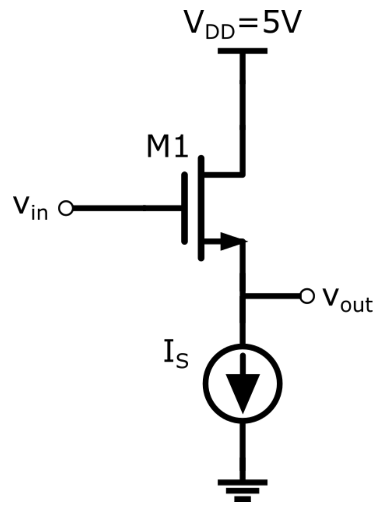

Figure 9: MOSFET Single Stage CD Amplifier

(a) What is the gate-to-source voltage of the transistor? State all necessary assumptions. [3 pts]

First, assume that M1 is operating in the saturation region. This gives us $I_{DS} = I_S = 0.82mA$. We can then use the equation for I_{DS} in saturation region to find V_{GS}.

$$I_{DS} = k \cdot (V_{GS} - V_{TH})^2 \cdot (1 + \lambda V_{DS}) \tag{70}$$

Since channel length modulation is ignored in biasing, we can approximate (70) and re-write it as,

$$I_{DS} = k \cdot (V_{GS} - V_{TH})^2 \tag{71}$$

Rewriting (71),

$$V_{GS} = \sqrt{\frac{I_{DS}}{k}} - V_{TH} = \sqrt{\frac{0.82mA}{400\mu A/V^2}} - 3V = 4.4317V \tag{72}$$

To check if the assumptions made are correct, we need to prove that $V_{DS} > V_{GS} - V_{TH}$. V_{GS} can be expressed as,

$$V_{GS} = V_{GG} - V_{SS} \tag{73}$$

Given a purely AC signal at the input ($V_{GG} = 0V$) and rewriting (73),

$$V_{SS} = V_{GG} - V_{GS} = 0V - 4.4317V = -4.4317 \tag{74}$$

Solving for V_{DS},

$$V_{DS} = V_{DD} - V_{SS} = 5V - (-4.4317V) = 9.4317V \tag{75}$$

Since $V_{GS} - V_{TH} = 1.4317$ and $V_{DS} > V_{GS} - V_{TH}$, then our assumption of the transistor being saturated is correct. Therefore, $V_{GS} = 4.4317V$ [3 pts]

(b) Draw the small-signal equivalent circuit. Properly label all parameters, voltages, and terminal names. [1 pt]

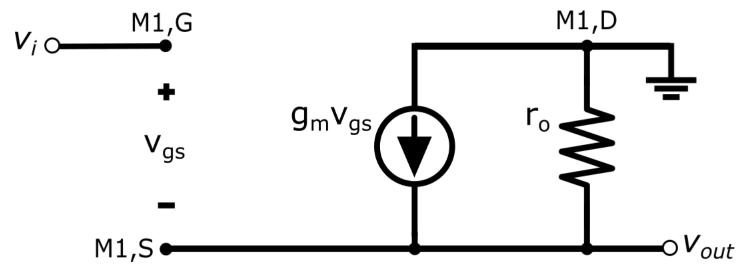

Figure 10: Small-signal model of the MOSFET Single Stage CD Amplifier

(c) Determine the expression for the circuit's transconductance G_m, input and output resistances R_i and R_o, and voltage gain A_V in terms of the small signal parameters. [2 pts]

Using KCL at the v_{out} node,

$$i_{out} = \frac{v_{out}}{r_{out}} - g_m v_{gs} = \frac{v_{out}}{r_{out}} - g_m(v_{in} - v_{out}) \tag{76}$$

Since $v_{out} = 0$ at no-load condition,

$$i_{out} = -g_m v_{in} \tag{77}$$

Solving for $G_m = \frac{i_{out}}{v_{in}}$ at no-load,

$$\boxed{G_m = -g_m \text{ [0.5 pt]}}$$

Since the r_π of a MOSFET approaches ∞, therefore,

$$\boxed{R_i \to \infty \text{ [0.5 pt]}}$$

Using the KCL equation from (76) and assuming a zero-input condition ($v_i = 0$), we get,

$$i_{out} = \frac{v_{out}}{r_{out}} + g_m v_{out} = v_{out}(\frac{1}{r_{out}} + g_m) \tag{78}$$

From this, we can solve for the expression for $R_o = \frac{v_{out}}{i_{out}}$ at zero-input which is,

$$\boxed{R_o = r_o \parallel \frac{1}{g_m} = \frac{r_o}{1+g_m r_o} \text{ [0.5 pt]}}$$

Computing for A_v, we use the formula,

$$A_v = -G_m R_o \tag{79}$$

Substituting R_o and G_m, we get,

$$\boxed{A_V = \frac{g_m r_o}{1+g_m r_o} \text{ [0.5 pt]}}$$

(d) Compute for G_m, R_o and A_v. Write your complete solution. [2 pts]
The small signal transconductance g_m can be solved using the equation,

$$g_m = \frac{2I_{DS}}{V_{GS} - V_{TH}} \tag{80}$$

Substituting the values, we get a value of $1.145mS$. Solving for G_m, we get,

$$\boxed{G_m = -1.145mS \text{ [0.75 pt]}}$$

For the small-signal output resistance r_o, this can be obtained using,

$$r_o = \frac{1}{\lambda I_{DS}} \tag{81}$$

Using $\lambda = 0.001V^{-1}$ and $I_{DS} = 0.82mA$, we get $r_o = 1.22M\Omega$. Using the expression for R_o from (c), we get a computed value of,

$$R_o = r_o \parallel \frac{1}{g_m} = \frac{ro}{1 + g_m r_o} = \frac{1.22M\Omega}{1 + (1.145mS)(1.22M\Omega)} = 872.36\Omega \tag{82}$$

$$\boxed{R_o = 872.36\Omega \text{ [0.75 pt]}}$$

For A_v, we also use the expression also obtained from (c). Substituting the values,

$$A_V = \frac{g_m r_o}{1 + g_m r_o} = \frac{(1.145mS)(1.22M\Omega)}{1 + (1.145mS)(1.22M\Omega)} = 0.99 \tag{83}$$

$$\boxed{A_v = 0.99 \text{ [0.5 pt]}}$$