2 Transistor Models

In EEE 41, you studied the fundamental concepts of how transistors can be realized using semiconductors, specifically, the two most popular transistors currently in use: the bipolar junction transistor (BJT) and the metal-oxide field-effect transistor (MOSFET).

In EEE 51, we want to be able to use these transistors (as well as other semiconductor devices such as diodes) to design and implement useful electronic circuits. This means we have to be able to describe, and eventually predict, the behavior of the terminal voltages and currents of these devices. We can accomplish this by using transistor models.

2.1 Large Signal Transistor Models

The large signal transistor models allow us to describe the electrical behavior of the transistor, when a voltage or current is varied over its allowable range. Here, we can see how varying the terminal voltages (and currents) determine the operating region of the transistor. Since there are many combinations of voltages and currents available for us to choose from, it is convenient to standardize which terminal voltages/currents we can use. This makes it easy to compare different transistors, as well as to systematically analyze and design electronic circuits based on these transistors.

The standard large signal transistor characteristics that we use are (1) the transfer characteristics, (2) the output characteristics, and (3) the input characteristics. Note that these models are normally considered as DC or low-frequency models, where we assume that the transistor parasitic capacitances are still negligible.

2.1.1 Transfer Characteristics

Transfer characteristics usually implies an input-output relationship, similar to the transfer function of a two-port network. But where are the input/output terminals or ports of a transistor? As we will see later on, one of the main goals of electronic circuits is to amplify signals (i.e. voltage, current, etc). Thus, it is convenient to choose the input-output terminal pair of a transistor that is commonly used to provide the largest amplification. In the BJT case, it is the common-emitter configuration, and in the MOSFET case, it is the common-source configuration.

The BJT Transfer Characteristic: The BJT common-emitter (CE) configuration is shown in Fig. 2.1a. We define the "input" signal as the base-emitter voltage, V_{BE}, and the "output" signal as the collector current I_C. Therefore, we can model the the BJT transfer characteristic using

$$I_C = I_S \cdot \left(e^{\frac{V_{BE}}{V_T}} - 1\right) \cdot \left(1 + \frac{V_{CE}}{V_A}\right) \tag{2.1}$$

for a transistor in the forward-active region[a]. Note that I_S is the reverse saturation current of the collector-base junction and dependent on the type and geometry of the transistor, V_A is the Early Voltage, and $V_T = \frac{kT}{q}$, the voltage equivalent of temperature ($V_T = 26\,\text{mV}$ at $T = 300\,\text{K}$). For typical discrete (not integrated) BJTs, the value of V_{BE} is around $0.6\,\text{mV}$, and since V_A is normally in the hundreds of volts, we can simplify Eq. 2.1 into

$$I_C = I_S \cdot e^{\frac{V_{BE}}{V_T}} \tag{2.2}$$

Eq. 2.2, shown in Fig. 2.1b, and superimposed with the transfer characteristic of a real BJT, allows us to approximate the behavior of the "output" signal, in this case the collector current, when we change the base-emitter voltage, our "input" signal.

The MOSFET Transfer Characteristic: The MOSFET common-source (CS) configuration is shown in Fig. 2.2a. As with the BJT, we define the "input" signal as the gate-source voltage, V_{GS}, and the "output" signal as the drain current, I_D. Thus, the MOSFET transfer characteristic can be modeled as

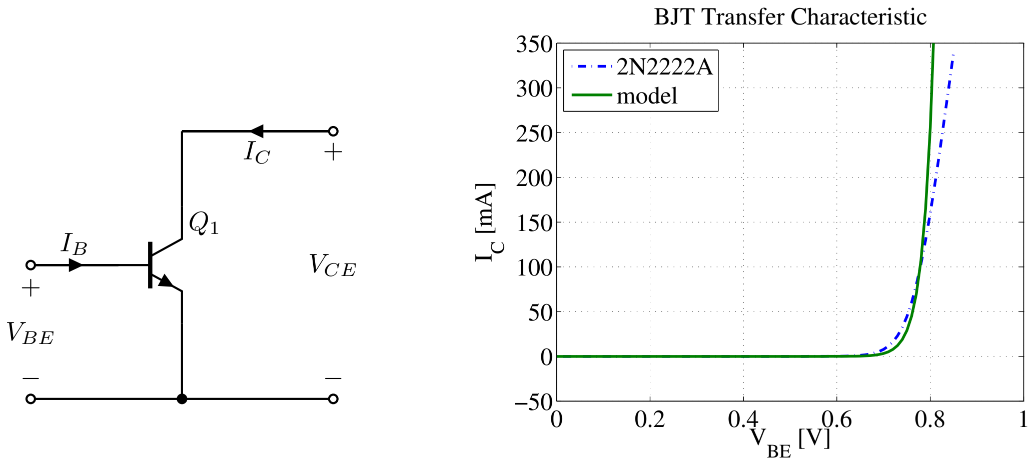
$$I_D = k \cdot (V_{GS} - V_{TH})^2 \cdot (1 + \lambda V_{DS}) \tag{2.3}$$

in the saturation region[b]. Note that k is a constant dependent on the technology used in the manufacture, and the geometry, of the transistor, V_{TH} is the transistor threshold voltage, and λ is the channel length modulation coefficient. For typical discrete (not integrated) MOSFETs, $\lambda \ll 1$, allowing us to approximate Eq. 2.3 with

$$I_D = k \cdot (V_{GS} - V_{TH})^2 \tag{2.4}$$

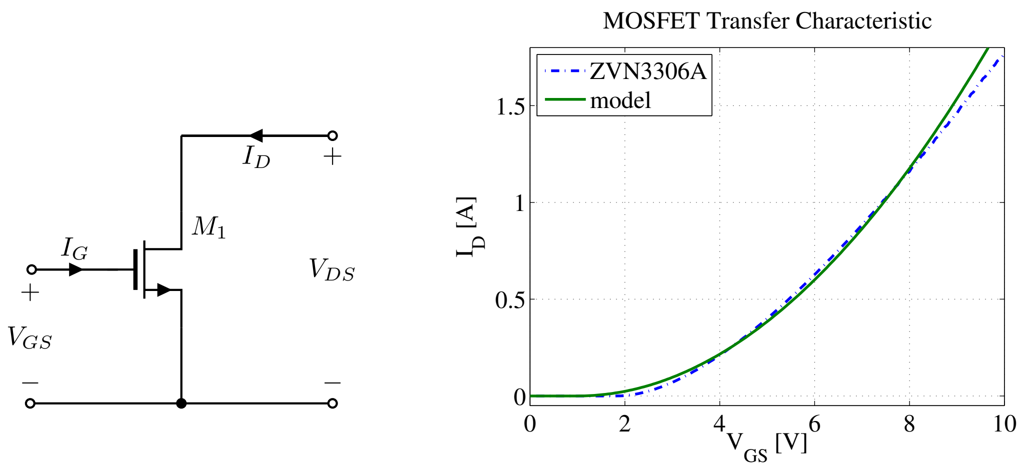
[a]In EEE 51, we will almost always use the BJT in the forward active region. However, in circuits where this is not the case, the transfer characteristic should be modeled with the appropriate relationship between V_{BE} and I_C.

[b]In EEE 51, we will almost always use the MOSFET in the saturation region. However, in circuits where this is not the case, the transfer characteristic should be modeled with the appropriate relationship between V_{GS} and I_D.



(a) Common-emitter configuration (b) Transfer characteristic

Figure 2.1: The NPN bipolar junction transistor (BJT).



(a) Common-source configuration (b) Transfer characteristic

Figure 2.2: The N-type metal-oxide field effect transistor (NMOSFET).

And just like the BJT transfer characteristic, Eq. 2.4 allows us to model the behavior of the "output" signal, in this case the drain current, when we change the gate-source voltage, our "input" signal. Fig. 2.2b shows the transfer characteristic of a real MOSFET, together with the plot of Eq. 2.4.

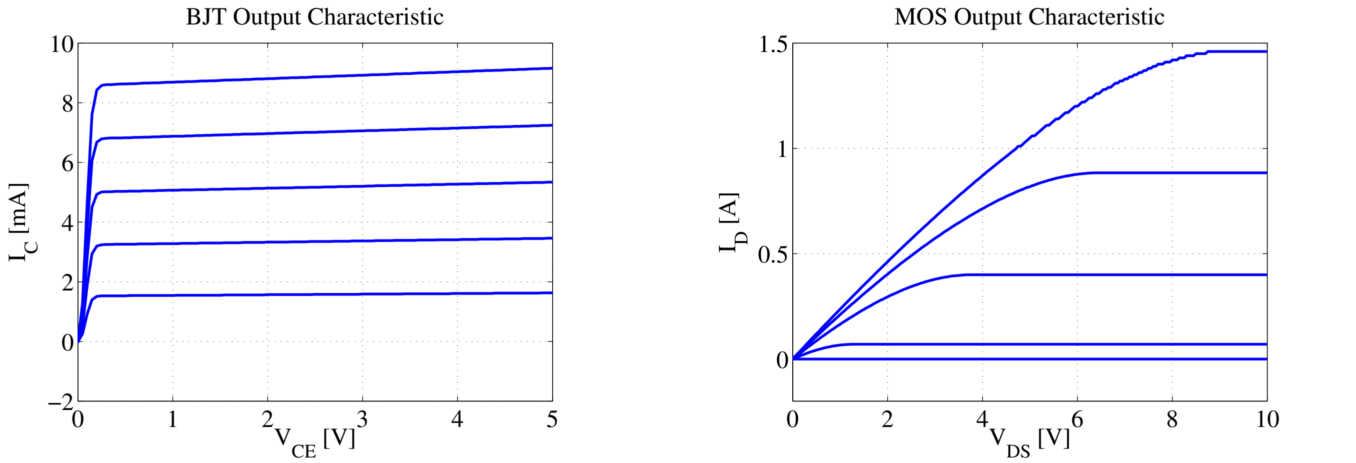
2.1.2 Output Characteristics

In both the BJT and MOSFET, we defined our "output" as either the collector current or drain current. However, from Eqs. 2.1 and 2.3, we can see that both these "output" currents are also dependent on their respective "output" voltages – the collector-emitter voltage, V_{CE} for the BJT and the drain-source voltage, V_{DS} for the MOSFET.

Since we want to be able to describe the transistor behavior completely, we need to take into account all the factors that affect the "output" current. Hence, we use the transistor output characteristic, I_C vs. V_{CE} for a BJT and I_D vs. V_{DS} for a MOSFET, to model the change in "output" current when the "output" voltage is changed.

The output characteristics of a typical BJT is shown in Fig. 2.3a, and Fig. 2.3b shows the output characteristic of a typical MOSFET. Note that in both cases, the output characteristics span several operating regions. In the case of the BJT, the saturation, forward-active and cut-off regions can be seen. The same is true for the MOSFET, where the linear or ohmic region, the saturation region, and the sub-threshold region[c] can be identified.

[c]It is a common simplification to assume that the drain current is zero when $V_{GS} < V_{TH}$, putting the MOSFET in the "cut-off" region. However, keep in mind that below the threshold voltage, the MOSFET behaves like a BJT with very small currents. In very low-power applications, some MOSFETs are intentionally used in this sub-threshold region.



(a) The 2N2222A NPN BJT output characteristic, where I_B is swept from $10\,\mu$A to $50\,\mu$A in steps of $10\,\mu$A.

(b) The ZVN3306A NMOSFET output characteristic, where V_{GS} is swept from 1 V to 9 V in 2 V steps.

Figure 2.3: Transistor output characteristics.

2.1.3 Input Characteristics

Since the BJT and MOSFET[d] can be considered a three-terminal device, to completely describe all the currents and voltages, we will need 2 out of 3 voltages, and 2 out of 3 node currents. This means that in a BJT, if we know V_{BE} and V_{CE}, we automatically know the value of V_{BC} by KVL. By knowing the transfer and output characteristics, we can determine all the voltages in the BJT. The same is true for a MOSFET.

However, if we want to determine all the currents in the transistor, we need another set of current-voltage (I-V) characteristics. It turns out that the most convenient set of I-V characteristics for designing linear electronic amplifiers, is the "input" current vs. the "input" voltage characteristic. In BJTs, this is the V_{BE} vs. I_B characteristic, and for the MOSFET, this is the V_{GS} vs. I_G characteristic. For a BJT, by knowing I_B and I_C, we can easily determine I_E using KCL. Again, the same is true for MOSFETs.

The input characteristics of a BJT can be derived from the BJT transfer characteristics. Since $I_B = \frac{I_C}{\beta}$, we can just divide the BJT transfer characteristic by the transistor β[e]. The input characteristic of a MOSFET is trivial, since $I_G = 0$[f].

In summary, these large signal models allow us to determine the resulting transistor "input" and "output" DC currents when we apply DC voltages across the transistor terminals. Since we can obtain the DC values of the currents and voltages in a transistor, the large signal transistor models are typically used in what is called "DC Analysis".

2.2 Small Signal Modeling

As we have just seen, semiconductor-based transistors are very nonlinear – exponential behavior for BJTs and quadratic behavior for MOSFETs, and these nonlinearities can vary significantly when going from one operating region to another. Analyzing circuits with these devices, by hand, using these large signal models, becomes very complex very quickly, especially when the number of transistors start to increase. Imagine writing your node or loop equations with exponentials and quadratic functions!

In order to analyze, and eventually design, linear electronic circuits, we use two very powerful tools: linearization, and two-port network reduction. These two tools will allow us to (1) use our linear circuit theory skills (taken up in EEE 31 and 33), and (2) break up complex circuits into smaller and simpler ones.

2.2.1 Linearizing the Transistor Transfer Characteristic

In many cases, and often by design, the input signal of an amplifier is made to change by a relatively small amount on top of its DC value. To determine the effect of this disturbance on the rest of the terminal voltages and currents, we can use the large signal models. However, this would result in rather complicated mathematical expressions, with

[d]In EEE 51, we will ignore the MOSFET body effect, and always assume that the body terminal is always connected to the source terminal.

[e]Note that this assumes β is constant over I_C. If the variation in β is significant, then divide the transfer characteristic point-by-point with $\beta(I_D)$.

[f]This ignores MOSFET gate leakage, which will not be taken up in EEE 51.

limited intuitive value. We will use the linearization process to reduce the complexity of the computation, as well as get a better intuitive grasp of the circuit implications, but at the cost of a certain amount of error.

Consider an NPN BJT, in the forward-active region, that is biased with a DC base-emitter voltage of $V_{BE,Q}$. This would result in a DC collector current $I_{C,Q}$, and from Eq. 2.2, is equal to

$$I_{C,Q} = I_S \cdot e^{\frac{V_{BE,Q}}{V_T}} \tag{2.5}$$

Note that we use the subscript Q to indicate that this is the quiescent[g] point DC bias, meaning that this is the purely DC voltage and current of the BJT when there are no disturbances present.

Suppose we add a signal v_{be} on top of $V_{BE,Q}$, such that the total base-emitter voltage is now

$$v_{BE} = V_{BE,Q} + v_{be} \tag{2.6}$$

Assuming that v_{be} is small enough that the transistor does not change its operating region, the collector current then becomes

$$i_C = I_{C,Q} + i_c = I_S \cdot e^{\frac{v_{BE}}{V_T}} = I_S \cdot e^{\frac{V_{BE,Q}+v_{be}}{V_T}} \tag{2.7}$$

where i_c is the collector current deviation away from its quiescent point, due to the addition of the "small" signal v_{be}. Simplifying Eq. 2.7, and using Eq. 2.5, we get

$$I_{C,Q} + i_c = I_S \cdot e^{\frac{V_{BE,Q}}{V_T}} \cdot e^{\frac{v_{be}}{V_T}} = I_{C,Q} \cdot e^{\frac{v_{be}}{V_T}} \tag{2.8}$$

As expected, i_c is a nonlinear function (exponential) of v_{be}. Now, let us try to linearize this relationship. Here we present two methods, but as we will see, they turn out to be equivalent.

Method 1: Recall, that any function that is infinitely differentiable, can be expressed as an infinite sum of terms, and that these terms are calculated from its derivatives at a certain point. We call this infinite sum the Taylor Series representation of a function, such that

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \ldots \tag{2.9}$$

where the function $f(x)$ is expanded about point a. Expanding $f(x) = e^x$, about $a = 0$, gives us the well known relationship

$$e^x = e^0 + \frac{e^0}{1!}x + \frac{e^0}{2!}x^2 + \frac{e^0}{3!}x^3 + \ldots = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots \tag{2.10}$$

Thus, expanding $e^{\frac{v_{be}}{V_T}}$, we get

$$e^{\frac{v_{be}}{V_T}} = 1 + \frac{v_{be}}{V_T} + \frac{1}{2!}\left(\frac{v_{be}}{V_T}\right)^2 + \frac{1}{3!}\left(\frac{v_{be}}{V_T}\right)^3 + \ldots \tag{2.11}$$

Substituting Eq. 2.11 into Eq. 2.8 gives us

$$I_{C,Q} + i_c = I_{C,Q} + \frac{I_{C,Q}}{V_T}v_{be} + \frac{1}{2!}I_{C,Q}\left(\frac{v_{be}}{V_T}\right)^2 + \frac{1}{3!}I_{C,Q}\left(\frac{v_{be}}{V_T}\right)^3 + \ldots \tag{2.12}$$

We can remove the DC quiescent current term $I_{C,Q}$ from both sides of Eq. 2.12, giving us

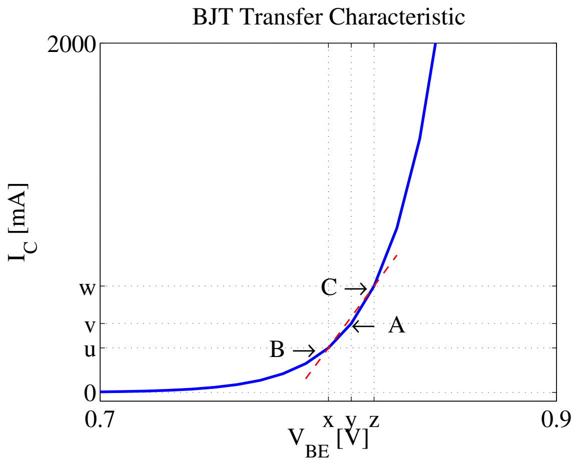
$$i_c = \frac{I_{C,Q}}{V_T}v_{be} + \frac{1}{2!}I_{C,Q}\left(\frac{v_{be}}{V_T}\right)^2 + \frac{1}{3!}I_{C,Q}\left(\frac{v_{be}}{V_T}\right)^3 + \ldots \tag{2.13}$$

Eq. 2.13 is a very important result. First, it gives us an alternative to Eq. 2.8 in computing for i_c, but more importantly, it shows us that for $\frac{v_{be}}{V_T} \ll 1$, all the terms of Eq. 2.13 become very much less than the first term. Thus, we can approximate i_c as

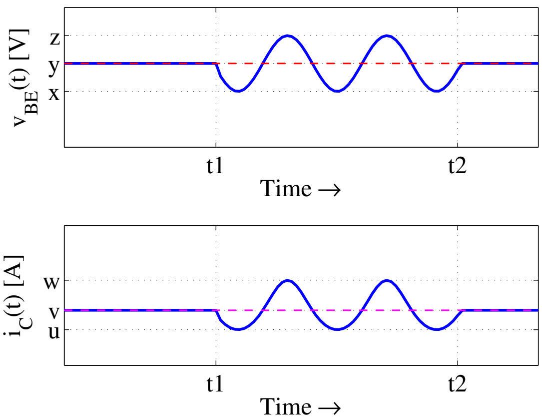
$$i_c = \frac{I_{C,Q}}{V_T} \cdot v_{be} \tag{2.14}$$

Note that Eq. 2.14 provides us with a "linear" relationship between i_c and v_{be}, and is a good approximation only when v_{be} is small enough, that is when $v_{be} \ll V_T$. In this case, we can assume that v_{be} is a "small signal". Eq. 2.13 also allows us to compute how much error we are incurring if we use Eq. 2.14, by summing up the remaining terms that we have ignored. Also, a very important observation here is that the relationship between i_c and v_{be} depends on $I_{C,Q}$.

[g] Synonymous to an idle, or rest state.



(a) The BJT transfer characteristic



(b) Time domain view

Figure 2.4: Linearization of the BJT transfer characteristic.

Method 2: Another approach to linearizing the relationship between i_c and v_{be} is by looking at this relationship graphically. Consider the graph of a BJT transfer characteristic shown in Fig. 2.4a.

If point A is the DC quiescent point, then $y = V_{BE,Q}$, and $v = I_{C,Q}$. Also, if $|v_{be}| = |z - x|$, then adding v_{be} on top of $V_{BE,Q}$ would result in the collector current also changing by an $|i_c| = |w - u|$. We are using the absolute value signs to indicate that both v_{be} and i_c can take on negative values. That is, when $v_{be} = 0$, the total base-emitter voltage is $v_{BE} = y = V_{BE,Q}$. When v_{be} is at its maximum value, $v_{BE} = z = V_{BE,Q} + v_{be,\max}$, and the transistor is operating at point C. When v_{be} is at its minimum (negative) value, $v_{BE} = x = V_{BE,Q} - |v_{be,\min}|$, corresponding to point B.

For clarity, a possible time domain view is given in Fig. 2.4b for $v_{BE}(t) = V_{BE,Q} + v_{be}(t)$ and $i_C(t) = I_{C,Q} + i_c(t)$. Note that for time $t < t_1$ and $t > t_2$, $v_{be}(t) = 0$, thus, $i_c(t) = 0$, which means that the transistor is in its quiescent point, that is $v_{BE}(t) = V_{BE,Q}$ and $i_C(t) = I_{C,Q}$.

We can approximate the relationship between v_{be} and i_c using a straight line (linear!) passing through points B and C. If we know the slope, m, of this line, we can estimate the magnitude i_c given v_{be} as $i_c = m \cdot v_{be}$. However, finding the slope of this line is not very straightforward, and as seen in Fig. 2.4a, using the dashed line will result in errors for points between B and C. On the other hand, if v_{be} is small, or equivalently if $x \to y$ and $z \to y$, then the error becomes smaller, and our linear approximation becomes more accurate. The slope of our linear approximation, if we make v_{be} approach zero, can be expressed as

$$m = \lim_{v_{be}\to 0} \frac{i_C\left(V_{BE,Q} + v_{be}\right) - i_C\left(V_{BE.Q}\right)}{V_{BE,Q} + v_{be} - V_{BE,Q}} \tag{2.15}$$

Recall that for a function, $f(x)$, its derivative at a point $x = X_0$ is given as

$$\left.\frac{\partial f(x)}{\partial x}\right|_{x=X_0} = \lim_{\Delta x\to 0} \frac{f(X_0 + \Delta x) - f(X_0)}{\Delta x} \tag{2.16}$$

Thus, for small values of v_{be}, we can approximate i_c as

$$i_c = \left.\frac{\partial I_C}{\partial V_{BE}}\right|_{V_{BE}=V_{BE,Q}} \cdot v_{be} = g_m \cdot v_{be} \tag{2.17}$$

where g_m is known as the device or transistor *transconductance*, since it relates small variations in the base-emitter voltage and collector current – the parameters of the transistor transfer characteristic. The transistor transconductance is formally defined as

$$g_m = \left.\frac{\partial I_C}{\partial V_{BE}}\right|_{V_{BE}=V_{BE,Q}} \tag{2.18}$$

Note that g_m has units of Siemens (S)[h], and is dependent on the quiescent point. Thus, a different $V_{BE,Q}$ would result in a different $I_{C,Q}$, and hence a different g_m.

By plugging in Eq. 2.2 into Eq. 2.18, we can calculate the value of the transconductance for a BJT as

[h] $1\,\mathrm{S} = 1\,\frac{1}{\Omega} = 1\,\mho$

$$g_m = \left.\frac{\partial I_C}{\partial V_{BE}}\right|_{V_{BE}=V_{BE,Q}} = \left.\frac{\partial}{\partial V_{BE}}\left(I_S \cdot e^{\frac{V_{BE}}{V_T}}\right)\right|_{V_{BE}=V_{BE,Q}} = \frac{1}{V_T} \cdot I_S \cdot e^{\frac{V_{BE,Q}}{V_T}} = \frac{I_{C,Q}}{V_T} \tag{2.19}$$

Thus, Eq. 2.19 makes Eq. 2.17 equivalent to Eq. 2.14, which should not be that surprising since the Taylor Series coefficients are obtained using derivatives and we are only approximating the transfer characteristic by the first derivative term.

We have just covered a very important concept: linearizing the BJT transfer function allows us to approximate the relationship of i_c and v_{be} using a linear function, given by Eq. 2.17, instead of using Eq. 2.8, an exponential equation. Also, if the signals we are dealing with are made smaller, then the errors we incur when using this linear approximation is reduced.

So far, we have used the BJT transfer characteristic. The process is exactly the same when dealing with MOSFETs. We can derive the Taylor Series expansion of Eq. 2.4 by just expanding the quadratic term

$$I_{D,Q} + i_d = k \cdot (V_{GS,Q} + v_{gs} - V_{TH})^2 = k \cdot (V_{GS,Q} - V_{TH})^2 + 2k \cdot (V_{GS,Q} - V_{TH}) \cdot v_{gs} + k \cdot v_{gs}^2 \tag{2.20}$$

Since Eq. 2.4 is already a polynomial, the Taylor Series representation shown in Eq. 2.20 is a finite series, and for our purposes, we group these terms into three. The first term in Eq. 2.20 is equal to $I_{D,Q}$. Therefore, we can write out the expression for i_d using the remaining two terms

$$i_d = 2k \cdot (V_{GS,Q} - V_{TH}) \cdot v_{gs} + k \cdot v_{gs}^2 \tag{2.21}$$

Again, if v_{gs} is small, then we can ignore the second term of Eq. 2.21, resulting in the linear relationship

$$i_d = 2k \cdot (V_{GS,Q} - V_{TH}) \cdot v_{gs} \tag{2.22}$$

If we use the definition of transconductance given in Eq. 2.18, and apply it to the MOSFET transfer characteristic given in Eq. 2.4, we would get

$$g_m = \left.\frac{\partial I_D}{\partial V_{GS}}\right|_{V_{GS}=V_{GS,Q}} = \left.\frac{\partial}{\partial V_{GS}}\left(k \cdot (V_{GS} - V_{TH})^2\right)\right|_{V_{GS}=V_{GS,Q}} = 2k \cdot (V_{GS,Q} - V_{TH}) \tag{2.23}$$

Using Eqs. 2.4 and 2.23, we can derive the other common forms of the MOSFET transconductance

$$g_m = 2k \cdot (V_{GS,Q} - V_{TH}) = \frac{2 \cdot I_{D,Q}}{V_{GS,Q} - V_{TH}} = \sqrt{4k \cdot I_{D,Q}} \tag{2.24}$$

Once again, we see that our two linearization methods are equivalent, thus

$$i_d = g_m \cdot v_{gs} \tag{2.25}$$

It is very important to remember that even though g_m is a function of the quiescent DC currents and voltages ($V_{BE,Q}$ and $I_{C,Q}$, or $V_{GS,Q}$ and $I_{D,Q}$), the linearized relationships using g_m in Eqs. 2.17 and 2.25 only relates the small signal disturbances superimposed on the DC signals, i_c and v_{be}, or i_d and v_{gs}. Also, the linearization methods presented here are general techniques, independent of the transistor operating region, and can be applied to the large signal characteristic of any device that we may want to linearize.

2.2.2 The Small Signal Equivalent Circuit

From the BJT output characteristic, we know that any small signal disturbance in the collector-emitter voltage will also result in a corresponding change in the collector current. In order to estimate this change in I_C due to a change in V_{CE}, we can easily extend our linearization of the BJT transfer characteristic to its output characteristic.

Consider a BJT in the forward active region, with quiescent current (using Eq. 2.1)

$$I_{C,Q} = I_S \cdot \left(e^{\frac{V_{BE,Q}}{V_T}} - 1\right) \cdot \left(1 + \frac{V_{CE,Q}}{V_A}\right) \tag{2.26}$$

Using our second linearization method, we define the *output conductance*[i], g_o, as

$$g_o = \left.\frac{\partial I_C}{\partial V_{CE}}\right|_{V_{CE}=V_{CE,Q}} \tag{2.27}$$

Thus,

[i]This term is called a conductance and not a transconductance since it relates the voltage across two terminals to the current flowing into one of the terminals.

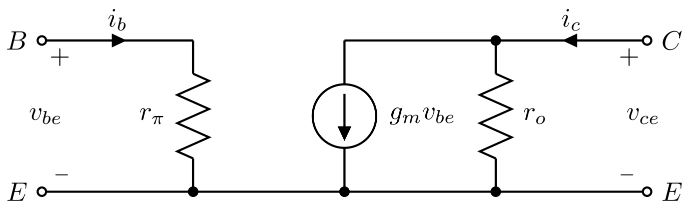


Figure 2.5: The BJT small signal equivalent circuit.

$$g_o = \frac{\partial}{\partial V_{CE}} \left(I_S \cdot \left(e^{\frac{V_{BE}}{V_T}} - 1 \right) \cdot \left(1 + \frac{V_{CE}}{V_A} \right) \right) \Bigg|_{V_{CE}=V_{CE,Q}} = I_S \cdot \left(e^{\frac{V_{BE,Q}}{V_T}} - 1 \right) \cdot \frac{1}{V_A} \approx \frac{I_{C,Q}}{V_A} \tag{2.28}$$

Often, it is convenient to express the output conductance of the BJT as an *output resistance*, r_o, given by

$$r_o = \frac{1}{g_o} = \frac{V_A}{I_{C,Q}} \tag{2.29}$$

Since g_o is the slope of the BJT output characteristic, we can then relate small changes in the collector-emitter voltage to the small changes in the collector current, using the linearized or small signal BJT output characteristic, given by

$$i_c = g_o \cdot v_{ce} = \frac{v_{ce}}{r_o} \tag{2.30}$$

By superposition, if we have small changes in the base-emitter voltage, and small changes in the collector-emitter voltage, we can combine Eqs. 2.17 and 2.30 to obtain the total small signal change in the collector current, resulting in

$$i_c = g_m \cdot v_{be} + \frac{v_{ce}}{r_o} \tag{2.31}$$

Lastly, if we add a small signal voltage on top of the base-emitter DC quiescent voltage, not only will there be changes in the collector current, we will also be a small signal disturbance in the base current. We can also linearize the relationship between v_{be} and i_b by extending our linearization process to the BJT input characteristic.

In the forward-active region, the quiescent base current is given by

$$I_{B,Q} = \frac{I_{C,Q}}{\beta} = \frac{I_S}{\beta} \cdot \left(e^{\frac{V_{BE,Q}}{V_T}} - 1 \right) \tag{2.32}$$

We define the BJT *input conductance*, g_π, as

$$g_\pi = \frac{\partial I_B}{\partial V_{BE}} \bigg|_{V_{BE}=V_{BE,Q}} = \frac{1}{\beta} \cdot \frac{\partial I_C}{\partial V_{BE}} \bigg|_{V_{BE}=V_{BE,Q}} = \frac{g_m}{\beta} = \frac{I_{C,Q}}{\beta \cdot V_T} \tag{2.33}$$

Again, it is often convenient to represent the input conductance as an *input resistance*, r_π, expressed as

$$r_\pi = \frac{1}{g_\pi} = \frac{\beta}{g_m} = \frac{\beta \cdot V_T}{I_{C,Q}} \tag{2.34}$$

Thus, we can approximate the small signal change in the base current as a linear function of v_{be}:

$$i_b = g_\pi \cdot v_{be} = \frac{v_{be}}{r_\pi} \tag{2.35}$$

Eqs. 2.31 and 2.35 are the linearized versions of the BJT transfer, input, and output characteristics, and are collectively known as the BJT *small signal model.* These equations give us a complete picture of what happens to a BJT when we superimpose small signals on top of the DC quiescent voltages and current.

We can create a circuit that can represent the BJT small signal model, called the BJT *small signal equivalent circuit*, as shown in Fig. 2.5. The small signal equivalent circuit represents the small signal model since by writing out the KCL and KVL equations for the equivalent circuit, we will get the BJT small signal model equations. This small signal equivalent circuit becomes extremely convenient when dealing with more complicated circuits containing multiple transistors.

The MOSFET small signal equivalent circuit can be derived using the same procedure used to derive the BJT small signal equivalent circuit. For a MOSFET in the saturation region, its quiescent DC drain current, from Eq. 2.3, is

$$I_{D,Q} = k \cdot (V_{GS,Q} - V_{TH})^2 \cdot (1 + \lambda V_{DS,Q}) \tag{2.36}$$

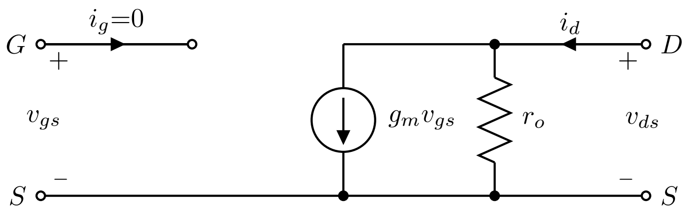


Figure 2.6: The MOSFET small signal equivalent circuit.

Thus, the MOSFET output conductance can be calculated as

$$g_o = \frac{\partial}{\partial V_{DS}} \left(k \cdot (V_{GS} - V_{TH})^2 \cdot (1 + \lambda V_{DS}) \right) \Big|_{V_{DS}=V_{DS,Q}} = k \cdot (V_{GS} - V_{TH})^2 \cdot \lambda \approx \lambda \cdot I_{D,Q} \tag{2.37}$$

and the corresponding MOSFET output resistance is

$$r_o = \frac{1}{g_o} = \frac{1}{\lambda \cdot I_{D,Q}} \tag{2.38}$$

Since the gate is insulated from the MOSFET channel, $I_G = 0$, therefore $i_g = 0$. This means that whatever v_{gs} we apply to the gate, there will be no change in gate current since it is always zero. If we calculate the MOSFET input conductance, we would get

$$g_\pi = \frac{\partial I_G}{\partial V_{GS}} \Big|_{V_{GS}=V_{GS,Q}} = 0 = \frac{1}{r_\pi} \tag{2.39}$$

resulting in a MOSFET input resistance of

$$r_\pi \to \infty \tag{2.40}$$

Therefore we can write the MOSFET small signal model, linearizing both its transfer and output characteristic, and with $i_g = 0$, as

$$i_d = g_m \cdot v_{gs} + \frac{v_{ds}}{r_o} \tag{2.41}$$

which leads us to the MOSFET small signal equivalent circuit is shown in Fig. 2.6. Note that the form of the BJT small signal equivalent circuit reduces to that of a MOSFET when $\beta \to \infty$, resulting in $r_\pi \to \infty$ and $i_b = 0$.

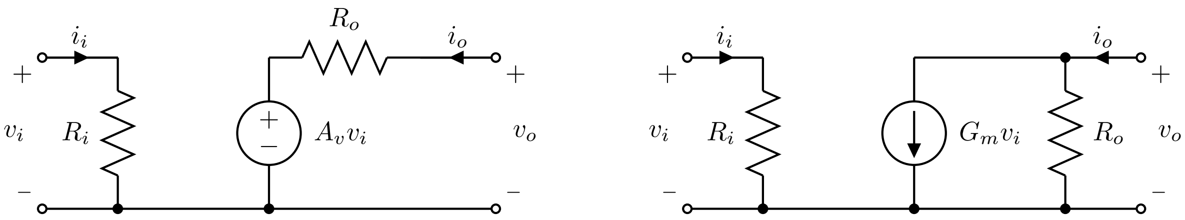
Both the BJT and MOSFET equivalent circuits are the linearized forms of the transistor's large signal characteristics. These small signal equivalent circuits (and models) have the following important characteristics:

- The small signal equivalent circuits are only valid for one operating or quiescent point. Note that the small signal parameters g_m, r_o, and r_π are dependent on the quiescent DC collector or drain current (and hence dependent on the DC voltages as well). If we change these currents, the parameter values of the small signal model (and circuit) will also change.
- These small signal circuits can only relate the "small signal" disturbances on top of the quiescent DC voltages and currents. Small signal analysis discards all the DC information once the linearization process is done. In order to compute the quiescent DC values, the large signal models have to be used.
- Since these circuits are already linear, all the linear circuit properties and analysis techniques (from EEE 31 and 33) are now applicable.

2.2.3 Two-Port Network Analysis

Since our transistor small signal models are linear, and these models have well-defined "input" and "output" ports in the BJT common-emitter and MOSFET common-source configurations, we can use one of the most powerful circuit analysis techniques: two-port network analysis. Two-port analysis allows us to reduce any linear circuit into four parameters, as long as we are only interested in the relationships among two pairs of voltages and currents. The most common two-port representations of circuits use the Z-, Y-, H-, S- and ABCD-parameters.

One of the most powerful uses of two-port network analysis is to enable circuit partitioning, for more intuitive analysis and design. In the case of EEE 51, we will use the unilateral hybrid-π two port representations, as shown in Fig. 2.7, since it will allow us to calculate the effects of adding loads to our amplifiers, such as speakers or other amplifiers. We are assuming unilateral operations for our transistors, meaning that the input voltage or current can



(a) The Thevenin model

(b) The Norton model

Figure 2.7: The unilateral hybrid-π two-port network equivalent circuits.

affect the output voltage or current, but the reverse is not true. Thus, for the unilateral hybrid-π equivalent circuit, we only need three two-port parameters. Fig. 2.7 also shows the polarity conventions that we will use in EEE 51.

Fig. 2.7a shows the Thevenin version of the hybrid-π two-port equivalent circuit. It has three parameters: the input resistance, R_i, the output resistance, R_o, and the voltage gain, A_v. The Norton equivalent hybrid-π is shown in Fig. 2.7b, with a circuit transconductance, G_m, instead of the voltage gain parameter. In order to calculate the hybrid-π parameters of any linear circuit, we need to define two circuit operating conditions: the no-load condition, and the zero-input condition.

The *no-load condition* is satisfied when the two-port network does not deliver any power from its output port to a load. Thus, for the Thevenin equivalent circuit in Fig. 2.7a (a voltage output circuit), the output current, i_o should be zero (i.e. an open circuit), and for the Norton equivalent circuit in Fig. 2.7b (a current output circuit), the output voltage, v_o, should be equal to zero (i.e. a short circuit).

The *zero-input condition* is satisfied when the input to the hybrid-π circuit is zero. This means that if a voltage source, v_s is driving the input, then $v_s = 0$. If the input port is being driven by a current source, i_s, then the zero-input condition is satisfied when $i_s = 0$. With these two conditions, we can now define the unilateral hybrid-π parameters.

The input resistance, R_i, is defined as the ratio of the input voltage, v_i, to the input current, i_i, at no load conditions, thus

$$R_i = \left.\frac{v_i}{i_i}\right|_{\text{no-load}} \tag{2.42}$$

Notice that at this point, it seems that the output no-load condition does not really affect the input resistance for the circuits in Fig. 2.7. However, in more complex circuits such as feedback amplifiers, this definition becomes a very important.

Output resistance, R_o, is defined as the ratio of the output voltage, v_o, to the output current, i_o, at zero-input conditions. We can write this as

$$R_o = \left.\frac{v_o}{i_o}\right|_{\text{zero-input}} \tag{2.43}$$

The reason why we take the output impedance at zero-input conditions should be easily seen from Fig. 2.7. Zeroing out the input (either voltage or current), results in $v_i = 0$. Thus, for the Thevenin circuit, $A_v \cdot v_i = 0$, shorting out the dependent voltage source, while for the Norton equivalent, $G_m \cdot v_i = 0$, resulting in the dependent current source acting like an open circuit.

The Thevenin circuit voltage gain, A_v, is defined as the ratio of the output voltage, v_o, to the input voltage, v_i, at no-load conditions:

$$A_v = \left.\frac{v_o}{v_i}\right|_{\text{no-load}} \tag{2.44}$$

For the Thevenin equivalent at no-load conditions, $i_o = 0$, therefore, the voltage drop across the output resistance, R_o, is zero, resulting in $A_v \cdot v_i = v_o$.

The Norton circuit transconductance, G_m, is defined as the ratio of the output current, i_o, to the input voltage, v_i, also at no-load conditions:

$$G_m = \left.\frac{i_o}{v_i}\right|_{\text{no-load}} \tag{2.45}$$

At no-load conditions, $v_o = 0$, thus, the current passing through the output resistance, R_o, is zero, which results in $G_m \cdot v_i = i_o$. Since the Thevenin and Norton circuits are equivalent, we can also express the circuit transconductance as

$$G_m = -\frac{A_v}{R_o} \tag{2.46}$$

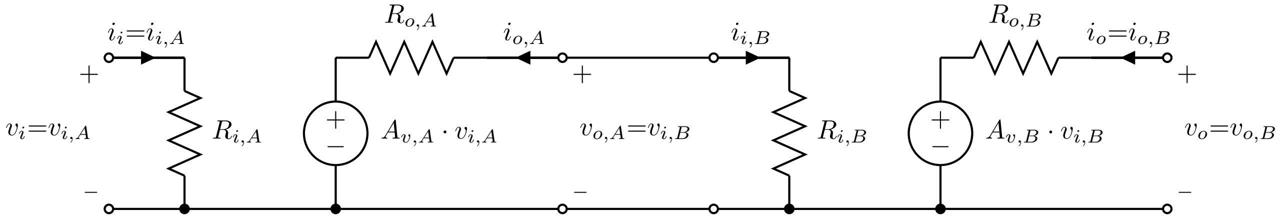


Figure 2.8: Cascading two linear circuits.

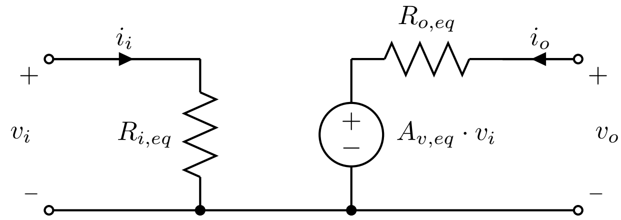


Figure 2.9: The quivalent hybrid-π circuit of the circuit in Fig. 2.8.

The negative sign is due to the polarity of the dependent current source in Fig. 2.7b.

Once we know the hybrid-π two-port equivalent circuit of two linear circuits, we can easily determine the overall behavior of the resulting circuit when these two linear circuits are cascaded. Fig. 2.8 shows two cascaded linear circuits A and B in their respective equivalent hybrid-π representations.

Using voltage division, we can compute the output voltage of circuit A, $v_{o,A}$ as

$$v_{o,A} = A_{v,A} \cdot v_i \cdot \frac{R_{i,B}}{R_{i,B} + R_{o,A}} = v_{i,B} \tag{2.47}$$

Thus, the equivalent no-load gain from v_i to v_o is

$$A_{v,eq} = \frac{v_o}{v_i} = A_{v,A} \cdot \frac{R_{i,B}}{R_{i,B} + R_{o,A}} \cdot A_{v,B} \tag{2.48}$$

Eq. 2.48 shows how circuit B loads circuit A, reducing the overall no-load voltage gain. If we used the Norton equivalent for circuit A, then we would have used current division instead of voltage division. By applying Eqs. 2.42 and 2.43, we can get the effective input resistance, $R_{i,eq} = R_{i,A}$, and output resistance, $R_{o,eq} = R_{o,B}$. Since the cascade combination of a linear circuit is also a linear circuit, we can derive the equivalent hybrid-π circuit of the circuit in Fig. 2.8, as shown in Fig. 2.9.

Now that we know how to linearize the behavior of an inherently non-linear device, such as a BJT or a MOSFET, when are inputs are small, as well as how to obtain the unilateral hybrid-π two-port quivalent circuit of any linear circuit, we can now apply these techniques to analyze the behavior of single-stage[j] transistor amplifiers.

[j] A single-stage amplifier is a transistor amplifier containing only one transistor as the amplifying element.