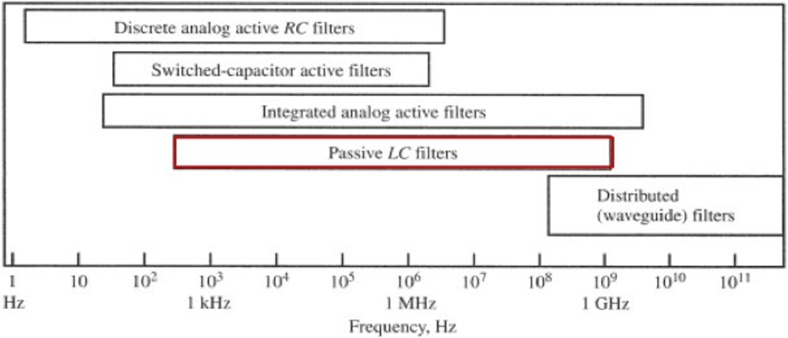
ECE 113: Communication Electronics

Meeting 10: Filter Design III

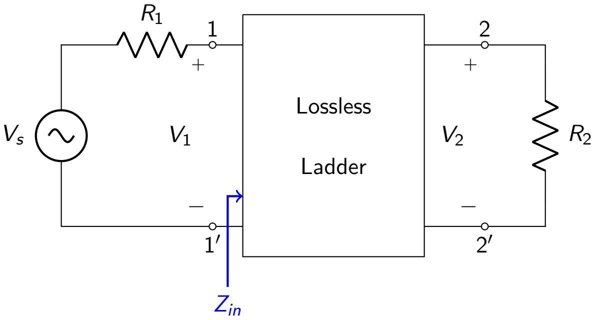
February 25, 2019

Filter Implementations



- LC Ladder Filters
- Lossless filters using lumped reactive components

LC Ladder Filters



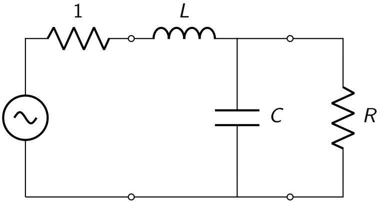
- Maximum power transfer $\rightarrow Z_{in} = R_1$

$$|H(j\omega)|^2 = \frac{P_2}{P_{1,max}} = 4\frac{R_1}{R_2}|\frac{V_2(j\omega)}{V_s(j\omega)}|^2 = 4\frac{R_1}{R_2}|T(j\omega)|^2 \leq 1$$

$$|H(j\omega)|^2 = \frac{4R_1R_{in}}{|R_1 + Z_{in}|^2} = 1 - \frac{|Z_{in} - R_1|^2}{|Z_{in} + R_1|^2} = 1 - |\Gamma|^2$$

Butterworth Filter Design

- Consider a 2^{nd} order lowpass filter



- For Butterworth response,

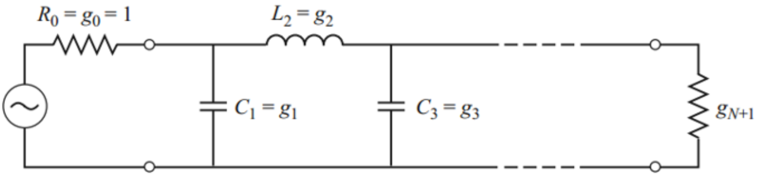
$$|H(j\omega)|^2 = \frac{1}{1+\omega^4} = 1 - |\Gamma|^2$$

$$\Gamma = \frac{Z_{in}-1}{Z_{in}+1} \qquad Z_{in} = j\omega L + \frac{R(1-j\omega RC)}{1+\omega^2 R^2 C^2}$$

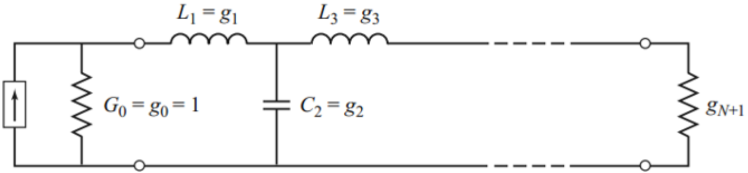
- $L = C = \sqrt{2} \qquad R = 1$

Lowpass Filter Prototype

- Derivation can be extended to any filter order **n**
- Derived LPF prototype has source impedance of 1Ω and normalized cut-off frequency of $1 rad/sec$



(a)



Butterworth LPF Prototype Table

TABLE 8.3 Element Values for Maximally Flat Low-Pass Filter Prototypes ($g_0 = 1$, $\omega_c = 1$, $N = 1$ to 10)

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	2.0000	1.0000									
2	1.4142	1.4142	1.0000								
3	1.0000	2.0000	1.0000	1.0000							
4	0.7654	1.8478	1.8478	0.7654	1.0000						
5	0.6180	1.6180	2.0000	1.6180	0.6180	1.0000					
6	0.5176	1.4142	1.9318	1.9318	1.4142	0.5176	1.0000				
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450	1.0000			
8	0.3902	1.1111	1.6629	1.9615	1.9615	1.6629	1.1111	0.3902	1.0000		
9	0.3473	1.0000	1.5321	1.8794	2.0000	1.8794	1.5321	1.0000	0.3473	1.0000	
10	0.3129	0.9080	1.4142	1.7820	1.9754	1.9754	1.7820	1.4142	0.9080	0.3129	1.0000

Source: Reprinted from G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures*, Artech House, Dedham, Mass., 1980, with permission.

TABLE 8.4 Element Values for Equal-Ripple Low-Pass Filter Prototypes ($g_0 = 1, \omega_c = 1, N = 1$ to 10, 0.5 dB and 3.0 dB ripple)

					0.5 dB Ripple						
N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	0.6986	1.0000									
2	1.4029	0.7071	1.9841								
3	1.5963	1.0967	1.5963	1.0000							
4	1.6703	1.1926	2.3661	0.8419	1.9841						
5	1.7058	1.2296	2.5408	1.2296	1.7058	1.0000					
6	1.7254	1.2479	2.6064	1.3137	2.4758	0.8696	1.9841				
7	1.7372	1.2583	2.6381	1.3444	2.6381	1.2583	1.7372	1.0000			
8	1.7451	1.2647	2.6564	1.3590	2.6964	1.3389	2.5093	0.8796	1.9841		
9	1.7504	1.2690	2.6678	1.3673	2.7239	1.3673	2.6678	1.2690	1.7504	1.0000	
10	1.7543	1.2721	2.6754	1.3725	2.7392	1.3806	2.7231	1.3485	2.5239	0.8842	1.9841

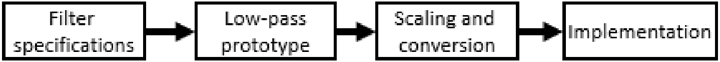
Chebyshev Type I LPF Prototype (3.0dB Ripple)

3.0 dB Ripple

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	1.9953	1.0000									
2	3.1013	0.5339	5.8095								
3	3.3487	0.7117	3.3487	1.0000							
4	3.4389	0.7483	4.3471	0.5920	5.8095						
5	3.4817	0.7618	4.5381	0.7618	3.4817	1.0000					
6	3.5045	0.7685	4.6061	0.7929	4.4641	0.6033	5.8095				
7	3.5182	0.7723	4.6386	0.8039	4.6386	0.7723	3.5182	1.0000			
8	3.5277	0.7745	4.6575	0.8089	4.6990	0.8018	4.4990	0.6073	5.8095		
9	3.5340	0.7760	4.6692	0.8118	4.7272	0.8118	4.6692	0.7760	3.5340	1.0000	
10	3.5384	0.7771	4.6768	0.8136	4.7425	0.8164	4.7260	0.8051	4.5142	0.6091	5.8095

Source: Reprinted from G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures*, Artech House, Dedham, Mass.,1980, with permission.

- Design procedures for any type of response are similar and follow broadly the same steps.



- First step is to determine the element values of the LPF Prototype from which all other filters are derived.

Filter Transformation

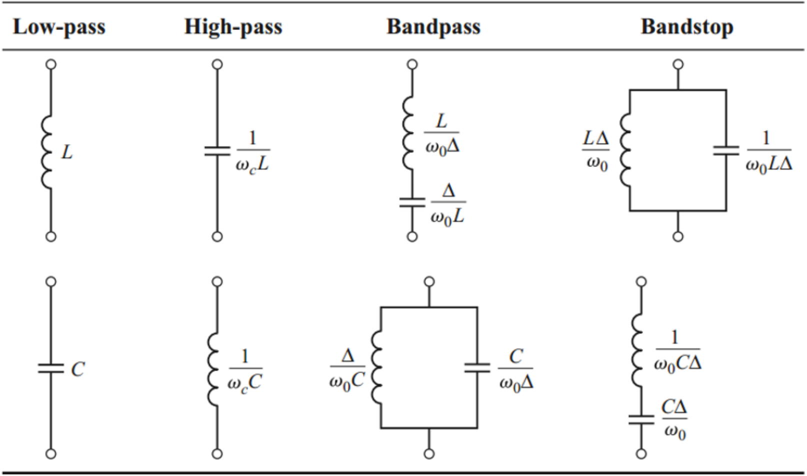
- Prototype has lowpass response
- Source Impedance/Admittance of 1Ω
- LPF cut-off frequency of $1 rad/sec$
- For practical use, we need to transform filter to work with actual source resistance AND desired cut-off frequency.

Filter Transformation

- Impedance of all components must be scaled by the same amount to have the same rescaled response.
  - Multiply series impedance by actual value of source resistance.
  - Divide shunt impedance by actual value of source resistance.
- ω becomes ω/ω_c (denormalize frequencies by dividing component values by ω_c
- Transform from lowpass response to other filter response (if necessary)
  - ie. inductors become capacitors, capacitors become inductors, etc.

Filter Transformation

TABLE 8.6 Summary of Prototype Filter Transformations $\left(\Delta = \frac{\omega_2 - \omega_1}{\omega_0}\right)$



Example

Design a 4^{th} order Butterworth lowpass filter with cut-off frequency of $50MHz$. The source impedance is 50Ω.

TABLE 8.3 Element Values for Maximally Flat Low-Pass Filter Prototypes ($g_0 = 1$, $\omega_c = 1$, $N = 1$ to 10)

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	2.0000	1.0000									
2	1.4142	1.4142	1.0000								
3	1.0000	2.0000	1.0000	1.0000							
4	0.7654	1.8478	1.8478	0.7654	1.0000						
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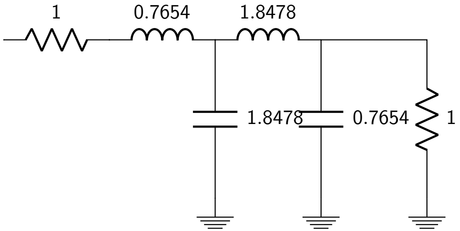
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450	1.0000			
8	0.3902	1.1111	1.6629	1.9615	1.9615	1.6629	1.1111	0.3902	1.0000		
9	0.3473	1.0000	1.5321	1.8794	2.0000	1.8794	1.5321	1.0000	0.3473	1.0000	
10	0.3129	0.9080	1.4142	1.7820	1.9754	1.9754	1.7820	1.4142	0.9080	0.3129	1.0000

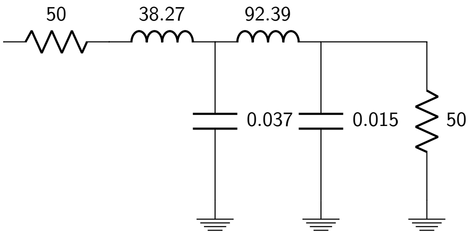
- The coefficients are

$$g1 = 0.7654 \qquad g2 = 1.8478$$
$$g3 = 1.8478 \qquad g4 = 0.7654$$

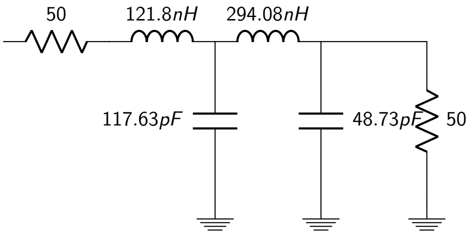
- The LPF prototype



- Impedance scaling



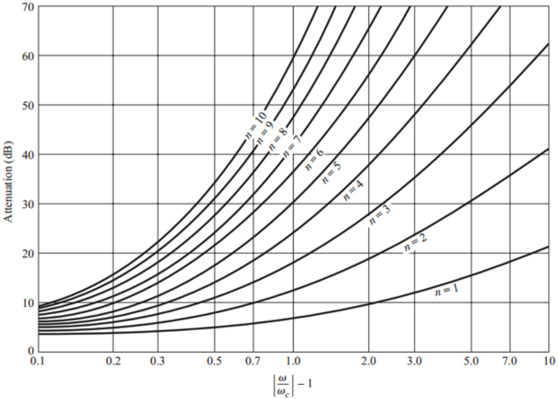
- Frequency scaling



Example

Design a Butterworth lowpass filter with cut-off frequency of 30MHz and has a minimum attenuation of 30dB at 65MHz. The source impedance is 100Ω.

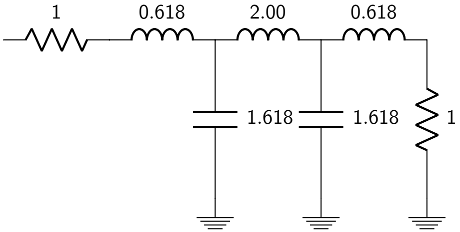
- Need to determine the filter order



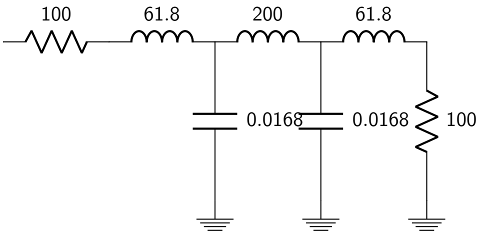
- From the graph, the minimum filter is $n = 5$
- The LPF coefficients are:

$$g1 = g5 = 0.618 \qquad g2 = g4 = 1.618 \qquad g3 = 2$$

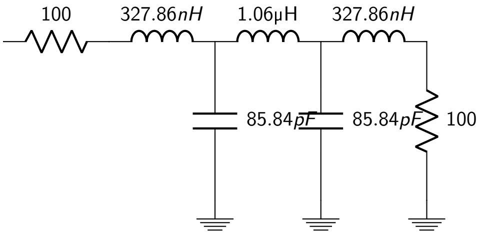
- LPF prototype



- Impedance scaling



- Frequency scaling



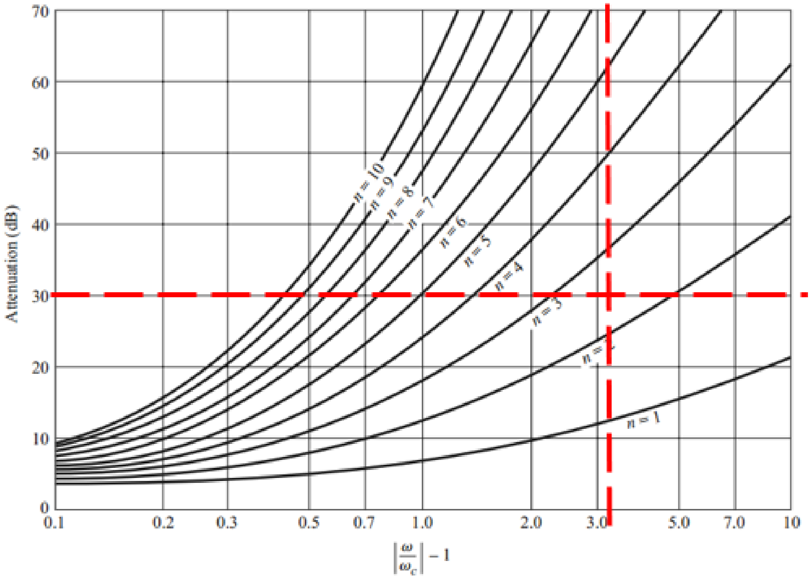
Example

Design a Butterworth bandpass filter with cut-off frequencies at $90MHz$ and $110MHz$. The minimum attenuation at $150MHz$ must be $30dB$. The source impedance is 100Ω.

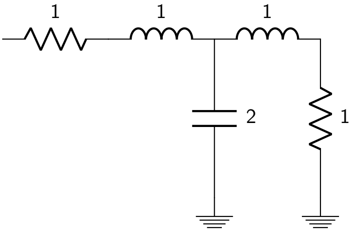
$$\Delta = \frac{\omega_2 - \omega_1}{\omega_o} = 0.201 \qquad \omega_o = \sqrt{\omega_1 \omega_2} = 625.17 Mrad/sec$$

- Transform $150MHz$ to prototype frequency

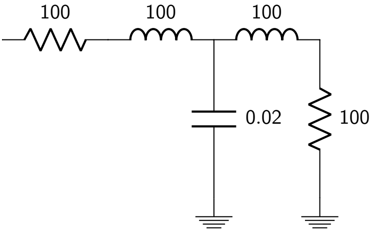
$$\omega' = \frac{1}{\Delta}(|\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}|) = 4.2$$


70
60
50
40
30
20
10
0
0.1
0.2
0.3
0.5
0.7
1.0
2.0
3.0
5.0
7.0
10
Attenuation (dB)
$\left|\frac{\omega}{\omega_c}\right| - 1$
n = 10
n = 9
n = 8
n = 7
n = 6
n = 5
n = 4
n = 3
n = 1

- LPF Prototype



- Impedance Scaling



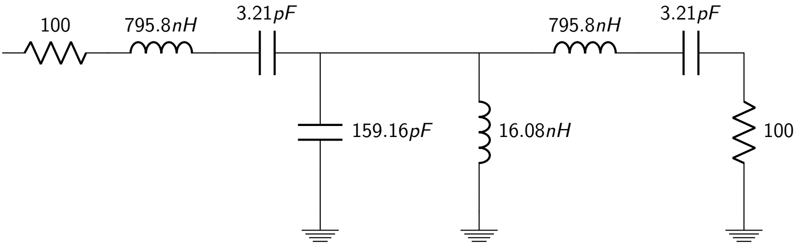
- Transform series elements to series LC and shunt elements to shunt LC

  - For series leg:

$$L'_k = \frac{L_k}{\Delta\omega_o} \qquad C'_k = \frac{\Delta}{\omega_o L_k}$$

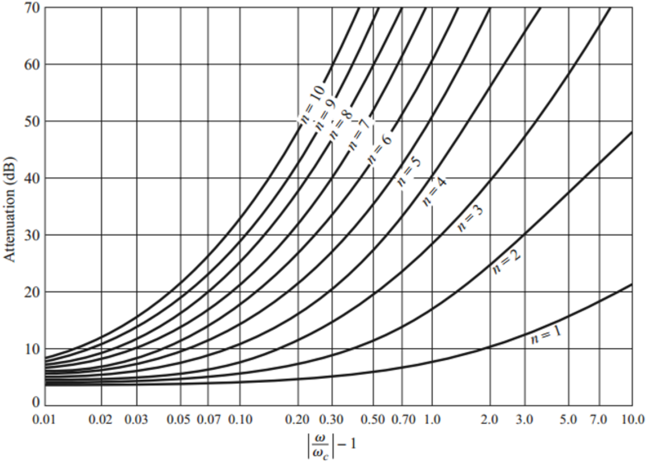
  - For shunt leg:

$$L'_k = \frac{\Delta}{\omega_o C_k} \qquad C'_k = \frac{C_k}{\Delta\omega_o}$$



Example

Design a 3dB Chebyshev highpass filter with cutoff frequency of $100MHz$. The minimum attenuation at $50MHz$ should be $25dB$. The source impedance is 75Ω



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