EEE 51: Second Semester 2017 - 2018

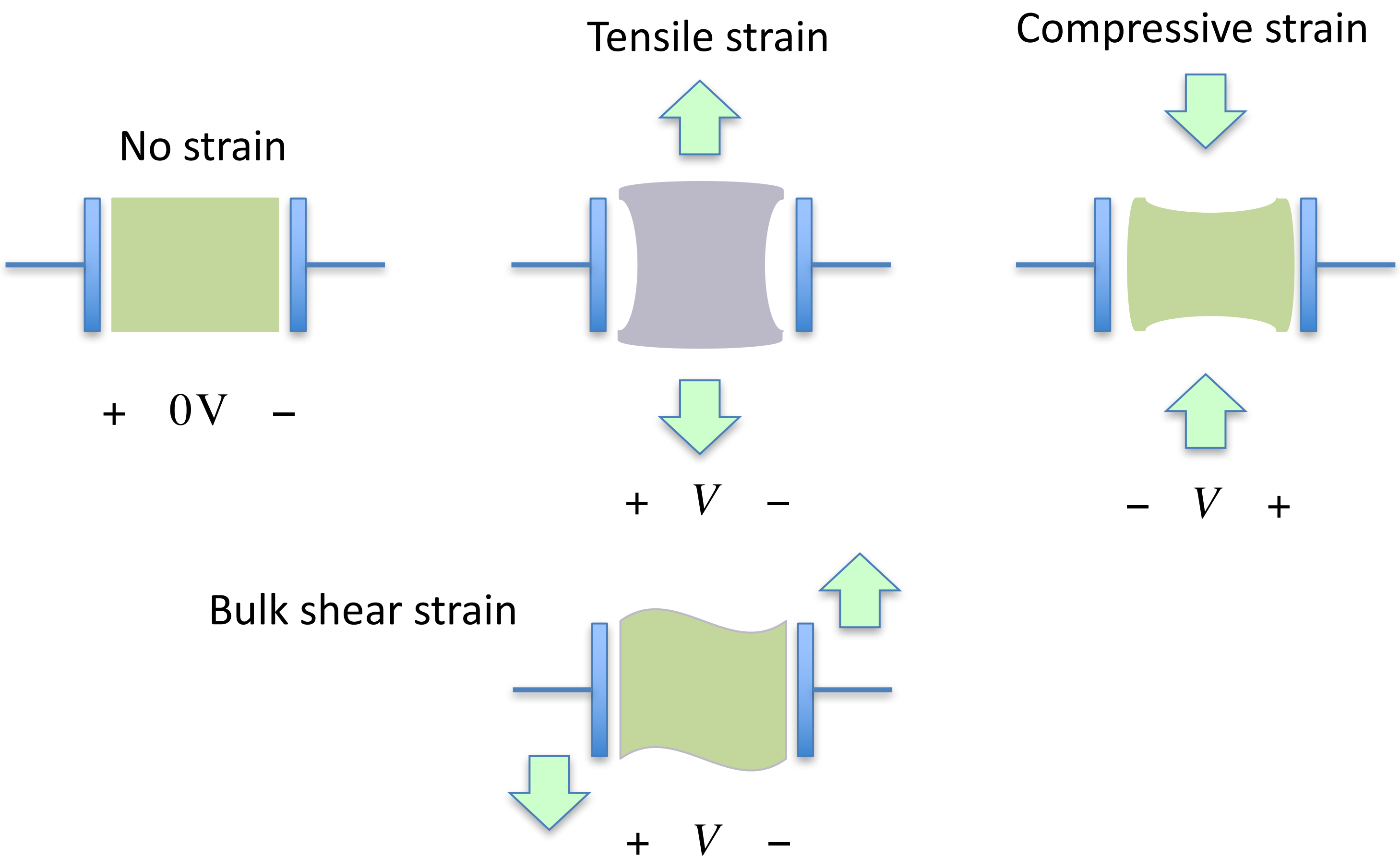
Lecture 24

Oscillators

Crystal Oscillators

- Crystals are materials that exhibits the piezoelectric effect
  - When stress is applied, voltage is generated between opposite faces of the crystal
  - When a voltage is applied, the crystal deforms at the frequency of the applied voltage
- Mechanical to electrical energy conversion
- Provides stable frequency of oscillation
  - Resonant frequency dependent on physical crystal size
  - 1 ppm/°C or 0.0001%/°C
  - Compare with LC oscillator: ~1% drift

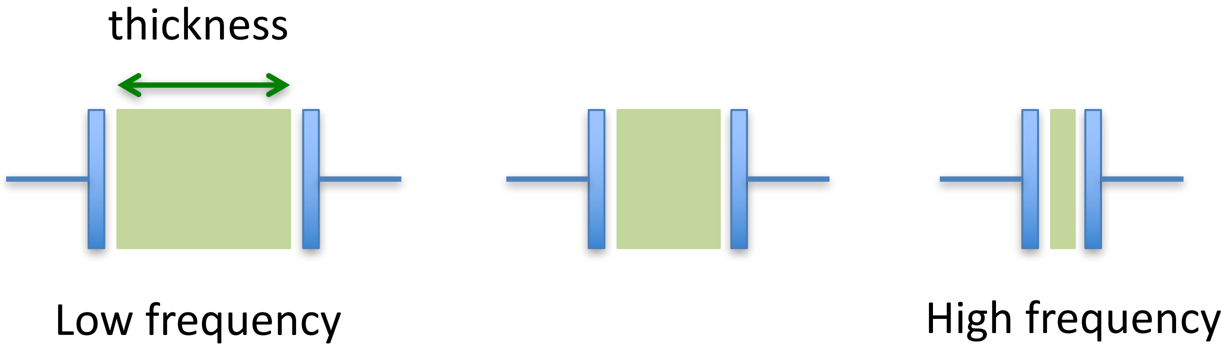
Crystal Strain



Natural Crystal Frequency

- Proportional to crystal thickness

$$f \propto \frac{1}{\text{thickness}}$$

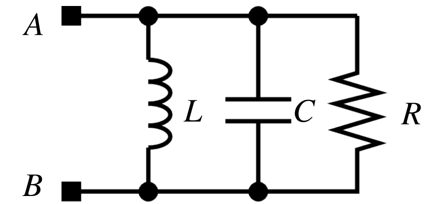


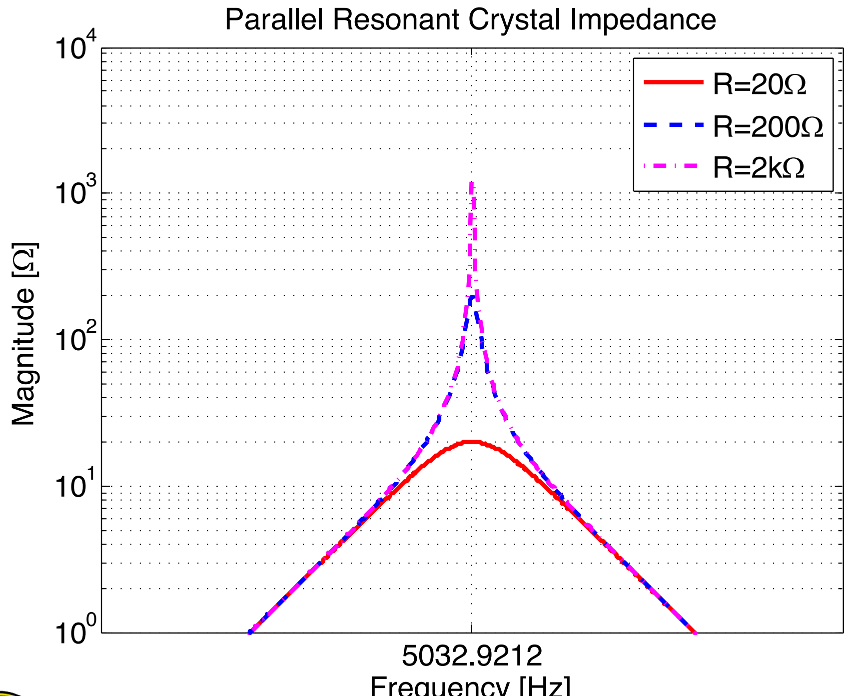
Typical natural frequencies below 20-30 MHz

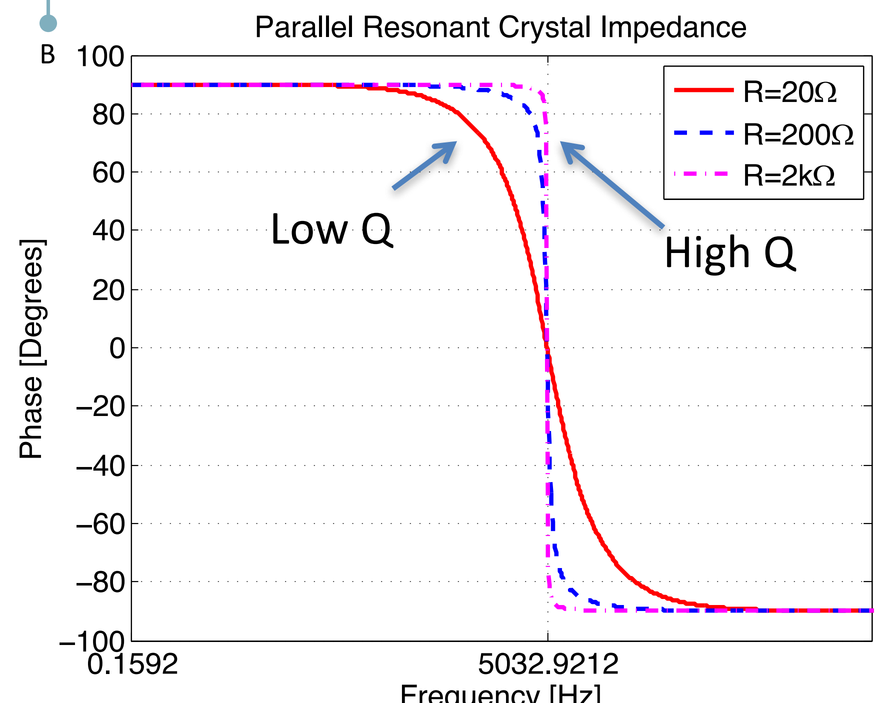
- For 100 MHz, thickness ~ 17µm thick

The Parallel Resonant Mode

$$L = 1\text{mH} \qquad C = 1\mu\text{F}$$

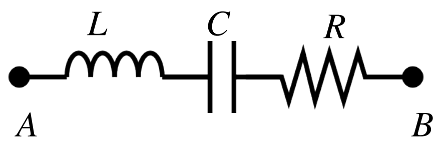


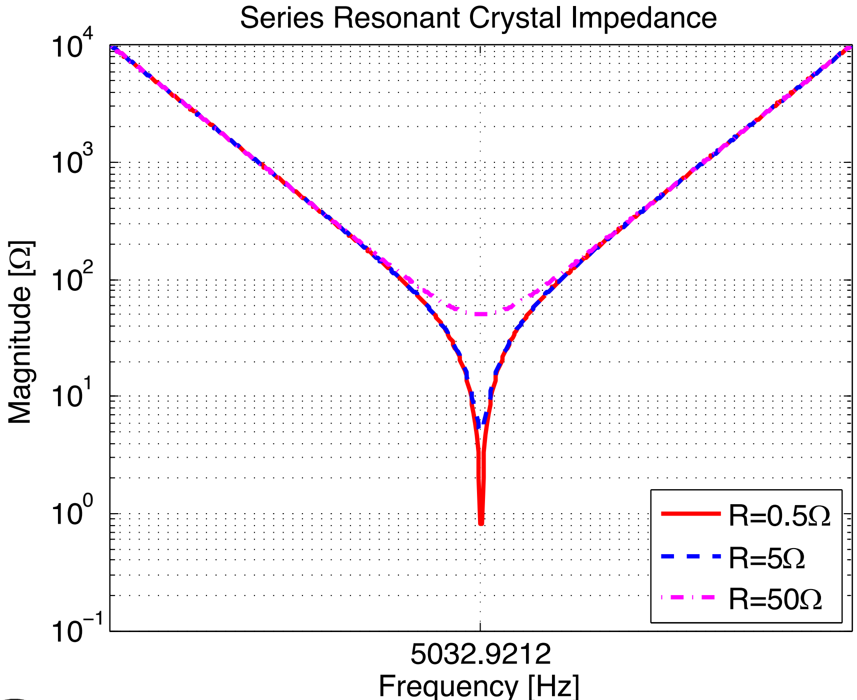


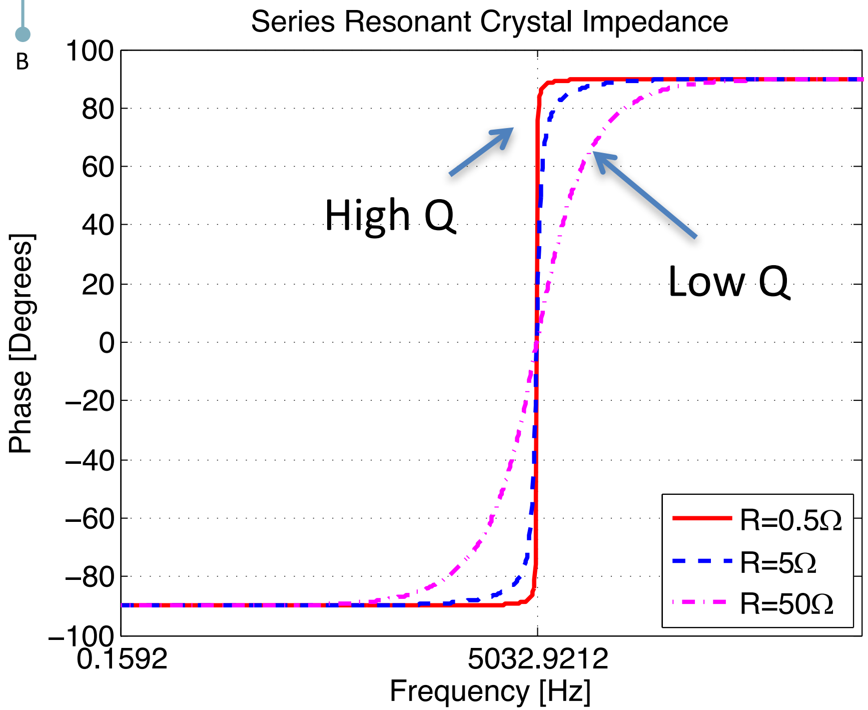


The Series Resonant Mode

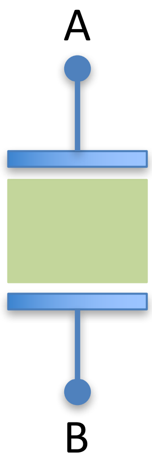
$L = 1\text{mH} \quad C = 1\mu\text{F}$

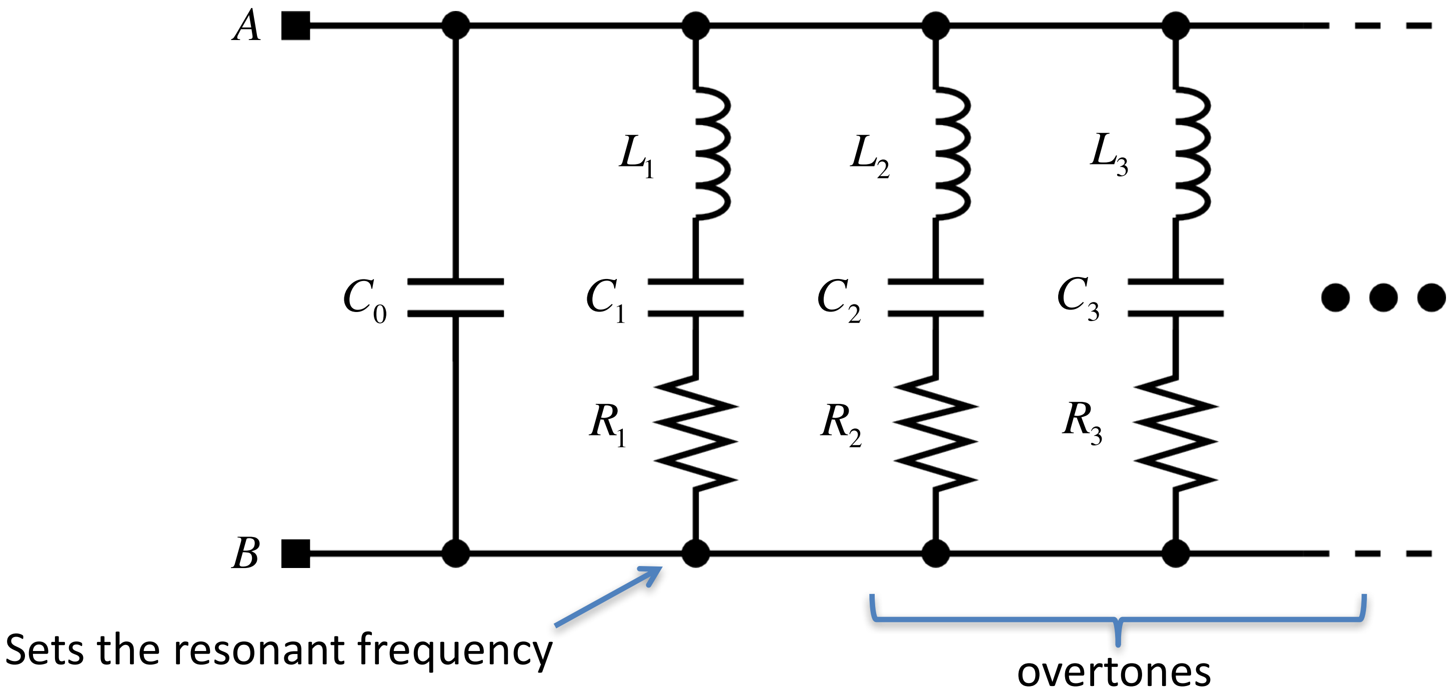






Electrical Equivalent Circuit



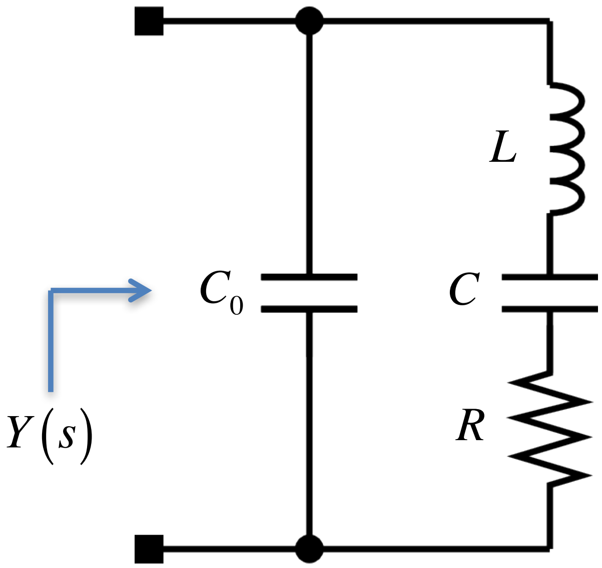


$C_0 \equiv$ parallel capacitances due to contacts and wires

$L_i, C_i \equiv$ mechanical energy storage (mass & spring effects)

$R_i \equiv$ electrical losses due to mechanical effects (e.g. friction)

Crystal Equivalent Circuit

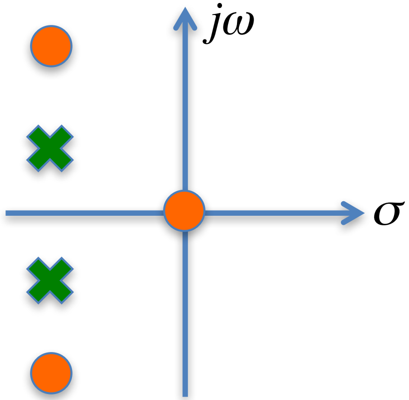


$$Y(s) = sC_0 + \frac{1}{sL + \frac{1}{sC} + R} = sC_0 + \frac{sC}{s^2LC + sRC + 1}$$

$$= \frac{sC_0\left[s^2 + s\frac{R}{L} + \left(\frac{C}{C_0} + 1\right)\omega_0^2\right]}{s^2 + s\frac{R}{L} + \omega_0^2}$$

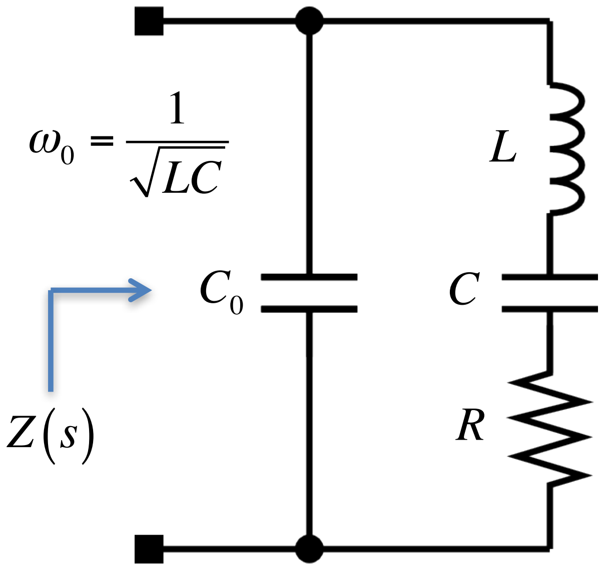
$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad Z(s) = \frac{1}{Y(s)}$$

p-z plot:



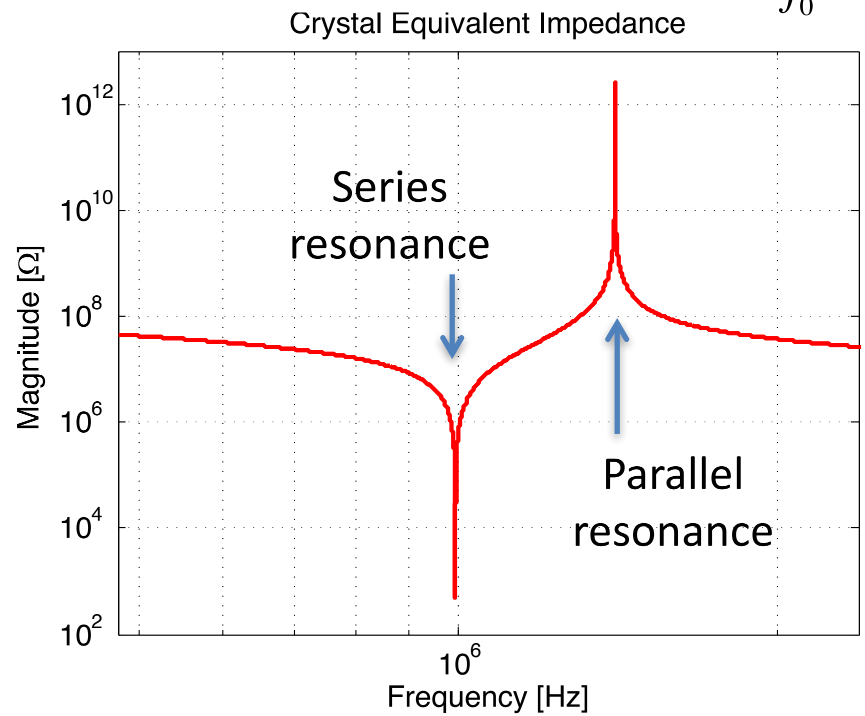
Crystal Equivalent Circuit

$$\omega_0 = \frac{1}{\sqrt{LC}}$$



$$Z(s) = \frac{s^2 + s\frac{R}{L} + \omega_0^2}{sC_0\left[s^2 + s\frac{R}{L} + \left(\frac{C}{C_0} + 1\right)\omega_0^2\right]}$$

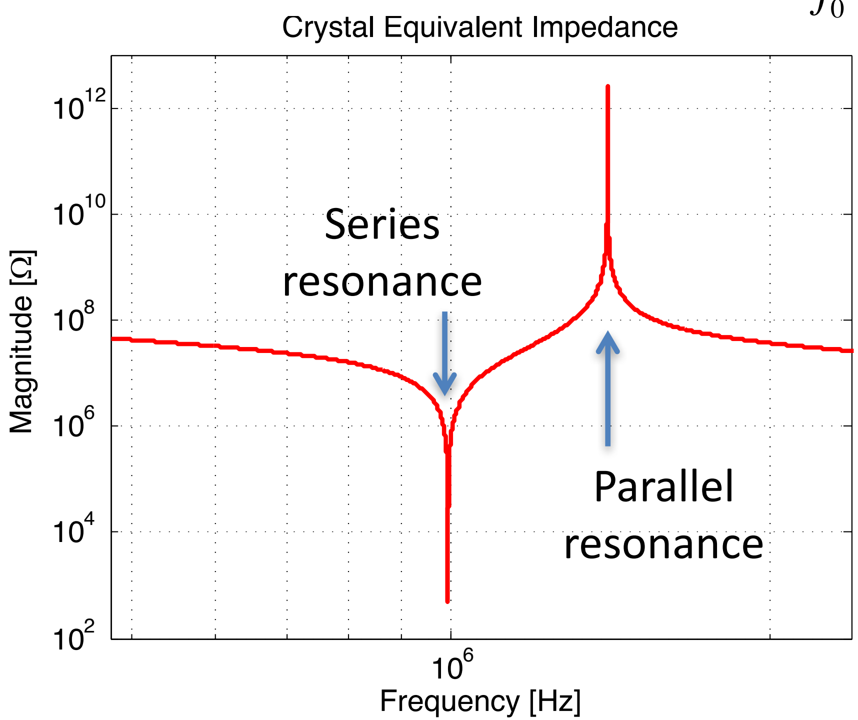
$L = 10\text{H}$ $\quad C = C_0 = 3.2\text{fF}$ $\quad R \approx \frac{5\times 10^8}{f_0}$

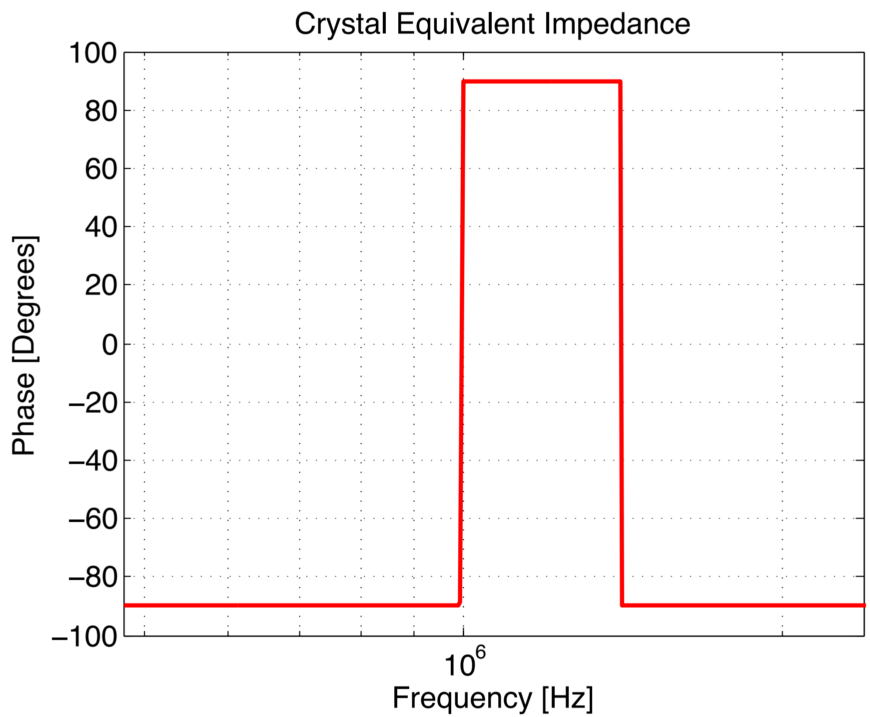


How can we use this to create an oscillator?

Crystal Equivalent Circuit

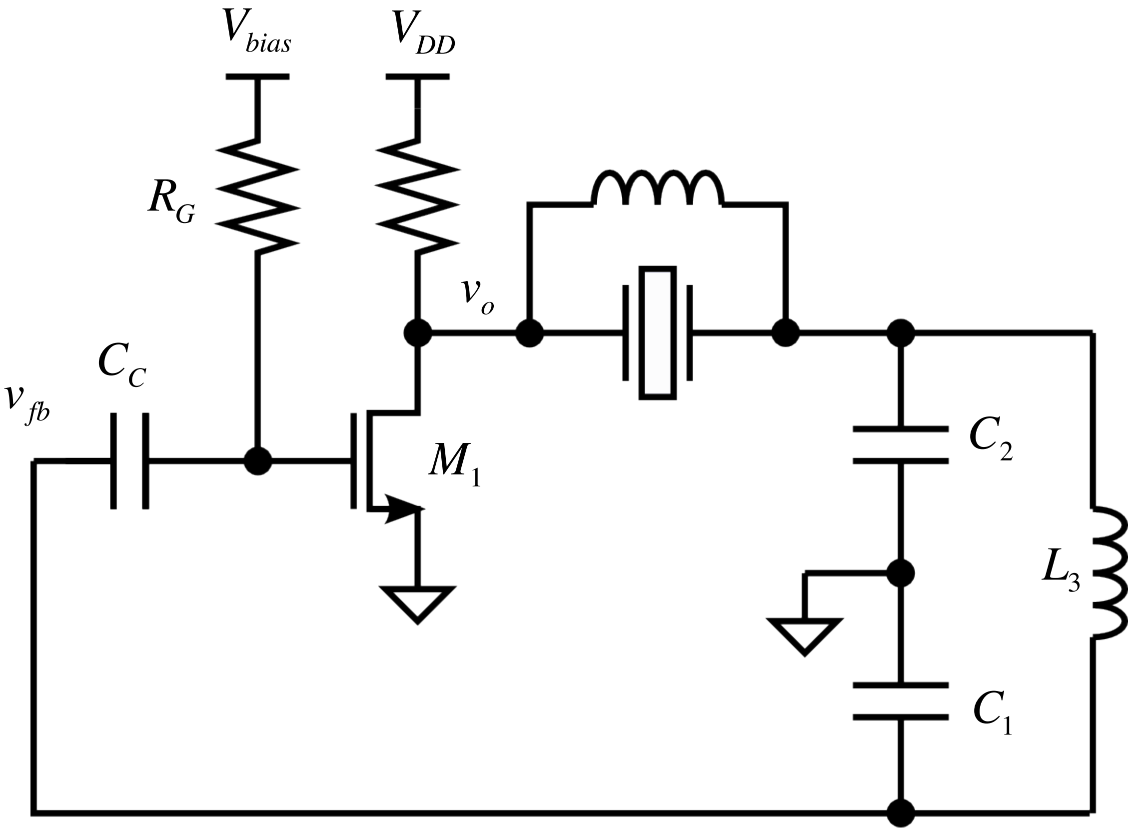
$$L = 10\text{H} \qquad C = C_0 = 3.2\text{fF} \qquad R \approx \frac{5\times10^8}{f_0}$$





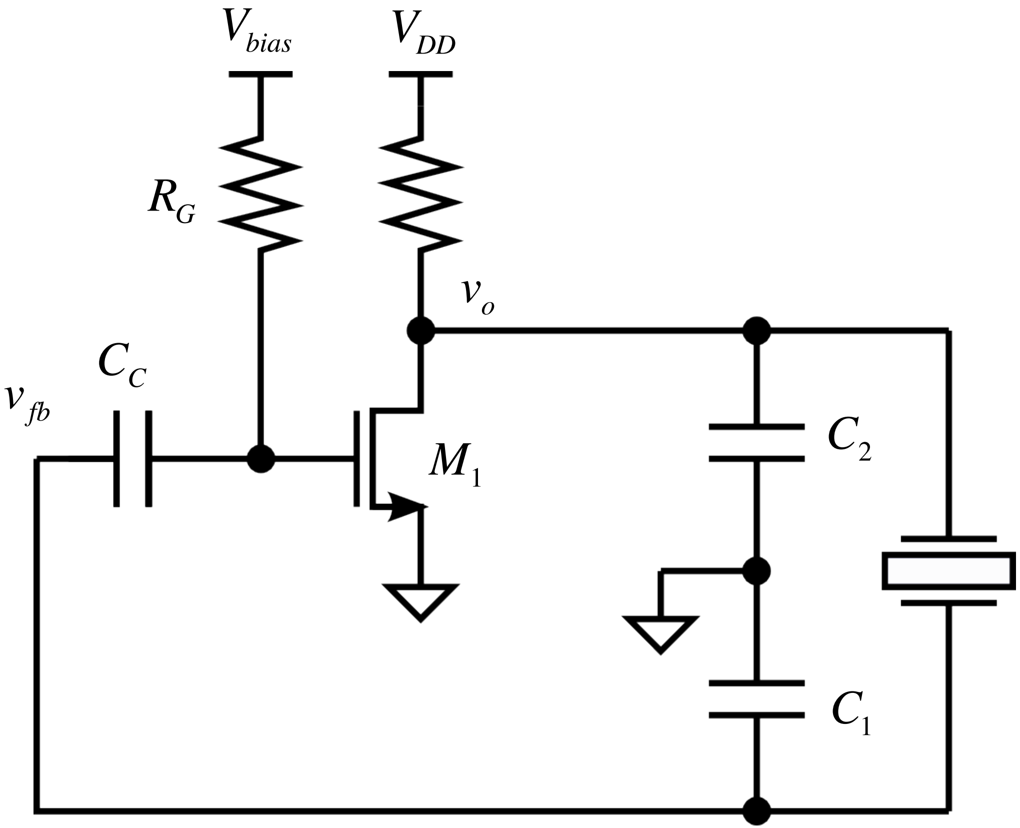
How can we use this to create an oscillator?

Direct Application: A Colpitts Crystal Oscillator



- The crystal is used to close the feedback loop only at the desired frequency
- The parallel inductance is used to cancel out the crystal parallel capacitance C_0
    - Only the series RLC branch controls the feedback path

Another Colpitts Crystal Oscillator

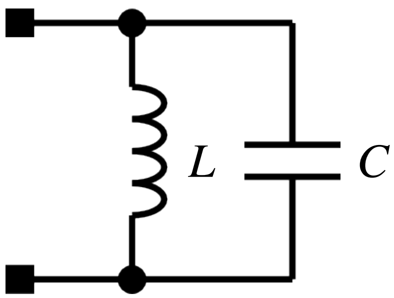


- The crystal is used as an inductance
- The oscillation frequency is set to be a little above series resonance
  - The crystal impedance is inductive
- Note that the crystal series resonant frequency is not the same as the output oscillation frequency
  - Crystal is cut to oscillate at a specified load capacitance

Negative Resistance Oscillators

- What happens in an ideal LC circuit?

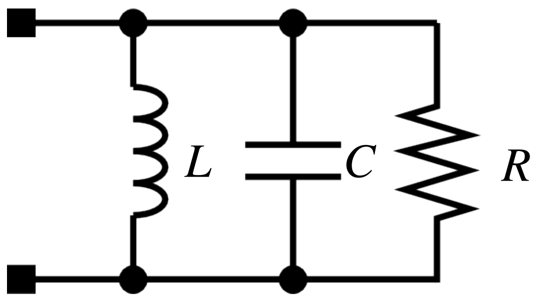
Ideal



The circuit will oscillate at

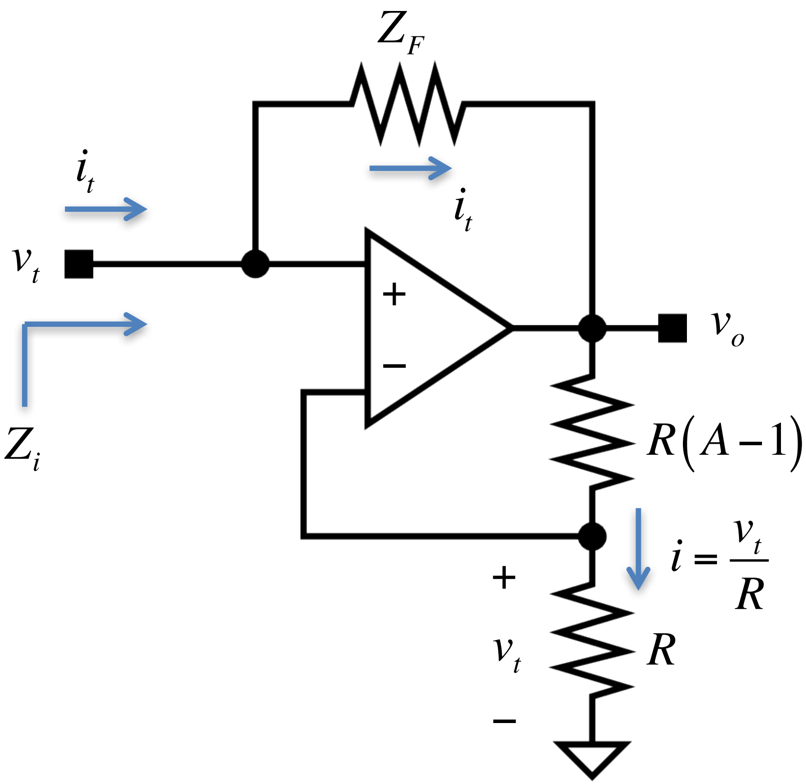
$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Real



This circuit will also oscillate but the oscillations will die out due to the resistance R

Example:



$$v_o = v_t + i \cdot R(A-1) = v_t + \frac{v_t}{R} \cdot R(A-1) = v_t \cdot A$$

Thus,

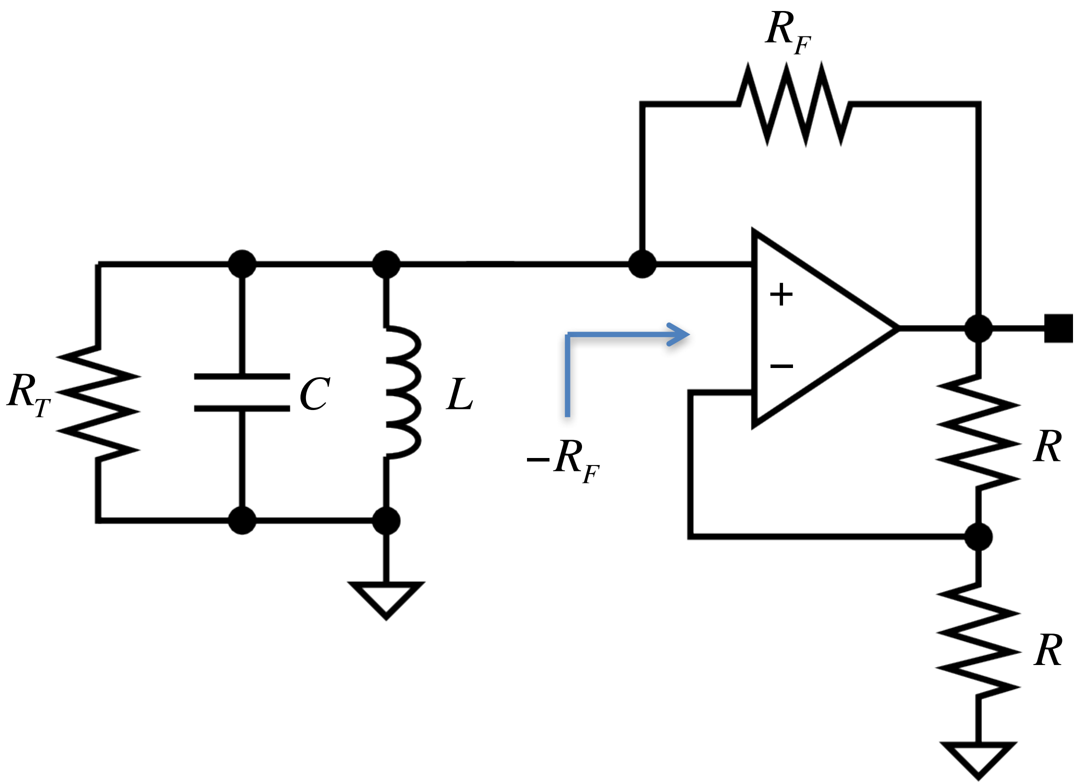
$$i_t = \frac{v_t - A \cdot v_t}{Z_F}$$

$$Z_i = \frac{v_t}{i_t} = \frac{Z_F}{1-A}$$

For A = 2:

$$Z_i = -Z_F$$

Negative Resistance Oscillators

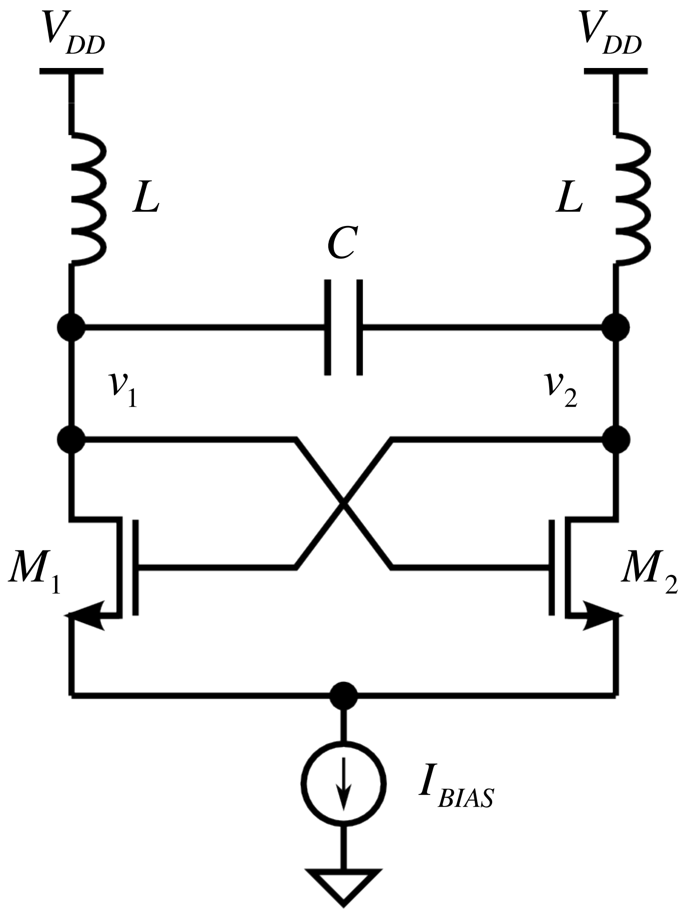


$$R_T > R_F$$

$$\frac{1}{R_{eff}} = \frac{1}{R_T} - \frac{1}{R_F}$$

The effective resistance seen by the LC circuit is negative

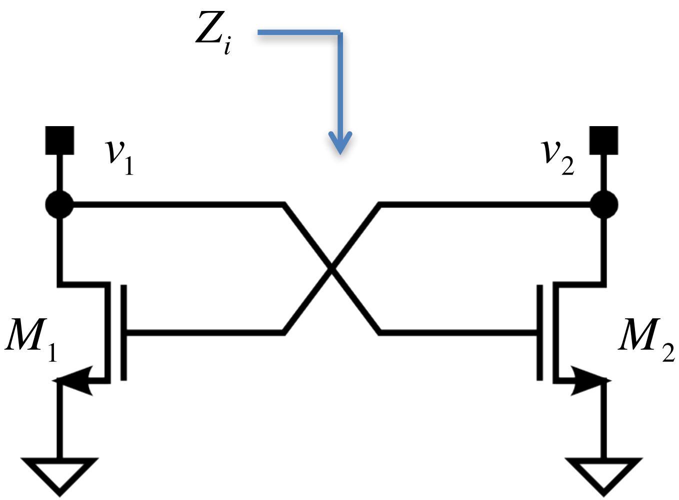
Differential Negative Resistance Oscillator

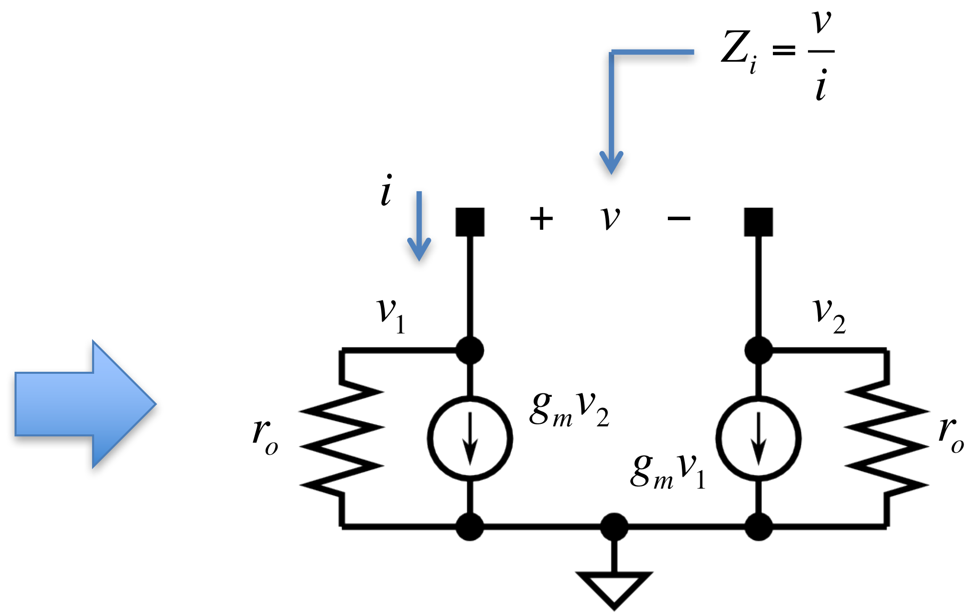


How do we analyze this?

- Get the small signal impedance between v1 and v2 without the LC network
- Convert to the single ended equivalent

MOS Differential Cross-Coupled Pair

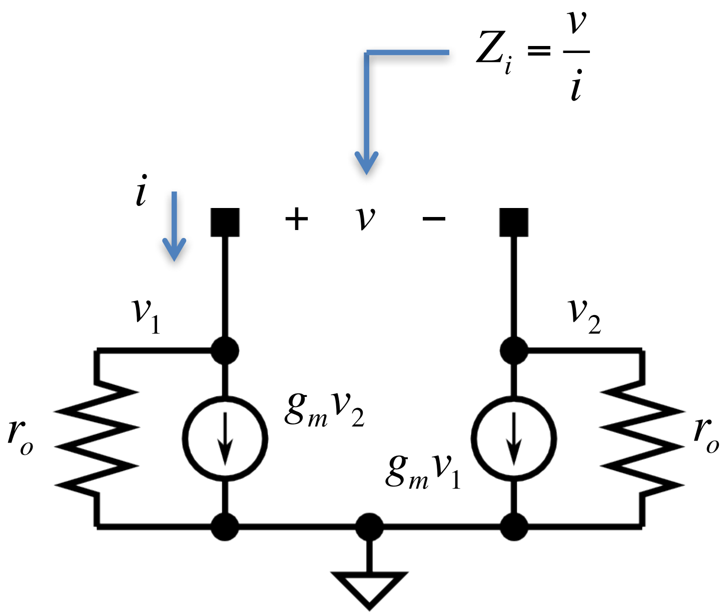




$$i = g_m v_2 + \frac{v_1}{r_o} = -g_m v_1 - \frac{v_2}{r_o} \quad \Rightarrow \quad v_1\left(g_m + \frac{1}{r_o}\right) = -v_2\left(g_m + \frac{1}{r_o}\right)$$

$$v_1 = -v_2$$

MOS Differential Cross-Coupled Pair



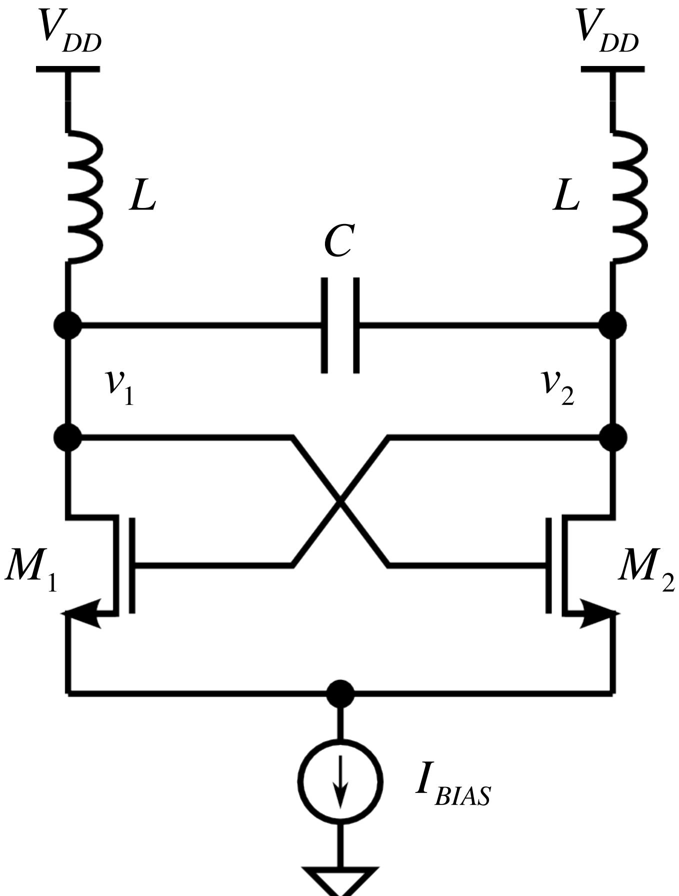
Since $v_1 = -v_2$ and $v = v_1 - v_2 = 2v_1$

$$i = -g_m v_1 + \frac{v_1}{r_o} = v_1\left(-g_m + \frac{1}{r_o}\right)$$

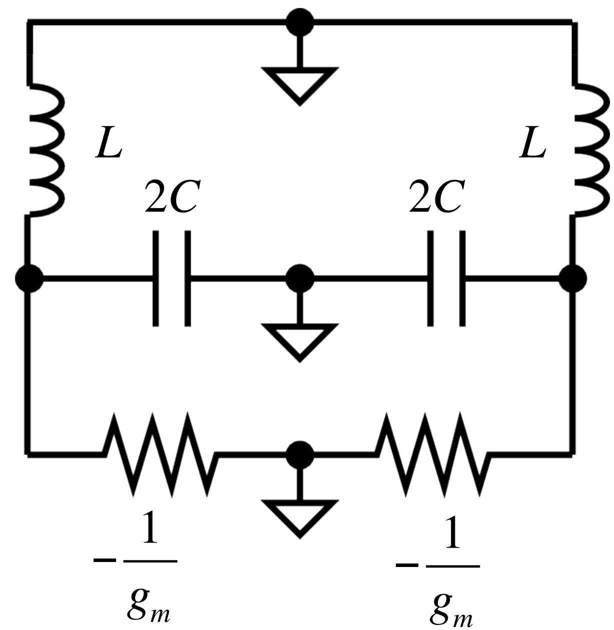
$$= \frac{v}{2}\left(-g_m + \frac{1}{r_o}\right)$$

$$Z_i = \frac{v}{i} = \frac{2}{-g_m + \frac{1}{r_o}} \approx -\frac{2}{g_m}$$

Differential Negative Resistance Oscillator



The differential half circuit:



$$\omega_0 = \frac{1}{\sqrt{2LC}}$$

Reminder

- Final Exam:
  - May 24, 2018 (THURSDAY)
  - 1 – 4 pm
  - Bring: pen, calculator