Lecture 13
TWO WINDING TRANSFORMER MODEL

Agenda

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Lecture Outcomes

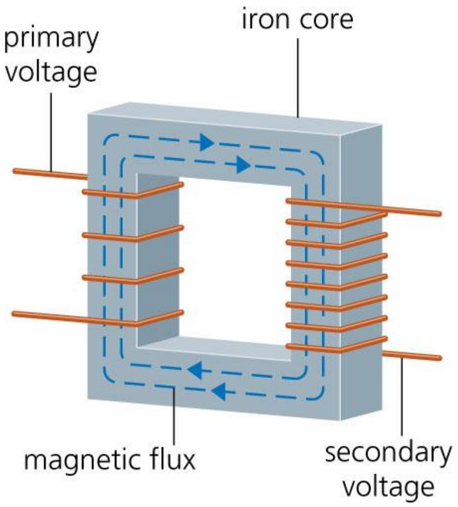
at the end of the lecture, the student must be able to …

- Explain how a transformer works
- List the characteristics of a practical transformer

Transformer Model

Two-Winding Transformer

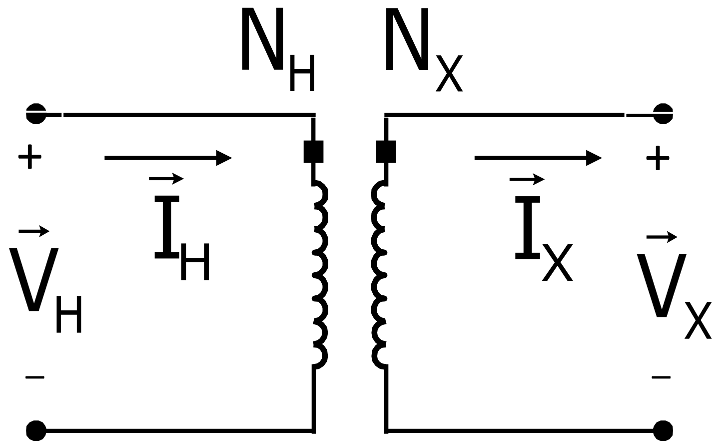
Ideal Transformer



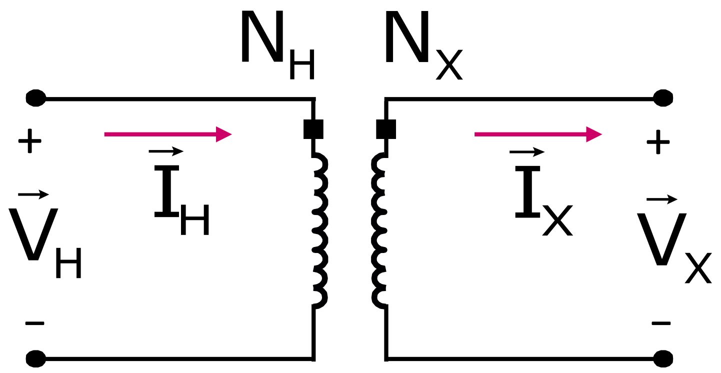
The voltage drop from the polarity-marked terminal to the non-polarity-marked terminal of the H winding is in phase with the voltage drop from the polarity-marked terminal to the non-polarity-marked terminal of the X winding.

Voltage Equation:

$$\frac{\vec{V}_H}{\vec{V}_X} = \frac{N_H}{N_X}$$



Two-Winding Transformer



Current Equation:

$$N_H \vec{I}_H = N_X \vec{I}_X$$

The current that enters the H winding through the polarity-marked terminal is in phase with the current that leaves the X winding through the polarity-marked terminal.

Note: Balancing ampere-turns must be satisfied at all times.

Two-Winding Transformer

Referred Values

From the Transformation Ratio,

$$a = \frac{\vec{V}_H}{\vec{V}_X} \qquad \vec{V}_H = a\vec{V}_X$$

$$a = \frac{\vec{I}_X}{\vec{I}_H} \qquad \vec{I}_H = \frac{\vec{I}_X}{a}$$

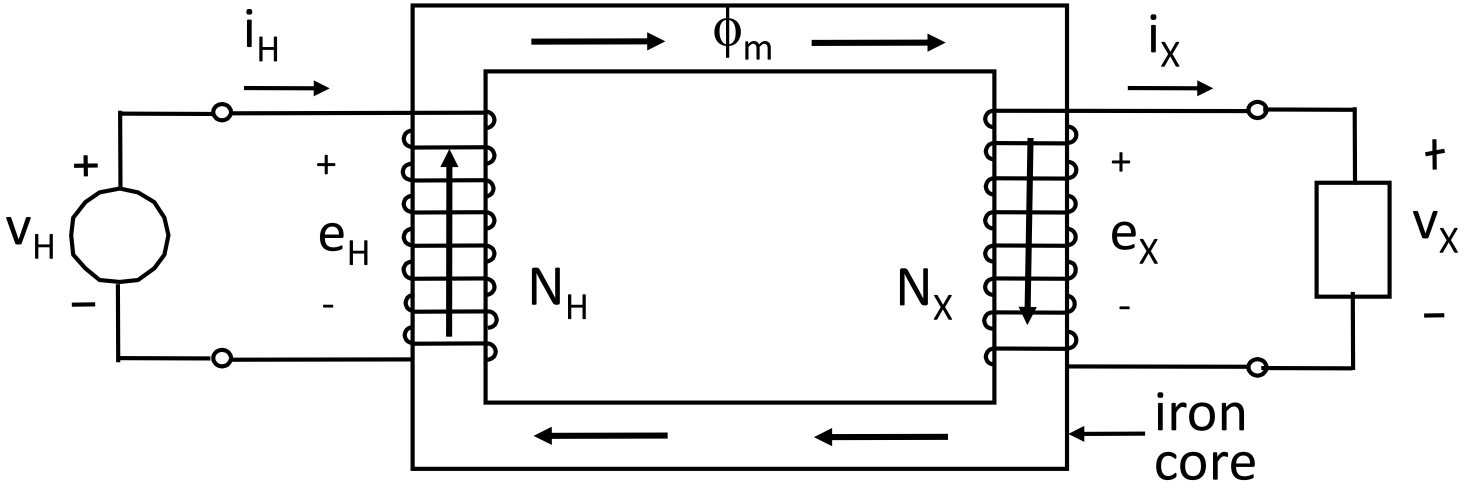
Dividing V_H by I_H,

$$\frac{\vec{V}_H}{\vec{I}_H} = a^2 \frac{\vec{V}_X}{\vec{I}_X} \qquad Z_H = a^2 Z_X$$

Two-Winding Transformer

Practical Transformer

1. The H and X coils have a small resistance.
2. There are leakage fluxes in the H and X coils.
3. There is resistance loss in the iron core.
4. The permeability of the iron is not infinite.



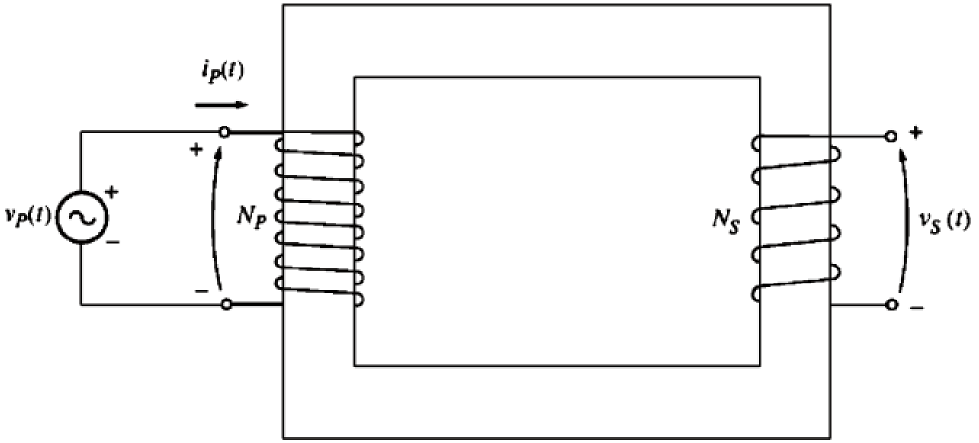


FIGURE 2–8
Sketch of a real transformer with no load attached to its secondary.

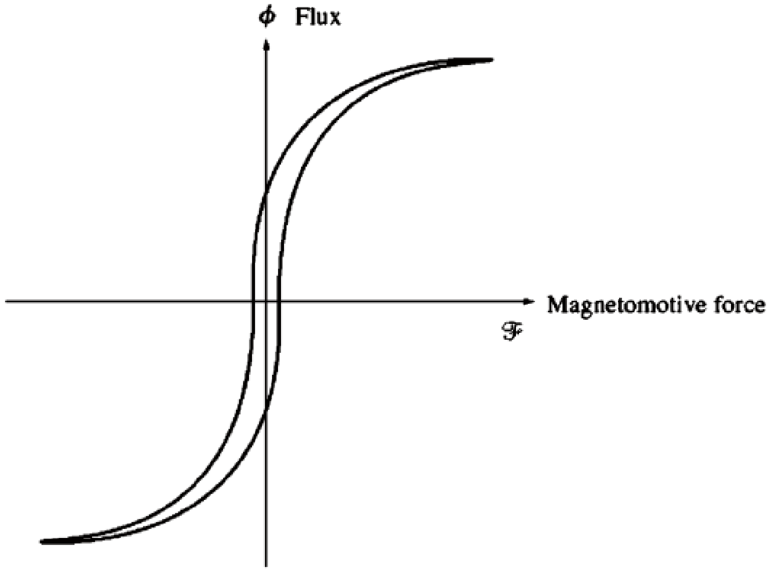


FIGURE 2–9
The hysteresis curve of the transformer.

Theory of operation of Real single Phase Transformer

1. $e_{ind} = \frac{d\lambda}{dt}$
2. $\lambda = \sum_{i=1}^{N} \phi_i \rightarrow \overline{\phi} = \frac{\lambda}{N}$
3. $e_{ind} = N \frac{d\overline{\phi}}{dt}$
4. $\overline{\phi} = \frac{1}{N_p} \int v_p(t)dt$ -> ave flux in primary. What effect on secondary?
5. $\overline{\phi}_p = \phi_M + \phi_{LP}$
6. Similarly $\overline{\phi}_s = \phi_M + \phi_{LS}$

Theory of Operation of Real Single Phase Transformer

- Expressed in terms of faradays law
  - $v_p(t) = e_p(t) + e_{LP}(t)$
  - $v_s(t) = e_s(t) + e_{LS}(t)$
- We can derive
  - $\frac{e_p(t)}{N_p} = \frac{d\phi_M}{dt} = \frac{e_s(t)}{N_s}$
- Therefore
  - $\frac{e_p(t)}{e_s(t)} = \frac{N_p}{N_s} = a \rightarrow if\ ideal,\ \frac{v_p(t)}{v_s(t)} = \frac{N_p}{N_s} = a$

Magnetization Current in a real transformer

- Consider an open circuited secondary of transformer
- There is exists a no load current on the primary that consists of:
  - Magnetization current, i_m
  - Core Loss Current, i_{h+e}

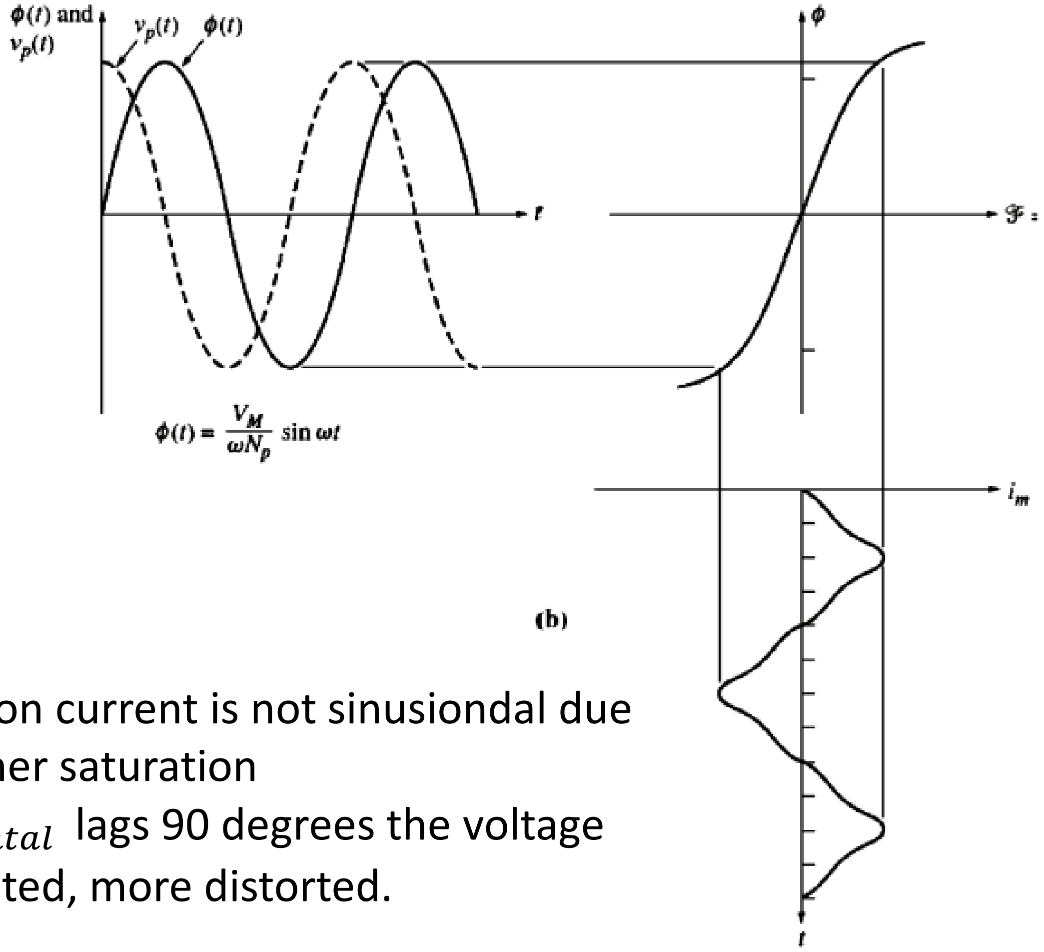


FIGURE 2–11
(a) The magnetization curve of the transformer core. (b) The magnetization current caused by the flux in the transformer core.

$$\overline{\phi} = \frac{1}{N_P}\int v_P(t)dt$$

$$\overline{\phi} = \frac{1}{N_P}\int V_M \cos \omega t \, dt$$

$$= \frac{V_M}{\omega N_P}\sin \omega t \quad \text{Wb}$$

Neglect leakage reactance for the mean time

I. Magnetization current is not sinusiondal due to transformer saturation
II. $i_{m,fundamental}$ lags 90 degrees the voltage
III. More saturated, more distorted.

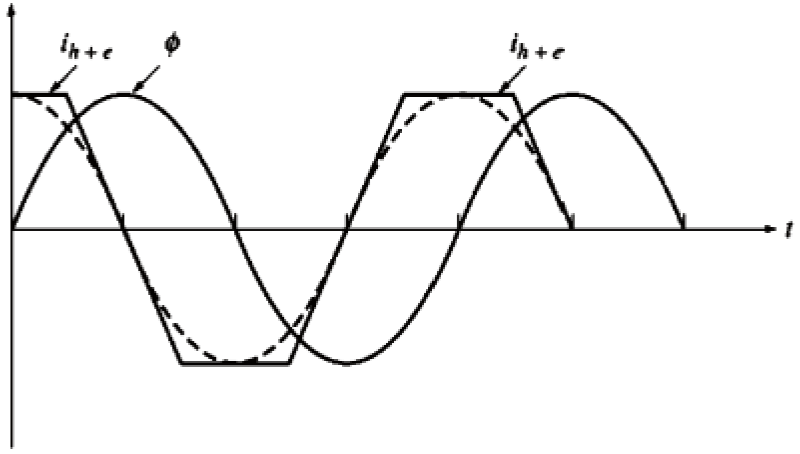


FIGURE 2–12
The core-loss current in a transformer.

I. Eddy currents in core are proportional to the derivative of the flux
II. There the eddy current are highest when the flux passes through zero.
III. Non-linear because of non-linear effects of hysteresis.
IV. Eddy current is in phase with voltage

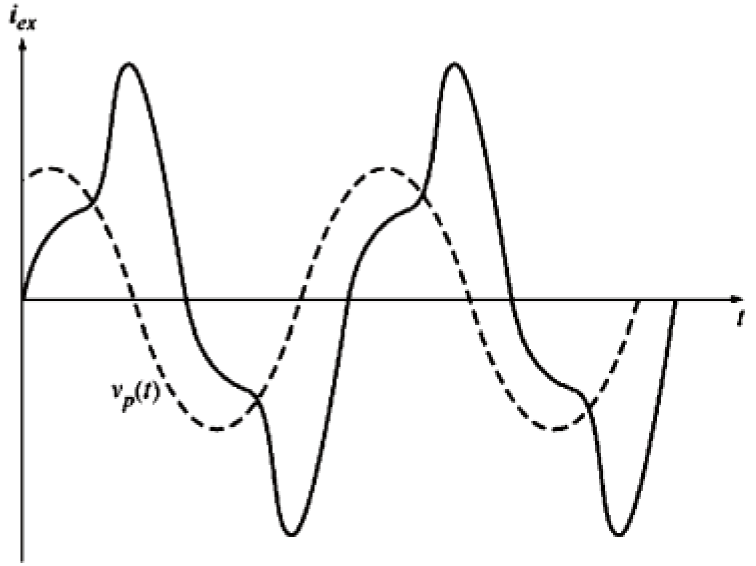


FIGURE 2–13
The total excitation current in a transformer.

- Total excitation current consists of the two no load components

The current ration on a transformer and the Dot Convention

- Dot Convention – the current flowing in the dotted end of a winding produces a positive magnetomotive force, while a current flowing into the undotted end of a winding produce a negative magnetomotive force.

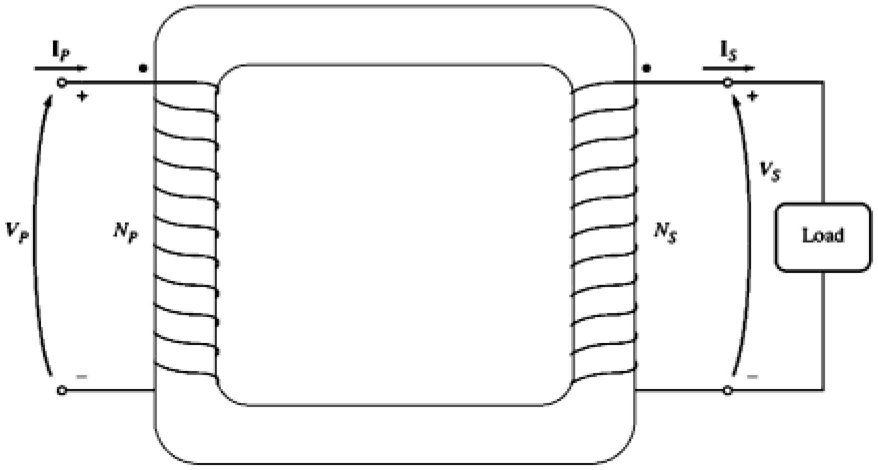


FIGURE 2–14
A real transformer with a load connected to its secondary.

Therefore,

- $F_{net} = N_p i_p - N_s i_s = \phi R \approx 0 (well\ designed)$

Hence

- $N_p i_p = N_s i_s$

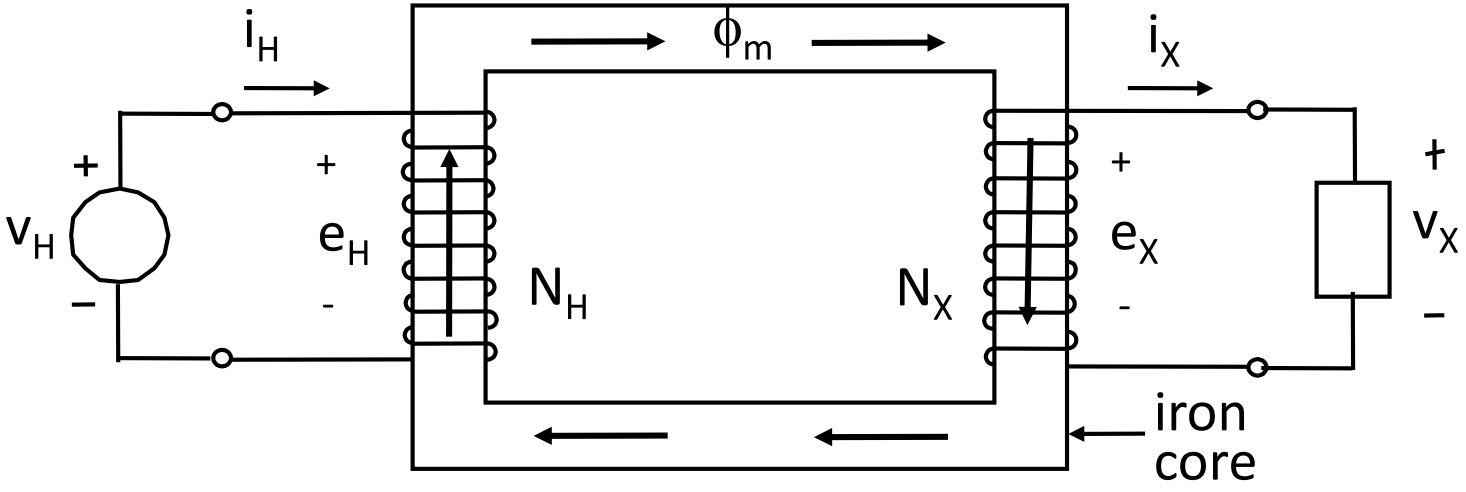
What assumptions are required to convert a real transformer into the ideal ransformer described previously? They are as follows:

1. The core must have no hysteresis or eddy currents.
2. The magnetization curve must have the shape shown in Figure 2–15. Notice that for an unsaturated core the net magnetomotive force $\mathscr{F}_{net} = 0$, implying that $N_P i_P = N_S i_S$.
3. The leakage flux in the core must be zero, implying that all the flux in the core couples both windings.
4. The resistance of the transformer windings must be zero.

Two-Winding Transformer

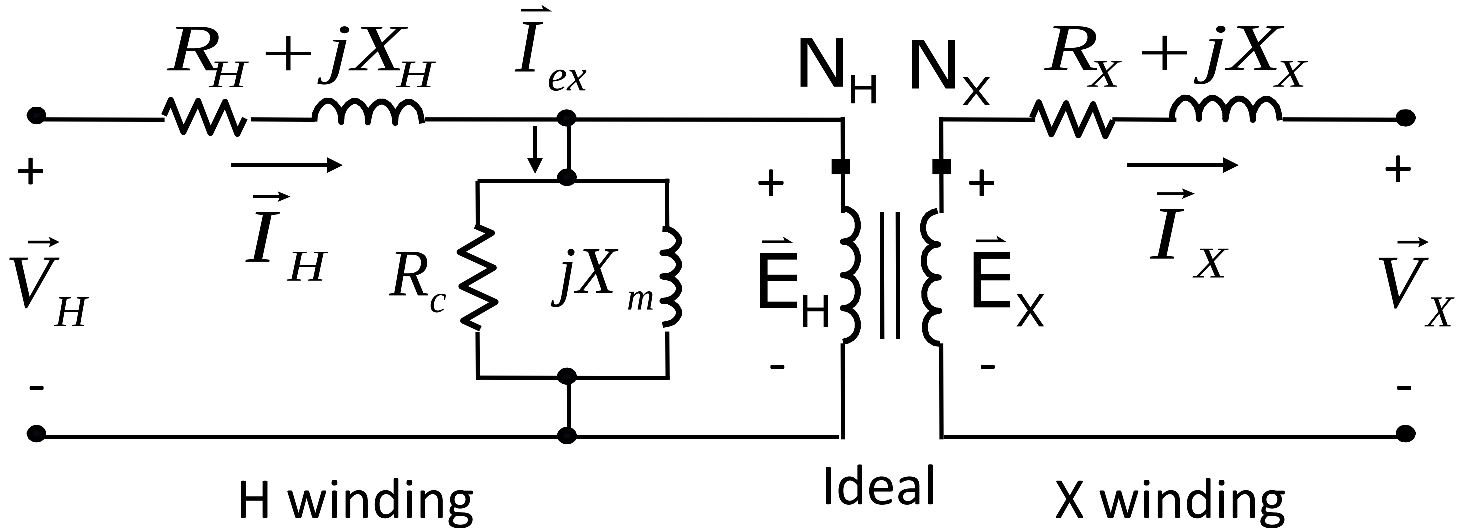
Practical Transformer

1. The H and X coils have a small resistance.
2. There are leakage fluxes in the H and X coils.
3. There is resistance loss in the iron core.
4. The permeability of the iron is not infinite.



Two-Winding Transformer

Equivalent Circuit



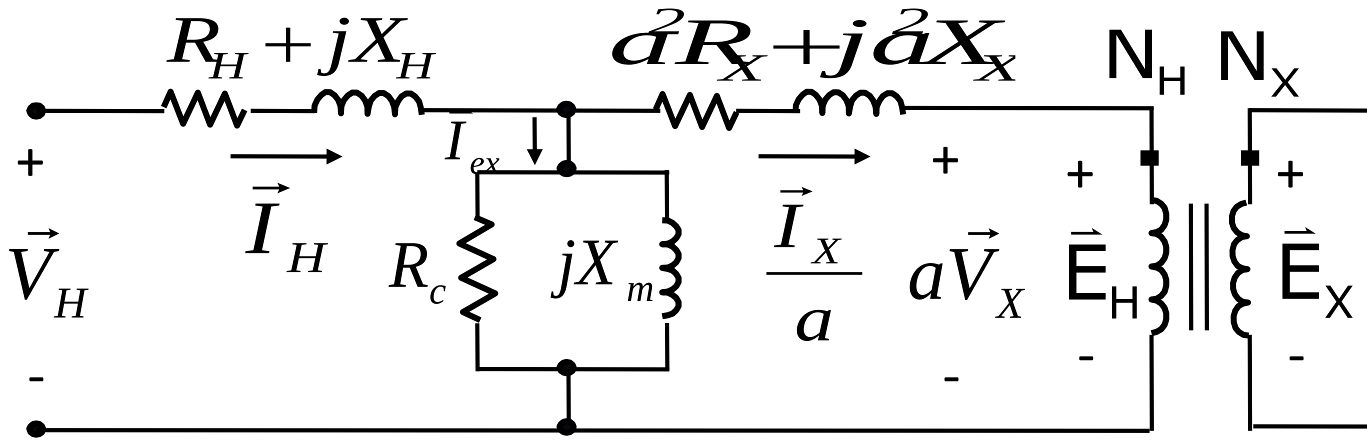
R_H, X_H = resistance and leakage reactance of H coil

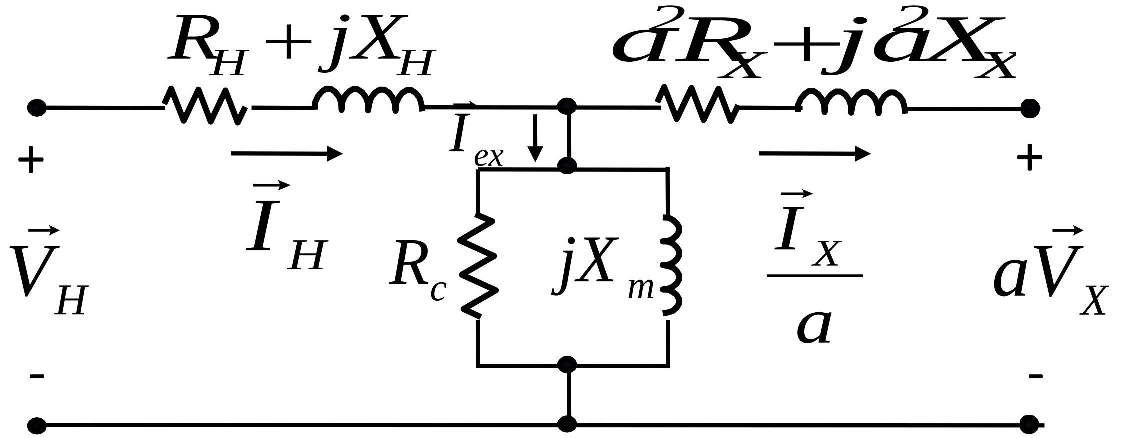
R_X, X_X = resistance and leakage reactance of X coil

R_c, X_m = core resistance and magnetizing reactance

Two-Winding Transformer

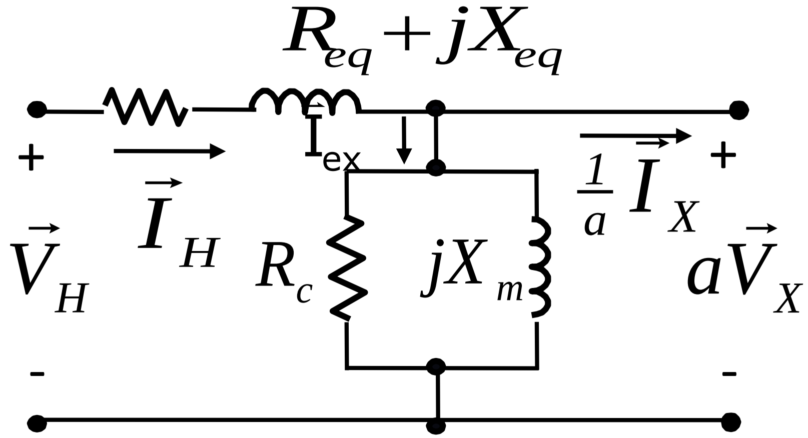
Referring secondary quantities to the primary side,





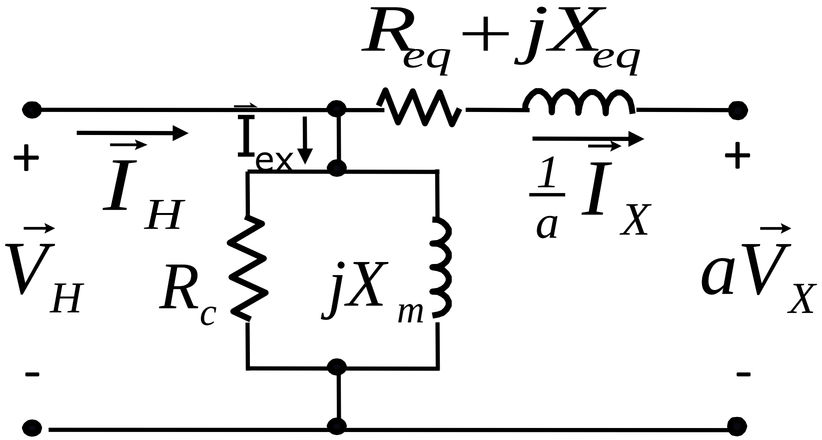
Two-Winding Transformer

The transformer equivalent circuit can be approximated by



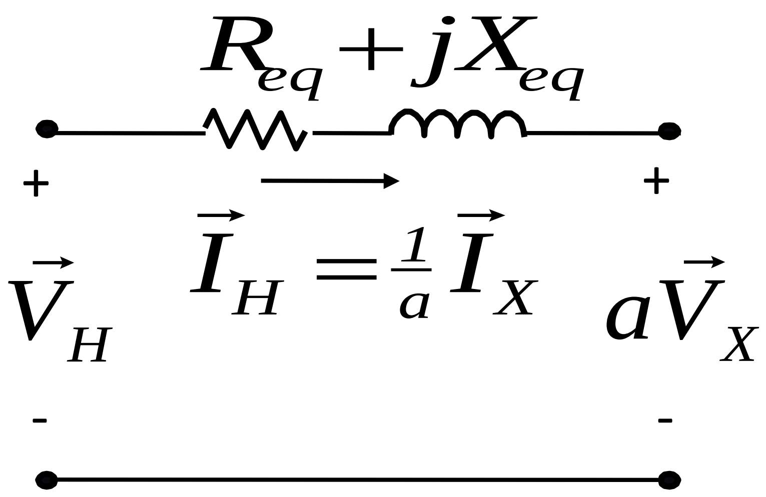
$$R_{eq} = R_H + a^2 R_X$$

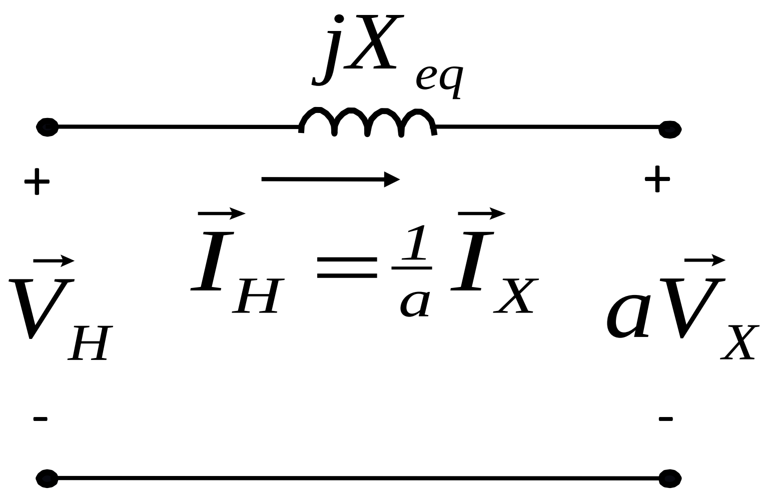
$$X_{eq} = X_H + a^2 X_X$$



Two-Winding Transformer

For large power transformers, shunt impedance and resistance can be neglected





Problem 1

- A transformer is to be used to transform the impedance of a 8Ω resistor to an impedance of 75 Ω. Calculate the Required Turns ration assuming the transformer to be ideal.

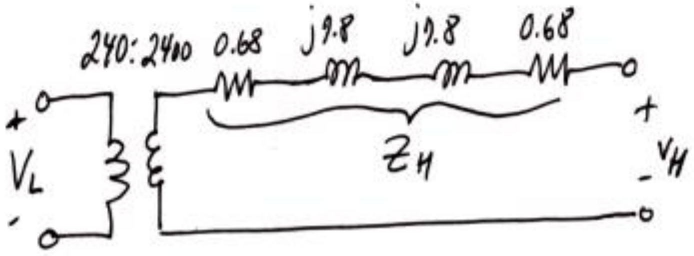
Answer N = 3
Turns

Problem 2

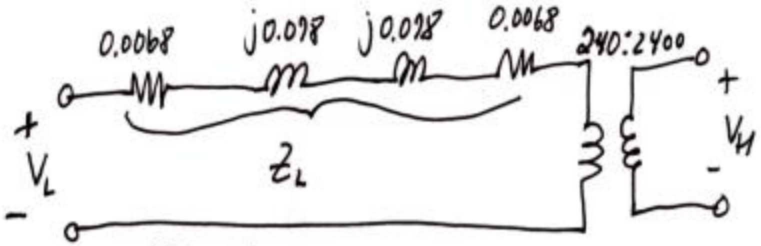
- The resistances and leakage reactances of 30-kVA, 60 Hz, 2400-V:240V distribution transformer are
  - $R1 = 0.68\ \Omega$
  - $X1,h = 7.8\ \Omega$
  - $R2 = 0.0068\ \Omega$
  - $X1,l = 0.0780\ \Omega$
- Draw the equivalent circuit referred to the high voltages side
- Consider the transformer to deliver its rated KVA to a load with 230 V across the load
  - Find the voltage at the high side for a load power factor of 0.85 pf lag
  - Find the voltage at the high side for a load power factor of 0.85 pf lead

Problem 2 SCRATCH SLIDE

Problem 2 SCRATCH SLIDE



(i) HV SIDE



(ii) LV SIDE

part (b):

$$\hat{I}_{\text{load}} = \frac{30 \text{ kW}}{230 \text{ V}} e^{j\phi} = 93.8\, e^{j\phi} \quad \text{A}$$

where ϕ is the power-factor angle. Referred to the high voltage side, $\hat{I}_{\text{H}} = 9.38\, e^{j\phi}$A.

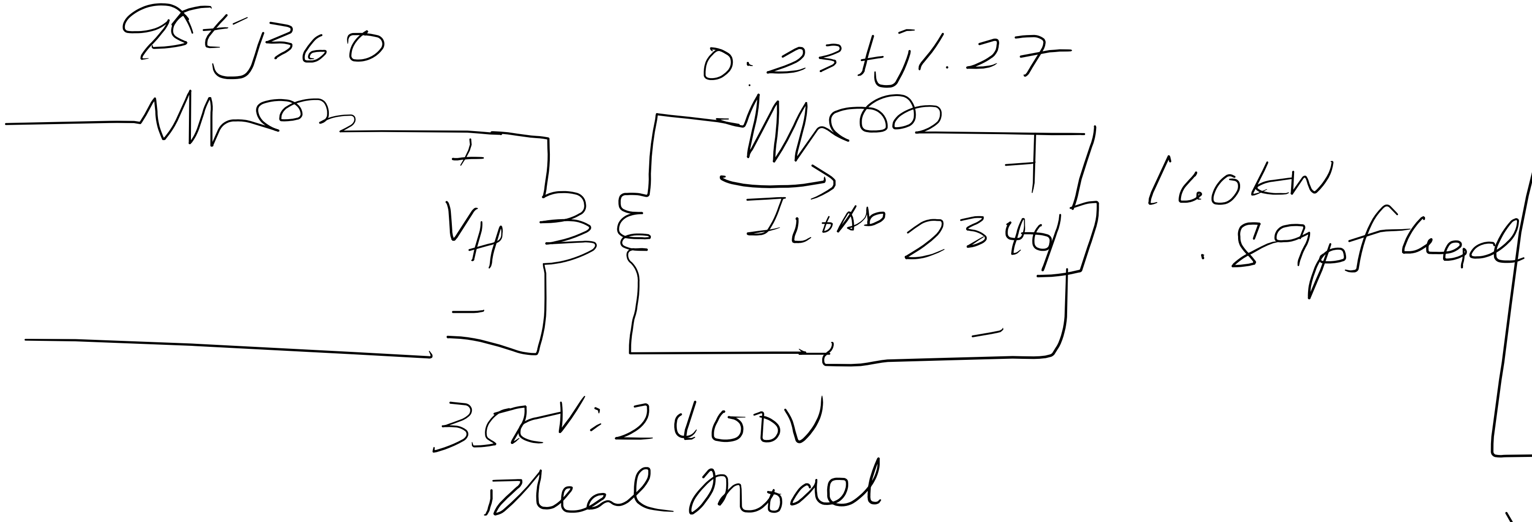
$$\hat{V}_{\text{H}} = Z_{\text{H}} \hat{I}_{\text{H}}$$

Thus, (i) for a power factor of 0.85 lagging, $V_{\text{H}} = 2413$ V and (ii) for a power factor of 0.85 leading, $V_{\text{H}} = 2199$ V.

Concept Test

- A Single-phase load is supplied through a 35kV feeder whose impedance is 95 + j360 Ω and a 35kV:2400V transformer whose equivalent impedance is 0.23 + j1.27 referred to its low voltage side. The load is 160 kW at 0.89 lead pf and 2340 V.
  - (CONCEPT TEST)
  - Compute the voltage at the high-voltage terminals of the transformer
  - Compute the voltage at the sending end of the feeder
  - Compute the power and reactive power input at the send end of the feeder.

CONCEPT TEST SCRATCH SLIDE



$P = 164\ kW$

$Q = -69.5\ kvars$

$S = VI^*$

$$V_H = \frac{35kV}{2400V}\left(2340\angle 0^\circ + (0.23+j1.27)(I_{LOAD})\right)$$

$$I_{LOAD} = \frac{160kW}{.89(2340)} = 76.827\angle 27.126$$

$$\boxed{\overline{V_H} = 33.7kV} \qquad \overrightarrow{V_H} = 33.733\ \angle 2.3511^\circ kV$$

$$I_p = \frac{N_s}{N_p} I_{LOAD}$$

$$I_p = 5.268$$

$$V_s = 33.733\angle 2.3511^\circ + (95+j360)(5.268\angle 27.126^\circ)$$

$$\boxed{= 33.4kV}$$

part (a): $I_{\mathrm{load}} = 160\,\mathrm{kW}/2340\,\mathrm{V} = 68.4$ A at $\angle = \cos^{-1}(0.89) = 27.1°$

$$\hat{V}_{\mathrm{t,H}} = N(\hat{V}_{\mathrm{L}} + Z_{\mathrm{t}} I_{\mathrm{L}})$$

which gives $V_{\mathrm{H}} = 33.7$ kV.

part (b):

$$\hat{V}_{\mathrm{send}} = N(\hat{V}_{\mathrm{L}} + (Z_{\mathrm{t}} + Z_{\mathrm{f}}) I_{\mathrm{L}})$$

which gives $V_{\mathrm{send}} = 33.4$ kV.

part (c):

$$S_{\mathrm{send}} = P_{\mathrm{send}} + jQ_{\mathrm{send}} = \hat{V}_{\mathrm{send}} \hat{I}^*_{\mathrm{send}} = 164 \text{ kW} - j64.5 \text{ kVAR}$$

Thus $P_{\mathrm{send}} = 164$ kW and $Q_{\mathrm{send}} = -64.5$ kVAR.