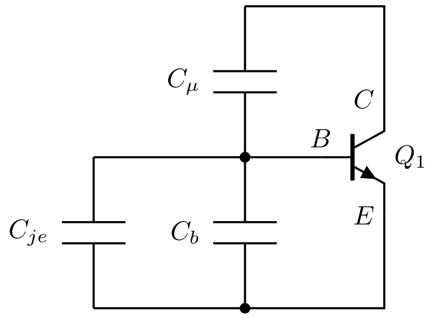


Figure 7.1: BJT parasitic capacitances.

7 Amplifier Frequency Response

In our analysis so far, we have considered only "low-frequency" small signals, or signals with frequencies such that we can ignore all frequency-dependent circuit elements such as capacitors and inductors. As we increase the frequency of the amplifier input signals, the impedances of these frequency-dependent elements can start to dominate, thus changing the amplifier's small signal behavior. In order to determine how capacitances and inductances affect our electronic circuits, we first review the sources of these parasitic[a] elements.

7.1 BJT Parasitic Capacitances

A BJT has two junctions, and each junction has a parasitic capacitance associated with it, as shown in Fig. 7.1. In the forward-active region, the forward-biased base-emitter junction capacitance is dominated by the (1) base-charging capacitance, and (2) the base-emitter junction capacitance. The reversed-bias base-collector capacitance is mostly due to the base-collector junction capacitance.

The *base-charging capacitance*, C_b, models the change in majority carriers in the base due to the change in base-emitter voltage. These majority carriers recombine with the injected minority carriers from the emitter, and the amount of minority charge injected is dependent on V_{BE}. Thus

$$C_b = \frac{\partial Q_b}{\partial V_{BE}} = g_m \tau_F \tag{7.1}$$

where τ_F is the forward base transit time, or the average time needed by a carrier to cross the transistor base region. Typically, C_b is in the hundreds of femtofarads.

The *base-emitter junction capacitance*, C_{je} represents the change in diffusion charge of the forward-biased base-emitter junction as V_{BE} is varied. Being a junction capacitance, C_{je} is nonlinear, and if we assume that the base-emitter junction is a step junction, C_{je} can be expressed as

$$C_{je} = \frac{\partial Q_{BE}}{\partial V_{BE}} = \frac{C_{je0}}{\sqrt{1 - \frac{V_{BE}}{V_{j,BE}}}} \tag{7.2}$$

where C_{je0} is the capacitance when $V_{BE} = 0$, and $V_{j,BE}$ is the built-in potential of the base-emitter junction. For $V_{BE} \approx V_{j,BE}$, $C_{je} \approx 2 \cdot C_{je0}$, and with typical values in the tens of femtofarads.

The base-collector junction capacitance, C_μ, represents the depletion capacitance of the reverse-biased collector-base junction, and for a step junction, we get

$$C_\mu = \frac{\partial Q_{CB}}{\partial V_{CB}} = \frac{C_{\mu 0}}{\sqrt{1 + \frac{V_{CB}}{V_{j,CB}}}} \tag{7.3}$$

where $C_{\mu 0}$ is the capacitance when $V_{CB} = 0$ and$V_{j,CB}$ is the built-in potential of the collector-base junction. Typically, C_μ is less than 10 fF. However due to the *Miller effect*[b], this capacitance can seem bigger to circuits driving the transistor.

Note that these small signal capacitances are dependent on the quiescent DC operating point of the transistor. Thus, the small signal equivalent circuit of the BJT, including these capacitances, is shown in Fig. 7.2, where $C_\pi = C_b + C_{je}$.

[a]These elements are called *parasitic* since they are not built on purpose, but exist only as artifacts due to (1) the fundamental structure of the device and (2) how the device is built.

[b]This is something we will discuss in detail later in this handout.

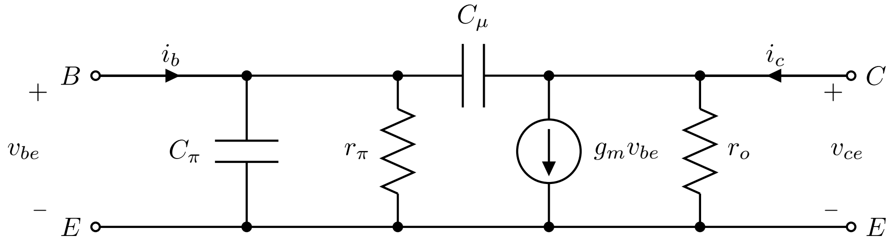


Figure 7.2: The BJT small signal equivalent circuit including the small signal capacitances.

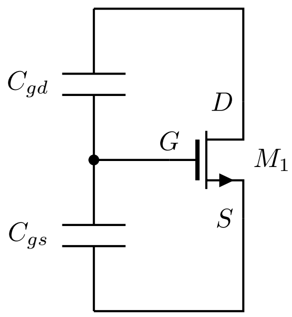


Figure 7.3: MOSFET parasitic capacitances.

7.2 MOSFET Parasitic Capacitances

In planar MOSFETs, the main parasitic capacitances are the gate-to-source capacitance, C_{gs}, and the gate-to-drain capacitace, C_{gd}.

Gate-to-drain capacitance can be considered linear, and dependent on the dimensions of the transistor...

Gate-to-source capacitance is nonlinear, MOS capacitor, but in EEE 51, the MOSFET will almost always be, by design, in the super-threshold, saturation region... thus, in most cases, we can express C_{gs} as

$$C_{gs} \approx \frac{2}{3} \cdot C_{ox} \cdot W \cdot L \tag{7.4}$$

The small signal equivalent circuit of the MOSFET, including the parasitic capacitances, is shown in Fig. 7.4.

7.3 Frequency Response of Resistor-Capacitor Circuits

Consider a simple RC circuit, shown in Fig. 7.5a. Solving for the output voltage in the Fourier domain, we get

$$v_o(\omega) = v_i(\omega) \cdot \frac{Z_C}{R + Z_C} = v_i(\omega) \cdot \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = v_i(\omega) \cdot \frac{1}{1 + j\omega RC} = v_i(\omega) \cdot \frac{1}{1 + j\frac{\omega}{\omega_o}} = A_v \cdot v_i(\omega) \tag{7.5}$$

Thus, the voltage gain becomes

$$A_v(\omega) = \frac{v_o}{v_i}(\omega) = \frac{1}{1 + j\frac{\omega}{\omega_o}} \tag{7.6}$$

where $\omega_o = \frac{1}{RC}$.

Note that A_v is a complex number, and therefore we need two components to fully specify it. Therefore, A_v can be represented by either its real and imaginary components, $\Re(A_v)$ and $\Im(A_v)$, or its magnitude and phase, $|A_v|$ and

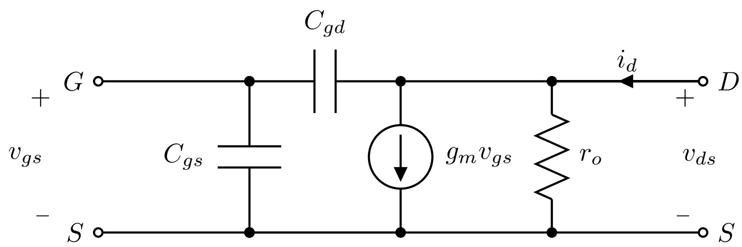
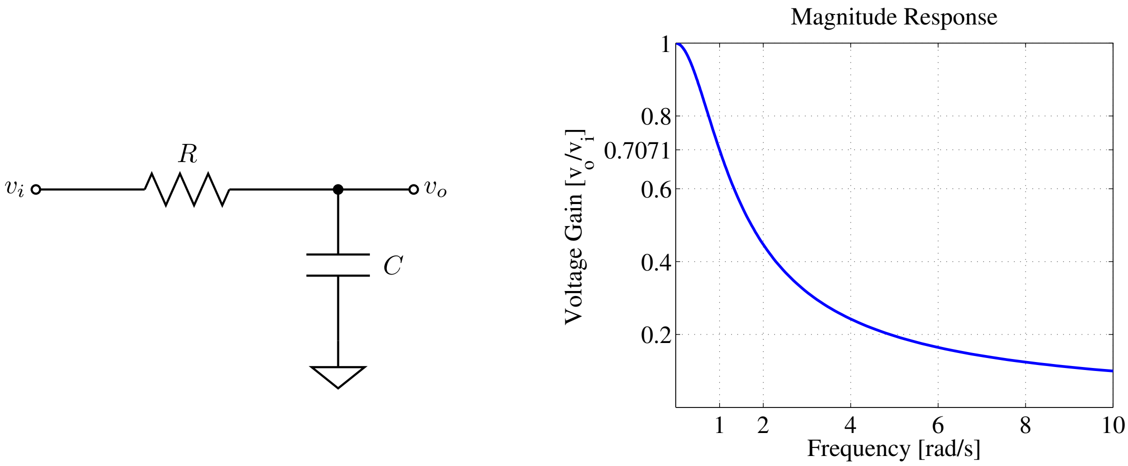


Figure 7.4: The MOSFET small signal equivalent circuit including the small signal capacitances.



(a) Schematic (b) Magnitude response

Figure 7.5: A simple RC circuit example with $R = 1\,\Omega$ and $C = 1\,\text{F}$.

$\angle A_v$, thus

$$A_v = \Re(A_v) + j \cdot \Im(A_v) = |A_v| \cdot e^{j \cdot \angle A_v} \tag{7.7}$$

where

$$|A_v| = \sqrt{A_v \cdot A_v^*} = \sqrt{(\Re(A_v) + j \cdot \Im(A_v)) \cdot (\Re(A_v) - j \cdot \Im(A_v))} = \sqrt{\Re(A_v)^2 + \Im(A_v)^2} \tag{7.8}$$

and

$$\angle A_v = \tan^{-1} \frac{\Im(A_v)}{\Re(A_v)} \tag{7.9}$$

If we assume that the inputs are purely sinusoidal signals, then we can represent these inputs as phasors, with both magnitude and phase,

$$|v_o| \cdot e^{j \cdot \angle v_o} = |A_v| \cdot e^{j \cdot \angle A_v} \cdot |v_i| \cdot e^{j \cdot \angle v_i} = |A_v| \cdot |v_i| \cdot e^{j \cdot (\angle A_v + \angle v_i)} \tag{7.10}$$

Thus, in the time domain, for $v_i(t) = V_i \cdot \sin(\omega t)$, the general form of the output is

$$v_o(t) = V_o \cdot \sin(\omega t + \phi) \tag{7.11}$$

Therefore, in order to get the complete time domain response for a purely sinusoidal input, we need to determine V_o and ϕ. Recall that in linear circuits, the output frequency will always be the same as the input frequency.

7.3.1 Magnitude Response

The magnitude of A_v can be expressed as

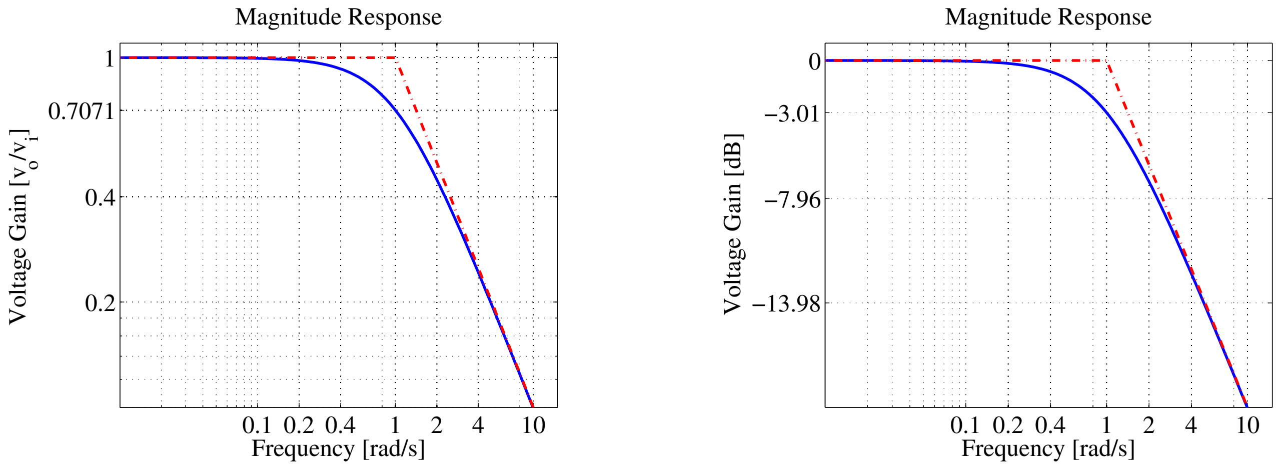
$$|A_v(\omega)| = \sqrt{A_v(\omega) \cdot A_v^*(\omega)} = \sqrt{\frac{1}{1 + j\frac{\omega}{\omega_o}} \cdot \frac{1}{1 - j\frac{\omega}{\omega_o}}} = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_o}\right)^2}} \tag{7.12}$$

and is plotted in Fig. 7.5b as a function of the frequency ω.

Eq. 7.12 describes the behavior of the voltage gain when a sinusoid voltage, with frequency[c] ω, is applied at the input of the simple RC circuit in Fig. 7.5a. As expected, at DC (when $\omega = 0$), the output voltage is equal to the input voltage, since the capacitor acts like an open circuit. The voltage gain drops as the frequency of the input sinusoid increases. This is due to the decrease in the capacitor impedance, $Z_C = \frac{1}{j\omega C}$, reducing the output voltage. At very high frequencies (when $\omega \to \infty$), the capacitor starts behaving like a short circuit, or $Z_C \to 0$, thus, the output voltage also approaches zero.

The frequency ω_o represents the boundary between these two cases. When $\omega \ll \omega_o$, Eq. 7.12 becomes $|A_v(\omega)| \approx 1$. On the other hand, when $\omega \gg \omega_o$, $|A_v(\omega)| \approx \frac{\omega_o}{\omega}$. Thus, in this case, ω_o is called the *bandwidth* or *corner frequency*.

[c] In calculations, we use radians per second instead of Hertz, but in plots, the x-axis can either be in radians per second or Hertz. Note that $1\,\frac{\text{rad}}{\text{s}} = 2\pi\,\text{Hz}$.



(a) Magnitude (b) Log-magnitude

Figure 7.6: Logarithmic frequency response plots.

Below this frequency, $Z_C \gg R$, and by voltage division, the output voltage is approximately equal to the input voltage. At frequencies above ω_o, $Z_C \ll R$, resulting in an output voltage that is inversely proportional to the input frequency.

At exactly ω_o, we get

$$|A_v(\omega_o)| = \left|\frac{v_o}{v_i}(\omega_o)\right| = \frac{1}{\sqrt{2}} \tag{7.13}$$

Recall that power is proportional to the square of the voltage. Thus, if we assume that the input and output voltages are impressed on the same resistance, we can define the power gain as

$$P_v(\omega_o) = \left|\frac{v_o^2}{v_i^2}(\omega_o)\right| = \frac{1}{2} \tag{7.14}$$

Thus, ω_o is also called the *half-power point frequency.*

The magnitude response plot in Fig. 7.5b is relatively hard to interpret when looking at a broad range of (1) frequencies, ranging from a few radians per second, to billions of radians per second, and (2) gains ranging from very small fractions to millions. These are the ranges which we normally consider when designing amplifiers. Thus, the magnitude frequency response behavior is typically plotted on a *logarithmic* scale.

Redrawing Fig. 7.5b using logarithmic frequency and gain axes, we get Fig. 7.6a. Note that in a logarithmic axis, we usually measure distances in factors of 10, or in *decades*. Thus, the $10\,\frac{\text{rad}}{\text{s}}$ point is a decade above from the $1\,\frac{\text{rad}}{\text{s}}$ point, which is also a decade above from the $0.1\,\frac{\text{rad}}{\text{s}}$ point. Also notice that the $0.2\,\frac{\text{rad}}{\text{s}}$ point is also a decade below from the $2\,\frac{\text{rad}}{\text{s}}$ point. Another measure of distance in a logarithmic axis is in factors of 2, or in *octaves*. Thus, the $1\,\frac{\text{rad}}{\text{s}}$ point is an octave below the $2\,\frac{\text{rad}}{\text{s}}$ point, which is an octave below the $4\,\frac{\text{rad}}{\text{s}}$ point.

Two important characteristics of logarithmic plots... 1) exponential functions appear as straight lines, therefore easy to approximate... 2) products appear as sums... multiple poles and zeros... their behavior is additive!

Thus, for $v_i(t) = V_i \cdot \sin(\omega t)$,

$$|v_o(\omega)| = V_o = |A_v(\omega)| \cdot |v_i(\omega)| = |A_v(\omega)| \cdot V_i \tag{7.15}$$

The dB scale...

Note that

$$20 \cdot \log(|A_v(\omega_o)|) = 20 \cdot \log\left(\frac{1}{\sqrt{2}}\right) \approx -3\,\text{dB} \tag{7.16}$$

Thus, ω_o is also called the $-3\,\text{dB}$ *frequency.*

7.3.2 Phase Response

Aside from the magnitude response, phase response describes the shift in a sinusoid at the output relative to the input...

$$\angle A_v(\omega) = \tan^{-1}\frac{\Im(A_v(\omega))}{\Re(A_v(\omega))} = \tan^{-1}\left(\frac{0}{1}\right) - \tan^{-1}\left(\frac{\omega}{\omega_o}\right) = -\tan^{-1}\left(\frac{\omega}{\omega_o}\right) \tag{7.17}$$

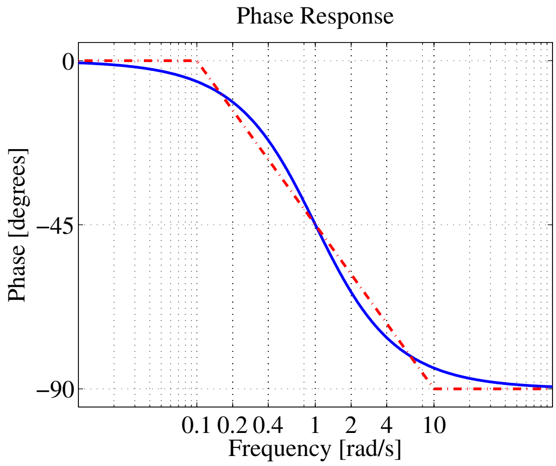


Figure 7.7: Phase response plot.

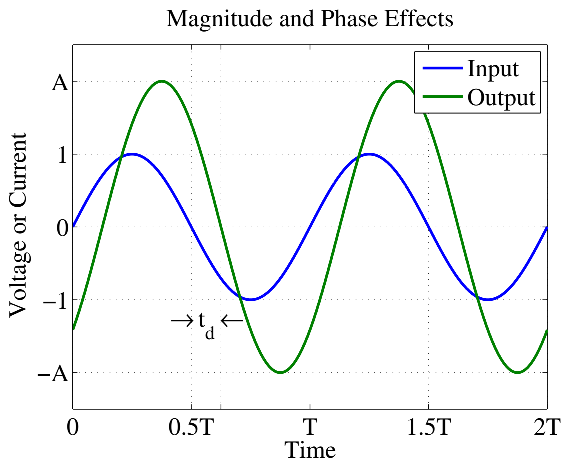


Figure 7.8: Magnitude and phase effects on an input sinusoid.

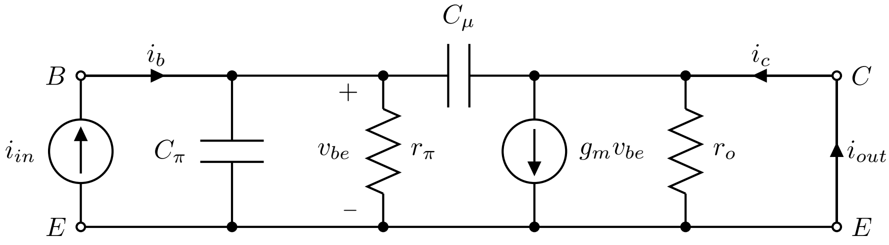


Figure 7.9: Small signal equivalent circuit for calculating f_T.

7.4 Poles and Zeros

In the Laplace domain, any transfer function, like the ratio of input and output voltages of a two-port network, can be expressed as

$$A(s) = A_0 \cdot \frac{\left(1+\frac{s}{z_1}\right)\cdot\left(1+\frac{s}{z_2}\right)\cdots\left(1+\frac{s}{z_m}\right)}{\left(1+\frac{s}{p_1}\right)\cdot\left(1+\frac{s}{p_2}\right)\cdots\left(1+\frac{s}{p_n}\right)} \tag{7.18}$$

where A_0 is the DC value, or the value of $A(s)$ when $s = 0$, and z_i and p_j are the poles and zeros of $A(s)$. Recall that z_i is a zero if $A(s = -z_i) = 0$. Similarly, p_j is a pole if $A(s = -p_j) \to \infty$.

Laplace vs. Fourier... more information in s... Fourier only takes the complex part of s...

7.4.1 Pole Frequency Response

Consider the simple RC circuit in Fig. 7.5a. Writing out the voltage gain in the Laplace domain gives us

$$A_v(s) = \frac{v_o}{v_i}(s) = \frac{1}{1+s\cdot RC} \tag{7.19}$$

By inspection, we can see we have a real pole, $p = -\frac{1}{RC}$. In the complex s-plane, we can plot $A_v(s)$ as a surface, as shown in Fig. , where $\sigma = \Re(s)$ and $\omega = \Im(s)$, such that $s = \sigma + j\omega$.

7.4.2 Zero Frequency Response

7.5 The Transistor Transition Frequency

One metric we can use to compare the behavior of transistors as input frequency is increased, is the transistor's transition frequency, f_T. Transition frequency is defined as the frequency at which the short-circuit common-emitter or common-source current gain falls to unity.

Thus, for a BJT, using the circuit in Fig. 7.9, and noting that at very high frequencies, the impedance of the capacitors, in the Laplace domain, will be very much less than r_π, we can express v_{be} as

$$v_{be} = i_{in}\cdot\left(\frac{1}{sC_\pi} \parallel \frac{1}{sC_\mu} \parallel r_\pi\right) \approx i_{in}\left(\frac{1}{sC_\pi} \parallel \frac{1}{sC_\mu}\right) = \frac{i_{in}}{s(C_\pi + C_\mu)} \tag{7.20}$$

Also, if we assume that the output short-circuit current, i_{out} is mainly due to the transconductance, we get

$$i_{out} = g_m v_{be} = g_m \cdot \frac{i_{in}}{s(C_\pi + C_\mu)} \tag{7.21}$$

Therefore, at $s = j\omega$ and $\omega_T = 2\pi f_T$, and using the definition of the transition frquency, we get

$$\left|\frac{i_{out}}{i_{in}}\right| = 1 = \left|\frac{g_m}{j\omega_T(C_\pi + C_\mu)}\right| \tag{7.22}$$

In typical bipolar junction transistors, $C_\pi \gg C_\mu$. Thus, we get

$$\omega_T = \frac{g_m}{C_\pi + C_\mu} \approx \frac{g_m}{C_\pi} \tag{7.23}$$

For MOSFETs, the analysis is exactly the same, and we get a transition frequency equal to

$$\omega_T = \frac{g_m}{C_{gs} + C_{gd}} \approx \frac{g_m}{C_{gs}} \tag{7.24}$$

7.6 Frequency Response of Linear Amplifiers

Consider a common-emitter amplifier biased using a resistor, R_C, and driving a capacitive load, C_L, shown in Fig. ... small signal model including the BJT parasitic capacitances...

The small signal voltage gain...

7.7 Frequency Response of Differential Circuits