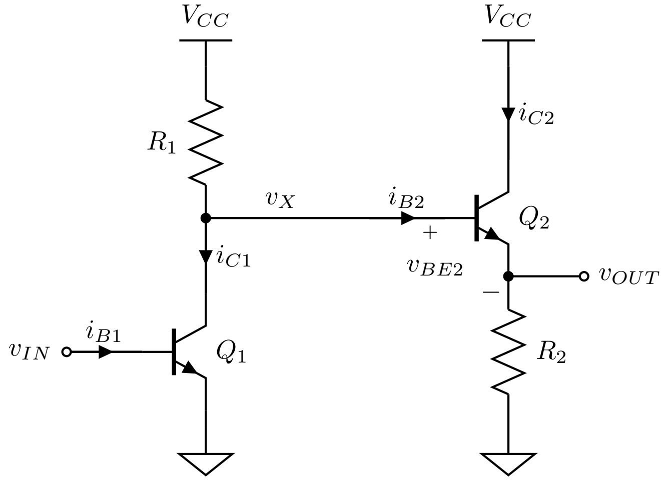

Figure 6.1: The CE-CC 2-stage amplifier.

6 Compound Amplifiers

In the single-stage and differential amplifiers we have seen so far, both the small signal characteristics, such as the voltage gain, transconductance, input and output resistances, and the large signal characteristics such as output swing, and input common-mode range, are very dependent on the amplifier topology used. In most cases, we want to "mix-and-match" these characteristics, and design an amplifier better suited for a given task.

For example, a common-emitter amplifier can have very large voltage gains, but this gain can only be realized if the output resistance is large. Thus, if we use the amplifier to drive very low resistance loads, the voltage gain is drastically reduced. But what if we want both high gain, and low output resistance? In this case, we can use several amplifiers, leveraging each others strengths, and avoiding the characteristics we do not want for a given application.

6.1 The Common-Emitter Common-Collector (CE-CC) Amplifier

One very common 2-stage amplifier topology is the common-emitter amplifier, followed by a common-collector amplifier, shown in Fig. 6.1.

In order to calculate the quiescent DC collector currents of both transistors, we write the KVL equation of the loop between the two stages as

$$V_{CC} - (I_{C1} + I_{B2}) \cdot R_1 - V_{BE2} - I_{E2}R_2 = 0 \tag{6.1}$$

Recall that

$$I_{C1} = I_{S1} \cdot e^{\frac{V_{IN}}{V_T}} \tag{6.2}$$

and

$$V_{BE2} = V_T \cdot \ln\left(\frac{I_{C2}}{I_{S2}}\right) \tag{6.3}$$

we can solve for I_{C2} using

$$V_{CC} - I_{S1} \cdot e^{\frac{V_{IN}}{V_T}} \cdot R_1 - \frac{I_{C2}}{\beta_2} \cdot R_1 - V_T \cdot \ln\left(\frac{I_{C2}}{I_{S2}}\right) - \frac{I_{C2}}{\alpha_2} \cdot R_2 = 0 \tag{6.4}$$

Note that the quiescent DC input voltage of the second stage is set by the quiescent output DC voltage of the first stage. Thus, it is important that the DC output voltage of the first stage be set to the voltage required by the second stage, as determined by the small signal specifications of the compound amplifier. If we assume that $\beta_2 \to \infty$, and $V_{BE2} \approx 0.7\,\mathrm{V}$, then

$$I_{C2} = \frac{V_{CC} - I_{S1} \cdot e^{\frac{V_{IN}}{V_T}} \cdot R_1 - 0.7\,\mathrm{V}}{R_2} \tag{6.5}$$

Once we have obtained the DC bias currents, we can now calculate the small signal parameters of the CE-CC amplifier by looking at the two component amplifier stages individually as shown in Fig. 6.2.

Recall that for the CE amplifier,

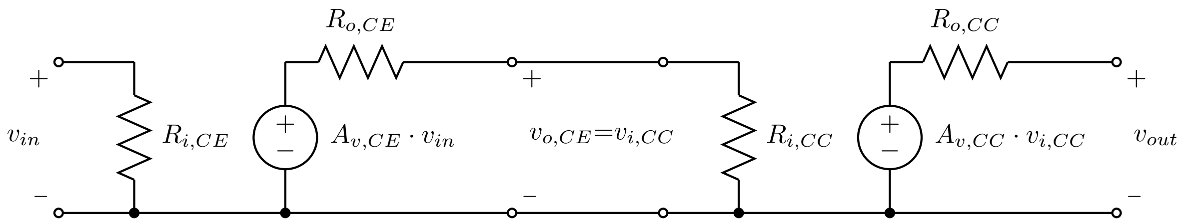

Figure 6.2: The CE-CC small signal equivalent circuit.

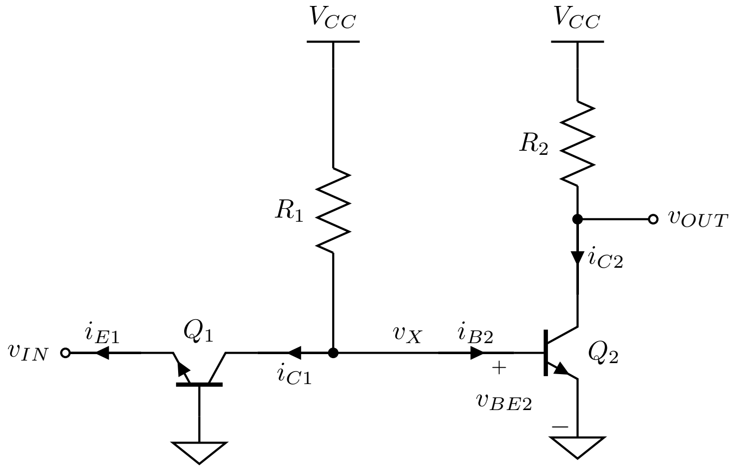

Figure 6.3: The CB-CE 2-stage amplifier.

$$A_{v,CE} = -g_{m1}\left(r_o \parallel R_1\right) \approx -g_{m1}R_1 \tag{6.6}$$

$$R_{o,CE} = \left(r_{o1} \parallel R_1\right) \approx R_1 \tag{6.7}$$

$$R_{i,CE} = r_{\pi 1} \tag{6.8}$$

and for the CC amplifier,

$$A_{v,CC} = \frac{g_{m2}R_2}{1+g_{m2}R_2} \approx 1 \tag{6.9}$$

$$R_{o,CC} = \frac{R_2}{1+g_{m2}R_2} \approx \frac{1}{g_{m2}} \tag{6.10}$$

$$R_{i,CC} = r_{\pi 2}\left(1+g_{m2}R_2\right) \tag{6.11}$$

Thus, the effective CE-CC amplifier two-port parameters are

$$A_{v,CE-CC} = A_{v,CE} \cdot \frac{R_{i,CC}}{R_{i,CC}+R_{o,CE}} \cdot A_{v,CC} = -g_{m1}R_1 \cdot \frac{r_{\pi 2}\left(1+g_{m2}R_2\right)}{r_{\pi 2}\left(1+g_{m2}R_2\right)+R_1} \approx -g_{m1}R_1 \tag{6.12}$$

$$R_{o,CE-CC} = R_{o,CC} = \frac{R_2}{1+g_{m2}R_2} \approx \frac{1}{g_{m2}} \tag{6.13}$$

$$R_{i,CE-CC} = R_{i,CE} = r_{\pi 1} \tag{6.14}$$

Notice that by cascading the two basic amplifier stages, we can create a new amplifier with approximately the same voltage gain and input resistance, but now has a significantly lower output resistance. This low output resistance is very useful in driving loads with small input resistances, such as $8\,\Omega$ speakers.

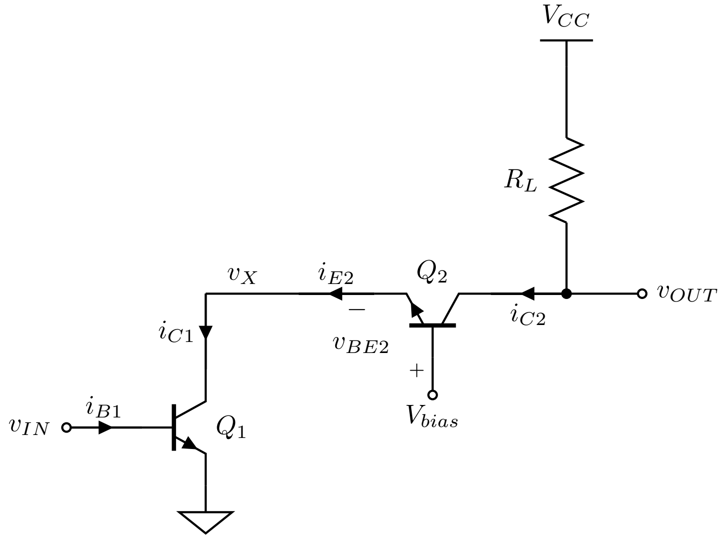

Figure 6.4: The cascode amplifier.

6.2 The Common-Base Common-Emitter (CB-CE) Amplifier

Another common topology is the common-base amplifier followed by a common-emitter amplifier, shown in Fig. 6.3.

Similar to the CE-CC amplifier, the input bias of the second stage is a function of the DC output voltage of the first stage. Writing the KVL equation around the loop in between the two stages, we get

$$V_{CC} - (I_{C1} + I_{B2}) \cdot R_1 - V_{BE2} = 0 \tag{6.15}$$

Recognizing that $V_{BE1} = -V_{IN}$, and that once again, $V_{BE2} = V_T \cdot \ln\left(\frac{I_{C2}}{I_S}\right)$, we get

$$V_{CC} - I_{S1} \cdot e^{\frac{-V_{IN}}{V_T}} \cdot R_1 - \frac{I_{C2}}{\beta_2} \cdot R_1 - V_T \cdot \ln\left(\frac{I_{C2}}{I_S}\right) \tag{6.16}$$

which we can then use to solve for I_{C2}.

Similar to the CE-CC amplifier, we can now calculate the overall gain of the CB-CE pair as

$$A_{v,CB-CE} = A_{v,CB} \cdot \frac{R_{i,CE}}{R_{i,CE} + R_{o,CB}} \cdot A_{v,CE} = g_{m1}R_1 \cdot \frac{r_{\pi 2}}{r_{\pi 2} + R_1} \cdot (-g_{m2}R_2) \approx -g_{m1}g_{m2}R_1R_2 \tag{6.17}$$

and the input and output resistances as

$$R_i = R_{i,CB} = \frac{1}{g_{m1}} + \frac{R_1 \parallel r_{\pi 2}}{g_{m1}r_{o1}} \approx \frac{1}{g_{m1}} \tag{6.18}$$

$$R_o = R_{o,CE} = r_{o2} \parallel R_2 \approx R_2 \tag{6.19}$$

Note that the input resistance of the CB-CE pair is a function of the transconductance, and thus, the collector current of the common-base stage. This dependency of the input resistance to I_{C1} allows us to match the input resistance of the CB-CE amplifier pair to the output resistance of a driving circuit. For example, in audio applications, most microphones will have an output resistance of around $10\,\mathrm{k\Omega}$. Thus, one way to achieve maximum power transfer between the microphone and the amplifier is to use the common-based input stage as a matching stage, with an input resistance of $10\,\mathrm{k\Omega}$.

6.3 The Common-Emitter Common-Base (Cascode) Amplifier

In the previous two compound amplifiers, two separate bias currents are needed, one for each stage. One strategy used to reduce the quiescent DC power of a multi-stage amplifier is to reuse the bias current. This allows multiple transistors to use the same bias current. The most popular example of an amplifier that takes advantage of this *current reuse* strategy is the common-emitter amplifier followed by a common-base amplifier, or otherwise known as a *cascode* pair, and is shown in Fig. 6.4.

The quiescent DC collector current of Q_1 can be expressed as

$$I_{C1} = I_{S1} \cdot e^{\frac{V_{IN}}{V_T}} \tag{6.20}$$

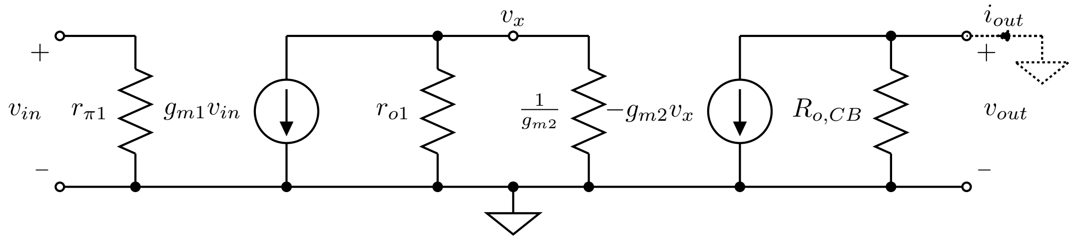

Figure 6.5: The small signal equivalent circuit of the cascode amplifier in Fig. 6.4.

Since Q_2 is in series with Q_1, then $I_{E2} = I_{C1}$, therefore

$$I_{C2} = \alpha_2 I_{E2} = \alpha_2 I_{C1} = \alpha_2 I_{S1} \cdot e^{\frac{V_{IN}}{V_T}} \tag{6.21}$$

In order to ensure that Q_1 is in the forward-active region, let us examine its collector-emitter voltage. We can express V_{CE1} as

$$V_{CE1} = V_{bias} - V_{BE2} = V_{bias} - V_T \ln\left(\frac{\alpha_2 I_{S1} \cdot e^{\frac{V_{IN}}{V_T}}}{I_{S2}}\right) = V_{bias} - V_T \ln\left(\alpha_2 \frac{I_{S1}}{I_{S2}}\right) - V_{IN} > V_{CE1,sat} \tag{6.22}$$

Thus, to ensure that Q_1 is in the forward-active region,

$$V_{bias} > V_{CE1,sat} + V_T \ln\left(\alpha_2 \frac{I_{S1}}{I_{S2}}\right) + V_{IN} \tag{6.23}$$

For Q_2, we can express V_{CE2} as

$$V_{CE2} = V_{CC} - I_{C2}R_L - V_{CE1} > V_{CE2,sat} \tag{6.24}$$

Therefore

$$V_{CC} - \alpha_2 I_{S1} \cdot e^{\frac{V_{IN}}{V_T}} \cdot R_L - V_{bias} + V_T \ln\left(\alpha_2 \frac{I_{S1}}{I_{S2}}\right) + V_{IN} > V_{CE2,sat} \tag{6.25}$$

Note that V_{CE1} and V_{CE2} can be set by (1) the DC input voltage, V_{IN}, (2) the DC bias voltage at the base of Q_2, V_{bias}, and (3) the value of the resistor, R_L.

In order to calculate the overall small signal parameters, we can draw the small signal equivalent circuit as a combination of the equivalent circuits of the CE and CB stages, as shown in Fig. 6.5.

As we have seen before, the common-base amplifier is not unilateral, i.e. its input resistance changes with load conditions. This means that its output resistance will also change if the amplifier driving the common-base amplifier has a finite output resistance[a]. Thus, we will need to solve for the overall two-port parameters in two steps: (1) first obtain the output resistance of the CB amplifier, $R_{o,CB}$, when driven by the CE amplifier, and then (2) calculate the effective transconductance, $G_{m,casc}$, of the CE-CB pair. We can then calculate the overall voltage gain from these two quantities as $A_{v,casc} = -G_{m,casc} \cdot R_{o,CB}$.

To get the output resistance of the CB stage[b], $R_{o,CB}$, let us revisit the common-base small signal model shown in Fig. 6.6, but this time, it is driven by a common-emitter amplifier, modeled as dependent voltage source, $A_{v,CE} \cdot v_{in}$, with an output resistance, $R_{o,CE}$.

Note that zeroing out the input will result in a circuit that is the same as the small signal equivalent circuit of an emitter-degenerated amplifier. Thus, we can express the output resistance of the CB stage driven by a CE stage as

$$R_{o,CB} = R_L \parallel r_{o2}\left(1 + g_{m2}R_{o,CE}\right) = R_{o,casc} \tag{6.26}$$

The overall transconductance can be obtained by shorting the output of the circuit in Fig. 6.5 to ground. Notice that this step is done independent of the output resistance of the CB amplifier since no current will flow through $R_{o,CB}$. By inspection, we can see that the effective transconductance is

$$G_{m,casc} = \frac{i_{out}}{v_{in}} = -g_{m1}\left(r_{o1} \parallel \frac{1}{g_{m2}}\right) \cdot (-g_{m2}) \approx g_{m1} \cdot \frac{g_{m2}}{g_{m2}} = g_{m1} \tag{6.27}$$

[a]The zero input condition is different since even if we zero out the driving source, we will still be left with the driving source's output resistance, and this will affect the output resistance of the common-base amplifier.

[b]Note that the output resistance of the cascode amplifier is equal to the output resistance of the CB stage, as long as it takes into account the fact that the CB stage is being driven by the CE amplifier.

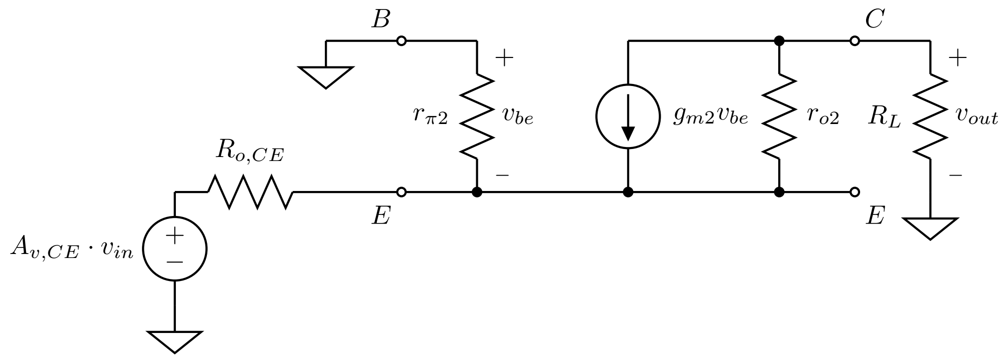

Figure 6.6: The small signal model of the CB amplifier driven by a CE amplifier.

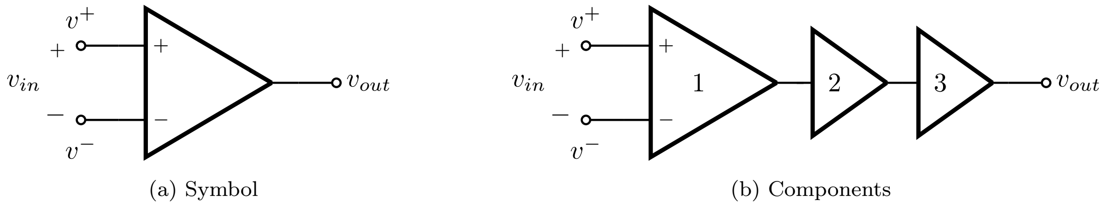

Figure 6.7: The operational amplifier.

Therefore, the oveall gain of the cascode amplifier is

$$A_{v,casc} = -G_{m,casc}R_{o,casc} = -g_{m1} \cdot \left(R_L \parallel r_{o2}\left(1 + g_{m2}R_{o,CE}\right)\right) = -g_{m1} \cdot \left(R_L \parallel r_{o2}\left(1 + g_{m2}r_{o1}\right)\right) \tag{6.28}$$

For the case when $R_L \ll r_{o2}$, then $A_{v,casc} \approx -g_{m1} \cdot R_L$. However, if we replace R_L by an ideal current source, i.e. $R_L \to \infty$, the maximum voltage gain we can get out of the cascode is

$$a_{o,casc} \approx -g_{m1}r_{o1}g_{m2}r_{o2} \approx -g_m^2 r_o^2 \tag{6.29}$$

The input resistance is equal to the input resistance of the common-emitter stage, thus

$$R_{i,casc} = r_{\pi 1} \tag{6.30}$$

One very important characteristic of the cascode amplifier is that it can achieve gains in the order of $g_m^2 r_o^2$, similar to the CB-CE amplifier, but with just one current branch. Having only one current branch can potentially lead to lower DC power consumption. However, since there are two transistors in series, the minimum output voltage is increased to support the two saturation voltages, thus reducing the output voltage swing.

6.4 The Operational Amplifier

The operational amplifier is one of the most important building blocks in modern electronic circuit design. Many larger and more complex electronic circuits such as filters, analog-to-digital converters, and motor drives, are based on the premise that we can build these operational amplifiers.

An ideal operational amplifier has (1) differential inputs, (2) single-ended outputs, (3) infinite gain, (4) infinite input resistance, and (5) zero output resistance. The symbol for an operational amplifier is shown in Fig. 6.7a.

We know we can create amplifiers with differential inputs and single-ended outputs, and we can approximate infinite input resistance amplifiers using MOSFET differential pairs, but we cannot really create infinite gain amplifiers, nor can we build zero output resistance amplifiers. However, in most cases, building amplifiers with voltage gains close to 1×10^6 is doable, as well as obtaining output resistances in the mΩ range. To build these types of circuits, we need to use several amplifier stages.

A typical operational amplifier is composed of three main blocks: (1) a differential-to-single-ended amplifier, (2) a gain stage, and (3) an output stage, as shown in Fig. 6.7b. A BJT implementation of an operational amplifier is shown in Fig. 6.8.

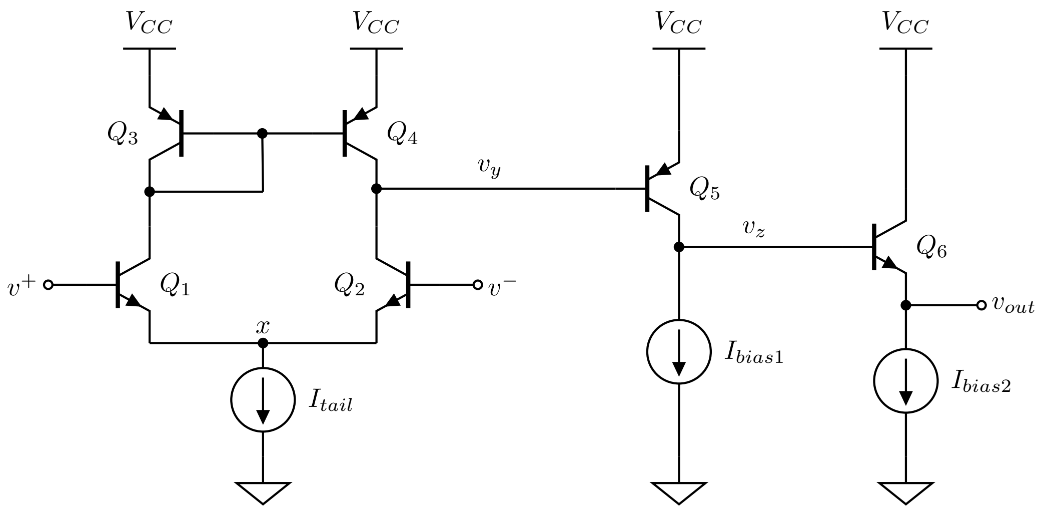

Figure 6.8: A simple BJT operational amplifier.