Lecture 9
PER UNIT SYSTEMS

Agenda

- **Reminders**
- **Lecture**

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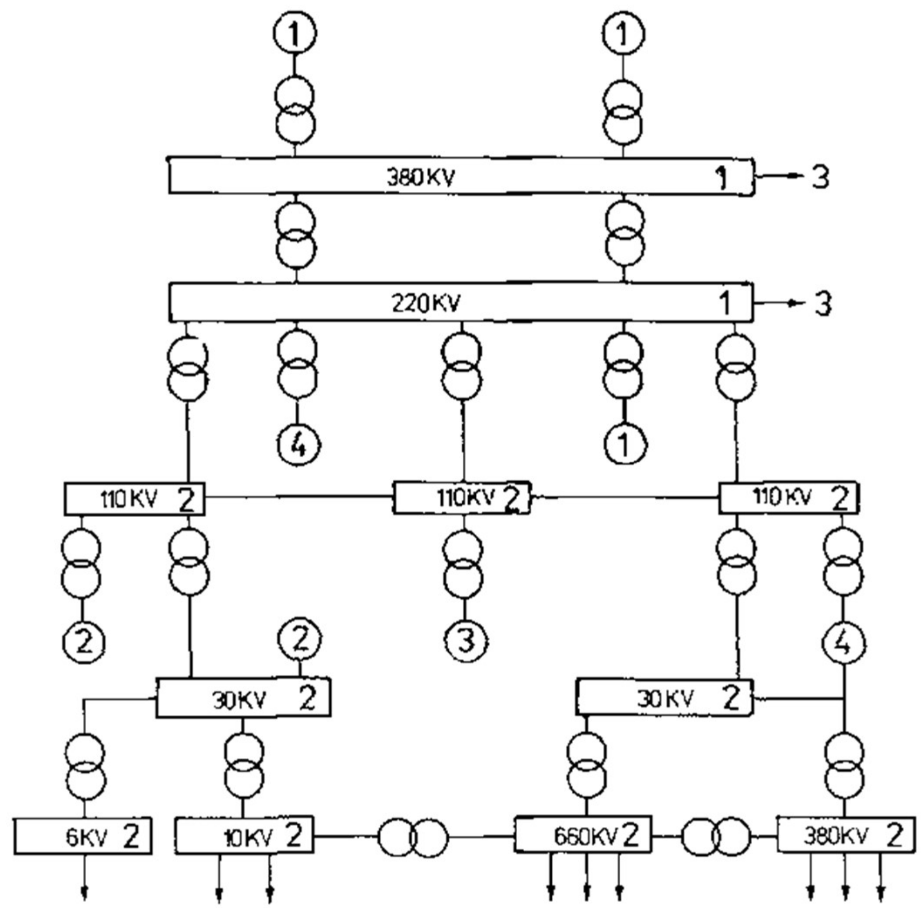
Per Unit – Simplifies Power System Analysis



Lecture Outcomes

at the end of the lecture, the student must be able to …

- Compute the equivalent per-unit value from the actual values and base quantities
- Derive the per-unit representation of a Small Electric power system

Things to Watch out For

- Base Voltage, Current, Power, and Impedance
- Change of Base

EEE 103
Introduction to Power Systems

Power System Modelling
Part 1

Per-Unit Quantities

Per Unit Quantities

Per-Unit Quantity

$$Per - Unit\ Value = \frac{Actual\ Value}{Base\ Value}$$

Per-Unit Value is a dimensionless real quantity.

$$Percent = \frac{Actual\ Value}{100}$$

Other Per Unit Definitions

$$Per - Unit\ Power = \frac{Actual\ Value\ of\ Power}{Base\ Power}$$

$$Per - Unit\ Voltage = \frac{Actual\ Value\ of\ Voltage}{Base\ Voltage}$$

$$Per - Unit\ Current = \frac{Actual\ Value\ of\ Current}{Base\ Current}$$

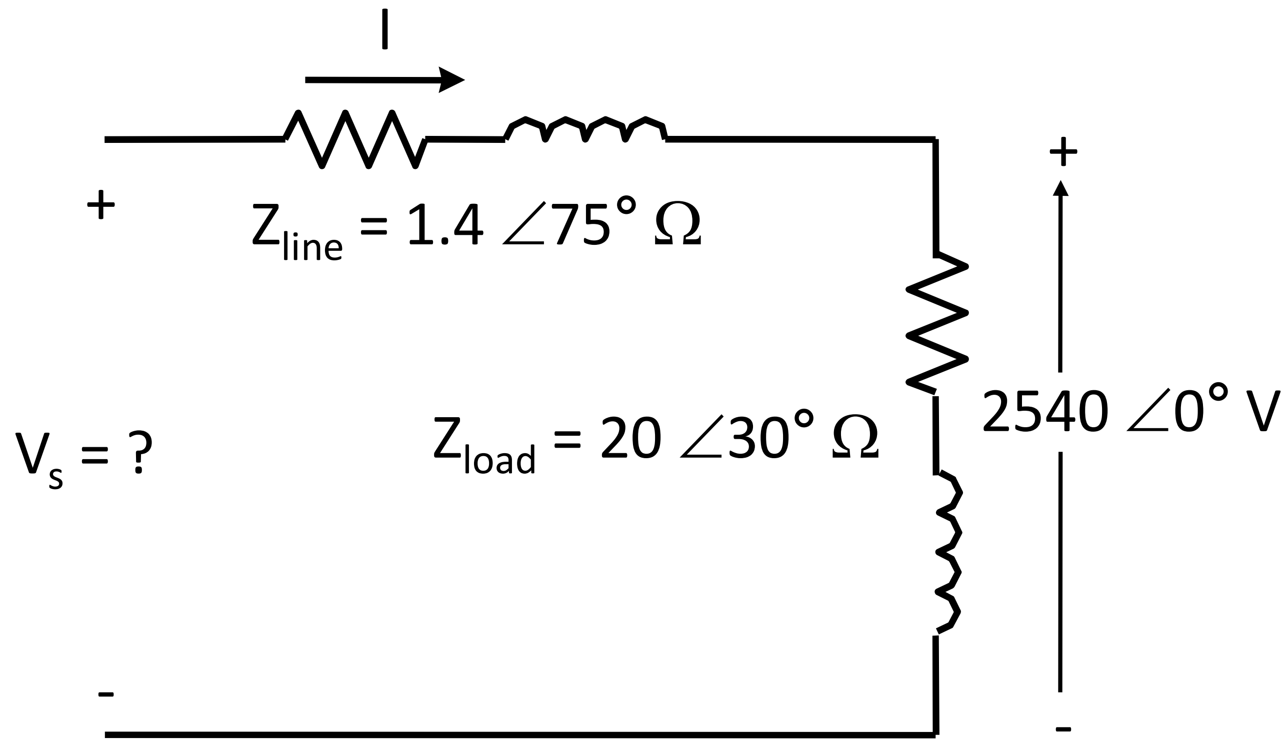
$$Per - Unit\ Impedance = \frac{Actual\ Value\ of\ Impedance}{Base\ Impedance}$$

Relationships between Per Unit Quantities – PU Ohm's Law

$$Per - Unit\ Current = \frac{PU\ Voltage}{PU\ Impedance}$$

$$Per - Unit\ Power = PU\ Voltage\ x\ PU\ Current$$

Example



Determine V_s using the per-unit system. Use 20 ohms as base impedance and 2540 V as base voltage

Example

Choose: Base Impedance = 20 Ω (single phase)

Base Voltage = 2540 V (single phase)

PU Impedance of the load = $20\angle 30°/20$ = ______ p.u.

PU Impedance of the line = $1.4\angle 75°/20$ = ______ p.u.

PU Voltage at the load = $2540\angle 0°$ /2540 = ______ p.u.

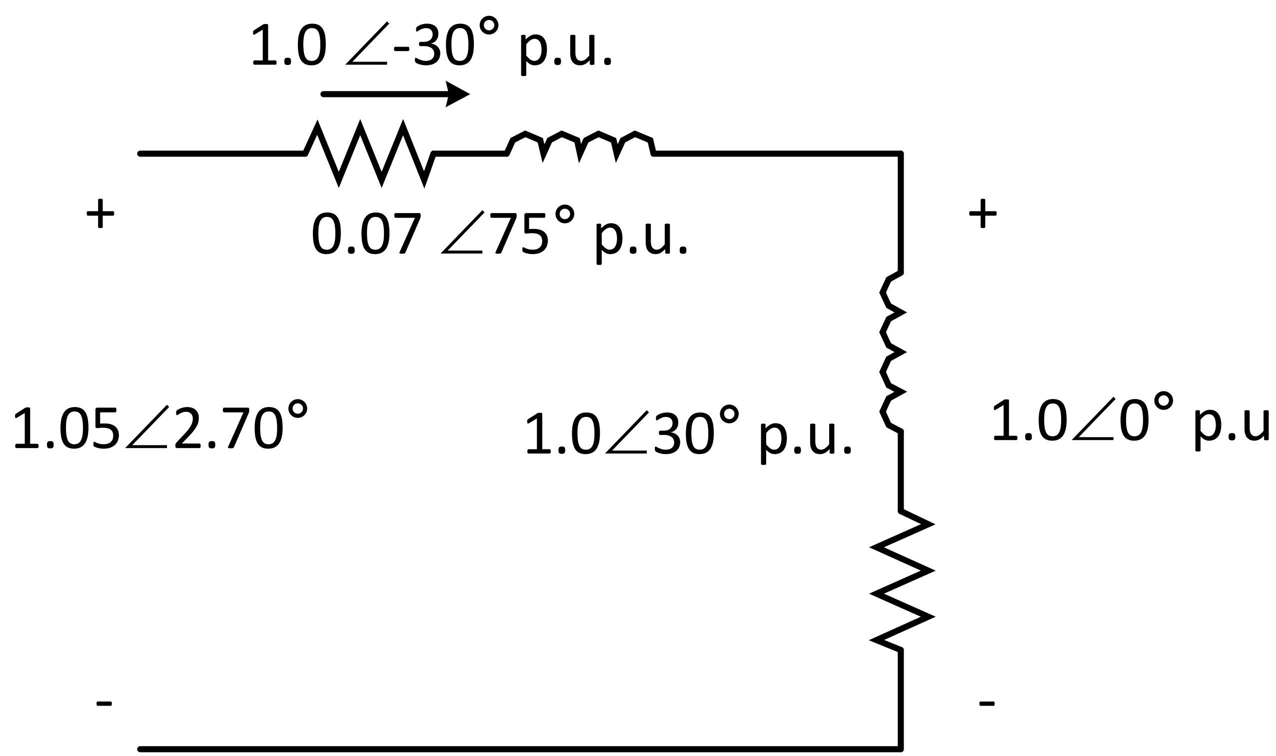
Line Current in PU = PU voltage / PU impedance of the load

= ______ / ______ = ______ p.u.

PU Voltage at the Substation = $V_{load(pu)} + I_{pu}Z_{Line(pu)}$

= ________ + _______ x _______ = _______ p.u.

Example



The magnitude of the voltage at the substation is

1.05 p.u. x 2540 Volts = _______ Volts

How to Establish Base Values

- Must comply with fundamental electrical laws (e.g., Ohm's Law and Kirchoff's Laws).

- V_{base}, I_{base}, S_{base}, and Z_{base} are related such that selection of base values for **any two of them** determines the base values for the remaining two.

Normally, the base voltage (V_{base}) and power (S_{base}) are specified, and the base current (I_{base}) and base impedance (Z_{base}) are derived.

Establishing Base Values from Power and Voltage

$$Base\ Current = \frac{Base\ Power}{Base\ Voltage}$$

$$Base\ Impedance = \frac{Base\ Voltage}{Base\ Current} = \frac{(Base\ Voltage)^2}{Base\ Power}$$

$$\boxed{= \frac{KV^2}{MVA}}$$

Establishing Base Values – Single and Three Phase Definitions

$Single - Phase\ System$

$$I_{base} = \frac{P_{base,1\phi}}{V_{base,1\phi}}$$

$$Z_{base} = \frac{V_{base,1\phi}}{I_{base,1\phi}}$$

$$Z_{base} = \frac{(V_{base,1\phi})^2}{P_{base,1\phi}}$$

$Three - Phase\ System$

$$I_{base} = \frac{S_{base,3\phi}}{\sqrt{3}V_{base,3\phi}}$$

$$Z_{base} = \frac{V_{base,LN}}{I_{base,L}}$$

$$Z_{base} = \frac{(V_{base,LL})^2}{P_{base,3\phi}}$$

Establishing Base Values

Base values can be established from either single-phase or three-phase quantities:

$$Base\,MVA_{1\phi} = \frac{1}{3} Base\,MVA_{3\phi}$$

$$Base\,kV_{1\phi} = \frac{Base\,kV_{LL}}{\sqrt{3}}$$

- The Base MVA is the same value for the Apparent (S), Active (P), and Reactive (Q) power.
- The Base Z is the same value for the Impedance (Z), Resistance (R), and Reactance (X).

Establishing Base Values

Example:

Base $kVA_{3\phi}$	= 30,000 kVA = 30 MVA	Base $kVA_{1\phi}$	= 10,000 kVA = 10 MVA
Base kV_{LL}	= 120 kV	Base kV_{LN}	= 69.282 kV

Establishing Base Values

Example:

Base $kVA_{3\phi}$	= 30,000 kVA = 30 MVA	Base $kVA_{1\phi}$	= 10,000 kVA = 10 MVA
Base kV_{LL}	= 120 kV	Base kV_{LN}	= 69.282 kV
Base Z	$= (120)^2/30$ $= 480$ ohms	*Base Z*	$= (69.282)^2/10$ $= 480$ ohms

$$BaseCurrent = \frac{30x1000}{\sqrt{3}(120)}$$

$$= 144.34 Amps$$

$$BaseCurrent = \frac{10x1000}{69.282}$$

$$= 144.34 Amps$$

Review

- What are the four base quantities?

Per Unit Impedance

Generators

- Manufacturers specify the following impedances in per unit:
  1. Armature Resistance, Ra
  2. Direct-axis Reactances, Xd”, Xd’, and Xd
  3. Quadrature-axis Reactances, Xq”, Xq’, and Xq

     Positive Sequence Impedances
  4. Negative Sequence Reactance, X2
  5. Zero Sequence Reactance, X0
- The Base Values used by manufacturers are:
  1. Rated Capacity (MVA, KVA, or VA)
  2. Rated Voltage (kV or V)

Per Unit Impedance

Transmission and Distribution Lines

$$R_{(pu)} = \frac{R_{(\Omega)}}{Z_{base}}$$

$$X_{L(pu)} = \frac{X_{L(\Omega)}}{Z_{base}}$$

$$X_{C(pu)} = \frac{X_{C(\Omega)}}{Z_{base}}$$

Per Unit Impedance

Transformers

- The ohmic values of resistance and leakage reactance of a transformer depends on whether they are measured on the high- or low-tension side of the transformer.
- The impedance of the transformer is in percent or per unit with the Rated Capacity and Rated Voltages taken as base Power and Base Voltages, respectively.
- The per unit impedance of the transformer is the same regardless of whether it is referred to the high-voltage or low-voltage side.
- The per unit impedance of the three-phase transformer is the same regardless of the connection.

Per Unit Impedance

Example

A single-phase transformer is rated 110/440 V, 2.5 kVA. The impedance of the transformer measured from the low-voltage side is 0.06 ohms. Determine the impedance in per unit (a) when referred to low-voltage side and (b) when referred to high-voltage side

Solution

Low-voltage Z_{base} = $\dfrac{110^2}{2500} = 4.84\Omega$

PU Impedance, $Z_{pu} = \dfrac{0.06}{4.84} = 0.0124$ p.u.

Per Unit Impedance

If impedance had been measured on the high-voltage side, the ohmic value would be

$$Z = 0.06\left(\frac{440}{110}\right)^2 = 0.96\Omega$$

$$\text{High voltage } Z_{base} = \frac{440^2}{2500} = 77.44\Omega$$

$$\text{per unit impedance} = \frac{0.96}{77.44} = 0.0124 \text{ p.u.}$$

Note: PU value of impedance referred to any side of the transformer is the same

Per Unit Impedance

Example:

Consider a three-phase transformer rated 20 MVA, 69 kV/13.2 kV voltage ratio and a reactance of 7%. The resistance is negligible.

a) What is the equivalent reactance in ohms referred to the high voltage side?
b) What is the equivalent reactance in ohms referred to the low voltage side?
c) Calculate the per unit values both in the high voltage and low voltage side at 100 MVA.

Ans: (a) 16.66 Ω, (b) 0.610 Ω, (c) 0.35 p.u., 0.35 p.u.

Change in Base Values

- Usually, the nameplate of each component gives the component impedance in per unit, using rated power and rated voltage as base power and base voltage.
- Different components will have different ratings, hence, different base values.
- To perform per unit calculations that will be consistent with calculations using actual values, there must be a consistent per unit representation of the power system.

Change in Base Values

The actual value of impedance for any power system component is constant, i.e., it is independent of the chosen base values.

Using old base values: $Z_{base}^{old} = \dfrac{(V_{base}^{old})^2}{S_{base}^{old}} \rightarrow Z_{pu,old} = \dfrac{Z_{actual}}{Z_{base}^{old}}$

Using new base values: $Z_{base}^{new} = \dfrac{(V_{base}^{new})^2}{S_{base}^{new}} \rightarrow Z_{pu,new} = \dfrac{Z_{actual}}{Z_{base}^{new}}$

$\therefore$

$$Z_{pu,new} = Z_{pu,old} \times \frac{Z_{base,old}}{Z_{base,new}} = Z_{pu,old} \times \left(\frac{(V_{base}^{old})^2}{S_{base}^{old}}\right) \times \left(\frac{S_{base}^{new}}{(V_{base}^{new})^2}\right) = Z_{pu,old} \times \left(\frac{V_{base}^{old}}{V_{base}^{new}}\right)^2 \times \frac{S_{base,new}}{S_{base,old}}$$

If base voltages are the same:

Remember:

$$\boxed{\frac{MVA_{new}}{MVA_{old}}}$$

Per-Unit Representation

<u>Procedure:</u>

a) Establish Base Power and Base Voltages

- Declare Base Power for the whole Power System
- Declare Base Voltage for any one of the Power System components
- Compute the Base Voltages for the rest of the Power System Components using the voltage ratio of the transformers

Note: Define each subsystem with unique Base Voltage based on separation due to magnetic coupling, i.e., by transformer windings.

Per-Unit Representation

b) Compute Base Impedance and Base Current

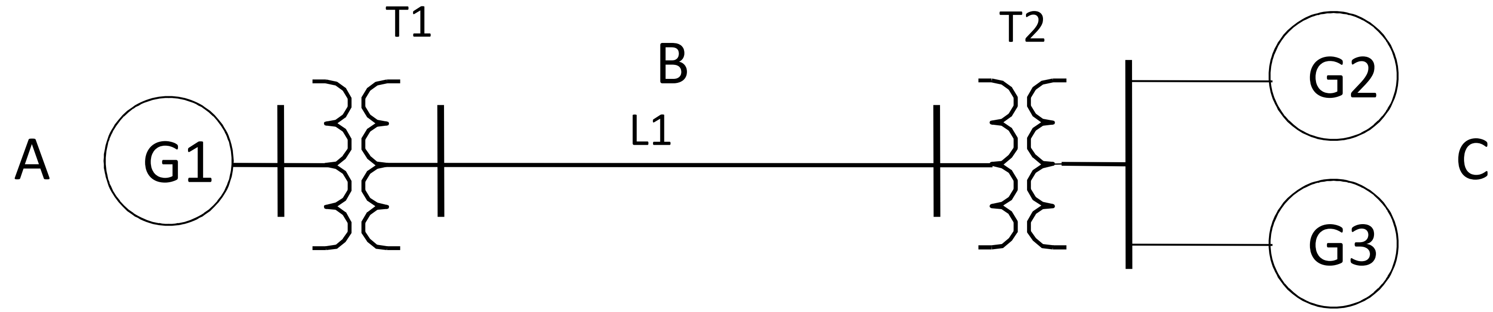
- Using the Declared Base Power and Base Voltages, compute the Base Impedances and Base Currents for each Subsystem

c) Compute Per Unit Impedance

- Using the declared and computed Base Values, compute the Per Unit values of the impedance by:
  - *Dividing Actual Values by Base Values*
  - *Changing Per Unit Impedance with change in Base Values*

Per-Unit Representation

Use Base Power = 100 MVA



Generator 1 (G1): 300 MVA; 20 kV; 3ϕ; X_d" = 20 %

Transmission Line(L1): 64 km; X_L = 0.5 Ω / km

Transformer 1 (T1): 3ϕ; 350 MVA; 230 / 20 kV; X_{T1} = 10 %

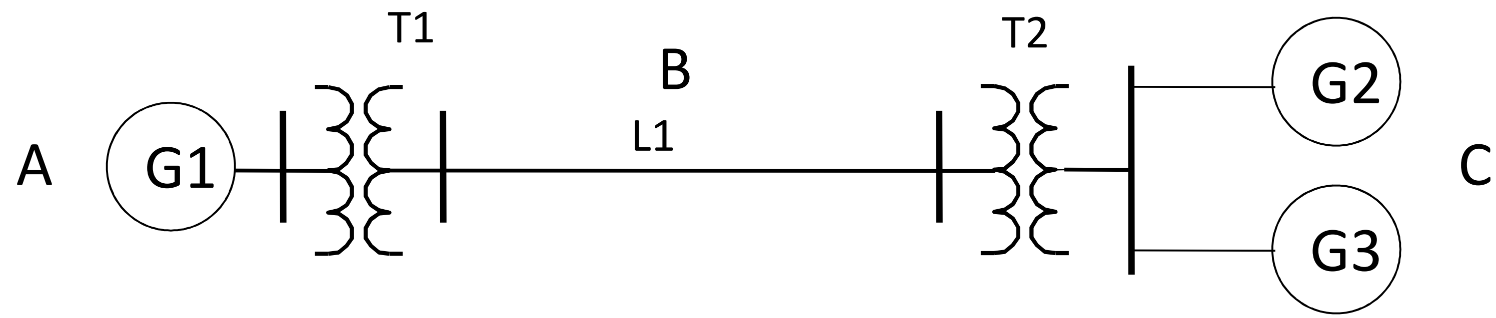
Transformer 2 (T2): 3-1ϕ; 100 MVA; 127 / 13.2 kV; X_{T2} = 10 %

Generator 2 (G2): 200 MVA; 13.8kV, X_d" = 20 %

Generator 3 (G3): 100 MVA; 13.8kV, X_d" = 20 %

Per-Unit Representation

a) Establish Base Power, Base Voltages, Base Impedance, and Base Current



Base Power:

Sub-System	V_{base} (kV)	Z_{base} (Ohm)	I_{base} (Amp)
A			
B			
C			

Per-Unit Representation

a) Establish Base Power, Base Voltages, Base Impedance, and Base Current

Base Power: 100 MVA

Sub-System	V_{base} (kV)	Z_{base} (Ohm)	I_{base} (Amp)
A	20	4	2886.8
B	230	529	251.0
C	13.8	1.9044	4183.7

Per-Unit Representation

b) Compute Per Unit Impedance

$$G1: 0.2 \times \left(\frac{20}{20}\right)^2 \frac{100}{300} = 0.0667 \text{ p.u.}$$

G3 : no change

$$G2: 0.2 \times \left(\frac{13.8}{13.8}\right)^2 \frac{100}{200} = 0.1 \text{p.u.}$$

$$Z_{\text{pu,new}} = Z_{\text{pu,old}} \times \left(\frac{V_{\text{base}}^{\text{old}}}{V_{\text{base}}^{\text{new}}}\right)^2 \frac{S_{\text{base}}^{\text{new}}}{S_{\text{base}}^{\text{old}}}$$

$$T1: 0.1 \times \left(\frac{20}{20}\right)^2 \frac{100}{350} = 0.0286 \text{ p.u.}$$

Refer to Next Slide for Detailed Explanation

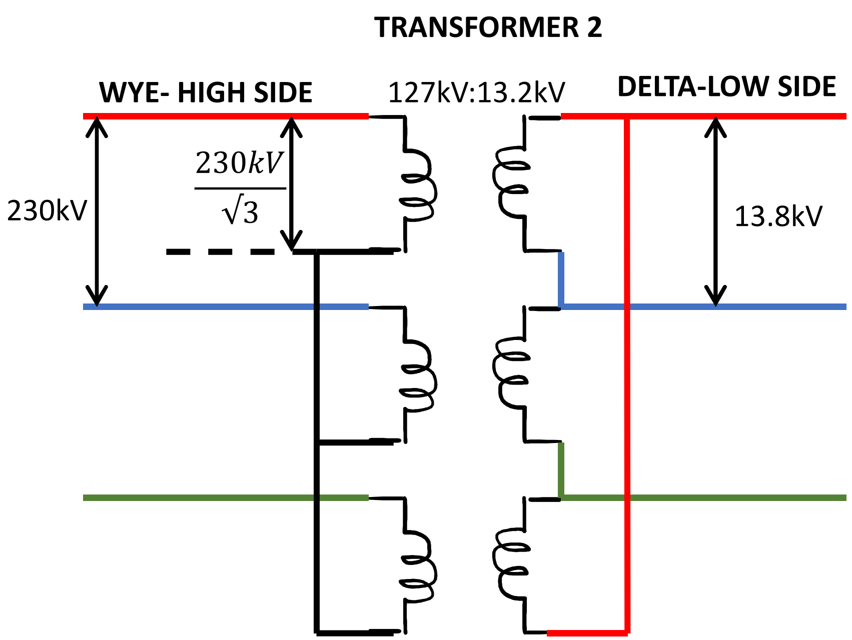
$$T2: 0.1 \times \left(\frac{127\sqrt{3}}{230}\right)^2 \frac{100}{300} = 0.0305 \text{ p.u.}$$

$$Line: 0.5 \frac{\Omega}{km} \times 64 km = 32\Omega \rightarrow Z_{pu} = \frac{32}{529} = 0.0605 \text{ p.u.}$$

Recall that transformer 2 has the following rating:
Transformer 2 (T2): 3-1ϕ; 100 MVA; 127 / 13.2 kV; X_{T2} = 10 %

3 Single Phase Transformers rated at…

- Based on our actual voltage levels, the voltage level (line to line) at section B is 230kV; while at section C, it is 13.8kV. This means there is a ratio of 230kV/13.8kV = **16.67** between the line to line voltages of the high and low sides.
- This means the transformer 2 setup *must be connected* in a way that the voltage ratio between the line to line voltage has the same ratio (=16.67).
- To achieve this, transformer 2 must have a *Wye-Delta Configuration between the High voltage side and low voltage side*. This is shown below. We will then get the rating of the three single phase transformers connected as a three phase transformer.



The Single Phase Transformer Ratio is 127/13.2 = *9.621*
If the system is connected as wye-delta as shown, the phase voltage of the transformer at the high voltage side would be equivalent to the line to neutral voltage of the system which is 230/√3

The line to line voltage at the low voltage side is :

$$V_{LL,low} = \frac{230kV}{\sqrt{3}} * \frac{13.2kV}{127kV} = 13.8kV$$

Continued next slide…

The effective voltage ratio for the three phase transformer composed of three single phase transformers configured in a wye – delta arrangement between the high voltage side and the low voltage side is then:

$$\mathbf{127\sqrt{3}kV : 13.2kV}$$

That is the line to line voltage rating at high voltage side is $127\sqrt{3}kV$ and the line to line voltage rating at the low voltage side is 13.2kV

The Rated capacity is three times that of the single phase transformer.

$$S_{rated} = 3 * 100MVA$$

Hence, when we solve for the per unit representation in the system in the example

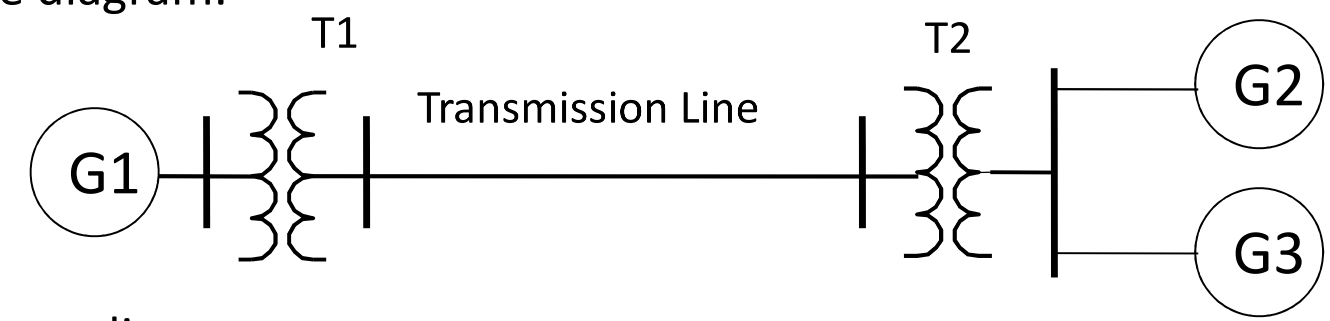
$$\text{T2}: 0.1 \times \left(\frac{\overset{V_{base,old}}{127\sqrt{3}}}{\underset{V_{base,new}}{230}} \right)^2 \frac{\overset{S_{base,new}}{100}}{\underset{S_{base,old}}{300}} = 0.0305 \text{ p.u.}$$

If we make use of the low voltage side ratings. The results will be consistent.

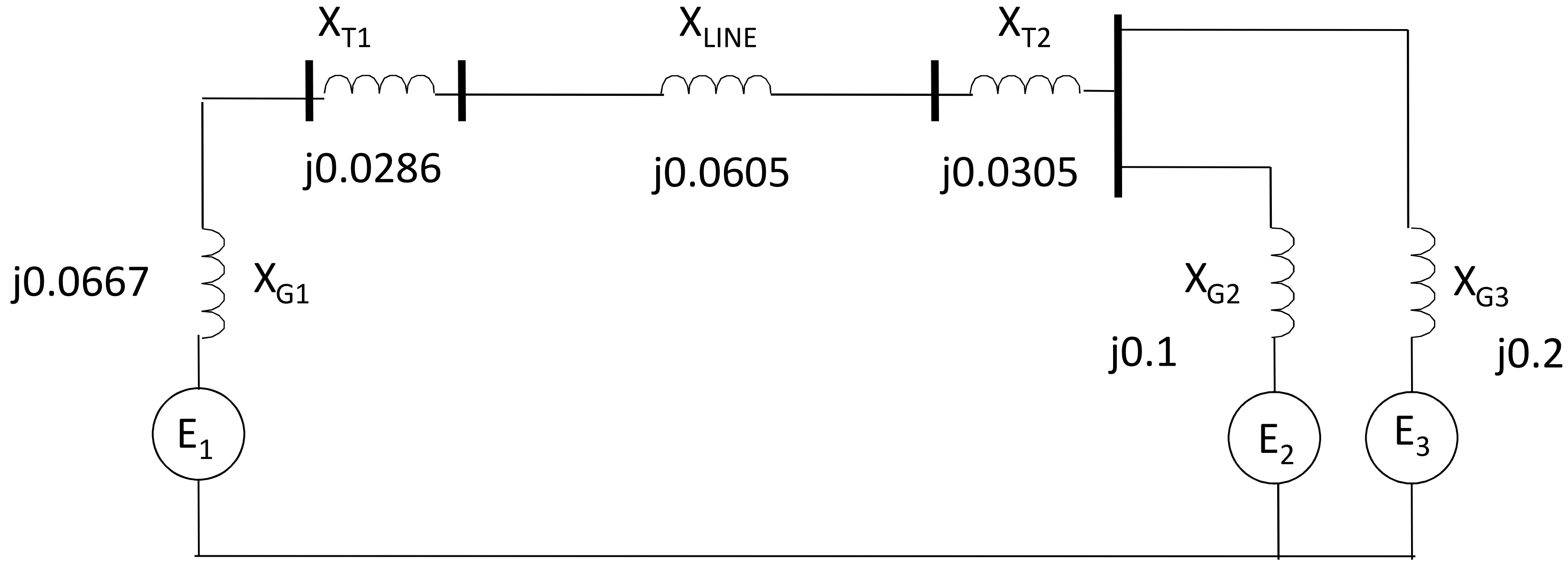
$$T2: 0.1 * \left(\frac{13.2kV}{13.8kV} \right)^2 * \frac{100}{300} = 0.0305\ pu$$

Per-Unit Representation

One-line diagram:



Impedance diagram:



Advantages of Per-Unit

1. Manufacturers usually specify impedances of equipment in percent or per-unit on the base of the nameplate rating.

1. The per-unit impedance of machines of the same type but widely different ratings lie within a narrow range. When the impedance is unknown, it is generally possible to select from tabulated average values.

1. When working in the per-unit system, we can select the base voltages such that the per-unit turns ratio of transformers in the system is 1:1.

Advantages of Per-Unit

1. Three-phase transformer connection does not affect the per-unit impedances of the equivalent circuit, although transformer connection does determine the relation between the voltage bases on the two sides of the transformer.

1. Per unit representation yields more meaningful and easily correlated data.

1. Network calculations are done in a much more handier fashion with less chance of mix-up:
    - between phase and line voltages
    - between single-phase and three-phase powers, and
    - between primary and secondary voltages.

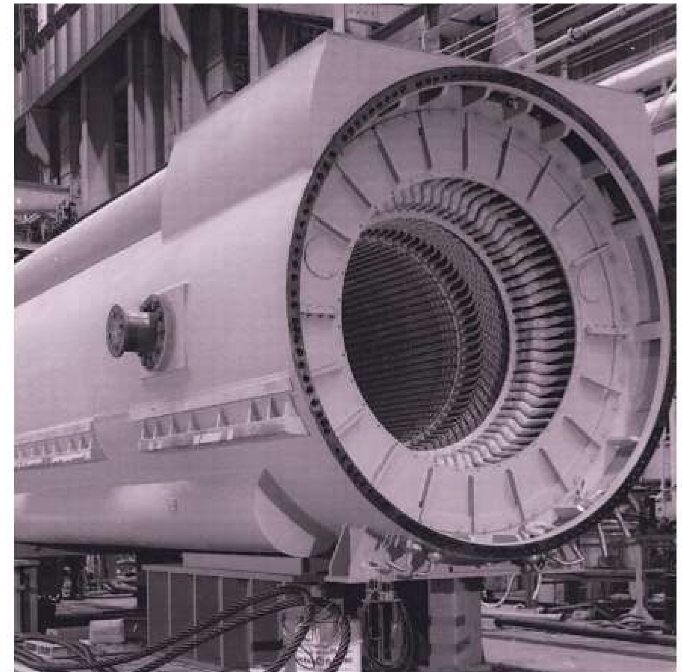
Summary

- Per Unit Concepts
- Change of Base Concept

READ

- Glover Chapter 8

Generator stator showing completed windings for a 757-MVA, 3600-RPM, 60-Hz synchronous generator (Courtesy of General Electric)



8

SYMMETRICAL COMPONENTS

The method of symmetrical components, first developed by C. L. Fortescue in 1918, is a powerful technique for analyzing unbalanced three-phase systems. Fortescue defined a linear transformation from phase components to a new set of components called *symmetrical components*. The advantage of this transformation is that for balanced three-phase networks the equivalent circuits obtained for the symmetrical components, called *sequence networks*, are separated into three uncoupled networks. Furthermore, for unbalanced three-phase systems, the three sequence networks are connected only at points of unbalance. As a result, sequence networks for many cases of unbalanced three-phase systems are relatively easy to analyze.

The symmetrical component method is basically a modeling technique that permits systematic analysis and design of three-phase systems. Decoupling a detailed three-phase network into three simpler sequence networks reveals complicated phenomena in more simplistic terms. Sequence network