Lecture
Three Phase Power Equations

Agenda

- **Lecture**

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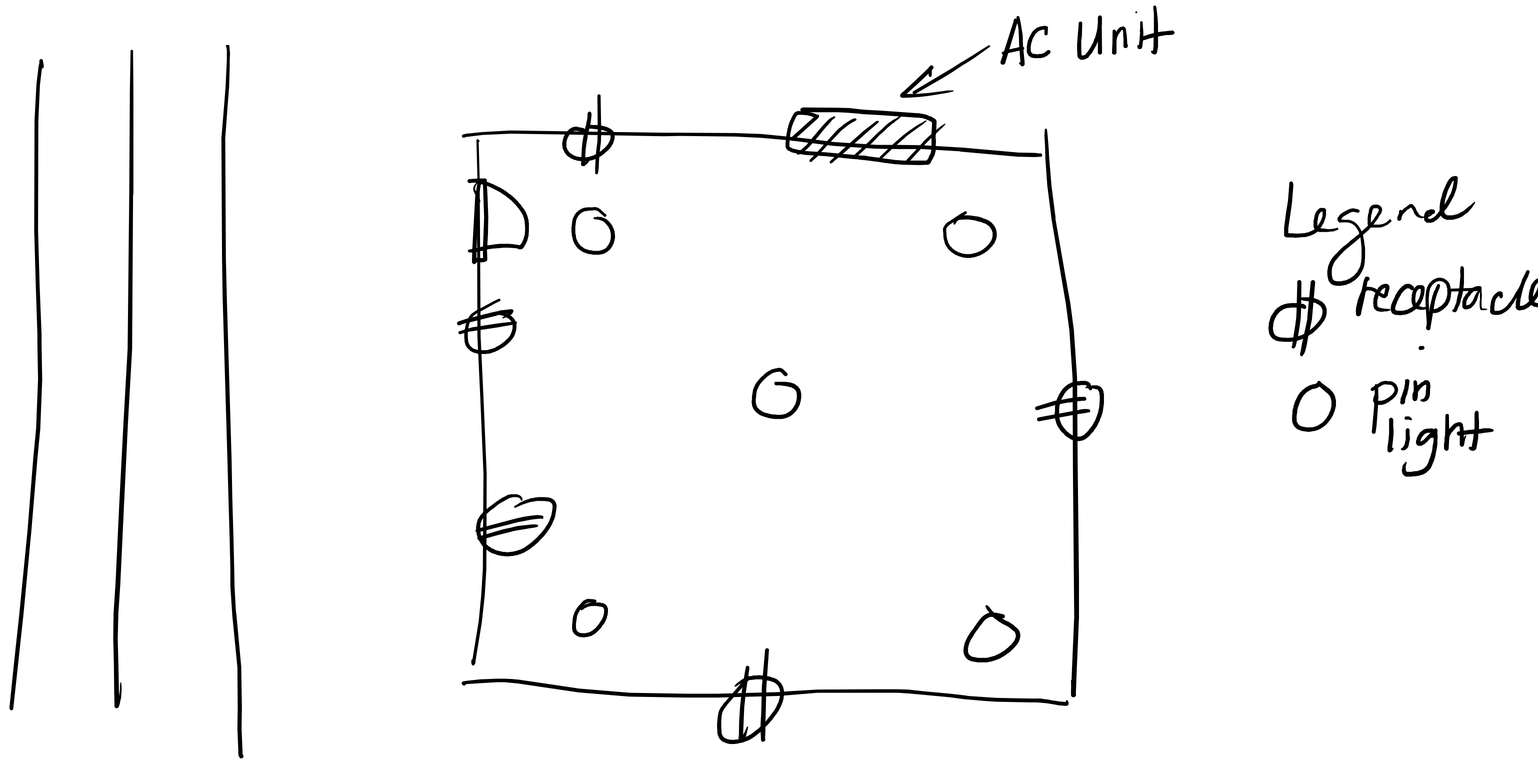
How is power in three phase systems distributed?



Lecture Outcomes

at the end of the lecture, the student must be able to …

- Formulate and Solve Complex Power Equations in Three-Phase AC Systems

Single-Phase Power Equations

For a single-phase system:

$$P_p = V_p I_p \mathrm{Cos}\theta$$

$$Q_p = V_p I_p \mathrm{Sin}\theta$$

$$S_p = V_p I_P^* = P_P + jQ_P$$

where:

V_p = magnitude of voltage per phase

I_p = magnitude of current per phase

θ = phase angle displacement between V_p and I_p

Three-Phase Power Equations

For a three-phase system:

$$P_{3\phi} = 3V_pI_p\cos\theta$$

$$Q_{3\phi} = 3V_pI_p\sin\theta$$

$$S_{3\phi} = 3V_pI_p^* = P_{3\phi} + jQ_{3\phi}$$

Note: V_p and I_p are per-phase (per leg) quantities; the equations here hold whether the system is wye- or delta-connected.

Three-Phase Power Equations

For a wye-connected system:

$$V_p = \frac{V_{LL}}{\sqrt{3}}$$

$$I_p = I_L$$

For a delta-connected system:

$$V_p = V_{LL}$$

$$I_p = \frac{I_L}{\sqrt{3}}$$

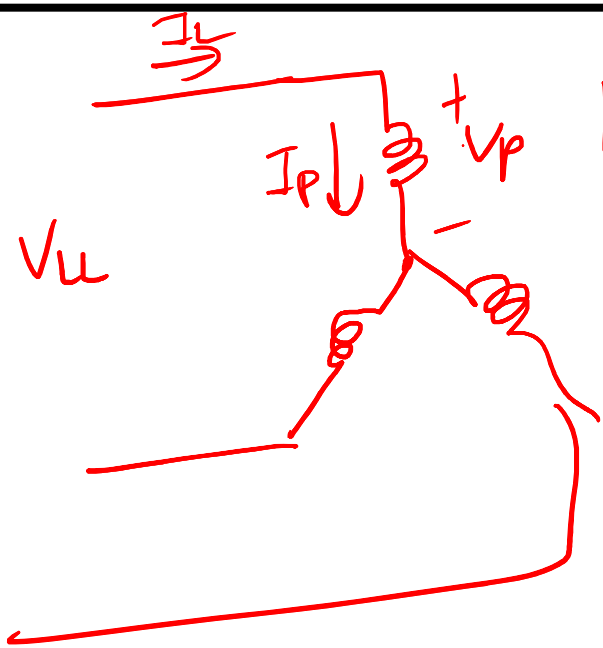
Whether wye- or delta-connected:

$$P_{3\phi} = \sqrt{3} V_L I_L \cos\theta$$

$$Q_{3\phi} = \sqrt{3} V_L I_L \sin\theta$$

$$S_{3\phi} == P_{3\phi} + jQ_{3\phi}$$

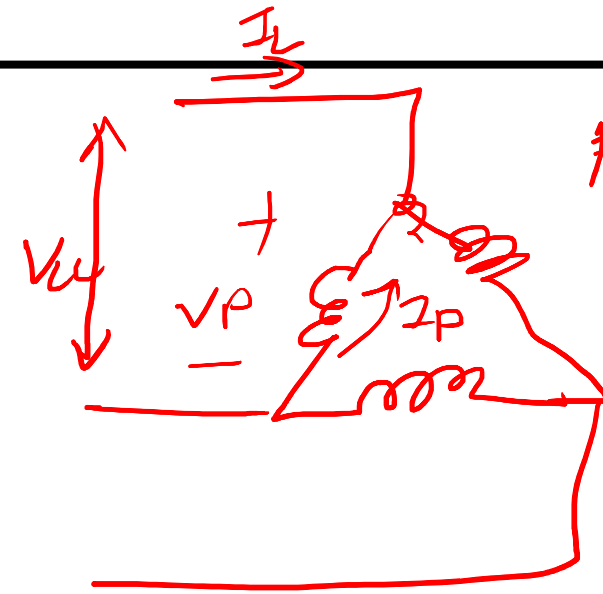
Note: θ is the angle between V_p and I_p, NOT between V_L and I_L.



For a wye

$$V_p = V_{LN} = \frac{V_{LL}}{\sqrt{3}}$$

$$I_L = I_p$$



For a Delta

$$V_{LL} = V_p$$

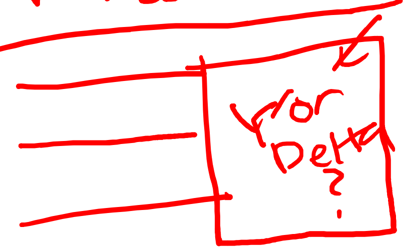
$$I_p = \frac{I_L}{\sqrt{3}}$$

$$P_{3\phi} = 3V_pI_p\cos\theta$$

$$\theta = \theta_{V_p} - \theta_{I_p}$$

$$\therefore P_{3\phi} = 3\frac{V_{LL}}{\sqrt{3}}I_L\cos\theta = \boxed{\sqrt{3}\,V_{LL}I_L\cos\theta}$$

$$P_{3\phi} = 3V_pI_p\cos\theta = 3V_{LL}\frac{I_L}{\sqrt{3}}\cos\theta = \boxed{\sqrt{3}\,V_{LL}I_L\cos\theta}$$



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Conventions for this Class

Unless otherwise stated, for our class:

- "Power" = Real Power.

  Take note of the context:

  ☞Total three-phase real power, if for three-phase systems.

  ☞Single-phase real power, if for single-phase systems.

- "Current" = Line current (rms value only).
- "Voltage" = Line-to-line voltage (rms value only).

  Note: These definitions will prove useful once we get to per unit systems.

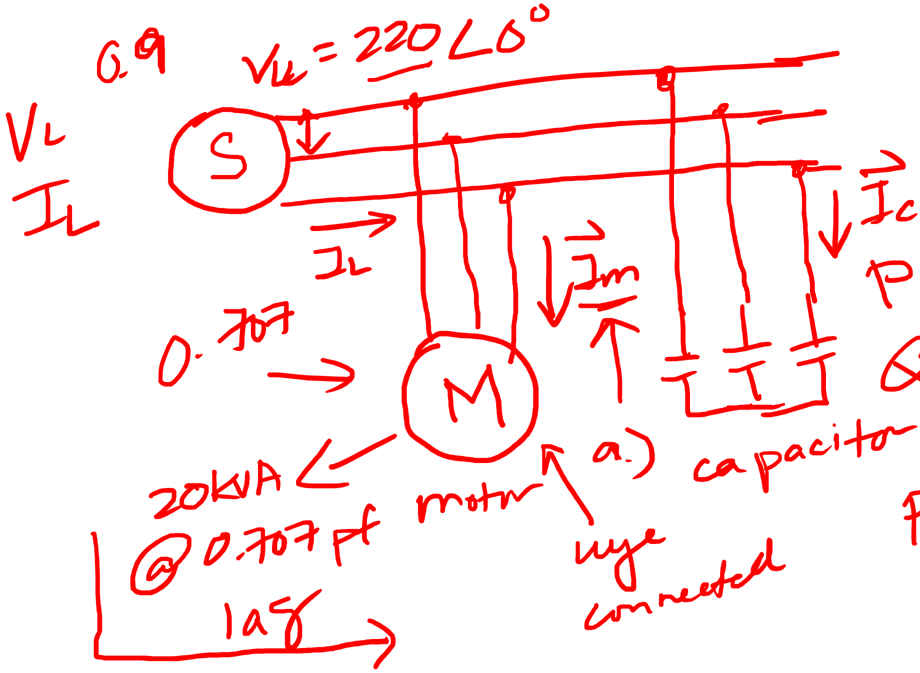
EXAMPLE

A three-phase motor draws 20-kVA at 0.707 pf lag from a 220-V source. A wye connected capacitor bank is connected in parallel with the motor to improve the power factor. Determine the following:

(a) Motor Line current (3 pts)

(b) The kVA rating of capacitors to make the combined load have pf = 0.90 lag.(4 pts)

(c) Line current of combined load (3 pts)



$P = 0$

$Q_{cap} = -Q$

$P_m = \sqrt{3} V_{LL} I_M \cos\theta_{pf}$

$$I_M = \frac{P_m}{\sqrt{3} V_{LL} \cos\theta_{pf}}$$

$$= \frac{14.14\text{ kW}}{\sqrt{3}(220)(0.707)}$$

$$= 52.49\text{ A}$$

method 2

$S = 3VI^*$

$$I = \left(\frac{S}{3V}\right)^*$$

a.) Motor Line current

method 1

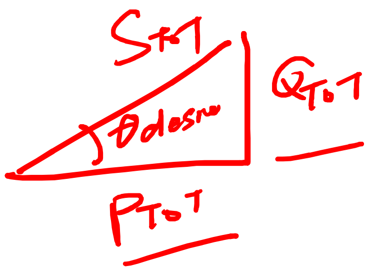
$P_m = 20\text{kVA}(0.707) = 14.14\text{ kW}$

3ϕ $Q_m = 20\text{kVA} \sin\cos^{-1} 0.707$

$= 14.14\text{ kvar}$

Motor	Capacitor	Total
$P_M = 14.14\,kW$	$P_C = 0$	$P_{Tot} = 14.14\,kW$
$Q_M = 14.14\,kVar$	$Q_C = -x$	$Q_{tot} = 14.14\,kvars - x$

$$\cos\theta_{desired} = 0.9\,pf$$



$$\tan\theta_{desired} = \frac{Q_{tot}}{P_{tot}}$$

$$\tan\cos^{-1}0.9 = \frac{14.14k - x}{14.14\,kW}$$

$$x = ? = \underline{7.43\,kvars}$$

$$\text{Rating of capacitors} = \underline{\underline{7.43\,k\,vars}}$$

$$\vec{I}_L = \vec{I}_M + \vec{I}_C = 52.49\angle -45^\circ + 19.486\angle 90^\circ$$

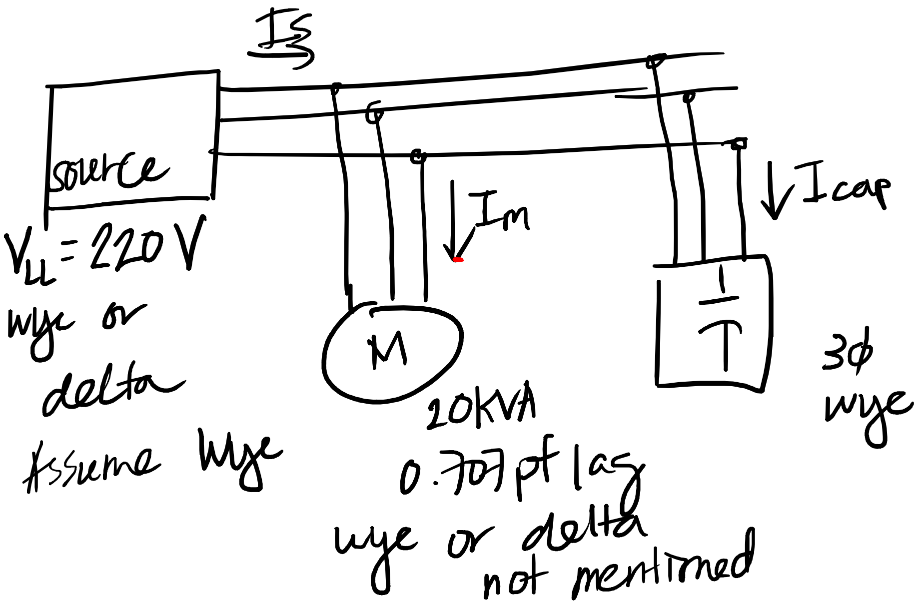
$$\vec{I}_m = 52.49\angle -45^\circ = \_\_\_\_\_\_ \checkmark$$

$$\vec{I}_c = ?$$

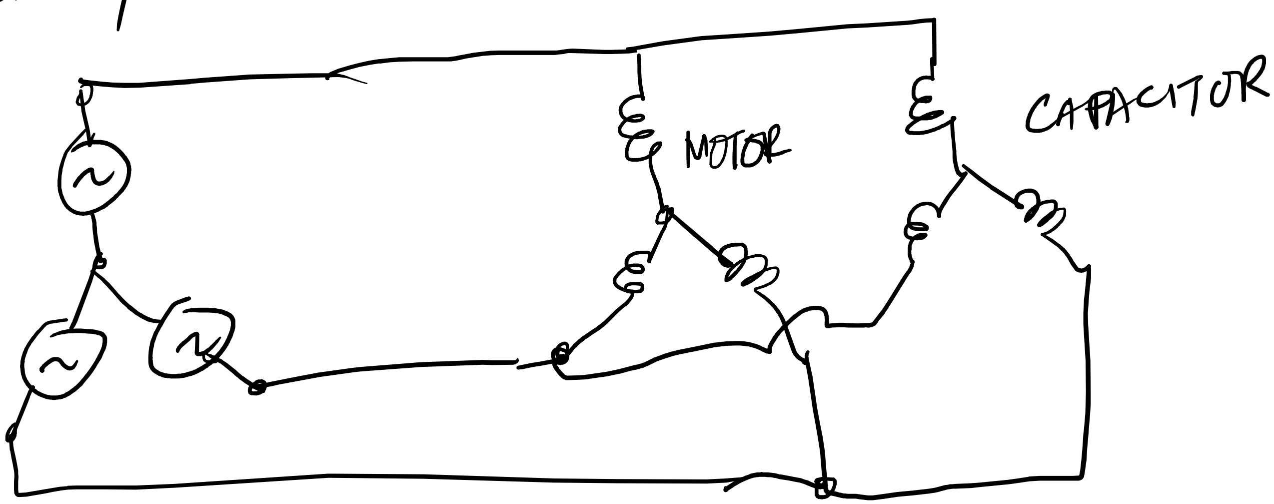
$$Q = \sqrt{3}\, V_{LL} I_L = -7.43\ \text{kvars}$$

$$|I_c| = \frac{-7.43\ \text{kvars}}{-\sqrt{3}(220)} = 19.486$$

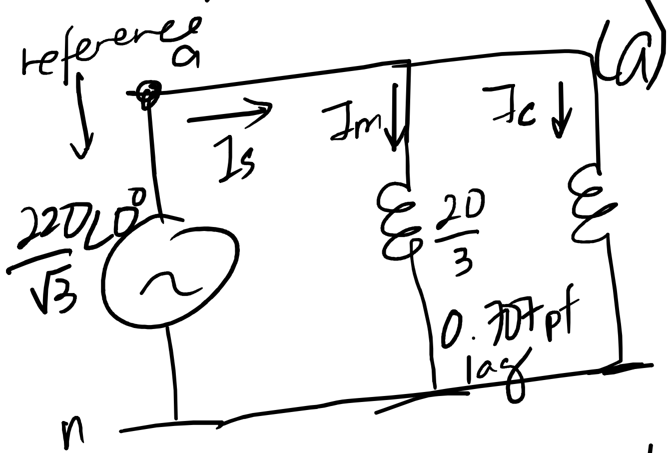
$$\vec{I}_c = 19.486\angle 90^\circ$$



Assumed 3φ circuit



1φ Equivalent circuit

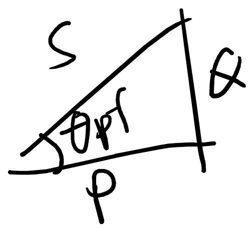


Recall $S = VI^*$

(a) $I_m = \left(\frac{S_m}{V}\right)^*$ $\quad S_m = \frac{20}{3}\angle\cos^{-1}0.707$ kVA

$$I_m = \frac{\frac{20}{3}\angle -\cos^{-1}0.707 \text{ kVA}}{\frac{220}{\sqrt{3}}\angle 0^\circ \text{ V}} \qquad |I_m| = 52.49\text{A}$$

(b) $pf = \cos\theta_{pf} = \cos\left(\tan^{-1}\frac{Q_s}{P_s}\right) = 0.9$



$Q_s = Q_m + Q_c = Q_m + Q_c$

$P_s = P_m + \cancel{P_c} = P_s$

$\cos\left(\tan^{-1}\left(\frac{Q_m+Q_c}{P_c}\right)\right) = 0.9 \longrightarrow \tan\cos^{-1}0.9 = \frac{Q_m+Q_c}{P_s}$

$Q_c = P_c \tan\cos^{-1}0.9 + Q_m$

$Q_c = \frac{20}{3}(0.707)\tan\cos^{-1}0.9 + \frac{20}{3}\tan\cos^{-1}0.707$

HOMEWORK 2

A three-phase line, which has an impedance of $(2 + j4)\ \Omega$ per phase, feeds two balanced three-phase loads that are connected in parallel. One of the loads is Y-connected with an impedance of $(30 + j40)\ \Omega$ per phase, and the other is Δ-connected with an impedance of $(60 - j45)\ \Omega$ per phase. The line is energized at the sending end from a 60-Hz, three-phase, balanced voltage source of $120\sqrt{3}$ V (rms, line-to-line). Determine (a) the current, real power, and reactive power delivered by the sending-end source; (b) the line-to-line voltage at the load; (c) the current per phase in each load; and (d) the total three-phase real and reactive powers absorbed by each load and by the line. Check that the total three-phase complex power delivered by the source equals the total three-phase power absorbed by the line and loads.

SUMMARY

- Formulating Three Phase Power Equations