# EEE 51 Exam 1

Electrical and Electronics Engineering Institute
University of the Philippines Diliman
2nd Semester SY 2017 - 2018
Saturday, March 24, 2018 1PM - 4PM
L. Alarcón, M. T. de Leon, R. J. Maestro, C. Santos

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Instructions: THINK before you answer. Your score will be based solely on what you have written. Write legibly and avoid erasures. Anything we cannot understand will be marked wrong. You may use the back of the exam sheets as extra scratch space, but your final answer, with the correct units, should be placed in the designated areas. DO NOT separate the exam sheets. All parts (I, II, III, and IV) carry the same weight.

**Name:** Part 1 solutions

**Student No.:**

**Section:**

*Encircle one:*

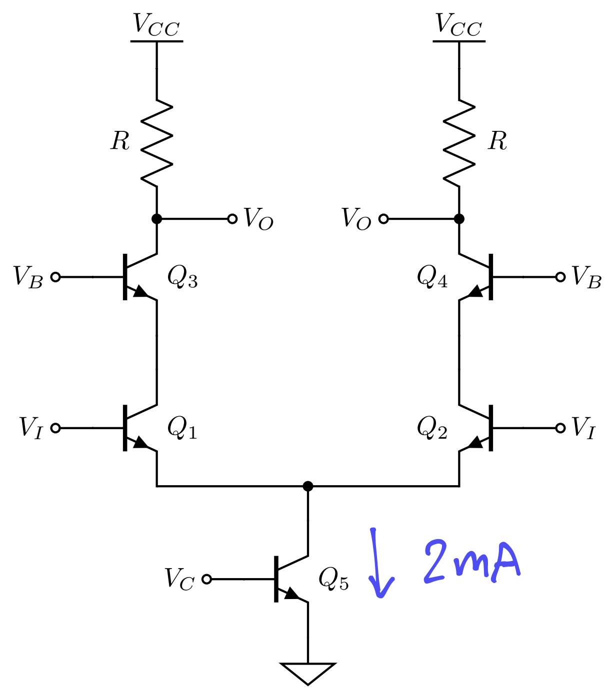
| | |
|---|---|
| Alarcón: | THQ (7:00am-8:30am) |
| de Leon: | THR (8:30am-10:00am) |
| Santos: | THU (10:00am-11:30am) |
| de Leon: | THX (2:30pm-4:00pm) |
| Maestro: | WFX (2:30pm-4:00pm) |

**Scores:**

| | |
|---|---|
| Part I: | |
| Part II: | |
| Part III: | |
| Part IV: | |
| Total: | |

## Part I:

*(12 points)* You are tasked to setup the quiescent operating point of all the transistors in the cascoded differential amplifier below. The transistors are identical, with $I_S = 5\,\text{fA}$, $\beta \to \infty$, $|V_A| \to \infty$, and $V_{CE,\text{sat}} = 0.2\,\text{V}$. The supply voltage, $V_{CC}$, is $5\,\text{V}$, and you need to make sure that the collector current of transistor $Q_5$ is $2\,\text{mA}$ at a temperature of $300\,\text{K}$. From your trusty notes, you see that the charge of an electron is $1.602 \times 10^{-19}\,\text{C}$, and Boltzmann's constant is $1.381 \times 10^{-23}\,\text{kg}\frac{\text{m}^2}{\text{s}^2}\frac{1}{\text{K}}$.




1. Determine the DC bias voltage $V_C$. *(2 points)*

$$V_C = V_{CEs} = \frac{KT}{q} \ln\left(\frac{2\text{mA}}{I_s}\right) = (25.86\,\text{mV}) \ln\left(\frac{2\text{mA}}{5\,\text{fA}}\right)$$

$$V_C = 0.691\text{V}$$

$V_C =$ 0.691V

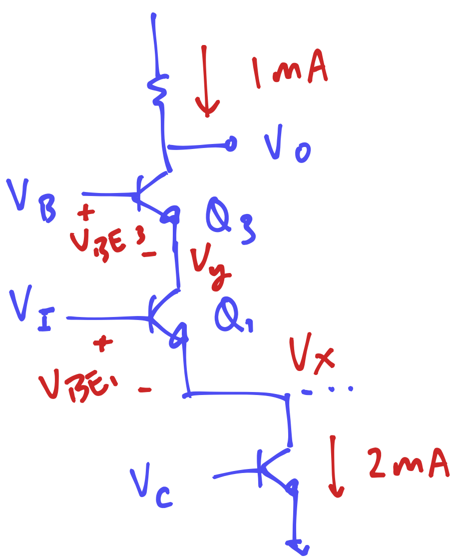
2. Your required output common-mode voltage is half the supply voltage. What is the value of the resistor, $R$? *(2 points)*

$$V_O = \frac{V_{CC}}{2} = 2.5\text{V} \qquad R = \frac{V_R}{1\text{mA}} = \frac{V_{CC} - V_O}{1\text{mA}} = \frac{2.5\,\text{V}}{1\text{mA}}$$

$$R = 2.5\text{K}\Omega$$

$R =$ 2.5 k$\Omega$

3. You want to get the largest possible symmetric differential output swing from the cascoded differential amplifier, given that the output common-mode voltage is half the supply voltage. What values would you choose for the DC bias voltage at the input, $V_I$, and the cascode bias, $V_B$? *(5 points)*




$V_{BE_1} = V_T \ln\left(\frac{1mA}{5fA}\right) = 0.673\ V$

$V_{x\,min} = V_{CE,sat} = 0.2\ V$

$V_I = V_{BE_1} + V_x$

$= 0.673V + 0.2V$

$= 0.873V$

$V_{BE_3} = V_T \ln\left(\frac{1mA}{5fA}\right) = 0.673V$

$V_{y\,min} = V_{CE,sat} + V_{x\,min}$

$= 0.2V + 0.2V = 0.4V$

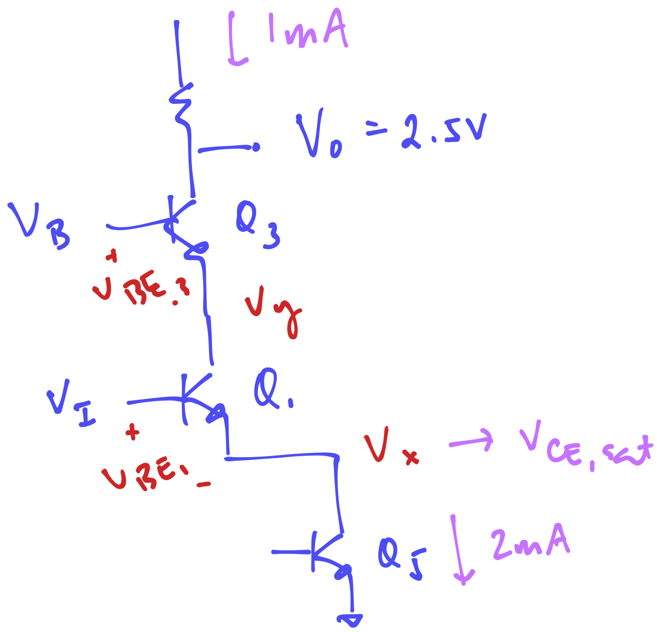
$V_B = V_{BE_3} + V_{y,min}$

$= 1.073\ V$

$V_I =$ 0.873 V

$V_B =$ 1.073 V

4. Instead of trying to maximize output swing, you want to maximize input common-mode range instead. What DC voltage would you use for $V_B$? Note that you are still required to maintain the output common-mode voltage at half the supply voltage. *(3 points)*




As $V_I$ increases,

$V_y = V_{CE_1} + V_x$

$= V_{CE_1} + V_I - V_{BE_1}$

$V_{y\,max} = V_{CE_1,min} + V_{I\,max} - V_{BE_1}$

To accommodate the max. possible $V_I$, $V_y$ should be as high as possible

$\therefore V_{y.max} = V_o - V_{CE_3,sat}$

$= 2.5V - 0.2V = 2.3V$

$\therefore V_B = V_{y\,max} + V_{BE_3}$

$= 2.3V + 0.673V$

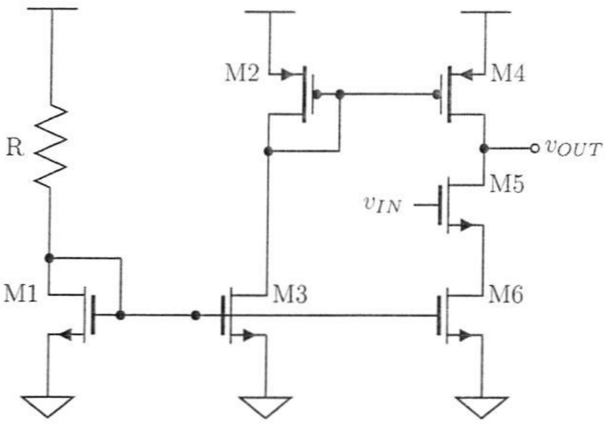
$= 2.973V$

$V_B =$ 2.973 V

| Name: | Student #: | Section: |
|---|---|---|

## Part II:

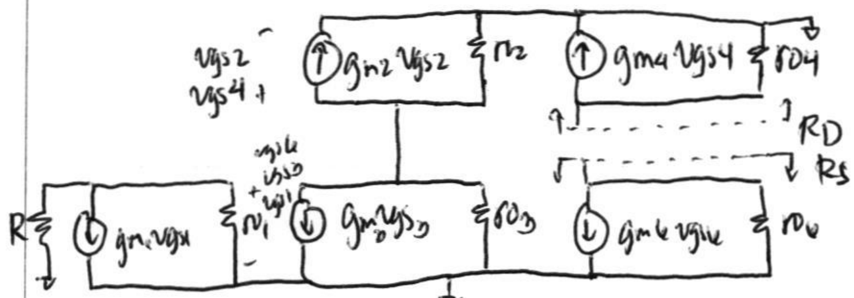
Consider the amplifier below and the table of DC parameters. Assume that the voltage supply is $3.3V$, $R = 1.8k\Omega$ and that the circuit is operating at room temperature. *(20 points)*

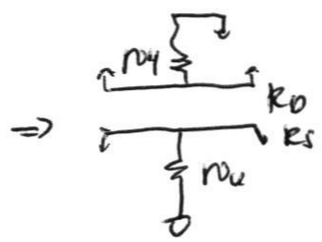



| | $M1$ | $M2$ | $M3$ | $M4$ | $M5$ | $M6$ |
|---|---|---|---|---|---|---|
| $\lambda$ | $0.025V^{-1}$ | $0.025V^{-1}$ | $0.01V^{-1}$ | $0.01V^{-1}$ | $0.2V^{-1}$ | $0.05V^{-1}$ |
| $\lvert I_D \rvert$ | $1mA$ | $1mA$ | $1mA$ | $3mA$ | $3mA$ | $3mA$ |
| $\lvert V_{GS} \rvert$ | $1.5V$ | $1.5V$ | $2.3V$ | $2.3V$ | $1.5V$ | $1.5V$ |
| $\lvert V_{DS} \rvert$ | $1.5V$ | $1V$ | $2.3V$ | $1.65V$ | $0.85V$ | $0.8V$ |
| $\lvert V_{th} \rvert$ | $0.8V$ | $0.8V$ | $1.55V$ | $1.55V$ | $0.8V$ | $0.8V$ |

Table 1: Transistor Parameters

1. Determine the expression and actual values of the small signal resistances seen by drain terminal and by the source terminal of transistor $M_5$. *(5 points)*



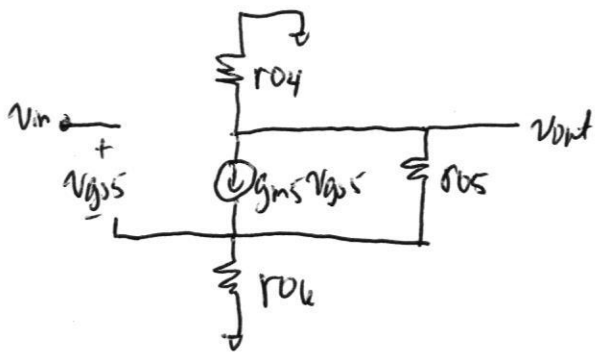




$r_{o4} = \frac{1}{\lambda_4 I_{D4}}$ $\quad r_{o6} = \frac{1}{\lambda_6 I_{D6}}$

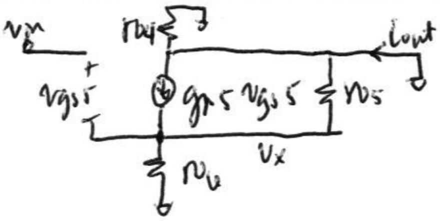
No small-signal $v_{gsx}$, therefore all current sources will be 0.

| | |
|---|---|
| $R_D(exp) =$ $r_{o4}$ | $R_S(exp) =$ $r_{o6}$ |
| $R_D(value) =$ $33.33k\Omega$ | $R_S(value) =$ $6.66K\Omega$ |

2. Using your answers in #1, draw the **simplified** small signal circuit of the amplifier. Use small-signal parameters as labels for the components. No need to compute for the actual values. *(3 points)*




3. Derive an expression and compute for the actual value for the $G_M$ of the circuit. *(4 points)*




$$i_{out} = g_{m5} v_{gs5} - \frac{v_x}{r_{o5}}$$

$$v_x = i_{out} r_{o6} = v_{in} - v_{gs5}$$

$$v_{gs5} = v_{in} - i_{out} r_{o6}$$

$$i_{out} = g_{m5} v_{in} - g_{m5} i_{out} r_{o6} - i_{out} \frac{r_{o6}}{r_{o5}}$$

$$i_{out}\left(1 + g_{m5} r_{o6} + \frac{r_{o6}}{r_{o5}}\right) = g_{m5} v_{in}$$

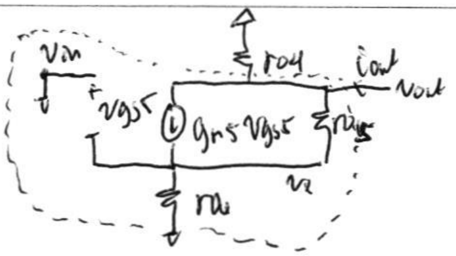
$$G_M = \frac{i_{out}}{v_{in}} = \frac{g_{m5}}{1 + g_{m5} r_{o6} + \frac{r_{o6}}{r_{o5}}}$$

$$g_{m5} = \frac{2 I_{DS5}}{V_{DSAT5}} = 8.57\ mS$$

$$r_{o5} = \frac{1}{\lambda_5 I_{DS5}} = 1.66\ k\Omega$$

| | |
|---|---|
| $G_M(expression) =$ | $\frac{g_{m5}}{1 + g_{m5} r_{o6} + \frac{r_{o6}}{r_{o5}}}$ |
| $G_M(value) =$ | $0.138\ mS$ |

4. Derive an expression and compute for the actual value for the $R_O$ of the circuit. *(4 points)*




$$R_{dotted} = \frac{v_{out}}{i_{dotted}}$$

$$i_{dotted} = g_{m5} v_{gs5} + \frac{v_{out} - v_x}{r_{o5}}$$

$$v_x = -v_{gs5} = i_{dotted} r_{o6}$$

$$i_{dotted} = -g_{m5} i_{dotted} r_{o6} + \frac{v_{out}}{r_{o5}} - \frac{i_{dotted} r_{o6}}{r_{o5}}$$

$$i_{dotted}\left(1 + g_{m5} r_{o6} + \frac{r_{o6}}{r_{o5}}\right) = \frac{v_{out}}{r_{o5}}$$

$$\frac{v_{out}}{i_{dotted}} = r_{o5}\left(1 + g_{m5} r_{o6} + \frac{r_{o6}}{r_{o5}}\right)$$

| | |
|---|---|
| $R_O(expression) =$ | $r_{o4} // r_{o5}\left(1 + g_{m5} r_{o6} + \frac{r_{o6}}{r_{o5}}\right)$ |
| $R_O(value) =$ | $25.19\ k\Omega$ |

$R_O = R_{dotted} // r_{o4}$

$R_{dotted} = 103.07\ k\Omega$

5. Derive an expression and compute for the actual value for the $A_V$ of the circuit. *(4 points)*

$$A_V = -G_m R_O$$

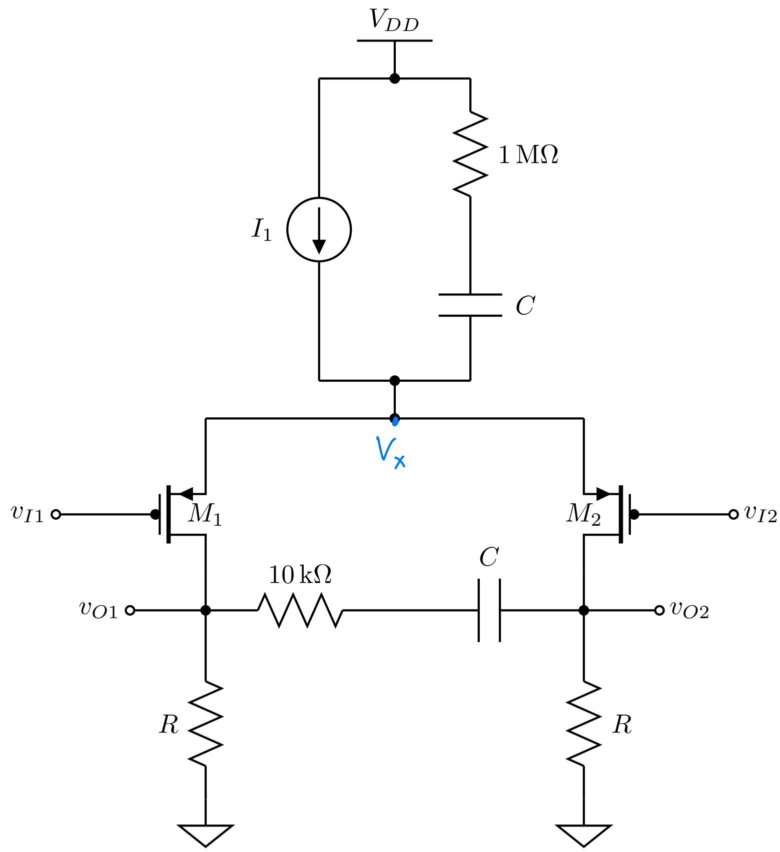
$$= \frac{-g_{m5}}{\left(1 + g_{m5} r_{o6} + \frac{r_{o6}}{r_{o5}}\right)} \cdot r_{o4} // r_{o5}\left(1 + g_{m5} r_{o6} + \frac{r_{o6}}{r_{o5}}\right)$$

$$= \frac{-g_{m5} r_{o4} r_{o5}}{r_{o4} + r_{o5}\left(1 + g_{m5} r_{o6} + \frac{r_{o6}}{r_{o5}}\right)}$$

| | |
|---|---|
| $A_V(expression) =$ | $-\frac{g_{m5} r_{o4} r_{o5}}{r_{o4} + r_{o5}\left(1 + g_{m5} r_{o6} + \frac{r_{o6}}{r_{o5}}\right)}$ |
| $A_V(value) =$ | $-3.48$ |

## Part III:

In the PMOS differential amplifier circuit below, $V_{DD} = 3\,V$, $I_1 = 2\,\text{mA}$, and $C \to \infty$. Note also that $M_1$ and $M_2$ are matched, with $V_{TH,P} = -1\,\text{V}$, $k = 4\,\frac{\text{mA}}{\text{V}^2}$, and $\lambda \to 0$. *(20 points total)*




1. If $V_{I1} = V_{I2} = 1\,V$, what value of $R$ will result in maximum output swing? *(8 points)*

at DC, $C \to \infty$ act as open circuits

since $I_1 = 2\,mA$, $I_{D1} = I_{D2} = 1\,mA$

$V_{SG1} = V_{SG2} = |V_{TH,P}| + \sqrt{\frac{I_D}{K}} = 1.5\,V$

$V_X = V_{I1} + V_{SG1} = 2.5\,V$

$V_{SD,SAT} = V_{SG1} - |V_{TH,P}| = 0.5\,V$ $\longrightarrow$ $V_{O1,max} = V_X - V_{SD,SAT} = 2\,V$

$V_{O1,min} = \emptyset\,V$

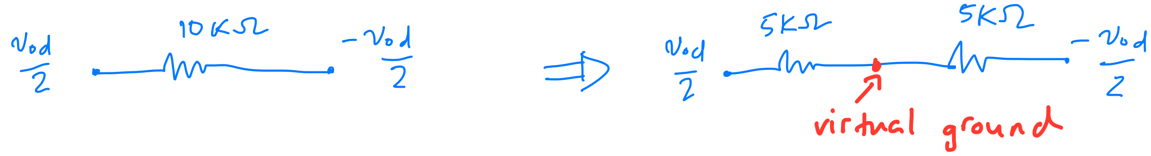
for max output swing, $V_{O1,DC} = 1\,V$

$R = \frac{V_{O1,DC}}{I_{D1}} = 1\,k\Omega$

$R =$ $1\,k\Omega$

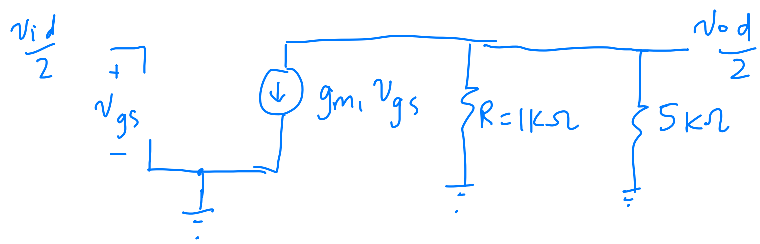
2. If $V_{I1} = V_{I2} = 1\,V$ and $R = 1\,\mathrm{k}\Omega$, draw the **differential-mode half circuit** below and determine the differential-mode gain. *(7 points)*

at AC, $C \to \infty$ will act as short circuits




$V_x$ will also be a virtual ground because of symmetry.

Hence, DM half-circuit would be:




$$g_{m1} = \sqrt{4K\,I_D} = 4\,mS$$

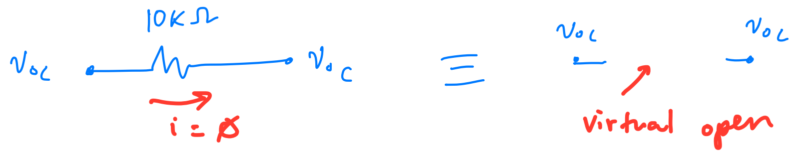
$$A_{DM} = \frac{v_{od}}{v_{id}} = -g_{m1}\,(5K \,//\, 1K)$$

$$= -3.33$$

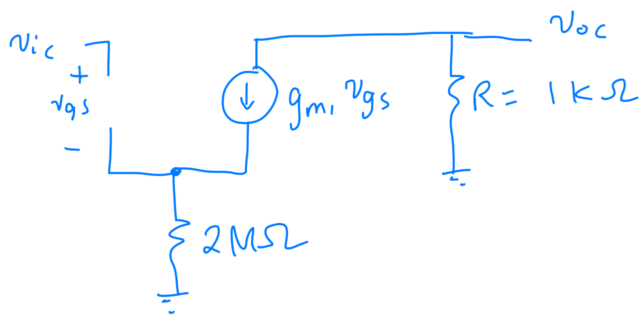
$A_{DM} = -3.33$

3. If $V_{I1} = V_{I2} = 1\,V$ and $R = 1\,\mathrm{k}\Omega$, draw the **common-mode half circuit** below and determine the common-mode gain. *(7 points)*

at common-mode,




Hence, CM half circuit would be:



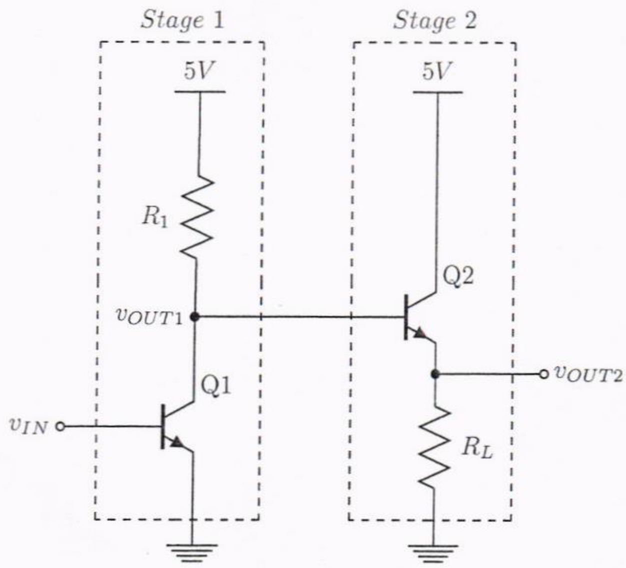

$$A_{CM} = \frac{v_{oc}}{v_{ic}} = \frac{-g_{m1}(1\,\mathrm{k}\Omega)}{1 + g_{m1}(2\,\mathrm{M}\Omega)} = -0.0005$$

$A_{CM} = -0.0005$

| Name: | Student #: | Section: |
|---|---|---|

## Part IV:

The compound BJT amplifier shown below is to be constructed with $R_L = 100\Omega$ and $V_{CC} = 5V$ using transistors with $\beta = 100$ and $V_A \to \infty$. The amplifier is operated at room temperature and the input DC bias voltage is set such that i) the output DC voltage is at half of the supply, ii) the collector current of the $Q_1$ is $100\times$ larger than the base current of $Q_2$, and iii) the base-emitter voltage of $Q_2$ is $0.7V$. *(20 points total)*




1. What is the DC collector current of $Q_2$? *(2 points)*

$$I_{E2} = I_{RL} = \frac{V_{CC}/2}{R_L}$$

$$I_{C2} = \frac{\beta}{\beta+1} I_{E2} = \frac{100}{101} \cdot \frac{2.5V}{100\Omega}$$

$I_{C2} =$ 24.752 mA

2. What should be the value of $R_1$ such that the conditions mentioned are met? *(2 points)*

$$I_{R1} = I_{C1} + I_{B2}$$
$$= 100 I_{B2} + I_{B2}$$
$$= 101 I_{B2}$$

$$V_{R1} = V_{CC} - V_{BE2} - V_{OUT2}$$
$$= V_{CC}/2 - V_{BE2}$$

$$R_1 = \frac{V_{R1}}{I_{R1}} = \frac{V_{CC}/2 - V_{BE2}}{101\,(I_{C2}/\beta)}$$
$$= 72.00138\ \Omega$$

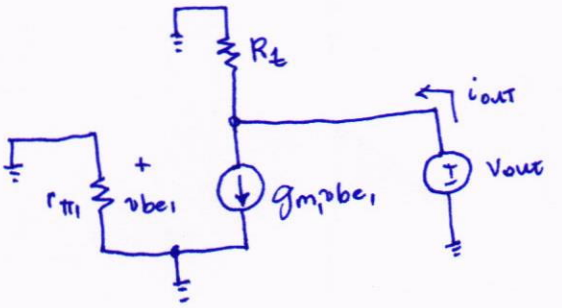
$R_1 =$ $72\ \Omega$

3. What is the DC voltage at the collector of $Q_1$? *(1 point)*

$$V_{C1} = V_{OUT1} = V_{BE2} + V_{OUT2}$$
$$= 0.7V + 2.5V$$

$V_{C1} =$ $\boxed{3.2V}$

4. The compound amplifier is to be analyzed in the small-signal domain as a cascade of two thevenin equivalent two-port networks with the division of stages as identified in the schematic. Provide the **expression** for the small-signal ouput resistance of the first stage to complete the table of two-port parameters. Draw the small signal circuit to be analyzed in the derivation. *(3 points)*




$$R_{OUT} = \left.\frac{v_{OUT}}{i_{OUT}}\right|_{v_i \to 0} \qquad r_o \to \infty$$

$$v_i = v_{be1} = 0$$

$$\text{KCL: } i_{out} = \frac{v_{out}}{R_1} + \cancel{g_{m1} v_{be1}}^{\,0}$$

$$\frac{v_{out}}{i_{out}} = R_1$$

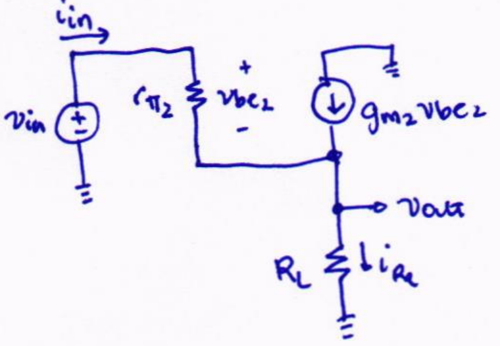
| Parameter | Expression for Stage 1 |
|---|---|
| $R_{IN}$ | $r_{\pi 1}$ |
| $A_V$ | $-g_{m1}R_1$ |
| $R_{OUT}$ | $R_1$ |

5. Provide the **expressions** for the small signal voltage gain and input resistance of the second stage to complete table of two port parameters. Draw the small signal circuits to be analyzed in the derivation. Use the alternate method for deriving the input impedance for bilateral circuits. *(10 points)*

$R_{in} = \left.\dfrac{v_{in}}{i_{in}}\right|$ output is open

$A_V = \left.\dfrac{v_{out}}{v_{in}}\right|$ no load — output is also open

$* \; r_o \to \infty$




$v_{be_2} = i_{in} r_{\pi_2}$

$i_{R_L} = i_{in} + g_{m_2} v_{be_2}$

$= i_{in}(1 + g_{m_2} r_{\pi_2})$

$= i_{in}(1 + \beta)$

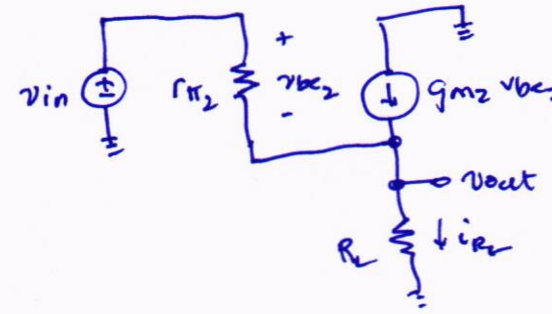
KVL: $v_{in} = v_{be_2} + v_{out}$

$= i_{in} r_{\pi_2} + i_{R_L} R_L$

$v_{in} = i_{in}(r_{\pi_2} + (1+\beta) R_L)$

$\dfrac{v_{in}}{i_{in}} = R_{IN} = r_{\pi_2} + (1+\beta) R_L$

$* \; r_o \to \infty$




$v_{out} = i_{R_L} \cdot R_L$

$= \left(\dfrac{v_{be2}}{r_{\pi 2}} + g_{m2} v_{be2}\right) \cdot R_L$

$= (v_{be2})\left(\dfrac{1}{r_{\pi 2}} + g_{m2}\right) \cdot R_L$

$= (v_{in} - v_{out})\left(\dfrac{1 + g_{m_2} r_{\pi 2}}{r_{\pi 2}}\right) R_L$

$v_{out} = (v_{in} - v_{out})\left(\dfrac{(1+\beta) R_L}{r_{\pi 2}}\right)$

$v_{out}\left(\dfrac{r_{\pi 2}}{(1+\beta) R_L} + 1\right) = v_{in}$

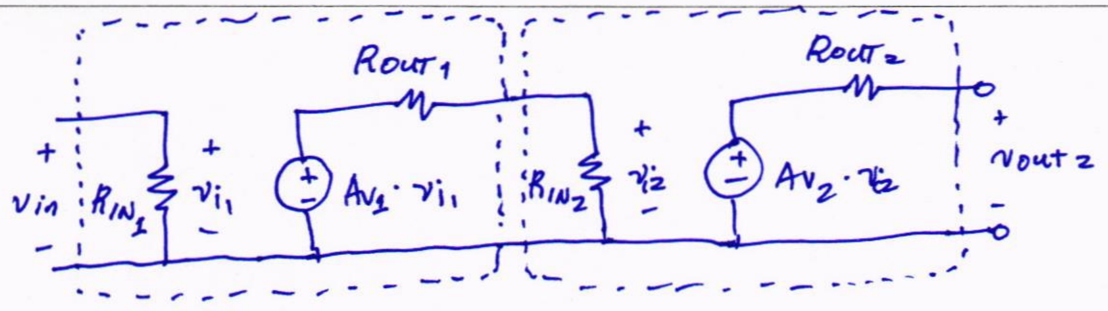
$v_{out}\left(\dfrac{r_{\pi 2} + (1+\beta) R_L}{(1+\beta) R_L}\right) = v_{in}$

$\dfrac{v_{out}}{v_{in}} = \dfrac{(1+\beta) R_L}{r_{\pi 2} + (1+\beta) R_L}$

$= \dfrac{g_{m_2} R_L}{\dfrac{\beta}{\beta+1} + g_{m2} R_L}$

| Parameter | Expression for Stage 2 |
|---|---|
| $R_{IN}$ | $r_{\pi 2} + (1+\beta) R_L$ |
| $A_V$ | $\dfrac{(1+\beta) R_L}{r_{\pi 2} + (1+\beta) R_L}$ |

6. Using the tables from the last 2 numbers, write the **expression** for the overall small signal voltage gain $A_{V,TOT} = v_{OUT2}/v_{IN}$ **in terms of the resistances and transistor small signal parameters**. The expression should be written in a way such that the propagation of the voltages from input to output should be the obvious. *(2 points)*




$$A_{V,TOT} = \left(-g_{m1} R_1\right)\left(\frac{r_{\pi 2} + (1+\beta)R_L}{R_1 + r_{\pi 2} + (1+\beta)R_L}\right)\left(\frac{(1+\beta)R_L}{r_{\pi 2} + (1+\beta)R_L}\right)$$