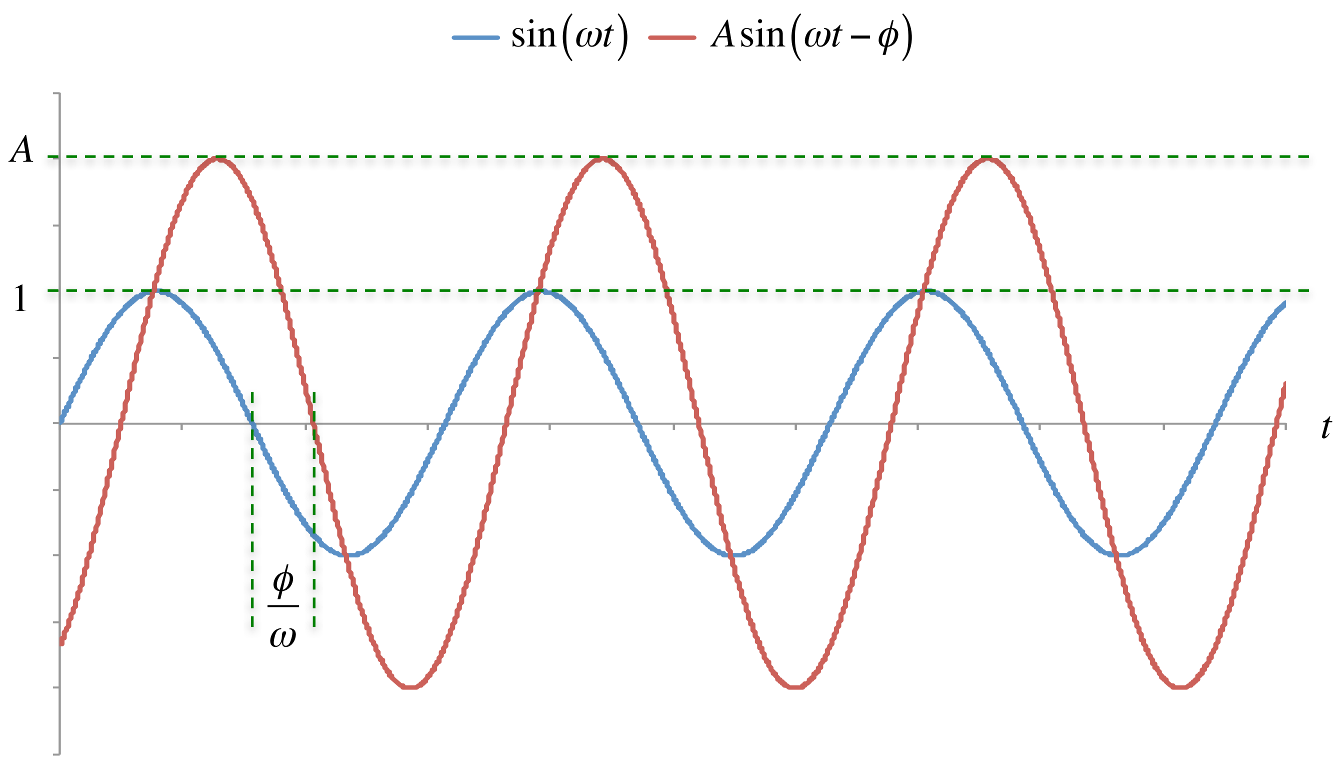
# EEE 51: Second Semester 2017 - 2018

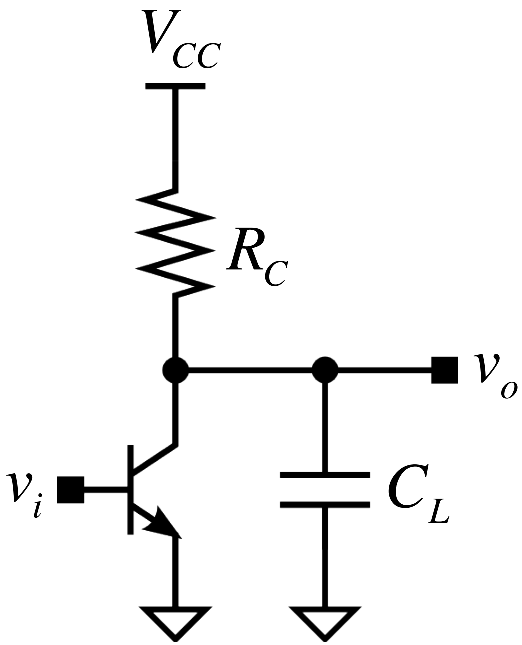
## Lecture 16

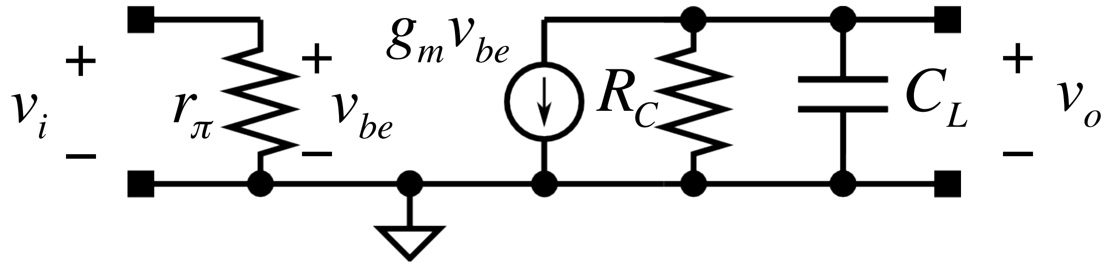
# Frequency Response

# Magnitude and Phase



# The Single-Pole Common Emitter Amplifier



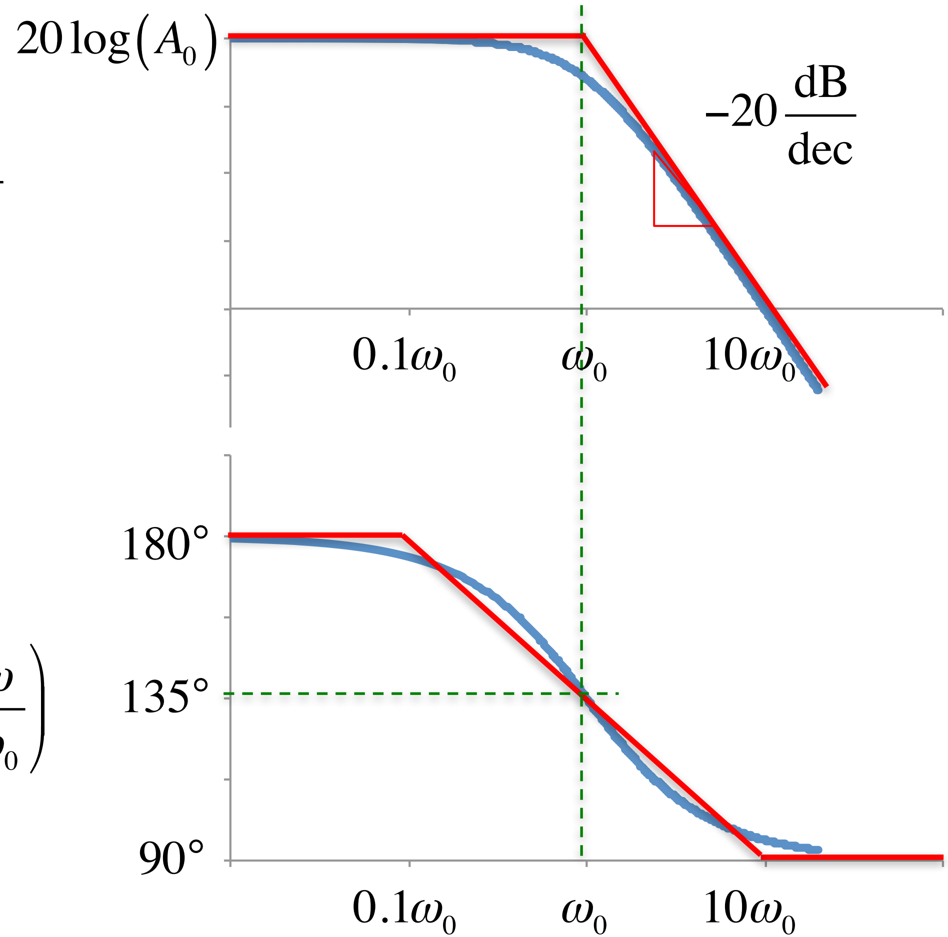

$$A_v = \frac{v_o}{v_i} = -g_m \frac{R_C}{1 + sR_C C_L}$$

# Magnitude and Phase Response

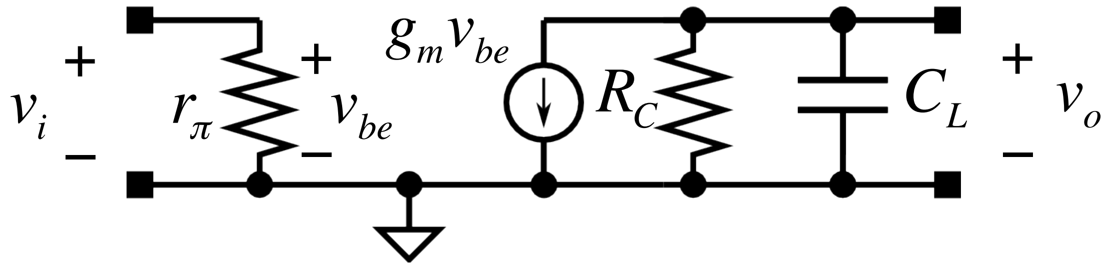
$$\left|A_v\right| = g_m R_C \frac{1}{\sqrt{1+\omega^2\left(R_C C_L\right)^2}} = \frac{A_0}{\sqrt{1+\frac{\omega^2}{\omega_0^2}}}$$

$$A_0 = g_m R_C \qquad \omega_0 = \frac{1}{R_C C_L}$$

$$\angle A_v = \pi - \tan^{-1}\left(\omega R_C C_L\right) = \pi - \tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

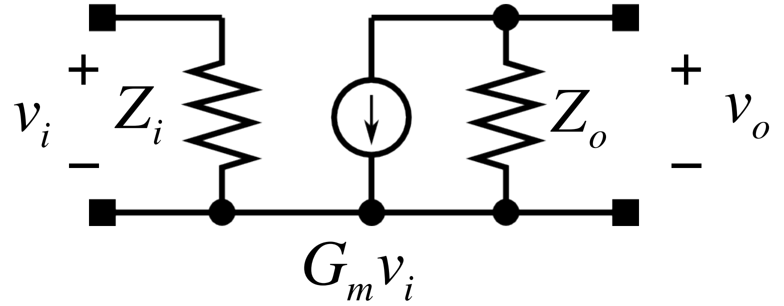


# CE Output Impedance

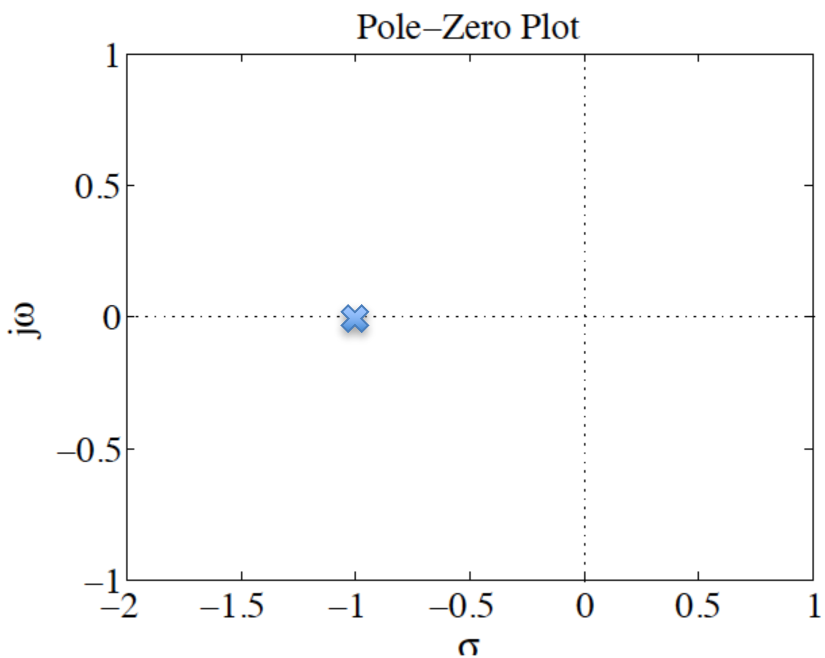



$$Z_o = R_C \,||\, \frac{1}{sC_L} = \frac{R_C}{1 + sR_C C_L}$$

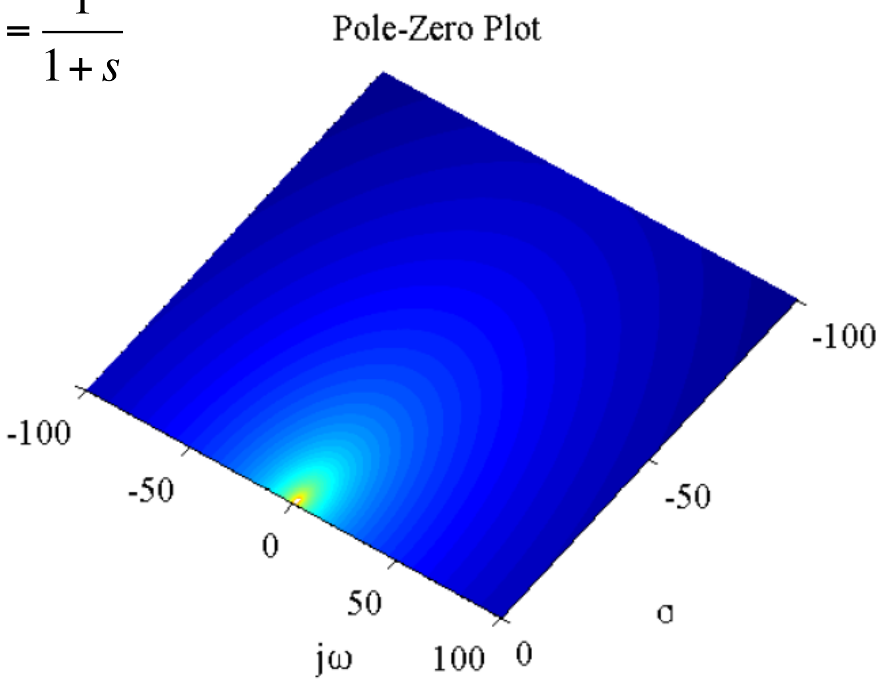
$$G_m = g_m$$



# Pole-Zero Plot: 1-Pole System

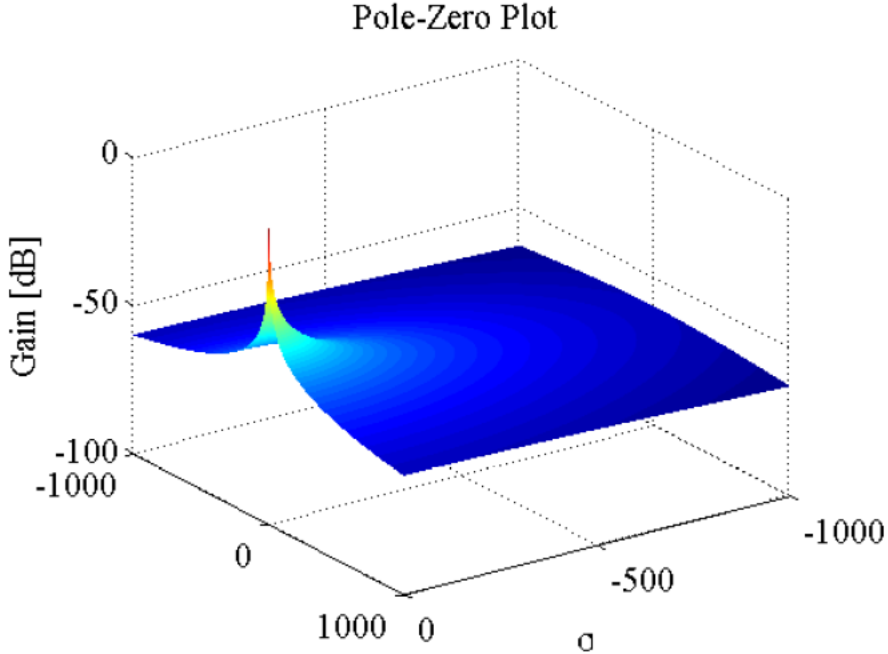


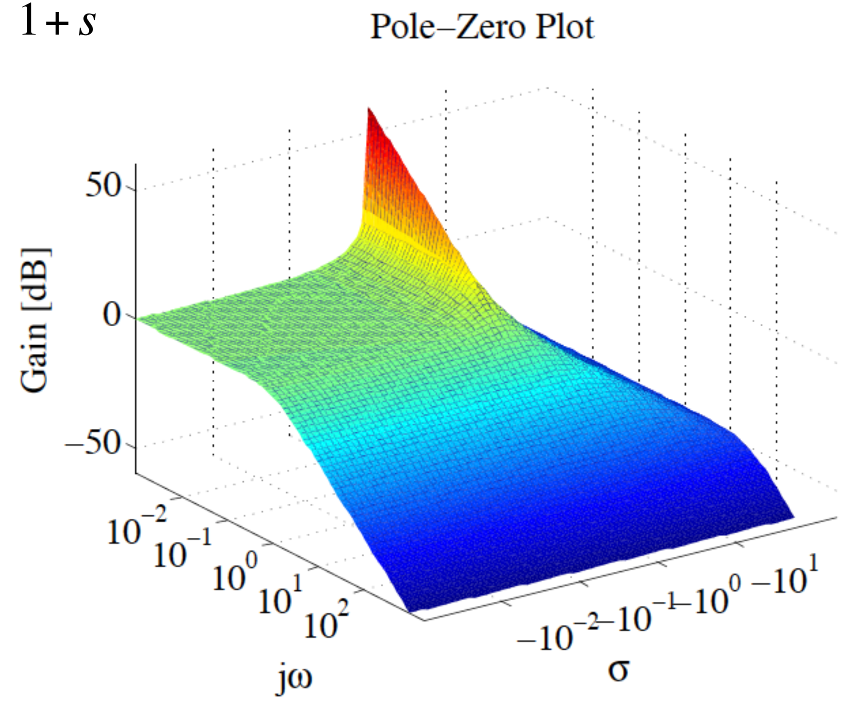

$$H(s) = \frac{1}{1+s}$$



# Pole-Zero Plot: 1-Pole System

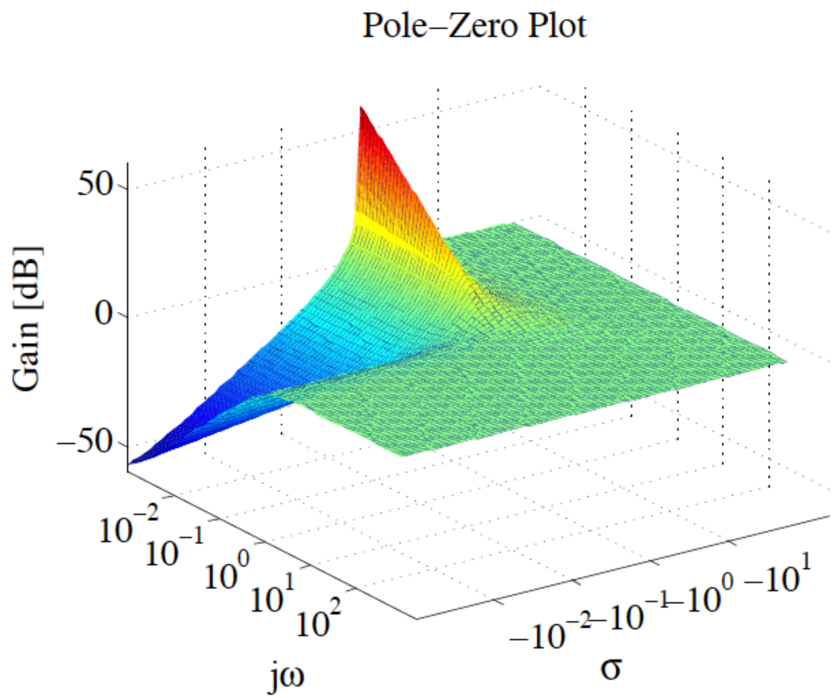
$$H(s) = \frac{1}{1+s}$$



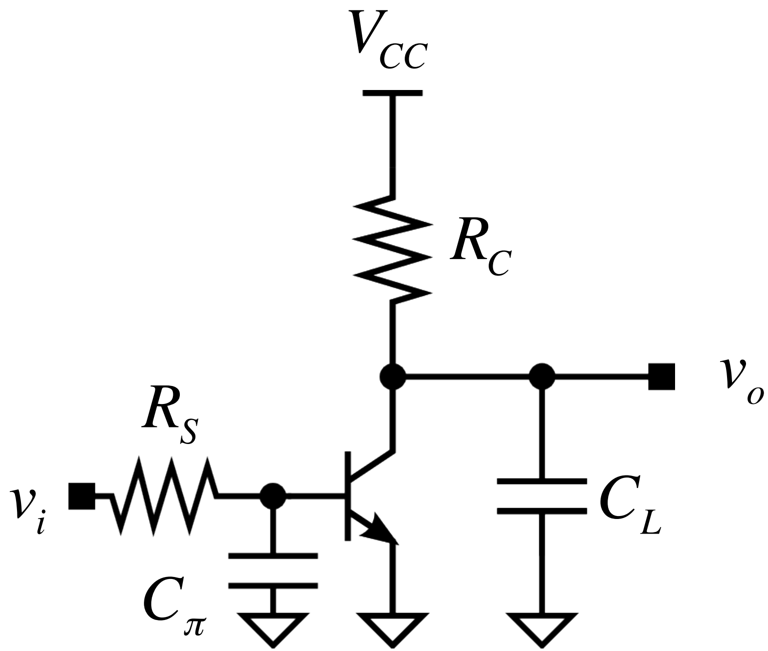



# A Pole and a Zero

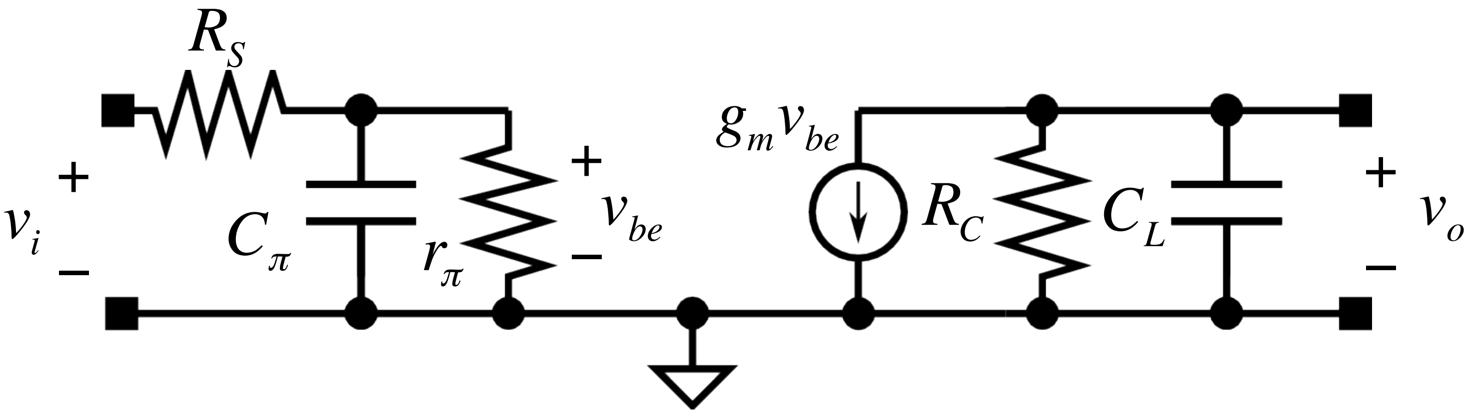
$$H(s) = \frac{s}{1+s}$$



# The CE Amplifier with an Input Pole



# The CE Amplifier with an Input Pole



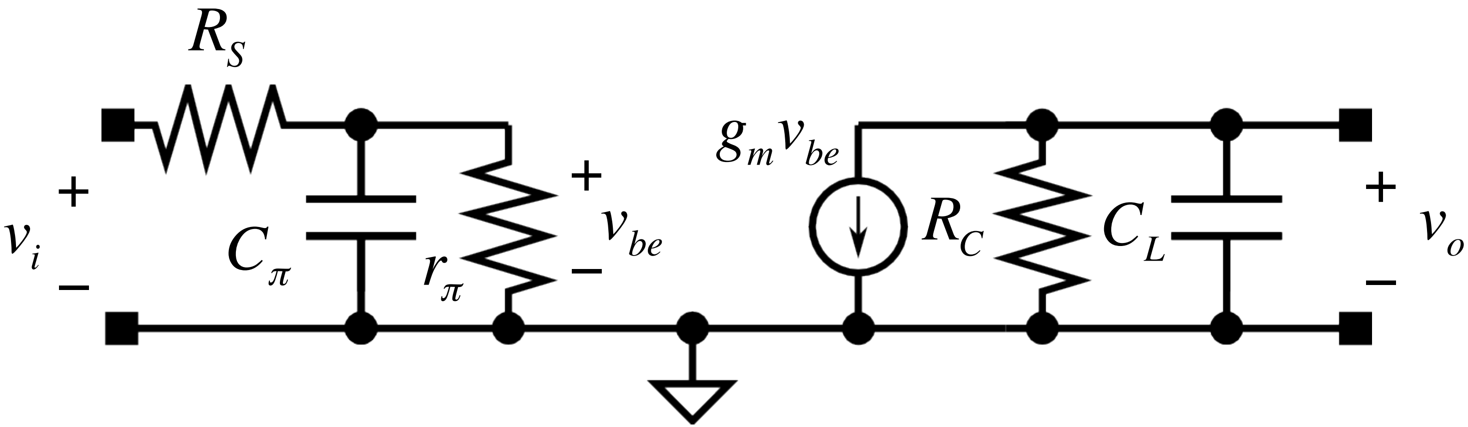

$$\frac{v_{be}}{v_i} = \frac{Z_\pi}{Z_\pi + R_S} = \frac{r_\pi \parallel \dfrac{1}{sC_\pi}}{r_\pi \parallel \dfrac{1}{sC_\pi} + R_S} = \frac{\dfrac{r_\pi}{1 + sr_\pi C_\pi}}{\dfrac{r_\pi}{1 + sr_\pi C_\pi} + R_S} = \frac{r_\pi}{r_\pi + R_S + sr_\pi R_S C_\pi}$$

$$= \frac{r_\pi}{r_\pi + R_S}\left(\frac{1}{1 + sC_\pi \dfrac{r_\pi R_S}{r_\pi + R_S}}\right) = A_1 \frac{1}{1 + j\dfrac{\omega}{\omega_{p1}}}$$

$$A_1 = \frac{r_\pi}{r_\pi + R_S}$$

$$\omega_{p1} = \frac{1}{C_\pi \left(r_\pi \parallel R_S\right)}$$

# The CE Amplifier with an Input Pole



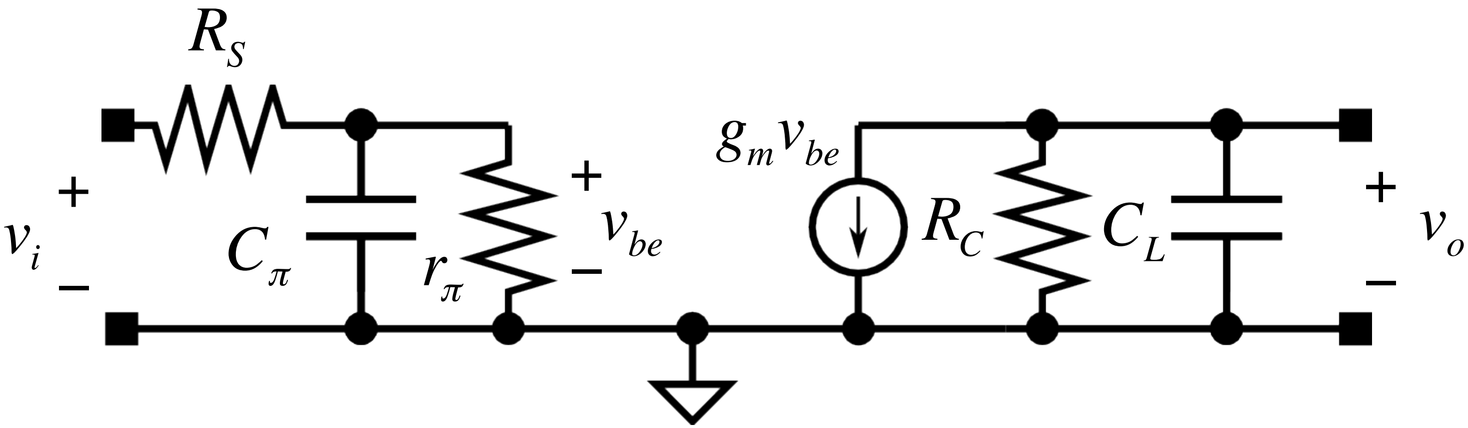

$$\frac{v_o}{v_{be}} = -g_m\left(R_C \parallel \frac{1}{sC_L}\right) = -g_m \frac{R_C \dfrac{1}{sC_L}}{\dfrac{1}{sC_L} + R_C} = \frac{R_C}{1 + sR_C C_L}$$

$$= g_m R_C \left(\frac{-1}{1 + sR_C C_L}\right) = A_2 \frac{-1}{1 + j\dfrac{\omega}{\omega_{p2}}}$$

$$A_2 = g_m R_C$$

$$\omega_{p2} = \frac{1}{R_C C_L}$$

# The CE Amplifier with an Input Pole




$$A_v = \frac{v_o}{v_i} = \frac{v_o}{v_{be}} \cdot \frac{v_{be}}{v_i} = A_1 A_2 \frac{-1}{\left(1 + j\frac{\omega}{\omega_{p1}}\right)\left(1 + j\frac{\omega}{\omega_{p2}}\right)}$$

$$= A_0 \frac{-1}{\left(1 + j\frac{\omega}{\omega_{p1}}\right)\left(1 + j\frac{\omega}{\omega_{p2}}\right)}$$

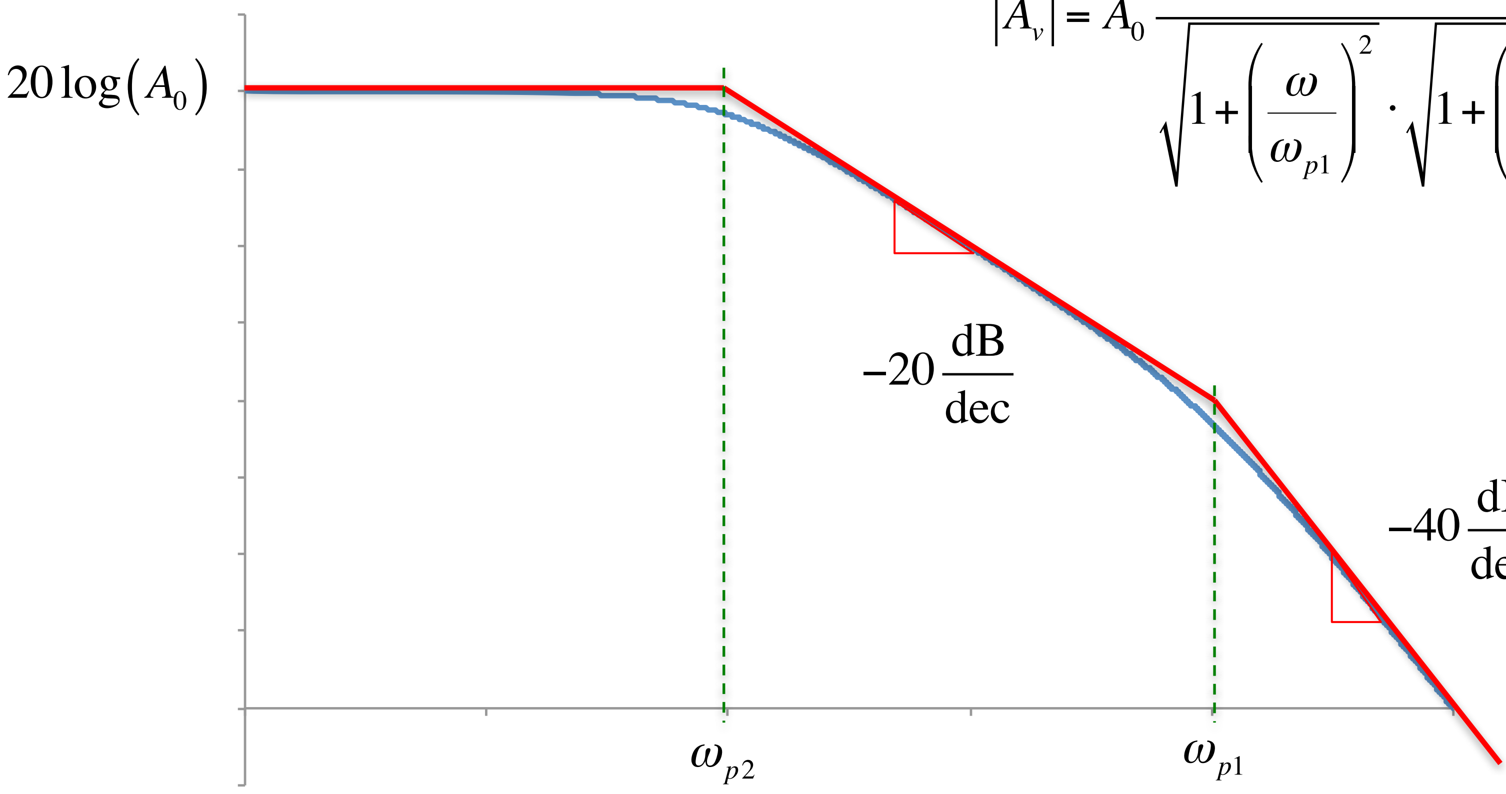
$$A_0 = A_1 A_2 = g_m R_C \frac{r_\pi}{r_\pi + R_S}$$

$$\omega_{p1} = \frac{1}{\left(r_\pi \parallel R_S\right) C_\pi}$$

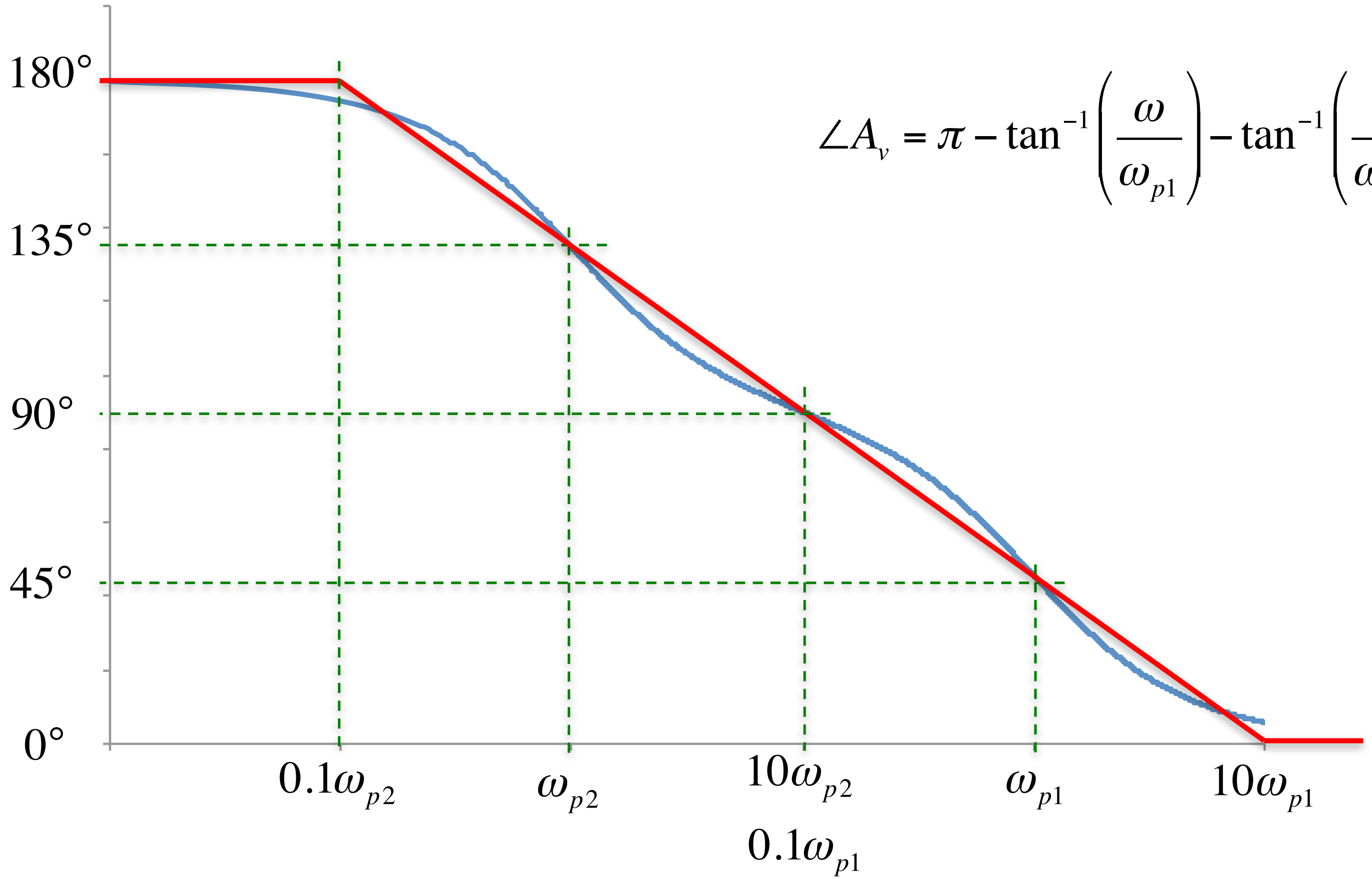
$$\omega_{p2} = \frac{1}{R_C C_L} < \omega_{p1}$$

# Magnitude Response

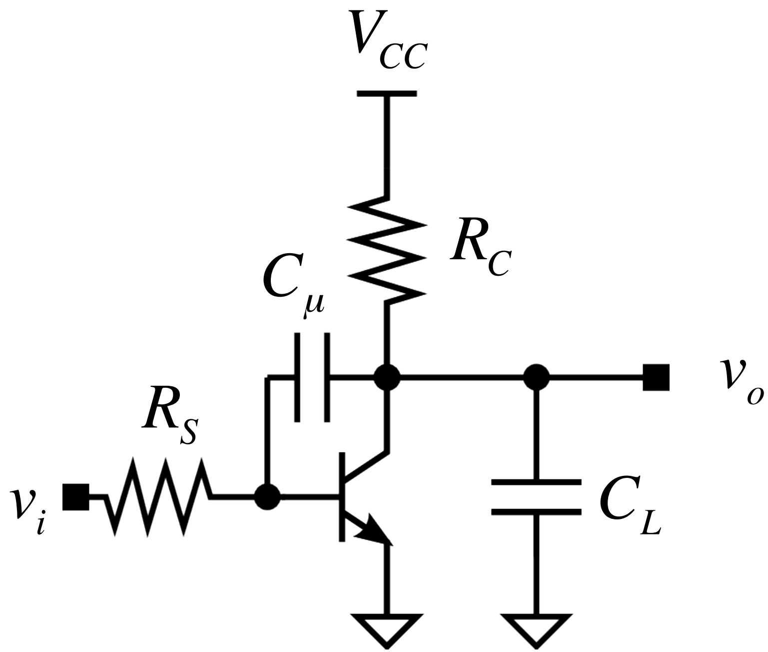
$$|A_v| = A_0 \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_{p1}}\right)^2} \cdot \sqrt{1+\left(\frac{\omega}{\omega_{p2}}\right)^2}}$$



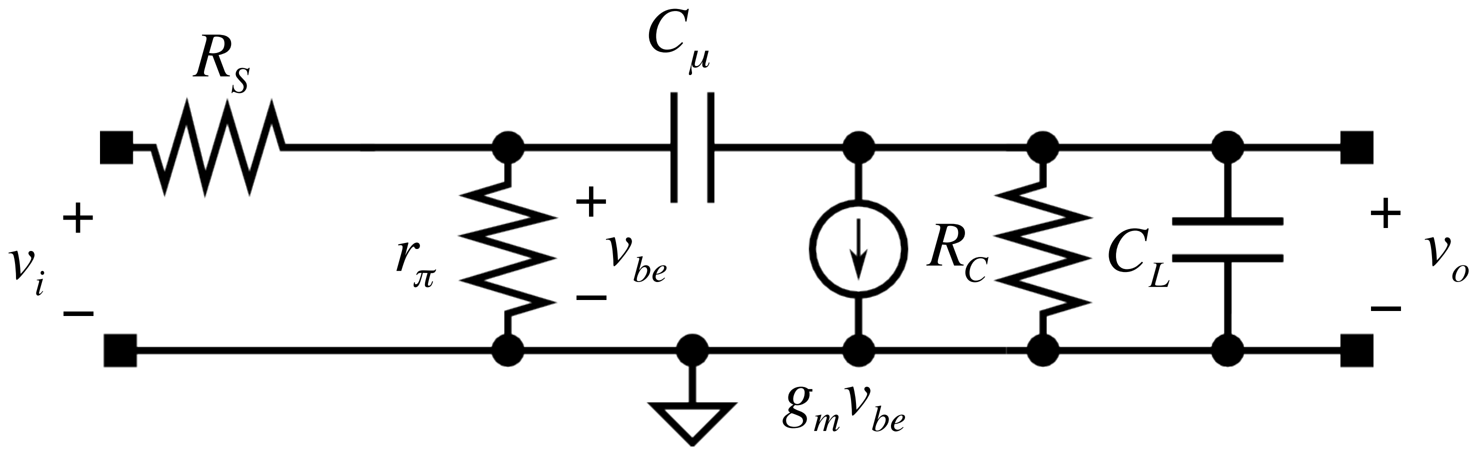
# Phase Response



# The Effect of the Miller Capacitance



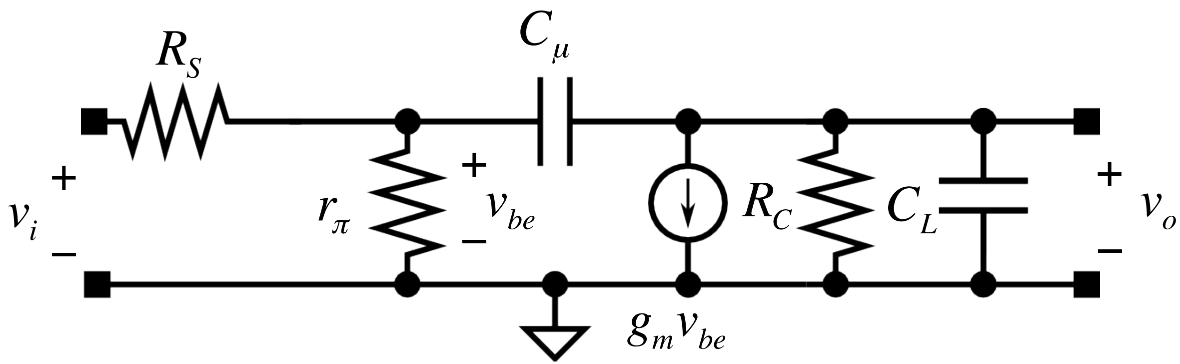
# Small Signal Model




$$v_{be}\left(\frac{1}{R_S}+\frac{1}{r_\pi}+sC_\mu\right)-v_o\left(sC_\mu\right)=v_i\left(\frac{1}{R_S}\right)$$

$$-v_{be}\left(sC_\mu-g_m\right)+v_o\left(sC_\mu+\frac{1}{R_C}+sC_L\right)=0$$

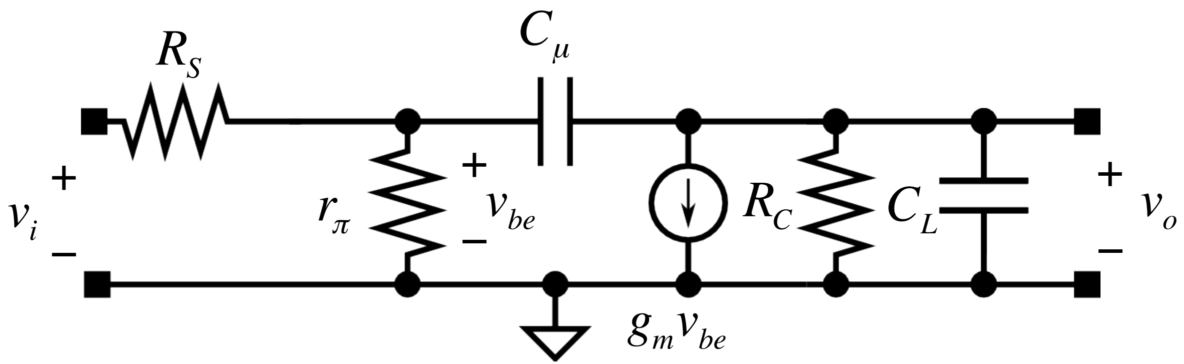
# Small Signal Model




$$-v_{be}\left(sC_\mu - g_m\right) + v_o\left(sC_\mu + \frac{1}{R_C} + sC_L\right) = 0$$

$$\frac{v_o}{v_{be}} = \frac{sC_\mu - g_m}{sC_\mu + \frac{1}{R_C} + sC_L} = \frac{sR_C C_\mu - g_m R_C}{1 + sR_C\left(C_\mu + C_L\right)} = -g_m R_C \frac{1 - s\frac{C_\mu}{g_m}}{1 + sR_C\left(C_\mu + C_L\right)}$$

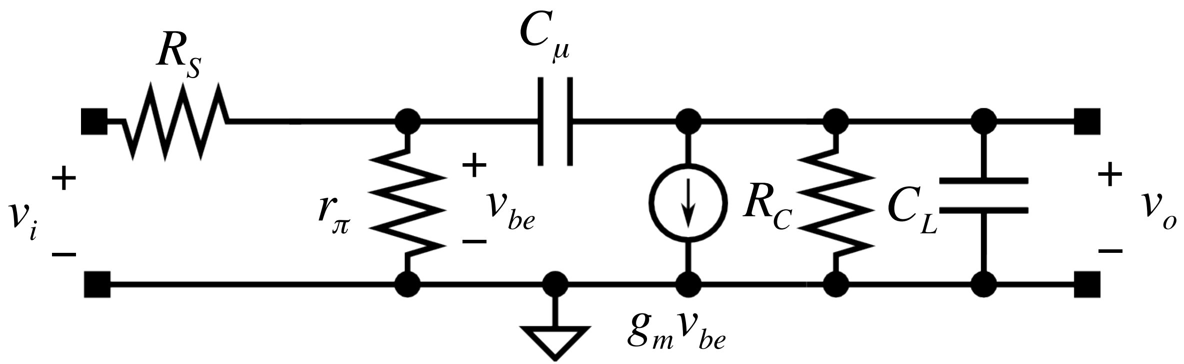
# Small Signal Model




$$v_{be}\left(\frac{1}{R_S}+\frac{1}{r_\pi}+sC_\mu\right)-v_o\left(sC_\mu\right)=v_i\left(\frac{1}{R_S}\right)\Rightarrow v_{be}\left(\frac{1+s\left(R_S \parallel r_\pi\right)C_\mu}{R_S \parallel r_\pi}\right)-v_o\left(sC_\mu\right)=v_i\left(\frac{1}{R_S}\right)$$

$$\left[-\frac{1+sR_C\left(C_\mu+C_L\right)}{g_m R_C\left(1-s\frac{C_\mu}{g_m}\right)}\left(\frac{1+s\left(R_S \parallel r_\pi\right)C_\mu}{R_S \parallel r_\pi}\right)-sC_\mu\right]v_o=v_i\left(\frac{1}{R_S}\right)$$

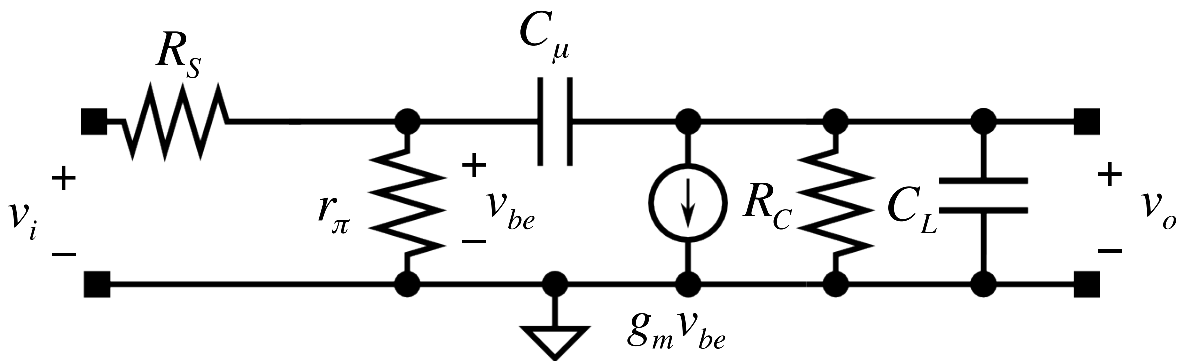
# Small Signal Model




$$\frac{v_o}{v_i}=\frac{R_S \parallel r_\pi}{R_S}\frac{-g_m R_C\left(1-s\frac{C_\mu}{g_m}\right)}{\left(1+sR_C\left(C_\mu+C_L\right)\right)\left(1+s\left(R_S \parallel r_\pi\right)C_\mu\right)+sC_\mu g_m R_C\left(1-s\frac{C_\mu}{g_m}\right)\left(R_S \parallel r_\pi\right)}$$

$$=\frac{-g_m R_C\frac{r_\pi}{r_\pi+R_S}\left(1-s\frac{C_\mu}{g_m}\right)}{\left(1+sR_C\left(C_\mu+C_L\right)\right)\left(1+s\left(R_S \parallel r_\pi\right)C_\mu\right)+sC_\mu g_m R_C\left(R_S \parallel r_\pi\right)-s^2C_\mu^2R_C\left(R_S \parallel r_\pi\right)}$$

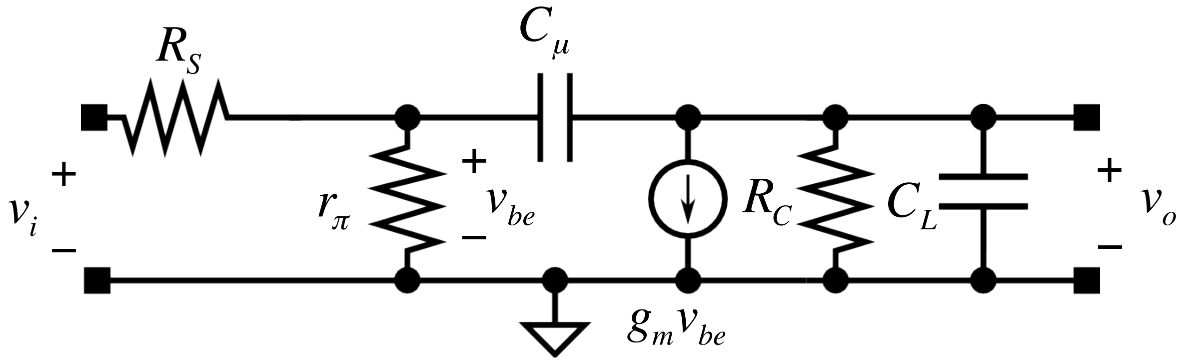
# Small Signal Model




$$\frac{v_o}{v_i} = \frac{-g_m R_C \frac{r_\pi}{r_\pi + R_S}\left(1 - s\frac{C_\mu}{g_m}\right)}{1 + s\left[R_C\left(C_\mu + C_L\right) + \left(R_S \parallel r_\pi\right)C_\mu\left(1 + g_m R_C\right)\right] + s^2\left[\left(R_S \parallel r_\pi\right)R_C C_L C_\mu\right]}$$

$$= \frac{A_0\left(1 - \frac{s}{z}\right)}{1 + sX_1 + s^2 X_2} = \frac{A_0\left(1 - \frac{s}{z}\right)}{\left(1 + \frac{s}{p_1}\right)\left(1 + \frac{s}{p_2}\right)}$$

# Small Signal Model




$$\frac{v_o}{v_i} = \frac{-g_m R_C \dfrac{r_\pi}{r_\pi + R_S}\left(1 - s\dfrac{C_\mu}{g_m}\right)}{1 + s\left[R_C\left(C_\mu + C_L\right) + \left(R_S \parallel r_\pi\right) C_\mu \left(1 + g_m R_C\right)\right] + s^2\left[\left(R_S \parallel r_\pi\right) R_C C_L C_\mu\right]}$$

$$= \frac{A_0\left(1 - \dfrac{s}{z}\right)}{\left(1 + \dfrac{s}{p_1}\right)\left(1 + \dfrac{s}{p_2}\right)} = \frac{A_0\left(1 - \dfrac{s}{z}\right)}{1 + s\left(\dfrac{1}{p_1} + \dfrac{1}{p_2}\right) + s^2\left(\dfrac{1}{p_1 p_2}\right)}$$

# Small Signal Model

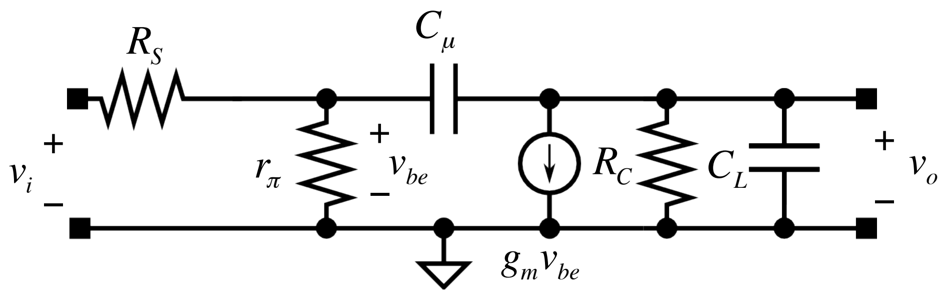
$$\frac{v_o}{v_i} = \frac{-A_0\left(1-\frac{s}{z}\right)}{\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)} = \frac{-A_0\left(1-\frac{s}{z}\right)}{1+s\left(\frac{1}{p_1}+\frac{1}{p_2}\right)+s^2\left(\frac{1}{p_1 p_2}\right)}$$

$$A_0 = g_m R_C \frac{r_\pi}{r_\pi + R_S}$$

$$\omega_{p1} \approx \frac{1}{R_C\left(C_\mu + C_L\right)} \approx \frac{1}{R_C C_L}$$

$$\omega_{p2} \approx \frac{1}{\left(R_S \parallel r_\pi\right) C_\mu \left(1+g_m R_C\right)}$$

$$\omega_z = \frac{g_m}{C_\mu}$$



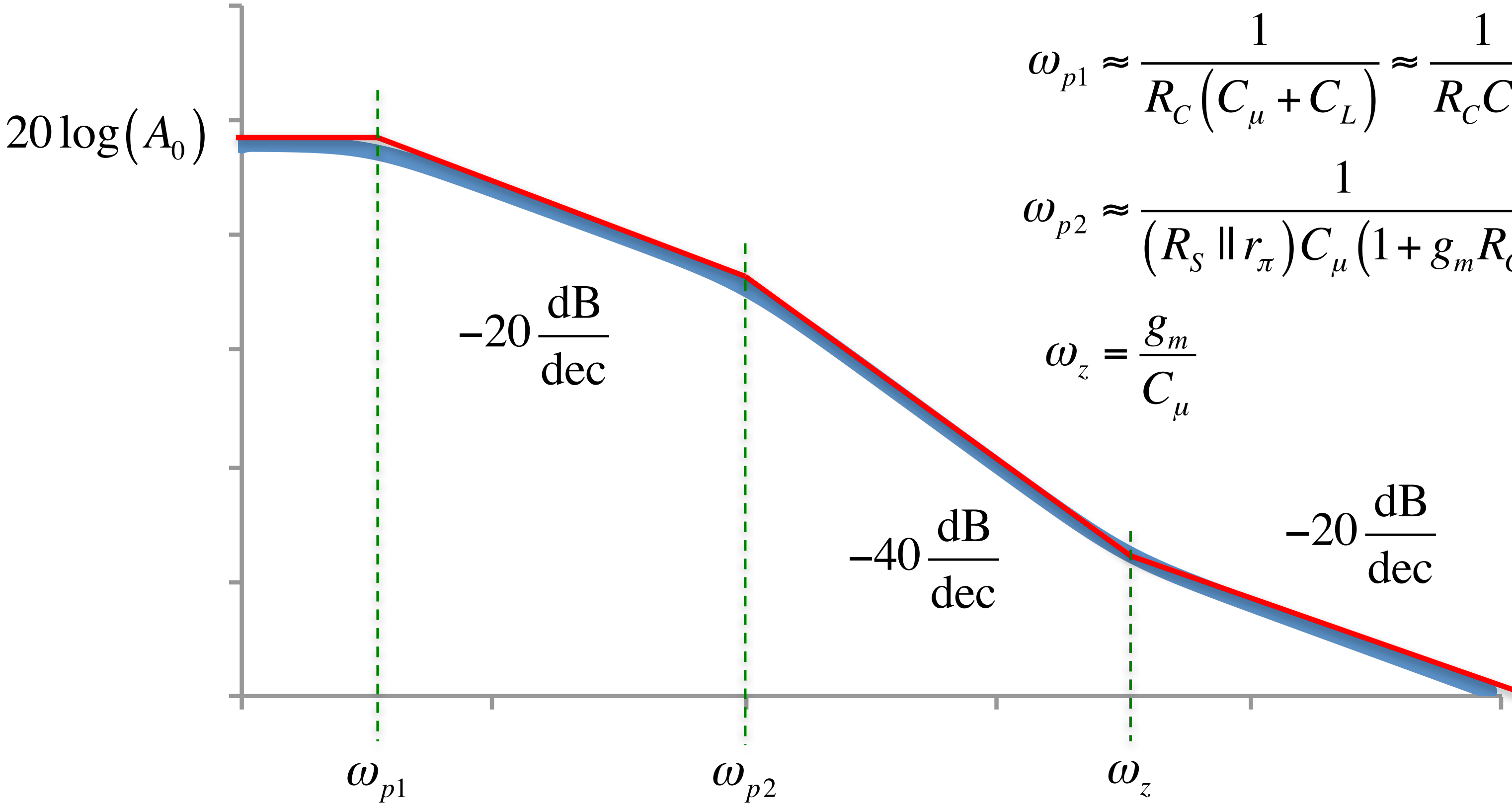
# Magnitude Response

$$A_0 = g_m R_C \frac{r_\pi}{r_\pi + R_S}$$

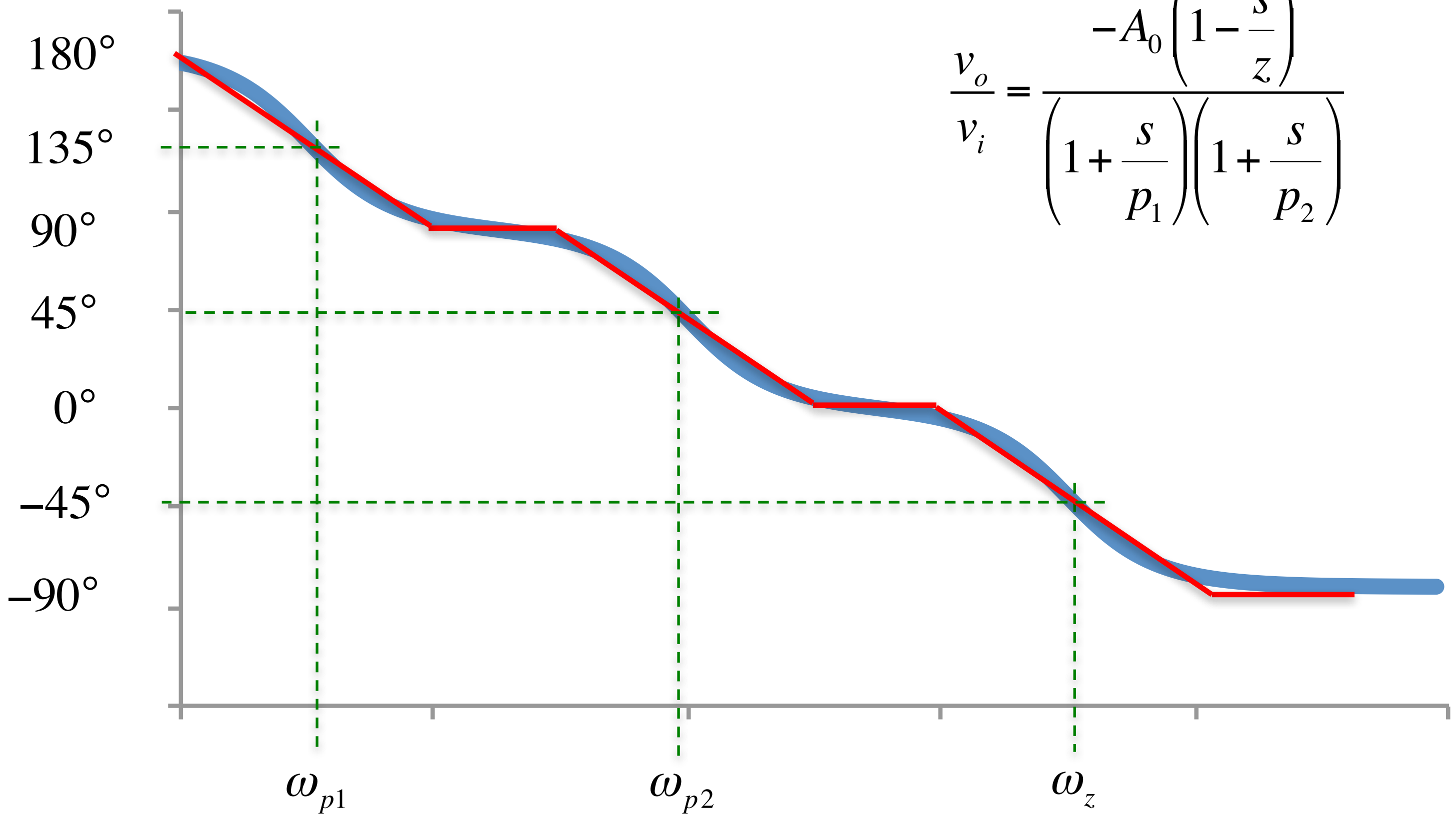
$$\omega_{p1} \approx \frac{1}{R_C\left(C_\mu + C_L\right)} \approx \frac{1}{R_C C_L}$$

$$\omega_{p2} \approx \frac{1}{\left(R_S \parallel r_\pi\right) C_\mu \left(1 + g_m R_C\right)}$$

$$\omega_z = \frac{g_m}{C_\mu}$$

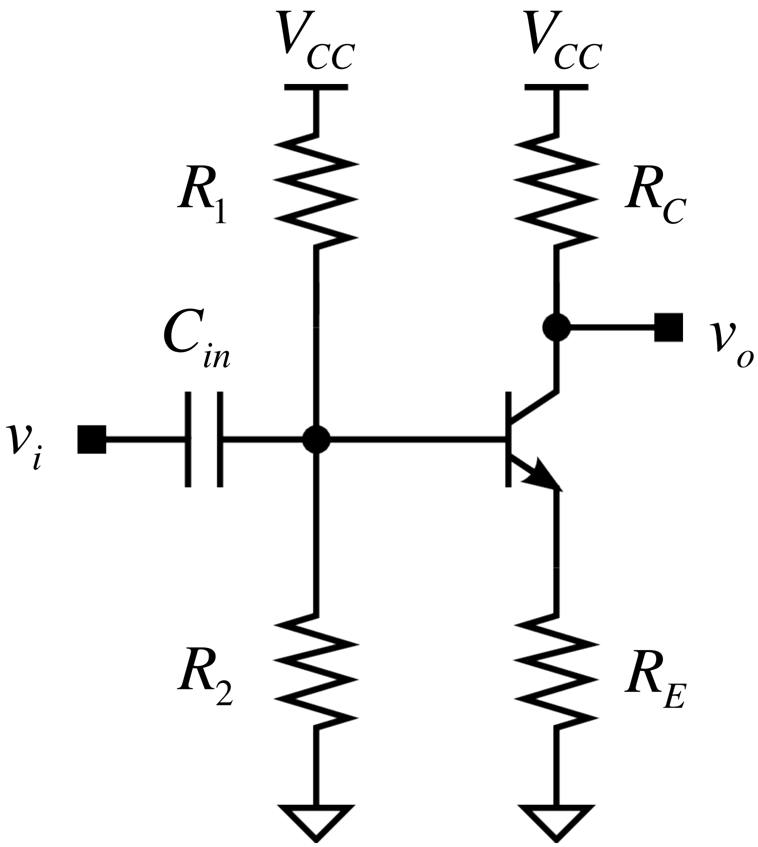


# Phase Response

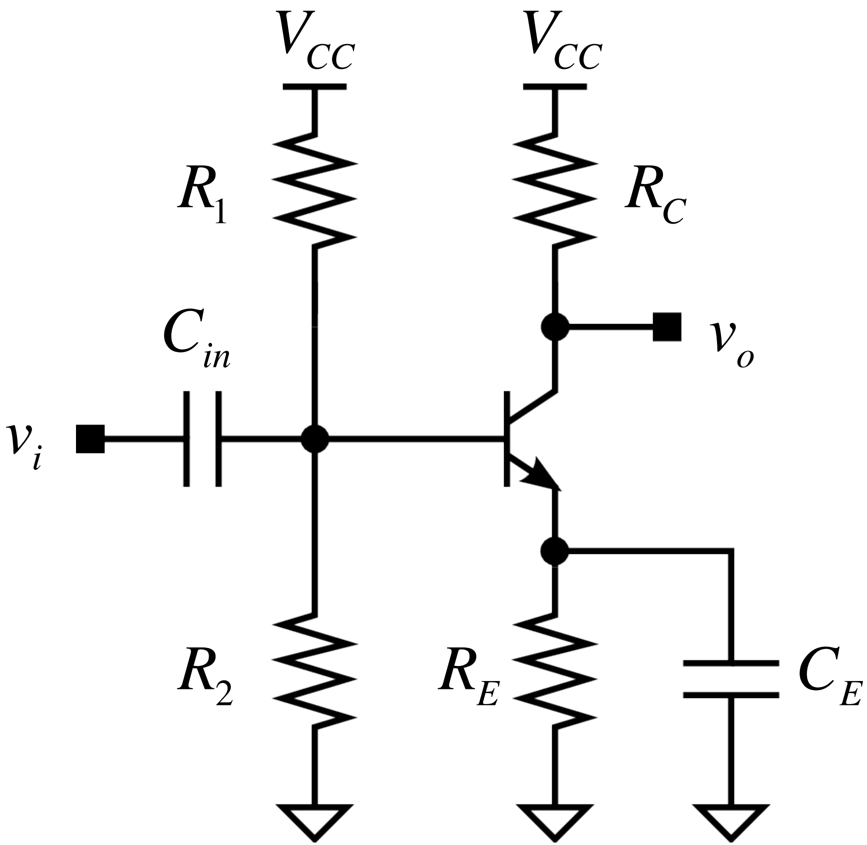



$$\frac{v_o}{v_i} = \frac{-A_0\left(1-\frac{s}{z}\right)}{\left(1+\frac{s}{p_1}\right)\left(1+\frac{s}{p_2}\right)}$$

# Using an AC-Coupling Capacitor



# Bypassing the Emitter Degeneration Resistor



# Next Meeting

- Frequency Response of Amplifiers