Lecture 16
TRANSMISSION LINE – SERIES IMPEDANCE

Agenda

Lecture

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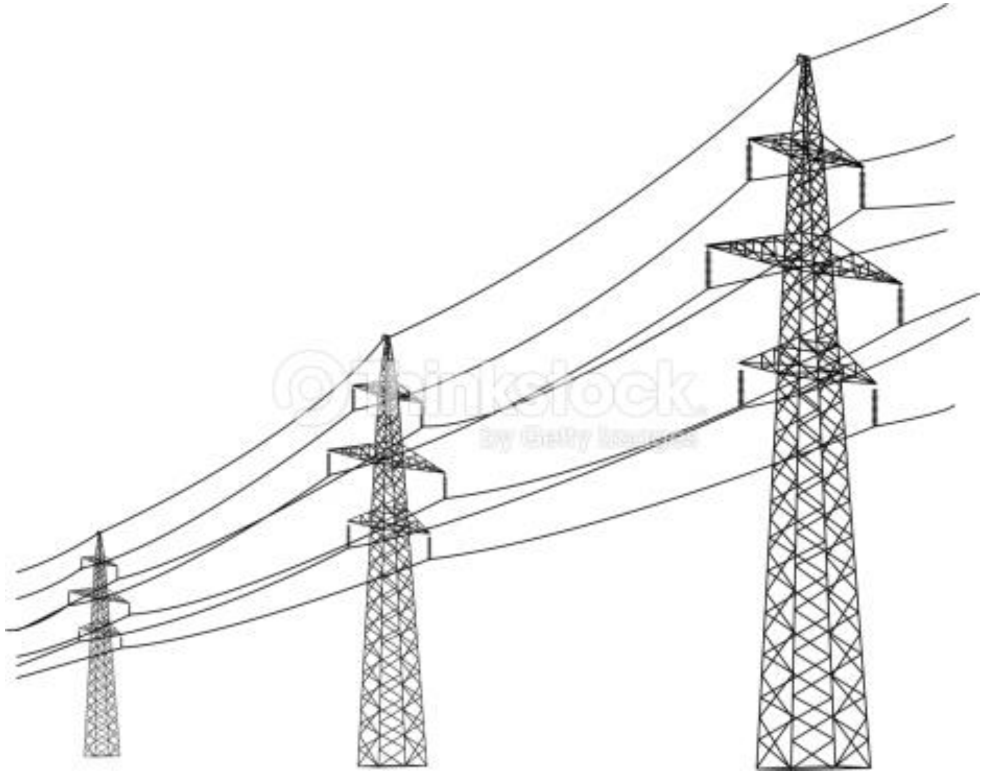
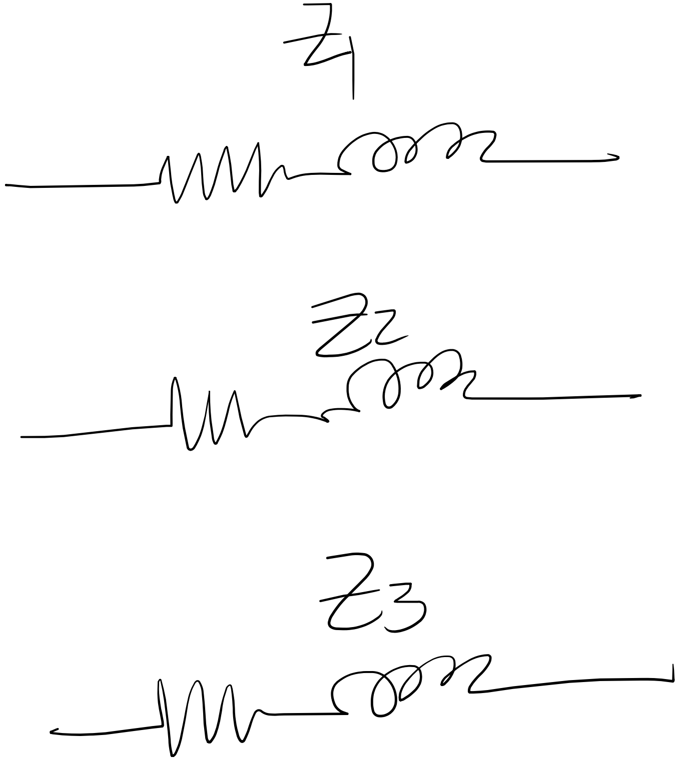
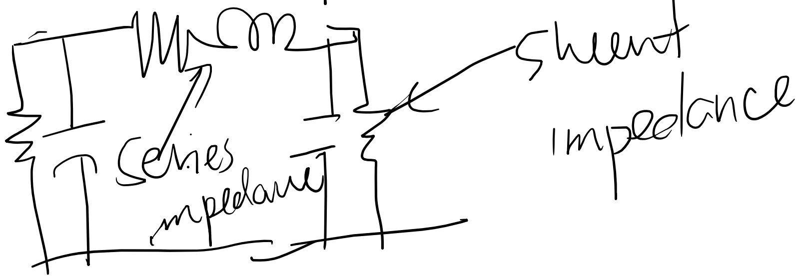
HOW DO WE MODEL TRANSMISSION LINES FOR CIRCUIT ANALYSIS?





how do we achieve

Generally, Any transmission line can be represented as



in reality, there are mutual inductances

$$\begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{bmatrix}$$


Why we use Bundled
conductors

Lecture Outcomes

at the end of the lecture, the student must be able to …

- Compute the series impedance of T&D lines
- Identify the variables that affect the series impedance of T&D Lines

EEE 103
Introduction to Power Systems

Power System Modeling

Transmission and Distribution Line Models

Line Parameters that we have to account

1. Resistance is inherent in any conductor material. The energy dissipated (losses) along the power cables is due to this resistance.

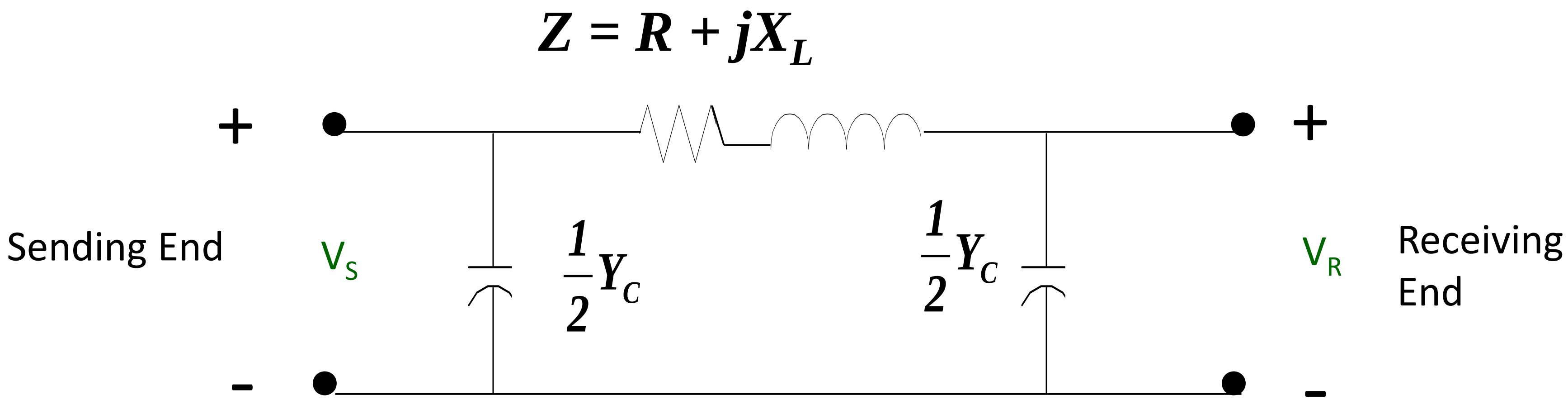
2. Inductance is the property of the electric circuit that relates the voltage induced by the changing flux to the rate of change of current. This are apparent due to different existing currents.

3. Capacitance exists between conductors and between conductors and the ground due to the potential difference between them.

4. Conductance accounts for the leakage current at the insulators of overhead lines and cables.

General Line Model

- Series Impedance: $Z_{LINE} = R + j\,X_L$
- Shunt Admittance: $Y_{LINE} = ½\,Y_C$

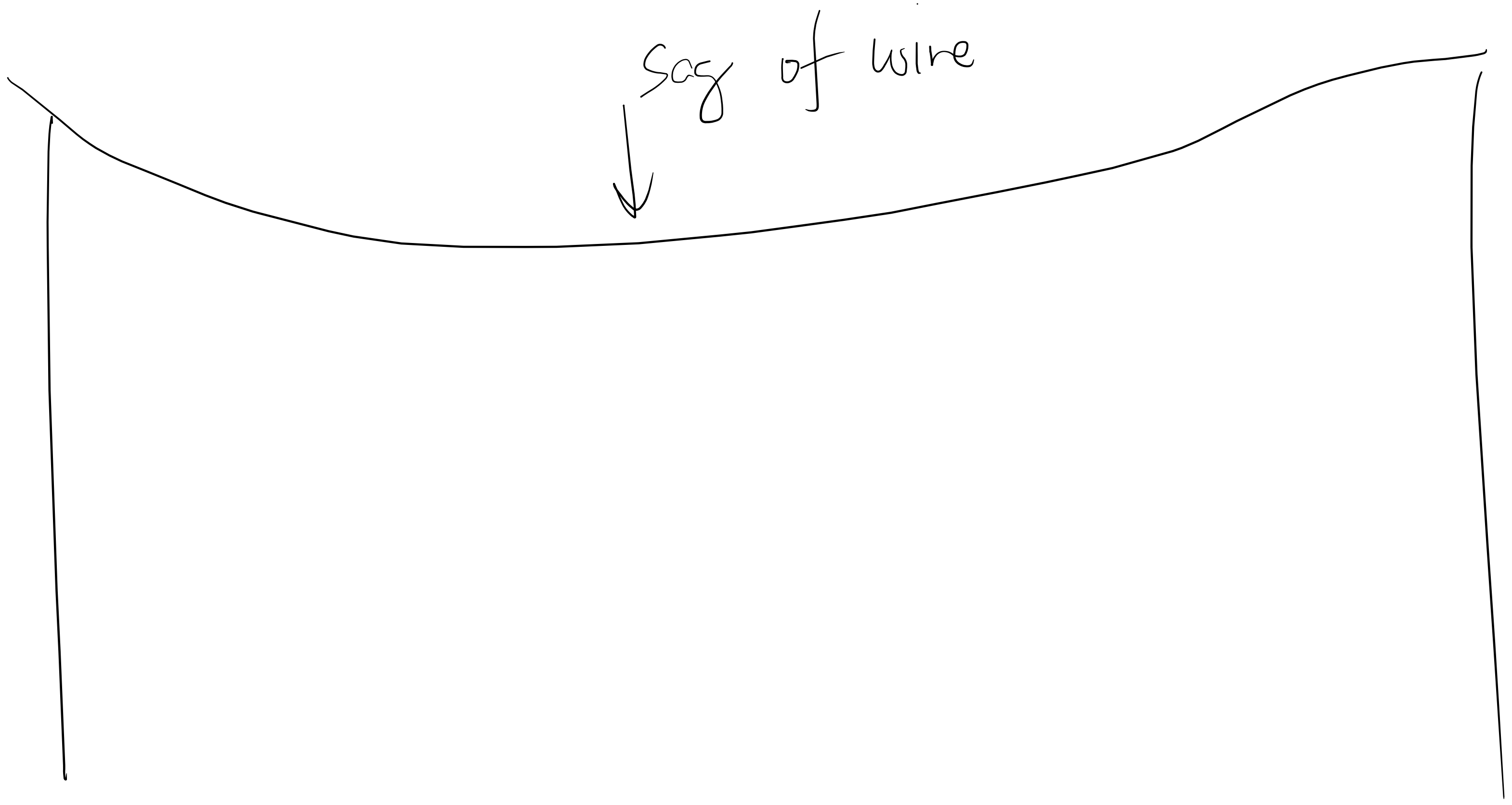


What are conductors made of?

- Aluminum is preferred over Copper as a material for transmission and lines due to:
  - lower cost
  - lighter weight
  - larger diameter for the same resistance*
    - **   This results in a lower voltage gradient at the conductor surface (less tendency for corona) because there is a large surface area.*
- Copper is preferred over Aluminum as a material for distribution lines due to lower resistance to reduce system losses.

Structural Matters

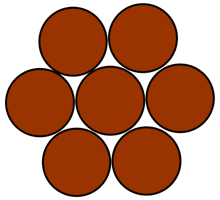
- Material Strength – How do we strengthen the wire?



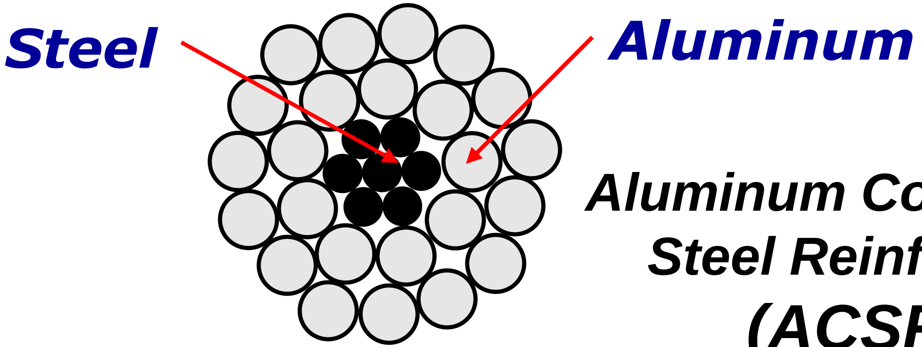
Stranding of Conductors

Alternate layers of wire of a stranded conductor are spiraled in opposite directions to prevent unwinding and make the outer radius of one layer coincide with the inner radius of the next.

The number of strands depends on the number of layers and on whether all the strands are of the same diameter. The total number of strands of uniform diameter in a concentrically stranded cable is 7, 19, 37, 61, 91, etc.



Hard-Drawn Copper
(Cu)



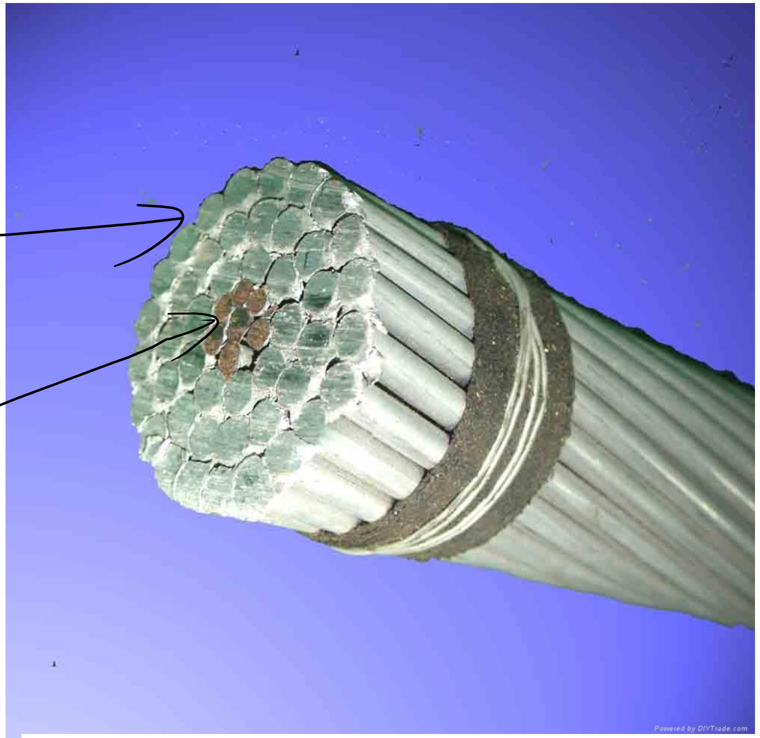
Aluminum Conductor Steel Reinforced
(ACSR)

Stranding of Conductors

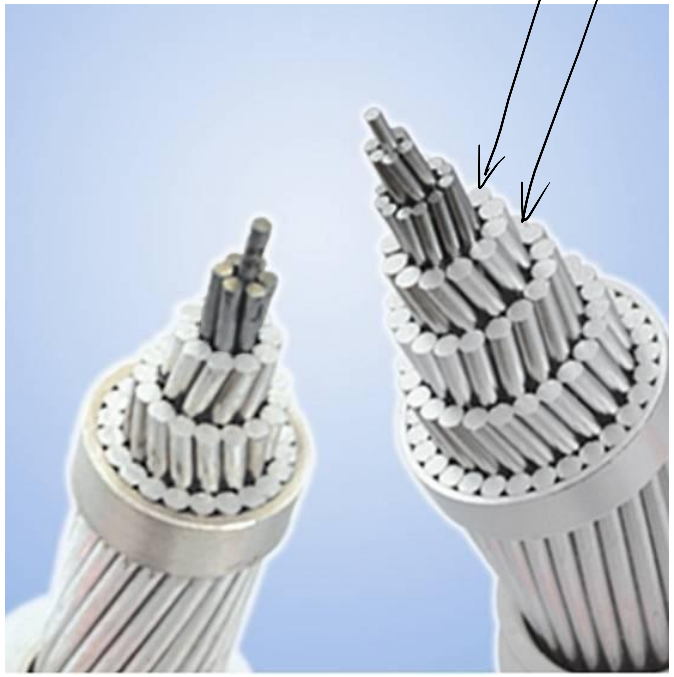
Aluminium

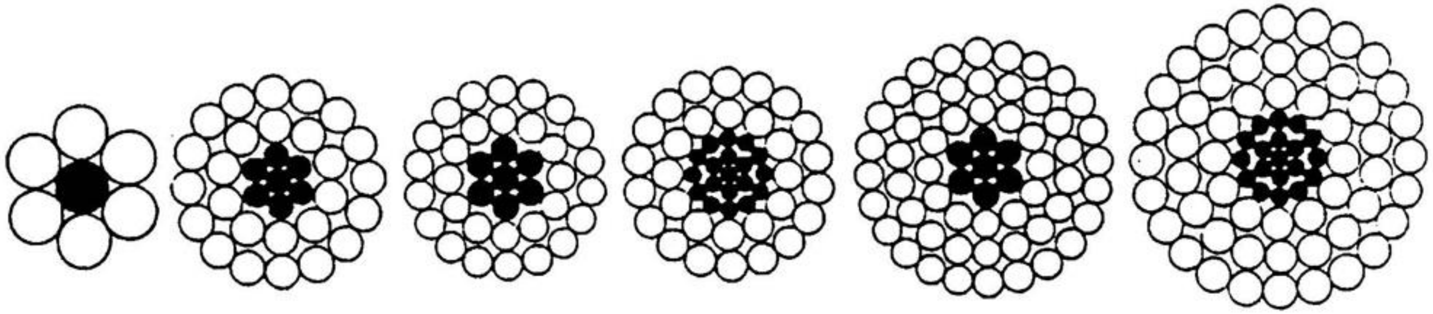
steel

- strong material



Increases Strength





SERIES IMPEDANCE
Resistance

Resistance of Conductors

- The Resistance of a Conductor depends on the material (e.g., Cu or Al)
- DC Resistance is directly proportional to Length and inversely proportional to cross-sectional area

$$\boxed{R = \rho \frac{L}{A}}$$

R – Resistance
ρ – Resistivity of Material (constant)
L – Length
A – Cross-Sectional Area

Resistance of Conductors

- The effective resistance of conductors can also be calculated by:

$$R = \frac{\text{Power loss in conductor}}{I^2}$$

- where Power Loss is in watts, and I is the RMS current in amperes flowing through the conductor.

Resistance of Conductors

- The variation of conductor resistance with temperature is linear, such that:

$$\frac{R_2}{R_1} = \frac{T + t_2}{T + t_1}$$

- where R_1 and R_2 are the conductor resistances at temperatures t_1 and t_2, respectively.
- T is the temperature, in Kelvins, at which the conductor resistance is zero.

Skin Effect

- The effective resistance of a conductor is equal to its DC resistance only if the distribution of current throughout the conductor is uniform.
- However, in AC systems, as the frequency increases, the charges have a greater tendency to concentrate at the outer surface of the conductor. The effective area for current flow decreases. Internal areas also see greater flux linkages than the outer areas, thus offering greater reactance to current flow.
- This phenomenon, called the <u>skin effect</u>, results in the increase in the effective resistance of the conductor.

Resistance of Conductors

INDEX	Conductor Type	Size		Strands	O.D. (Inches)	GMR (feet)	Resistance (Ohms/Mile)
		Value	Unit				
1	ACSR	6	AWG	6/1	0.19800	0.00394	3.98000
2	ACSR	5	AWG	6/1	0.22300	0.00416	3.18000
3	ACSR	4	AWG	7/1	0.25700	0.00452	2.55000
4	ACSR	4	AWG	6/1	0.25000	0.00437	2.57000
5	ACSR	3	AWG	6/1	0.28100	0.00430	2.07000
6	ACSR	2	AWG	7/1	0.32500	0.00504	1.65000
7	ACSR	2	AWG	6/1	0.31600	0.00418	1.69000
8	ACSR	1	AWG	6/1	0.35500	0.00418	1.38000
9	ACSR	1/0	AWG	6/1	0.39800	0.00446	1.12000
10	ACSR	2/0	AWG	6/1	0.44700	0.00510	0.89500

Source: Westinghouse T&D Handbook

Resistance of Conductors

ACSR bare conductor meets or exceeds BS 215 part 2 specifications as below:

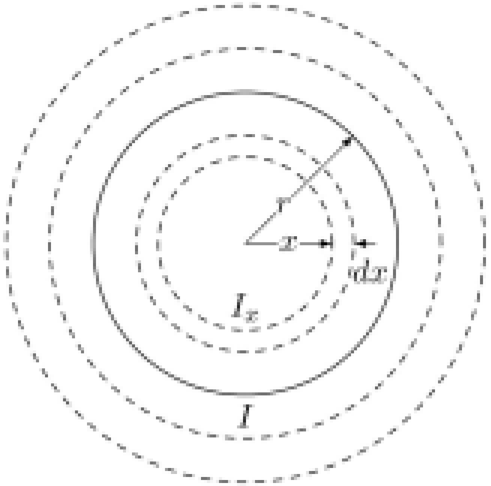
Code name	Cross section			Stranding&Wire Diameter		Overall diameter	Approx. weight	Rated strength	Max.DC resistance at 20C
	Al	Steel	Total	Al	Steel				
	mm^2	mm^2	mm^2	No./mm	No./mm	mm	kg/km	kN	Ohm/km
Mole	10.62	1.77	12.39	6/1.50	1/1.50	4.50	43	4.14	2.076
Squirrel	21.0	3.50	24.5	6/2.11	1/2.11	6.33	84.7	7.87	1.3659
Gopher	26.2	4.37	30.6	6/2.36	1/2.36	7.08	106.0	9.58	1.0919
Weasel	31.6	5.27	36.9	6/2.59	1/2.59	7.77	127.6	11.38	0.9065
Fox	36.7	6.11	42.8	6/2.79	1/2.79	8.37	148.1	13.21	0.7812
Ferret	42.41	7.07	49.48	6/3.00	1/3.00	9.00	172	15.20	0.6766
Rabbit	52.9	8.81	61.7	6/3.35	1/3.35	10.1	213.5	18.42	0.5419
Mink	63.1	10.5	73.6	6/3.66	1/3.66	11.0	254.9	21.67	0.4540
Skunk	63.27	36.93	100.30	12/2.59	7/2.59	12.95	465	53.00	0.4567
Beaver	75.0	12.5	87.5	6/3.99	1/3.99	12.0	302.9	25.76	0.3820
Horse	73.4	42.8	116.2	12/2.79	7/2.79	14.0	537.3	61.26	0.3936

Series Impedance - Resistance

- Straight Forward – Property of material
- Insight – Resistance of Conductors is given by manufacturers.
- Insight – Without manufacturers information, the resistance can still be modelled.

SERIES IMPEDANCE
Inductance

Inductance of a Single Conductor (SOLID)



Assumptions
1. Sufficiently long -> end effects are neglected
2. Non-magnetic – permeability is μ_o
3. Uniform current density -> skin effect is neglected

r – radius of conductor

Ampere's Law $\oint H dl = I_{enclosed}$

$H_x(2\pi x) = I_x$ for x < r

From current density uniform $\dfrac{I}{\pi r^2} = \dfrac{I_x}{\pi x^2}$

$$H_x = \frac{I}{2\pi r^2} x$$

Magnetic Flux Definition $B_x = \dfrac{\mu_0 I}{2\pi r^2} x$

$$d\emptyset_x = B_x dx \cdot l$$

Fraction of Flux linkage is

$$d\lambda_x = \left(\frac{x^2}{r^2}\right) d\emptyset_x$$

Total Flux linkage inside the conductor

$$\lambda_{int} = \frac{\mu_0 I}{2\pi r^4} \int_0^r x^3 dx$$

$$\boxed{\lambda_{int} = \frac{\mu_o I}{8\pi}}$$

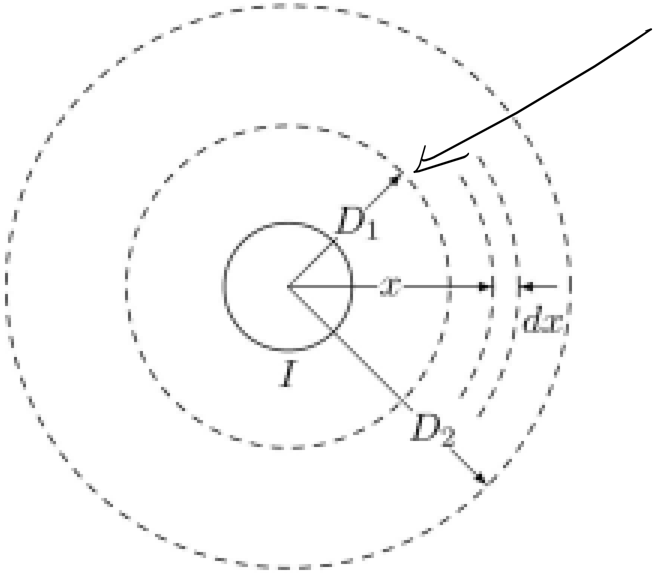
$$\boxed{L_{int} = \frac{\mu_0}{8\pi} = \frac{1}{2} \times 10^{-7} \quad \text{H/m}}$$

INTERNAL INDUCTANCE OF CONDUCTOR

Inductance of a Single Conductor (SOLID)

$$B_x = \frac{\mu_0 I}{2\pi x}$$

$$\lambda_{ext} = \frac{\mu_0 I}{2\pi} \int_{D_1}^{D_2} \frac{1}{x} dx$$



$$L_{ext} = 2\times10^{-7} \ln\frac{D_2}{D_1} \text{ H/m}$$

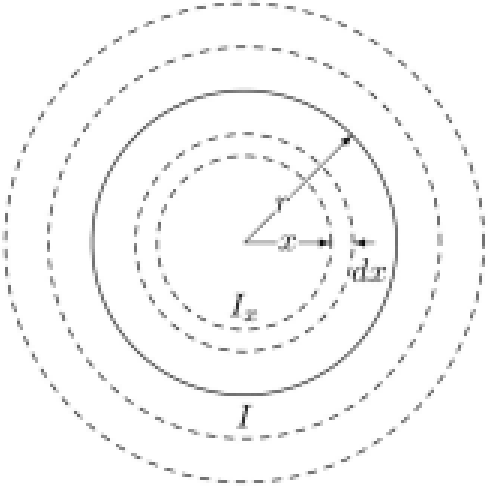
External Inductance of Conductor between 2 external points D1 and D2.

$$\boxed{L_{ext} = 2 \times 10^{-7}\, ln\frac{D}{r}}$$

D1 can be the radius of the conductor

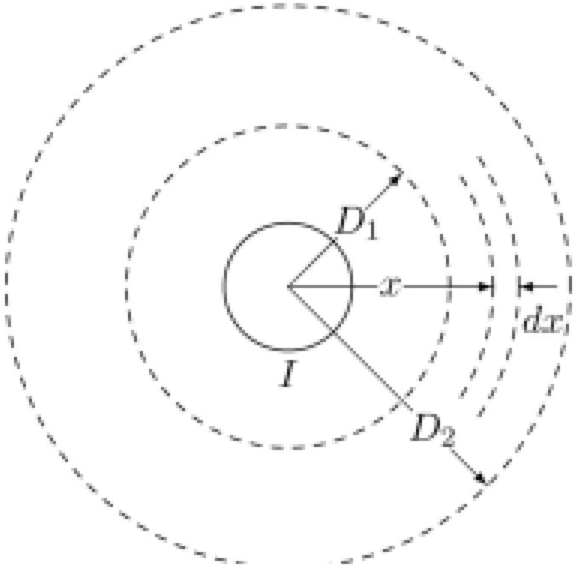
Inductance of a Single Conductor

$$L_{int} = \frac{\mu_0}{8\pi} = \frac{1}{2} \times 10^{-7} \text{ H/m}$$



D1 can be the radius of the conductor

$$L_{ext} = 2 \times 10^{-7} \ln \frac{D_2}{D_1} \text{ H/m}$$



INSIGHT – Inductance is affected by spacing of conductors

Using the identity $\frac{1}{2} = 2 \ln e^{1/4}$ in (4.4.18), a more convenient expression for λ_P is obtained:

$$\begin{aligned}\lambda_\mathrm{P} &= 2 \times 10^{-7} I \left(\ln e^{1/4} + \ln \frac{\mathrm{D}}{r} \right) \\ &= 2 \times 10^{-7} I \ln \frac{\mathrm{D}}{e^{-1/4} r} \\ &= 2 \times 10^{-7} I \ln \frac{\mathrm{D}}{r'} \quad \text{Wb-t/m}\end{aligned}$$

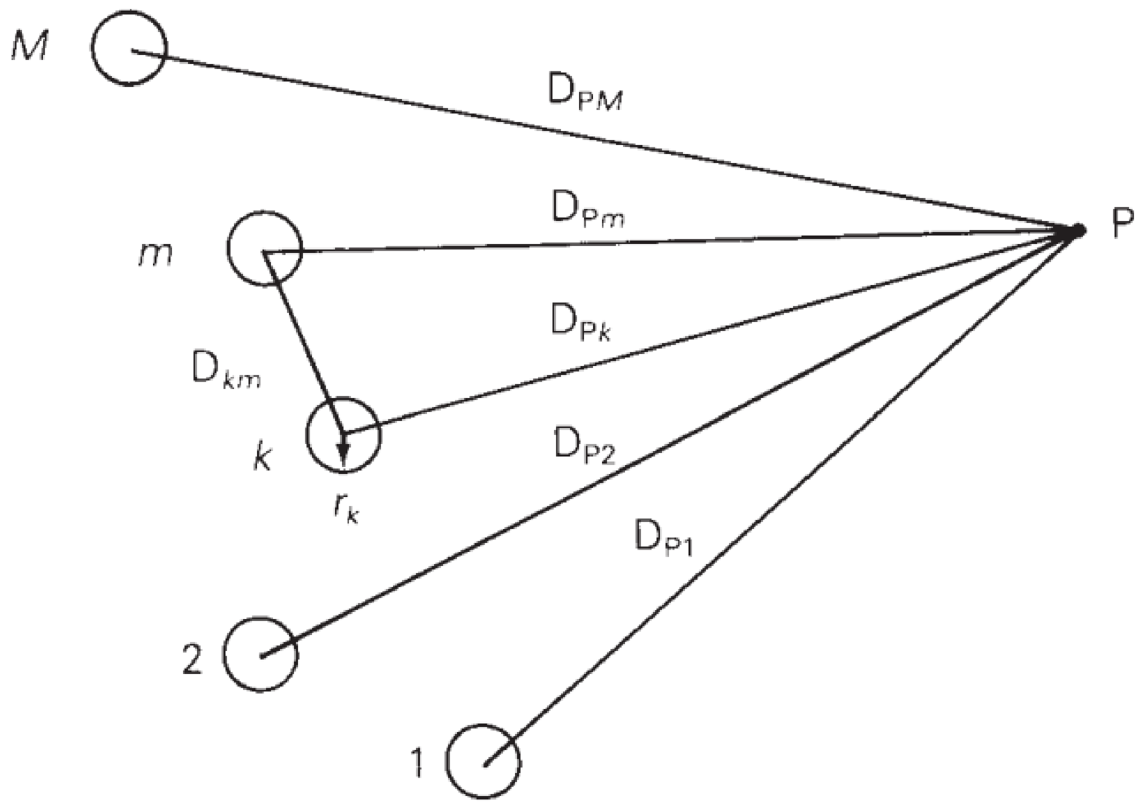
where

$$r' = e^{-1/4} r = 0.7788 r \tag{4.4.20}$$

Also, the total inductance $\mathrm{L_P}$ due to both internal and external flux linkages out to distance D is

$$\boxed{\mathrm{L_P} = \frac{\lambda_\mathrm{P}}{I} = 2 \times 10^{-7} \ln \left(\frac{\mathrm{D}}{r'} \right) \quad \mathrm{H/m}} \tag{4.4.21}$$

More Generally, Flux Linkage View for M Conductors



Assume that sum of conductor currents are zero.

$$I_1 + I_2 + \ldots + I_M = \sum I_M = 0$$

Flux Linkage λ_{kPk} which links conductor k out to point P due to current I_k

$$\lambda_{kPk} = 2 \times 10^{-7} \ln \frac{D_{Pk}}{r_k'}$$

Assuming that distances between conductors is substantially greater than the radius, and the distance is far away.

$$\lambda_k = 2 \times 10^{-7} \sum_{m=1}^{M} I_m \frac{1}{D_{km}}$$

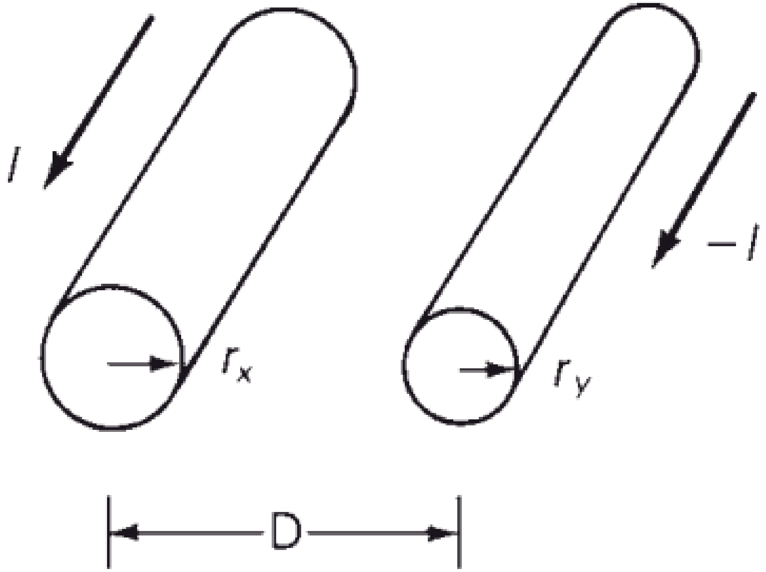
Where Dkm is distance between conductors.

SPECIFIC EXAMPLES OF TRANSMISSION LINES

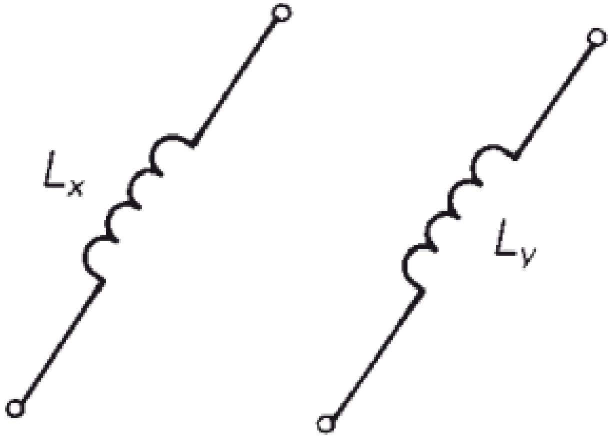
Examples

- Single Phase – Two Wire Line
- Three Phase – Three Wire Line Equally Space
- Three Phase – Three Wire Line Asymmetric Spacing

Inductance of Single Phase Lines



(a) Geometry



(b) Inductances

$$\lambda_x = 2 \times 10^{-7}\left(I_x \ln \frac{1}{D_{xx}} + I_y \ln \frac{1}{D_{xy}}\right)$$

$$= 2 \times 10^{-7}\left(I \ln \frac{1}{r'_x} - I \ln \frac{1}{D}\right)$$

$$= 2 \times 10^{-7} I \ln \frac{D}{r'_x} \quad \text{Wb-t/m}$$

where $r'_x = e^{-1/4} r_x = 0.7788 r_x$.

The inductance of conductor x is then

$$L_x = \frac{\lambda_x}{I_x} = \frac{\lambda_x}{I} = 2 \times 10^{-7} \ln \frac{D}{r'_x} \quad \text{H/m per condu}$$

Similarly, the total flux linking conductor y is

$$\lambda_y = 2 \times 10^{-7}\left(I_x \ln \frac{1}{D_{yx}} + I_y \ln \frac{1}{D_{yy}}\right)$$

$$= 2 \times 10^{-7}\left(I \ln \frac{1}{D} - I \ln \frac{1}{r'_y}\right)$$

$$= -2 \times 10^{-7} I \ln \frac{D}{r'_y}$$

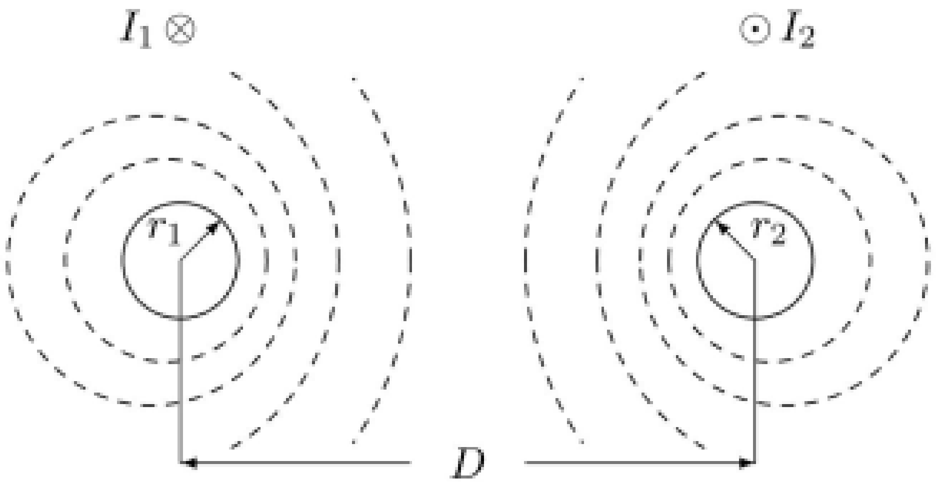
and

$$L_y = \frac{\lambda_y}{I_y} = \frac{\lambda_y}{-I} = 2 \times 10^{-7} \ln \frac{D}{r'_y} \quad \text{H/m per conductor}$$

Inductance of Single Phase Lines

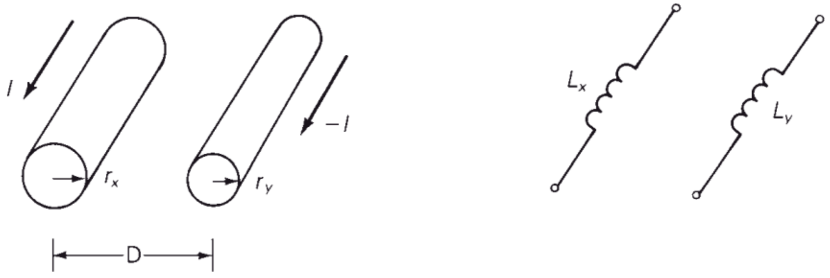
$$L = 2\times10^{-7}\ln\frac{1}{r'} + 2\times10^{-7}\ln\frac{D}{1}\,\text{H/m}$$

$$= 0.2\ln\frac{D}{D_s}\,\text{mH/km}$$



Ds = GMR or self Geometric Mean Distance

D = distance between the conductors



(a) Geometry (b) Inductances

OBSERVATIONS

1. Generally longer distance between conductors – Higher inductance
2. Larger Radius of the conductors – smaller inductance

Extending to Total Inductance also called the loop inductance

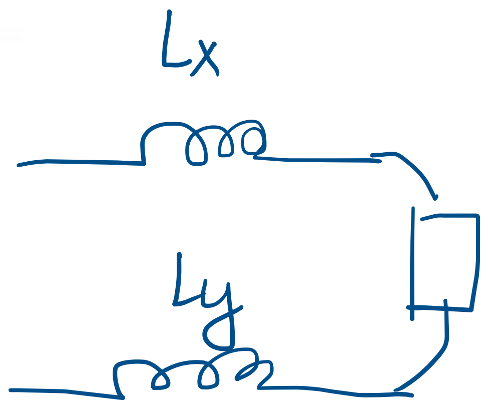
$$L = L_x + L_y = 2 \times 10^{-7}\left(\ln \frac{D}{r'_x} + \ln \frac{D}{r'_y}\right)$$

$$= 2 \times 10^{-7} \ln \frac{D^2}{r'_x r'_y}$$

$$= 4 \times 10^{-7} \ln \frac{D}{\sqrt{r'_x r'_y}} \quad \text{H/m per circuit}$$

Also, if $r'_x = r'_y = r'$, the total circuit inductance is

$$L = 4 \times 10^{-7} \ln \frac{D}{r'} \quad \text{H/m per circuit}$$



$$L = L_x + L_y$$

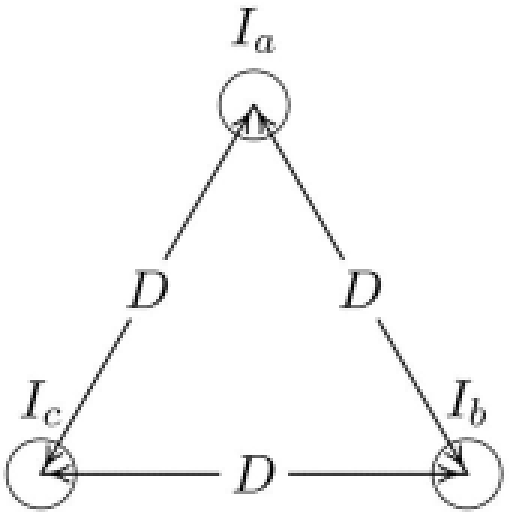
Inductance of Three-phase Transmission Lines

Symmetrical Spacing

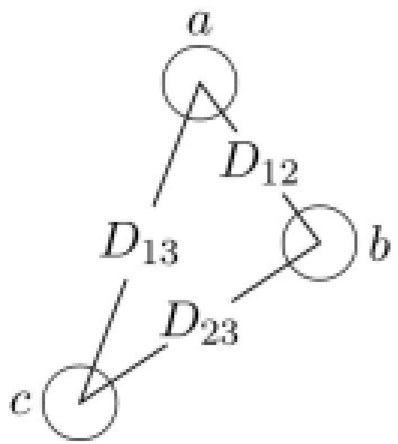
$$\lambda_a = 2\times10^{-7}\left(I_a \ln\frac{1}{r'} + I_b \ln\frac{1}{D} + I_c \ln\frac{1}{D} \right)$$

$$= 2\times10^{-7} I_a \ln\frac{D}{r'}$$

$$L = 0.2\ln\frac{D}{D_s}\ \text{mH/km}$$



Inductance of Three-phase Transmission Lines



Assymmetrical Spacing

$$\lambda_a = 2\times 10^{-7}\left(I_a \ln\frac{1}{r'} + I_b \ln\frac{1}{D_{12}} + I_c \ln\frac{1}{D_{13}} \right)$$

$$\lambda_b = 2\times 10^{-7}\left(I_a \ln\frac{1}{D_{12}} + I_b \ln\frac{1}{r'} + I_c \ln\frac{1}{D_{23}} \right)$$

$$\lambda_c = 2\times 10^{-7}\left(I_a \ln\frac{1}{D_{13}} + I_b \ln\frac{1}{D_{23}} + I_c \ln\frac{1}{r'} \right)$$

Inductance of Three-phase Transmission Lines

Assymmetrical Spacing

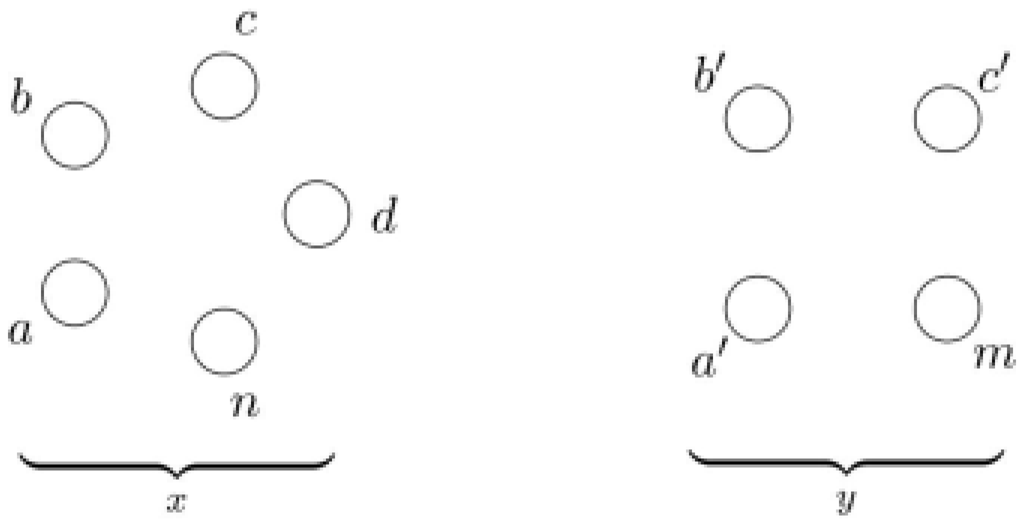
$$L_a = \frac{\lambda_a}{I_a} = 2\times 10^{-7}\left(\ln\frac{1}{r'} + a^2 \ln\frac{1}{D_{12}} + a \ln\frac{1}{D_{13}} \right)$$

$$L_b = \frac{\lambda_b}{I_b} = 2\times 10^{-7}\left(a \ln\frac{1}{D_{12}} + \ln\frac{1}{r'} + a^2 \ln\frac{1}{D_{23}} \right)$$

$$L_c = \frac{\lambda_c}{I_c} = 2\times 10^{-7}\left(a^2 \ln\frac{1}{D_{13}} + a \ln\frac{1}{D_{23}} + \ln\frac{1}{r'} \right)$$

Composite and Bundled Conductors

Inductance of Composite Conductors

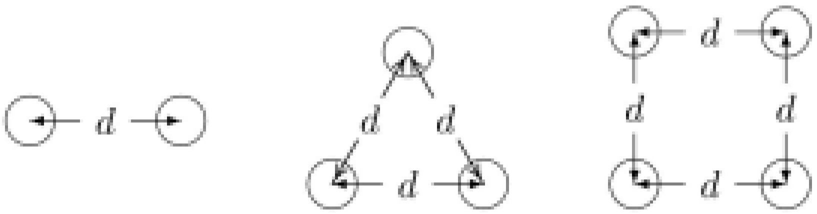


$$L_x = 2\times10^{-7}\ln\frac{GMD}{GMR_x} \qquad \text{H/m}$$

$$GMD = \sqrt[mn]{(D_{aa'}D_{ab'}\cdots D_{am})\cdots(D_{na'}D_{nb'}\cdots D_{nm})}$$

$$GMR_x = \sqrt[n^2]{(D_{aa}D_{ab}\cdots D_{an})\cdots(D_{na}D_{nb}\cdots D_{nn})} \quad D_{ii} = r_i'$$

GMR of Bundled Conductors



$$D_s^b = \sqrt[4]{(D_s \times d)^2}$$

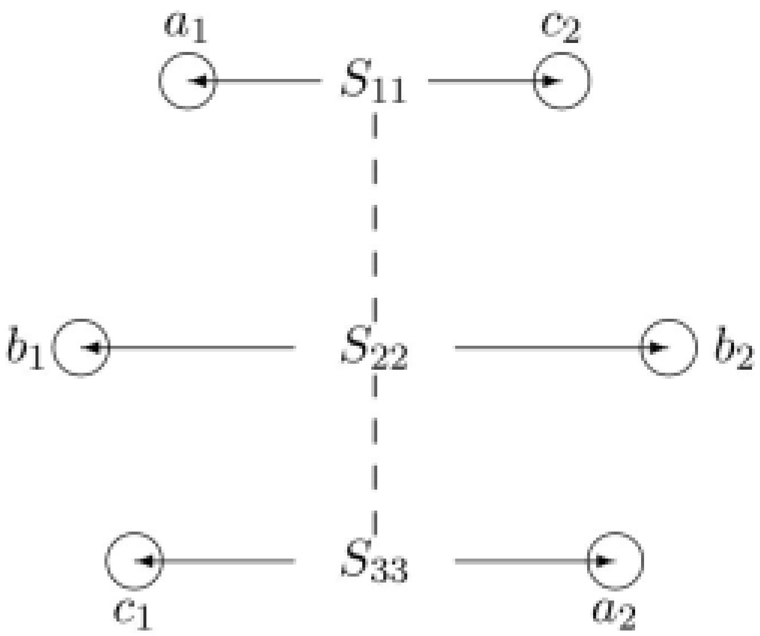
$$D_s^b = \sqrt[9]{\left(D_s \times d \times d\right)^3}$$

$$D_s^b = \sqrt[16]{\left(D_s \times d \times d \times \sqrt{2}d\right)^4}$$

Special Case

Inductance of Three-phase Double-circuit Lines

$$L = 2\times10^{-7}\ln\frac{GMD}{GMR_L} \quad \text{H/m}$$



$$GMD = \sqrt[3]{D_{AB}D_{BC}D_{CA}}$$

$$D_{AB} = \sqrt[4]{D_{a1b1}D_{a1b2}D_{a2b1}D_{a2b2}}$$

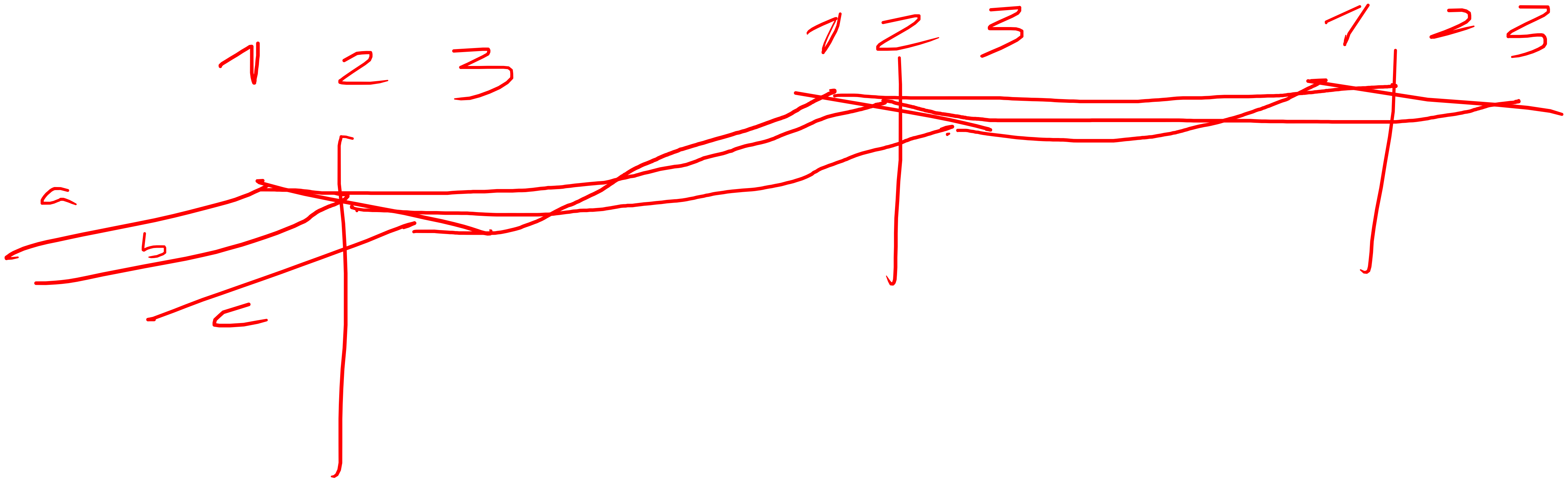
$$GMR_L = \sqrt[3]{D_{SA}D_{SB}D_{SC}}$$

$$D_{SA} = \sqrt[4]{(D_s D_{a1a2})^2}$$

AFTER EVERYTHING, THE EQUATION YOU ONLY NEED TO MEMORIZE IS

$$L_x = 2 \times 10^{-7} \ln \frac{GMD}{GMR_x} \; H/m$$

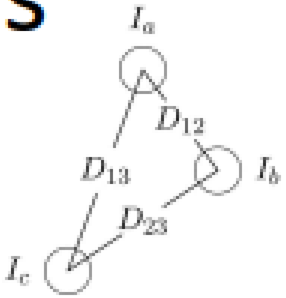
In the real world, configurations and conductors have their GMR and GMD specified by the Manufacturers.

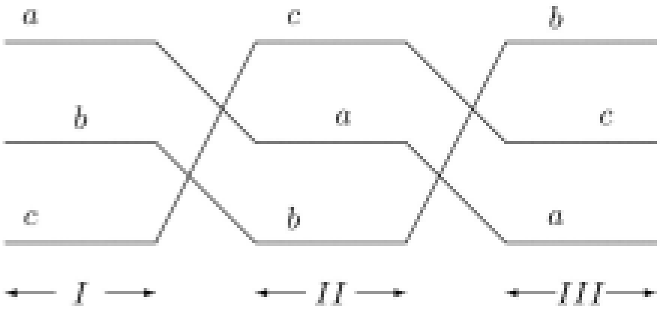


LINE TRANSPOSITION

"Fun isn't something one considers when balancing the universe. But this... does put a smile on my face."
—Thanos

Transposed Lines





TRANSPOSITION ALLOWS THIS FORMULA

$$L = \frac{L_a + L_b + L_c}{3}$$

$$L = \frac{2\times10^{-7}}{3}\left(3\ln\frac{1}{r'} - \ln\frac{1}{D_{12}} - \ln\frac{1}{D_{23}} - \ln\frac{1}{D_{13}}\right)$$

$$= 2\times10^{-7}\ln\frac{\left(D_{12}D_{23}D_{13}\right)^{1/3}}{r'}$$

$$= 0.2\ln\frac{GMD}{D_s}\ \text{mH/km}$$

Putting Things Together

END