EEE 51 Assignment 3 Answer Key

2nd Semester SY 2018-2019

1. **Single Stage Amplifier with Diode connected Load**

   (a) The amplifier configuration is a common source amplifier

   (b) From Figure 1, the diode connected PMOS M2's $|V_{GS_p}| = |V_{DS_p}|$

$$|V_{DS}| = |V_{GS_p}| = V_{DD} - V_{OUT} = 2.5V$$

   Using this, we can solve for the current $|I_{DS_p}| = I_{DS_n}$.

$$\begin{aligned}|I_{DS_p}| = |I_{DS_n}| &= k_p(|V_{GS}| - |V_{th_p}|)^2(1 + \lambda_p V_{DS_p}) \\ &\boxed{= 2.25mA}\end{aligned}$$

   With I_{DS} solved, $V_{BIAS} = V_{GS_n}$ can be solved using the equation

$$\begin{aligned}|I_{DS_n}| &= k_n(|V_{GS_n}| - |V_{th_n}|)^2(1 + \lambda_n V_{DS_n}) \\ V_{GS_n} = V_{BIAS} &= \sqrt{\frac{I_{DS_n}}{k_n(1 + \lambda_p V_{DS_n})}} + V_{th_n} \\ &\boxed{= 1.2477V}\end{aligned}$$

   (c) Small Signal Parameters

$$\begin{aligned}g_{m1} &= \frac{\partial I_{DS}}{\partial V_{GS_n}} \\ g_{m1} &= 2k_n(V_{GS_n} - V_{th_n})(1 + \lambda_n V_{DS_n}) \\ &\boxed{= 8.216mS}\end{aligned}$$

$$\begin{aligned}g_{o1} &= \frac{\partial I_{DS}}{\lambda_n V_{DS_n}} \\ g_{o1} &= \frac{1}{\lambda_n k_n (V_{GS_n} - V_{th_n})^2} \\ &\boxed{\frac{1}{g_{o1}} = r_{o1} = 5.556k\Omega}\end{aligned}$$

$$\begin{aligned}g_{m2} &= \frac{\partial I_{DS}}{\partial V_{GS_p}} \\ g_{m2} &= 2k_p(V_{GS_p} - V_{th_p})(1 + \lambda_p V_{DS_p}) \\ &\boxed{= 2.25mS}\end{aligned}$$

$$\begin{aligned}g_{o2} &= \frac{\partial I_{DS}}{\lambda_n V_{DS_p}} \\ g_{o2} &= \frac{1}{\lambda_p k_p (V_{GS_p} - V_{th_p})^2} \\ &\boxed{\frac{1}{g_{o1}} = r_{o1} = 10k\Omega}\end{aligned}$$

For the small signal circuit, notice that M1's drain is connected to M2's drain and M1's source is connected to M2's source. Also, M2 is diode connected so M2's gate is connected to M2's drain. The simplified small signal circuit is shown in Figure 1

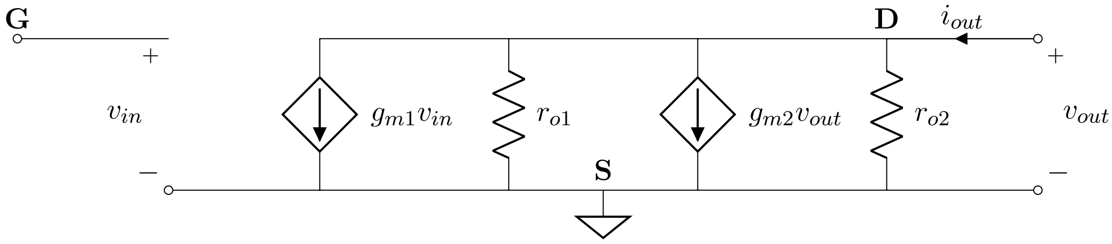

Figure 1: Small Signal Equivalent Circuit for Figure 1

(d) $R_{out} = \frac{v_{out}}{i_{out}}$ when v_{in}=0. Using KCL at the drain/v_{out} node

$$i_{out} = g_{m2}v_{out} + \frac{v_{out}}{r_{o2}} + \frac{v_{out}}{r_{o1}}$$

$$\boxed{R_{out} = \frac{v_{out}}{i_{out}} = \frac{1}{g_{m2} + \frac{1}{r_{o2}} + \frac{1}{r_{o1}}}}$$

$$\boxed{R_{out} = 395.229\Omega}$$

(e) $G_m = \frac{i_{out}}{v_{in}}$ when v_{out}=0. Since v_{out} is shorted, the expression for G_m is

$$i_{out} = g_{m1}v_{in} \boxed{G_m = \frac{i_{out}}{v_{in}} = g_{m1} = 8.216mS}$$

(f) For a common source amplifier, the small signal voltage gain is $a_v = \frac{v_{out}}{v_{in}} - G_m R_{out}$. Using answers in problems (d) and (e), the gain is

$$\boxed{a_v = -(8.216mS)(395.229\Omega) = -3.2472\frac{v}{v}}$$

2. **Single Stage Amplifier with Current Source Load**

(a) i. From Figure 2a the configuration type is a **Common Base Amplifier** with an external load resistance R_L

ii. The small signal equivalent circuit for Figure 2a is shown in Figure 2.

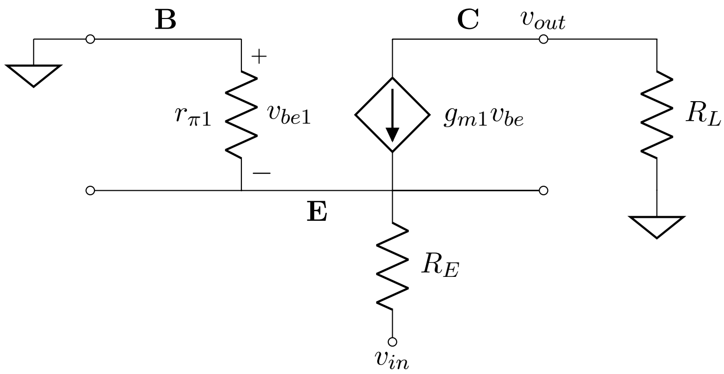

Figure 2: Small Signal Equivalent Circuit for Figure 2a

iii. KCL @ the Collector node:

$$g_{m1}v_{be1} + \frac{v_{out}}{R_L} = 0$$
$$v_{out} = -g_{m1}v_{be1}R_L$$

Nodal analysis @ the emitter node:

$$\frac{-v_{be1}}{r_{\pi 1}} - g_{m1}v_{be1} + \frac{-v_{be1} - v_{in}}{R_E} = 0$$
$$v_{be1}\left(\frac{1}{r_{\pi 1}} + g_{m1} + \frac{1}{R_E}\right) = \frac{-v_{in}}{R_E}$$

Since $r_{\pi 1} >> \frac{1}{g_{m1}}$, $(\frac{1}{r_{\pi 1}} + g_{m1}) \approx g_{m1}$.

$$v_{be1}\left(g_{m1} + \frac{1}{R_E}\right) = \frac{-v_{in}}{R_E}$$
$$v_{be1} = -v_{in}\left(\frac{\frac{1}{R_E}}{g_{m1} + \frac{1}{R_E}}\right)$$
$$v_{be1} = -v_{in}\left(\frac{1}{1 + g_{m1}R_E}\right)$$

Substituting v_{be1} to the v_{out} equation:

$$v_{out} = v_{in}\left(\frac{g_{m1}R_L}{1 + g_{m1}R_E}\right)$$
$$\boxed{A_{v1} = \frac{v_{out}}{v_{in}} = \frac{g_{m1}R_L}{1 + g_{m1}R_E}}$$

(b) i. It is still a Common Base Amplifier because $Q2$ acts as a load transistor current source. This is the practical implementation of a current source using transistors.

ii. The complete small signal equivalent circuit for Figure 2b is shown in Figure 3

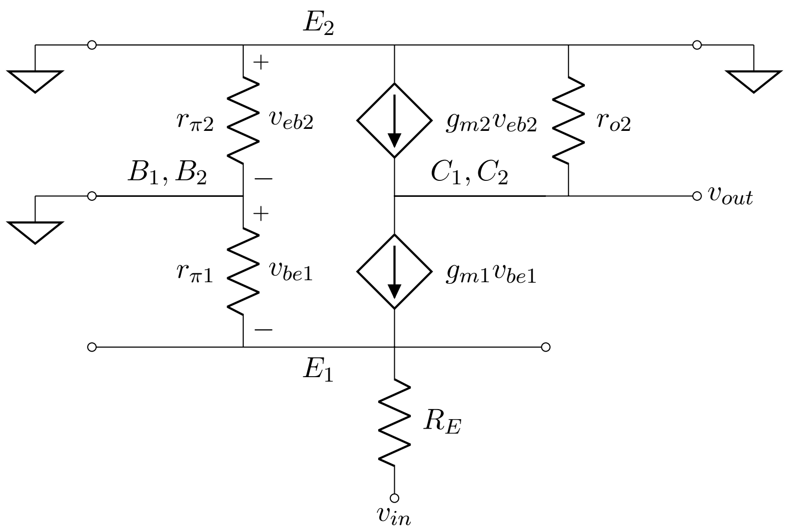

Figure 3: Complete Small Signal Equivalent Circuit for Figure 2b

Since $v_{eb2} = 0$, then $g_{m2}v_{eb2} = 0$, the simplified small signal equivalent circuit is shown in Figure 4.

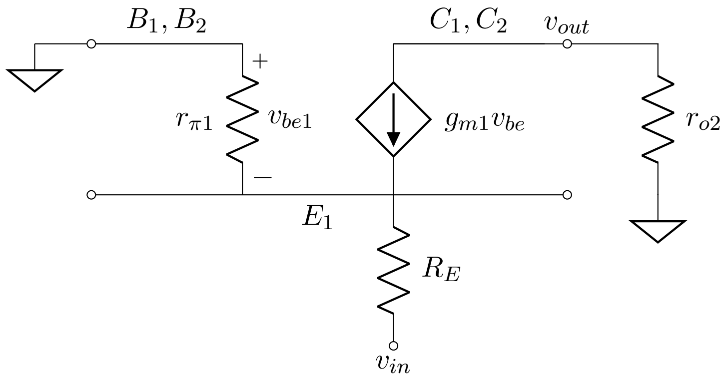

Figure 4: Simplified Small Signal Equivalent Circuit for Figure 2b

iii. The simplified small signal equivalent circuit (Figure 4) is similar to the small signal equivalent in Figure 2 and the only difference is the load resistor. Applying the same analysis from (a) iii. (KCL and Nodal Analysis) the resulting gain expression A_{v2} is:

$$\boxed{A_{v2} = \frac{v_{out}}{v_{in}} = \frac{g_{m1}r_{o2}}{1 + g_{m1}R_E}}$$

(c) The Givens are $V_{EE} = 8V$, $V_{OUT} = 3V$, $R_E = 80\Omega$, $V_{R_E} = 1.6V$, $V_{A_{Q2}} = 100V$, and $V_{A_{Q1}} \longrightarrow \infty$.

i. To solve for A_{v1}, we need the values for g_{m1}, R_L, and R_E. Using DC analysis from Figure 2a and the assumption that $I_{C1} = I_{E1}$:

$$I_{C1} = I_{E1} = \frac{V_{R_E}}{R_E} = \frac{1.6V}{80\Omega}$$
$$I_{C1} = 20mA$$

We can solve for R_L:

$$R_L = \frac{V_{EE} - V_{OUT}}{I_{C1}} = \frac{8V - 3V}{20mA}$$
$$R_L = 250\Omega$$

Solving for Small Signal Parameters:

The $Q1$ transconductance (g_{m1}):

$$g_{m1} = \frac{I_{C1}}{V_T} = \frac{20mA}{26mV}$$
$$g_{m1} = 769.23mS$$

We can now solve for A_{v1}:

$$A_{v1} = \frac{g_{m1}R_L}{1 + g_{m1}R_E} = \frac{(769.23mS)(250\Omega)}{1 + (769.23mS)(80\Omega)}$$
$$\boxed{A_{v1} = 3.08V/V}$$

To solve for A_{v2}, we need the values for g_{m1}, r_{o2}, and R_E. Using DC analysis from figure 2b and the assumption that $I_{C2} = I_{E2} = I_{C1} = I_{E1}$:

$$I_{C2} = I_{E2} = I_{C1} = I_{E1} = \frac{V_{R_E}}{R_E} = \frac{1.6V}{80\Omega}$$
$$I_{C1} = 20mA$$

Solving for Small Signal Parameters:

The transconductance (g_{m1}) for $Q1$ is equal to the previous g_{m1} for A_{v1} because of the same bias current which is $20mA$ so $g_{m1} = 769.23mS$ for A_{v2}.

We can solve for r_{o2}:

$$r_{o2} = \frac{V_{A_{Q2}}}{I_{C1}} = \frac{100V}{20mA}$$
$$r_{o2} = 5k\Omega$$

We can now solve for A_{v2}:

$$A_{v2} = \frac{g_{m1} r_{o2}}{1 + g_{m1} R_E} = \frac{(769.23mS)(5k\Omega)}{1 + (769.23mS)(80\Omega)}$$
$$\boxed{A_{v2} = 61.50V/V}$$

ii. Percentage increase/decrease between the two gains relative to A_{v1}:

$$\% = \left|\frac{A_{v2} - A_{v1}}{A_{v2}}\right| \cdot 100 = \left|\frac{61.50 - 3.08}{61.50}\right| \cdot 100$$
$$\boxed{\% = 95\%}$$

This concludes that there is a 95% **increase in unloaded open loop gain** when a resistive load was replaced by a transistor current source load.