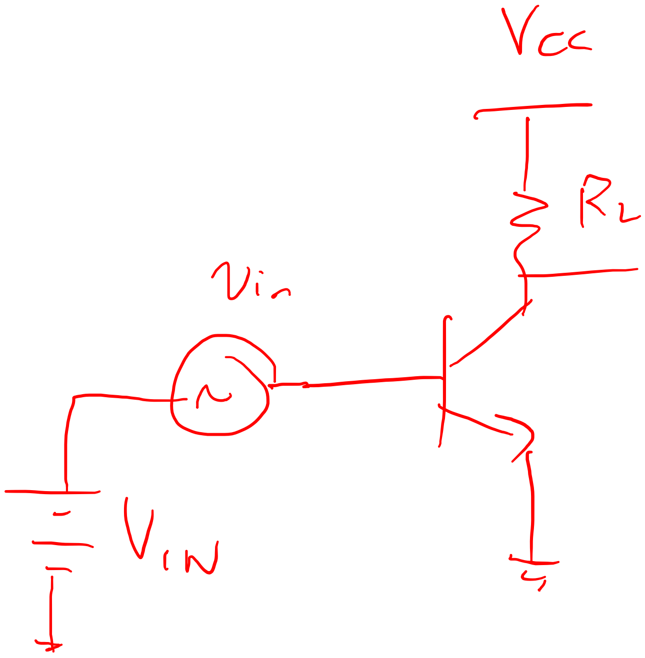
EEE 51: Second Semester 2017 - 2018

Lecture 4

Single-Stage Amplifiers

Today

- Single-Stage Amplifiers

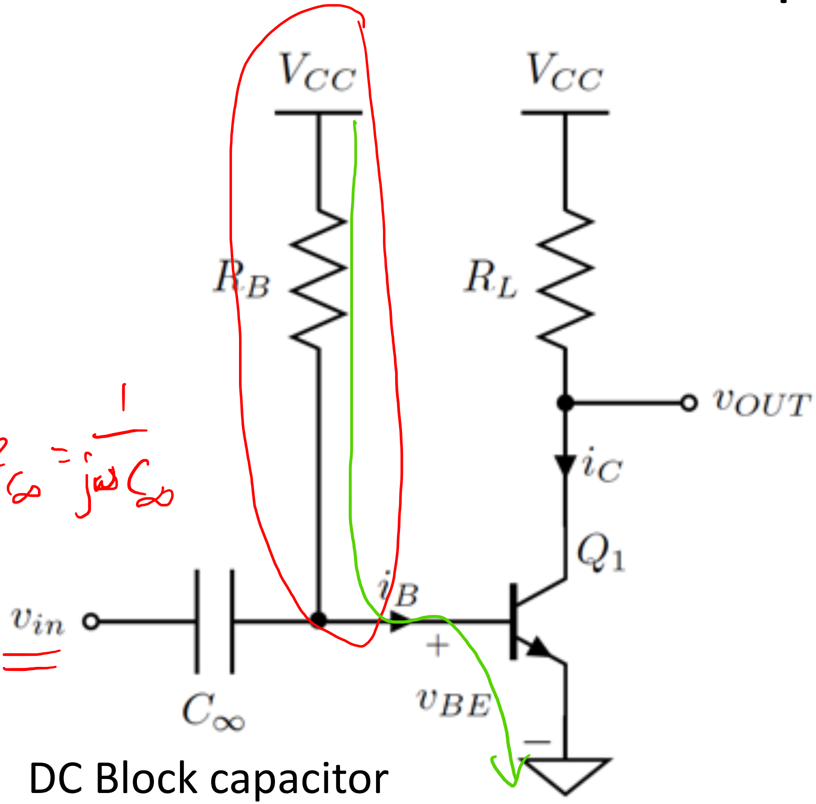


- requires 2 DC sources

A More Practical Common-Emitter Bias Strategy

- The Fixed-Bias CE Amplifier → only 1 DC source

$Z_{C_\infty} = \frac{1}{j\omega C_\infty}$



DC Block capacitor

$I_C = I_S e^{V_{BE}/V_T}$

$V_{BE} = V_T \ln\left(\frac{I_C}{I_S}\right)$

KVL at the input loop:

$$V_{CC} - I_{B,Q} R_B - V_{BE,Q} = 0$$

$$V_{CC} - \frac{I_{C,Q}}{\beta} R_B - V_T \ln\left(\frac{I_{C,Q}}{I_S}\right) = 0$$

Non-linear! How do we solve this?

- Graphical
- Numerical / iterative
  - put those EEE 11/13 skills to good use ☺

Iterative Solution

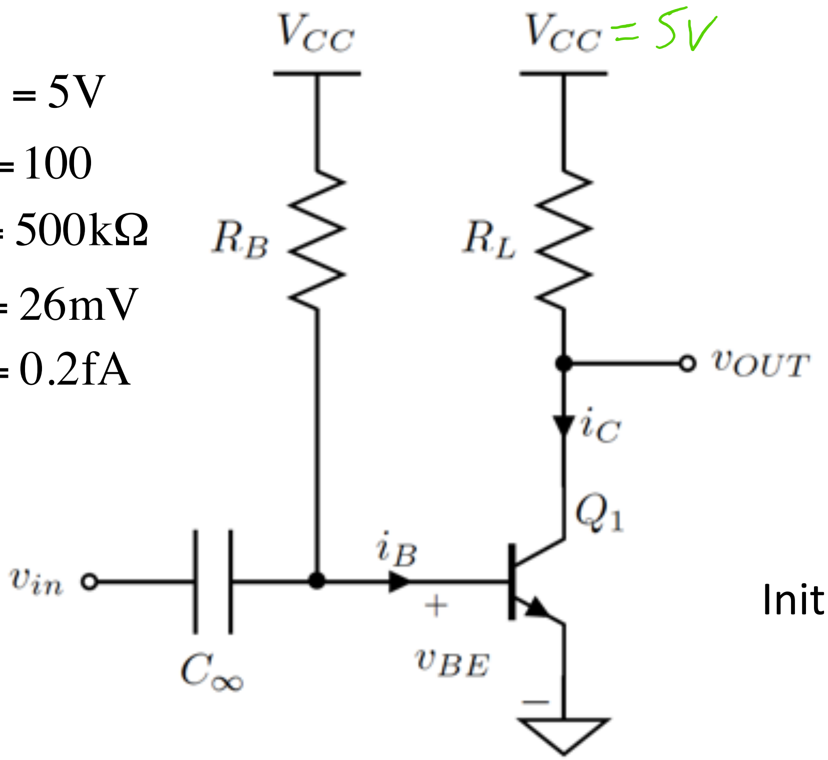
$V_{CC} = 5\text{V}$

$\beta = 100$

$R_B = 500\text{k}\Omega$

$V_T = 26\text{mV}$

$I_S = 0.2\text{fA}$



$$V_{CC} - \frac{I_{C,Q}^{A}}{\beta} R_B - V_T \ln\left(\frac{I_{C,Q}^{B}}{I_S}\right) = 0$$

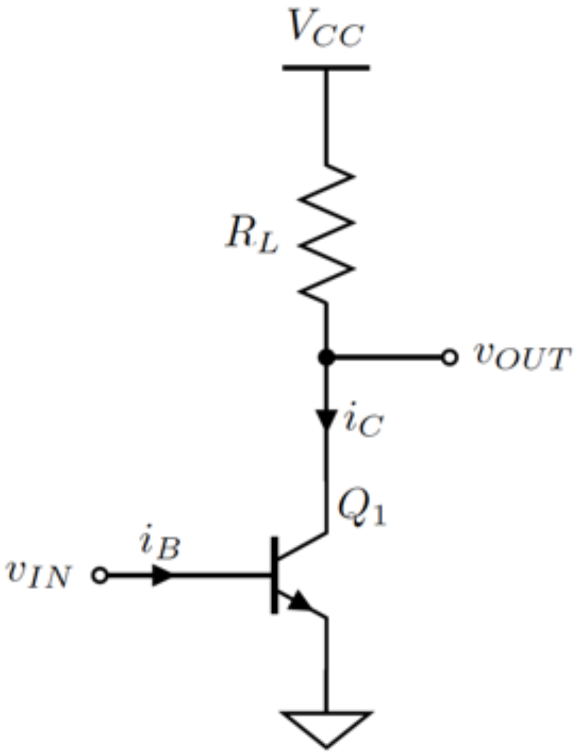
Rewrite:

$$I_{C,Q}^{A} = \frac{\beta}{R_B}\left[V_{CC} - V_T \ln\left(\frac{I_{C,Q}^{B}}{I_S}\right)\right]$$

	$I_{C,Q}^{B}$	$I_{C,Q}^{A}$
Initial guess	1mA	0.8479mA
	0.8479mA	0.8488mA
	0.8488mA	0.8488mA

$\boxed{I_{C,Q} = 848.8\mu\text{A}}$

Recall: Basic Common-Emitter Amplifier



	V_{IN} [mV]	$I_{C,Q}$ [mA]	V_{OUT} [V]	A_v $\left[\frac{\text{V}}{\text{V}}\right]$
Point A	672.5	1	4.5	−21.7
Point B	709.5	5	2.5	−108.7
Point C	718.9	7.5	1.25	−163.0

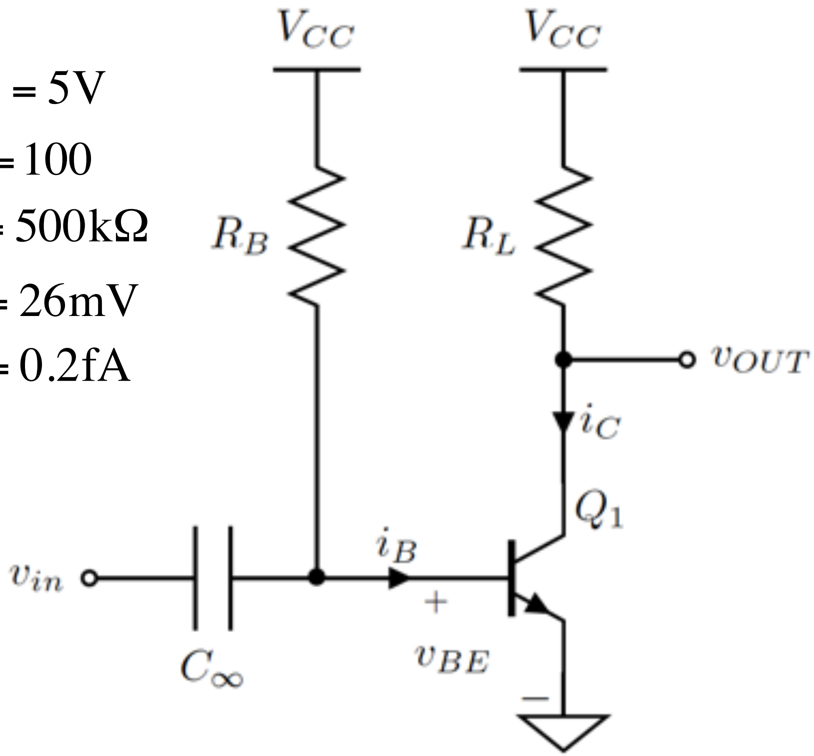
For a wide range of currents,

$$V_{BE} \approx 0.7\text{V}$$

What if we use this approximation?

Fixed-Bias Common-Emitter Amplifier Bias

$$V_{CC} = 5\text{V}$$
$$\beta = 100$$
$$R_B = 500\text{k}\Omega$$
$$V_T = 26\text{mV}$$
$$I_S = 0.2\text{fA}$$



$$V_{CC} - \frac{I_{C,Q}}{\beta} R_B - \underbrace{V_T \ln\left(\frac{I_{C,Q}}{I_S}\right)}_{V_{BE}} = 0$$

$$V_{CC} - \frac{I_{C,Q}}{\beta} R_B - 0.7\text{V} = 0$$

Thus,

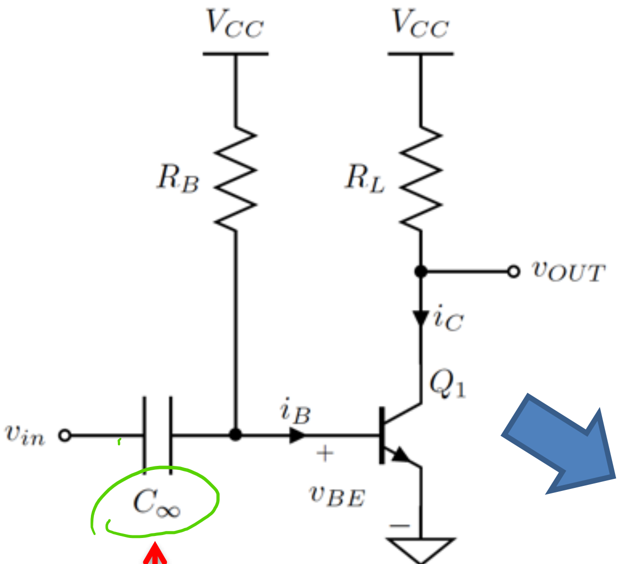
$$I_{C,Q} = \beta \cdot \frac{V_{CC} - 0.7\text{V}}{R_B}$$
$$= 860\mu\text{A}$$

Iterative solution:

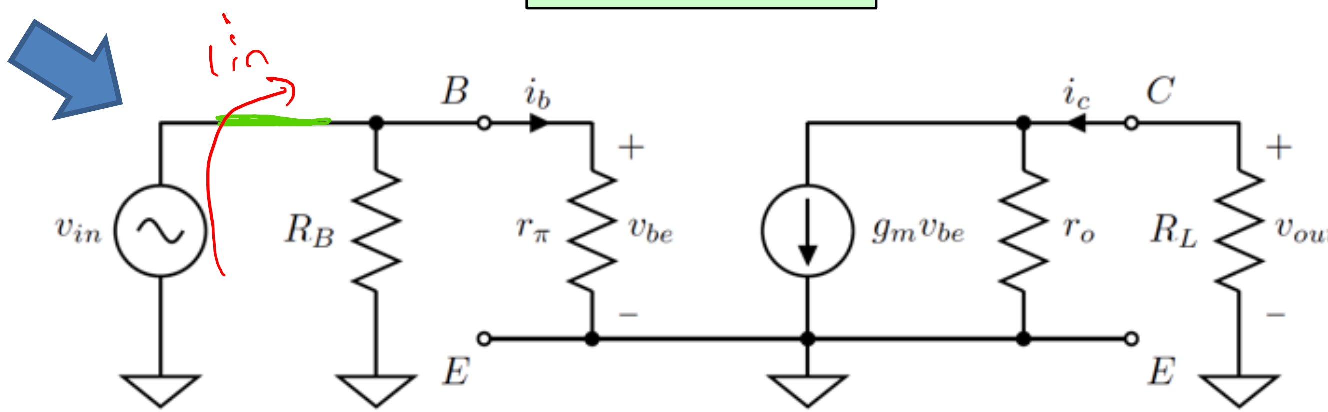
$$I_{C,Q} = 848.8\mu\text{A}$$ (error less than 2%)

Is this approximation good enough? → It depends on the application!

Small Signal Model



Open: at DC
Short: everywhere else



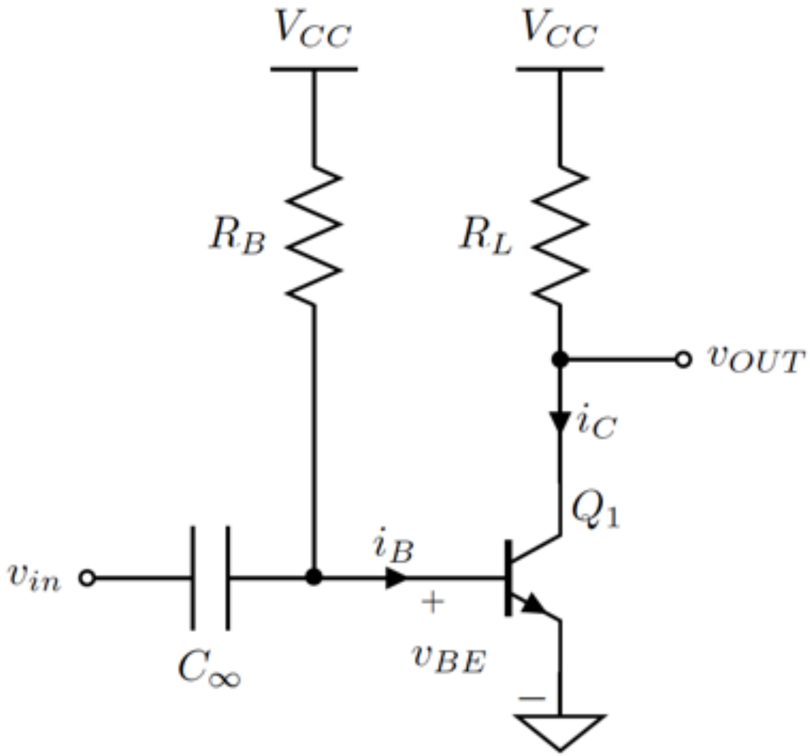
$$G_m = g_m$$
$$R_i = r_\pi \parallel R_B$$
$$R_o = r_o \parallel R_L$$
$$A_v = -G_m R_o$$
$$= -g_m \left(r_o \parallel R_L\right)$$

Input resistance changes

Common-Emitter → refers to the small signal model

Fixed-Bias Limitations

- β-variations



- Due to manufacturing imperfections

$$\beta = \beta_{\text{nominal}} \pm 50\%$$

- β doubles for every 80C° rise in temp

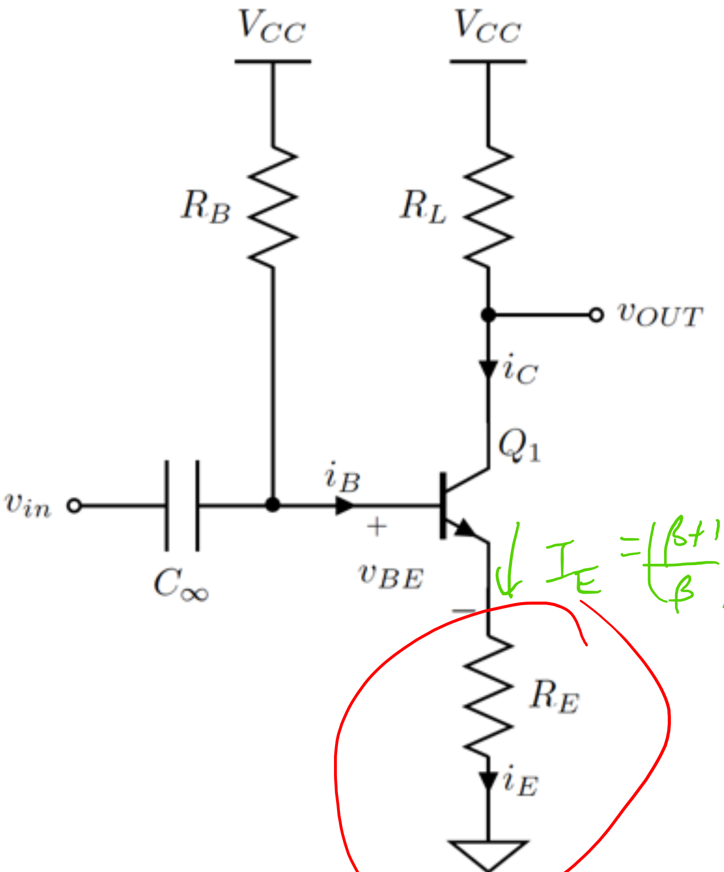
$V_{CC} - I_B R_B - V_{BE} = \emptyset$

Recall: $I_{C,Q} = \beta \cdot \dfrac{V_{CC} - 0.7\text{V}}{R_B}$

$I_{C,Q}$ can vary by a lot → due to β variations!

Can we do better than this?

Emitter-Degenerated Common-Emitter Amplifier



$I_E = \left(\frac{\beta+1}{\beta}\right) I_C$

KVL at the input loop:

$$V_{CC} - I_{B,Q}R_B - V_{BE,Q} - I_{E,Q}R_E = 0$$

$$V_{CC} - \frac{I_{C,Q}}{\beta}R_B - 0.7\text{V} - I_{C,Q}\left(1+\frac{1}{\beta}\right)R_E = 0$$

Solving for the collector current:

$$I_{C,Q} = \beta\frac{V_{CC} - 0.7\text{V}}{R_B + (\beta+1)R_E}$$

Is this bias scheme better?

For β → ∞:

$$I_{C,Q} \approx \frac{V_{CC} - 0.7\text{V}}{R_E}$$

Independent of β

Formalizing Parameter Effects

Define Sensitivity of X to Y as $S_Y^X = \frac{\partial X}{\partial Y}$ ⇨ $\Delta X = S_Y^X \cdot \Delta Y = \frac{\partial X}{\partial Y} \cdot \Delta Y$

Fixed-Bias

$$I_{C,Q} = \beta \cdot \frac{V_{CC} - 0.7\text{V}}{R_B}$$

$$S_\beta^{I_{C,Q}} = \frac{\partial I_{C,Q}}{\partial \beta} = \frac{\partial}{\partial \beta}\left(\beta \cdot \frac{V_{CC} - 0.7\text{V}}{R_B}\right)$$

$$= \frac{V_{CC} - 0.7\text{V}}{R_B}$$

Constant sensitivity

Emitter-Degeneration

$$I_{C,Q} = \beta \frac{V_{CC} - 0.7\text{V}}{R_B + (\beta + 1) R_E}$$

$$S_\beta^{I_{C,Q}} = \frac{\partial I_{C,Q}}{\partial \beta} = \frac{\partial}{\partial \beta}\left(\beta \frac{V_{CC} - 0.7\text{V}}{R_B + (\beta + 1) R_E}\right)$$

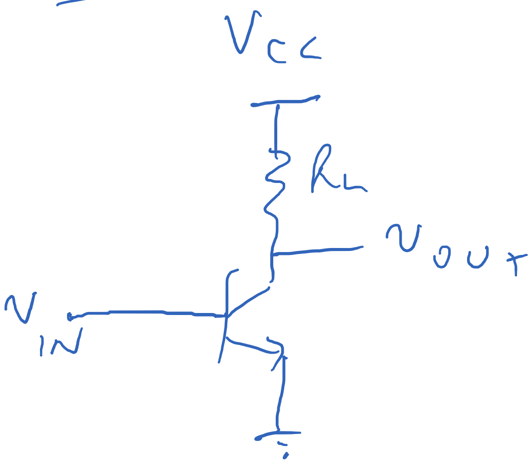
$$= \frac{V_{CC} - 0.7\text{V}}{R_B} \cdot \frac{1 + \frac{R_E}{R_B}}{\left(1 + (\beta + 1)\frac{R_E}{R_B}\right)^2}$$

< 1

Decreasing sensitivity as β increases

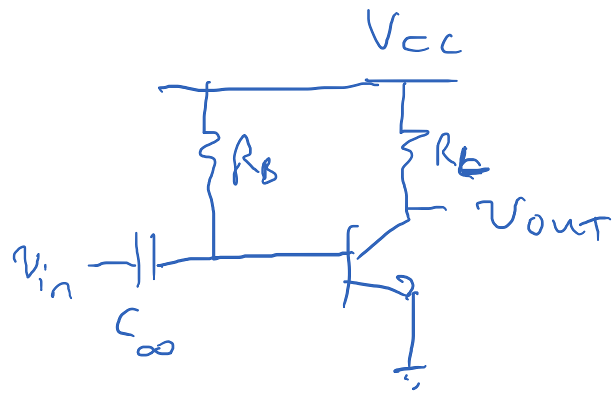
Common Emitter

BASIC



× uses 2 DC sources
– impractical

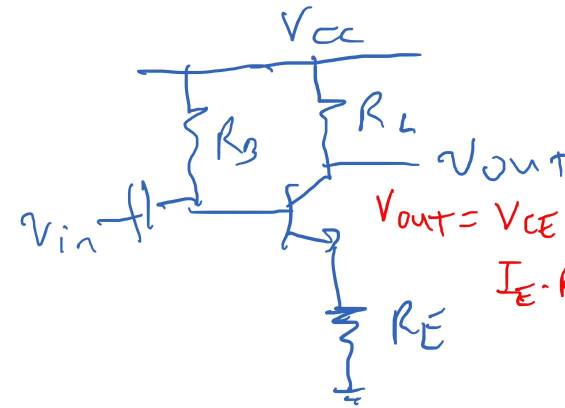
FIXED-BIAS



✓ uses only 1 DC source

× sensitive to β variations
→ unstable DC operating point

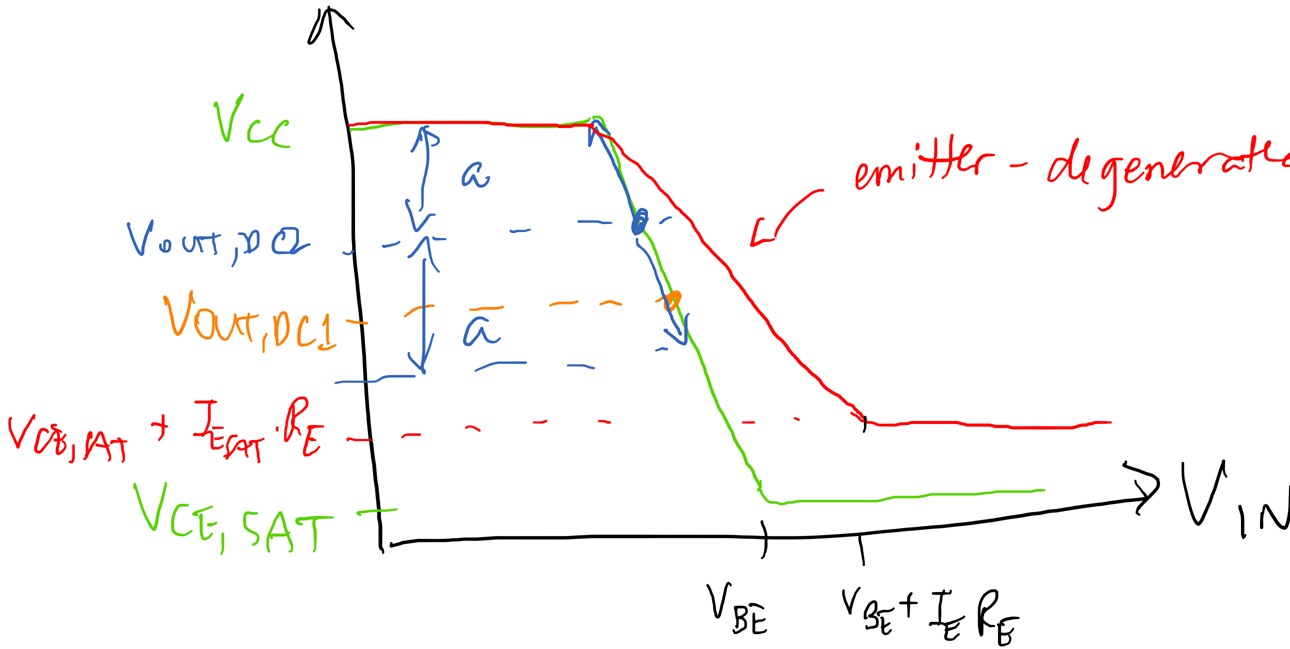
EMITTER-DEGENERATED



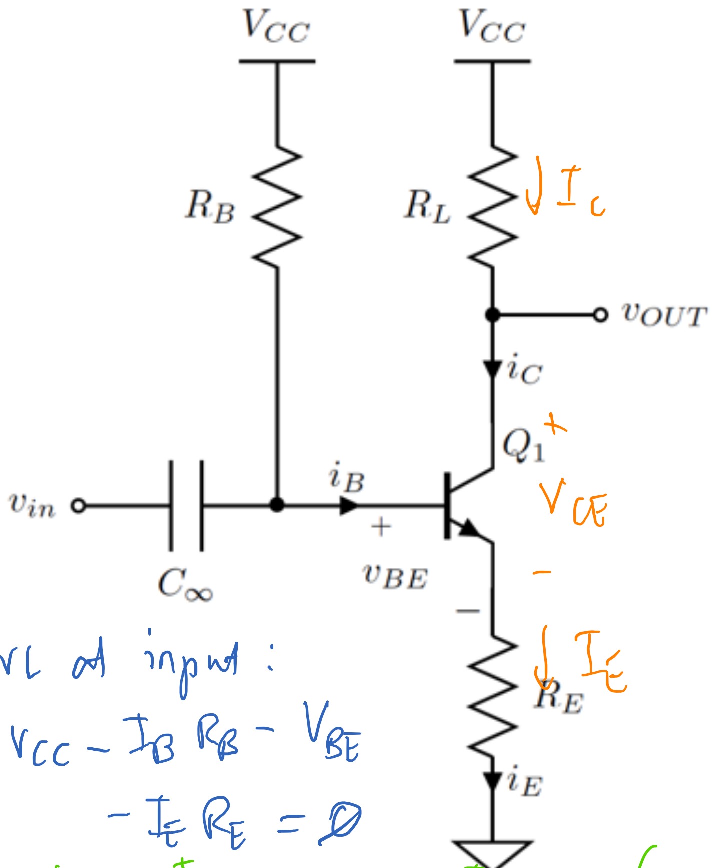
$V_{out} = V_{CE} + I_E \cdot R_E$

✓ 1 DC source
✓ less sensitive to β variations
× lower output swing

for maximum output swing, $V_{OUT,DC1} = \dfrac{V_{CC} + V_{CE,SAT}}{2}$



DC Effects of R_E



$I_E = I_C + I_B = I_C + \frac{I_C}{\beta}$

Output KVL:

$$V_{CC} - I_{C,Q}R_L - V_{CE,Q} - I_{E,Q}R_E = 0$$

$$V_{CC} - I_{C,Q}R_L - V_{CE,Q} - I_{C,Q}\left(1+\frac{1}{\beta}\right)R_E = 0$$

To keep Q_1 in the forward-active region:

$$\rightarrow \quad V_{CE,Q} > V_{CE,sat}$$

Thus, $V_{CE,sat} < V_{CC} - I_{C,Q}R_L - I_{C,Q}\left(1+\frac{1}{\beta}\right)R_E \quad \leftarrow$

$$\boxed{R_E < \frac{V_{CC} - I_{C,Q}R_L - V_{CE,sat}}{I_{C,Q}\left(1+\frac{1}{\beta}\right)}}$$

$V_{OUT} = ?$

$V_{CE} + I_E \cdot R_E$

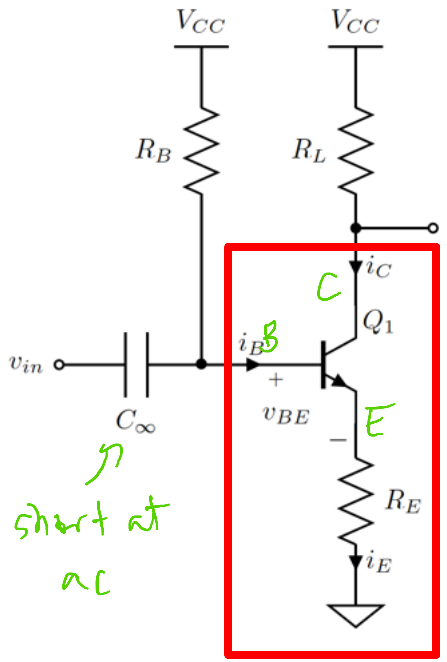
KVL at input:

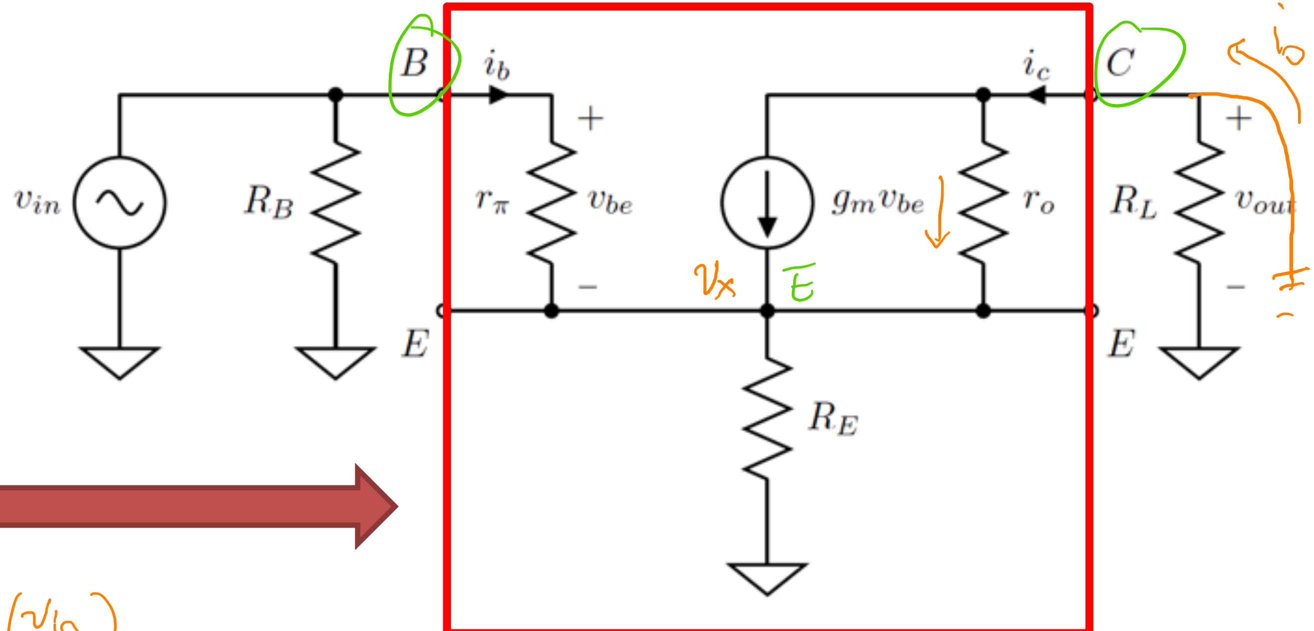
$V_{CC} - I_B R_B - V_{BE} - I_E R_E = 0$

$V_{CC} - \frac{I_C}{\beta} R_B - V_T \ln(I_C/I_S) - \left(1+\frac{1}{\beta}\right) I_C R_E = 0$

Small Signal Equivalent Circuit

$v_{be} = v_{in} - v_x$





KCL at v_x → $v_x = f(v_{in})$

KCL at C

$i_o = f(v_x, v_{in})$

$G_m = \frac{i_o}{v_{in}} \Big|_{v_o = 0}$

Let's look at the degenerated transistor first

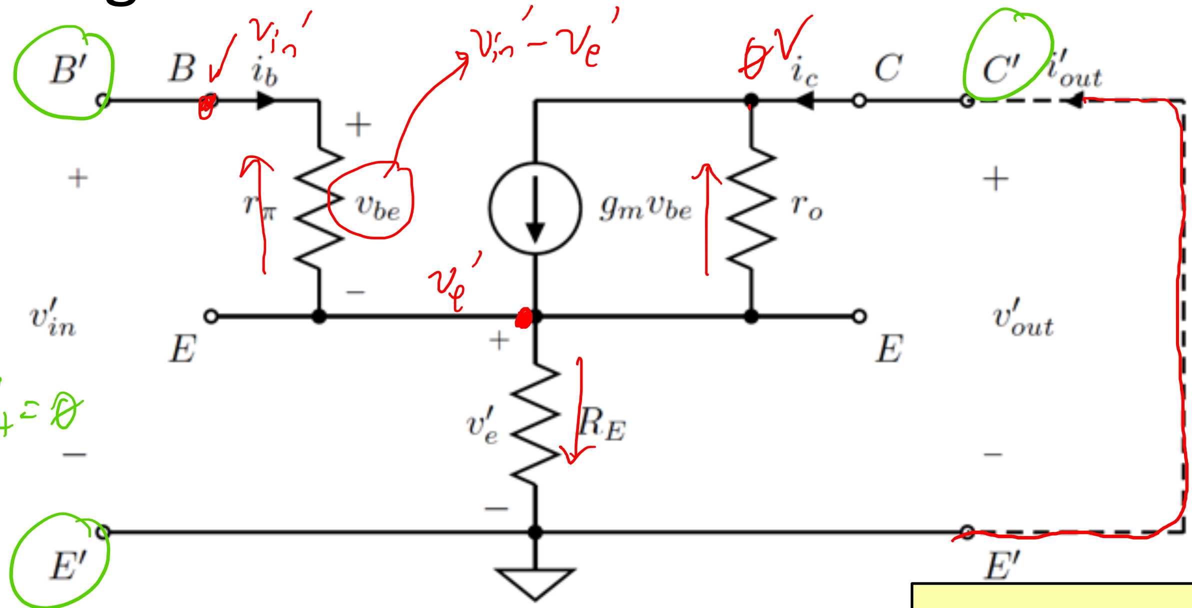
→ Derive the 2-port equivalent circuit

The Emitter-Degenerated Transistor (1)

- Calculating the transconductance

$$G_m' = \frac{i_{out}'}{v_{in}'}$$

$v_{out}' = 0$



no load

KCL at the emitter node:

$$\frac{v_e' - v_{in}'}{r_\pi} + \frac{v_e'}{R_E} + \frac{v_e'}{r_o} - g_m\left(v_{in}' - v_e'\right) = 0$$

$$v_e' = v_{in}' \cdot \frac{g_m + \frac{1}{r_\pi}}{g_m + \frac{1}{r_\pi} + \frac{1}{r_o} + \frac{1}{R_E}}$$

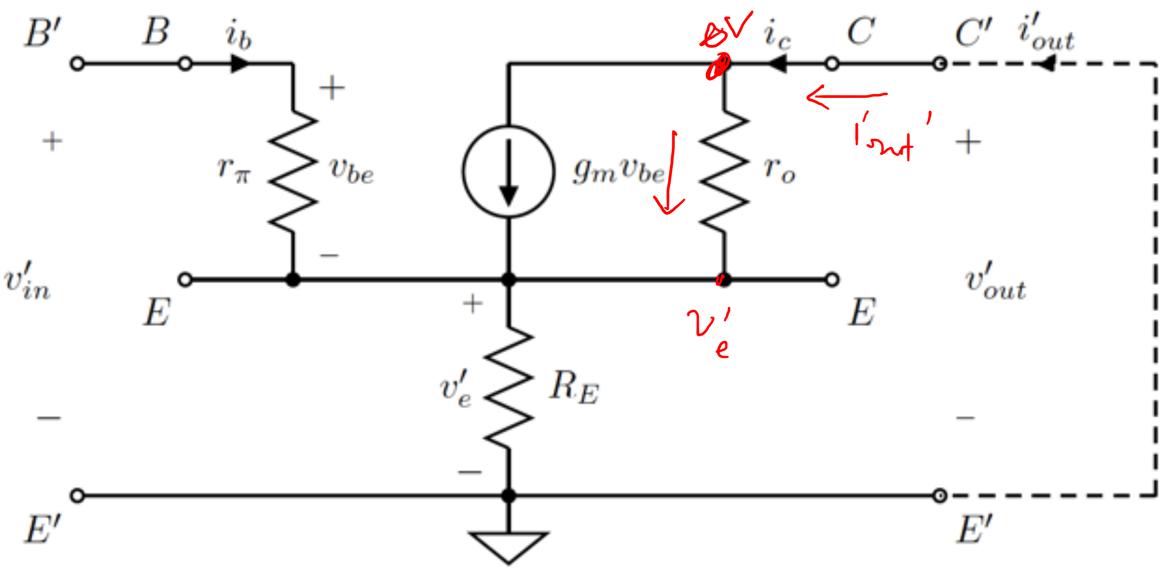
The Emitter-Degenerated Transistor (2)

KCL at the collector:

$$i'_{out} = g_m\left(v'_{in} - v'_e\right) - \frac{v'_e}{r_o}$$

Recall:

$$v'_e = v'_{in} \cdot \frac{g_m + \frac{1}{r_\pi}}{g_m + \frac{1}{r_\pi} + \frac{1}{r_o} + \frac{1}{R_E}}$$



$$i'_{out} = v'_{in} \cdot g_m \cdot \frac{\frac{1}{R_E} - \frac{1}{g_m r_o r_\pi}}{g_m + \frac{1}{r_\pi} + \frac{1}{r_o} + \frac{1}{R_E}}$$

$$G'_m = \frac{i'_{out}}{v'_{in}} = g_m \cdot \frac{\frac{1}{R_E} - \frac{1}{g_m r_o r_\pi}}{g_m + \frac{1}{r_\pi} + \frac{1}{r_o} + \frac{1}{R_E}}$$

The Emitter-Degenerated Transistor (3)

intrinsic gain

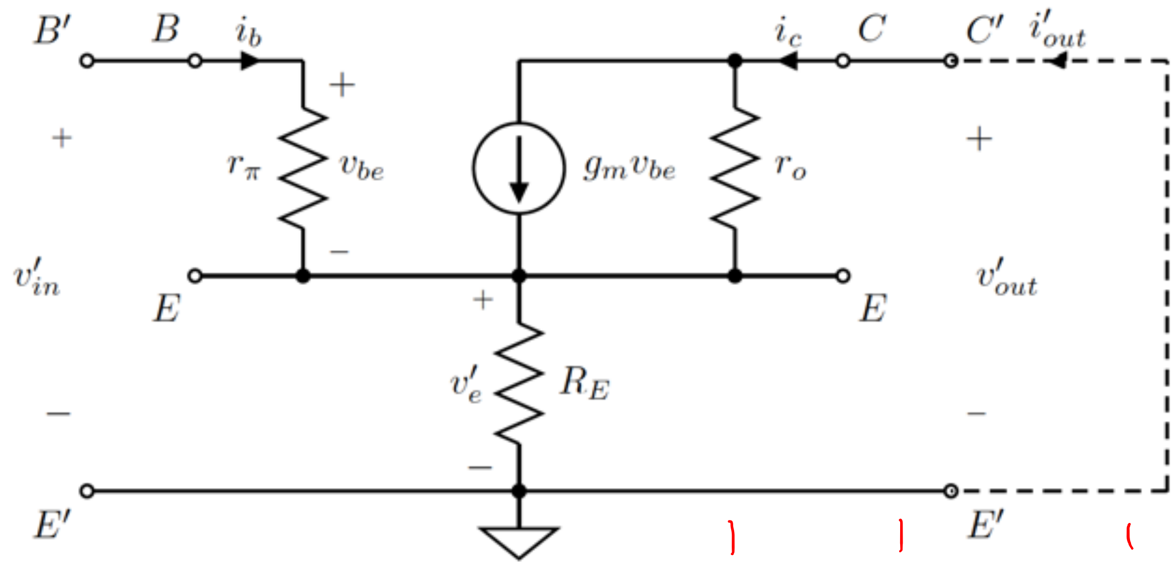
Assume:

$g_m r_o >> 1$

$g_m r_\pi = \beta >> 1$

$r_o >> R_E$

$r_\pi >> R_E$



$r_\pi = \frac{\beta}{g_m}$

$$G'_m = \frac{i'_{out}}{v'_{in}} = g_m \cdot \frac{\frac{1}{R_E} - \frac{1}{g_m r_o r_\pi}}{g_m + \frac{1}{r_\pi} + \frac{1}{r_o} + \frac{1}{R_E}} \approx \frac{g_m}{1 + g_m R_E} < g_m$$

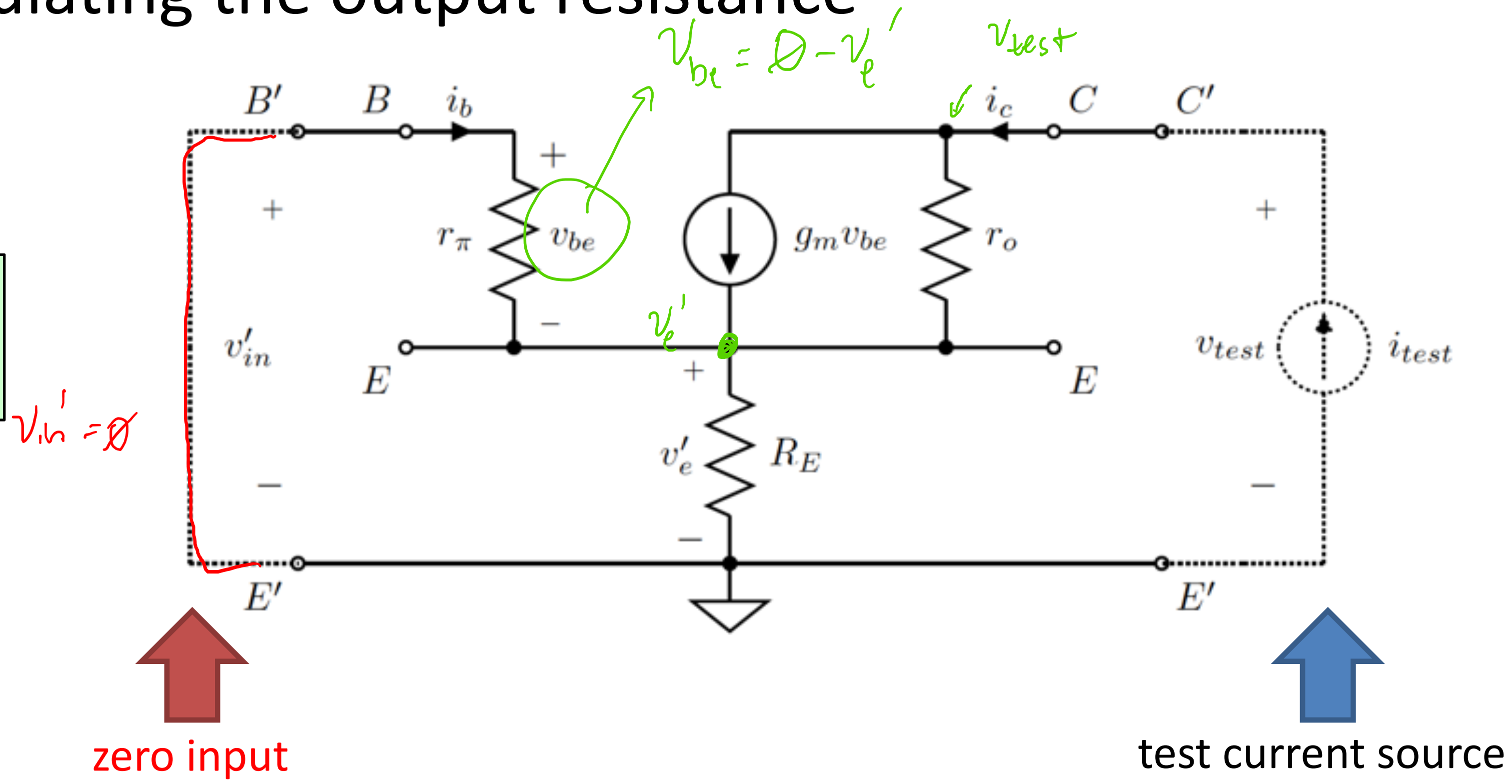
$\frac{1}{R_E} - \frac{1}{\beta \cdot r_o} \approx \frac{1}{R_E}$

$\approx g_m + \frac{1}{R_E} \approx \frac{g_m R_E + 1}{R_E}$

The transconductance is degenerated by R_E

- Calculating the output resistance

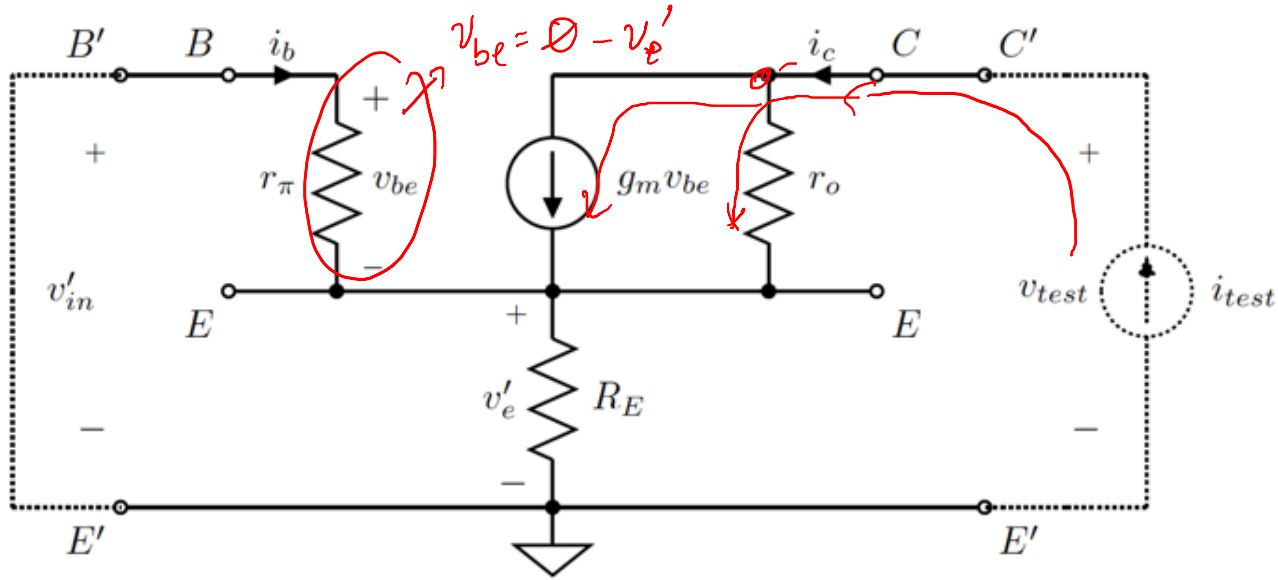
$$R_o' = \frac{v_{test}'}{i_{test}}$$



The Emitter-Degenerated Transistor (5)

By inspection:

$$\boxed{v_e' = i_{test} \cdot \left(r_\pi \parallel R_E \right) = -v_{be}}$$



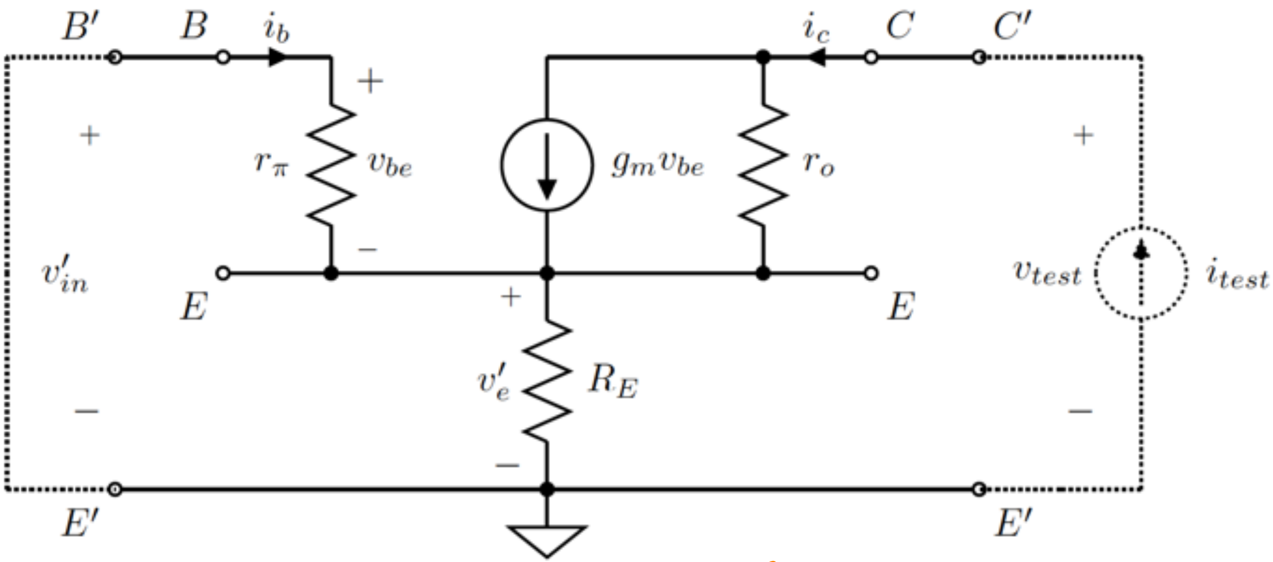
Current through r_o:

$$i_{r_o} = i_{test} - g_m v_{be} = i_{test} + g_m v_e'$$
$$= i_{test} \cdot \left(1 + g_m \left(r_\pi \parallel R_E \right) \right)$$

Solving for v_{test}:

$$\boxed{v_{test} = i_{r_o} r_o + v_e' = i_{test} \cdot \left[r_o + g_m r_o \left(r_\pi \parallel R_E \right) + \left(r_\pi \parallel R_E \right) \right]}$$

The Emitter-Degenerated Transistor (6)



Assume:

$$g_m r_o >> 1$$
$$g_m r_\pi = \beta >> 1$$
$$r_o >> R_E$$
$$r_\pi >> R_E$$

Recall:

$\approx R_E$ $\approx R_E$

$$\boxed{v_{test} = i_{test} \cdot \left[r_o + g_m r_o \left(r_\pi \parallel R_E \right) + \left(r_\pi \parallel R_E \right) \right]}$$

$= i_{test} \left[r_o + g_m r_o R_E + R_E \right]$

r_o

Solving for R'$_o$:

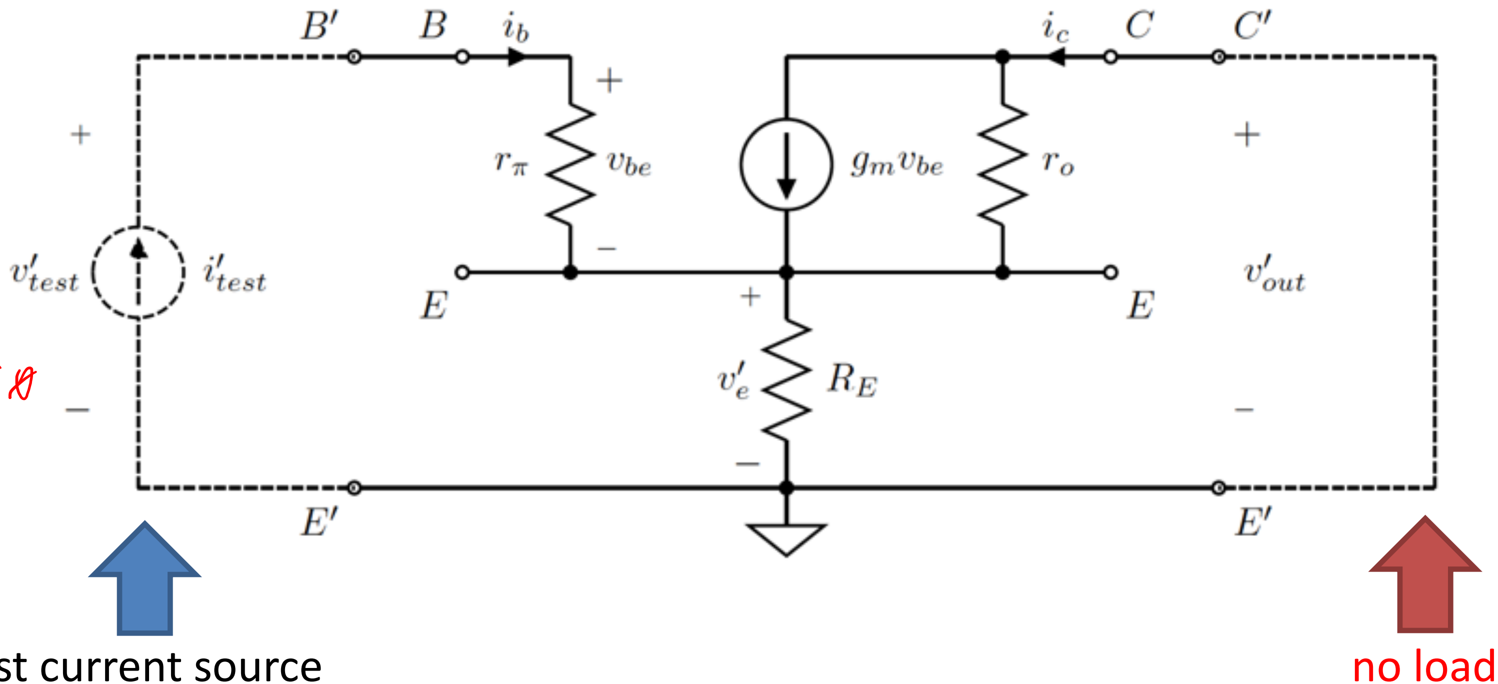
$$R_o' = \frac{v_{test}}{i_{test}} = \left[r_o + g_m r_o \left(r_\pi \parallel R_E \right) + \left(r_\pi \parallel R_E \right) \right]$$
$$\approx r_o + g_m r_o R_E \approx r_o \left(1 + g_m R_E \right) >> r_o$$

The Emitter-Degenerated Transistor (7)

- Calculating the input resistance

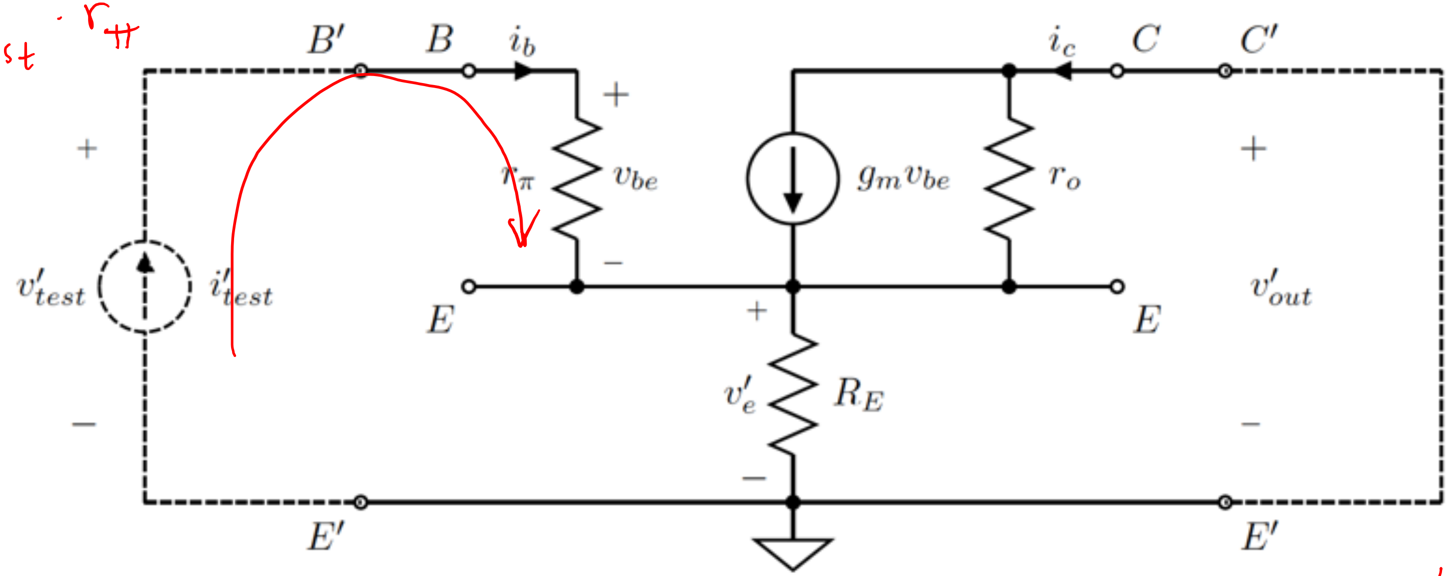
$$R_i' = \frac{v_{test}'}{i_{test}'}$$

$v_o = 0$



The Emitter-Degenerated Transistor (8)

$v_{be} = i'_{test} \cdot r_\pi$



By inspection:

$$v'_e = \left(i'_{test} + g_m v_{be}\right)\cdot\left(r_o \parallel R_E\right)$$
$$= i'_{test}\left(1+g_m r_\pi\right)\cdot\left(r_o \parallel R_E\right)$$

v'_e

$$v'_{test} = v_{be} + v'_e = i'_{test} r_\pi + i'_{test}\left(1+g_m r_\pi\right)\cdot\left(r_o \parallel R_E\right)$$
$$= i'_{test}\cdot\left[r_\pi + g_m r_\pi\left(r_o \parallel R_E\right) + \left(r_o \parallel R_E\right)\right]$$

The Emitter-Degenerated Transistor (9)

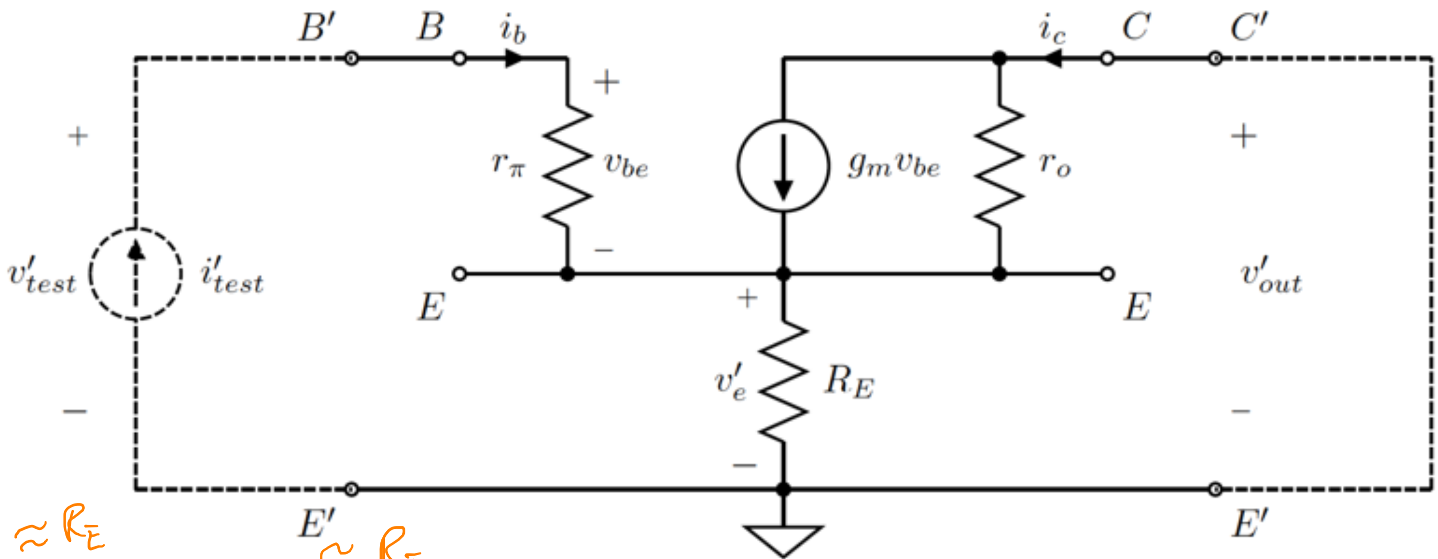
Assume:

$$g_m r_o >> 1$$

$$g_m r_\pi = \beta >> 1$$

$$r_o >> R_E$$

$$r_\pi >> R_E$$



Recall:

$\approx R_E$ $\approx R_E$

$$v'_{test} = i'_{test} \cdot \left[r_\pi + g_m r_\pi \left(r_o \parallel R_E \right) + \left(r_o \parallel R_E \right) \right]$$

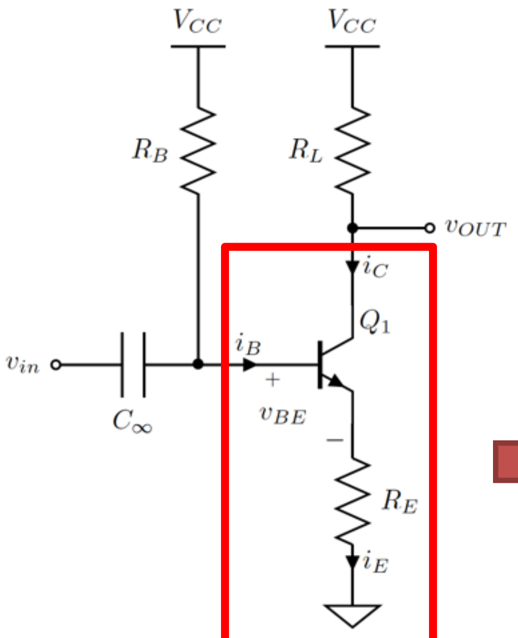
$$= i_{test} \left[\underbrace{r_\pi + g_m r_\pi R_E + R_E}_{\approx r_\pi} \right]$$

Solving for R'_i:

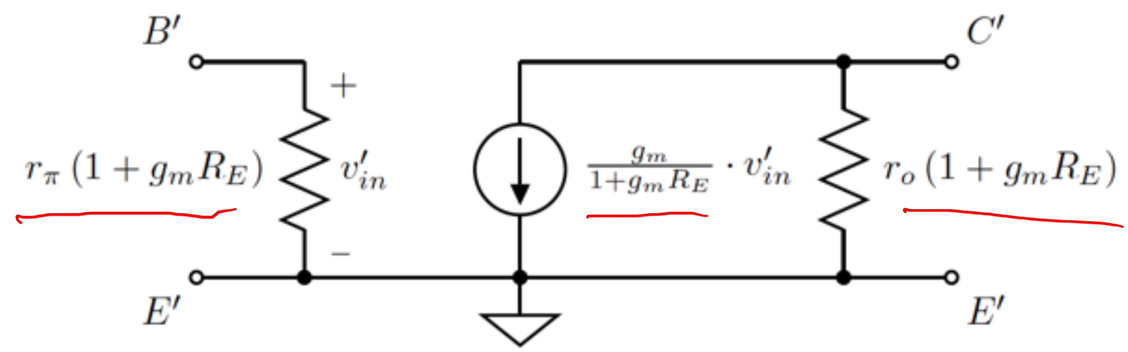
$$R'_i = \frac{v'_{test}}{i'_{test}} = \left[r_\pi + g_m r_\pi \left(r_o \parallel R_E \right) + \left(r_o \parallel R_E \right) \right]$$

$$\approx r_\pi \cdot \left(1 + g_m R_E \right) > r_\pi$$

The Emitter-Degenerated Transistor (10)



Degenerated transistor small signal equivalent

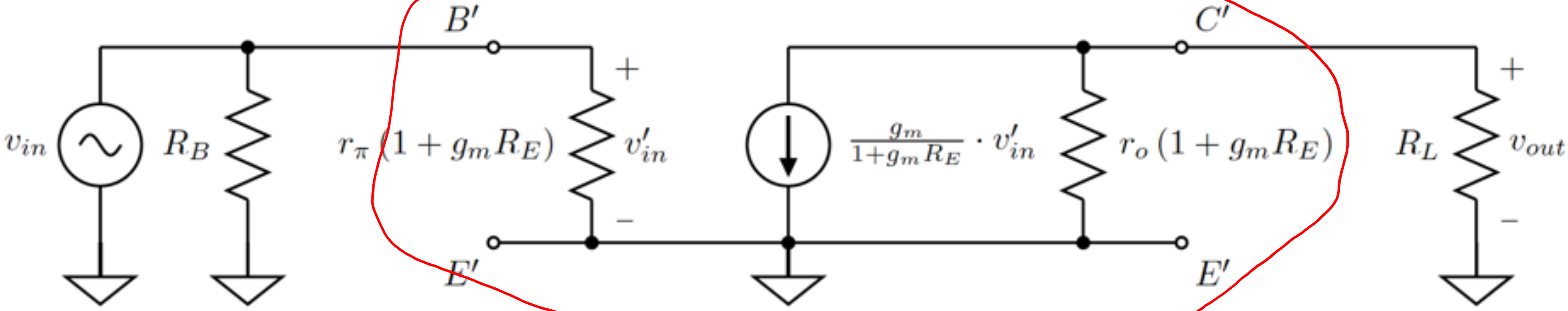


Transconductance → reduced (degenerated) by ($1+g_mR_E$)
Resistances → increased by ($1+g_mR_E$)

Is this good or bad?

Emitter-Degenerated Common-Emitter Amplifier

Overall analysis is now easy!



By inspection:

$$R_i = r_\pi \cdot (1+g_mR_E) \parallel R_B \approx R_B$$

$$G_m = \frac{g_m}{1+g_mR_E}$$

$$R_o = r_o(1+g_mR_E) \parallel R_L \approx R_L$$

Assume:

$$\left.\begin{array}{l} r_o >> R_L \\ r_\pi >> R_B \end{array}\right\}$$

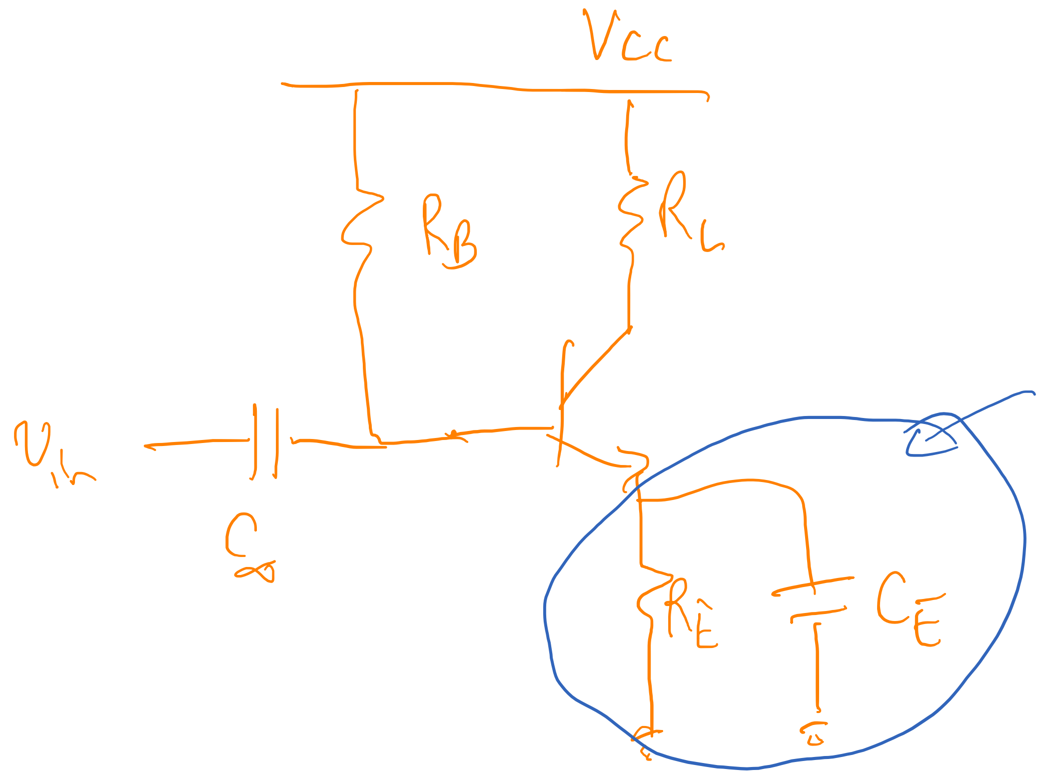
fixed bias:

$A_v = -g_m(r_o \parallel R_L)$

$\approx -g_m R_L$

Voltage gain:

$$A_v = -G_mR_o = -\frac{g_mR_L}{1+g_mR_E}$$



$$\left(R_E \,//\, \frac{1}{j\omega C_E}\right) \to \emptyset$$

Next Meeting

- Single-Stage Amplifiers
  - Common-Source Amplifier
  - Common-Base / Common-Gate Amplifier
  - Common-Collector / Common-Drain Amplifier