EEE 51 Assignment 8 Answer Key

2nd Semester SY 2018-2019
Due: 5pm Tuesday, April 23, 2019 (Rm. 220)

Instructions: Write legibly. Show all solutions and state all assumptions. Write your full name, student number, and section at the upper-right corner of each page. Start each problem on a new sheet of paper. Box or encircle your final answer.

Answer sheets should be color coded according to your lecture section. The color scheme is as follows:

THQ – yellow
THU – white
WFX – pink

1. **Operational Amplifier.**

   An operational amplifier with three blocks as shown in Figure 1 is being designed. The first block is a high gain block with a maximum gain of 2000 and with transfer function $H_1(s)$. The second block has a maximum gain of 50 and its transfer function is $H_2(s)$. This block is a high swing block. The last block is just a buffer.

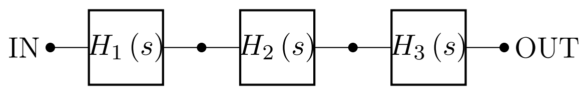


Figure 1: Blocks in an Operational Amplifier

The transfer functions are:

$$H_1(s) = k_1 \frac{s\left(1+\frac{s}{\omega_Z}\right)}{\left(1+\frac{s}{\omega_{P1}}\right)\left(1+\frac{s}{\omega_{ZP}}\right)}$$

$$H_2(s) = k_2 \frac{s}{\left(1+\frac{s}{\omega_{P3}}\right)\left(1+\frac{s}{\omega_{P4}}\right)}$$

$$H_3(s) = k_3 \frac{s}{\left(1+\frac{s}{\omega_{P5}}\right)\left(1+\frac{s}{\omega_{P6}}\right)}$$

with $f_Z = 100\,\text{kHz}$, $f_{P1} = 15\,\text{Hz}$, $f_{P2} = 25\,\text{kHz}$, $f_{P3} = 10\,\text{Hz}$, $f_{P4} = 100\,\text{kHz}$, $f_{P5} = 20\,\text{Hz}$, and $f_{P6} = 20\,\text{kHz}$.

(a) Give the values of:

i. k_1 (round to nearest whole number) (1 pt)

- Using the approximations for the Bode plot. and the fact that f_Z and f_{P2} are much greater than f_{P1}, then $f = 15\,\text{Hz}$ is the -3dB point of the first block.

$$\begin{aligned}
|H_1(s=j2\pi 15)| &= \left| k_1 \frac{(j2\pi 15)\left(1+\frac{j2\pi 15}{\omega_Z}\right)}{\left(1+\frac{j2\pi 15}{\omega_{P1}}\right)\left(1+\frac{j2\pi 15}{\omega_{P2}}\right)} \right| \\
\frac{2000}{\sqrt{2}} &= k_1 \frac{30\pi}{\sqrt{2}} \\
\frac{2000}{30\pi} &= k_1 \\
k_1 &\approx 21.22066 \\
&\approx \boxed{21}
\end{aligned}$$

- Or by using MATLAB or other software: $k_1 = 21.233 \rightarrow \boxed{k_1 \approx 21}$.

ii. k_2 (round to 3 decimal places) (1 pt)

- Just like the previous item, we use the Bode approximation:

$$
\begin{aligned}
\left|H_2\left(s=j2\pi 10\right)\right| &= \left|k_2 \frac{\left(j2\pi 10\right)}{\left(1+\frac{j2\pi 10}{\omega_{P3}}\right)\left(1+\frac{j2\pi 10}{\omega_{P4}}\right)}\right| \\
\frac{50}{\sqrt{2}} &= k_2 \frac{20\pi}{\sqrt{2}} \\
\frac{50}{20\pi} &= k_2 \\
k_2 &\approx 0.79577 \\
&\approx \boxed{0.796}
\end{aligned}
$$

- Or by using MATLAB or other software: $k_2 = 0.79585 \rightarrow \boxed{k_2 \approx 0.796}$.

iii. k_3 (round to 3 decimal places) (1 pt)

- by Bode approximation:

$$
\begin{aligned}
\left|H_3\left(s=j2\pi 20\right)\right| &= \left|k_3 \frac{\left(j2\pi 20\right)}{\left(1+\frac{j2\pi 20}{\omega_{P5}}\right)\left(1+\frac{j2\pi 20}{\omega_{P6}}\right)}\right| \\
\frac{1}{\sqrt{2}} &= k_3 \frac{40\pi}{\sqrt{2}} \\
\frac{1}{20\pi} &= k_3 \\
k_3 &\approx 0.0079577 \\
&\approx \boxed{0.008}
\end{aligned}
$$

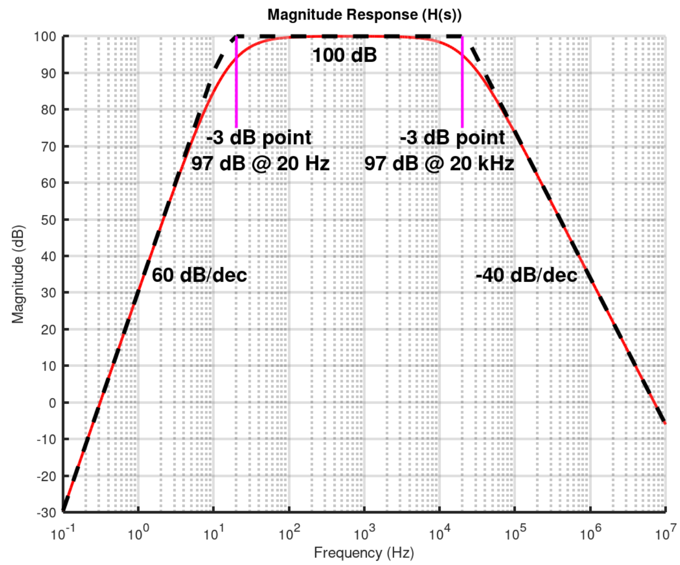
- Or by using MATLAB or other software: $k_3 = 0.0079657 \rightarrow \boxed{k_3 \approx 0.008}$.

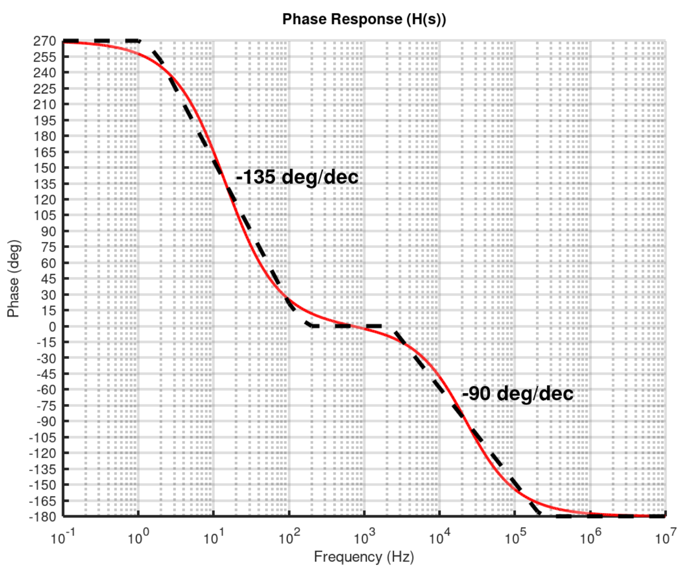
(b) Draw/Sketch the frequency response (magnitude and phase) of the op amp. You may attach a separate sheet for the plots should you wish to print them. You may use the approximations or the exact values for the Bode plot. Label the slopes, the -3dB points, and the maximum gain (in dB). Assume no loading effects between blocks. What is the transfer function $H(s)$ of the whole amplifier? (6 pts)

- For the overall transfer function, just multiply all of the given transfer functions:

$$
\begin{aligned}
H(s) &= H_1(s)\, H_2(s)\, H_3(s) \\
&= k_1 k_2 k_3 \frac{s\left(1+\frac{s}{\omega_Z}\right)}{\left(1+\frac{s}{\omega_{P1}}\right)\left(1+\frac{s}{\omega_{P2}}\right)} \frac{s}{\left(1+\frac{s}{\omega_{P3}}\right)\left(1+\frac{s}{\omega_{P4}}\right)} \frac{s}{\left(1+\frac{s}{\omega_{P5}}\right)\left(1+\frac{s}{\omega_{P6}}\right)} \\
&= (21)(0.796)(0.008) \frac{s\left(1+\frac{s}{2\pi 100\text{k}}\right)}{\left(1+\frac{s}{2\pi 15}\right)\left(1+\frac{s}{2\pi 25\text{k}}\right)} \frac{s}{\left(1+\frac{s}{2\pi 10}\right)\left(1+\frac{s}{2\pi 100\text{k}}\right)} \frac{s}{\left(1+\frac{s}{2\pi 20}\right)\left(1+\frac{s}{2\pi 20\text{k}}\right)} \\
&= \boxed{0.13 \frac{s^3}{\left(1+\frac{s}{2\pi 10}\right)\left(1+\frac{s}{2\pi 15}\right)\left(1+\frac{s}{2\pi 20}\right)\left(1+\frac{s}{2\pi 20\text{k}}\right)\left(1+\frac{s}{2\pi 25\text{k}}\right)}} \text{ or} \\
&= \boxed{0.13 \frac{s^3}{\left(1+\frac{s}{20\pi}\right)\left(1+\frac{s}{30\pi}\right)\left(1+\frac{s}{40\pi}\right)\left(1+\frac{s}{40\text{k}\pi}\right)\left(1+\frac{s}{50\text{k}\pi}\right)}}
\end{aligned}
$$

- For the Bode plots:





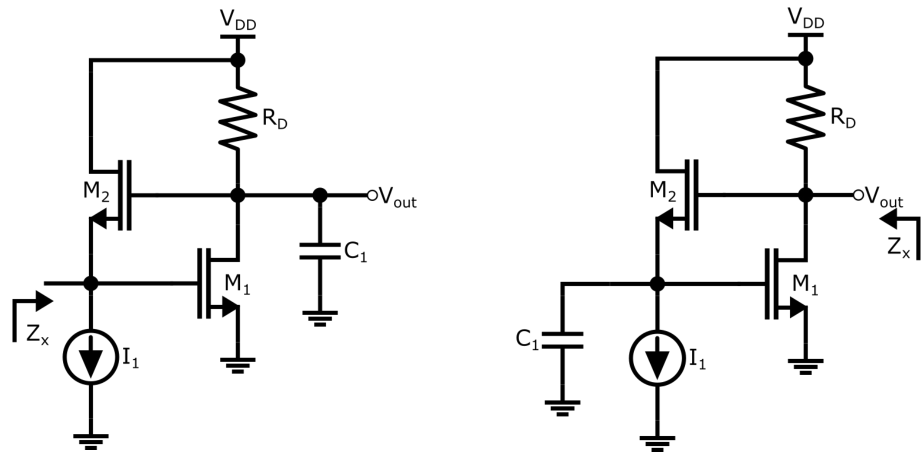
- **NOT SHOWN IN THE PLOT:**
  - **MAGNITUDE RESPONSE:** Between frequencies 10 Hz to 15 Hz, the slope is 40 dB/dec while it is 20 dB/dec between 15 Hz to 20 Hz. The slope is -20 dB/dec between 20 kHz to 25 kHz.
  - **PHASE RESPONSE:** The phase response has a slope of -45°/dec from 1 Hz to 1.5 Hz and -90°/dec from 1.5 Hz to 2 Hz. From 100 Hz to 150 Hz, the slope is -90°/dec, then from 150 Hz to 200 Hz it is -45°/dec. Starting from 2 kHz until 2.5 kHz, the slope is -45°/dec. This will change into -90°/dec from 2.5 kHz to 10 kHz. Lastly, the slope will be -45°/dec from 200 kHz to 250 kHz.
- If the above details are not included in the answers, full points will still be granted given that the rest of the plots are still correct. :)

(c) The amplifier will be used for an audio speaker. Will it be usable if the audible range of an average human is from 20 Hz to 20 kHz? Explain your answer. (1 pt)

- **YES**. Since the -3dB points are 20 Hz and 20 kHz, the audible range is within the bandwidth of the amplifier.

- Sub2Pewds

2. **Frequency analysis practice circuits.** Frequency-dependent components like capacitors and inductors could affect and vary the input and output impedances of amplifier circuits at certain operating frequencies. For the circuits shown below in Fig. 2 and ignoring all other intrinsic capacitances,



(a) Assume $\lambda = 0$ (b) Assume $\lambda \neq 0$

Figure 2: Impedance analysis for simple amplifier circuits

(a) Get the expression for Z_x. Express your answers in the form $Z_x(s) = A\frac{\left(1+\frac{s}{z}\right)}{\left(1+\frac{s}{p}\right)}$. [4 pts]

For Fig. 2a, the following is the small-signal circuit model in order to solve for Z_x,

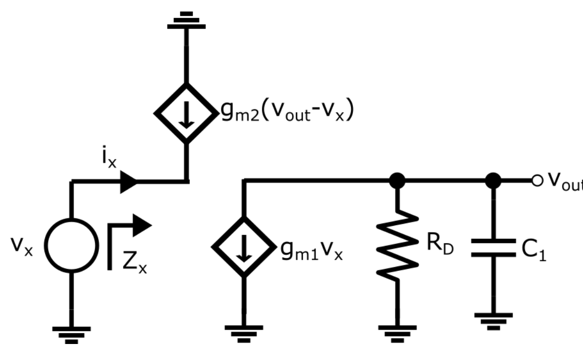


Figure 3: Small-signal model for Fig. 2a

$$v_{out} = -g_{m1}v_x\left(R_D//\frac{1}{sC_1}\right) \tag{1}$$

$$i_x = -g_{m2}(v_{out} - v_x) = -g_{m2}v_{out} + g_{m2}v_x = \left(g_{m2}g_{m1}\left(R_D//\frac{1}{sC_1}\right) + g_{m2}\right)v_x \tag{2}$$

Thus,

$$Z_x = \frac{v_x}{i_x} = \frac{1}{g_{m2}g_{m1}\left(R_D//\frac{1}{sC_1}\right) + g_{m2}} = \frac{1}{g_{m2}\left(\left(\frac{g_{m1}R_D}{1+sR_DC_1}\right)+1\right)} \tag{3}$$

$$\boxed{Z_x = \frac{1}{g_{m2}}\left(\frac{1+sR_DC_1}{1+g_{m1}R_D+sR_Dc_1}\right) = \frac{1}{g_{m2}(1+g_{m1}R_D)}\frac{\left(1+\frac{s}{\frac{1}{R_DC_1}}\right)}{\left(1+\frac{s}{\frac{1+g_{m1}R_D}{R_DC_1}}\right)}}$$

(0.5 pts). (1.5 pts) will be given for the solution.

For Fig. 2b, the following small-signal model is used,

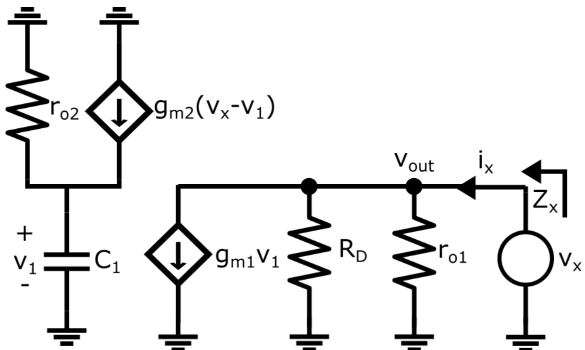


Figure 4: Small-signal model for Fig. 2a

At node v_1,

$$g_{m2}(v_x - v_1) - \frac{v_1}{r_{o2}} = sC_1 v_1 \tag{4}$$

$$v_1 = \frac{g_{m2}}{sC_1 + g_{m2} + \frac{1}{r_o 2}} v_x \tag{5}$$

At $v_o ut$,

$$i_x = \frac{v_x}{R_D} + \frac{v_x}{r_{o1}} + g_{m1}v_1 = \left(\frac{1}{R_D} + \frac{1}{r_{o1}} + \frac{g_{m1}g_{m2}}{sC_1 + g_{m2} + \frac{1}{r_o 2}}\right) v_x \tag{6}$$

$$Z_x = \frac{v_x}{i_x} = \left(\frac{1}{R_D} + \frac{1}{r_{o1}} + \frac{g_{m1}g_{m2}}{sC_1 + g_{m2} + \frac{1}{r_o 2}}\right)^{-1} = \left(\frac{(r_{01} + R_D)\left(g_{m2} + \frac{1}{r_{o2}} + sC_1\right) + g_{m1}g_{m2}R_D r_{o1}}{R_D r_{o1}\left(g_{m2} + \frac{1}{r_{o2}} + sC_1\right)}\right)^{-1} \tag{7}$$

$$Z_x = \left(\frac{R_D r_{o1} g_{m2} + R_D \frac{r_{o1}}{r_{o2}} + sR_D r_{o1} C_1}{(r_{01} + R_D)\left(g_{m2} + \frac{1}{r_{o2}}\right) + g_{m1}g_{m2}R_D r_{o1} + (r_{01} + R_D)sC_1}\right) \tag{8}$$

For now, let us set $X = r_{o1}g_{m2} + \frac{r_{o1}}{r_{o2}}$ and $Y = (r_{01} + R_D)\left(g_{m2} + \frac{1}{r_{o2}}\right) + g_{m1}g_{m2}R_D r_{o1}$,

$$Z_x = \frac{R_D(X + sr_{o1}C_1)}{Y + s(r_{o1} + R_D)C_1} = \frac{R_D X}{Y} \frac{\left(1 + \frac{s}{\frac{X}{r_{o1}C_1}}\right)}{\left(1 + \frac{s}{\frac{Y}{(r_{o1}+R_D)C_1}}\right)} \tag{9}$$

****GA Note:**

For the circuit in Fig. 2b, it was not foreseen that the expression for Z_x would be as complex and difficult to simplify as seen in the solution above, while originally intended for an "easier" method of checking the final answers, for Fig. 2b, expressions for Z_x that is not specified in the desired form of $Z_x(s) = A\frac{\left(1+\frac{s}{z}\right)}{\left(1+\frac{s}{p}\right)}$ **will still be considered and marked correct** provided that the derivation steps are completely shown in the solution. Thus, a final answer like,

$$Z_x = \left(\frac{1}{R_D} + \frac{1}{r_{o1}} + \frac{g_{m1}g_{m2}}{sC_1 + g_{m2} + \frac{1}{r_o 2}}\right)^{-1} \tag{10}$$

will still be considered.

(b) What is Z_x at $f = 0Hz$? [2 pts] at $f \to \infty Hz$? [2 pts]
For Fig. 2a,

$$Z_x(s=0) = \frac{1}{g_{m2}(1 + g_{m1}R_D)}$$

and

$$Z_x(s \to \infty) = \frac{1}{g_{m2}}$$

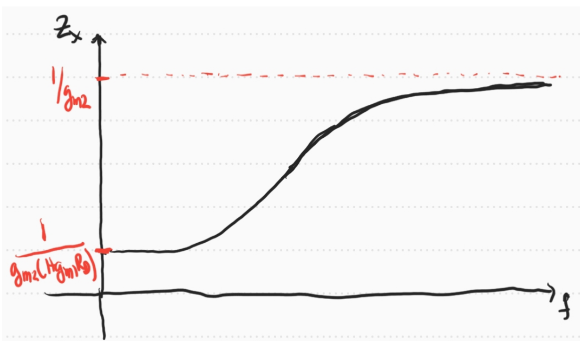
For Fig. 2b,

$$Z_x(s=0) = \frac{1}{\frac{1}{R_D} + \frac{1}{r_{o1}} + \frac{g_{m1}g_{m2}}{g_{m2}+\frac{1}{r_{o2}}}}$$

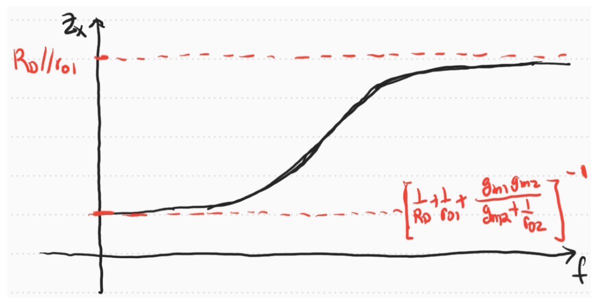
and

$$Z_x(s \to \infty) = \frac{r_{o1}R_D}{r_{o1} + R_D} = R_D//r_{o1}$$

(c) Roughly sketch $|Z_x|$ against frequency. Label the important points as solved from the previous item. [2 pts]



(a) For Fig. 2a



(b) For Fig. 2b

Figure 5: $|Z_x|$ against frequency plots

3. **Struggles from Miller Effect.** For this problem, assume the following when necessary: T=300K, $V_{BE,on} = 0.7V$, $V_{CE,sat} = 0.2V$, $\beta = 200$, $V_A = 100V$, $R_S = 50\Omega$, $I_C = 1mA$. Ignore the parasitic capacitances of the transistors.

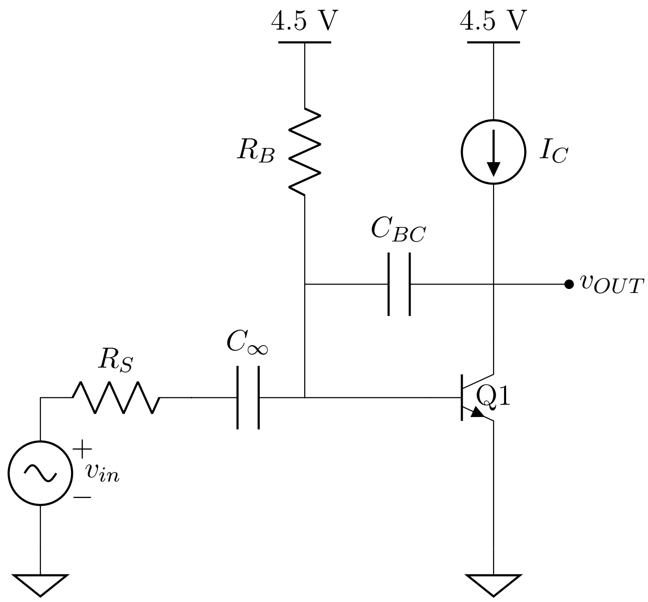


Figure 6: Common Emitter with Miller Capacitance

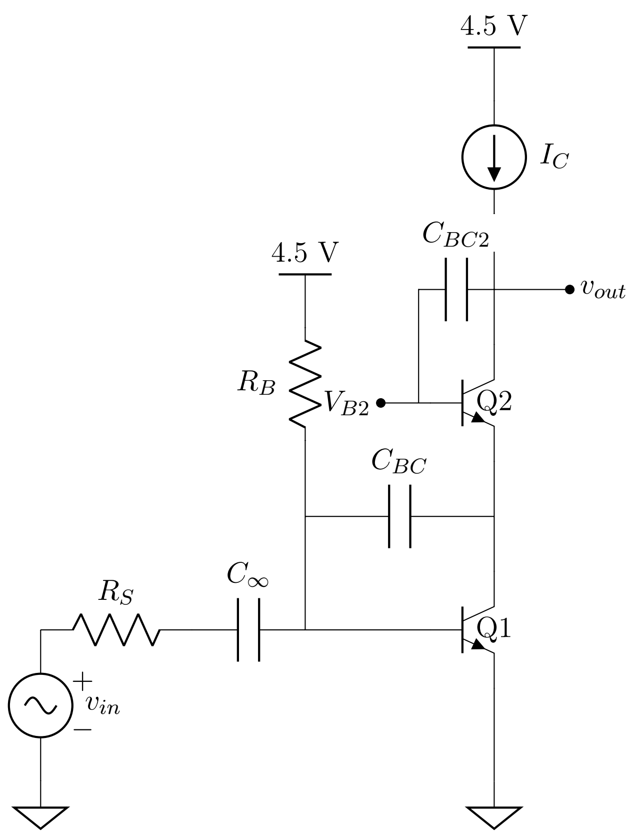


Figure 7: Cascode with Miller Capacitance

(a) Determine the DC gain of the amplifier in fig. 6. (1 pt)
It's a common-emitter amplifier with a source resistance R_S and an ideal current source load. So the DC gain is given by:

$$A_{V0} = -g_m r_o \frac{r_\pi || R_B}{R_S + (r_\pi || R_B)}$$

The small-signal parameters are:

$$g_m = \frac{I_C}{V_T} = \frac{1mA}{26mV} = 38.46mS$$
$$r_\pi = \frac{\beta}{g_m} = 5200\Omega$$
$$r_o = \frac{V_A}{I_C} = 100K\Omega$$

And R_B can be solved by using KVL on the bias loop:

$$4.5 - I_B R_B - V_{BE,on} = 0$$
$$I_B = \frac{I_C}{\beta} = \frac{1mA}{200} = 50\mu A$$
$$R_B = \frac{4.5 - V_{BE,on}}{I_B} = \frac{4.5 - 0.7}{50\mu A} = 76K\Omega$$

Thus, the DC gain is:

$$A_{V0} = -g_m r_o \frac{r_\pi || R_B}{R_S + (r_\pi || R_B)} = \frac{V_A}{V_T} \frac{r_\pi || R_B}{R_S + (r_\pi || R_B)} = \frac{100V}{26mV} \frac{5200||76000}{50 + (5200||76000)} = 3807.04V/V$$

$$\boxed{A_{V0} = 3807.04V/V}$$

(b) Solve for the transfer function of the amplifier in fig. 6. Write it in the form: $K\frac{(1+\frac{s}{\omega_{z1}})(1+\frac{s}{\omega_{z2}})...(1+\frac{s}{\omega_{zn}})}{(1+\frac{s}{\omega_{p1}})(1+\frac{s}{\omega_{p2}})...(1+\frac{s}{\omega_{pn}})}$ (1pt) The small-signal model is:

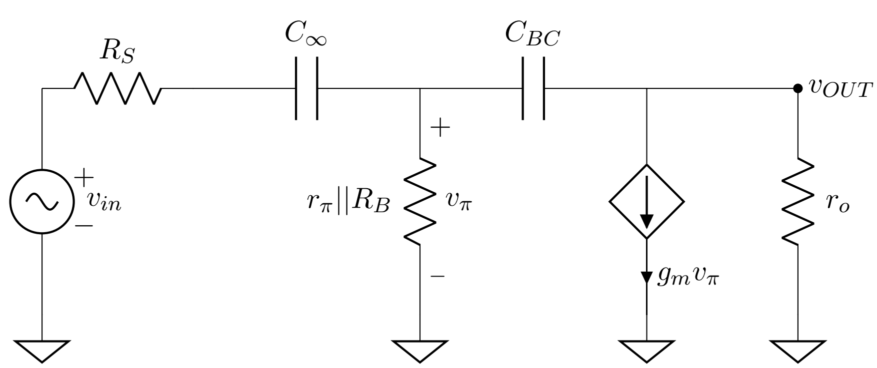


Figure 8: Common Emitter with Miller Capacitance

Thus, the transfer function is given by:

$$\boxed{A_V(s) = -g_m r_o \frac{r_\pi}{R_S + r_\pi}\left(\frac{1 - s\frac{C_{BC}}{g_m}}{1 + sC_{BC}\left(g_m r_o (R_S||r_\pi) + (R_S||r_\pi) + r_o\right)}\right)}$$

If you used simplifications while solving, you should get:

$$\boxed{A_V(s) = -g_m r_o \frac{r_\pi}{R_S + r_\pi}\left(\frac{1 - s\frac{C_{BC}}{g_m}}{1 + sC_{BC}\left(g_m r_o (R_S||r_\pi)\right)}\right)}$$

(c) What is the 3-dB bandwidth of the amplifier? Express your answer in rad/s. (1 pt)

$$\omega_{-3dB} = \frac{1}{C_{BC}\left(g_m r_o (R_S||r_\pi) + (R_S||r_\pi) + r_o\right)}$$

$$\boxed{\omega_{-3dB} = 9.63Mrad/s}$$

Suppose you added a cascode, as shown in fig. 8 such that $V_{CE1} = 0.3V$.

(d) Draw the small-signal model. Label the values of the small-signal parameters, and the components. Do NOT use the Miller approximation. (1 pt)

If you recognized that the current through Q1 and Q2 should be approximately the same, you may write $r_{\pi 2} = r_\pi$, $g_{m2} = g_m$, and $r_{o2} = r_o$. But this solution will follow the convention for writing them as if they are not approximately the same. You are challenged though, to try the solution while approximating them as the same. :)

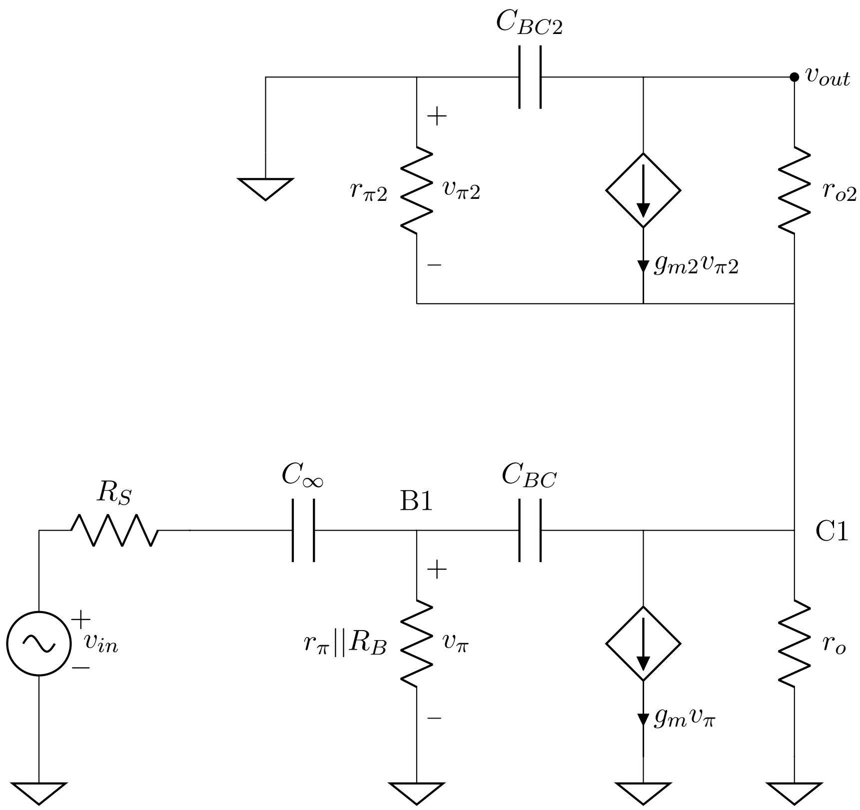


Figure 9: Cascode with Miller Capacitances

(e) Solve for the transfer function of the amplifier in fig. 8. Write it in the form: $K\frac{(1+\frac{s}{\omega_{z1}})(1+\frac{s}{\omega_{z2}})...(1+\frac{s}{\omega_{zn}})}{(1+\frac{s}{\omega_{p1}})(1+\frac{s}{\omega_{p2}})...(1+\frac{s}{\omega_{pn}})}$ (3pts) The derivation will be very similar to the one in homework 7, problem 2, though there will be slight differences. Let's start with the KCL equations:

KCL@node B1:

$$\frac{v_\pi - v_{in}}{R_S} + \frac{v_\pi}{r_\pi||R_B} + sC_{BC}(v_\pi - (-v_{\pi 2})) = 0$$

$$v_\pi\left(\frac{1}{R_S} + \frac{1}{r_\pi||R_B} + sC_{BC}\right) = \frac{v_{in}}{R_S} - sC_{BC}v_{\pi 2}$$

For convenience, let's write $R_1 = (r_\pi||R_B)||R_S$:

$$v_\pi\left(\frac{1}{R_1} + sC_{BC}\right) = \frac{v_{in}}{R_S} - sC_{BC}v_{\pi 2}$$

$$v_\pi = \frac{1}{\frac{1}{R_1} + sC_{BC}} \left(\frac{v_{in}}{R_S} - sC_{BC} v_{\pi 2} \right) \tag{11}$$

KCL@node C1:

$$\begin{aligned}
sC_{BC}(-v_{\pi 2} - v_\pi) + g_m v_\pi + \frac{-v_{\pi 2}}{r_o} + \frac{-v_{\pi 2}}{r_{\pi 2}} - g_{m2} v_{\pi 2} + \frac{-v_{\pi 2} - v_{out}}{r_{o2}} &= 0 \\
v_{\pi 2} \left(sC_{BC} + \frac{1}{r_o} + g_{m2} + \frac{1}{r_{\pi 2}} + \frac{1}{r_{o2}} \right) &= v_\pi \left(g_m - sC_{BC} \right) - \frac{v_{out}}{r_{o2}} \\
v_{\pi 2} &= \frac{1}{g_{m2} + \frac{1}{r_{\pi 2}} + \frac{1}{r_o} + \frac{1}{r_{o2}} + sC_{BC}} \left(v_\pi \left(g_m - sC_{BC} \right) - \frac{v_{out}}{r_{o2}} \right)
\end{aligned}$$

But we'll retain the second to the last form for convenience later:

$$v_{\pi 2} \left(g_{m2} + \frac{1}{r_{\pi 2}} + \frac{1}{r_o} + \frac{1}{r_{o2}} + sC_{BC} \right) = v_\pi \left(g_m - sC_{BC} \right) - \frac{v_{out}}{r_{o2}} \tag{12}$$

KCL@node v_{out}:

$$\begin{aligned}
sC_{BC2} v_{out} + g_{m2} v_{\pi 2} + \frac{v_{out} - (-v_{\pi 2})}{r_{o2}} &= 0 \\
v_{out} \left(\frac{1}{r_{o2}} + sC_{BC2} \right) &= -v_{\pi 2} \left(g_{m2} + \frac{1}{r_{o2}} \right)
\end{aligned}$$

$$v_{\pi 2} = -v_{out} \left(\frac{\frac{1}{r_{o2}} + sC_{BC2}}{g_{m2} + \frac{1}{r_{o2}}} \right) \tag{13}$$

Substituting eq. (11) to eq. (12), we get:

$$v_{\pi 2} \left(g_{m2} + \frac{1}{r_{\pi 2}} + \frac{1}{r_o} + \frac{1}{r_{o2}} + sC_{BC} \right) = \left(\frac{1}{\frac{1}{R_1} + sC_{BC}} \left[\frac{v_{in}}{R_S} - sC_{BC} v_{\pi 2} \right] \right) \left(g_m - sC_{BC} \right) - \frac{v_{out}}{r_{o2}}$$

$$\begin{aligned}
v_{\pi 2} \left(g_{m2} + \frac{1}{r_{\pi 2}} + \frac{1}{r_o} + \frac{1}{r_{o2}} + sC_{BC} \right) \left(\frac{1}{R_1} + sC_{BC} \right) = \left(\frac{v_{in}}{R_S} - sC_{BC} v_{\pi 2} \right) \left(g_m - sC_{BC} \right) \\
- \left(\frac{1}{R_1} + sC_{BC} \right) \frac{v_{out}}{r_{o2}}
\end{aligned}$$

$$\begin{aligned}
v_{\pi 2} \left[\underline{\left(g_{m2} + \frac{1}{r_{\pi 2}} + \frac{1}{r_o} + \frac{1}{r_{o2}} + sC_{BC} \right) \left(\frac{1}{R_1} + sC_{BC} \right)} + sC_{BC} \left(g_m - sC_{BC} \right) \right] \\
= \left(\frac{v_{in}}{R_S} \right) \left(g_m - sC_{BC} \right) - \left(\frac{1}{R_1} + sC_{BC} \right) \frac{v_{out}}{r_{o2}}
\end{aligned}$$

In distributing terms here, let's group terms without s, and terms with s. So we can easily see that the underlined term simply makes a second-order expression. Notice that the terms underlined in the next expression cancel out, and we can factor in the remaining $g_m sC_{BC}$ to the group with sC_{BC}.

$$\begin{aligned}v_{\pi 2}\Bigg[\left(g_{m2}+\frac{1}{r_{\pi 2}}+\frac{1}{r_o}+\frac{1}{r_{o2}}\right)\left(\frac{1}{R_1}\right)+sC_{BC}\left((g_{m2}+\frac{1}{r_{\pi 2}}+\frac{1}{r_o}+\frac{1}{r_{o2}})+(\frac{1}{R_1})\right)\\ +(sC_{BC})^2+sC_{BC}\left(g_m-sC_{BC}\right)\Bigg]=\left(\frac{v_{in}}{R_S}\right)(g_m-sC_{BC})-\left(\frac{1}{R_1}+sC_{BC}\right)\frac{v_{out}}{r_{o2}}\end{aligned}$$

$$\begin{aligned}v_{\pi 2}\left[\left(g_{m2}+\frac{1}{r_{\pi 2}}+\frac{1}{r_o}+\frac{1}{r_{o2}}\right)\left(\frac{1}{R_1}\right)+sC_{BC}\left((g_{m2}+\frac{1}{r_{\pi 2}}+\frac{1}{r_o}+\frac{1}{r_{o2}})+(\frac{1}{R_1})+g_m\right)\right]\\ =\left(\frac{v_{in}}{R_S}\right)(g_m-sC_{BC})-\left(\frac{1}{R_1}+sC_{BC}\right)\frac{v_{out}}{r_{o2}}\end{aligned}\tag{14}$$

We'll do a segue here to get some intuition on what we can expect.
START SEGUE
Notice in this part that if we assume negligible the $\frac{v_{out}}{r_{o2}}$ term, you substitute eq. (13) to eq. (14), and reorder a few terms, you'll get:

$$\frac{v_{out}}{v_{in}}=-\frac{\left(\frac{1}{R_S}\right)\left(g_{m2}+\frac{1}{r_{o2}}\right)(g_m-sC_{BC})}{\left[\left(g_{m2}+\frac{1}{r_{\pi 2}}+\frac{1}{r_o}\right)\left(\frac{1}{R_1}\right)+sC_{BC}\left((g_{m2}+\frac{1}{r_{\pi 2}}+\frac{1}{r_o})+(\frac{1}{R_1})+g_m\right)\right]\left(\frac{1}{r_{o2}}+sC_{BC2}\right)}$$

Assuming $g_m >> \frac{1}{r_\pi}$, and $g_m >> \frac{1}{r_o}$, this can be simplied to:

$$\begin{aligned}\frac{v_{out}}{v_{in}}&=-g_m r_{o2}\frac{R_1}{R_S}\frac{1-s\frac{C_{BC}}{g_m}}{\left(1+sC_{BC}\left(\frac{g_m}{g_{m2}}R_1+R_1+\frac{1}{g_{m2}}\right)\right)(1+sC_{BC2}r_{o2})}\\ &=-g_m r_{o2}\frac{(r_\pi||R_B)}{(r_\pi||R_B)+R_S}\frac{1-s\frac{C_{BC}}{g_m}}{\left(1+sC_{BC}\left(\frac{g_m}{g_{m2}}R_1+R_1+\frac{1}{g_{m2}}\right)\right)(1+sC_{BC2}r_{o2})}\end{aligned}$$

This initial result shows the following properties:

$$\begin{aligned}A_{V0}&=-g_m r_{o2}\frac{(r_\pi||R_B)}{(r_\pi||R_B)+R_S}\\ \omega_z&=\frac{g_m}{C_{BC}}\\ \omega_{p1}&=\frac{1}{C_{BC}\left(\frac{g_m}{g_{m2}}R_1+R_1+\frac{1}{g_{m2}}\right)}\\ \omega_{p2}&=\frac{1}{C_{BC2}r_{o2}}\end{aligned}$$

All of the parameters can be expected intuitively except for ω_{p1} which is due to an interaction between C_{BC} and resistances and transconductances at nodes B1 and C1. Things should be a little different when we finally factor in the $\frac{v_{out}}{r_{o2}}$ term that we neglected, but we expect some of these to be similar.
END SEGUE

Substituting eq. (13) to eq. (14), we get:

$$-v_{out}\left(\frac{\frac{1}{r_{o2}}+sC_{BC2}}{g_{m2}+\frac{1}{r_{o2}}}\right)\left[\left(g_{m2}+\frac{1}{r_{\pi2}}+\frac{1}{r_o}+\frac{1}{r_{o2}}\right)\left(\frac{1}{R_1}\right)+sC_{BC}\left((g_{m2}+\frac{1}{r_{\pi2}}+\frac{1}{r_o}+\frac{1}{r_{o2}})+(\frac{1}{R_1})+g_m\right)\right]$$
$$=\left(\frac{v_{in}}{R_S}\right)(g_m-sC_{BC})-\left(\frac{1}{R_1}+sC_{BC}\right)\frac{v_{out}}{r_{o2}}$$

$$-v_{out}\left(\frac{1}{r_{o2}}+sC_{BC2}\right)\left[\left(g_{m2}+\frac{1}{r_{\pi2}}+\frac{1}{r_o}+\frac{1}{r_{o2}}\right)\left(\frac{1}{R_1}\right)+sC_{BC}\left((g_{m2}+\frac{1}{r_{\pi2}}+\frac{1}{r_o}+\frac{1}{r_{o2}})+(\frac{1}{R_1})+g_m\right)\right]$$
$$=\left(\frac{v_{in}}{R_S}\right)(g_m-sC_{BC})\left(g_{m2}+\frac{1}{r_{o2}}\right)-\left(\frac{1}{R_1}+sC_{BC}\right)\frac{v_{out}}{r_{o2}}\left(g_{m2}+\frac{1}{r_{o2}}\right)$$

$$v_{out}\left[\left(\frac{1}{r_{o2}}+sC_{BC2}\right)\left[\left(g_{m2}+\frac{1}{r_{\pi2}}+\frac{1}{r_o}+\frac{1}{r_{o2}}\right)\left(\frac{1}{R_1}\right)+sC_{BC}\left((g_{m2}+\frac{1}{r_{\pi2}}+\frac{1}{r_o}+\frac{1}{r_{o2}})+(\frac{1}{R_1})+g_m\right)\right]\right.$$
$$\left.-\frac{1}{r_{o2}}\left(\frac{1}{R_1}+sC_{BC}\right)\left(g_{m2}+\frac{1}{r_{o2}}\right)\right]=-\left(\frac{v_{in}}{R_S}\right)\left(g_{m2}+\frac{1}{r_{o2}}\right)(g_m-sC_{BC}) \tag{15}$$

Let's recognize a few things to not get confused. The term in the dotted underline expands to a 2nd order expression of s, while the term in the dashed underline is just a first order expression of s. If we recognize them of the form:

$$(A+sB)(C+sD)-K(E+sF)(G)$$

we can easily see that it would simplify into:

$$(AC-KEG)+s(AD+BC-KFG)+s^2BD$$

Equation (15) would simplify into:

$$v_{out}\left[\left(\frac{1}{r_{o2}}\frac{1}{R_1}(\frac{1}{r_{\pi2}}+\frac{1}{r_o})\right)+s\left(C_{BC2}\frac{1}{R_1}\left(g_{m2}+\frac{1}{r_{\pi2}}+\frac{1}{r_o}+\frac{1}{r_{o2}}\right)+C_{BC}\frac{1}{r_{o2}}\left(g_m+\frac{1}{r_{\pi2}}+\frac{1}{r_o}+\frac{1}{R_1}\right)\right)\right.$$
$$\left.+s^2C_{BC}C_{BC2}\left(g_m+g_{m2}+\frac{1}{r_{\pi2}}+\frac{1}{r_o}+\frac{1}{r_{o2}}+\frac{1}{R_1}\right)\right]=-\left(\frac{v_{in}}{R_S}\right)\left(g_{m2}+\frac{1}{r_{o2}}\right)(g_m-sC_{BC}) \tag{16}$$

We note that the current through the common-emitter stage, and the common-base stage are the same, and $\beta = 200$. Therefore, $\frac{1}{r_{\pi2}} >> \frac{1}{r_{o2}}$, $\frac{1}{r_{\pi2}} >> \frac{1}{r_o}$, $g_{m2} >> \frac{1}{r_{\pi2}}$. Equation (16) simplifies into:

$$v_{out}\left[\left(\frac{1}{r_{o2}}\frac{1}{R_1}\frac{1}{r_{\pi2}}\right)+s\left(\frac{C_{BC2}g_{m2}}{R_1}+\frac{C_{BC}(g_m+\frac{1}{R_1})}{r_{o2}}\right)+s^2C_{BC}C_{BC2}\left(g_m+g_{m2}+\frac{1}{R_1}\right)\right]$$
$$=-\left(\frac{v_{in}}{R_S}\right)\left(g_{m2}+\frac{1}{r_{o2}}\right)(g_m-sC_{BC})$$

$$v_{out}\left[1+s\left((C_{BC2}r_{o2})(g_{m2}r_{\pi2})+(C_{BC}(g_mR_1+1))r_{\pi2}\right)+s^2C_{BC}C_{BC2}\left(g_m+g_{m2}+\frac{1}{R_1}\right)r_{o2}r_{\pi2}R_1\right]$$
$$=-v_{in}r_{\pi2}\left(\frac{R_1}{R_S}\right)(g_{m2}r_{o2}+1)(g_m-sC_{BC})$$

$$v_{out}\left[1+s\left((C_{BC2}r_{o2})(g_{m2}r_{\pi2})+(C_{BC}(g_mR_1+1))r_{\pi2}\right)+s^2C_{BC}C_{BC2}\left(g_m+g_{m2}+\frac{1}{R_1}\right)r_{o2}r_{\pi2}R_1\right]$$
$$=-v_{in}g_mr_{\pi2}\left(\frac{R_1}{R_S}\right)(g_{m2}r_{o2}+1)\left(1-s\frac{C_{BC}}{g_m}\right)$$

$$\frac{v_{out}}{v_{in}}=A_{V0}\frac{\left(1-s\frac{C_{BC}}{g_m}\right)}{1+s\left((C_{BC2}r_{o2})(g_{m2}r_{\pi2})+(C_{BC}(g_mR_1+1))r_{\pi2}\right)+s^2C_{BC}C_{BC2}\left(g_m+g_{m2}+\frac{1}{R_1}\right)r_{o2}r_{\pi2}R_1}$$
$$A_{V0}=-g_mr_{\pi2}\left(\frac{R_1}{R_S}\right)(g_{m2}r_{o2}+1) \quad (17)$$

Recognizing that the second-order expression in the denominator is of the form: $c+sb+s^2a$, we can try several factoring techniques. To ease the process, let's look at the determinant b^2-4ac. The b^2 term will have a $(g_mr_o)^2$ expression, while the $4ac$ term will not, making $b^2>>4ac$. In such cases, the roots of a quadratic expression can be approximated by $-\frac{b}{a}$ and $-\frac{c}{b}$. [You can prove this using the Taylor series expansion of $sqrt(b^2-4ac)$ around b^2] Therefore, eq. (17) can be approximated as:

$$\boxed{\begin{aligned}
\frac{v_{out}}{v_{in}}&=A_{V0}\frac{\left(1-\frac{s}{\omega_z}\right)}{(1+\frac{1}{\omega_{p1}})(1+\frac{1}{\omega_{p2}})}\\
A_{V0}&=-g_mr_{\pi2}\left(\frac{R_1}{R_S}\right)(g_{m2}r_{o2}+1)=-g_mr_{\pi2}\frac{(r_\pi||R_B)}{(r_\pi||R_B)+R_S}(g_{m2}r_{o2}+1)\\
\omega_z&=\frac{g_m}{C_{BC}}\\
\omega_{p1}&=-\frac{1}{C_{BC}\left(\frac{g_m}{g_{m2}}R_1+R_1+\frac{1}{g_{m2}}\right)}\\
\omega_{p2}&=-\frac{1}{g_{m2}r_{\pi2}C_{BC2}r_{o2}}=-\frac{1}{\beta C_{BC2}r_{o2}}
\end{aligned}}$$

This result is similar to the one we got earlier by neglecting the $\frac{v_{out}}{r_{o2}}$ term. The noticeable changes are in A_{V0}, and ω_{p2}, notably, ω_{p2} is β times lower than expected.

(f) Determine the I_C needed such that the circuit in fig. 8 will have the same DC gain as in fig. 6. (1 pt) To do so, we need to equate the DC gain of the cascode with the DC gain of the common-emitter. Let $r_{\pi B}=r_\pi$ of the cascode.

$$A_{V0,CE}=-g_mr_o\frac{r_\pi||R_B}{R_S+(r_\pi||R_B)}=-\frac{V_A}{V_T}\frac{r_\pi||R_B}{R_S+(r_\pi||R_B)}$$
$$A_{V0,cascode}=-g_mr_{\pi2}\frac{(r_\pi||R_{BB})}{(r_\pi||R_{BB})+R_S}(g_{m2}r_{o2}+1)\approx-\frac{V_A}{V_T}\frac{(r_\pi||R_{BB})}{(r_\pi||R_{BB})+R_S}g_mr_{\pi2}$$

$$-\frac{V_A}{V_T}\frac{r_\pi||R_B}{R_S+(r_\pi||R_B)}=-\frac{V_A}{V_T}\frac{(r_\pi||R_{BB})}{(r_\pi||R_{BB})+R_S}g_mr_{\pi2}$$
$$\frac{r_\pi||R_B}{R_S+(r_\pi||R_B)}=\frac{(r_{\pi B}||R_{BB})}{(r_{\pi B}||R_{BB})+R_S}g_mr_{\pi2}$$

For this case, $r_\pi<<R_B$ and $r_{\pi B}<<R_{BB}$ are both applicable. Let's use it to simplify calculations.

Also, we recognize that $g_m r_{\pi 2} = g_{m2} r_{\pi 2} = \beta$ for the cascode.

$$\begin{aligned}\frac{r_\pi}{R_S + r_\pi} &= \frac{r_{\pi B}}{r_{\pi B} + R_S}\beta \\ r_{\pi B} &= \frac{r_\pi R_S}{\beta(r_\pi + R_S) - r_\pi} \\ I_{CB} &= \frac{(\beta - 1)r_\pi + \beta R_S}{R_S} I_C \\ &= \frac{(\beta - 1)\beta V_T}{R_S} + \beta I_C\end{aligned}$$

$$\boxed{I_{CB} = 20.9A}$$

For this case, reducing the gain of the cascode required more current since it is in open-loop configuration, and the gain relies heavily on the output resistance, and the voltage division due to the input resistance.

(g) What is the 3-dB bandwidth of the amplifier for the calculated I_C? Express your answer in rad/s. (1 pt) For the calculated I_C, the small-signal parameters of the cascode are:

$$\begin{aligned}g_m = g_{m2} = \frac{I_C}{V_T} = \frac{20.9A}{26mV} &= 803.85mS \\ r_\pi = r_{\pi 2} = \frac{\beta}{g_m} &= 0.249\Omega \\ r_o = r_{o2} = \frac{V_A}{I_C} &= 4.78\Omega\end{aligned}$$

Thus,

$$\begin{aligned}\omega_z &= \frac{g_m}{C_{BC}} = 803.85Trad/s \\ \omega_{p1} &= -\frac{1}{C_{BC}\left(\frac{g_m}{g_{m2}}R_1 + R_1 + \frac{1}{g_{m2}}\right)} = -2.12Trad/s \\ \omega_{p2} &= -\frac{1}{g_{m2} r_{\pi 2} C_{BC2} r_{o2}} = -\frac{1}{\beta C_{BC2} r_{o2}} = -1.05Grad/s\end{aligned}$$

The GBW is approximately 4 Trad/s. Since the zero is much much greater than the GBW, it would have no significant effect at the 3-dB point. Therefore,

$$\boxed{\omega_{-3dB} = 1.05Grad/s}$$

(h) Did the cascode increase, decrease, or not affect the bandwidth of the amplifier? Explain this in relation to the Miller capacitance/effect. (1 pt) The cascode increases the bandwidth of the amplifier. The cascode divides the needed gain per stage, reducing the effect of the Miller capacitance. Therefore, for the same DC gain, the cascode still has a higher bandwidth.

TOTAL: 30 points.