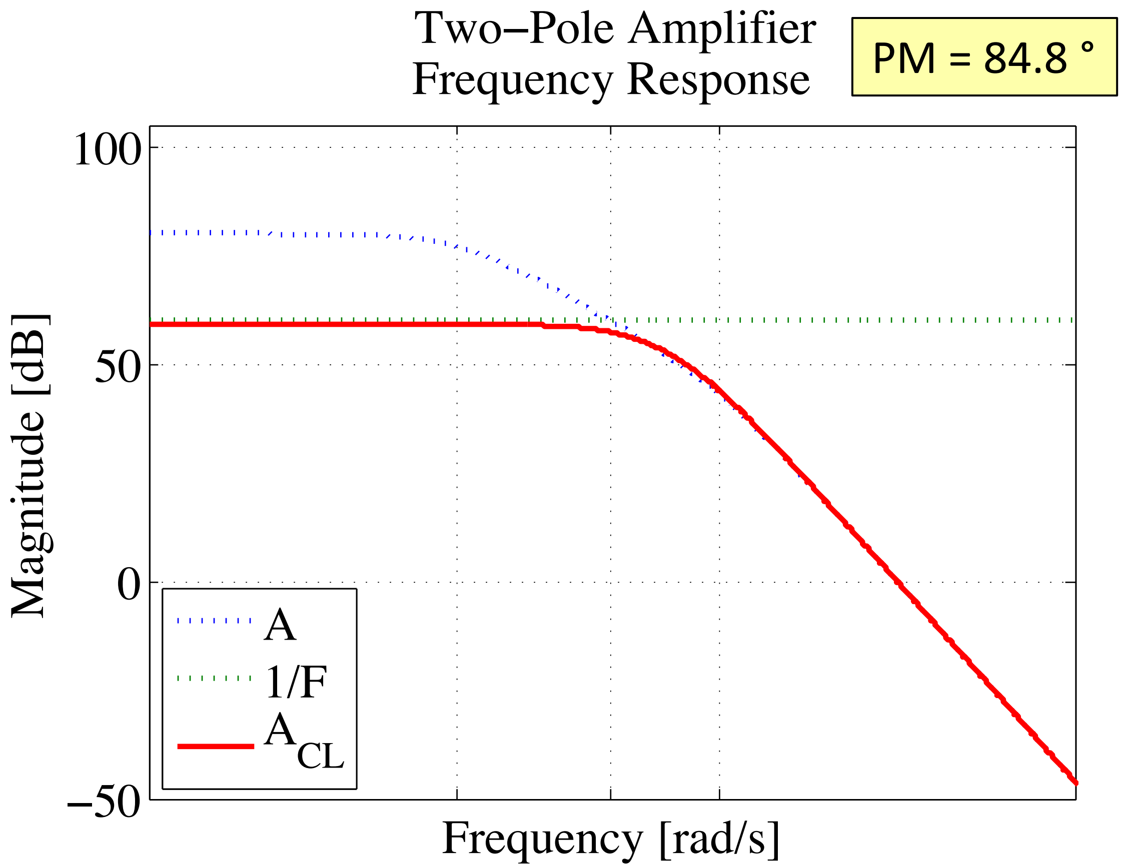
EEE 51: Second Semester 2017 - 2018

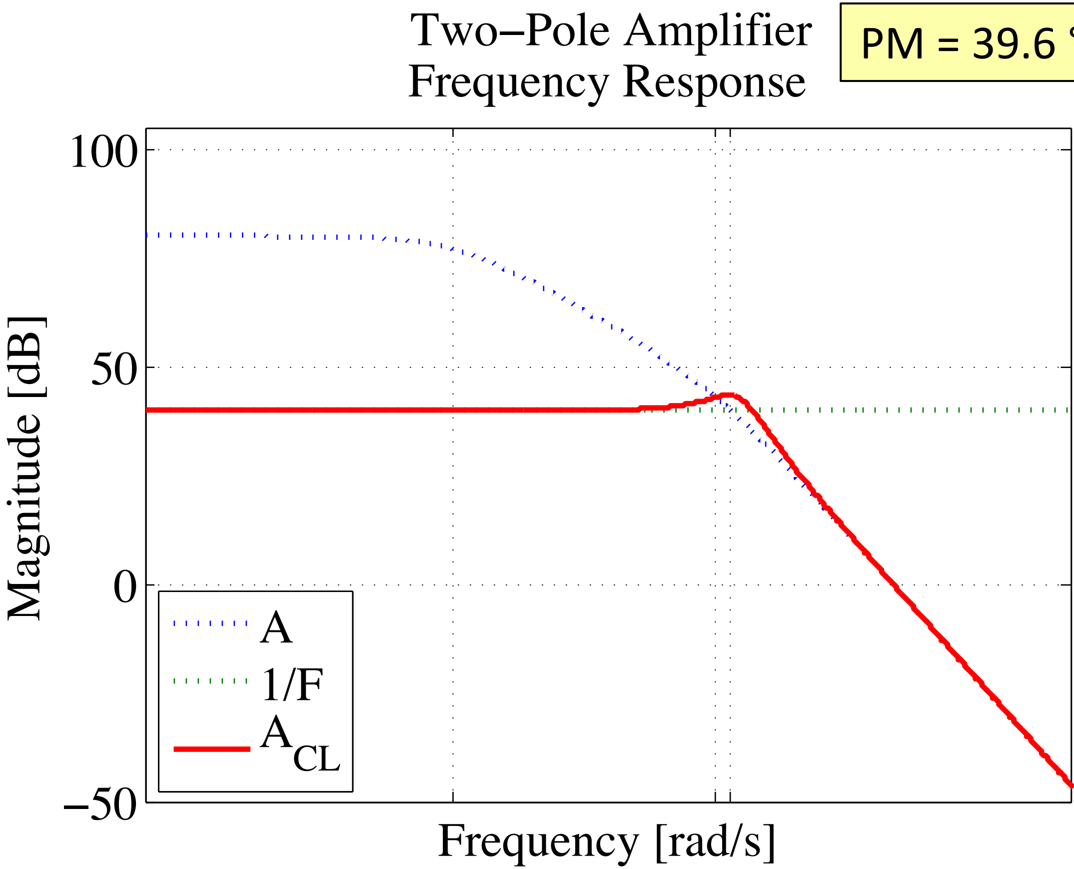
Lecture 22

Feedback Frequency Response and Oscillators

Varying Loop Gain: f = 0.001

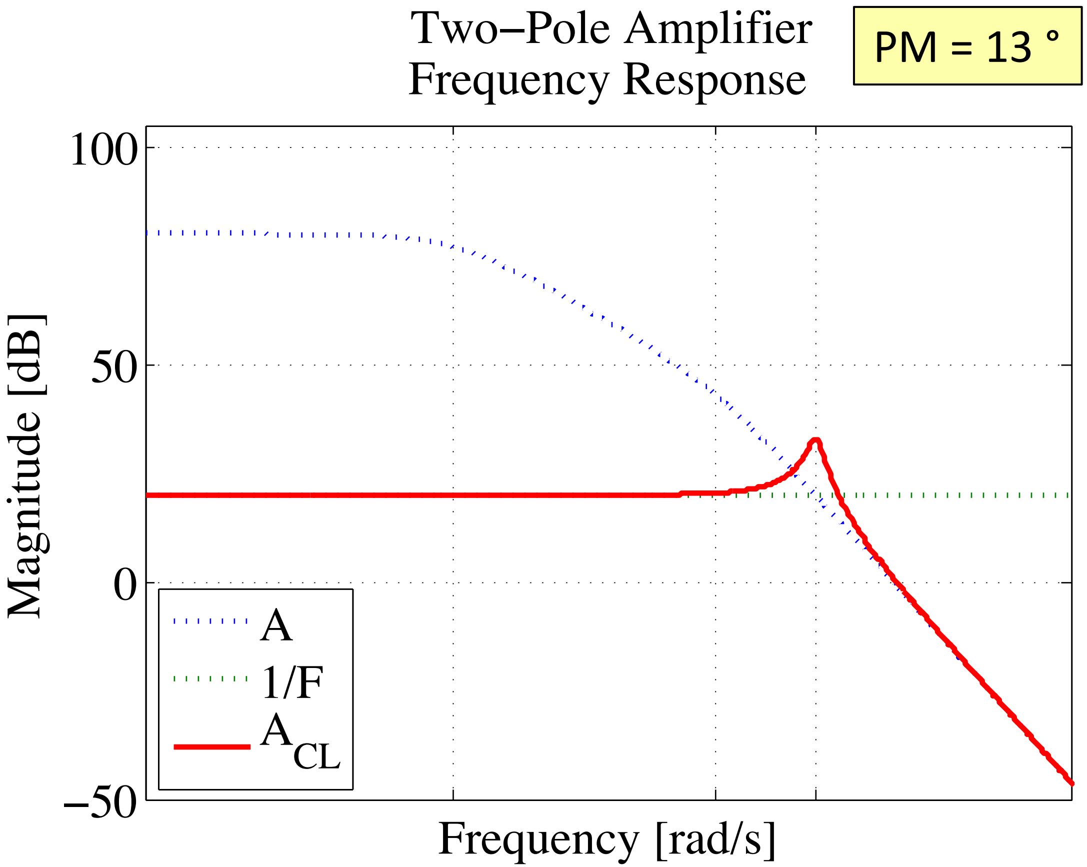


Varying Loop Gain: f = 0.01

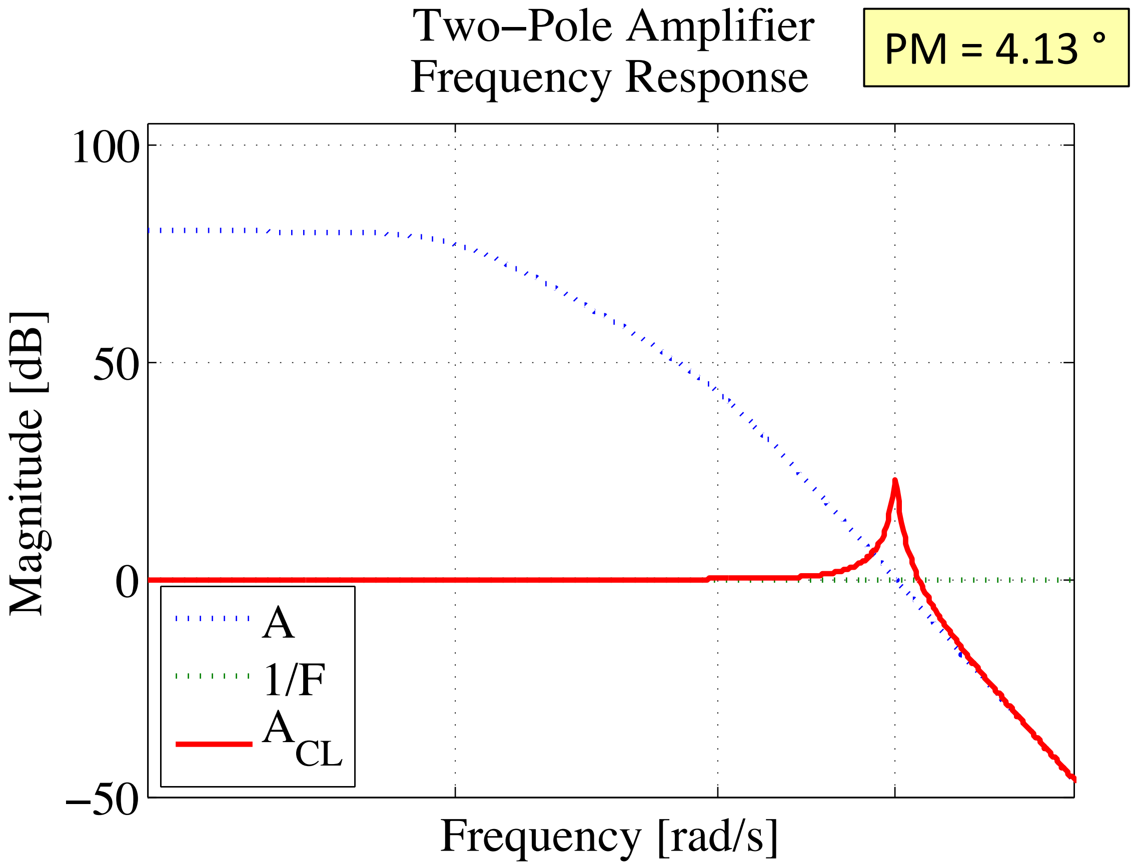


PM = 39.6 °

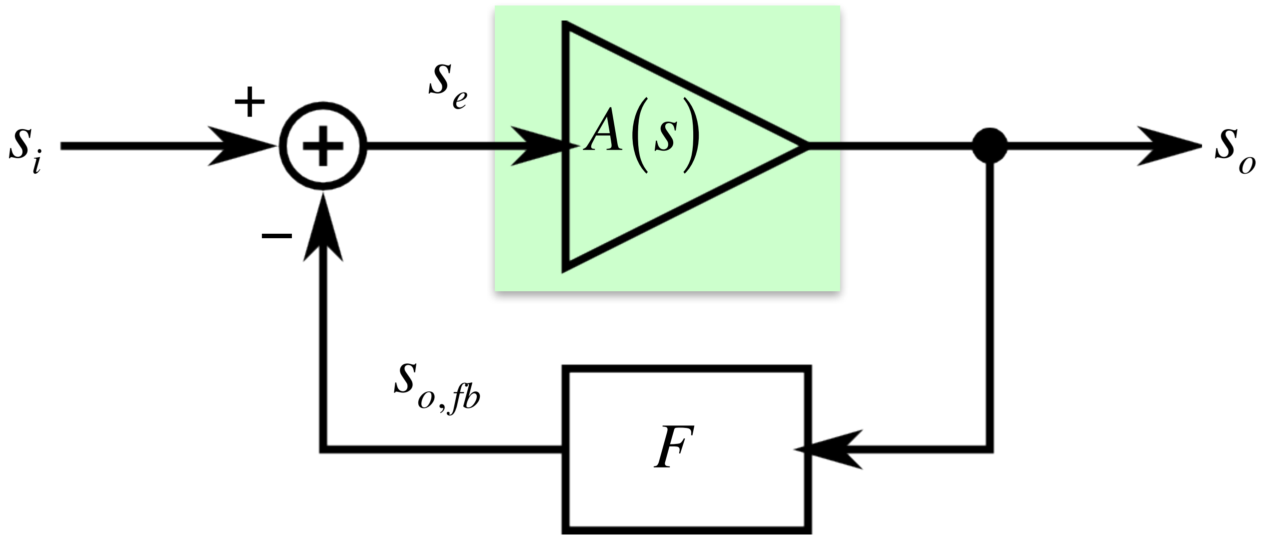
Varying Loop Gain: f = 0.1



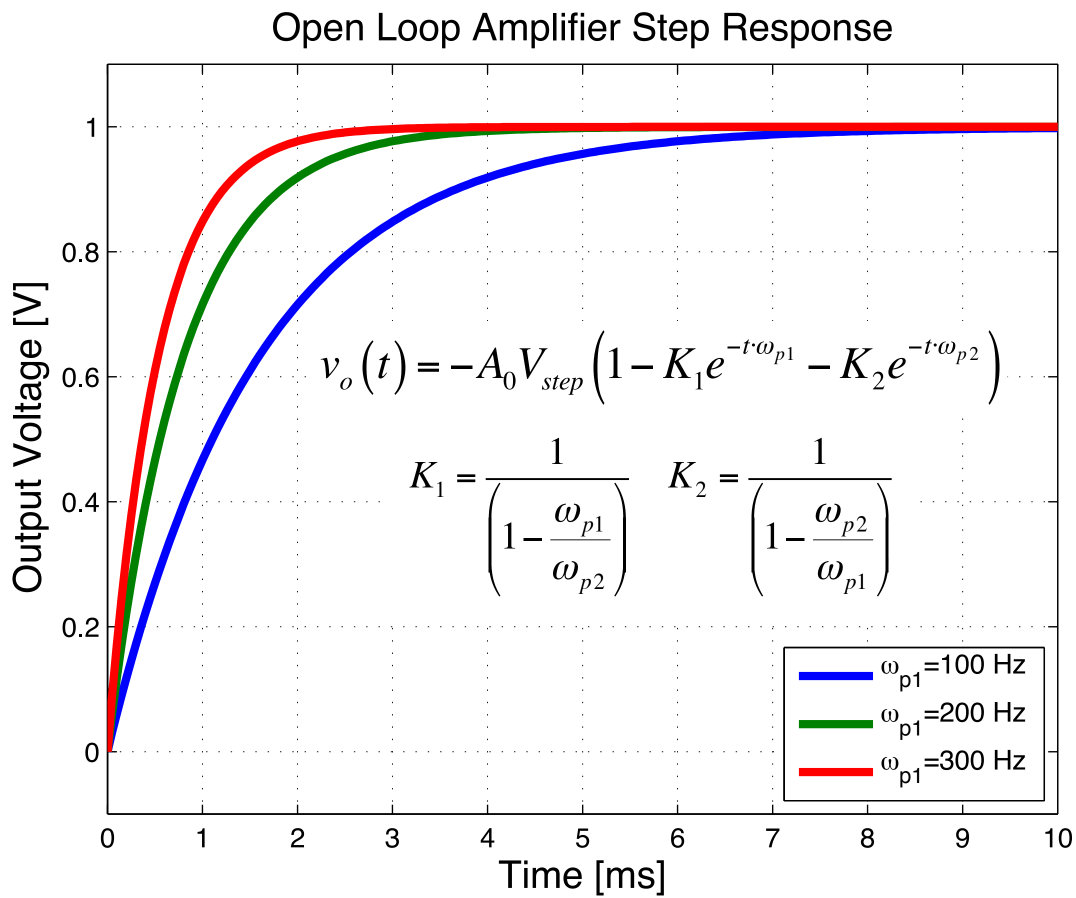
Varying Loop Gain: f = 1



Open Loop Amplifier Step Response

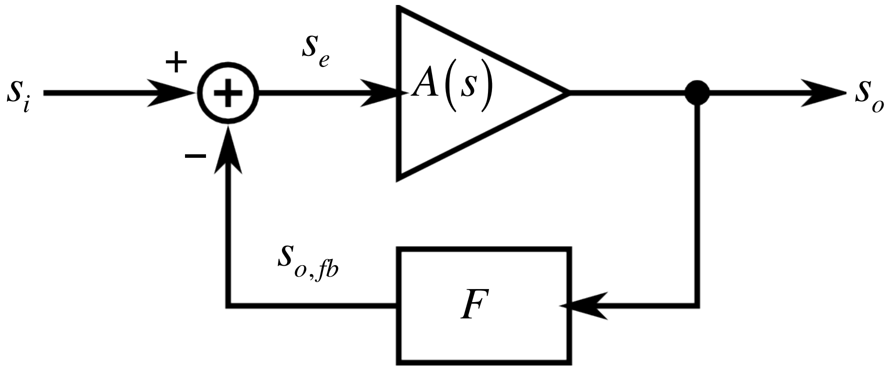


$$v_o(s) = \frac{-A_0}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)} \cdot \frac{V_{step}}{s}$$



Open Loop Amplifier Step Response
Output Voltage [V]
Time [ms]
0
0.2
0.4
0.6
0.8
1
0 1 2 3 4 5 6 7 8 9 10
$v_o(t) = -A_0 V_{step}\left(1 - K_1 e^{-t\cdot\omega_{p1}} - K_2 e^{-t\cdot\omega_{p2}}\right)$
$K_1 = \frac{1}{\left(1 - \frac{\omega_{p1}}{\omega_{p2}}\right)}$ $K_2 = \frac{1}{\left(1 - \frac{\omega_{p2}}{\omega_{p1}}\right)}$
ω_{p1}=100 Hz
ω_{p1}=200 Hz
ω_{p1}=300 Hz

Closed Loop Amplifier Step Response



$$T(s) = \frac{A_0 F}{\left(1 + \frac{s}{\omega_{p1}}\right)\left(1 + \frac{s}{\omega_{p2}}\right)}$$

$$\frac{v_o(s)}{v_i(s)} = -\frac{1}{F} \cdot \frac{T(s)}{1 + T(s)} = -\frac{1}{F} \cdot \frac{T_0}{1 + T_0 + s\left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p1}}\right) + s^2\left(\frac{1}{\omega_{p1}\omega_{p2}}\right)}$$

Closed Loop Amplifier Step Response

$$\frac{v_o(s)}{v_i(s)} = -\frac{1}{F}\cdot\frac{T(s)}{1+T(s)} = -\frac{1}{F}\cdot\frac{T_0}{1+T_0+s\left(\frac{1}{\omega_{p1}}+\frac{1}{\omega_{p2}}\right)+s^2\left(\frac{1}{\omega_{p1}\omega_{p2}}\right)}$$

$$= -\frac{1}{F}\cdot\frac{T_0}{1+T_0}\cdot\frac{1}{1+s\left(\frac{1}{\omega_{p1}}+\frac{1}{\omega_{p2}}\right)\frac{1}{1+T_0}+s^2\left(\frac{1}{\omega_{p1}\omega_{p2}}\right)\frac{1}{1+T_0}}$$

$$= -A_0\omega_{p1}\omega_{p2}\cdot\frac{1}{s^2+s\left(\omega_{p1}+\omega_{p2}\right)+\omega_{p1}\omega_{p2}\left(1+T_0\right)}$$

Closed Loop Amplifier Step Response

$$\frac{v_o(s)}{v_i(s)} = -A_0\omega_{p1}\omega_{p2}\cdot\frac{1}{s^2+s(\omega_{p1}+\omega_{p2})+\omega_{p1}\omega_{p2}(1+T_0)}$$

Roots of the denominator:

$$s = -\frac{(\omega_{p1}+\omega_{p2})}{2} \pm \frac{\sqrt{(\omega_{p1}+\omega_{p2})^2-4\omega_{p1}\omega_{p2}(1+T_0)}}{2}$$

$$= -\frac{(\omega_{p1}+\omega_{p2})}{2} \pm \frac{\sqrt{\omega_{p1}^2+2\omega_{p1}\omega_{p2}+\omega_{p2}^2-4\omega_{p1}\omega_{p2}-4\omega_{p1}\omega_{p2}T_0}}{2}$$

$$= -\frac{(\omega_{p1}+\omega_{p2})}{2} \pm \frac{\sqrt{\omega_{p1}^2-2\omega_{p1}\omega_{p2}+\omega_{p2}^2-4\omega_{p1}\omega_{p2}T_0}}{2}$$

$$= -\frac{(\omega_{p1}+\omega_{p2})}{2} \pm \frac{\sqrt{(\omega_{p1}-\omega_{p2})^2-4\omega_{p1}\omega_{p2}T_0}}{2}$$

Closed Loop Amplifier Step Response

$$s = -\frac{\left(\omega_{p1}+\omega_{p2}\right)}{2} \pm \frac{\sqrt{\left(\omega_{p1}-\omega_{p2}\right)^2 - 4\omega_{p1}\omega_{p2}T_0}}{2}$$

$$= -\frac{\left(\omega_{p1}+\omega_{p2}\right)}{2} \pm j\frac{\sqrt{4\omega_{p1}\omega_{p2}T_0 - \left(\omega_{p1}-\omega_{p2}\right)^2}}{2}$$

$$= -\rho \pm j\mu$$

$$\rho = \frac{\left(\omega_{p1}+\omega_{p2}\right)}{2} \qquad \mu = \frac{\sqrt{4\omega_{p1}\omega_{p2}T_0 - \left(\omega_{p1}-\omega_{p2}\right)^2}}{2}$$

Step Response

$$v_o(s) = -A_0\omega_{p1}\omega_{p2} \cdot \frac{1}{s^2 + s\left(\omega_{p1} + \omega_{p2}\right) + \omega_{p1}\omega_{p2}\left(1 + T_0\right)} \cdot \frac{V_{step}}{s}$$

$$= -\frac{A_0\omega_{p1}\omega_{p2}}{\left(s + [\rho - j\mu]\right)\left(s + [\rho + j\mu]\right)} \cdot \frac{V_{step}}{s}$$

$$= -\frac{A_0\omega_{p1}\omega_{p2}}{\left(\rho^2 + \mu^2\right)\left(1 + \frac{s}{\rho - j\mu}\right)\left(1 + \frac{s}{\rho + j\mu}\right)} \cdot \frac{V_{step}}{s}$$

$$= -\frac{A_0}{1 + T_0} \cdot \frac{1}{\left(1 + \frac{s}{\rho - j\mu}\right)\left(1 + \frac{s}{\rho + j\mu}\right)} \cdot \frac{V_{step}}{s}$$

For Real Roots:

$$\left(\omega_{p1}-\omega_{p2}\right)^2 \geq 4\omega_{p1}\omega_{p2}T_0$$

$$0=\left(\omega_{p1}-\omega_{p2}\right)^2-4\omega_{p1}\omega_{p2}T_0$$

$$0=\omega_{p1}^2-2\omega_{p1}\omega_{p2}+\omega_{p2}^2-4\omega_{p1}\omega_{p2}T_0$$

$$0=\omega_{p1}^2-2\left(1+2T_0\right)\omega_{p1}\omega_{p2}+\omega_{p2}^2$$

$$\omega_{p2,c}=\left(1+2T_0\right)\omega_{p1}\pm\sqrt{\left(1+2T_0\right)^2\omega_{p1}^2-\omega_{p1}^2}$$

$$=\left(1+2T_0\right)\omega_{p1}\pm\sqrt{\left(1+2T_0\right)^2\omega_{p1}^2-\omega_{p1}^2}$$

$$\approx\left(2T_0\pm 2T_0\right)\omega_{p1}=4T_0\omega_{p1}$$

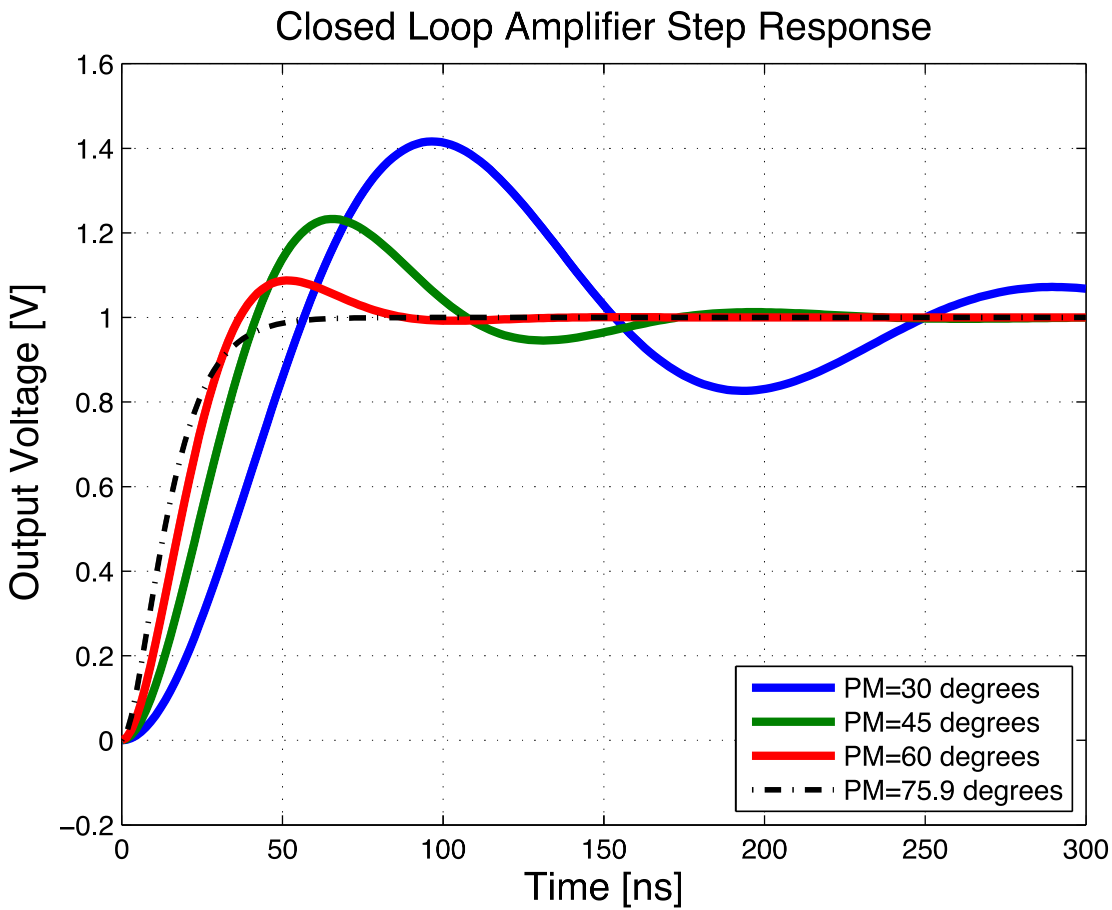
For Real Roots:

$$\omega_{p2,c} = 4T_0\omega_{p1}$$

$$\omega_u \cong T_0\omega_{p1}$$

$$\begin{aligned} PM_c &= \angle T(s) - (-180^\circ) \\ &= -\tan^{-1}(T_0) - \tan^{-1}\left(\frac{1}{4}\right) - (-180^\circ) \\ &\approx 90^\circ - \tan^{-1}\left(\frac{1}{4}\right) = \tan^{-1}(4) = 75.96^\circ \end{aligned}$$

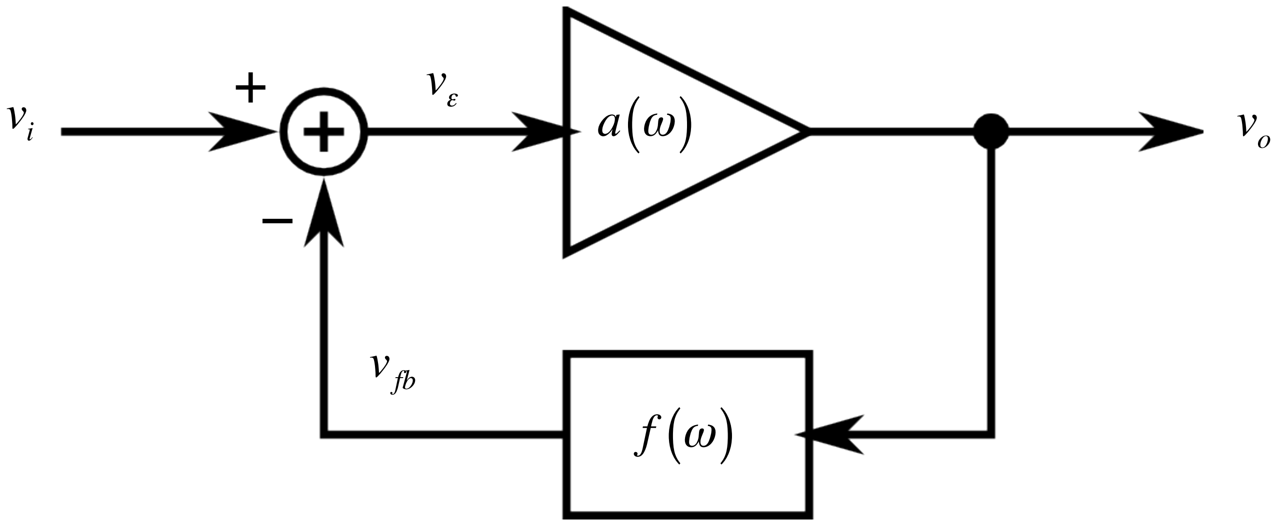
Closed Loop Step Response



Oscillators

- Sinusoidal Oscillators
  - General Structure
    - RC Oscillators
    - LC Oscillators
    - Crystal Oscillators

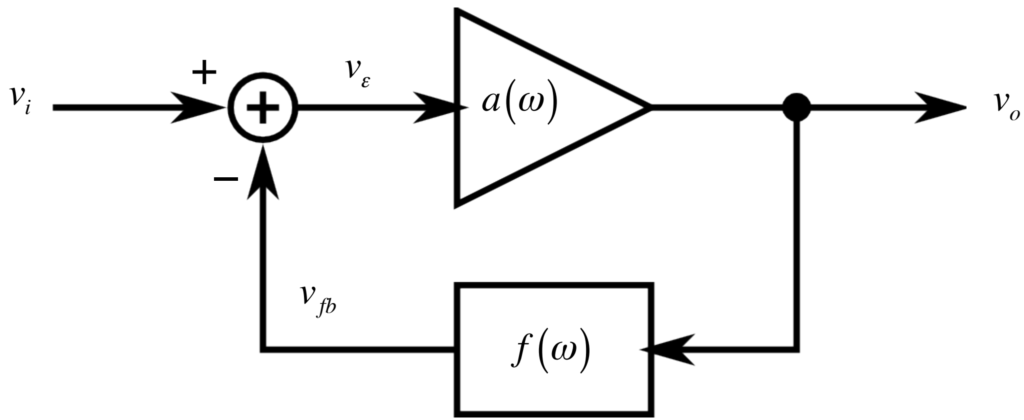
- A Feedback Network:



$$A_{CL}(\omega) = \frac{v_o}{v_i} = \frac{a(\omega)}{1 + a(\omega)\cdot f(\omega)} = \frac{a(\omega)}{1 + T(\omega)}$$

$$T(\omega) = a(\omega)\cdot f(\omega)$$

Negative Feedback



$$A_{CL}(\omega) = \frac{v_o}{v_i} = \frac{a(\omega)}{1 + a(\omega)\cdot f(\omega)} = \frac{a(\omega)}{1 + T(\omega)}$$

Take the case when

$$T(\omega) = a(\omega)\cdot f(\omega) = -1 = 1\angle 180°$$

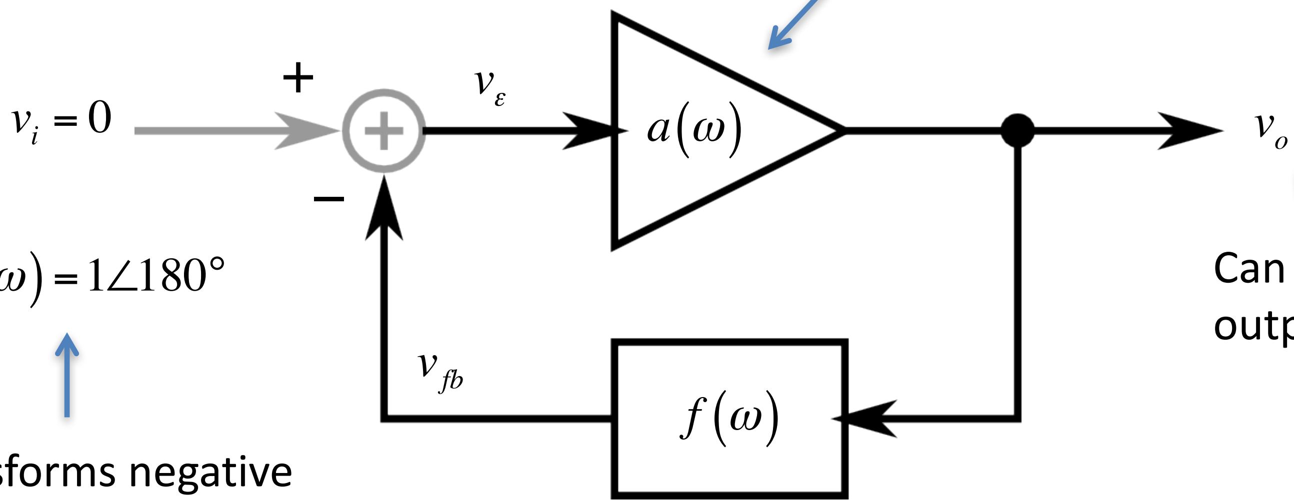
The closed-loop gain will be:

$$A_{CL}(\omega) = \frac{v_o}{v_i} = \frac{a(\omega)}{1 + T(\omega)} \to \infty$$

Thus, for $v_i = 0$, v_o can be non-zero!

Case: T(ω) = -1

Energy must be provided to the amplifier



Can have a finite output voltage

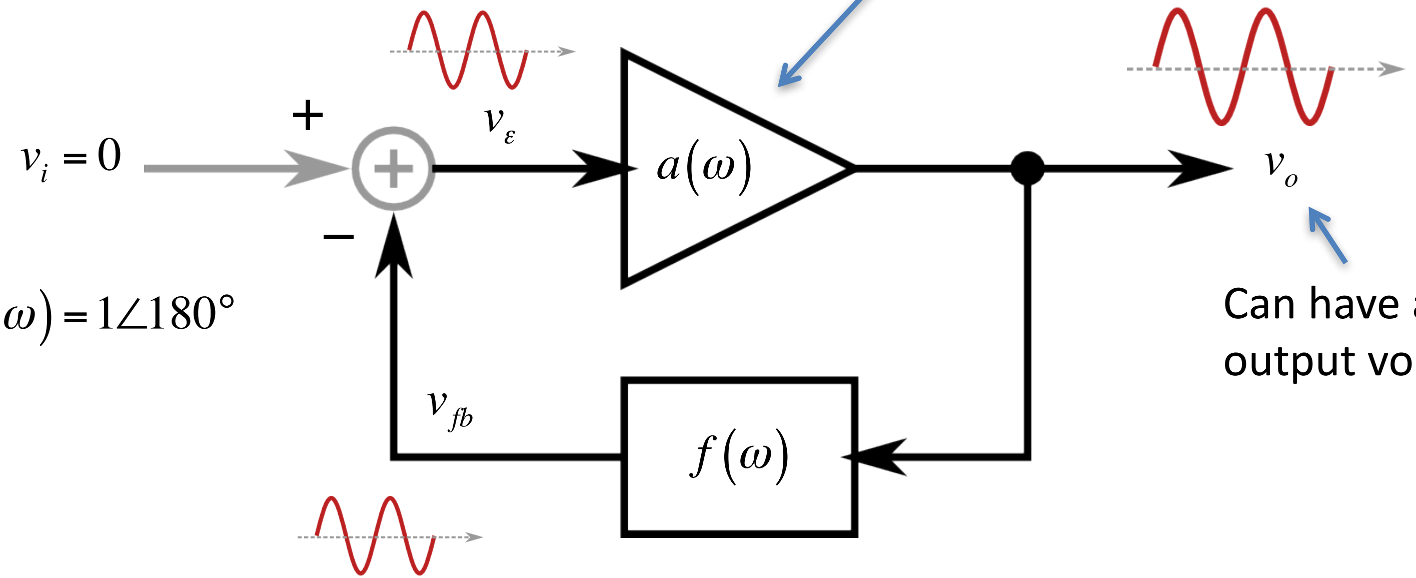
$$T(\omega) = 1\angle 180°$$

Transforms negative feedback to positive feedback

$$A_{CL}(\omega) = \frac{v_o}{v_i} = \frac{a(\omega)}{1+T(\omega)} \rightarrow \infty$$

Case: T(ω) = -1

Energy must be provided to the amplifier

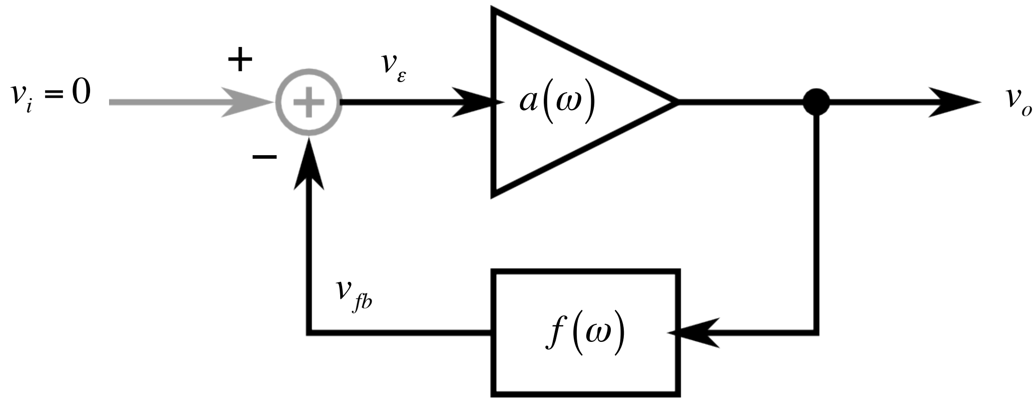


$$T(\omega) = 1\angle 180°$$

Can have a finite output voltage

$$A_{CL}(\omega) = \frac{v_o}{v_i} = \frac{a(\omega)}{1+T(\omega)} \rightarrow \infty$$

Barkhausen's Criterion

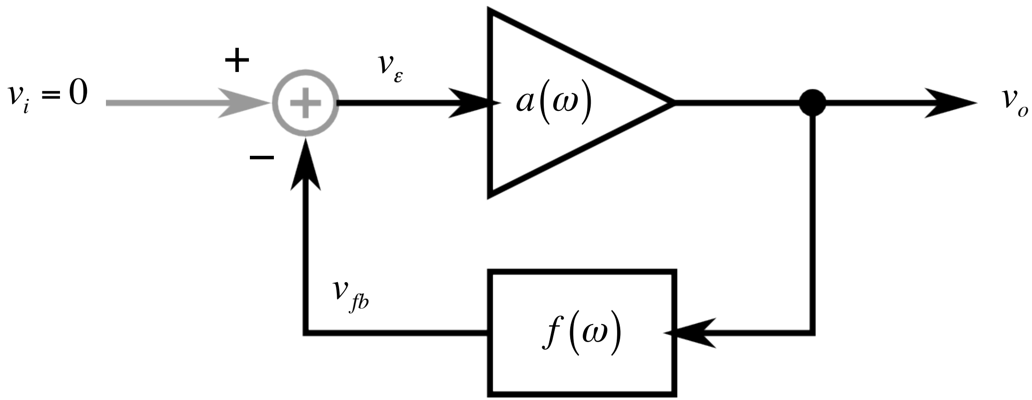


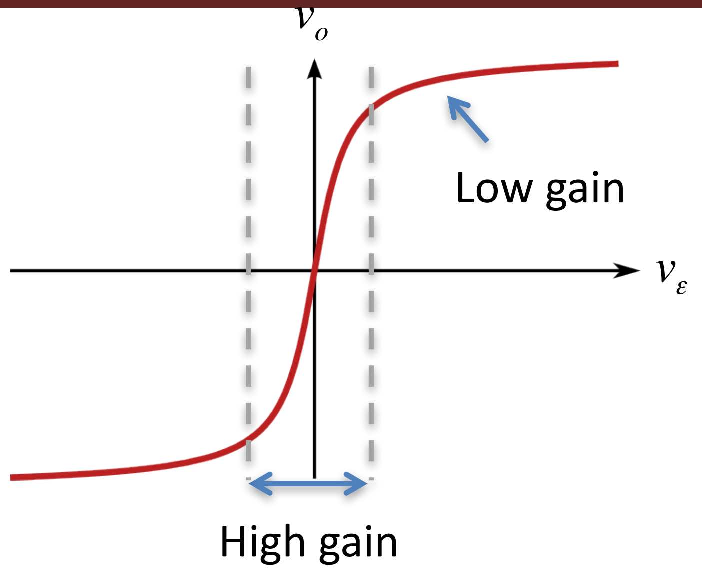
The circuit will be able to sustain steady state oscillations only at frequencies where:

$$|T(\omega)| = |a(\omega) \cdot f(\omega)| = 1$$

$$\angle T(\omega) = 180^\circ \cdot (2n+1) \quad n \in 0,1,2,3\ldots$$

Barkhausen's Criterion



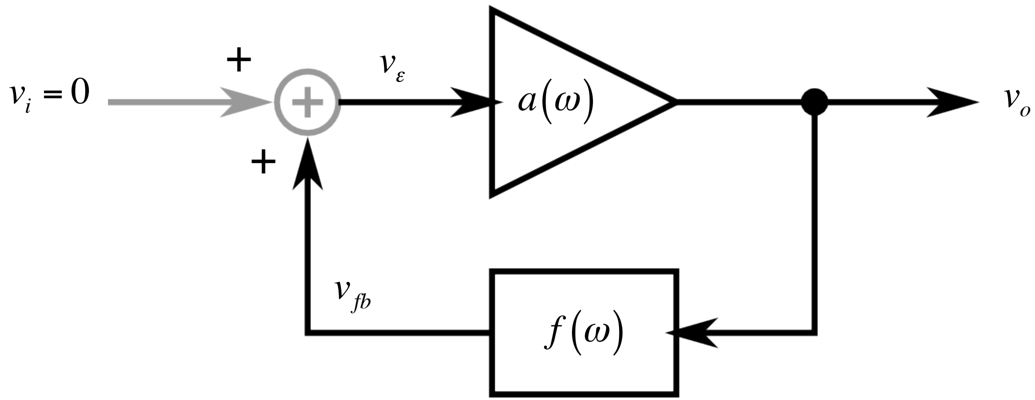


Note: If $|T(\omega)| = |a(\omega) \cdot f(\omega)| \geq 1$ and $\angle T(\omega) = 180° \cdot (2n+1)$ $n \in 0,1,2,3\ldots$

The circuit will meet |T(ω)| = 1 by going nonlinear

- Resulting in reduced amplifier gain

Barkhausen's Criterion (Positive Feedback)



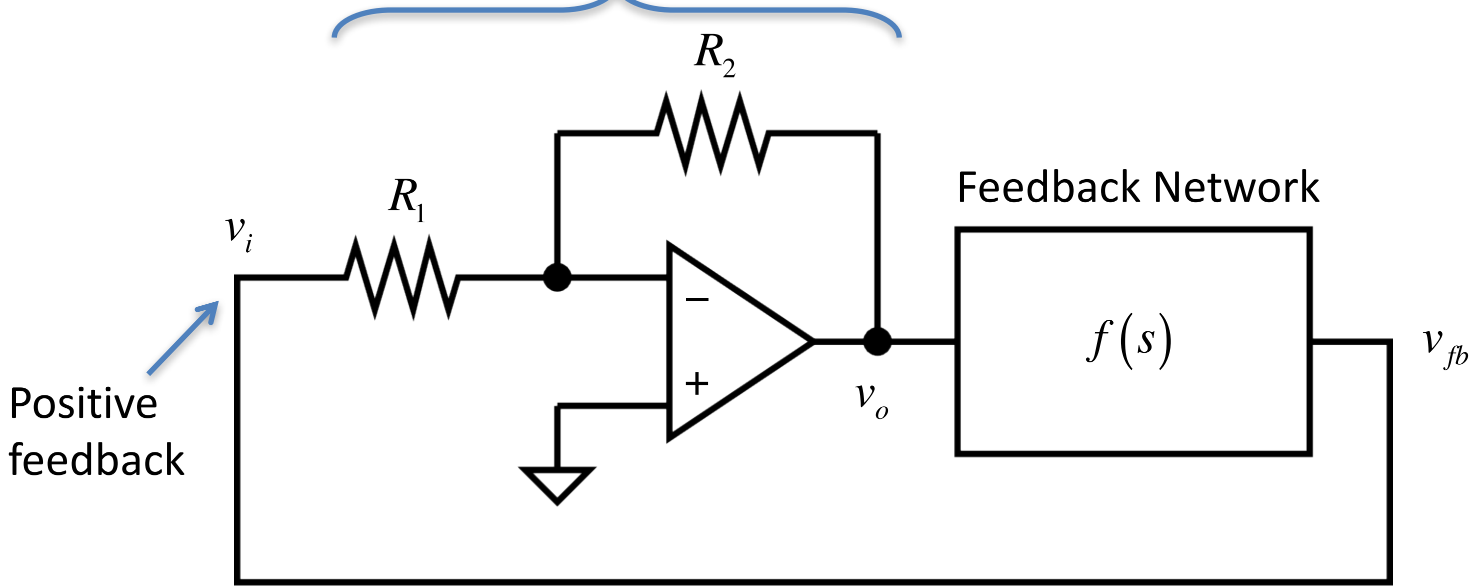
The circuit will be able to sustain steady state oscillations only at frequencies where:

$$|T(\omega)| = |a(\omega) \cdot f(\omega)| = 1$$

$$\angle T(\omega) = 360^\circ \cdot n \quad n \in 0,1,2,3\ldots$$

Example: The Phase Shift Oscillator

Inverting Amplifier $a(\omega) = -\frac{R_2}{R_1}$



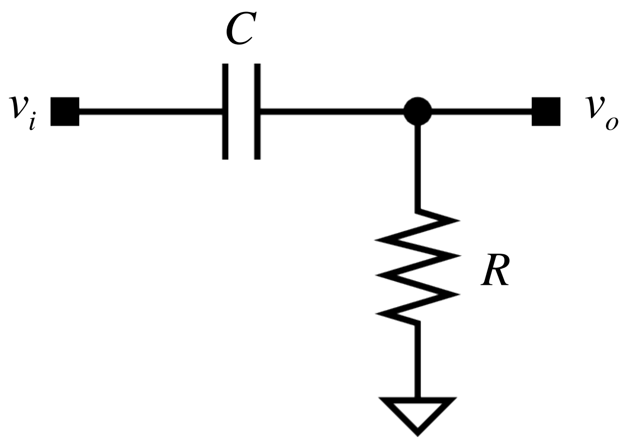
Loop gain needed to oscillate:

$$T(\omega) = a(\omega) \cdot f(\omega) = -\frac{R_2}{R_1} \cdot f(\omega) = 1\angle 0^\circ$$

$$\angle f(s) = -180^\circ$$

Phase Shift Feedback Network

- Consider the RC circuit



$$\frac{v_o}{v_i} = \frac{sRC}{1+sRC}$$

Magnitude: $$\left|\frac{v_o}{v_i}\right| = \frac{\omega RC}{\sqrt{1+\omega^2 (RC)^2}}$$

Phase: $$\angle\frac{v_o}{v_i} = 90° - \tan^{-1}\omega RC$$

Low frequencies:

$$\omega \to 0 \qquad \angle\frac{v_o}{v_i} \to 90°$$

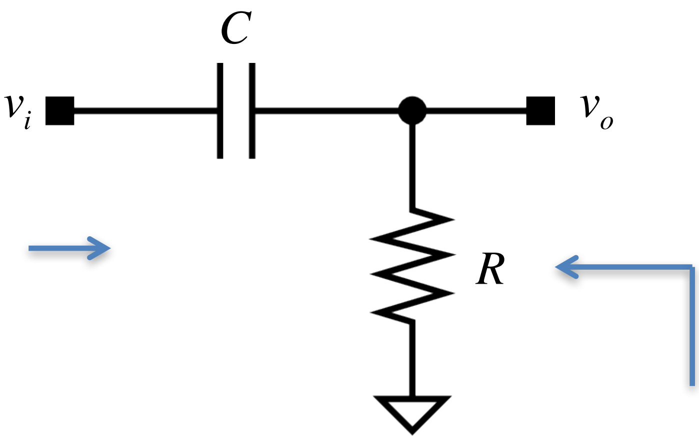
High frequencies:

$$\omega \to \infty \qquad \angle\frac{v_o}{v_i} \to 0°$$

Need at least 3 stages to get to -180°

Phase Shift Feedback Network

- Consider the RC circuit



$$Z_i = \frac{1}{sC} + R$$

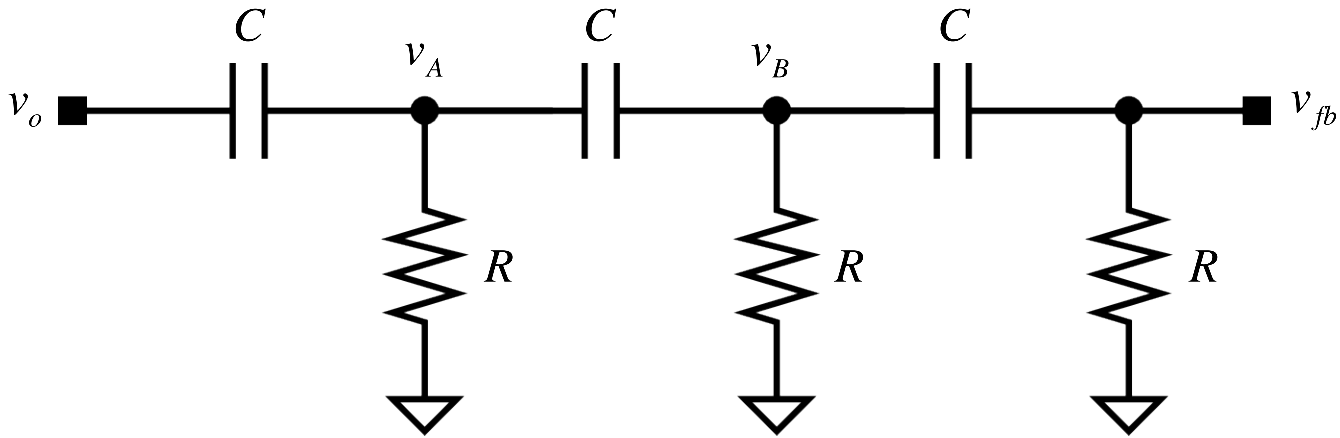
$$Z_o = \frac{R}{1+sRC}$$

$$\frac{v_o}{v_i} = \frac{sRC}{1+sRC}$$

$$\left|\frac{v_o}{v_i}\right| = \frac{\omega RC}{\sqrt{1+\omega^2 (RC)^2}}$$

$$\angle \frac{v_o}{v_i} = 90° - \tan^{-1} \omega RC$$

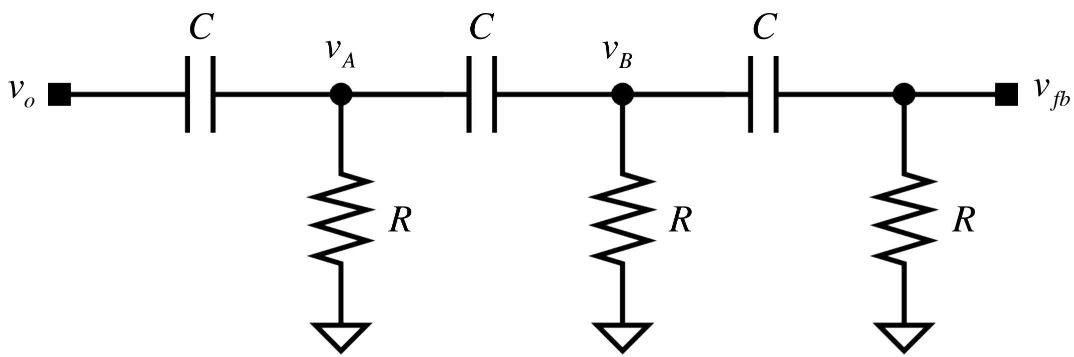
Phase Shift Feedback Network



$$f(s) = \frac{v_{fb}}{v_o} = \frac{1}{1 + \frac{6}{sRC} + \frac{5}{s^2(RC)^2} + \frac{1}{s^3(RC)^3}}$$

$$f(\omega) = \frac{v_{fb}}{v_o} = \frac{1}{1 - j\frac{6}{\omega RC} - \frac{5}{\omega^2(RC)^2} + j\frac{1}{\omega^3(RC)^3}} = \frac{1}{\left(1 - \frac{5}{\omega^2(RC)^2}\right) + j\left(\frac{1}{\omega^3(RC)^3} - \frac{6}{\omega RC}\right)}$$

Phase Shift Feedback Network



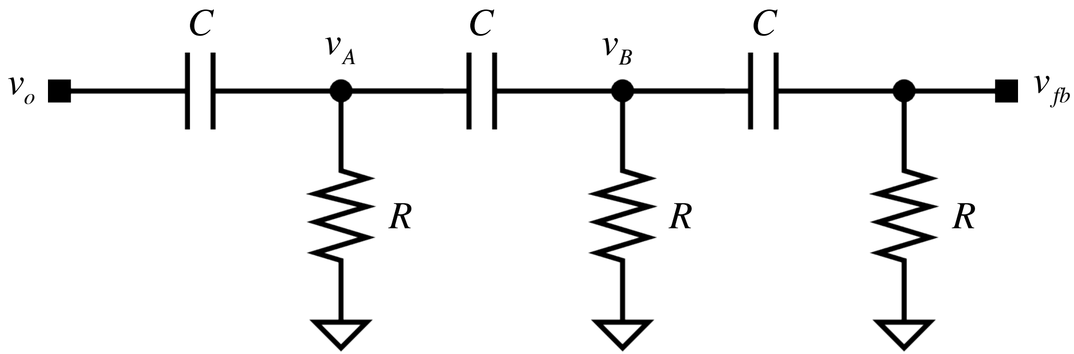
Need: $\angle f(s) = -180°$

$$f(\omega) = \frac{v_{fb}}{v_o} = \frac{1}{\left(1 - \frac{5}{\omega^2 (RC)^2}\right) + j\left(\frac{1}{\omega^3 (RC)^3} - \frac{6}{\omega RC}\right)}$$

$$\frac{1}{\omega_0^3 (RC)^3} - \frac{6}{\omega_0 RC} = 0 \quad \Rightarrow \quad \frac{1}{\omega_0 RC} = \sqrt{6}$$

For frequency ω_0, the phase shift of the feedback network will either be 0 or 180° depending on the sign of the real component

Phase Shift Feedback Network



Need: $\angle f(s) = -180°$

$$\frac{1}{\omega_0 RC} = \sqrt{6}$$

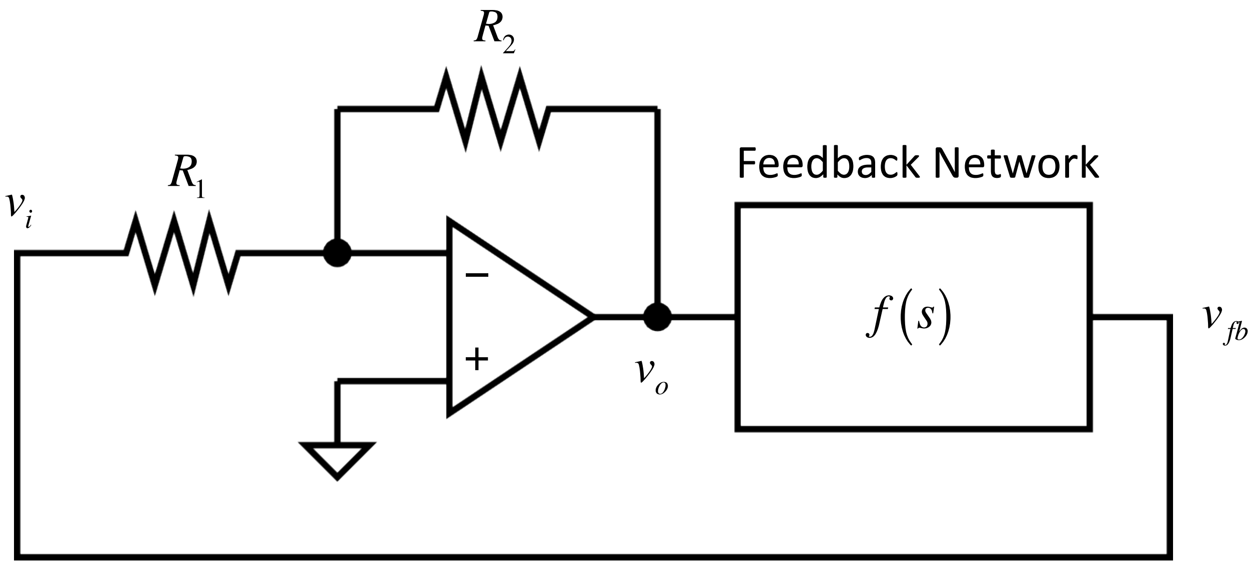
$$f(\omega) = \frac{v_{fb}}{v_o} = \frac{1}{\left(1 - \frac{5}{\omega^2 (RC)^2}\right) + j\left(\frac{1}{\omega^3 (RC)^3} - \frac{6}{\omega RC}\right)}$$

$$f(\omega_0) = \frac{v_{fb}}{v_o} = \frac{1}{\left(1 - \frac{5}{\omega_0^2 (RC)^2}\right)} = \frac{1}{1 - 5(6)} = -\frac{1}{29}$$

- Phase of the feedback network is 180°
- The oscillator can oscillate at frequency w_0

$$\omega_0 = \frac{1}{\sqrt{6}(RC)}$$

Phase Shift Feedback Network



$$f(\omega_0) = -\frac{1}{29}$$

$$\omega_0 = \frac{1}{\sqrt{6}(RC)}$$

What is the required amplifier gain at ω_0?

$$T(\omega_0) = a(\omega_0)\cdot f(\omega_0) = -\frac{R_2}{R_1}\cdot\left(-\frac{1}{29}\right) = 1\angle 0°$$

$$\frac{R_2}{R_1} = 29$$

Or is it $\frac{R_2}{R_1} \geq 29$?

Next Meeting

- Oscillators