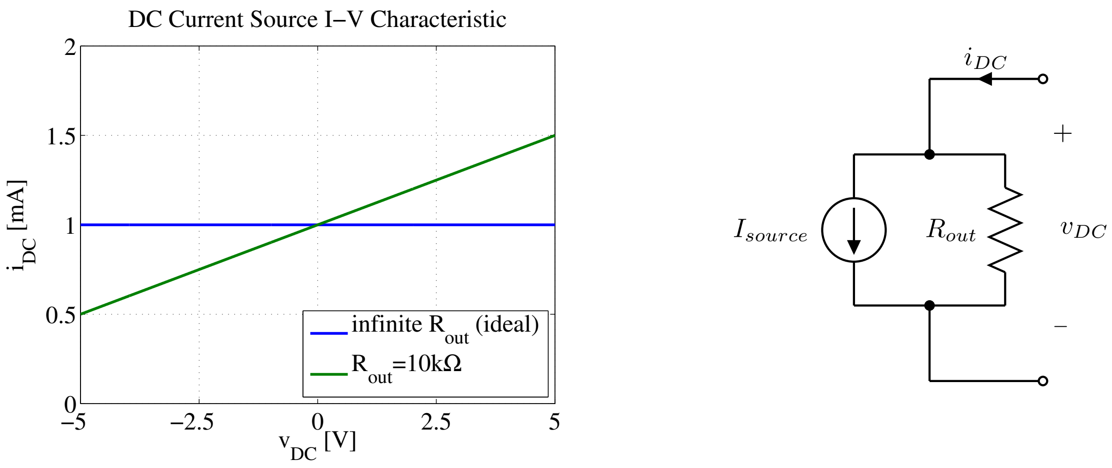


(a) I-V characteristic

(b) Circuit model

Figure 4.1: Current source characteristics.

4 Current Sources

The largest gain we can get out of a transistor is its intrinsic gain, $|a_o| = |g_m r_o|$. As we have seen previously, this intrinsic gain can be achieved if the transistor is biased by an ideal current source. In this section, we will look at how we build current sources, their characteristics, and how current source-biased amplifiers compare with their resistor-biased counterparts.

4.1 Non-Ideal Current Sources

An ideal current source maintains a constant current through its terminals, independent of the voltage applied across these terminals. However, in a real current source, the current through its terminals can vary due to an applied voltage, as seen from the I-V characteristic in Fig. 4.1a. We can model this non-ideality as the output resistance of a current source, as shown in Fig. 4.1b, where the slope of the I-V characteristic is equal to $\frac{1}{R_{out}}$.

The total current, i_{DC}, can then be expressed as

$$i_{DC} = I_{source} + \frac{v_{DC}}{R_{out}} \tag{4.1}$$

Thus, as the output resistance, R_{out}, is increased, the closer the current source is to an ideal current source.

4.1.1 Biasing Transistor Amplifiers

Let us examine the implications of using a current source, instead of a resistor, to bias the common-emitter amplifier. Recall that the small signal gain of the common-source amplifier is

$$A_v = -g_m \left(r_o \parallel R_L\right) \tag{4.2}$$

where R_L is the load resistor we use to deliver $I_{C,Q}$ from the supply voltage, V_{CC}, to the transistor. To keep the transistor in the forward active region,

$$V_{OUT} = V_{CC} - I_{C,Q} R_L > V_{CE,sat} \tag{4.3}$$

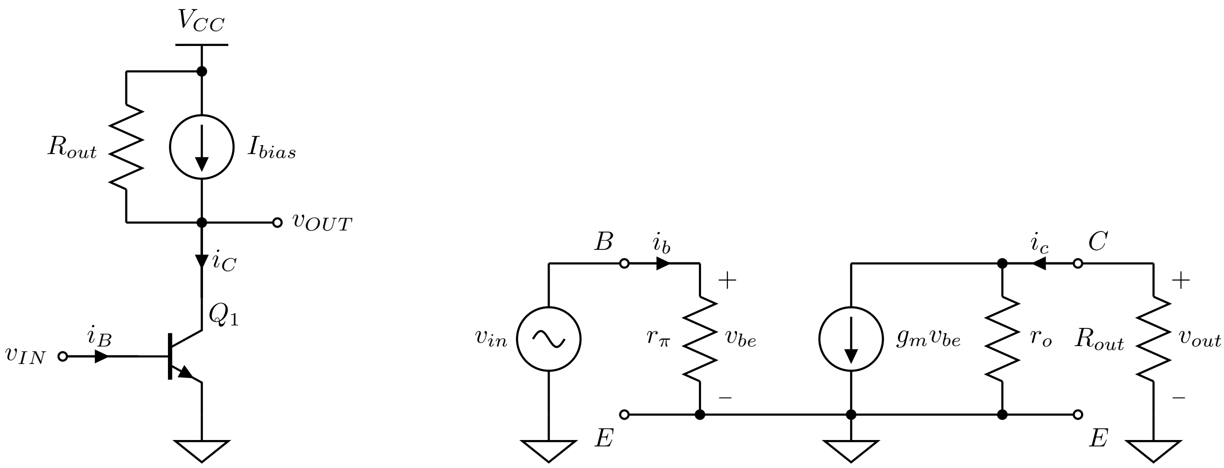
If we want a gain close to the intrinsic transistor gain, a_o, we want to make R_L as large as possible, as seen in Eq. 4.2. However, if we want to keep $I_{C,Q}$ constant[a], then from Eq. 4.3, increasing R_L will result in a smaller output voltage that can potentially drive the transistor into saturation. One way to resolve this problem is to increase the supply voltage, V_{CC}. In some cases, such as for high-power applications, the supply voltage can be increased to around 48 V or higher.

In most applications, however, we are limited to using supply voltages of around 1 V to 12 V. An alternative way to provide the quiescent DC collector current is to use a current source, as seen in Fig. 4.2a.

From the small signal equivalent circuit in Fig. 4.2b, we can see that the gain is now

$$A_v = -g_m \left(r_o \parallel R_{out}\right) \tag{4.4}$$

[a] Most amplifiers need a certain transconductance value in order to achieve a certain specification. Since the quiescent DC collector (or drain) current determines the transconductance, g_m, a specific value of $I_{C,Q}$ (or $I_{D,Q}$) is often required.



(a) Schematic (b) Small signal equivalent circuit

Figure 4.2: A common-emitter amplifier biased using a current source.

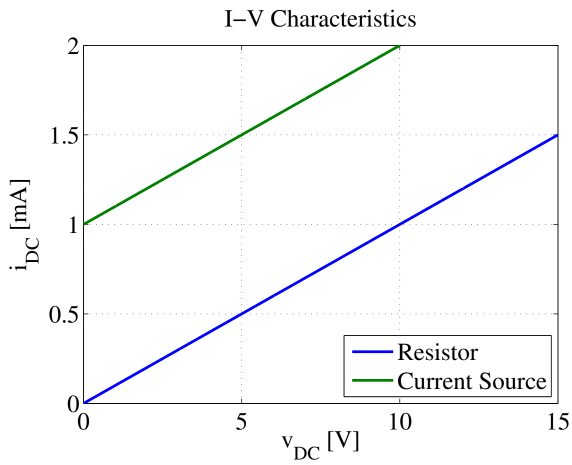


Figure 4.3: The I-V characteristics of a $10\,\text{k}\Omega$ resistor and a $1\,\text{mA}$ current source with $R_{out} = 10\,\text{k}\Omega$.

where R_{out} is the output resistance of the current source. Again, to get the a small signal voltage gain close to a_o, we want R_{out} to be as large as possible. The quiescent DC collector current can then be expressed as

$$I_{C,Q} = I_{bias} + \frac{V_{CC} - V_{OUT}}{R_{out}} \tag{4.5}$$

Thus, as $R_{out} \to \infty$, $I_{C,Q} \to I_{bias}$. The output voltage is then equal to

$$V_{OUT} = V_{CC} - R_{out}\left(I_{C,Q} - I_{bias}\right) \tag{4.6}$$

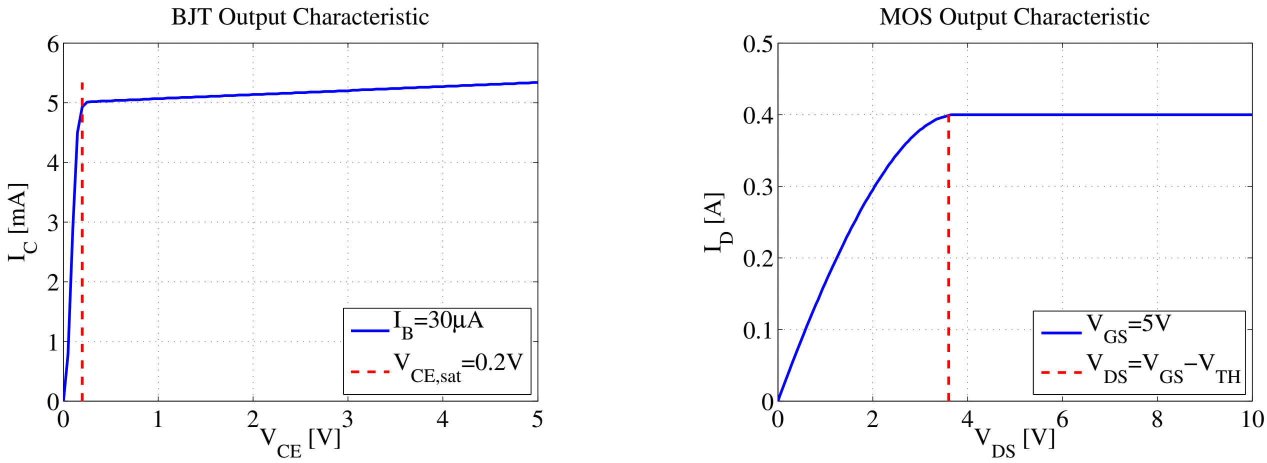
Looking Eq. 4.6, we can see that by making R_{out} large, $(I_{C,Q} - I_{bias})$ becomes smaller, removing the need for higher supply voltages. If we can manage to create a current source with $R_{out} \to \infty$, we can make $A_v \to a_o$.

Another way of looking at this is by looking at the I-V charcteristics of both the resistor and the current source, as seen in Fig. 4.3. In order to obtain a current of $1.5\,\text{mA}$, the current source just needs a voltage of $5\,\text{V}$, while the resistor needs $15\,\text{V}$. This is due to the fact that being linear, the resistor has to pass through the point when both the current and voltage is zero. The current source, on the other hand, is non-linear, and thus, does not have to pass through the same zero voltage and current point.

Thus, using current sources for biasing transistor amplifiers allow us to remove the relationship between the DC output voltage and the small signal voltage gain, unlike the resistor-biased amplifier, which uses R_L to control both V_{OUT} and A_v.

4.1.2 Transistor Current Sources

How do we build current sources? Let us examine the output characteristic of a typical BJT, as shown in Fig. 4.4a. In the forward-active region, the BJT output characteristic looks very much like a current source with an output resistance. Recall that the slope of the BJT output characteristic is equal to $\frac{1}{r_o}$, thus, if we think of the transistor as



(a) BJT (b) MOSFET

Figure 4.4: Transistor output characterisics.

a current source, we will have

$$I_{source} = I_C = I_S \cdot e^{\frac{V_{BE}}{V_T}} \cdot \left(1 + \frac{V_{CE}}{V_A}\right) = \beta \cdot I_B \tag{4.7}$$

and its output resistance can be expressed as

$$R_{out} = \left(\frac{\partial I_C}{\partial V_{CE}}\right)^{-1} = r_o = \frac{V_A}{I_C} \tag{4.8}$$

Note that this output resistance remains relatively large as long as the transistor is in its forward-active region. We can see from Fig. 4.4a that as soon as the transistor enters the saturation region, the slope increases, thus decreasing the output resistance. Thus, in current sources built using transistors, we can define a minimum voltage, $V_{\min}$, such that below this minimum voltage, the output resistance drops significantly. For the case of a BJT used as a current source,

$$V_{\min} = V_{CE,sat} \tag{4.9}$$

Similar to the BJT, in the saturation region, the output characteristic of a MOSFET, shown in Fig. 4.4b, looks very similar to the I-V curve of a current source. Thus, if we use a MOSFET as a current source, we get

$$I_{source} = I_D = k \cdot (V_{GS} - V_{TH})^2 \cdot (1 + \lambda V_{DS}) \tag{4.10}$$

$$R_{out} = \left(\frac{\partial I_D}{\partial V_{GS}}\right)^{-1} = r_o = \frac{1}{\lambda \cdot I_D} \tag{4.11}$$

$$V_{\min} = V_{DSAT} = V_{GS} - V_{TH} \tag{4.12}$$

We can therefore create a simple current source out of a transistor by providing the appropriate input DC signal, that is, V_{BE} (or I_B), for a BJT, or V_{GS} for a MOSFET. Fig. 4.5 shows a simple resistive voltage divider bias that can be used to realize a BJT current source. Is this a good idea?

Let us take a closer look at the current source in Fig. 4.5. In order to get a collector current of 1 mA, and assuming $I_S = 2 \times 10^{-16}$ A, and that the output voltage is negligible compared to V_A[b] and larger than $V_{CE,sat}$, we need a base-emitter voltage of

$$V_{BE} = V_T \ln\left(\frac{I_C}{I_S}\right) = 26\,\mathrm{mV} \cdot \ln\left(\frac{1\,\mathrm{mA}}{2 \times 10^{-16}\,\mathrm{A}}\right) = 0.7603\,\mathrm{V} \tag{4.13}$$

If we increase the base-to-emitter voltage by just 10 mV (or 1.32%), and use $V_{BE} = 0.7703$ V, we would get a collector current of 1.5 mA, which is a 50% increase! Thus, we have to be able to set V_{BE} to a certain precision, which is usually hard to do.

[b] In the context of current sources, base-width modulation and channel length modulation play a big role. However, including V_A or λ in every equation results in very long and cumbersome expressions that could prevent us from obtaining an intuitive grasp of the ideas. Thus, in most cases, we will assume that $V_A \to \infty$ or $\lambda = 0$, and just point out situations when these parameters become significant.

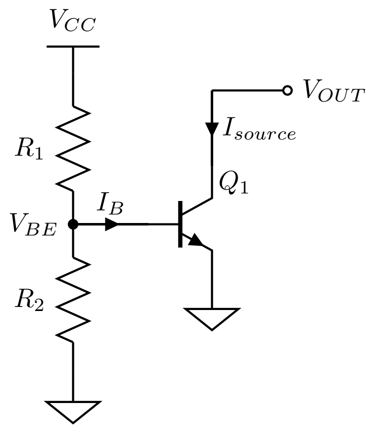


Figure 4.5: Resistive voltage divider BJT current source.

If we want a maximum current error of $\pm 1\%$ (or $\pm 10\,\mu$A), the we would require that V_{BE} be within $0.7603\,\text{V} \pm 260\,\mu$V. This is due to the exponential relationship between V_{BE} and I_C. We can express the V_{BE} as

$$V_{BE} = I_2 R_2 = (I_1 - I_B) \cdot R_2 = \left(\frac{V_{CC} - V_{BE}}{R_1} - \frac{I_S}{\beta} \cdot e^{\frac{V_{BE}}{V_T}} \right) \cdot R_2 \tag{4.14}$$

For example, if $\beta = 200$ and $V_{CC} = 5\,\text{V}$, and if we select $R_1 = 10\,\text{k}\Omega$, then

$$R_2 = \frac{V_{BE}}{\frac{V_{CC} - V_{BE}}{R_1} - \frac{I_C}{\beta}} = 1.8146\,\text{k}\Omega \tag{4.15}$$

Solving for β, we get

$$\beta = \frac{I_S \cdot e^{\frac{V_{BE}}{V_T}}}{\frac{V_{CC}}{R_1} - V_{BE}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)} \tag{4.16}$$

Thus, if we want a maximum $\Delta V_{BE} = 260\,\mu$V, from Eq. 4.16 we can calculate the maximum and minimum β we can tolerate as

$$\beta_{\max} = \frac{I_S \cdot e^{\frac{V_{BE} + \Delta V_{BE}}{V_T}}}{\frac{V_{CC}}{R_1} - (V_{BE} + \Delta V_{BE})\left(\frac{1}{R_1} + \frac{1}{R_2}\right)} = 209 \tag{4.17}$$

$$\beta_{\min} = \frac{I_S \cdot e^{\frac{V_{BE} - \Delta V_{BE}}{V_T}}}{\frac{V_{CC}}{R_1} - (V_{BE} - \Delta V_{BE})\left(\frac{1}{R_1} + \frac{1}{R_2}\right)} = 191 \tag{4.18}$$

Note that this means the required tolerance for the transistor β is $\frac{\beta_{\max} - \beta}{\beta} = 4.52\%$. As we have mentioned before, this is very hard to obtain since typical β-variations could reach up to 50%.

For MOSFETs, the main problem turns out to be the variations in the value of the threshold voltage, V_{TH}. In typical MOSFETs, the threshold voltage could also vary by as much as 50% as well. Thus, with even perfect resistors, it is very hard to control the drain current due to the quadratic relationship between V_{TH} and I_D, as seen in Eq. 4.10.

Thus, to get a usable current source using a resistive divider, we need to (1) tightly control the the values of the supply voltage, the resistors, and the transistor β or V_{TH}, as well as (2) be able specify the values of these components to a very high degree of precision. In most cases, using very high quality components would be very costly, and most of the time, components with these specifications do not exist commercially.

4.2 The BJT Current Mirror

An alternative biasing strategy is to use another transistor to generate the required V_{BE}, instead of resistors. Consider the circuit in Fig. 4.6. Notice that since $V_{CB} = 0$, the transistor will never go into the saturation region as long as the base-emitter junction is forward-biased, since $V_{BE} = V_{CE} > V_{CE,sat}$. If we apply an input current I_o, the collector current would be

$$I_C = I_o - I_B = I_o - \frac{I_C}{\beta} \tag{4.19}$$

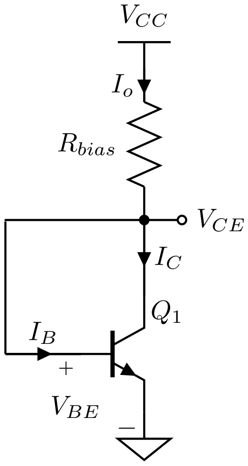


Figure 4.6: A diode-connected BJT.

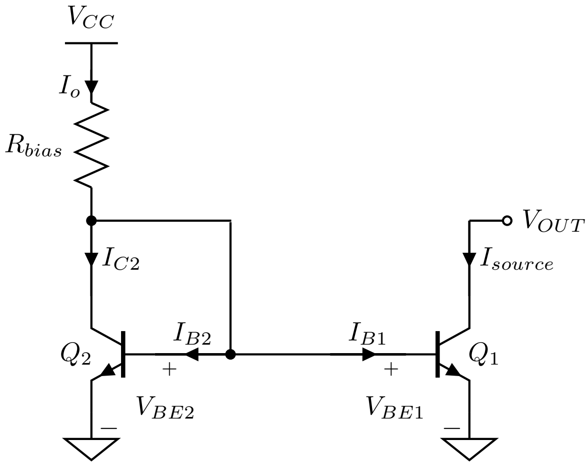


Figure 4.7: A resistor-biased BJT current mirror.

Thus, assuming $V_A \to \infty$, the base-emitter voltage can be written as

$$V_{BE} = V_T \ln\left(\frac{I_C}{I_S}\right) = V_T \ln\left(\frac{I_o}{I_S} \cdot \frac{\beta}{\beta+1}\right) \tag{4.20}$$

Since the transistor is now effectively a two-terminal device, and follows the I-V relationship similar to a diode, as given in Eq. 4.20, the transistor in Fig. 4.6 is often referred to as a *diode-connected* transistor.

Writing the KVL equation at the for the base-emitter loop, we get

$$V_{CC} - I_o R_{bias} - V_{BE} = 0 \tag{4.21}$$

Combining Eqs. 4.21 and 4.20, we get an expression for the R_{bias} needed to generate a specific V_{BE}, as given by

$$R_{bias} = \frac{V_{CC} - V_{BE}}{\left(1 + \frac{1}{\beta}\right) \cdot I_S \cdot e^{\frac{V_{BE}}{V_T}} \cdot \left(1 + \frac{V_{BE}}{V_A}\right)} \tag{4.22}$$

We can then use this diode connected transistor and resistor combination to generate the required V_{BE} of a transistor used as a current source, as seen in Fig. 4.7. Since $V_{BE1} = V_{BE2} = V_{BE}$, and assuming that Q_1 and Q_2 are identical transistors, giving us $\beta_1 = \beta_2 = \beta$ and $I_{S1} = I_{S2} = I_S$, we get

$$V_T \ln\left(\frac{I_{C1}}{I_{S1} \cdot \left(1 + \frac{V_{CE1}}{V_A}\right)}\right) = V_T \ln\left(\frac{I_{C2}}{I_{S2} \cdot \left(1 + \frac{V_{CE2}}{V_A}\right)}\right) \tag{4.23}$$

which tells us that

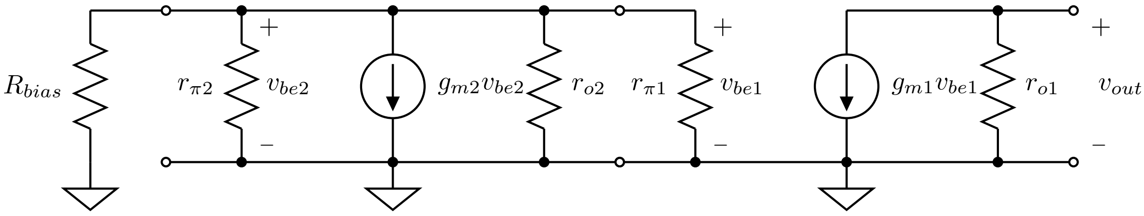


Figure 4.8: The BJT current mirror small signal equivalent circuit.

$$I_{source} = I_{C1} = I_{C2} \cdot \frac{\left(1 + \frac{V_{CE1}}{V_A}\right)}{\left(1 + \frac{V_{CE2}}{V_A}\right)} = I_{C2} \cdot \frac{V_A + V_{CE1}}{V_A + V_{CE2}} \tag{4.24}$$

We can express I_{C2} as

$$I_{C2} = I_o - I_{B1} - I_{B2} = I_o - \frac{I_{source}}{\beta} - \frac{I_{C2}}{\beta} \tag{4.25}$$

and solving for I_{C2} in terms of I_o, we get

$$I_{C2} = I_o \frac{\beta}{\beta + 1} - I_{source} \cdot \frac{1}{\beta + 1} \tag{4.26}$$

Thus, the output current of the BJT current source is

$$I_{source} = I_o \frac{\frac{\beta}{\beta+1} \cdot \frac{V_A+V_{CE1}}{V_A+V_{CE2}}}{1 + \frac{1}{\beta+1} \cdot \frac{V_A+V_{CE1}}{V_A+V_{CE2}}} = I_o \frac{\frac{\beta}{\beta+1} \cdot \frac{V_A+V_{OUT}}{V_A+V_{BE}}}{1 + \frac{1}{\beta+1} \cdot \frac{V_A+V_{OUT}}{V_A+V_{BE}}} \tag{4.27}$$

For $V_A \to \infty$, we get

$$I_{source} = I_o \frac{1}{1 + \frac{2}{\beta}} \tag{4.28}$$

and if $\beta \gg 1$ and if $V_A \to \infty$, then $I_{source} \approx I_o$. Since the output current, I_{source} is a replica of the input current, I_o, the transistor pair in Fig. 4.7 is called a *current mirror*.

There exists a mirroring error due to the fact that I_o also has to supply the base currents of the two transistors, and that any mismatch between the collector-emitter voltages of the two transistors would cause a current difference due to base-width modulation.

Assuming $V_A \to \infty$, and again, solving for β, we get

$$\beta = \frac{2}{\frac{I_o}{I_{source}} - 1} \tag{4.29}$$

Thus, to get a maximum output current variation of $\Delta I_{source} = 10\,\mu\text{A}$ for $I_{source} = 1\,\text{mA}$, resulting in an $I_o = 1.01\,\text{mA}$, we can then calculate $\beta_{\min}$ as

$$\beta_{\min} = \frac{2}{\frac{I_o}{I_{source} - \Delta I_{source}} - 1} = 99 \tag{4.30}$$

This makes the BJT current mirror very robust to β-variations since we can now tolerate a 50.5% reduction in β. Note that the higher β is, the lower the error, and as $\beta \to \infty$, $I_{source} \to I_o$, which is within our $\pm 1\%$ limit.

Using Eq. 4.27 with Eq. 4.21, and assuming that $V_A \to \infty$, the required R_{bias} needed to generate I_{source} is then

$$R_{bias} = \frac{V_{CC} - V_{BE}}{\left(1 + \frac{2}{\beta}\right) \cdot I_S \cdot e^{\frac{V_{BE}}{V_T}}} = \frac{V_{CC} - V_T \ln\left(\frac{I_{source}}{I_S}\right)}{I_{source}\left(1 + \frac{2}{\beta}\right)} \tag{4.31}$$

For the current mirror, we are using R_{bias} to generate I_o linearly, and since the conversion from I_o to V_{BE} and back again to I_{source} is effectively linear, as seen in Eq. 4.27, the required degree variation control of the resistor values is linear with the output current tolerance requirements, instead of exponential, as shown for the resistive voltage divider bias current source. This allows us to use lower quality resistors and still obtain the same output current variation.

To get the small signal output resistance of the current mirror, we set all small signal inputs to zero. Since there are no small signal inputs, we can directly obtain the small signal equivalent circuit in Fig. 4.8.

By inspection, we can see that $v_{be1} = v_{be2} = 0$, thus, the output resistance of the current mirror is

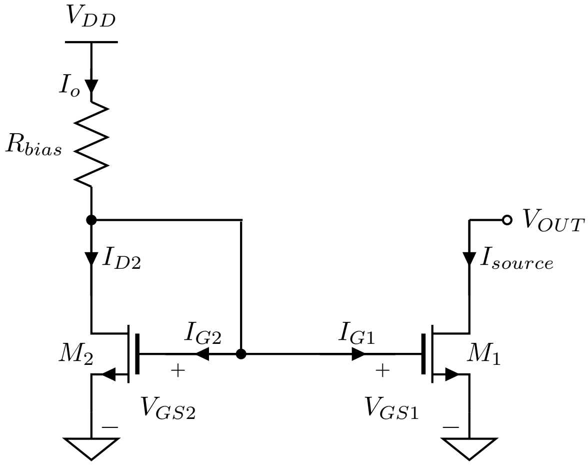


Figure 4.9: The MOSFET Current Mirror.

$$R_{out} = r_{o1} \tag{4.32}$$

The minimum output voltage of the current mirror is still

$$V_{\min} = V_{CE,sat} \tag{4.33}$$

4.3 The MOSFET Current Mirror

A MOSFET current mirror is shown in Fig. 4.9. Assuming that M_1 and M_2 are identical, and since the gate-source voltages are the same, we get

$$V_{GS1} = V_{TH} + \sqrt{\frac{I_{D1}}{k \cdot (1 + \lambda V_{DS1})}} = V_{GS2} = V_{TH} + \sqrt{\frac{I_{D2}}{k \cdot (1 + \lambda V_{DS2})}} \tag{4.34}$$

We can then compute for the output current as

$$I_{source} = I_{D1} = I_{D2} \cdot \frac{1 + \lambda V_{DS1}}{1 + \lambda V_{DS2}} = I_o \cdot \frac{1 + \lambda V_{OUT}}{1 + \lambda V_{GS}} \tag{4.35}$$

Note that since the gate currents are zero, the only source of error between I_o and I_{source} is the mismatch between the drain-source voltages of the two transistors due to channel length modulation. Thus, if $\lambda \to 0$, then $I_{source} \approx I_o$.

Calculating the bias resistor needed to generate I_{source}, and assuming $\lambda = 0$, we get

$$R_{bias} = \frac{V_{DD} - V_{GS}}{I_o} = \frac{V_{DD} - V_{TH} - \sqrt{\frac{I_{source}}{k}}}{I_{source}} \tag{4.36}$$

Similar to the BJT current mirror, the output resistance is still r_{o1}, and $V_{\min} = V_{GS} - V_{TH}$.

4.4 Current Mirror Loads

Now that we have a way to build current sources using current mirrors, let us use these current sources to bias our amplifiers. Consider the common-emitter amplifier biased using current mirrors shown in Fig. 4.10. In this instance, we are using PNP transistors for our current mirror and an NPN transistor for the gain stage.

Using a current source greatly simplifies the DC analysis since if we assume that I_{bias} represents some other circuit that provides an input current to the current mirror, and that $V_A \to \infty$ and $\beta \to \infty$, we then get $I_{C1,Q} = I_{bias}$. However, since in the forward-active region, Q_1 and Q_2 behave like high-impedance current sources, in order to obtain the quiescent DC output voltage, we cannot assume that the Early voltage is infinite. Thus, to compute V_{OUT}, and recognizing the fact the the collector currents of Q_1 and Q_2 are the same, we get

$$I_{C1,Q} = I_{S1} \cdot e^{\frac{V_{IN}}{V_T}} \cdot \left(1 + \frac{V_{OUT}}{V_{A1}}\right) = I_{C2,Q} = I_{S2} \cdot e^{\frac{|V_{BE2}|}{V_T}} \cdot \left(1 + \frac{|V_{CC} - V_{OUT}|}{V_{A2}}\right) \tag{4.37}$$

Ignoring the mirroring errors from Q_3 to Q_2, we can express V_{EB2} as

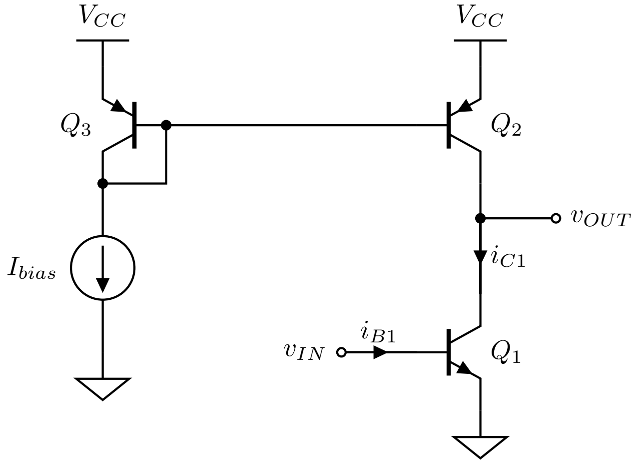


Figure 4.10: A common-emitter amplifier biased using a current mirror.

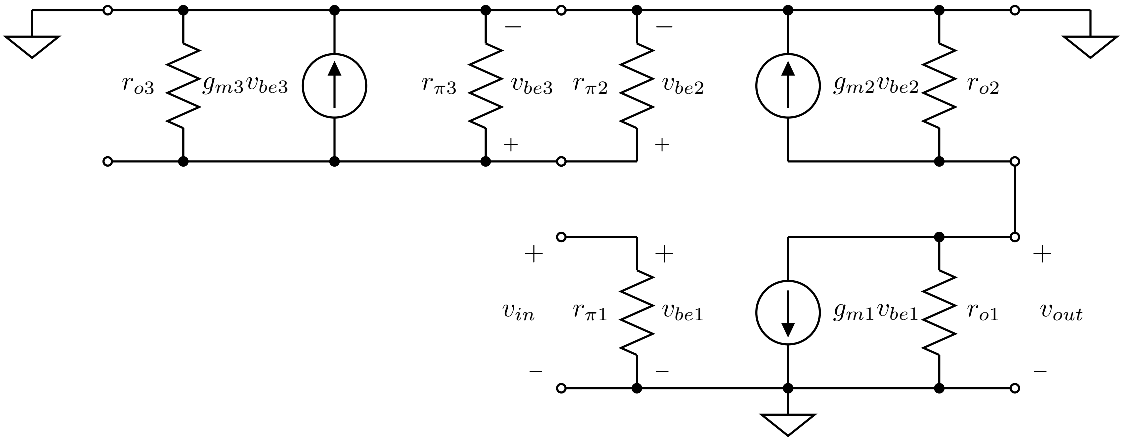


Figure 4.11: The small signal equivalent circuit of the amplifier in Fig. 4.10.

$$V_{BE2} \approx \left| V_T \ln\left(\frac{I_{bias}}{I_{S2}}\right)\right| \tag{4.38}$$

Once we know the quiescent collector currents, we can now determine the small signal parameters of each transistor. The small signal equivalent circuit of the amplifier is shown in Fig. 4.11.

Recognizing that Q_3 is a diode-connected transistor driven by an ideal current source, the resulting $V_{BE3} = V_{BE2}$ is a constant. Thus, there can be no small signal change in V_{BE3} and V_{BE2}. We can then conclude that $v_{be3} = v_{be2} = 0$. By inspection, we can see that

$$G_m = g_m \tag{4.39}$$

$$R_o = r_{o1} \parallel r_{o2} \tag{4.40}$$

$$R_i = r_{\pi1} \tag{4.41}$$

$$A_v = -g_m \cdot (r_{o1} \parallel r_{o2}) \tag{4.42}$$

Note that if $V_{A1} = V_{A2}$, then $r_{o1} = r_{o2} = r_o$, then $A_v = -\frac{g_m r_o}{2}$, which is half the intrinsic transistor gain.