EEE 51 Exam 2

Electrical and Electronics Engineering Institute
University of the Philippines Diliman
2nd Semester SY 2017 - 2018
Thursday, May 24, 2018 1PM - 4PM
L. Alarcón, M. T. de Leon, R. J. Maestro, C. Santos

Instructions: THINK before you answer. Your score will be based solely on what you have written. Write legibly and avoid erasures. Anything we cannot understand will be marked wrong. You may use the back of the exam sheets as extra scratch space, but your final answer, with the correct units, should be placed in the designated areas. DO NOT separate the exam sheets. All parts (I, II, and III) carry the same weight.

Name:

Student No.:

Section:

Encircle one:

Alarcón:	THQ (7:00am-8:30am)
de Leon:	THR (8:30am-10:00am)
Santos:	THU (10:00am-11:30am)
de Leon:	THX (2:30pm-4:00pm)
Maestro:	WFX (2:30pm-4:00pm)

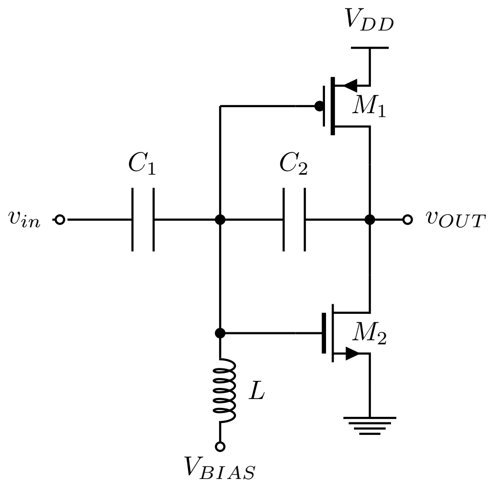
Scores:

Part I:	
Part II:	
Part III:	

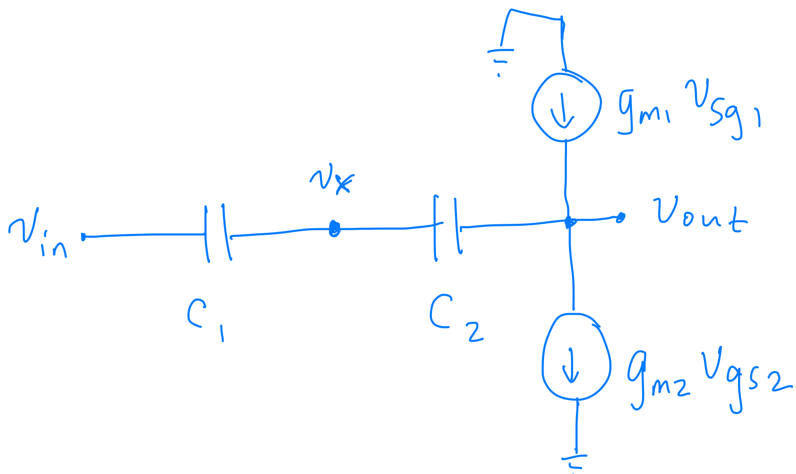
Total:

Part I:

(*20 points*) In the given circuit below, the voltage V_{BIAS} is set at a certain DC voltage to keep M_1 and M_2 in saturation. Assume that all transistors have $\lambda = 0$, $L \to \infty$, and ignore all other capacitances.



1. (*4 points*) Draw the small-signal equivalent circuit. Label all components clearly.



$v_x = v_{sg1} = v_{gs2}$

2. (*6 points*) Derive the expression for the gain, $\frac{v_{out}}{v_{in}}(s)$, in terms of g_{m1}, g_{m2}, C_1, and C_2.

KCL at v_{out}: $g_{m1} v_x + g_{m2} v_x + (v_{out} - v_x) sC_2 = 0$

$$v_x = \frac{-sC_2}{g_{m1} + g_{m2} - sC_2} v_{out}$$

KCL at v_x: $(v_{in} - v_x) sC_1 = (v_x - v_{out}) sC_2$

$$sC_1 v_{in} = s(C_1 + C_2)\left[\frac{-sC_2}{g_{m1} + g_{m2} - sC_2} v_{out}\right] - sC_2 v_{out}$$

$$\frac{v_{out}}{v_{in}} = \frac{-s^{\cancel{2}} C_1 C_2 + \cancel{s} C_1 (g_{m1} + g_{m2})}{-s^{\cancel{2}}(C_1 C_2 + C_2^2 - \cancel{C_2^2}) - \cancel{s} C_2 (g_{m1} + g_{m2})}$$

$$= \frac{sC_1C_2 - C_1(g_{m1} + g_{m2})}{sC_1C_2 + C_2(g_{m1} + g_{m2})}$$

$$\boxed{\frac{v_{out}}{v_{in}}(s) = \frac{sC_1C_2 - C_1(g_{m1} + g_{m2})}{sC_1C_2 + C_2(g_{m1} + g_{m2})}}$$

3. (*2 points*) Determine the gain at very low frequencies if $g_{m1} = g_{m2} = 10\,\text{mS}$, $C_1 = 10\,\text{pF}$, and $C_2 = 1\,\text{pF}$.

as $\omega \to 0$, $\frac{v_{out}}{v_{in}} = \frac{-C_1 \cancel{(g_{m1} + g_{m2})}}{C_2 \cancel{(g_{m1} + g_{m2})}} = \frac{-C_1}{C_2}$

$$= \frac{-10\,pF}{1\,pF} = -10$$

$$\boxed{\frac{v_{out}}{v_{in}} \text{ at low frequencies} = -10}$$

4. (*2 points*) Determine the gain at very high frequencies if $g_{m1} = g_{m2} = 10\,\text{mS}$, $C_1 = 10\,\text{pF}$, and $C_2 = 1\,\text{pF}$.

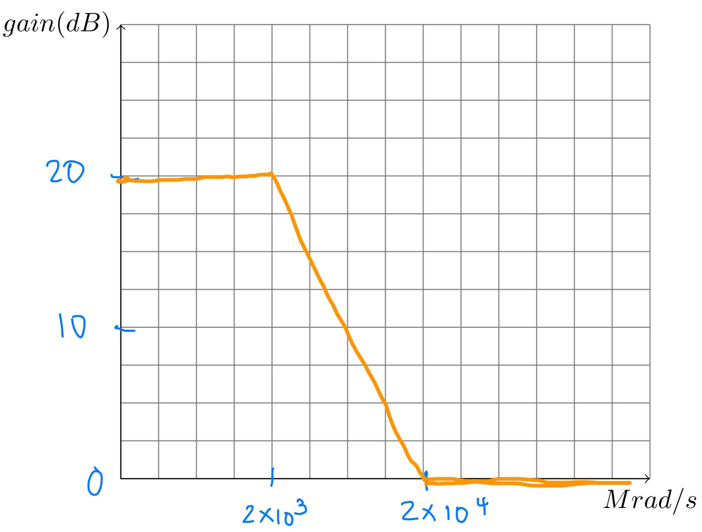
as $\omega \to \infty$, $\dfrac{v_{out}}{v_{in}} = 1$

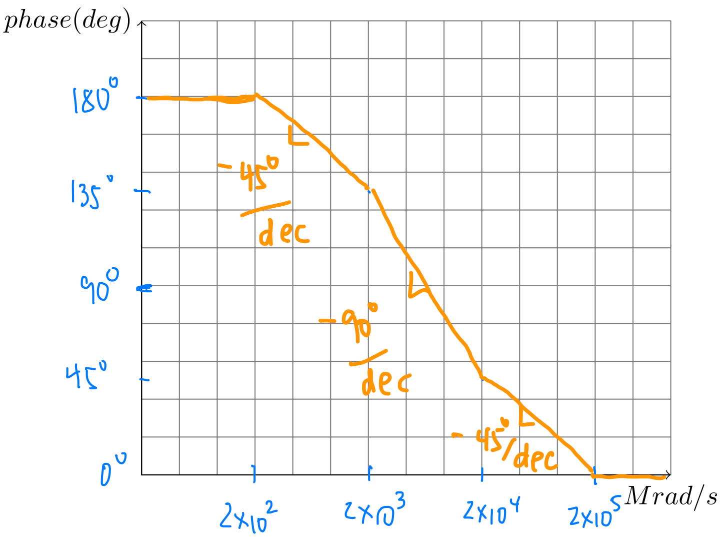
$\frac{v_{out}}{v_{in}}$ at high frequencies = 1

5. (*6 points*) Sketch the magnitude and phase response of the circuit for $g_{m1} = g_{m2} = 10\,\text{mS}$, $C_1 = 10\,\text{pF}$, and $C_2 = 1\,\text{pF}$. Label all pole and/or zero locations accordingly.

$$\omega_z = \frac{g_{m1} + g_{m2}}{C_2} = 2\times10^4 \ \frac{\text{Mrad}}{s}$$

$$\omega_p = \frac{g_{m1} + g_{m2}}{C_1} = 2\times10^3 \ \frac{\text{Mrad}}{s}$$





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Part II-A:

(*6 points*) Write your answers **in CAPITAL LETTERS** at the left side of each number, **enclosed in a BOX**. Correct answers will be given a credit of 1.5 points each.

A

1. In a negative feedback system where $\angle\mathbf{F}$ is always $0°$ and $|\mathbf{F}| > 0$, the poles of the loop gain $\mathbf{T}(s)$ is

   (a) also the poles of the forward gain $\mathbf{A}(s)$

   (b) also the poles of the closed-loop gain $\mathbf{A_{CL}}(s)$

   (c) also the poles of $1 + \mathbf{A}(s)\mathbf{F}(s)$

   (d) none of the above

C

or D

2. If the feedback factor is 1 and the forward gain is $|A_0| / \left(1 + j\frac{\omega}{\omega_p}\right)$ where $|A_0|$ is a constant greater than 1 and ω_p is greater than $0Hz$, then in negative feedback

   (a) $0° < PM \leq 45°$

   (b) $45° < PM \leq 90°$

   (c) $90° < PM \leq 135°$

   (d) $135° < PM \leq 180°$

B

3. Which among the following statements is true?

   (a) The phase margin of a negative feedback system (with no zeros) can be determined given the relative distance of all the poles of $\mathbf{T}(s)$.

   (b) The gain margin is $0dB - T_{dB}(\omega_x)$ where ω_x is the frequency where $\angle\mathbf{T} = -180°$.

   (c) A peaking at the output of the plot of $|\mathbf{A_{CL}}(\omega)|$ indicates that the system is unstable.

   (d) The system is stable if an ideal amplifier (with ideal input impedance, ideal gain, and ideal output impedance) for the forward gain is used.

A

4. Which is always true regarding $\omega_{u,T}$ or the unity gain frequency of $\mathbf{T}(s)$ for negative feedback?

   (a) $PM = \angle\mathbf{T}(\omega_{u,T}) + 180°$

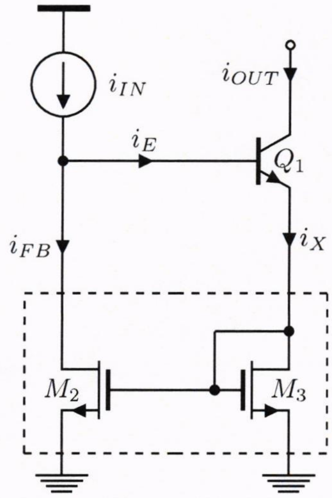
   (b) $\mathbf{A_{CL}}(\omega_{u,T}) = \frac{\mathbf{A}(\omega_{u,T})}{2}$

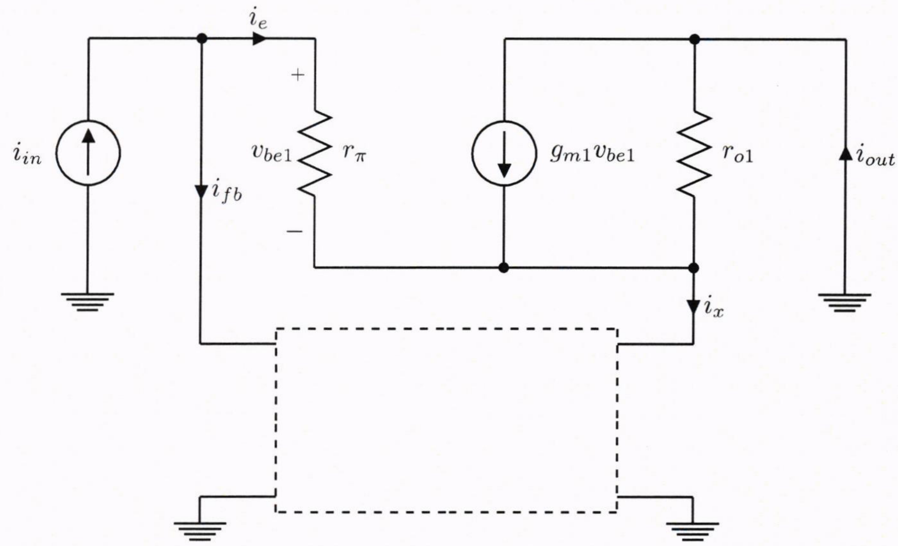
   (c) $\mathbf{A_{CL}}(\omega_{u,T}) = \frac{1}{2F}$

   (d) $\omega_{u,T}$ is also the GBP of $\mathbf{T}(s)$

Part II-B:

(*14 points*) The circuit shown below is a Bi-CMOS implementation of a Wilson Current source where the current mirror below acts as the feedback network. Assume that the small-signal parameters $r_{\pi 1}$, g_{m1}, r_{o1}, g_{m2}, and g_{m3} are known finite quantities and that r_{o2} and r_{o3} approaches ∞. A partial small-signal model of the circuit is also provided. Again, for multiple choice type questions, write your answers **in CAPITAL LETTERS** at the left side of each number, **enclosed in a BOX**.



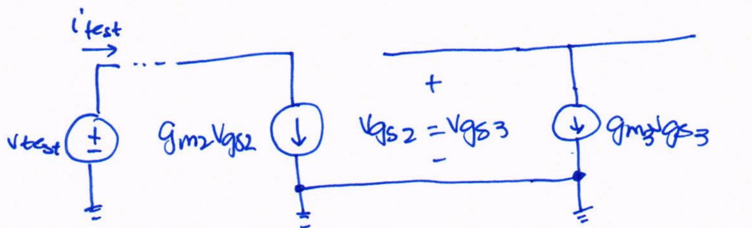


1. (*1 point*) What is i_x in terms of the given current variables in the figure (not in terms of transistor parameters)?

$i_x =$ $i_e + i_{out}$

2. (*1 point*) What is the output impedance of the feedback network/block?

$R_o = \left.\dfrac{v_{test}}{i_{test}}\right|_{\text{no-input}}$



* no input $\rightarrow$ $v_{gs2/3} = 0$ $\rightarrow$ $i_{test} = 0$

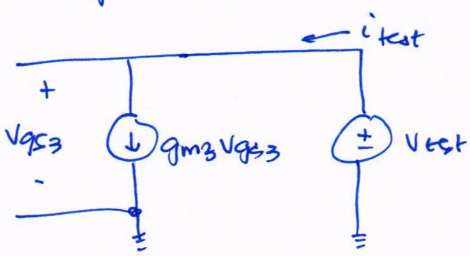
* $r_{o2} \rightarrow \infty$

$R_{O,FB} =$ ∞

3. (*2 points*) What is the input impedance of the feedback network/block?

$$R_1 = \frac{v_{test}}{i_{test}}$$

* current mirror is unilateral at the input side...



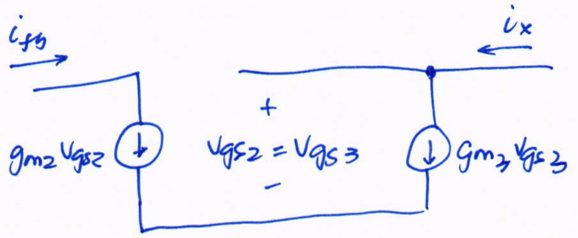
$$V_{gs3} = V_{test}$$

$$g_{m3}V_{gs3} = i_{test}$$

$$\frac{V_{test}}{i_{test}} = \frac{V_{gs3}}{g_{m3}V_{gs3}} = \frac{1}{g_{m3}}$$

$$R_{I,FB} = \boxed{\frac{1}{g_{m3}}}$$

4. (*2 points*) What is i_{fb}/i_x?



$$i_x = g_{m3}V_{gs3}$$

$$V_{gs3} = V_{gs2} = i_x/g_{m3}$$

$$i_{fb} = g_{m2}V_{gs2} = g_{m2}V_{gs3}$$

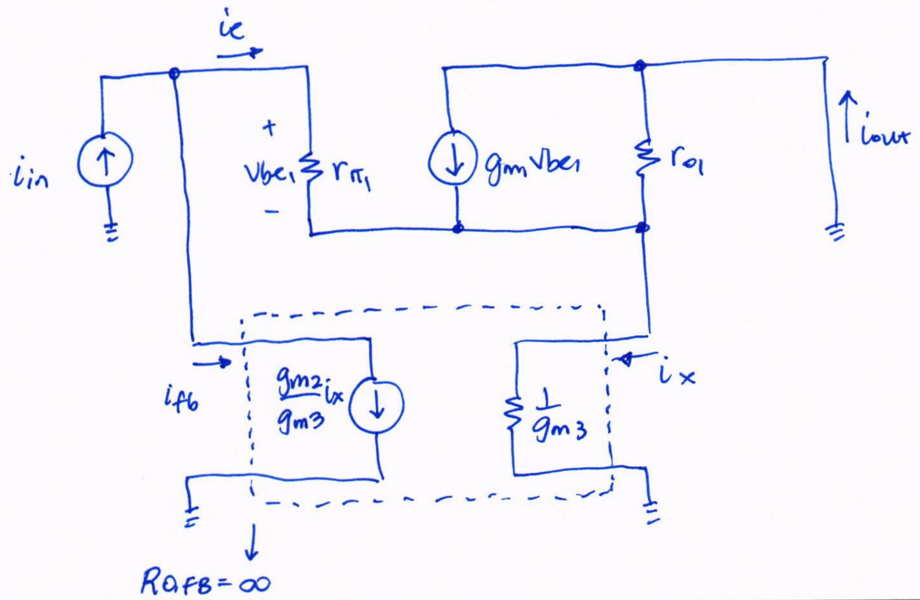
$$i_{fb} = g_{m2}(i_x/g_{m3})$$

$$\frac{i_{fb}}{i_x} = \frac{g_{m2}}{g_{m3}}$$

$$i_{fb}/i_x = \boxed{\frac{g_{m2}}{g_{m3}}}$$

→ current gain

5. (*1 point*) Draw the *complete* small-signal model of the Wilson current source by incorporating your previous answers and subsituting them on the dashed box.



6. (*4 points*) What is the forward gain $A = i_{out}/i_e$? Note that this should consider the loading due to the feedback block. Do NOT do approximations in deriving. The answer must be written in the form $\frac{XY-1}{Y+1}$ where X and Y can be composed of 1 or more variables.

let V_x be the voltage at the emitter

$$V_x = i_x \frac{1}{g_{m3}} = \frac{i_e + i_{out}}{g_{m3}}$$

KCL at the output :

$$g_{m1} V_{be1} + \frac{0 - V_x}{r_{o1}} = i_{out}$$

$$g_{m1}(r_{\pi 1} i_e) - \frac{V_x}{r_{o1}} = i_{out}$$

$$g_{m1} r_{\pi 1} i_e - \left(\frac{i_e}{g_{m3} r_{o1}} + \frac{i_{out}}{g_{m3} r_{o1}}\right) = i_{out}$$

$$i_{out}\left(1 + \frac{1}{g_{m3} r_{o1}}\right) = i_e\left(g_{m1} r_{\pi 1} - \frac{1}{g_{m3} r_{o1}}\right)$$

$$i_{out}\left(\frac{g_{m3} r_{o1} + 1}{g_{m3} r_{o1}}\right) = i_e\left(\frac{g_{m1} r_{\pi 1} g_{m3} r_{o1} - 1}{g_{m3} r_{o1}}\right)$$

$$\frac{i_{out}}{i_e} = \frac{g_{m1} r_{\pi 1} g_{m3} r_{o1} - 1}{g_{m3} r_{o1} + 1} = \frac{\beta g_{m3} r_{o1} - 1}{g_{m3} r_{o1} + 1}$$

$* \approx \beta$ when $g_{m3} r_{o1} \gg 1$

$$A = \frac{g_{m1} r_{\pi 1} g_{m3} r_{o1} - 1}{g_{m3} r_{o1} + 1}$$

7. (*3 points*) What is the feedback factor $F = i_{fb}/i_{out}$? Take note of your answer in #1. Do NOT do approximations in deriving. The answer can be written in the form $Z\frac{XY+Y}{Y-1}$ where X, Y, and Z can be composed of 1 or more variables.

$$\frac{i_{fb}}{i_{out}} = \left(\frac{i_{fb}}{i_x}\right)\cdot\left(\frac{i_x}{i_{out}}\right)$$

$$= \frac{g_{m2}}{g_{m3}} \cdot \left(\frac{i_e + i_{out}}{i_{out}}\right)$$

$$= \frac{g_{m2}}{g_{m3}} \cdot \left(\frac{g_{m3} r_{o1} + 1}{g_{m1} r_{\pi 1} g_{m3} r_{o1} - 1} + 1\right)$$

$$= \frac{g_{m2}}{g_{m3}} \left(\frac{g_{m3} r_{o1} + 1 + g_{m1} r_{\pi} g_{m3} r_{o1} - 1}{g_{m1} r_{\pi 1} g_{m3} r_{o1} - 1}\right)$$

$$= \frac{g_{m2}}{g_{m3}} \left(\frac{g_{m1} r_{\pi 1} g_{m3} r_{o1} + g_{m3} r_{o1}}{g_{m1} r_{\pi} g_{m3} r_{o1} - 1}\right)$$

add'l notes :

$$= \frac{g_{m2}}{g_{m1}} \left(\frac{\beta g_{m3} r_{o1} + g_{m3} r_{o1}}{\beta g_{m3} r_{o1} - 1}\right)$$

$$= \frac{g_{m2}}{g_{m1}} \left(\frac{\cancel{\beta} (\beta + 1) g_{m3} r_{o1}}{\beta g_{m3} r_{o1} - 1}\right)$$

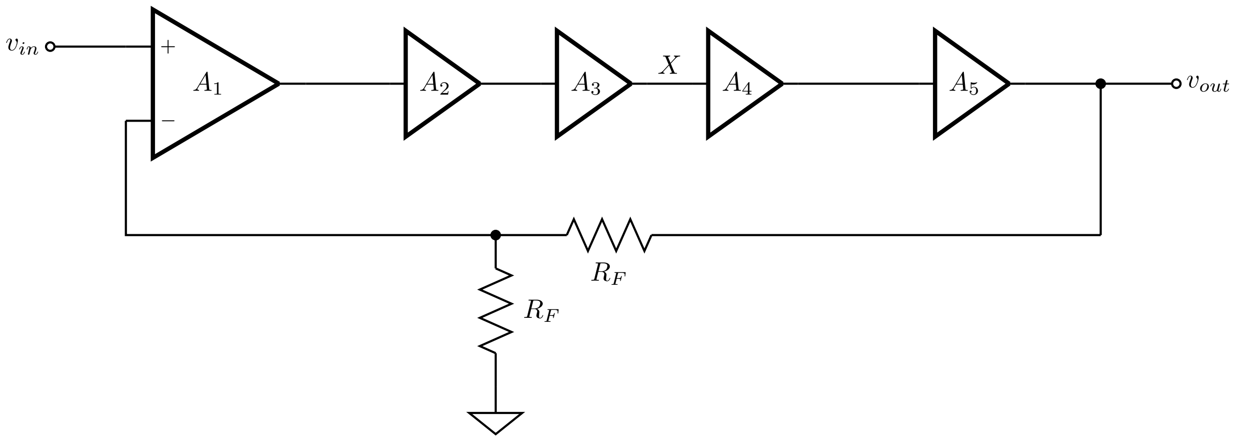
$$\approx \frac{g_{m2}}{g_{m1}} \left(\frac{\beta + 1}{\beta}\right) \qquad g_{m3} r_{o1} \gg 1$$

$$\approx \frac{g_{m2}}{g_{m1}} \frac{1}{\alpha}$$

$$\boxed{F = \frac{g_{m2}}{g_{m3}} \left(\frac{g_{m1} r_{\pi 1} g_{m3} r_{o1} + g_{m3} r_{o1}}{g_{m1} r_{\pi} g_{m3} r_{o1} - 1}\right)}$$

Part III:

(19 points) You are given the feedback amplifier below. The amplifier A_1 has a voltage gain of 10, an output impedance of $100\,\Omega$, and an infinite input impedance. The amplifier A_5 is an ideal voltage buffer with voltage gain of 1, zero output impedance, and infinite input impedance. Amplifiers A_2, A_3, and A_4 are identical, with the following characteristics: $A_v = 10\,\frac{\mathrm{V}}{\mathrm{V}}$, $Z_o = 100\,\Omega$, and $Z_i = 900 \parallel \frac{1}{j\omega C_i}\,\Omega$, where $C_i = 1\,\mathrm{pF}$.



1. Determine the feedback factor, f. *(1 point)*

$$f = \frac{R_F}{R_F + R_F} = 0.5\ \mathrm{V/V}$$

$f =$ 0.5 V/V

2. Determine the value of the forward gain at DC, a_0. *(2 points)*

$$a_0 = A_{v_1} \cdot \frac{R_i}{R_o + R_i} \cdot A_{v_2} \cdot \frac{R_i}{R_o + R_i} \cdot A_{v_3} \cdot \frac{R_i}{R_o + R_i} \cdot A_{v_4} \cdot A_{v_5}$$

$$= 10^4 \cdot (0.9)^3 \cdot 1 = 7290\ \mathrm{V/V}$$

$a_0 =$ 7290 V/V

3. Determine the value of the loop gain at DC, T_0. *(1 point)*

$$T_0 = a_0 \cdot f$$
$$= 7290 \cdot (0.5)$$
$$= 3645$$

$T_0 =$ 3645

4. Determine the value of the closed-loop gain at DC, A_{CL0}. *(1 point)*

$$A_{CL0} = \frac{1}{f} \cdot \frac{T_0}{1+T_0} = 2 \cdot \frac{3645}{3646}$$

$$= 1.9995 \text{ V/V}$$

$A_{CL0} = \boxed{1.9995 \text{ V/V}}$

5. Find the unity-gain frequency of the loop gain, ω_u. *(5 points)*

$$Z_i = R_i \parallel \frac{1}{sC_i} = \frac{R_i}{1+sR_iC_i}$$

$$\frac{Z_i}{Z_i+R_o} = \frac{R_i}{R_i+R_o+sR_iR_oC_i} = \frac{R_i}{R_i+R_o} \cdot \left(\frac{1}{1+s(R_i \parallel R_o)C_i}\right)$$

$$\therefore T(s) = f \cdot a(s) = f \cdot A_{v_1} \cdot \left[10 \cdot \frac{Z_i}{Z_i+R_o}\right]^3 \cdot 1$$

$$= 0.5 \cdot 10 \cdot 10^3 \cdot \left[\frac{R_i}{R_i+R_o}\right]^3 \cdot \left(\frac{1}{1+s\tau}\right)^3$$

$$T(j\omega) = \frac{3645}{(1+s\tau)^3}, \quad \tau = (R_i \parallel R_o)C_i = 90\text{ ps}$$

$$|T(j\omega_u)| = 1 = \left|\frac{3645}{(1+j\omega_u\tau)^3}\right| = \frac{3645}{\left(\sqrt{1+\omega_u^2\tau^2}\right)^3}$$

$$\therefore \omega_u = \sqrt{\frac{\left(\sqrt[3]{3645}\right)^2 - 1}{(90p)^2}} = 1.7064 \times 10^{11} \text{ rad/s}$$

$$= 170.64 \text{ Grad/s}$$

$\omega_u = \boxed{170.64 \text{ Grad/s}}$

6. What is the feedback amplifier's phase margin? *(4 points)*

$$\angle T(\omega) = -\tan^{-1}(\omega\tau) - \tan^{-1}(\omega\tau) - \tan^{-1}(\omega\tau)$$

$$= -3\tan^{-1}(\omega \cdot 90ps)$$

@ ω_u,

$$\angle T(\omega_u) = -3\tan^{-1}(\omega_u \cdot 90ps)$$

$$= -3\tan^{-1}(170.64\ Grad/s \cdot 90ps)$$

$$= -4.5173\ rad$$

$$= -258.82^\circ$$

$$PM = \angle T(\omega_u) - (-180^\circ)$$

$$= -258.82^\circ + 180^\circ$$

$$= -78.82^\circ$$

$\rightarrow$ unstable since this must have passed -180° with $|T| > 1$

$PM =$ -78.82°

7. In order to set the phase margin to 45°, what capacitance value, C_C, must be placed between node X and ground? *(5 points)*

with C_c,

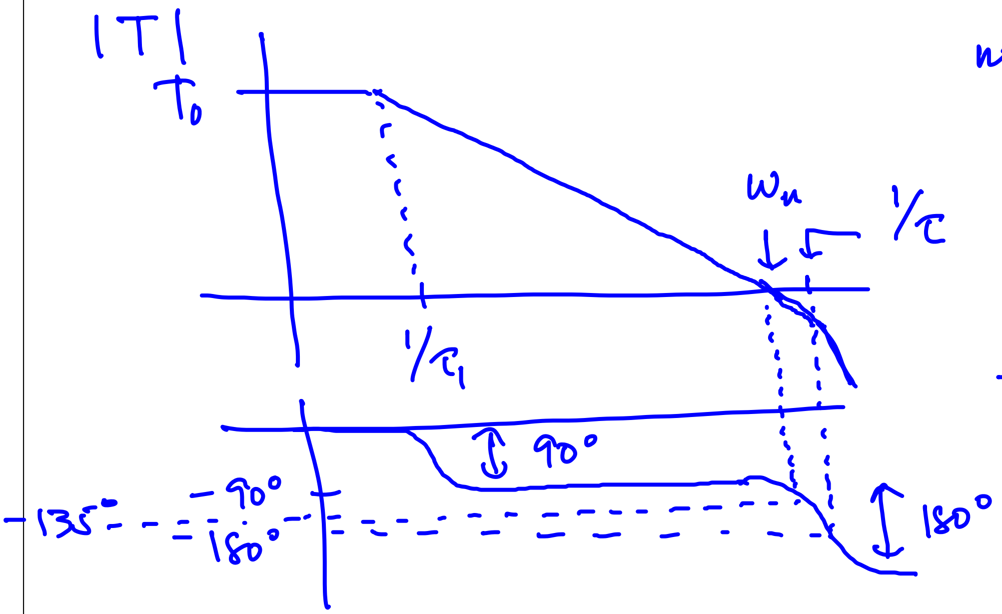
$$T(s) = \frac{T_o}{(1+s\tau_1)(1+s\tau)^2}$$

$$\tau = (R_i \| R_o) C_i$$

$$\tau_1 = (R_i \| R_o)(C_i + C_c)$$

$$PM = \angle T(\omega_u) - (-180^\circ) = 45^\circ \Rightarrow \angle T(\omega_u) = -135^\circ$$

$$\angle T(\omega_u) = -2\tan^{-1}\omega_u\tau - \tan^{-1}\omega_u\tau_1 = -135^\circ$$



note: if $\omega_u = 1/\tau$ then PM = 0° due to the double pole!

$\therefore$ around ω_u

$\tan^{-1}\omega_u\tau_1 \cong 90^\circ$

$$\therefore -2\tan^{-1}\omega_u\tau = +90^\circ - 135^\circ = -45^\circ$$

$$\omega_u = \frac{\tan 22.5^\circ}{\tau} = \frac{\tan 22.5^\circ}{90\text{ ps}} = 4.6\text{ Grad/s}$$

$$\therefore T_o \cdot 1/\tau_1 = 4.6\text{ Grad/s}$$

$$\tau_1 = \frac{T_o}{4.6\text{ Grad/s}} = \frac{3645}{4.6\text{ Grad/s}} = 791.98\text{ ps} = (R_i \| R_o)(C_i + C_c)$$

$$= 90(1\text{pF} + C_c)$$

$$C_c = 7.8\text{ pF}$$

$C_C =$ 7.8 pF