EEE 51 Assignment 7 Answer Key

2nd Semester SY 2018-2019

1. **Frequency Response Exercise.** For the circuit in Figure 1,

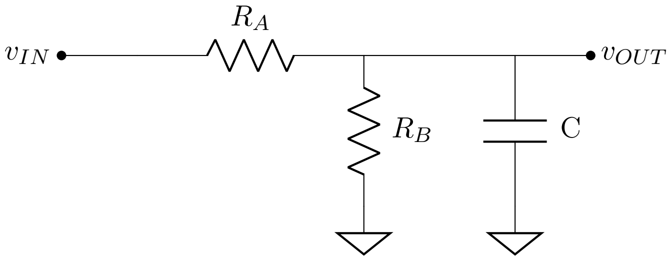

Figure 1: Voltage Divider with Output Capacitance.

(a) Solve for the transfer function $\frac{v_{out}(s)}{v_{in}(s)}$. Write it in the form: $K\frac{(1+\frac{s}{\omega_{z1}})(1+\frac{s}{\omega_{z2}})...(1+\frac{s}{\omega_{zn}})}{(1+\frac{s}{\omega_{p1}})(1+\frac{s}{\omega_{p2}})...(1+\frac{s}{\omega_{pn}})}$ (1pt)

$$\begin{aligned}
\frac{v_{out}(s)}{v_{in}(s)} &= \frac{R_B||\frac{1}{sC}}{R_A + (R_B||\frac{1}{sC})} \\
&= \frac{R_B}{R_A(1+sCR_B)+R_B} \\
&= \frac{R_B}{R_A+R_B+sCR_AR_B} \\
&= \boxed{\frac{R_B}{R_A+R_B}\frac{1}{1+sC\frac{R_AR_B}{R_A+R_B}}} \\
&= \boxed{\frac{R_B}{R_A+R_B}\frac{1}{1+sC(R_A||R_B)}}
\end{aligned}$$

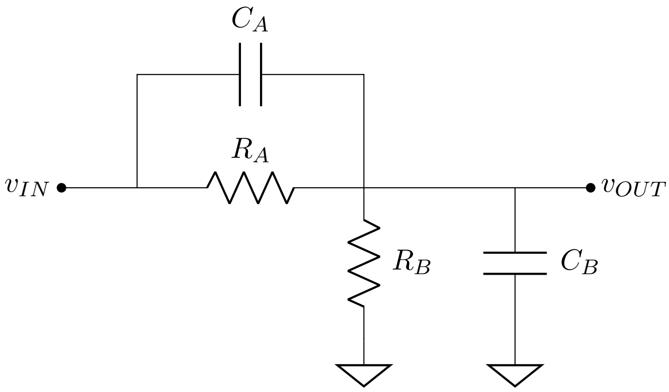

Figure 2: Frequency Compensated Voltage Divider.

For the succeeding problems, refer to Figure 2.

(b) Solve for the transfer function $\frac{v_{out}(s)}{v_{in}(s)}$. Write it in the form: $K\frac{(1+\frac{s}{\omega_{z1}})(1+\frac{s}{\omega_{z2}})...(1+\frac{s}{\omega_{zn}})}{(1+\frac{s}{\omega_{p1}})(1+\frac{s}{\omega_{p2}})...(1+\frac{s}{\omega_{pn}})}$ (1pt)

$$
\begin{aligned}
\frac{v_{out}(s)}{v_{in}(s)} &= \frac{R_B||\frac{1}{sC_B}}{(R_A||\frac{1}{sC_A}) + (R_B||\frac{1}{sC_B})} \\
&= \frac{\frac{R_B}{1+sC_BR_B}}{\frac{R_A}{1+sC_AR_A} + \frac{R_B}{1+sC_BR_B}} \\
&= \frac{R_B(1+sC_AR_A)}{R_A(1+sC_BR_B) + R_B(1+sC_AR_A)} \\
&= \frac{R_B(1+sC_AR_A)}{R_A + R_B + sC_BR_AR_B + sC_AR_AR_B} \\
&= \frac{R_B(1+sC_AR_A)}{R_A + R_B + s(C_B+C_A)R_AR_B} \\
&= \frac{R_B}{R_A+R_B}\frac{1+sC_AR_A}{1+s(C_B+C_A)\frac{R_AR_B}{R_A+R_B}} \\
&= \boxed{\frac{R_B}{R_A+R_B}\frac{1+sC_AR_A}{1+s(C_B+C_A)(R_A||R_B)}}
\end{aligned}
$$

(c) Plot the pole-zero diagram. Label the axes, and the pole(s)/zero(s). (1pt)
Figure 3 shows the pole-zero diagram of the frequency-compensated voltage divider in fig. 2, where:

$$
\begin{aligned}
\omega_Z &= -\frac{1}{R_AC_A} \\
\omega_P &= -\frac{1}{(C_A+C_B)(R_A||R_B)}
\end{aligned}
$$

Note: The case shown here is when $|\omega_P| < |\omega_Z|$. It's possible to have cases where $|\omega_P| > |\omega_Z|$, depending on the ratio of the resistances and capacitances.

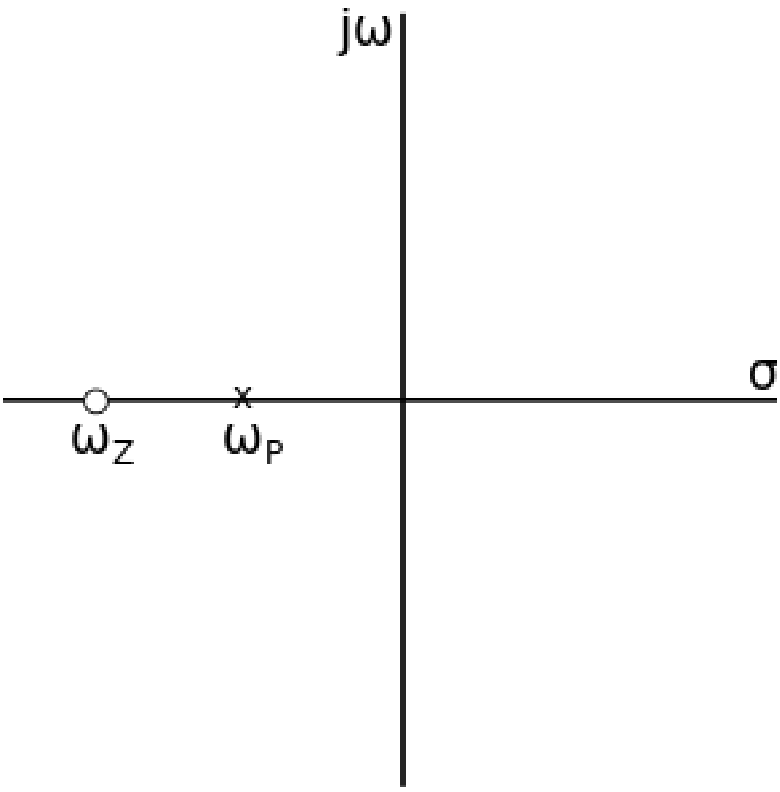

Figure 3: Pole Zero Diagram of Frequency Compensated Voltage Divider.

Suppose $R_A = 159K\Omega$, $R_B = 1.59M\Omega$, $C_A = 1pF$, $C_B = 10pF$, and $v_{IN} = sin(2\pi(100KHz)t)$ V.

(d) Sketch the magnitude response (in dB) of the circuit on a log-log plot. Label the pole/zero location/s, DC gain, slopes, and the axes. (2pts)

$$\omega_Z = -\frac{1}{R_A C_A} = -6.29Mrad/s$$
$$\omega_P = -\frac{1}{(C_A + C_B)(R_A || R_B)} = -0.629Mrad/s$$

OR

$$f_Z = -1MHz$$
$$f_P = -100KHz$$

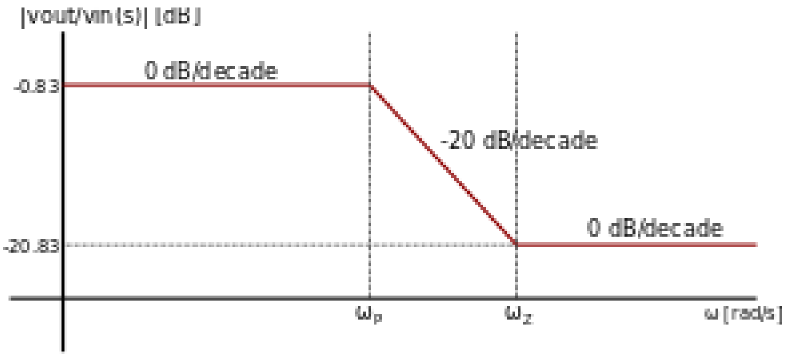

Figure 4: Magnitude Response of Frequency Compensated Voltage Divider.

(e) Sketch the phase response (in degrees) of the circuit on a semi-log plot. Label the pole/zero location/s, phase at 0 Degrees, slopes, the axes, and important frequencies. (2pts)

$$\omega_Z = -\frac{1}{R_A C_A} = -6.29Mrad/s$$
$$\omega_P = -\frac{1}{(C_A + C_B)(R_A || R_B)} = -0.629Mrad/s$$

OR

$$f_Z = -1MHz$$
$$f_P = -100KHz$$

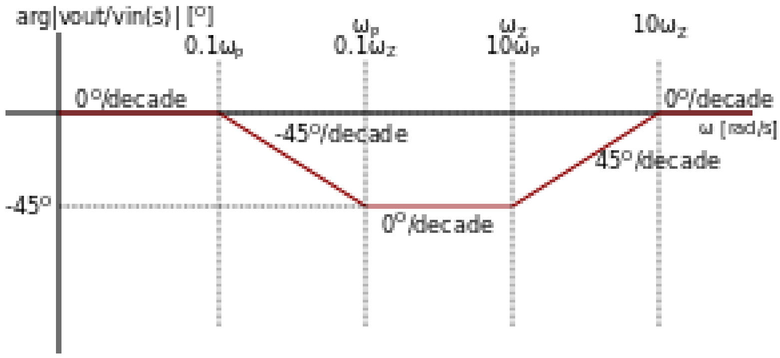

Figure 5: Magnitude Response of Frequency Compensated Voltage Divider.

(f) Write the expression for v_{OUT}. (1pt)
Let $A(s) = \frac{v_{out}(s)}{v_{in}(s)}$. v_{IN} is a single-tone sinusoid. That means, we can simply multiply its magnitude by $|A(\omega)|$, and add $\arg|A(\omega)|$ to its phase. The expression for v_{out} will be given by:

$$v_{out} = |A(\omega)| sin(2\pi(100KHz)t + \arg|A(\omega)|)$$

At $\omega = 2\pi(100KHz)$,

$$|A(\omega)| = -3.83dB = 0.64$$

$$arg|A(\omega)| = -45^o = -\frac{\pi}{4} rad$$

Thus,

$$v_{out} = \boxed{0.64 sin(2\pi(100KHz)t + \frac{\pi}{4} rad)}$$

(g) The frequency-compensated voltage divider is usually used to increase the bandwidth of a voltage divider with an output parasitic capacitance, like in oscilloscope probes. Given the chance to change C_A, should you? If yes, what C_A would maximize the filter's bandwidth? Else, why? (2pts)

With the current values, the zero is located a decade above the pole. To maximize the bandwidth of the filter, the pole and the zero should be at the same location. (i.e. use the pole-zero cancellation technique) To do that, we equate the location of the pole to the location of the zero, and solve for C_A.

$$\omega_Z = \omega_P$$

$$\frac{1}{R_A C_A} = \frac{1}{(C_A + C_B)(R_A || R_B)}$$

$$R_A C_A = (C_A + C_B)(R_A || R_B)$$

$$\frac{R_A}{R_A || R_B} = \frac{C_A + C_B}{C_A}$$

$$R_A(\frac{1}{R_A} + \frac{1}{R_B}) = 1 + \frac{C_B}{C_A}$$

$$1 + \frac{R_A}{R_B} = 1 + \frac{C_B}{C_A}$$

$$\frac{R_A}{R_B} = \frac{C_B}{C_A}$$

$$\frac{R_B}{R_A} = \frac{C_A}{C_B}$$

$$C_B \frac{R_B}{R_A} = C_A$$

Thus, $\boxed{C_A = 100pF}$

2. **Transit Frequency, and Intrinsic Capacitances on Cascode Amplifier**

(a) The equation for V_{BIAS} is given below:

$$V_{BIAS,min} = V_{be_{Q2}} + V_{ce,sat_{Q1}} + V_{I_{source}}$$

where $V_{I_{source}}$ equation is:

$$V_{I_{source}} = V_{CC} - (I_{C2})(R_L) - V_{ce,sat_{Q2}} - V_{ce,sat_{Q1}}$$

To get I_{C2} we have $I_{source} = 1.5mA$ and $\beta = 100$ for both transistors.

$$\begin{aligned} I_{C2} &= \frac{I_{E2}}{\frac{1}{\beta} + 1} \\ &= \frac{I_{source}}{\left(\frac{1}{\beta}+1\right)\left(\frac{1}{\beta}+1\right)} \\ I_{C2} &= 1.470mA \end{aligned}$$

We now know I_{C2} so we can get $V_{I_{source}}$:

$$\begin{aligned} V_{I_{source}} &= 20V - (1.470mA)(8.6k\Omega) - 0.2V - 0.2V \\ V_{I_{source}} &= 6.95V \end{aligned}$$

We can now solve for V_{BIAS}:

$$\begin{aligned} V_{BIAS,min} &= 0.7V + 0.2V + 6.95V \\ &\boxed{V_{BIAS,min} = 7.85V} \end{aligned}$$

(b) The formula for f_T is given and manipulating to get $C_{\mu 1}$ and $C_{\mu 2}$ are shown below:

$$\begin{aligned} 2\pi f_T &= \frac{g_m}{C_\pi + C_\mu} \\ C_{\mu 1} &= \frac{g_{m1}}{2\pi f_T} - C_{\pi 1} \\ C_{\mu 2} &= \frac{g_{m2}}{2\pi f_T} - C_{\pi 2} \end{aligned}$$

We already know I_{C2}, and I_{C1} is just $\frac{I_{source}}{\left(\frac{1}{\beta}+1\right)}$. Therefore, $I_{C1} = 1.485mA$.

$$\begin{aligned} C_{\mu 1} &= \frac{\frac{I_{C1}}{V_T}}{2\pi f_T} - C_{\pi 1} \\ C_{\mu 1} &= \frac{\frac{1.485mA}{26mV}}{2\pi \cdot 450MHz} - 2pF \\ &\boxed{C_{\mu 1} = 18.20pF} \\ C_{\mu 2} &= \frac{\frac{I_{C2}}{V_T}}{2\pi f_T} - C_{\pi 2} \\ C_{\mu 2} &= \frac{\frac{1.470mA}{26mV}}{2\pi \cdot 450MHz} - 2pF \\ &\boxed{C_{\mu 2} = 18.00pF} \end{aligned}$$

(c) Shown in Figure 6 is the simplified Small-Signal Equivalent Circuit for the Cascode Amplifier with Intrinsic Capacitances.

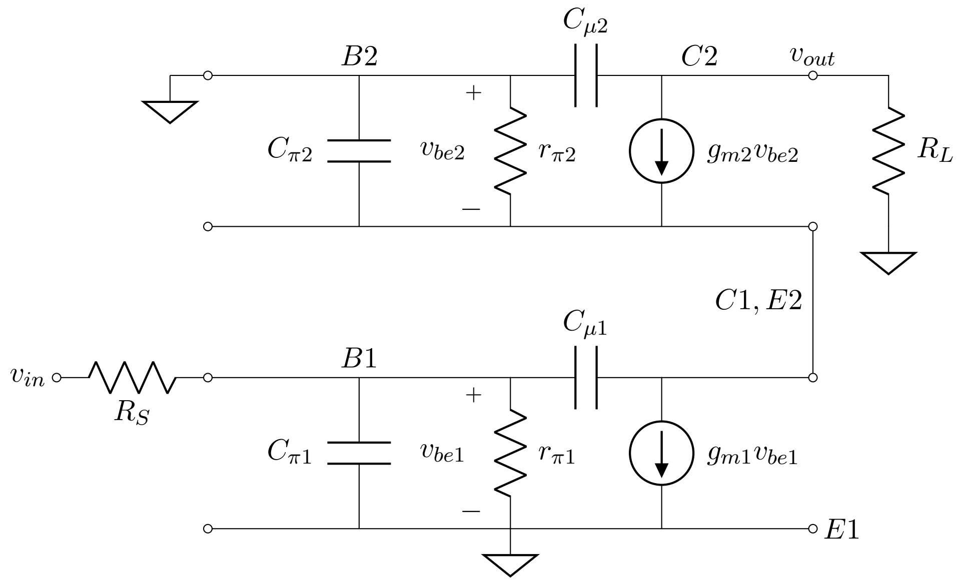

Figure 6: Small Signal Equivalent of the Cascode Amplifier with Intrinsic Capacitances

(d) The effect of C_E under DC operation is an open terminal. If operated under any operating frequency, it will have an impedance which will be present when obtaining the small-signal equivalent. But C_E's value is much larger than the internal capacitance present in the circuit (C_π, C_μ). therefore it's effect will be dominant on low frequencies. So typically it is used as a bypass capacitor to eliminate the degenerated effects if I_{source} is not ideal meaning has a finite output impedance.

(e) To derive the transfer function $\frac{v_{out}}{v_{in}}(s)$, simply use KCL equations from nodes $B1$, $(C1, E2)$, and $C2$. The miller approximation was prohibited to use because the analysis must be the effect of total capacitance connected on output to input. This affects both the input and output impedance of the whole amplifier depending on its total gain. The case for this problem, it does not apply with individual $C_{\mu 1}$, and $C_{\mu 2}$ because of the presence of $C_{\pi 2}$. Beware that even if the miller approximation method is not applied, the cascode amplifier still exhibits miller effect.

KCL @ $B1$:

$$\frac{v_{be1} - v_{in}}{R_S} + \frac{v_{be1}}{\frac{1}{sC_{\pi 1}}||r_{\pi 1}} + \frac{v_{be1} + v_{be2}}{\frac{1}{sC_{\mu 1}}} = 0 \tag{1}$$

KCL @ $C1, E2$:

$$g_{m1}v_{be1} - g_{m2}v_{be2} + \frac{-v_{be2} - v_{be1}}{\frac{1}{sC_{\mu 1}}} + \frac{-v_{be2}}{\frac{1}{sC_{\pi 2}}||r_{\pi 2}} = 0 \tag{2}$$

KCL @ $C2$:

$$g_{m2}v_{be2} + \frac{v_{out}}{R_L} + \frac{v_{out}}{\frac{1}{sC_{\mu 2}}} = 0 \tag{3}$$

Rearranging each KCL equations:

$$v_{be1}\left(\frac{1}{R_S} + \frac{1 + sr_{\pi 1}C_{\pi 1}}{r_{\pi 1}} + sC_{\mu 1}\right) + v_{be2}(sC_{\mu 1}) = v_{in}\left(\frac{1}{R_S}\right) \tag{4}$$

$$v_{be2}\left(\frac{1 + r_{\pi 2}C_{\pi 2}}{r_{\pi 2}} + g_{m2} + sC_{\mu 1}\right) = v_{be1}(g_{m1} - sC_{\mu 1}) \tag{5}$$

$$v_{be2}(-g_{m2}) = v_{out}\left(sC_{\mu 2} + \frac{1}{R_L}\right) \tag{6}$$

Substituting (5) to (4) via v_{be1}:

$$v_{be2}\left[\left(\frac{\left(\frac{1}{R_S}+\frac{1+sr_{\pi1}C_{\pi1}}{r_{\pi1}}+sC_{\mu1}\right)\left(\frac{1+sr_{\pi2}C_{\pi2}}{r_{\pi2}}+g_{m2}+sC_{\mu1}\right)}{g_{m1}-sC_{\mu1}}\right)+sC_{\mu1}\right]=v_{in}\left(\frac{1}{R_S}\right)$$

$$v_{be2}\left[\left(\frac{\left(1+\frac{R_S+sr_{\pi1}R_SC_{\pi1}}{r_{\pi1}}+sR_SC_{\mu1}\right)\left(\frac{1+sr_{\pi2}C_{\pi2}}{r_{\pi2}}+g_{m2}+sC_{\mu1}\right)}{g_{m1}-sC_{\mu1}}\right)+sR_SC_{\mu1}\right]=v_{in} \tag{7}$$

$$\frac{v_{in}(g_{m1}-sC_{\mu1})}{\left(1+\frac{R_S+sr_{\pi1}R_SC_{\pi1}}{r_{\pi1}}+sR_SC_{\mu1}\right)\left(\frac{1+sr_{\pi2}C_{\pi2}}{r_{\pi2}}+g_{m2}+sC_{\mu1}\right)+(sR_SC_{\mu1})(g_{m1}-sC_{\mu1})}=v_{be2}$$

Substituting (7) to (6) via v_{be2}:

$$\frac{-v_{in}g_{m2}(g_{m1}-sC_{\mu1})}{\left[\left(1+\frac{R_S+sr_{\pi1}R_SC_{\pi1}}{r_{\pi1}}+sR_SC_{\mu1}\right)\left(\frac{1+sr_{\pi2}C_{\pi2}}{r_{\pi2}}+g_{m2}+sC_{\mu1}\right)+sR_SC_{\mu1}(g_{m1}-sC_{\mu1})\right]}=v_{out}\left(sC_{\mu2}+\frac{1}{R_L}\right) \tag{8}$$

Rearranging to $\frac{v_{out}}{v_{in}}(s)$ and reinforcing the form given:

$$\frac{v_{out}}{v_{in}}(s)=\frac{-\left(\frac{g_{m1}g_{m2}r_{\pi1}r_{\pi2}R_L}{(r_{\pi1}+R_S)(1+g_{m2}r_{\pi2})}\right)\left(1-s\frac{C_{\mu1}}{g_{m1}}\right)}{\left(1+s\frac{r_{\pi1}R_S(C_{\pi1}+C_{\mu1})}{r_{\pi1}+R_S}\right)\left(1+s\frac{r_{\pi2}(C_{\pi2}+C_{\mu1})}{1+g_{m2}r_{\pi2}}\right)(1+sR_LC_{\mu2})+\frac{g_{m1}r_{\pi1}r_{\pi2}\left(1-s\frac{C_{\mu1}}{g_{m1}}\right)(1+sR_LC_{\mu2})(sR_SC_{\mu1})}{(r_{\pi1}+R_S)(1+g_{m2}r_{\pi2})}} \tag{9}$$

Ignoring the term $\left[\frac{g_{m1}r_{\pi1}r_{\pi2}}{(r_{\pi1}+R_S)(1+g_{m2}r_{\pi2})}\left(1-s\frac{C_{\mu1}}{g_{m1}}\right)(1+sR_LC_{\mu2})(sR_SC_{\mu1})\right]$ which is similar to the given form $a_1\left(1+\frac{s}{\omega_{z1}}\right)\left(1+\frac{s}{\omega_{p1}}\right)(s\tau_1)$, the final transfer function is:

$$\boxed{\frac{v_{out}}{v_{in}}(s)=-\left(\frac{g_{m1}g_{m2}r_{\pi1}r_{\pi2}R_L}{(r_{\pi1}+R_S)(1+g_{m2}r_{\pi2})}\right)\frac{\left(1-s\frac{C_{\mu1}}{g_{m1}}\right)}{\left(1+s\frac{r_{\pi1}R_S(C_{\pi1}+C_{\mu1})}{r_{\pi1}+R_S}\right)\left(1+s\frac{r_{\pi2}(C_{\pi2}+C_{\mu1})}{1+g_{m2}r_{\pi2}}\right)(1+sR_LC_{\mu2})}}$$

(f) From the derived $\frac{v_{out}}{v_{in}}(s)$, a_o, ω_{z1}, ω_{p1}, ω_{p2}, ω_{p3} can be extracted. Since f_z and f_p are needed, it is simply $f=\frac{\omega}{2\pi}$.
For the DC gain a_o:

$$a_o=-\frac{g_{m1}g_{m2}r_{\pi1}r_{\pi2}R_L}{(r_{\pi1}+R_S)(1+g_{m2}r_{\pi2})}$$
$$\boxed{a_o=-49.08\,V/V}$$

For zero and poles ω_z and ω_p:

$$\omega_{z1}=\frac{g_{m1}}{C_{\mu1}}$$
$$\omega_{p1}=-\frac{r_{\pi1}+R_S}{r_{\pi1}R_S(C_{\pi1}+C_{\mu1})}$$
$$\omega_{p2}=-\frac{1+g_{m2}r_{\pi2}}{r_{\pi2}(C_{\pi2}+C_{\mu1})}$$
$$\omega_{p3}=-\frac{1}{R_LC_{\mu2}}$$

The corresponding values of ω (which is in rad/sec) are:

$$\omega_{z1}=3.139\,Grad/sec$$
$$\omega_{p1}=-31.451\,Mrad/sec$$
$$\omega_{p2}=-2.828\,Grad/sec$$
$$\omega_{p3}=-6.460\,Mrad/sec$$

Getting f (which is in Hz):

$$f_{z1} = 499.51\, MHz$$
$$|f_{p1}| = 5.01\, MHz$$
$$|f_{p2}| = 450.05\, MHz$$
$$|f_{p3}| = 1.03\, MHz$$

(g) Shown in Figures 7, and 8 are the Magnitude and Phase Response of the simplified transfer function of the cascode amplifier.

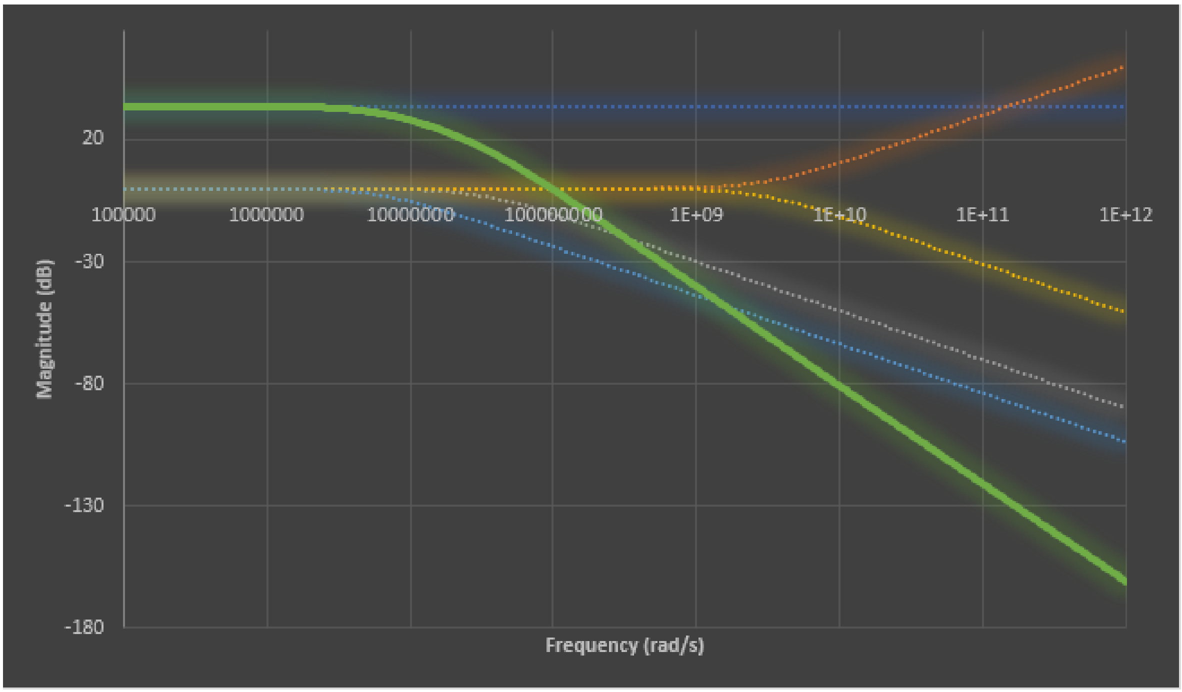

Figure 7: Magnitude Response of the Cascode Amplifier

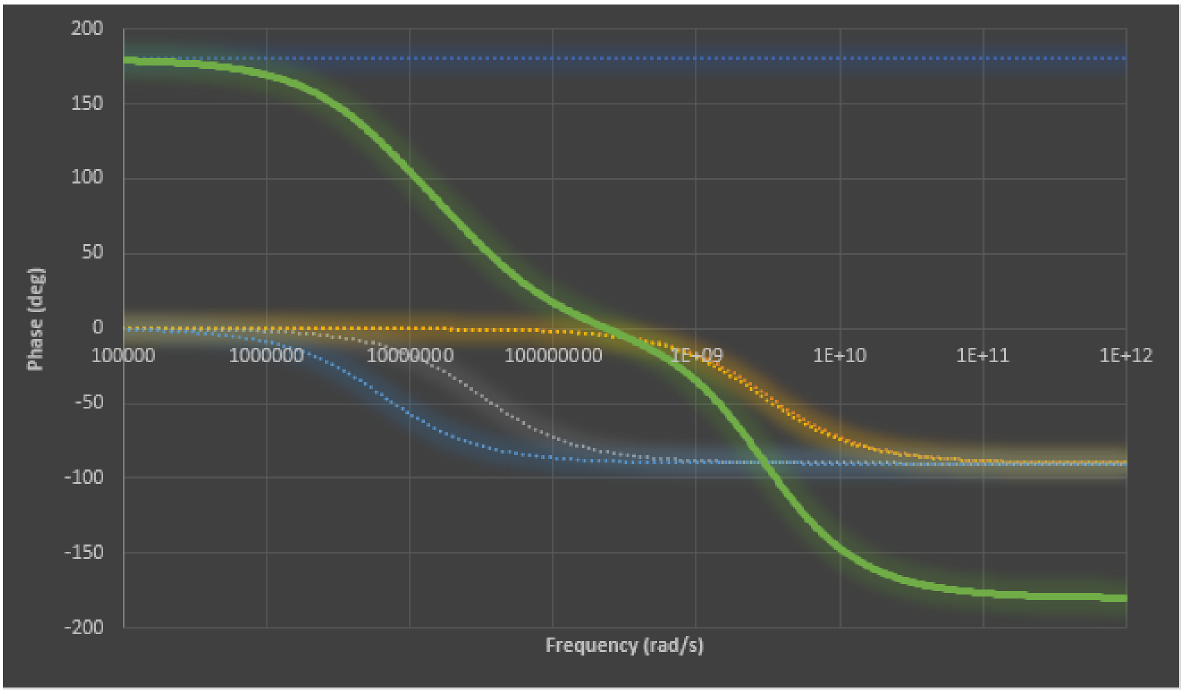

Figure 8: Phase Response of the Cascode Amplifier

The dashed lines from both magnitude and phase response are the contributions of a_o(navy blue), ω_{z1}(orange), ω_{p1}(gray), ω_{p2}(yellow), and ω_{p3}(light blue). The green line is the total of all the dashed line representing the response of the cascode amplifier.

(h) From the magnitude response, the unity gain frequency is around $96Mrad/s$ or $f_U = 15.38MHz$ which is far from $f_T = 450MHz$. Yes that one of the pole (f_{p2}) corresponds to f_T but keep in mind that f_T is the unity current gain of the transistor and not the whole amplifier. This is because f_T is calculated using the ratio of shorted collector current with respect to the base current of each transistor while for the whole amplifier's voltage gain/transfer function, it accounts of the resistive components in the circuit which mainly affects the positioning of the poles. Therefore, f_T does not directly translates to the poles/zeros, unity gain frequency f_U, and circuit level performance parameters of the whole amplifier but is a good metric for the performance of the device itself which can be seen mainly in the datasheet of the device.

3. **The Danger in Miller approximation.** Assume T=300K when necessary. Also assume that the external capacitances are much much greater than the parasitic capacitances. For the CE amplifier in fig. 9,, $V_{BE,on} = 0.7V$, $V_{CE,sat} = 0.2V$, $\beta = 200$, $V_A \to \infty$, $R_B = 95K\Omega$, $R_C = 204\Omega$, and $R_L = 794\Omega$:

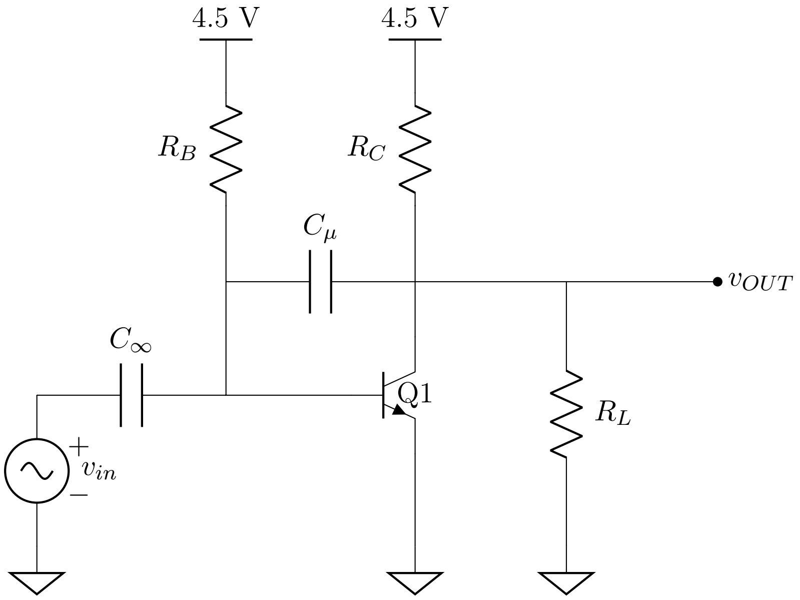

Figure 9: Common Emitter with capacitances

(a) Draw the small-signal model without using the Miller approximation for C_μ. Label the values of all components, v_{in}, and v_{out}. (1pt)

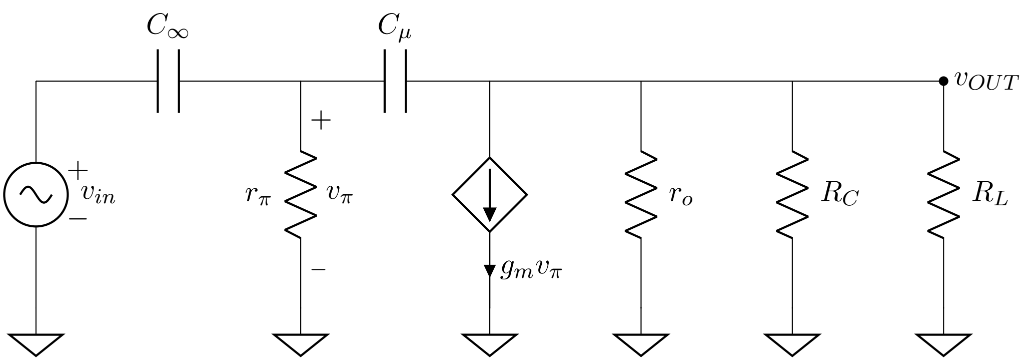

Figure 10: Common Emitter with capacitances small-signal model

This is the same circuit as in Homework 2, but with a capacitance C_μ across the base and the collector of Q1. Capacitors are effectively open at DC, so the circuit is the same as in Homework 2 at DC. Thus, the current $I_C = 8mA$. The following are the values of the components of the small-signal model:

$$
\begin{aligned}
g_m &= \frac{I_C}{V_T} = 307.692mS \\
r_\pi &= \frac{g_m}{\beta} = 650\Omega \\
r_o &= \frac{V_A}{I_C} = 12.5K\Omega \\
C_\mu &= 10nF(\text{was given in Piazza}) \\
R_C &= 204\Omega \\
R_L &= 794\Omega
\end{aligned}
$$

(b) Solve for the transfer function $\frac{v_{out}(s)}{v_{in}(s)}$. Write it in the form: $K\frac{(1+\frac{s}{\omega_{z1}})(1+\frac{s}{\omega_{z2}})...(1+\frac{s}{\omega_{zn}})}{(1+\frac{s}{\omega_{p1}})(1+\frac{s}{\omega_{p2}})...(1+\frac{s}{\omega_{pn}})}$ (1pt) The problem would be more insightful if we solve with v_{in} having a series resistance R_S first. So, we'll do that first. KCL at v_π node yields:

$$\frac{v_\pi - v_{in}}{R_S} + \frac{v_\pi}{r_\pi} + sC_\mu(v_\pi - v_{out}) = 0$$

$$v_\pi(\frac{1}{R_S} + \frac{1}{r_\pi} + sC_\mu) = \frac{v_{in}}{R_S} + sC_\mu v_{out}$$

If we let R_1 be the parallel combination of r_π and R_S,

$$v_\pi(\frac{1}{R_1} + sC_\mu) = \frac{v_{in}}{R_S} + sC_\mu v_{out}$$

$$v_\pi = \frac{1}{\frac{1}{R_1} + sC_\mu}(\frac{v_{in}}{R_S} + sC_\mu v_{out})$$

KCL at v_{out} node then yields:

$$sC_\mu(v_{out} - v_\pi) + g_m v_\pi + \frac{v_{out}}{r_o} + \frac{v_{out}}{R_C} + \frac{v_{out}}{R_L} = 0$$

If we let R_2 be the parallel combination of r_o, R_C, and R_L,

$$sC_\mu(v_{out} - v_\pi) + g_m v_\pi + \frac{v_{out}}{R_2} = 0$$

$$v_{out}(sC_\mu + \frac{1}{R_2}) + v_\pi(g_m - sC_\mu) = 0$$

$$v_{out}(sC_\mu + \frac{1}{R_2}) + \frac{g_m - sC_\mu}{\frac{1}{R_1} + sC_\mu}(\frac{v_{in}}{R_S} + sC_\mu v_{out}) = 0$$

$$v_{out}(sC_\mu(1 + \frac{g_m - sC_\mu}{\frac{1}{R_1} + sC_\mu}) + \frac{1}{R_2}(\frac{1}{R_1} + sC_\mu)) = -\frac{g_m - sC_\mu}{\frac{1}{R_1} + sC_\mu}(\frac{v_{in}}{R_S})$$

$$v_{out}(sC_\mu((\frac{1}{R_1} + sC_\mu) + (g_m - sC_\mu)) + \frac{1}{R_2}(\frac{1}{R_1} + sC_\mu)) = -(g_m - sC_\mu)\frac{v_{in}}{R_S}$$

$$v_{out}(sC_\mu(\frac{1}{R_1} + g_m) + \frac{1}{R_2}(\frac{1}{R_1} + sC_\mu)) = -(g_m - sC_\mu)\frac{v_{in}}{R_S}$$

$$-\frac{(g_m - sC_\mu)\frac{1}{R_S}}{sC_\mu(\frac{1}{R_1} + g_m) + \frac{1}{R_2}(\frac{1}{R_1} + sC_\mu)} = \frac{v_{out}}{v_{in}}$$

$$-\frac{(g_m - sC_\mu)\frac{1}{R_S}}{sC_\mu(\frac{1}{R_1} + \frac{1}{R_2} + g_m) + \frac{1}{R_1 R_2}} = \frac{v_{out}}{v_{in}}$$

$$-\frac{(g_m - sC_\mu)\frac{1}{R_S}}{\frac{1}{R_1 R_2} + sC_\mu(\frac{1}{R_1} + \frac{1}{R_2} + g_m)} = \frac{v_{out}}{v_{in}}$$

$$-\frac{\frac{g_m}{R_S}}{\frac{1}{R_1 R_2}}\frac{(1 - s\frac{C_\mu}{g_m})}{1 + sC_\mu\frac{(\frac{1}{R_1} + \frac{1}{R_2} + g_m)}{\frac{1}{R_1 R_2}}} = \frac{v_{out}}{v_{in}}$$

$$-\frac{g_m R_2 R_1}{R_S}\frac{(1 - s\frac{C_\mu}{g_m})}{1 + sC_\mu(R_2 + R_1 + g_m R_1 R_2)} = \frac{v_{out}}{v_{in}}$$

$$-g_m R_2\frac{R_1}{R_S}\frac{(1 - s\frac{C_\mu}{g_m})}{1 + sC_\mu(R_2 + R_1 + g_m R_1 R_2)} = \frac{v_{out}}{v_{in}}$$

Recall that R_1 is the parallel combination of r_π and R_S, $R_1 = \frac{r_\pi R_S}{r_\pi + R_S}$,

$$-g_m R_2\frac{r_\pi}{r_\pi + R_S}\frac{(1 - s\frac{C_\mu}{g_m})}{1 + sC_\mu(R_2 + R_1 + g_m R_1 R_2)} = \frac{v_{out}}{v_{in}}$$

From here, we can see that there is a zero at $\omega_z = \frac{g_m}{C_\mu}$, and a pole at $\omega_p = -\frac{1}{C_\mu(R_2+R_1+g_m R_1 R_2)}$ where R_1 is the parallel combination of R_S and r_π, while R_2 is the parallel combination of r_o, R_C, and R_L. It is noticeable that ω_p can be approximated as $\omega_p = \frac{1}{C_\mu R_1(1+g_m R_2)}$ where $g_m R_2$ is the DC gain of the amplifier, verifying the Miller effect.

For the case where $R_S = 0\Omega$,

$$\boxed{\frac{v_{out}}{v_{in}} = -g_m R_2 \frac{1 - s\frac{C_\mu}{g_m}}{1 + sC_\mu R_2} = -49.3 \frac{1 - \frac{s}{30.77Mrad/s}}{1 + \frac{s}{624.14Krad/s}}}$$

(c) If there are finite pole(s)/zero(s) in the transfer function, write their location/s. (0.5pt)

$$\boxed{\omega_z = \frac{g_m}{C_\mu} = 30.77Mrad/s}$$

$$\boxed{\omega_p = -\frac{1}{C_\mu R_2} = 624.14Krad/s}$$

(R_2 is the parallel combination of r_o, R_C, and R_L)

(d) Draw the small-signal model, using the Miller approximation for C_μ. Label the values of all components, v_{in}, and v_{out}. (1pt)
Generally, the miller approximation at the input would yield a lower pole, but the input here is shorted to v_{in} at AC. So we'll look at the Miller approximation at the output.

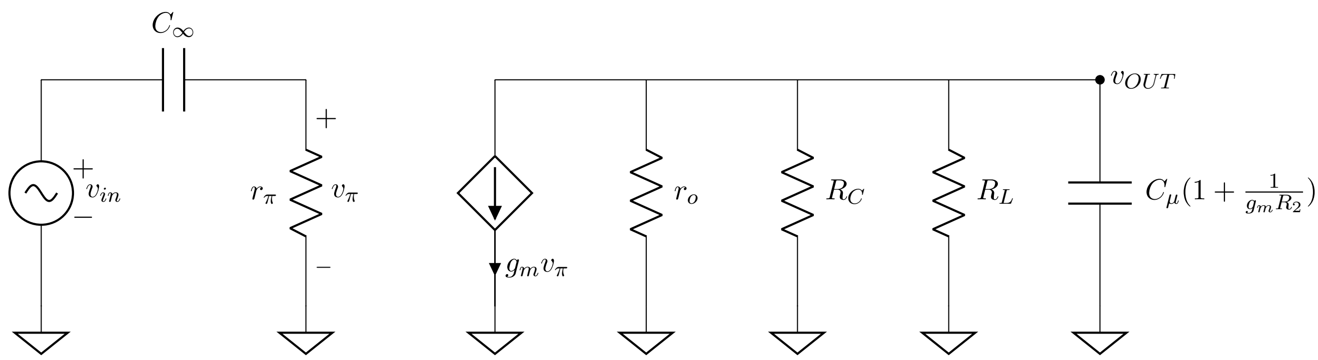

Figure 11: Common Emitter with capacitances small-signal model using Miller approximation at the output

(e) Solve for the transfer function $\frac{v_{out}(s)}{v_{in}(s)}$. Write it in the form: $K\frac{(1+\frac{s}{\omega_{z1}})(1+\frac{s}{\omega_{z2}})...(1+\frac{s}{\omega_{zn}})}{(1+\frac{s}{\omega_{p1}})(1+\frac{s}{\omega_{p2}})...(1+\frac{s}{\omega_{pn}})}$ (1pt)

$$v_\pi = v_{in}$$

$$\frac{v_{out}}{v_{in}} = -g_m(r_o||R_C||R_L||\frac{1}{sC_\mu(1+\frac{1}{g_m R_2})})$$

Let R_2 be the parallel combination of r_o, R_C, and R_L,

$$= -\frac{g_m}{\frac{1}{R_2} + sC_\mu(1+\frac{1}{g_m R_2})}$$

$$= -g_m R_2 \frac{1}{1 + sC_\mu(R_2 + \frac{1}{g_m})}$$

$$= \boxed{-g_m R_2 \frac{1}{1 + sC_\mu(R_2 + \frac{1}{g_m})}}$$

(f) If there are finite pole(s)/zero(s) in the transfer function, write their location/s. (0.5pt)

$\boxed{\text{Based on part E, there is a pole at } \omega_p = -\frac{1}{C_\mu(R_2+\frac{1}{g_m})} = 611.73Krad/s.}$ Though if we look at the case where the input is not shorted to v_{in} at AC, i.e. there is a source resistance R_S, and solve with the Miller approximation at the input, we would draw the small-signal model as:

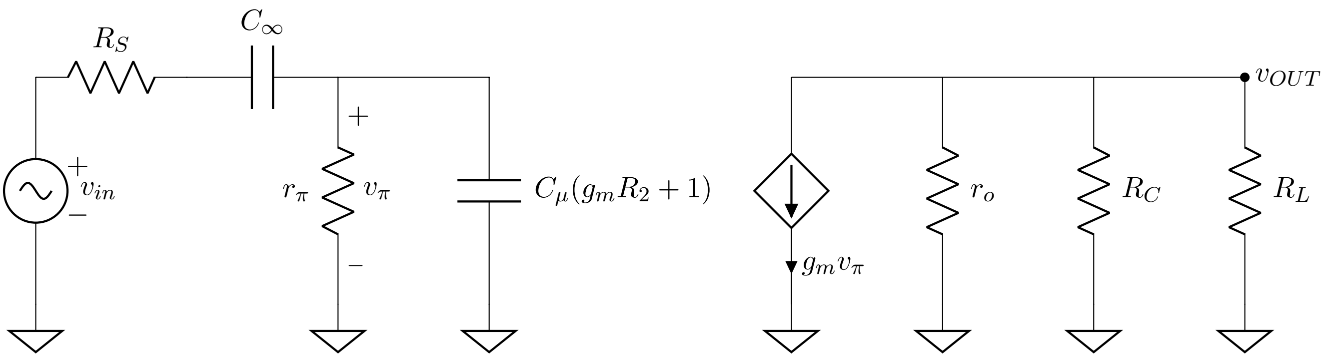

Figure 12: Common Emitter with capacitances small-signal model using Miller approximation at the input

And we would solve,

$$v_\pi = \frac{r_\pi}{R_S+r_\pi}\frac{1}{1+sC_\mu(g_mR_2+1)(r_\pi||R_S)}v_{in}$$

$$v_{out} = -g_m(r_o||R_C||R_L)v_\pi$$

Let R_2 be the parallel combination of r_o, R_C, and R_L,

$$\begin{aligned} v_{out} &= -g_mR_2v_\pi \\ &= -g_mR_2\frac{r_\pi}{R_S+r_\pi}\frac{1}{1+sC_\mu(g_mR_2+1)(r_\pi||R_S)}v_{in} \\ \frac{v_{out}}{v_{in}} &= -g_mR_2\frac{r_\pi}{R_S+r_\pi}\frac{1}{1+sC_\mu(g_mR_2+1)(r_\pi||R_S)} \end{aligned}$$

Let R_1 be the parallel combination of r_π and R_S,

$$= \boxed{-g_mR_2\frac{r_\pi}{R_S+r_\pi}\frac{1}{1+sC_\mu(g_mR_2+1)R_1}}$$

Thus, if we had used a non-ideal source, we would verify the Miller effect, inducing a pole at $\omega_p = -\frac{1}{C_\mu(g_mR_2+1)R_1}$ where R_1 is the parallel combination of r_π and R_S.

(g) Is there a difference/s between the transfer functions in parts (b) and (e)? If yes, enumerate it/them. (1pt)
Yes. There are differences. The original transfer function has a zero at $\omega_z = \frac{g_m}{C_\mu}$, while the Miller approximated one doesn't. Also, for the general case of a non-zero R_S, the pole of the original function is at $\omega_p = -\frac{1}{C_\mu(R_2+R_1+g_mR_1R_2)}$, whereas using the Miller approximation at the input or the output yields $\omega_p = -\frac{1}{C_\mu(R_1+g_mR_1R_2)}$ or $\omega_p = -\frac{1}{C_\mu R_2}$ respectively.

For the remainder of the problem, refer to the transfer function in part (b).

(h) Plot the pole-zero diagram. Label the axes, and the pole(s)/zero(s). (1pt)
Figure 13 shows the pole-zero diagram, with the pole and zero locations as:

$$\omega_z = \frac{g_m}{C_\mu} = 30.77Mrad/s$$

$$\omega_p = -\frac{1}{C_\mu R_2} = 624.14Krad/s$$

(R_2 is the parallel combination of r_o, R_C, and R_L)

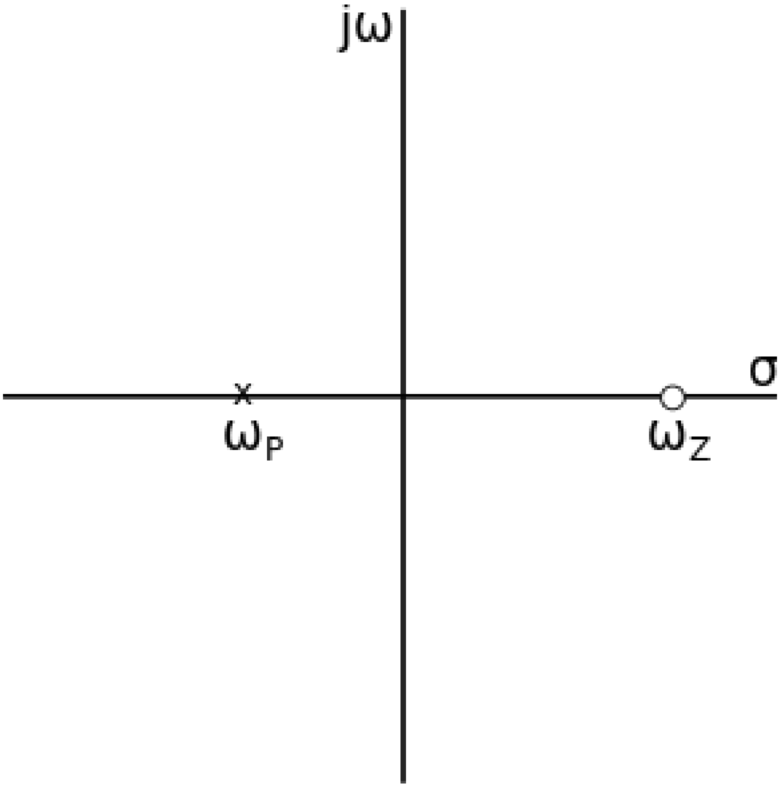

Figure 13: Pole-Zero Diagram of Common-emitter with Miller capacitance.

(i) Sketch the magnitude response (in dB) of the circuit on a log-log plot. Label the pole/zero location/s, DC gain, slopes, and the axes. (1pt)

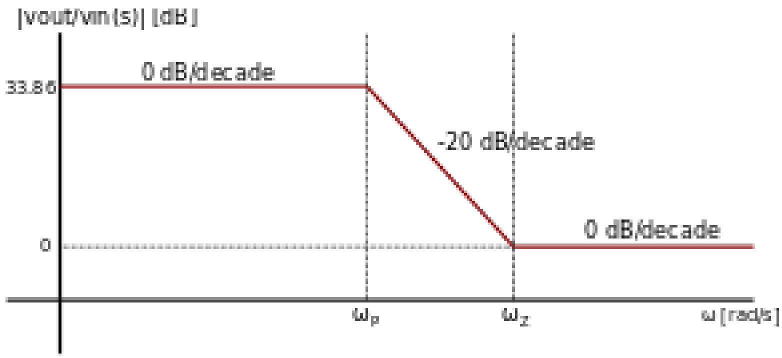

Figure 14: Magnitude Response of Common-emitter with Miller capacitance.

(j) Sketch the phase response (in degrees) of the circuit on a semi-log plot. Label the pole/zero location/s, phase at 0 Degrees, slopes, the axes, and important frequencies. (1pt)

Note that the zero is in the right-half plane. Therefore, the magnitude contribution of the zero is still $+20$dB/decade, while its phase contribution becomes -45^O/decade.

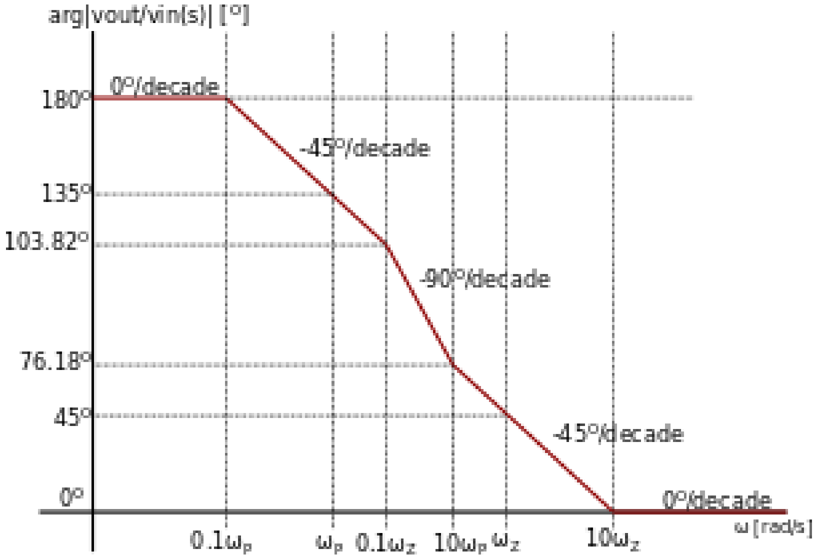

Figure 15: Phase Response of Common-emitter with Miller capacitance.

(k) What is the unity-gain frequency (in rad/s), ω_u? (1pt)
From the magnitude plot (fig. 14, we can see that the magnitude is asymptotic to 1 at very high frequencies. From this, we can say that $\boxed{\text{the unity-gain frequency is "at infinity."}}$