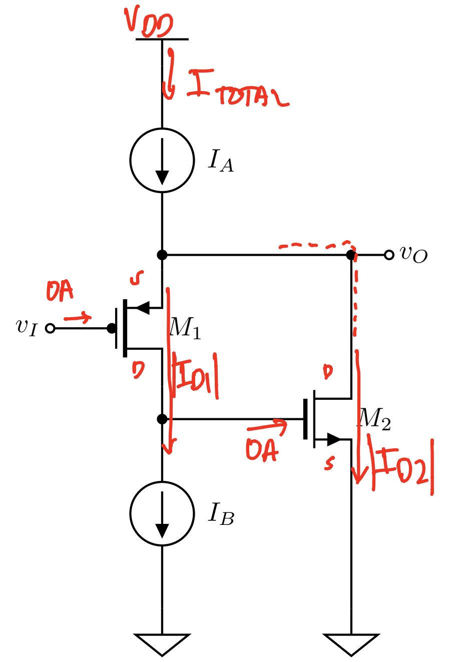
Name: SOLUTION	Student No.:

Part II:

(20 points) The transistors in the MOSFET amplifier shown have identical properties: $|k_1| = |k_2| = 125\frac{\mu A}{V^2}$ and $|\lambda_1| = |\lambda_2| = 0.01V^{-1}$. Both transistors are biased in the saturation region. The circuit draws a total of $180\mu W$ from a $1.8V$ supply. One current source is known to have a current of $20\mu A$ more than the other. Assume that the current sources are ideal.

open circuit in small signal



* $M_1 \rightarrow$ PMOS
* $M_2 \rightarrow$ NMOS

1. Determine the current through the two current sources. *(1 point)*

$P = V_{DD} \cdot I_{TOTAL}$

$I_{TOTAL} = \frac{P}{V_{DD}} = \frac{180\mu W}{1.8V} = 100\mu A$

$I_{TOTAL} = I_A = 100\mu A$

by KCL: $I_A = I_B + I_{D2}$

$\rightarrow I_A > I_B$

$I_B = I_A - 20\mu A$

$I_B = 80\mu A$

$I_A = 100\mu A$	$I_B = 80\mu A$

2. What are the drain currents of the two transistors? *(1 point)*

$|I_{D1}| = I_B = 80\mu A$

$|I_{D2}| = I_A - I_B = 100\mu A - 80\mu A = 20\mu A$

$\lvert I_{D1}\rvert = 80\mu A$	$\lvert I_{D2}\rvert = 20\mu A$

3. Determine the small signal parameters of the two transistors. *(2 points)*

$g_{m1} = \sqrt{4kI_{D1}} = \sqrt{4\cdot 125\,\frac{\mu A}{V^2}\cdot 80\mu A} = 200\,\mu S$

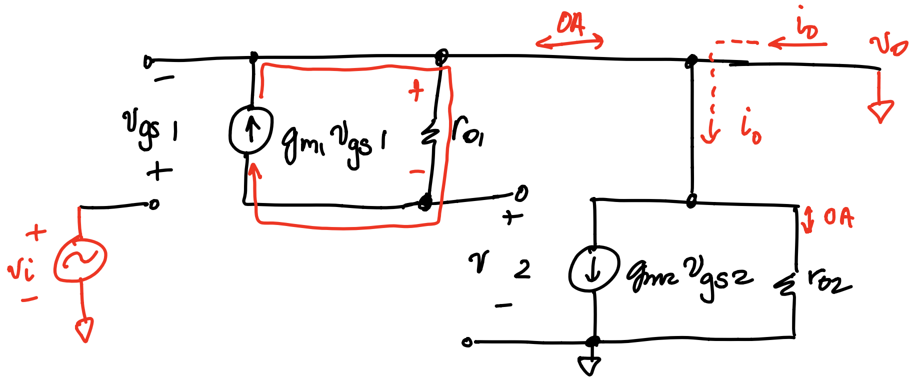
$g_{m2} = \sqrt{4kI_{D2}} = \sqrt{4\cdot 125\,\frac{\mu A}{V^2}\cdot 20\mu A} = 100\,\mu S$

$r_{o1} = \frac{1}{\lambda I_{D1}} = \frac{1}{0.01\,V^{-1}\cdot 80\mu A} = 1.25\,M\Omega$

$r_{o2} = \frac{1}{\lambda I_{D2}} = \frac{1}{0.01\,V^{-1}\cdot 20\mu A} = 5\,M\Omega$

$g_{m1} =$ $200\,\mu S$	$g_{m2} =$ $100\,\mu S$	$r_{o1} =$ $1.25\,M\Omega$	$r_{o2} =$ $5\,M\Omega$

4. Derive the approximate expression for the transconductance of the amplifier. Simplify the expression by assuming that $g_x \gg \frac{1}{r_x}$ where g_x can be any of g_{m1} or g_{m2} and r_x can be any of r_{o1} or r_{o2}. Include the small signal circuit needed in deriving the expression. *(6 points)*



* r_{o2} is shorted to ground
→ no voltage, no current

* current $g_{m1}v_{gs1}$ flows only through r_{o1} (can be shown by KCL at drain of M_1)
→ no current from M_1 to M_2 or output node

→ by KCL: $i_o = g_{m2}v_{gs2}$

→ $v_{gs1} = v_i - \cancel{v_o}^{0} = v_i$

→ $v_{r_{o1}} = g_{m1}v_{gs1}\cdot r_{o1} = \cancel{v_o}^{0} - v_{gs2}$

$g_{m1}r_{o1}v_i = -v_{gs2}$

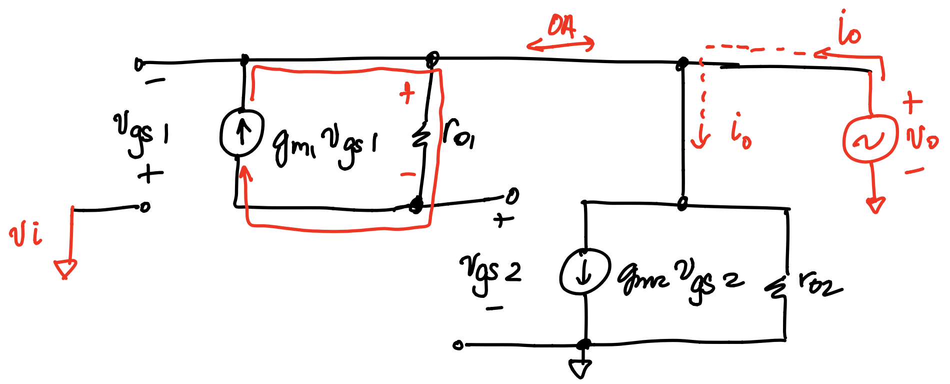
$v_{gs2} = -g_{m1}r_{o1}v_i$

→ $i_o = g_{m2}v_{gs2} = g_{m2}(-g_{m1}r_{o1}v_i)$

$\Rightarrow \frac{i_o}{v_i}\Big|_{v_o=0} = G_M = -g_{m2}g_{m1}r_{o1}$

$G_{M(\text{expression})} = -g_{m2}g_{m1}r_{o1}$

5. Derive the approximate expression for the output resistance of the amplifier. Simplify the expression by assuming that $g_x \gg \frac{1}{r_x}$ where g_x can be any of g_{m1} or g_{m2} and r_x can be any of r_{o1} or r_{o2}. Include the small signal circuit needed in deriving the expression. *(6 points)*



$\rightarrow v_{gs1} = \cancel{v_i}^{\,0} - v_o = -v_o$

$g_x \gg \frac{1}{r_x}$

$g_x r_x \gg 1$

$\rightarrow v_{ro1} = g_{m1} v_{gs1} \cdot r_{o1} = v_o - v_{gs2}$

$g_{m1} r_{o1} \cdot (-v_o) = v_o - v_{gs2}$

$v_{gs2} = v_o \cdot (1 + g_{m1} r_{o1})$

$v_{gs2} \approx v_o (g_{m1} r_{o1})$

$\rightarrow i_o = g_{m2} v_{gs2} + \frac{v_o}{r_{o2}} \approx g_{m2} \cdot [v_o (g_{m1} r_{o1})] + \frac{v_o}{r_{o2}} = v_o \left(g_{m2} g_{m1} r_{o1} + \frac{1}{r_{o2}} \right)$

$\left.\frac{v_o}{i_o}\right|_{v_i=0} = R_O \approx \frac{1}{g_{m2} g_{m1} r_{o1} + \frac{1}{r_{o2}}} \approx \frac{1}{g_{m2} g_{m1} r_{o1}}$

$g_{m2} \gg \frac{1}{r_{o2}}$ and

$g_{m1} r_{o1} \gg 1$

$\therefore g_{m2} g_{m1} r_{o1} \ggg \frac{1}{r_{o2}}$

$R_{O(\text{expression})} = \frac{1}{g_{m2} g_{m1} r_{o1}}$

6. What is the value of the output resistance? *(1 point)*

$R_O = \frac{1}{g_{m2} g_{m1} r_{o1}} = \frac{1}{100\mu S \cdot 200\mu S \cdot 1.25 M\Omega} = 40\Omega$

$R_O = 40\Omega$

7. Based on the derived expressions, to which *single transistor amplifier* is the circuit analyzed closest to? What is the *expression* for the output resistance of this single transistor amplifier when biased with an ideal current source at its output? Do NOT assume that $g_m r_o \gg 1$. *(2 points)*

$$A_V = -G_M R_O \approx -(-g_{m2} g_{m1} r_{o1}) \cdot \frac{1}{g_{m2} g_{m1} r_{o1}}$$

$A_V \approx 1 \rightarrow$ unity

$R_O \approx \frac{1}{g_{m2} g_{m1} r_{o1}} \longrightarrow$ small $\qquad R_I \rightarrow \infty$ (gate)

Common DRAIN	$R_{O(\text{single transistor amplifier})} = \frac{r_o}{1+g_m r_o}$ or $\frac{1}{g_m} \parallel r_o$

8. Suppose that the single transistor amplifier identified in the previous number was biased to achieve the same output resistance as the main amplifier analyzed in this problem. How much current is needed by the single transistor amplifier? Assume that the same MOSFET is used ($k = 125\frac{\mu A}{V^2}$ and $\lambda = 0.01V^{-1}$). *(1 point)*

$$R_{O(\text{target})} = 40\Omega = \frac{1}{g_m} \parallel r_o = \frac{1}{g_m + \frac{1}{r_o}}$$

$$40\Omega = \frac{1}{\sqrt{4kI_D} + \lambda I_D}$$

$$40\Omega = \frac{1}{\sqrt{4 \cdot 125\frac{\mu A}{V^2} \cdot I_D} + 0.01V^{-1} \cdot I_D} \Rightarrow I_D \approx 0.67A$$

$\lvert I_{D(\text{single transistor amplifier})} \rvert =$ 670 mA	