ECE 113: Communication Electronics

Lecture 6: Resonant Circuits

February 4, 2019

Resonance

Definition

- the quality in a sound of being deep, full, and reverberating
- the condition in which an object or system is subjected to an oscillating force having a frequency close to its own natural frequency
- the occurrence of a simple ratio between the periods of revolution of two bodies about a single primary

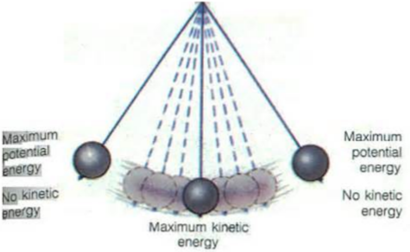
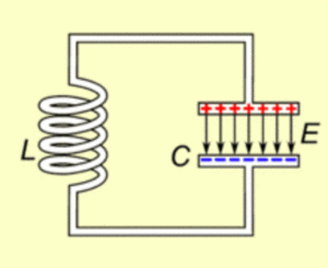
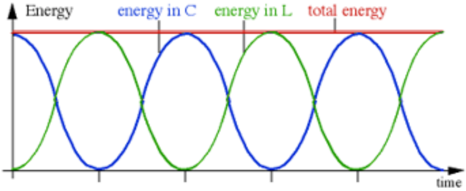


Image retrieved from [1] http://www.delawareright.com/wp-content/uploads/2014/12/pendulum.jpg

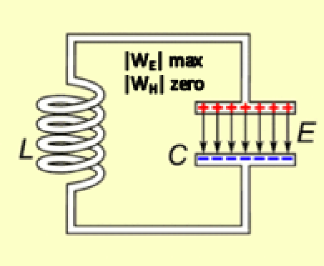
Electromagnetic Resonance

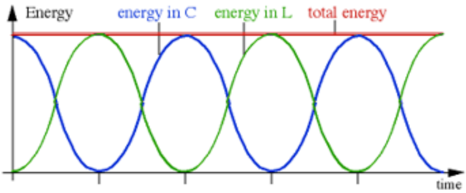
- Occurs when Capacitors and Inductors (reactive components that store energy) are connected together in a circuit





- Oscillates between 2 states:
  - Discharging capacitor produces current that sets up H-field in inductor
  - Collapsing H-field produces current that charges capacitor

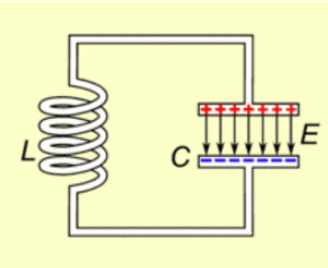


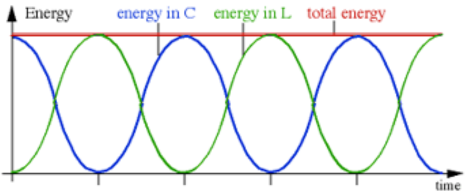


- In resonant circuits, this oscillation happens very fast
  - Resonant frequency - frequency in which

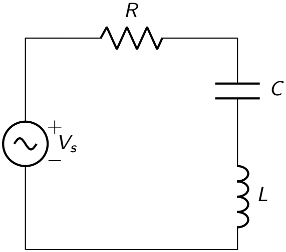
$$\text{Ave. energy stored in L} = \text{Ave. energy stored in C}$$

- Ideal LC tank circuit (lossless)
  - In reality, damping will introduced





Series Resonant Circuit



- Input Impedance

$$Z_{in} = R + j\omega L - j\frac{1}{\omega C}$$

- Complex Power

$$S_{in} = \frac{1}{2}VI^* = \frac{1}{2}|I|^2 Z_{in} = \frac{1}{2}I^2(R + j\omega L - j\frac{1}{\omega C})$$

$$S_{in} = P_{loss} + j\omega(W_M - W_E)$$

At Resonance

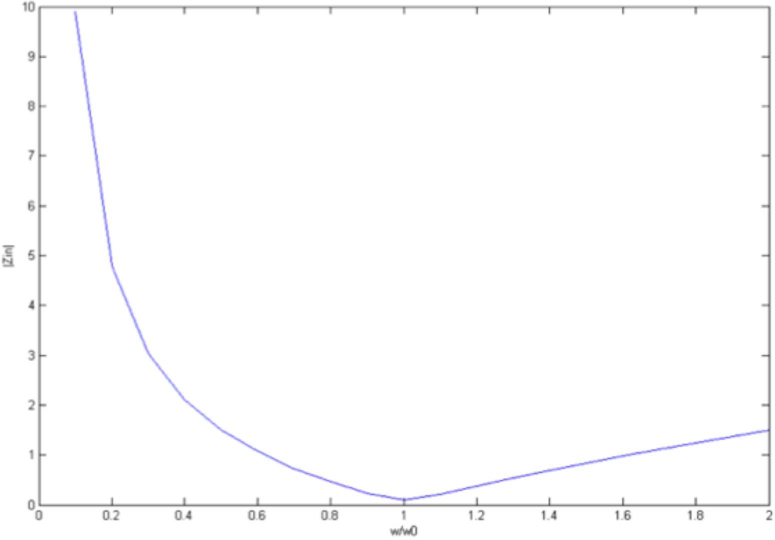
- Resonance occurs when average energy stored in E-field and H-field are equal

$$Z_{in} = \frac{P_{loss} + j\omega(W_M - W_E)}{\frac{|I|^2}{2}} = \frac{P_{loss}}{\frac{|I|^2}{2}} = R$$

- Resonant frequency, $W_M = W_E$

$$\frac{1}{2}|I|^2 L = \frac{1}{2}|I|^2 \frac{1}{\omega^2 C} \rightarrow \omega_o = \frac{1}{\sqrt{LC}}$$

Series Resonant Circuit



Circuit Q (Series Resonant Circuit)

- Different from component Q
- Measure of loss in the circuit
- Measure of selectivity (for tuning circuits)
- $\uparrow$Q translates to $\downarrow$loss

$$Q = 2\pi \times \frac{Max\ Energy\ Stored}{Energy\ Lost\ Per\ Cycle}$$

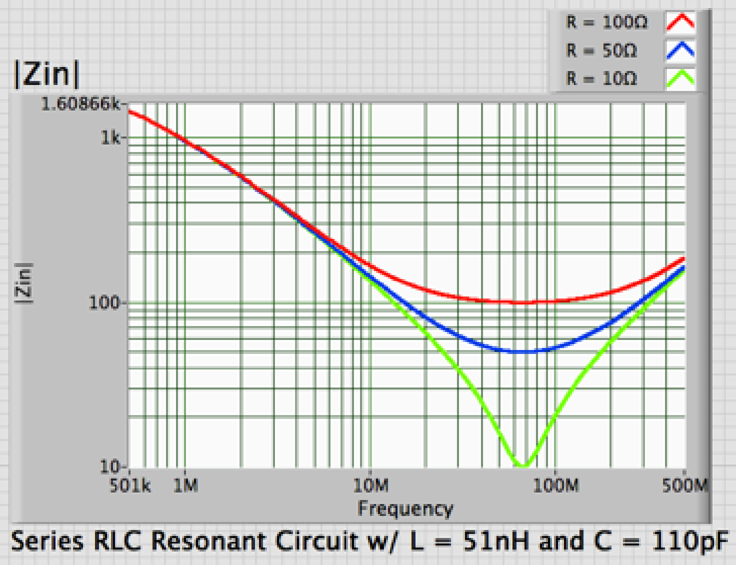
Circuit Q (Series Resonant Circuit)

- At resonance:

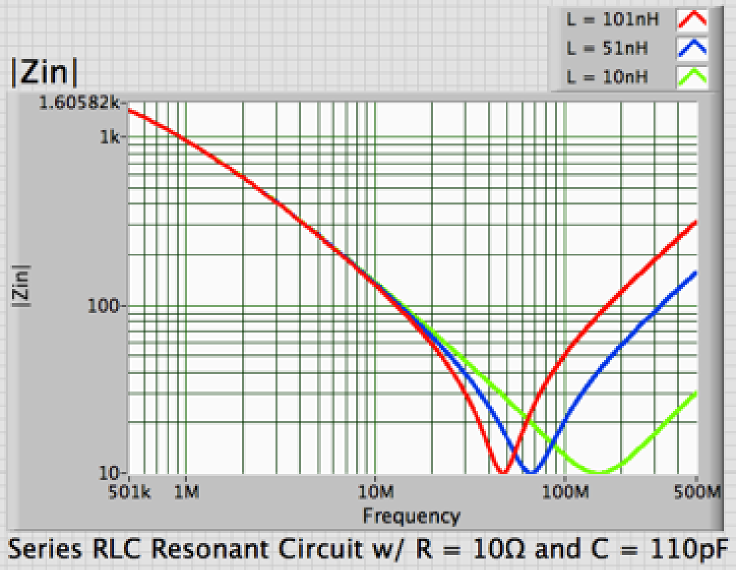
$$Q = 2\pi \times \frac{W_M \; or \; W_E}{\frac{P_{loss}}{f_o}}$$

$$Q_s = \frac{\omega_o L}{R} \qquad\qquad Q_s = \frac{1}{\omega_o RC}$$

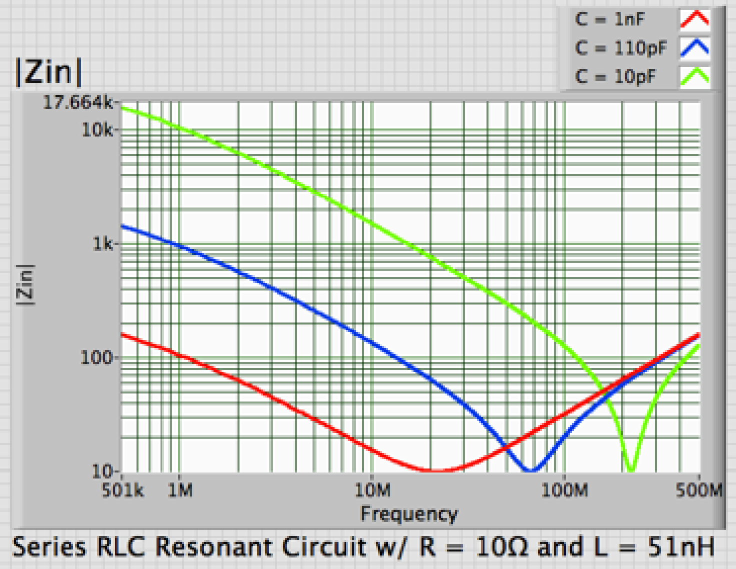
Example: Varying Resistance



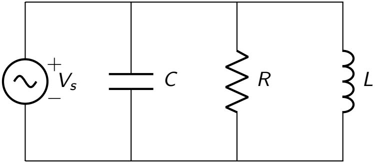
Example: Varying Inductance



Example: Varying Capacitance



Parallel Resonant Circuit



- Input impedance

$$Z_{in} = (\frac{1}{R} - \frac{j}{\omega L} + j\omega C)^{-1}$$

- Complex Power

$$S_{in} = \frac{1}{2}VI^* = \frac{1}{2}V(\frac{V}{Z_{in}})^* = \frac{1}{2}\frac{|V|^2}{Z_{in}^*}$$

$$S_{in} = P_{loss} + j\omega(W_M - W_E)$$

At Resonance

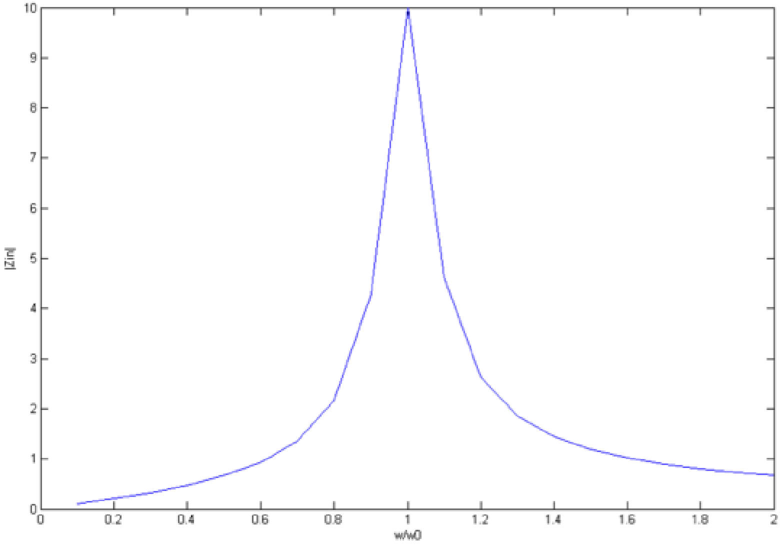
- Input impedance is still purely real (purely resistive) at resonance

$$Z_{in} = \frac{P_{loss}}{\frac{|I|^2}{2}} = R$$

- Resonant frequency is still

$$\omega_o = \frac{1}{\sqrt{LC}}$$

Parallel Resonant Circuit



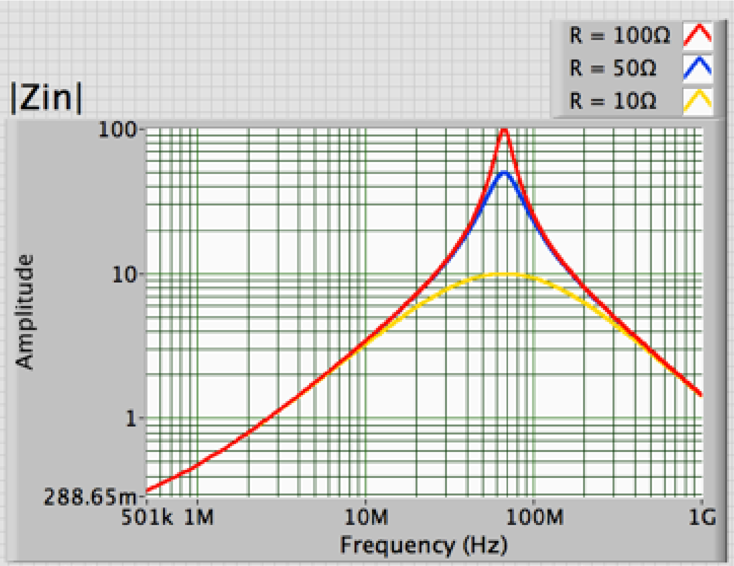
Circuit Q (Parallel Resonant Circuit)

- At resonance:

$$Q = 2\pi \times \frac{W_M \text{ or } W_E}{\frac{P_{loss}}{f_o}}$$

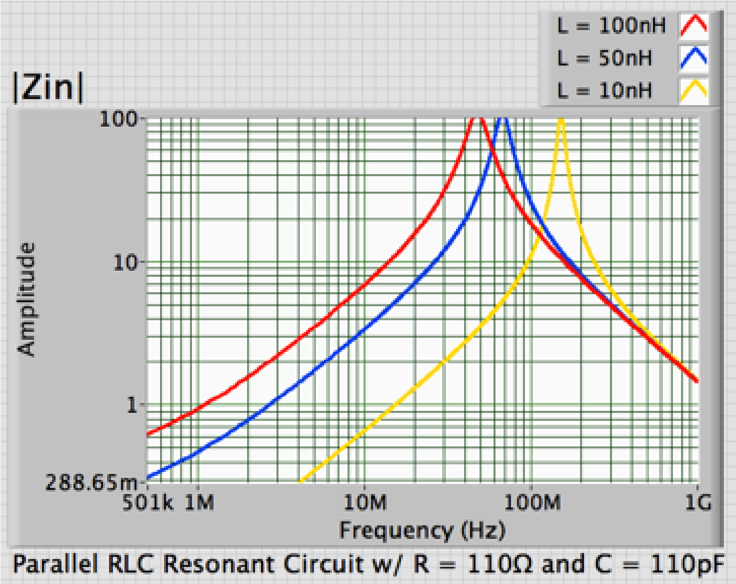
$$Q_p = \frac{R}{\omega_o L} \qquad Q_p = \omega_o RC$$

Example: Varying Resistance

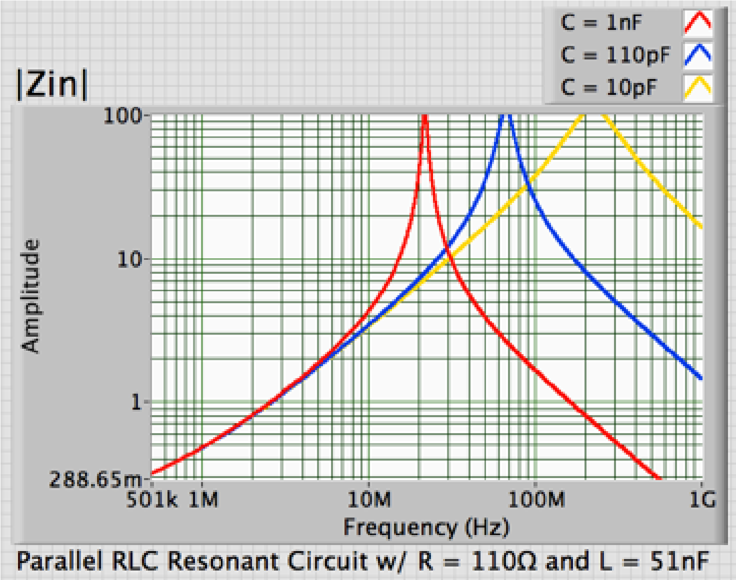


Parallel RLC Resonant Circuit w/ L = 51nH and C = 110pF

Example: Varying Inductance



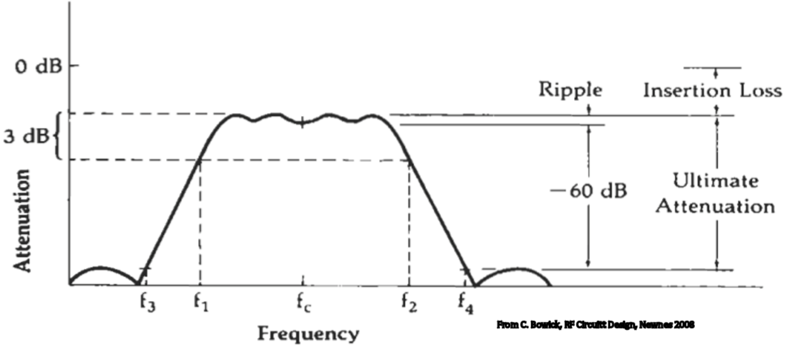
Example: Varying Capacitance



Resonant Circuits in RF Applications

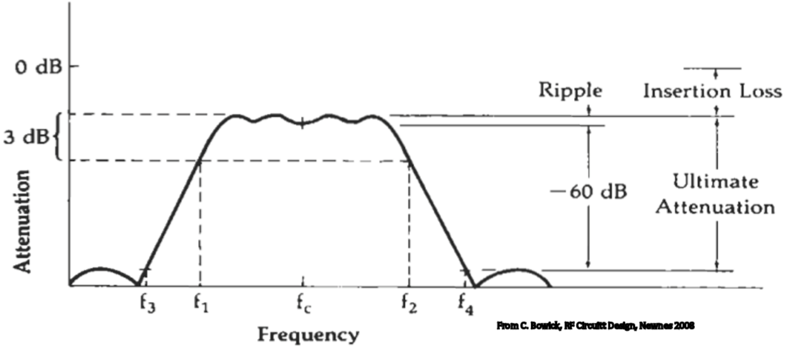
- Used extensively to **selectively pass** certain frequency components from a source to a load while attenuating other undesired frequencies.
- Lowpass, Highpass, Bandpass, Bandstop Filters

Resonant Circuit Response Characteristics



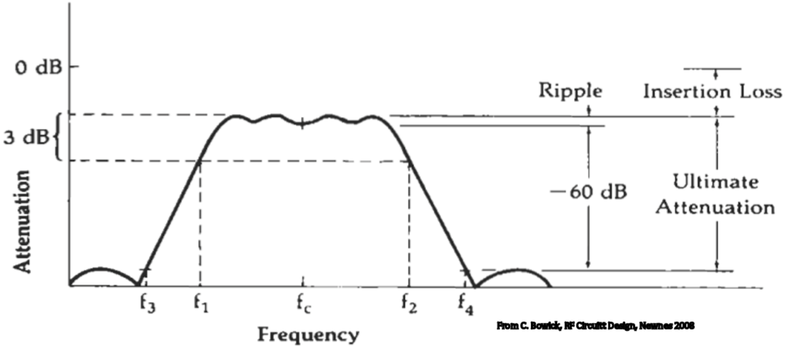
- Insertion Loss
  - The attenuation that is introduced into the passband to the resonant circuit.

Resonant Circuit Response Characteristics



- Ripple
  - Measure of flatness of the passband
  - The difference between minimum and maximum attenuation at the passband of the resonant circuit

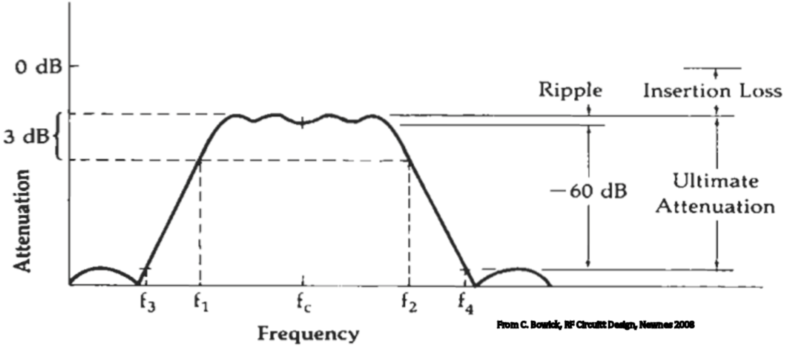
Resonant Circuit Response Characteristics



- Bandwidth
  - Difference between the upper and lower frequency of the circuit at which the magnitude response is 3dB below the passband response

$$BW = \omega_H - \omega_L$$

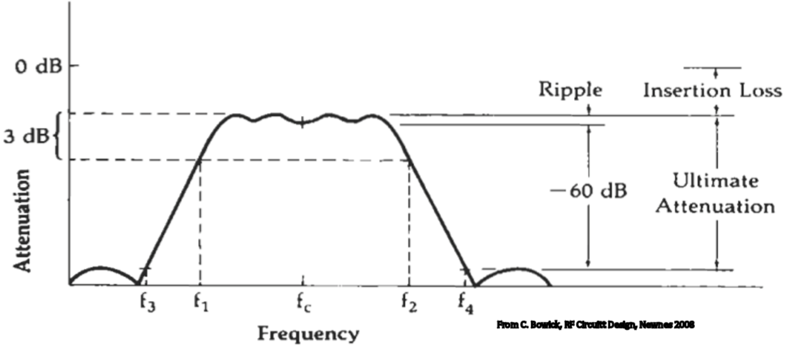
Resonant Circuit Response Characteristics



- Fractional Bandwidth
  - Bandwidth divided by the center frequency ω_o

$$fBW = \frac{\omega_H - \omega_L}{\omega_o}$$

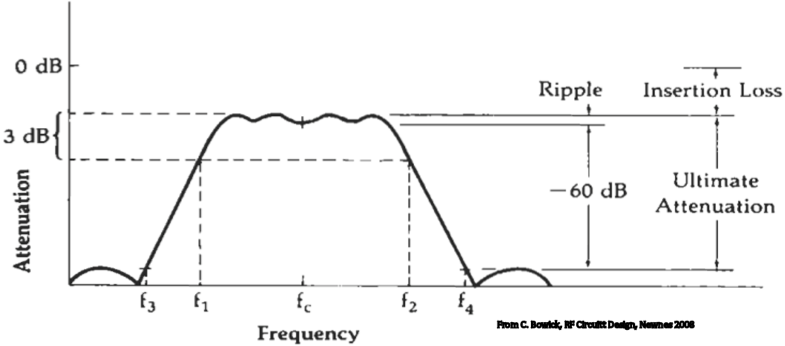
Resonant Circuit Response Characteristics



- Circuit Quality Factor (Q)
  - Ratio between energy stored and energy dissipated in a resonant circuit
  - Measure of selectivity of the resonant circuit

$$Q = \frac{\omega_o}{\omega_H - \omega_L} = \frac{\omega_o}{BW}$$

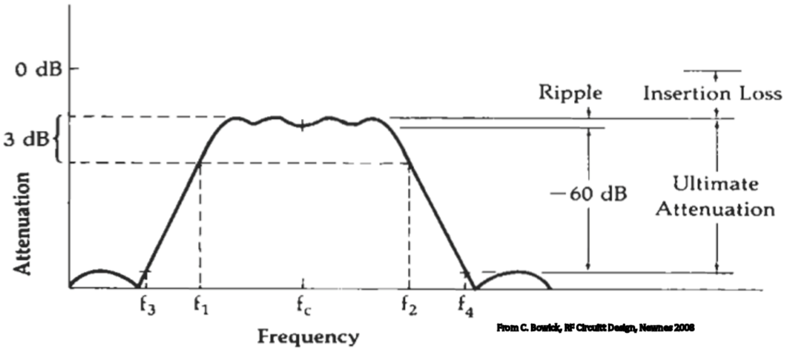
Resonant Circuit Response Characteristics



- Shape Factor
  - Measures the steepness of the "skirts" of a resonant circuit
  - Ratio between 60dB and 3dB bandwidth of a resonant circuit

$$\text{SF} = \frac{\omega_{H60} - \omega_{L60}}{\omega_H - \omega_L}$$

Resonant Circuit Response Characteristics



- Ultimate Attenuation
  - Minimum attenuation of the resonant circuit outside of the pass band

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