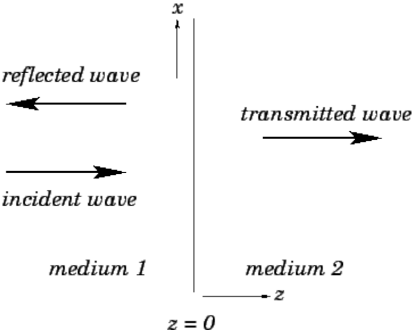
ECE 113: Communication Electronics

Meeting 5: Network Analysis II

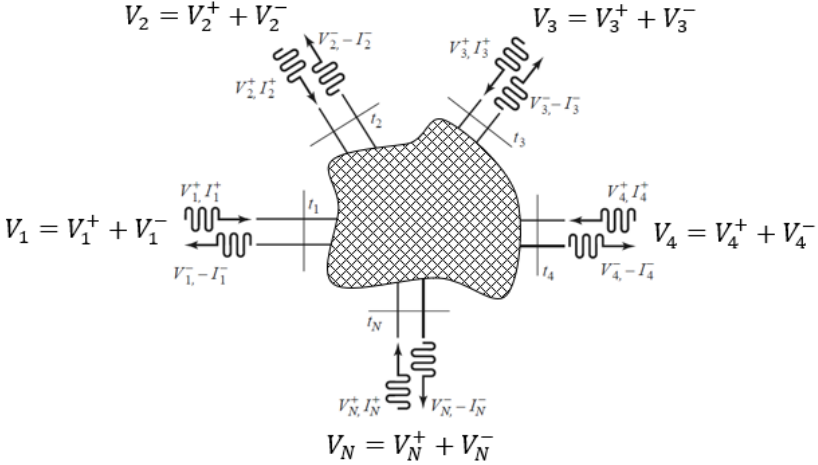
February 4, 2019

Traveling Waves



- Voltages and Currents at a specified terminal (port) can be identified as its incident/reflected wave
- At any given terminal/port n, the voltage and current is given by:
  - $V_n = V_n^+ + V_n^-$
  - $I_n = I_n^+ + I_n^-$

N-Port Network



Reflection Coefficient

- Relates incident and reflected waves at a certain point of the circuit.
- Given the characteristic impedance Z_O, the reflection coefficient of a load impedance Z_L is given by

$$\Gamma = \frac{V^-}{V^+} = \frac{Z_L - Z_O}{Z_L + Z_O}$$

The Scattering Parameters

- For high frequencies, it is more natural to work with forward and reverse propagating waves.
- The scattering parameters (S-parameters) relates the incident and reflected voltages/currents in an N-port network
- Provides a complete description of the network as seen at its N-ports

Scattering Parameters

- The scattering matrixor the **[S]** matrix is defined in terms of the incident V_n^+ and reflected V_n^-

$$\begin{bmatrix} V_1^- \\ V_2^- \\ \cdot \\ \cdot \\ V_N^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdot & \cdot & S_{1N} \\ S_{21} & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ S_{N1} & \cdot & \cdot & \cdot & S_{NN} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ \cdot \\ \cdot \\ V_N^+ \end{bmatrix}$$

- In matrix form:

$$[V^-] = [S][V^+]$$

S-matrix Elements

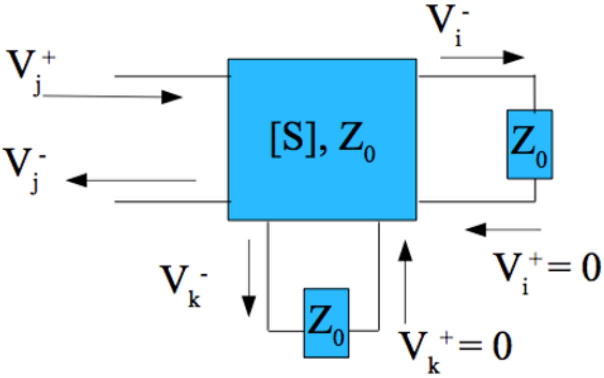
- The elements of the S-matrix S_{ij} can be evaluated

$$S_{ij} = \frac{V_i^-}{V_j^+}\Big|_{V_k^+=0} \text{ for } k \neq j$$

- S_{ij} is found by driving port j with incident voltage V_j^+ and measuring the reflected voltage V_i^- coming out of port i
- How do we set $V_k^+ = 0$ for $k \neq j$?

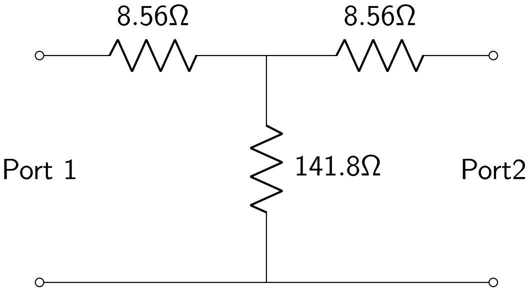
S-matrix Elements

- $V_k^+ = 0$ holds when the ports are terminated with characteristic impedance Z_O

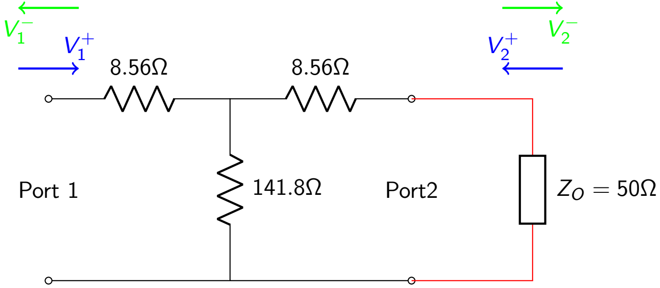


Example

- Find the S-parameters of the circuit below. The characteristic impedance $Z_O = 50\Omega$.



- Solving for S_{11}

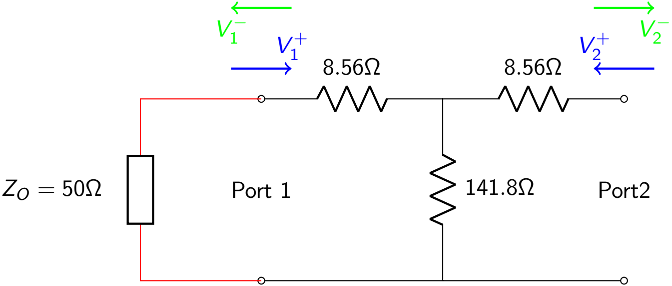


$$S_{11} = \frac{V_1^-}{V_1^+}\Big|_{V_k=0 \ \ for \ \ k\neq 1}$$

$$Z_{in1} = [(50 + 8.56) \parallel 141.8] + 8.56 = 50\Omega$$

$$S_{11} = \frac{V_1^-}{V_1^+} = \Gamma_1 = \frac{Z_{in1} - Z_O}{Z_{in1} + Z_O} = \frac{50 - 50}{50 + 50} = 0$$

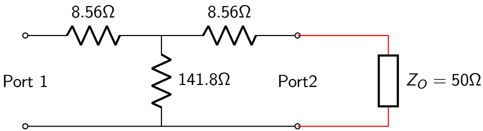
- Solving for S_{22}



$$S_{22} = \frac{V_2^-}{V_2^+}$$

$$S_{22} = S_{11} = 0$$

- Solving for S_{21}



$$S_{21} = \frac{V_2^-}{V_1^+} | V_k^+ = 0 \ for \ k \neq 1$$

$V_2^+ = 0 \rightarrow V_2 = V_2^-$ matched $\rightarrow$ no reflections at driving port

$$V_1^- = 0 \rightarrow V_1 = V_1^+$$

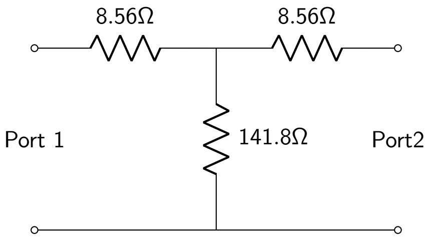
$$V_2^- = V_1^+ \left(\frac{(50 + 8.56) \parallel 141.8}{(50 + 8.56) \parallel 141.8 + 8.56}\right)\left(\frac{50}{50 + 8.56}\right)$$

$$S_{21} = \frac{V_2^-}{V_1^+} = 0.707 = S_{12}$$

- Consolidating the matrix elements

$$[S]=\begin{bmatrix} 0 & 0.707 \\ 0.707 & 0 \end{bmatrix}$$

- This network is a 3-dB attenuator and is a reciprocal network



Reciprocal and Lossless Networks

- For a reciprocal network, the [S] matrix is symmetric:

$$[S] = [S]^T$$

- For a lossless network, the [S] matrix is unitary:

$\sum_{k=1}^{N} S_{ki} S_{kj}^* = 1$	for all $i = j$	$[S][S]^H = [S]^H[S] = I$ H $\rightarrow$ conjugate
$\sum_{k=1}^{N} S_{ki} S_{kj}^* = 0$	for all $i \neq j$	transpose

Conversion Between Parameters

- 2-port parameters relate voltages and currents at the ports of the network.
- One 2-port network can be represented by any of the different parameters discussed previously
- It is possible to transform from one parameter to another
  - Convenient when cascading networks represented by parameters other than ABCD parameters.

Conversion from S to ABCD Parameters

$$A = \frac{(1+S_{11})(1-S_{22})+S_{12}S_{21}}{2S_{21}} \qquad B = Z_O\frac{(1+S_{11})(1+S_{22})-S_{12}S_{21}}{2S_{21}}$$

$$C = \frac{1}{Z_O}\frac{(1-S_{11})(1-S_{22})-S_{12}S_{21}}{2S_{21}} \qquad D = \frac{(1-S_{11})(1+S_{22})+S_{12}S_{21}}{2S_{21}}$$

Conversion from ABCD to S Parameters

$$S_{11} = \frac{A + Y_O B - Z_O C - D}{A + Y_O B + Z_O C + D} \qquad S_{12} = \frac{2(AD - BC)}{A + Y_O B + Z_O C + D}$$

$$S_{21} = \frac{2}{A + Y_O B + Z_O C + D} \qquad S_{22} = \frac{-A + Y_O B - Z_O C + D}{A + Y_O B + Z_O C + D}$$

Use of Conversion Between Parameters

- Most of the circuits at high frequencies are expressed in terms of their S-parameters
- Most of these complex circuits can be considered as a cascade of several 2 port networks
- Conversion from S to ABCD and ABCD to S makes analysis of these circuits easier
- Long process of analyzing complex circuits is reduced to conversion from S to ABCD, matrix multiplication, and conversion back from ABCD to S parameters

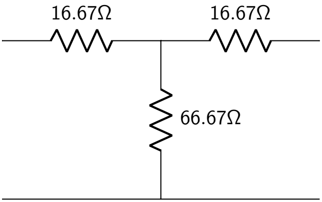
Example

Consider 2 networks that are cascaded together.

Network A

$$S = \begin{bmatrix} 0 & 0.89 \\ 0.89 & 0 \end{bmatrix}$$

Network B



1. Obtain the S-parameters for Network B.
2. Derive the cascaded S-parameters.
3. Describe the function of each network and how it relates to the cascaded network.