



EEE 51: Second Semester 2017 - 2018

Lecture 9

Differential Circuits

Today

- Differential Circuits



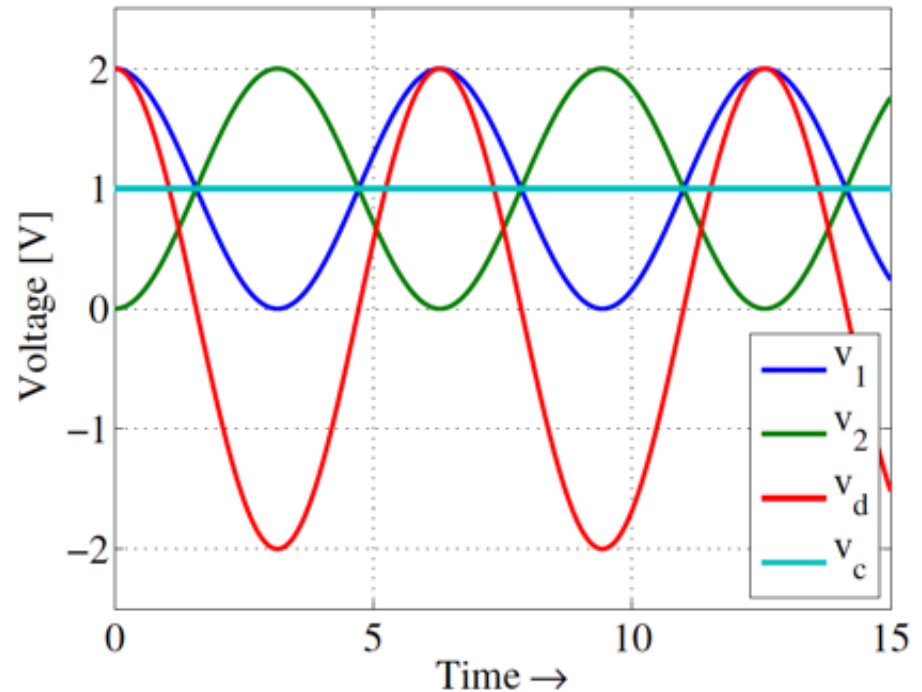
Differential Signals Revisited

- Limit analysis to sinusoids → easier analysis

$$v_{i1} = \frac{\hat{V}}{2} \cos(\omega t) + V_{cm}$$
$$v_{i2} = -\frac{\hat{V}}{2} \cos(\omega t) + V_{cm}$$

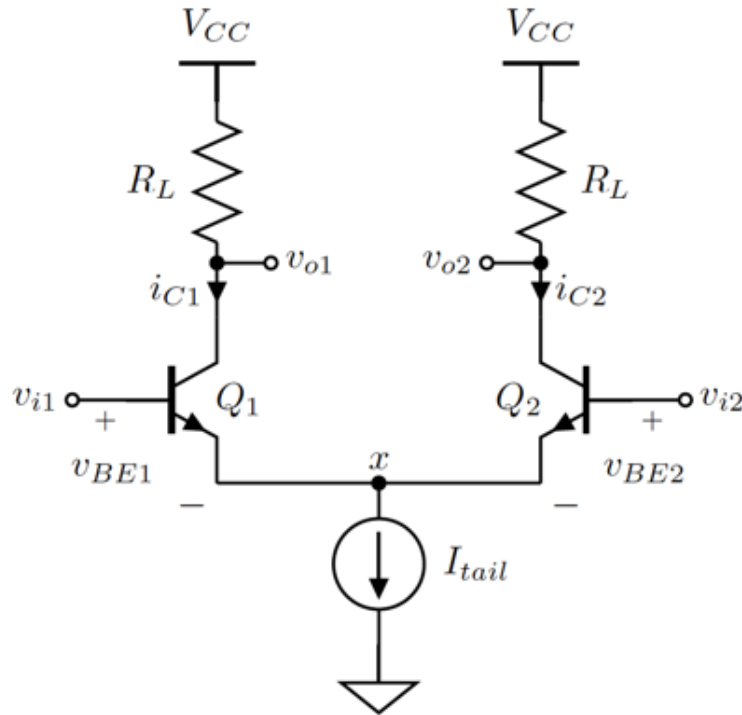
$$v_{id} = v_{i1} - v_{i2} = \hat{V} \cos(\omega t)$$
$$v_{ic} = \frac{v_{i1} + v_{i2}}{2} = V_{cm}$$

Common-mode \approx DC Offset



Recall: The Differential Pair (1)

- BJT: The emitter-coupled pair DC Analysis



KVL at the input loop: $V_{i1} - V_{BE1} + V_{BE2} - V_{i2} = 0$

$$V_{i1} - V_{i2} = V_{BE1} - V_{BE2}$$

$$V_{id} = V_T \ln\left(\frac{I_{C1}}{I_S}\right) - V_T \ln\left(\frac{I_{C2}}{I_S}\right) = V_T \ln\left(\frac{I_{C1}}{I_{C2}}\right)$$

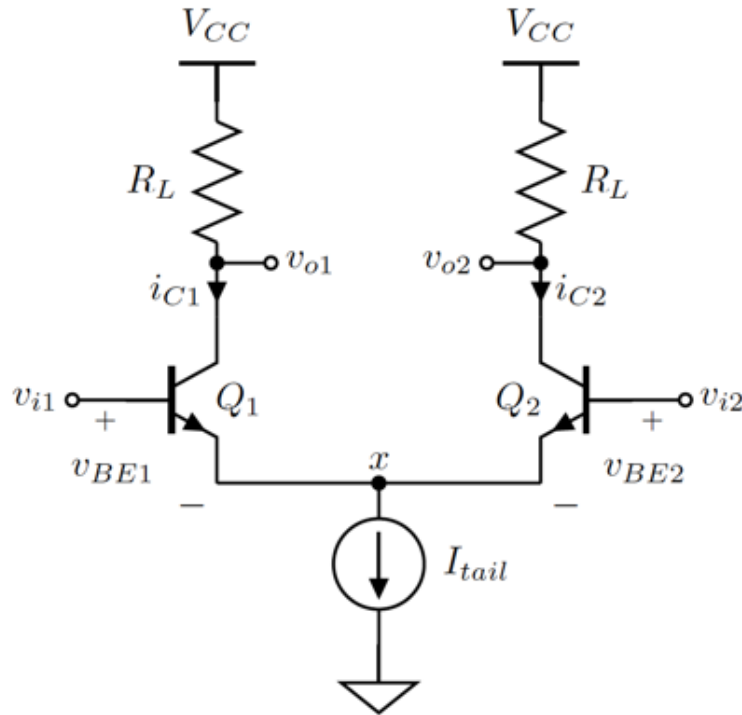


$$I_{C1} = I_{C2} \cdot e^{\frac{V_{id}}{V_T}}$$



Recall: The Differential Pair (2)

- BJT: The emitter-coupled pair DC Analysis



KCL at node x:
$$I_{tail} = I_{E1} + I_{E2} = \frac{I_{C1} + I_{C2}}{\alpha}$$

Recall:

$$I_{C1} = I_{C2} \cdot e^{\frac{V_{id}}{V_T}}$$



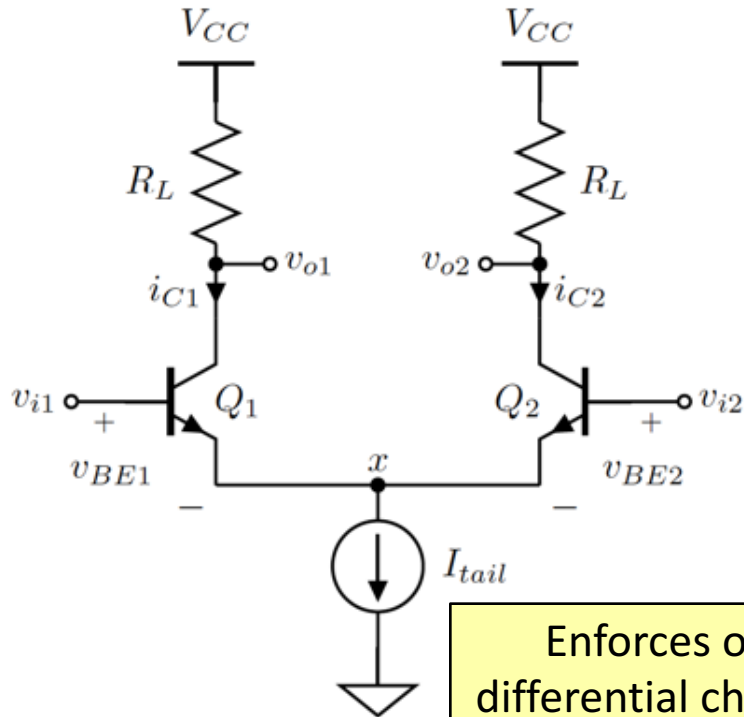
$$I_{C1} = \frac{\alpha \cdot I_{tail}}{1 + e^{-\frac{V_{id}}{V_T}}}$$

$$I_{C2} = \frac{\alpha \cdot I_{tail}}{1 + e^{+\frac{V_{id}}{V_T}}}$$

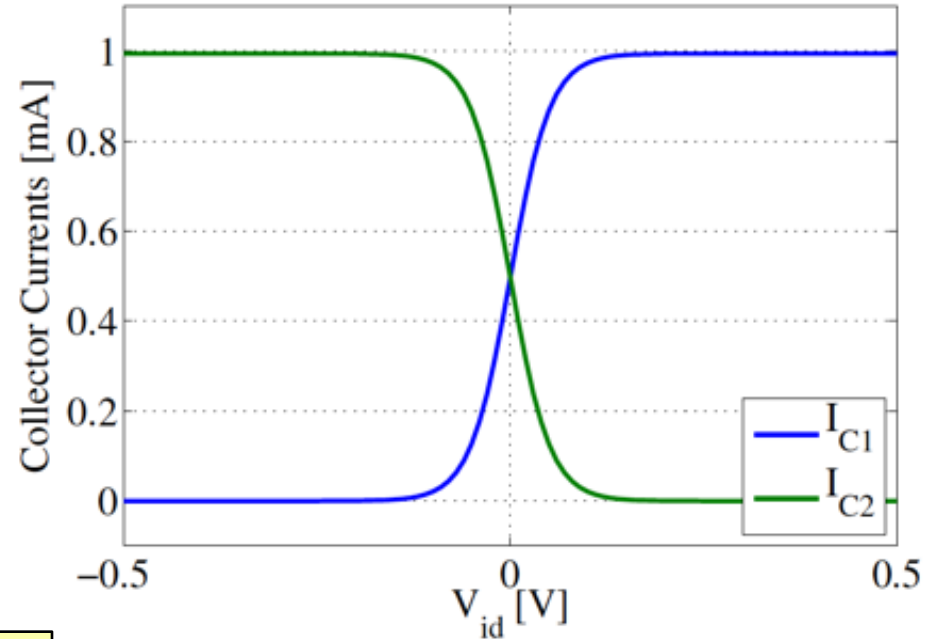


Recall: The Differential Pair (3)

- BJT: The emitter-coupled pair DC Analysis



Enforces only differential changes!



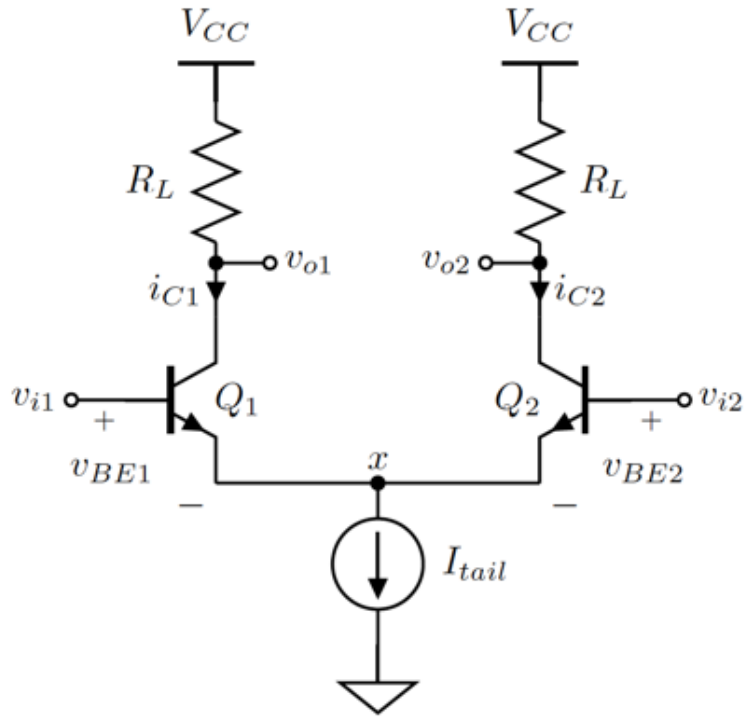
$$I_{C1} = \frac{\alpha \cdot I_{tail}}{1 + e^{\frac{V_{id}}{V_T}}}$$

$$I_{C2} = \frac{\alpha \cdot I_{tail}}{1 + e^{-\frac{V_{id}}{V_T}}}$$



Recall: The Differential Pair (4)

- Output Voltage



$$I_{C1} = \frac{\alpha \cdot I_{tail}}{1 + e^{-\frac{V_{id}}{V_T}}}$$

$$I_{C2} = \frac{\alpha \cdot I_{tail}}{1 + e^{+\frac{V_{id}}{V_T}}}$$

KVL at the output loop:

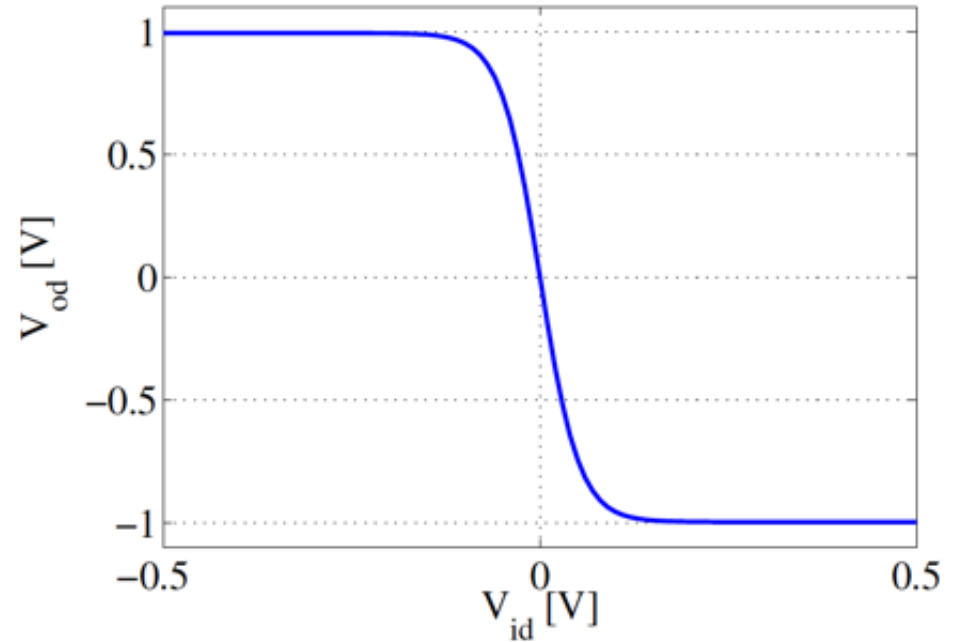
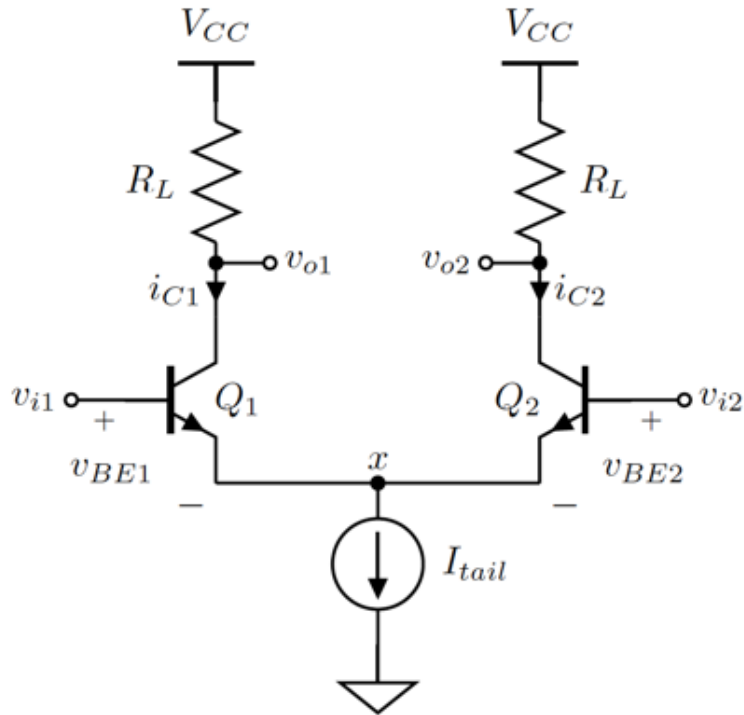
$$V_{od} = V_{o1} - V_{o2} = R_L (I_{C2} - I_{C1})$$

$$\begin{aligned} V_{od} &= \alpha \cdot I_{tail} \cdot R_L \cdot \left(\frac{1}{1 + e^{+\frac{V_{id}}{V_T}}} - \frac{1}{1 + e^{-\frac{V_{id}}{V_T}}} \right) \\ &= \alpha \cdot I_{tail} \cdot R_L \cdot \tanh\left(-\frac{V_{id}}{2 \cdot V_T}\right) \end{aligned}$$



Recall: The Differential Pair (5)

- Transfer Characteristic

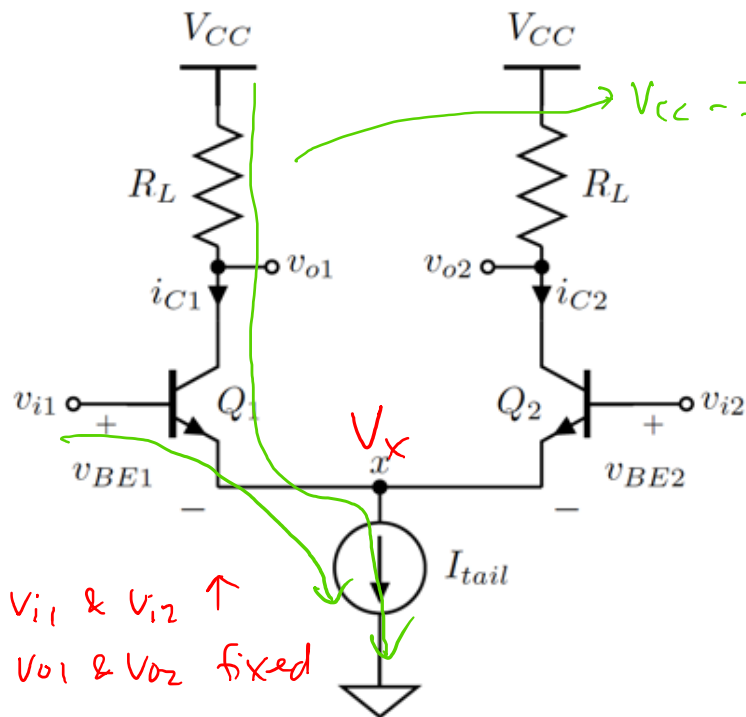


$$V_{od} = \alpha \cdot I_{tail} \cdot R_L \cdot \tanh\left(-\frac{V_{id}}{2 \cdot V_T}\right)$$



Common-Mode Input Range (1)

- BJT Operating Region?



Assume zero differential input

$\rightarrow V_{i1} = V_{i2}$

KVL: $V_{CE1} = V_{CC} - I_{C1}R_L - V_X > V_{CE,sat}$

Note: $V_X = V_{I1} - V_{BE1}$

$$= V_{I1} - V_T \ln\left(\frac{I_{C1}}{I_S}\right)$$

to keep Q₁ in f.a.

V_X is controlled by the input common-mode (DC input)!

$$V_{CE1} = V_{CC} - I_{C1}R_L - V_{I1} + V_T \ln\left(\frac{I_{C1}}{I_S}\right) > V_{CE,sat}$$

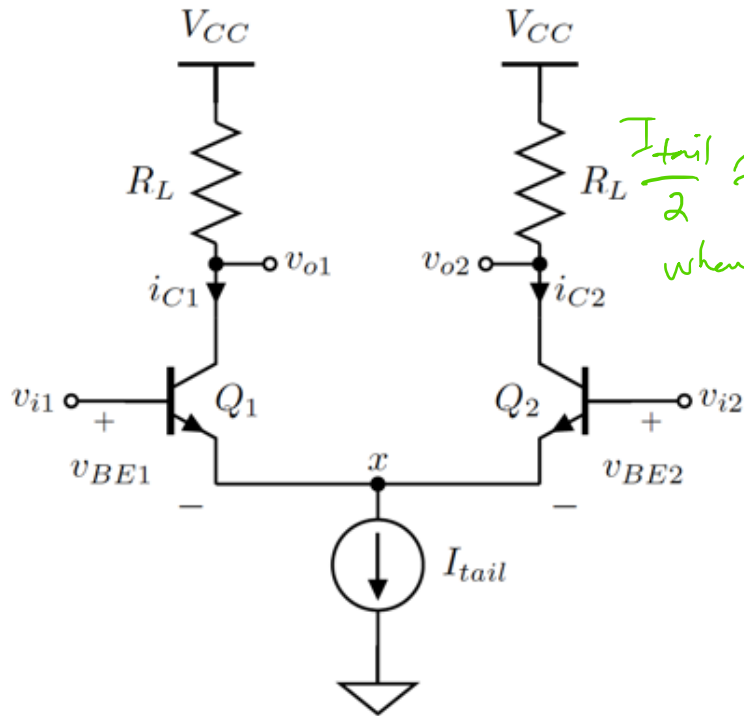
as V_{i1} & V_{i2} ↑
 V_{o1} & V_{o2} fixed

V_X ↑ $V_{CE1} = V_{o1} - V_X$, $V_{CE2} = V_{o2} - V_X$



Common-Mode Input Range (2)

- BJT Operating Region?



For zero differential input: $v_{ic} = V_{I1} = V_{I2} = V_{cm}$

$$I_{C1} = I_{C2} = \frac{I_{tail}}{2}$$

$\frac{I_{tail}}{2} \approx$
when β is large

For a range of common-mode inputs!

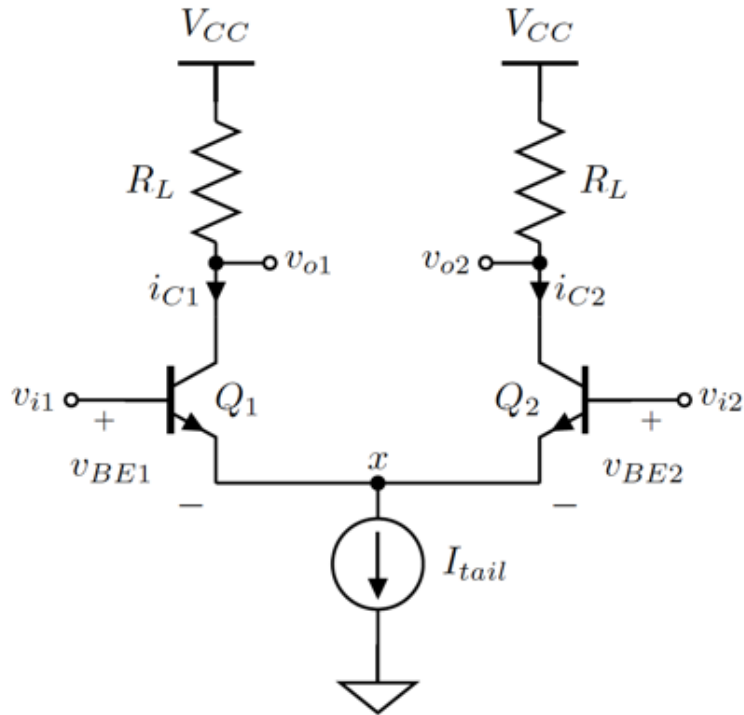
$$V_{CE1} = V_{CC} - \underbrace{I_{C1} R_L}_{\frac{I_{tail}}{2}} - \underbrace{V_{I1}}_{V_{cm}} + V_T \ln\left(\frac{\underbrace{I_{C1}}_{I_{tail}/2}}{I_S}\right)$$

$$V_{CE1} = V_{CC} - \frac{I_{tail} R_L}{2} - V_{cm} + V_T \ln\left(\frac{I_{tail}}{2 \cdot I_S}\right) > V_{CE,sat}$$



Common-Mode Input Range (3)

- BJT Operating Region?



$$V_{CE1} = V_{CC} - \frac{I_{tail}R_L}{2} - V_{cm} + V_T \ln\left(\frac{I_{tail}}{2 \cdot I_S}\right) > V_{CE,sat}$$



$$V_{cm} < V_{CC} - \frac{I_{tail}R_L}{2} + V_T \ln\left(\frac{I_{tail}}{2 \cdot I_S}\right) - V_{CE,sat}$$

Maximum common-mode input voltage:

$$V_{cm,max} = V_{CC} - \frac{I_{tail}R_L}{2} + V_T \ln\left(\frac{I_{tail}}{2 \cdot I_S}\right) - V_{CE,sat}$$



Common-Mode Input Range (4)

- BJT Operating Region?

If the tail current is not ideal $\rightarrow V_{\min}$

$$V_X = V_{I1} - V_{BE1}$$

$$= V_{I1} - V_T \ln\left(\frac{I_{C1}}{I_S}\right) = V_{cm} - V_T \ln\left(\frac{I_{tail}}{2 \cdot I_S}\right) > V_{\min}$$

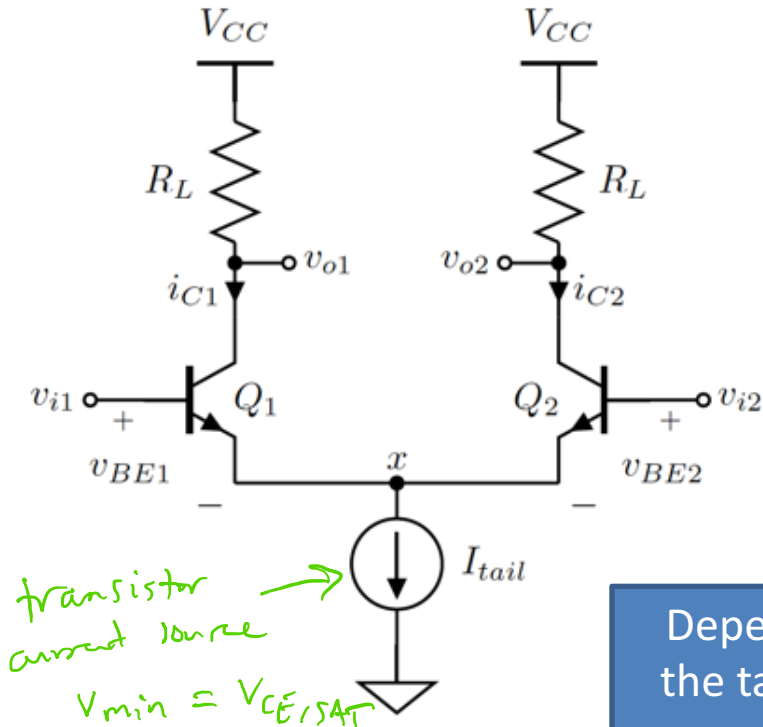


$$V_{cm} > V_{\min} + V_T \ln\left(\frac{I_{tail}}{2 \cdot I_S}\right)$$

Minimum common-mode input voltage:

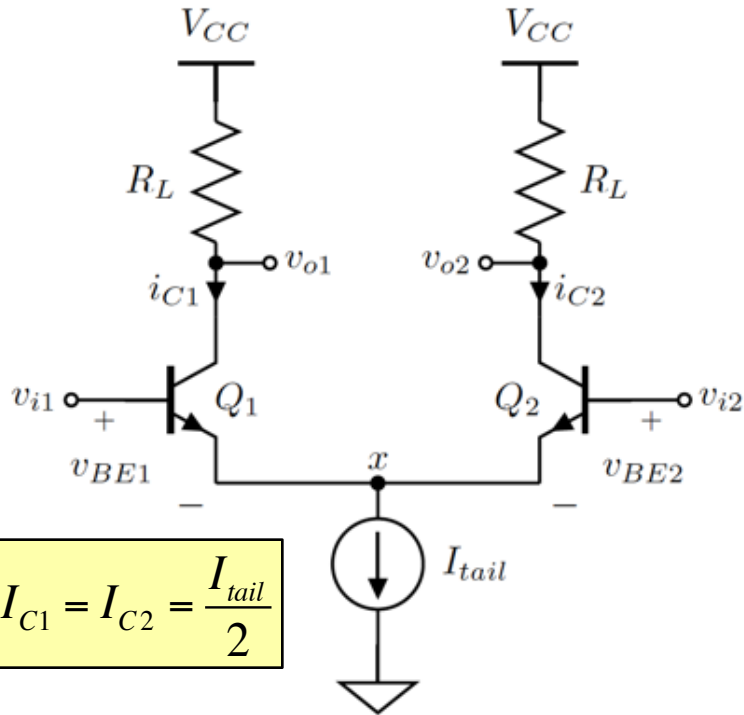
$$V_{cm,\min} = V_{\min} + V_T \ln\left(\frac{I_{tail}}{2 \cdot I_S}\right)$$

Dependent on
the tail current
source



Common-Mode Input Range (5)

- BJT Operating Region \rightarrow set by the input common-mode!



$$V_{cm,max} = V_{CC} - \frac{I_{tail}R_L}{2} + V_T \ln\left(\frac{I_{tail}}{2 \cdot I_S}\right) - V_{CE,sat}$$

Common-mode input range

$$V_{cm,min} = V_{min} + V_T \ln\left(\frac{I_{tail}}{2 \cdot I_S}\right)$$



Next Meeting

- Continue with Differential Circuits

