An Efficient Double Auction Mechanism for Job Allocation

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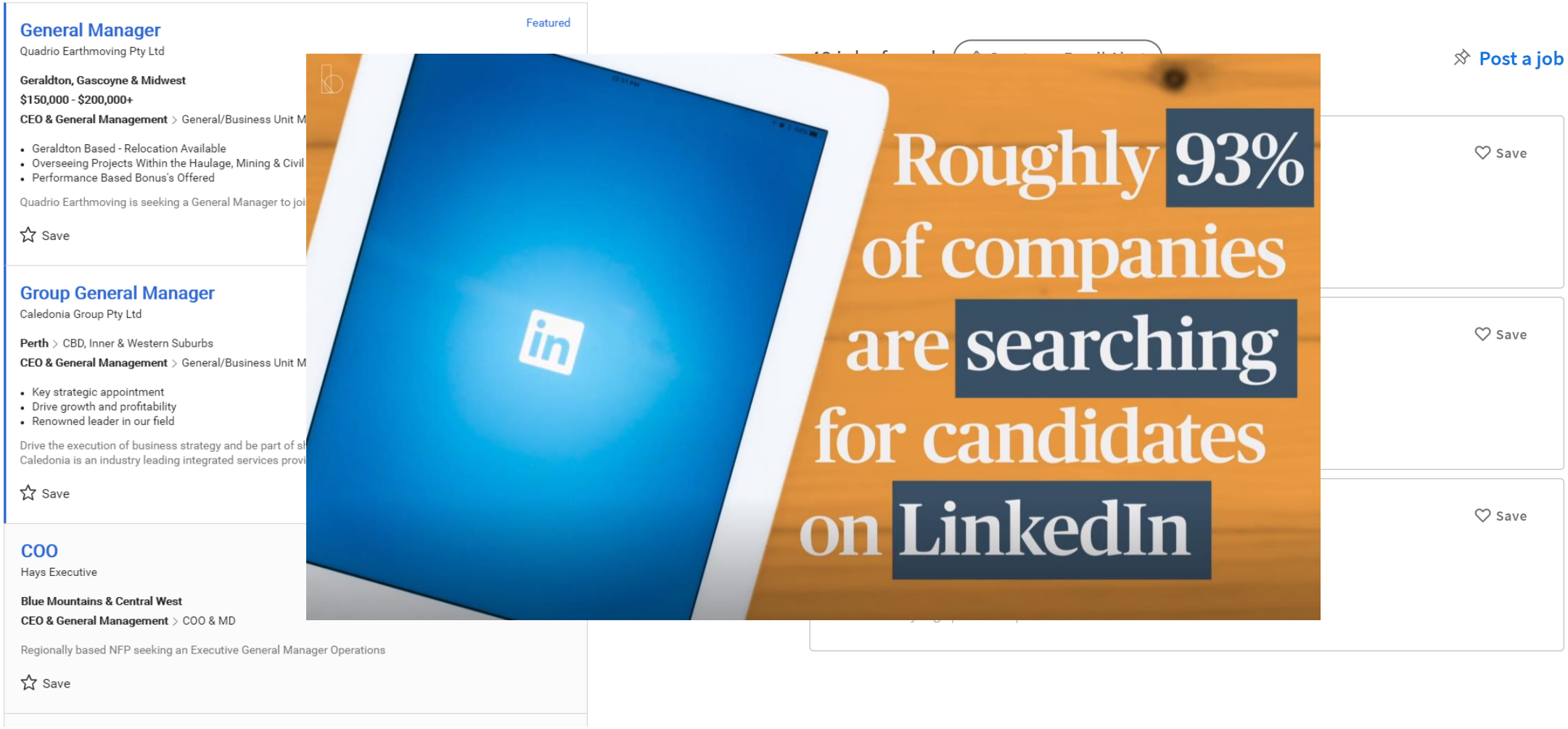
Job Supply and Demand Market



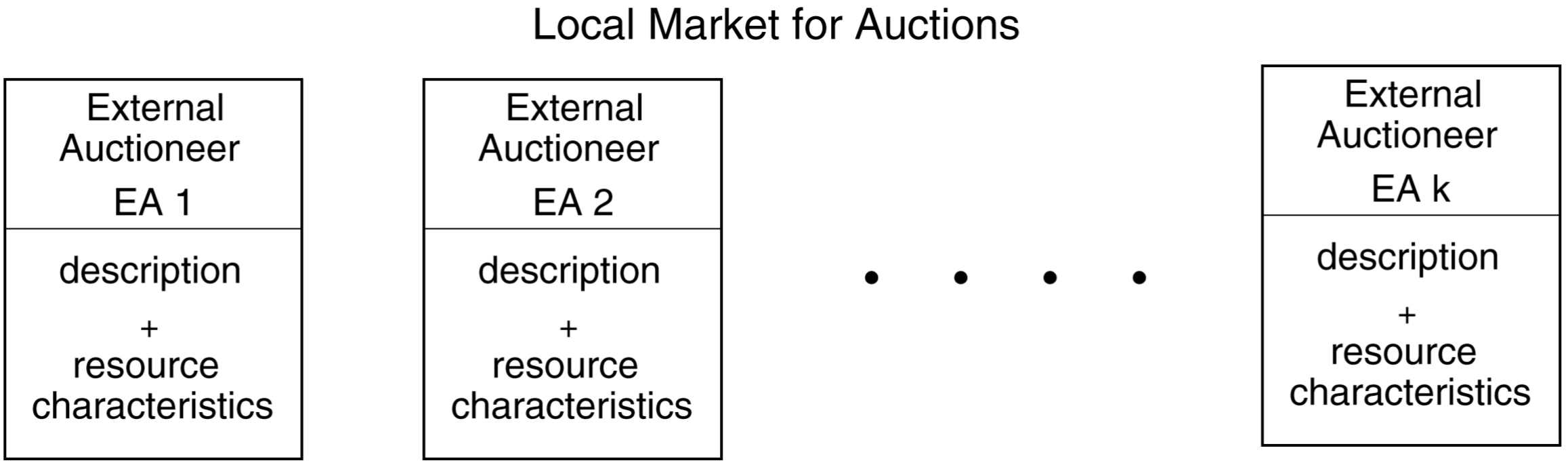
Market Based Models

Two categories of market based models:

- Commodities Market Model
- Auction Model
  - one-sided auctions
  - two-sided auctions (double auctions)

The advantage of using auctions for resource allocation is that they require little global information, have decentralized structure and are easy to implement.

Auction Model on Job Allocation



If the existing auction mechanism is directly applied to solve the problem, it may not achieve expected results.

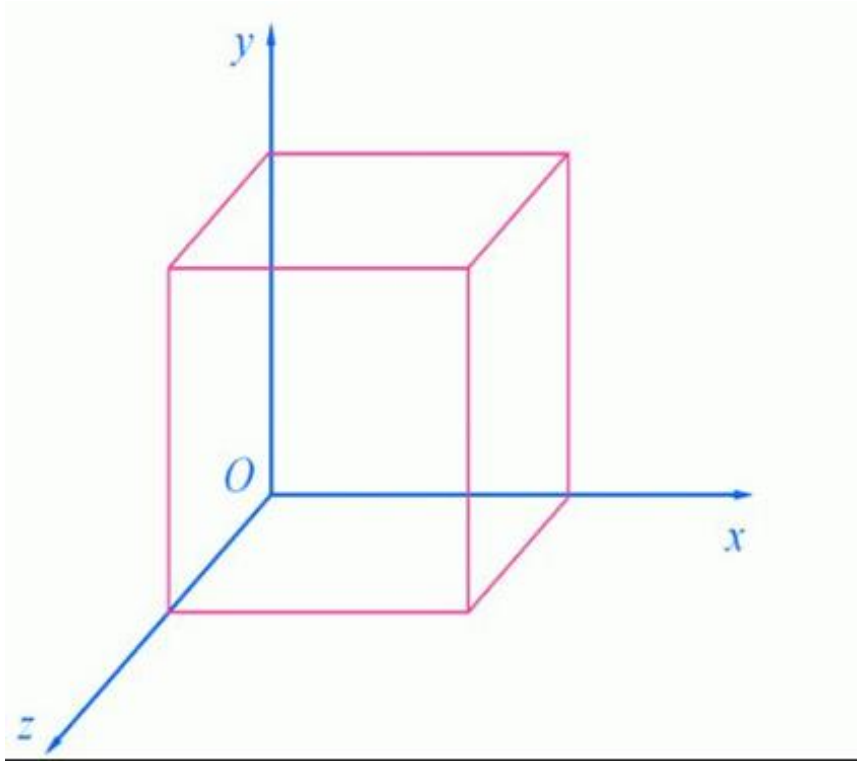
General Double Auction Model

- **Time First:** When the buyer and the seller have the same bid price, the buyer who bids earlier will prioritize transactions
- **Price First:** The buyer who bids higher will prioritize transactions. In contrast, the seller who asks lower price will prioritize transactions
- **Transaction Price:** The transaction price is generally the average of bidding and asking price. The premise of the transaction is that the seller's asking price is lower than the buyer's bid

Greedy Double Auction Mechanism(GDA)

- The auctioned goods come from various positions offered by different companies are heterogeneous
- There is a one-to-one correspondence between the bidder and the auction goods
- Third-party platforms have access to all product (or job) as well as bid information, however, this information is private to both parties

Winner Decision Problem



$$Max \sum_{1}^{|X_{win}|} X_{ij}^{k}\{[p_{ij} - \frac{p_{ij} - b_{ij}^{k}}{2}] + [\frac{p_{ij} - b_{ij}^{k}}{2} - b_{ij}^{k}]\}$$

$$X_{ij}^{k} \in \{0,\ 1\}$$

X_{win} represents the solution which maximizes the social welfare in auction and meets the constrains.

GDA : Find Valid Bids

Algorithm 1: Valid Bids

Input: J, S, B
Output: $J_{potential}, J_{value}$

```
for t = 1 → |B| do
    if B_t is marked with i, j then
        Put job_ij into J_potential
    end
end
for t = 1 → |J_potential| do
    Read J_t get i, j
end
index ← 0; count ← 0;
for s = 1 → |B| do
    Search b marked with i, j
    if b_ij^x ≤ p_ij then
        count=count+1;
        Put b_ij^x into bid_index
    end
end
if count > 0 then
    Put job_ij into J_value
end
```

- Statistic the jobs we are competing, exclude jobs that are not competed, and put them into the set $J_{potential}$
- Compare the bid of company p_{ij}. If $p_{ij} \geq b_{ij}^{k}$ the transaction can be completed

GDA : D-value Sort

Algorithm 2: D-value Sort

Input: J_{value}, Bid, S, J

Output: $NewBids$

for $s = 0 \rightarrow |Bid| - 1$ **do**

  Read $bid[s]'s, j, kindex$ $D_{ij}^k = p_{ij} - b_{ij}^k$

end

Sort all D_{ij}^k in Bid while max put forward

Put $result$ into $NewBids$

- Sort valid bids according to their difference between the value of position

GDA : Greedy Allocation

Algorithm 3: Greedy Allocation

Input: J_{value}, Bid, S, J
Output: $Solution$

for $s = 0 \rightarrow (|Bid| - 1)$ **do**
  Read bid[s]'s index i, j, k ;
  $D_{ij}^{k} = p_{ij} - b_{ij}^{k}$
end
Sort all D_{ij}^{k} in Bid while maximum put forward;
Put result into $NewBids$
$J_w = \emptyset, S_w = \emptyset$
for $s = 1 \rightarrow |NeewBids|$ **do**
  if $|NewBids| > 0$ **then**
    Read $NewBids$ and get marked i, j, k;
    Put s_k into S_w, put job_{ij} into J_w;
    Remove all bids in $NewBids$ marked with i or j and k;
  end
end

Based on the greedy mechanism, we give priority to the transaction where quotation and the bidding with large price difference, until all the positions are distributed.

Time Complexity

The time complexity of the greedy double auction (GDA) mechanism is **$O(m*n+2n^2)$**.

Valid Bids		D-value Sort		Greedy Allocation		
$O(m*n)$	+	$O(n^2)$	+	$O(n^2)$	=	$O(m*n+2n^2)$

Individual Rational

The greedy double auction (GDA) mechanism is **individual rational**.

Proof : In algorithmic design,

- if a candidate lies about his true desire to apply for a job, he is unable to compete for a position. At this point, $p_{ij}^{S} = 0, \mathrm{U}^{S} = 0$.
- When the position job_{ij} has only one valid bidder, if and only if $\mathrm{p}_{ij} \geq b_{ij}^{k}$, then $\mathrm{U}^{S} = \frac{\left(\mathrm{p}_{ij} + b_{ij}^{k}\right)}{2} - b_{ij}^{k}$. Therefore there must be $\mathrm{U}^{S} \geq 0$.
- When the position job_{ij} has more than one effective bidder, there must be a lowest bidder. If the applicant s_{k} has lowest bid, then $U^{S} \geq 0$. If the bid is not the lowest, then $\mathrm{U}^{S} = 0$.

Budget-balanced

Greedy double auction (GDA) mechanism is **budget-balanced**.

Proof : the transaction price between the buyer and the seller is the average of the buyer's bid and the seller's asking price.

$$\sum_{job_{ij} \in J_w} p_{ij} - \sum_{s_j \in S_w} p_{ij}^S = \sum_{s_j \in S_w} p_{ij} - p_{\sigma(ij)}^s = 0$$

Truthful

Greedy double auction (GDA) mechanism is **truthful**.

Proof :

- the bid b'^{k}_{ij} lower than true bid:

Job seeker wins bid, his new utility is $U^{S'}$. $U = (b + p)/2 - b$. Due to $b'^{k}_{ij} \leq b^{k}_{ij}$, so $\mathrm{U}^{S'} \leq \mathrm{U}^{S}$. In this case, the false bid leads to a lower utility.

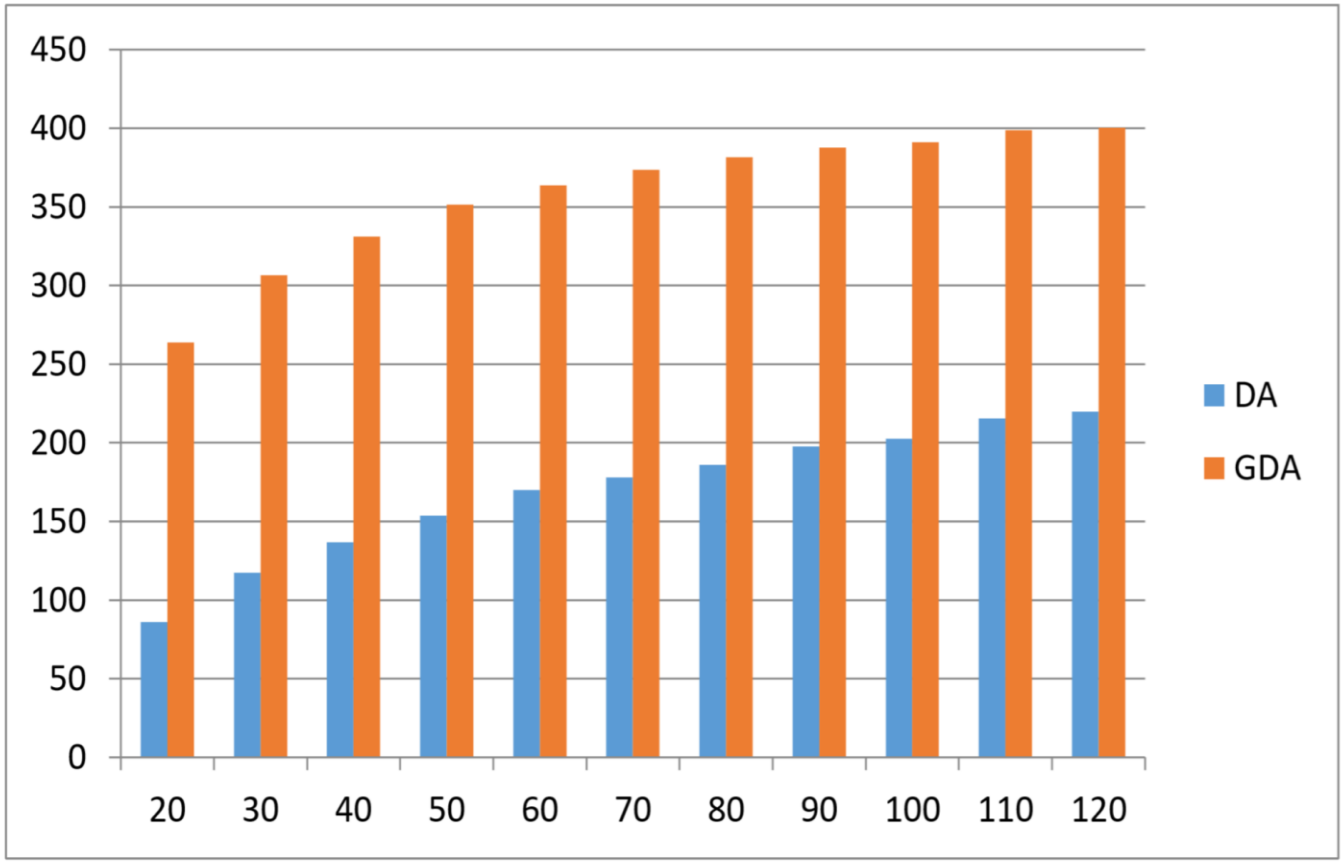
- the bid b'^{k}_{ij} higher than true bid:

1). job seekers fails the competition, $\mathrm{U}^{S'} = 0$.

2). job seeker wins the bid, due to the utility function: $U = \frac{b+p}{2} - b = (p - b)/2$.

The price of the position remains unchanged with the raise of bid b provided by the seeker. As a result, job seekers' utility declines.

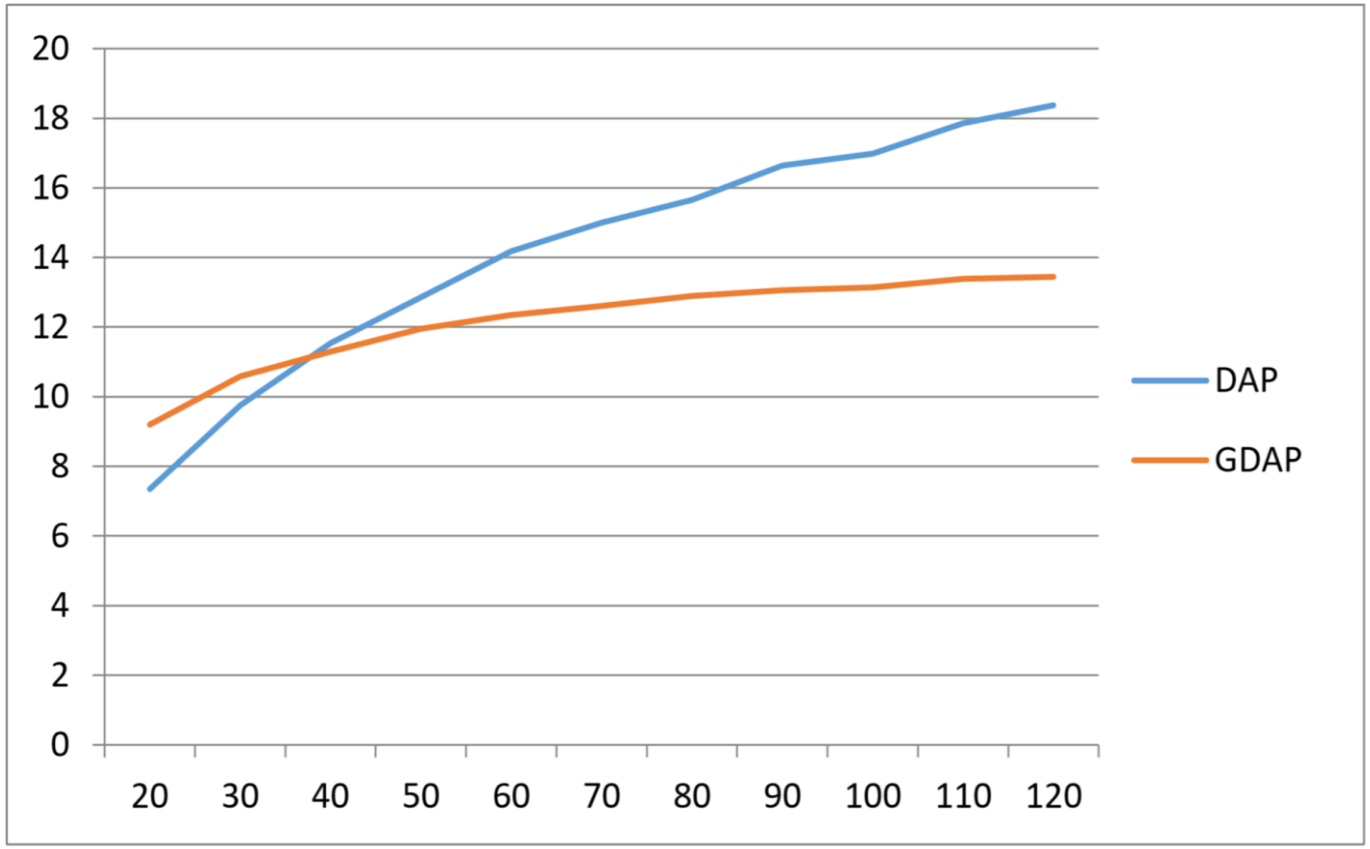
DA & GDA Results on Social Welfare



Parameters	Descriptions	Range
m	the number of positions	50
n	the number of job seekers	$[20, 100]$
A	the value of positions	$[4500, 7500]$
B	the bid of seekers for positions	$[4000, 8000]$
d	step length	10

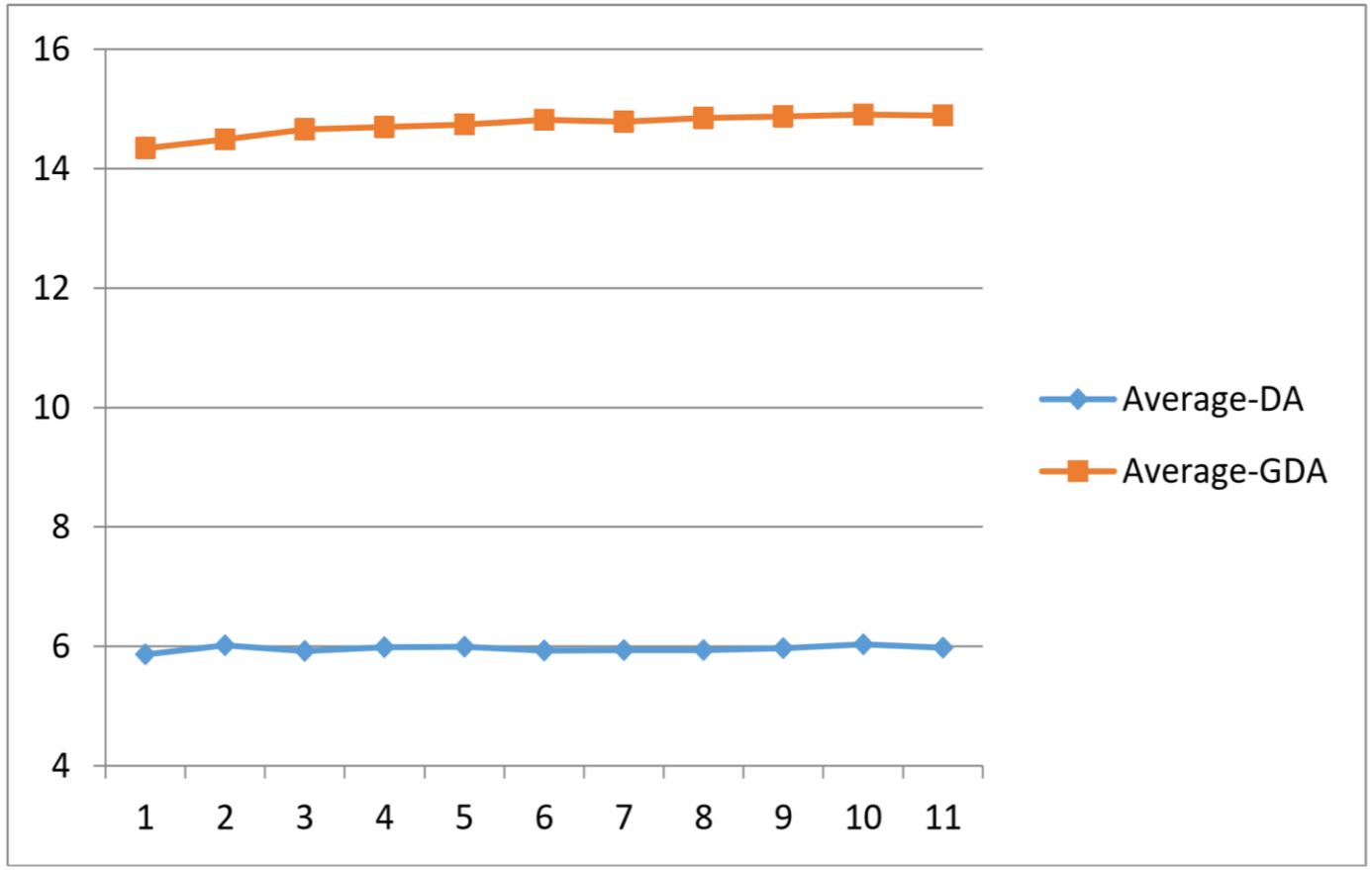
- When the number of jobs remain unchanged, the increase in the number of job seekers is conducive to the increase of social welfare.
- The welfare will be reduced after reaching a certain value.
- GDA has higher social welfare than DA under the same conditions.

Job matching rate



- With the increase of job seekers, the matching rate of DA and GDA has both increased.
- The matching rate of GDA is higher when there are fewer people in the early stage, but the matching rate of DA is higher than that of GDA when there are more people in the later stage.

Average utility of seeker



- The average utility of GDA is higher than that of DA.
- With the increasing number of job seekers, the average seeker's utility of DA and GDA are basically stable and occasionally fluctuate.

Conclusion

- GDA has higher social welfare than DA under the same conditions.
- The matching rate of GDA is higher when there are fewer people in the early stage, but the matching rate of DA is higher than that of GDA when there are more people in the later stage.
- The average utility of GDA is higher than that of DA.

The mechanism has effectively improved social welfare and the average utility of job seekers.