선형대수 (liner algebra)

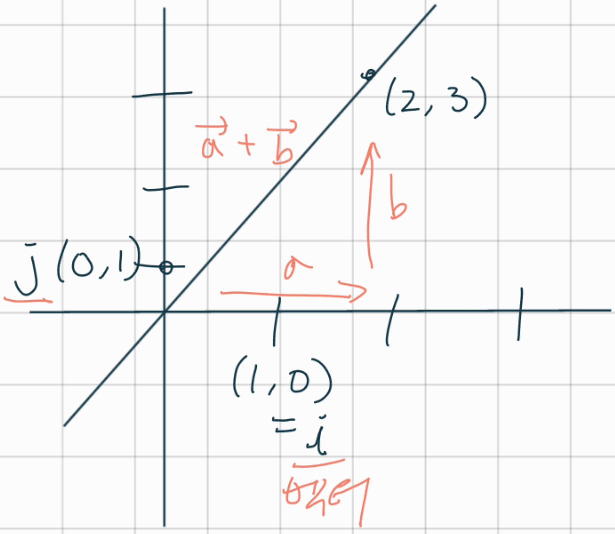
ㄴ벡터 . 행렬 . 연산

벡터 scalar

크기+방향

ㄴ> 한 공간의 점

크기와 방향이 같으면 같은 벡터



$$(2,3) = \underbrace{2}_{\text{스칼라}}\underbrace{(1,0)}_{\text{벡터}} + 3(0,1)$$

평행이동 함

$$= (2,0) + (0,3)$$

$$= 2i + 3j$$

i, j가 벡타

Q

$\vec{c} - \vec{f} =$

$\vec{c} = (3, 2, 7)$

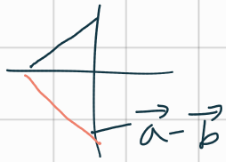
$f = (1, 0, -3)$

$= (3-1, 2-0, 7+3)$

$= (2, 2, 10)$

if. 빼기 (음수는 방향 바꾸는 것)

$\vec{a} - \vec{b}$



$= \vec{a} + (-\vec{b})$

$= (2, -3)$

$3\vec{a}+7\vec{b} = (3,14,14,14)$

$\vec{a} = (1,0,0,7)$

$\vec{b} = (0,2,2,-1)$

$(2,3,7)$의 벡터 크기는 $\sqrt{2^2+3^2+7^2}$

피타고라스 정리 3차원판

$(a_1, a_2, \cdots a_n)$

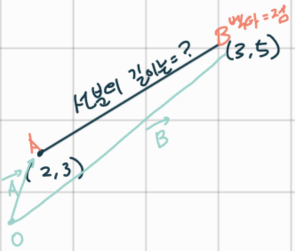
$\sqrt{a_1^2+a_2^2\cdots+a_n^2} = a_1+a_2\cdots\overset{??}{a_n}$

$\sqrt{\sum_{j=1}^{n}(a_j)^2} \neq \sum_{j=1}^{n}\sqrt{(a_j)^2}$

$= \sqrt{a_1^2}+\sqrt{a_2^2}+\cdots\sqrt{a_n^2}$

$= a_1+a_2+\cdots a_n$

Σ
합=시그마



O(원점)

$\overline{AB} = \overline{OB} - \overline{OA}$

$\vec{b} - \vec{a}$

$= \vec{b} + (\vec{-a})$

$(3-2)+(5-3)$

$= (1,2)$

$\sqrt{1^2+2^2} = \sqrt{5}$

Geometry (기하학)

$\vec{a} = (a_1, a_2, \cdots a_n)$

$\vec{b} = (b_1, b_2, \cdots b_n)$

$|\vec{b}-\vec{a}|$

절댓값 쓰는게 벡터의 거리

두 벡터의 선 길이

$(b_1-a_1, b_2-a_2\cdots b_n-a_2)$

$\sqrt{(b_1-a_1)^2+(b_2-a_2)^2\cdots(b_n-a_n)^2}$

벡터 간의 곱(product)

- 내적(점곱) dot Product $\boxed{\vec{a}\cdot\vec{b}}$
- 외적 Cross Product × $\vec{a}\times\vec{b}$ (로봇 설계자들)

$\vec{a} = (a_1, a_2)$

$\vec{b} = (b_1, b_2)$

$\vec{a}\cdot\vec{b} = (a_1b_1 + a_2b_2) =$ 스칼라

$\vec{a}\cdot\vec{b} = |\vec{a}|\cdot|\vec{b}|\cos\theta = (1\times2+2\times(-1)+3\times2) = 6$

$\vec{a}\cdot\vec{b}$ 이루는 각

$\vec{a}(1,2,3) = \sqrt{1+4+9} = \sqrt{14}$

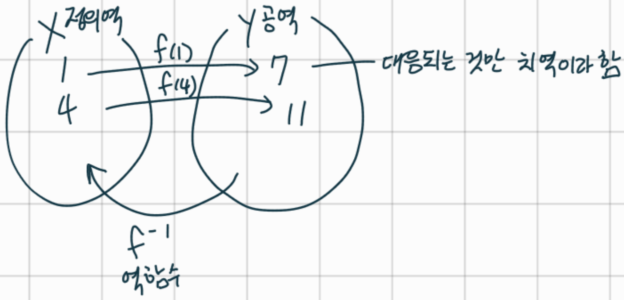
$\vec{b}(2,-1,2) = \sqrt{4+1+4} = 3$

$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \frac{6}{\sqrt{14}\cdot3} = \frac{2}{\sqrt{14}}$

$\sqrt{14} = \frac{2}{\sqrt{2}\sqrt{7}} = \frac{\sqrt{2}}{\sqrt{7}} = \sqrt{\frac{2}{7}}$

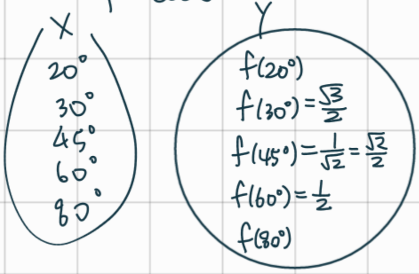
$\frac{2}{\sqrt{14}} = \frac{\sqrt{4}}{\sqrt{14}} = \sqrt{\frac{4}{14}} = \sqrt{\frac{2}{7}}$

$\cos\theta = \sqrt{\frac{2}{7}}$



$f^{-1}(7) = 1$

$f = \cos\theta$



$f^{-1}\left(\frac{\sqrt{3}}{2}\right) = 30°$

$\cos\theta = \sqrt{\frac{2}{7}}$

$\cos^{-1}\sqrt{\frac{2}{7}} = \theta$ Arccosine

np.arccos($\sqrt{\frac{2}{7}}$)

$\vec{a}\cdot\vec{b}$

$= |\vec{a}|\cdot|\vec{b}|\cos\theta$

직선의 방정식

$2x+3y=5 \quad \times 3 \qquad 6x+9y=15$

$y = \frac{2}{3}x + \frac{5}{3}$

$3x+2y=4 \quad \times 2 \qquad 6x+4y=8 \quad -$

$5y = 7$

$y = \frac{7}{5}$

$2x + 3\times\frac{7}{5} = 5$

$2x + \frac{21}{5} = \frac{25}{5}$

$2x = \frac{4}{5}$

$x = \frac{4}{10}$

$\left(\frac{4}{10}, \frac{7}{5}\right)$ = 교점

각각요소끼리 곱하고 더함 (내적)

$$\begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$$

$2\times x + 3\times y = 5$

$(2,3)\cdot(x,y)$

$\begin{pmatrix} a \\ b \end{pmatrix} = \vec{a}$

$\vec{a} = (2,3)$

$\vec{x} = (x,y)$

행(가로)과 열(세로) 바꿈 (transpose)

$\vec{a}^{T}\cdot\vec{x} = \vec{b}$

$x\vec{a}_1 + y\vec{a}_2 = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$

선형결합
liner Combination
빅데이터 하는 일
거 찾기

$\frac{4}{10}\begin{pmatrix} 2 \\ 3 \end{pmatrix} + \frac{7}{5}\begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$

$\alpha\vec{a} + \beta\vec{b} \rightarrow \vec{c}$

벡터 → 독립이냐 → 수직이면 독립O, 독립이라고 수직X

→ 종속이냐 (서로가 배(x)해서 나오면 종속)

R^2 real 2차원 평면

독립이어야 표현할 수 있다 (span)

3차원은 독립 3개는 필요.

표준 직교 basis

수직 (span, 독립) ⊕ → 그 집합을 basis 기저 라고 한다.

$\{(1,0)\ (0,1)\}$

basis = 기저

$\{(1,2)\ (3,2)\}$

얘도 basis 가 무지 많음.

$3x+2y+3z=5$
$1x+y-z=7$

$$\begin{pmatrix} 3 & 2 & 3 \\ 1 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \\ 0 \end{pmatrix}$$

주요공부!

1) liner Combination
2) 독립 / 종속 / 수직이냐
3) span의 기능
4) basis가 무엇이냐

transpos

$3x+y-z=6$
$2x+y-2z=8$
$x+2y+7z=3$

$$\begin{array}{cc} & \begin{array}{ccc} c_1 & c_2 & c_3 \end{array} \\ \begin{array}{c} a_1 \\ a_2 \\ a_3 \end{array} & \begin{pmatrix} 3 & 1 & -1 \\ 2 & 1 & -2 \\ 1 & 2 & 7 \end{pmatrix} \end{array} \underbrace{\begin{pmatrix} x \\ y \\ z \end{pmatrix}}_{x} = \underbrace{\begin{pmatrix} 6 \\ 8 \\ 3 \end{pmatrix}}_{b}$$

$$\cos\theta = \frac{(A\text{벡터} \times B\text{벡터})}{(|A| \times |B|)}$$

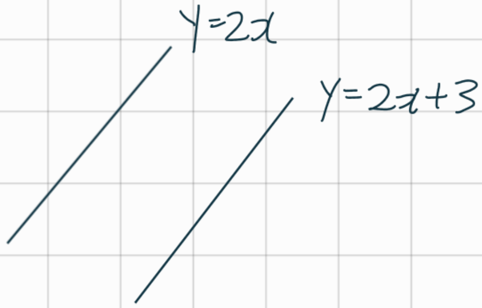
$$a_1^T \cdot x = b$$

$$x\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} + y\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} + z\begin{pmatrix} -1 \\ 2 \\ 7 \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \\ 3 \end{pmatrix}$$

$$A^{-1}\underbrace{A}_{\text{행렬}}\ \underbrace{x}_{\text{벡터}} = bA^{-1}$$

\- 단위행렬 -

$$\underbrace{\underset{3\times 3}{\underset{A}{\begin{pmatrix} 1 & 3 & 7 \\ 1 & -1 & 8 \\ 2 & 4 & 9 \end{pmatrix}}}\ \underset{3\times 3}{\underset{I}{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}}}_{\text{같아야함}} = \underset{3\times 3}{\begin{pmatrix} (0,0) & (0,1) & \\ & & \\ & & \end{pmatrix}}$$



$2x-y=0$
$2x-y+3=0$

$$\underset{A}{\begin{pmatrix} 2 & -1 \\ 2 & -1 \end{pmatrix}}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ -3 \end{pmatrix} = \frac{X}{ab-bc}$$

역행렬 못 구한다는 얘기.

$\cancel{A^{-1}}\cancel{A}x = \underbrace{bA^{-1}}_{x}$

np.dot(np.linalg.inv(A), b)

$5^{-1}5x = 7 \times 5^{-1}$

$\underbrace{bA^{-1}}_{x} \times A^{-1} = b$???

np.dot(A, B)

$(A) \times (B)$