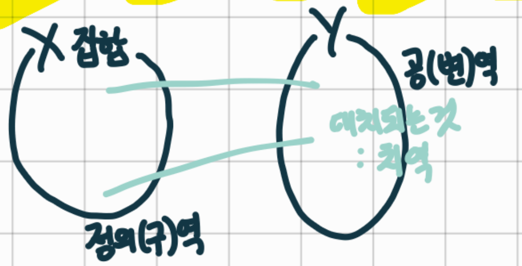
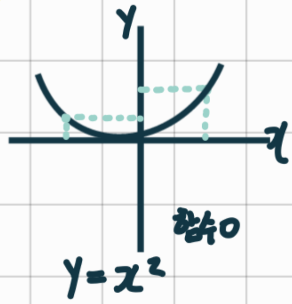
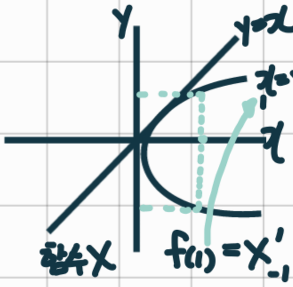
수학: 집합과 함수를 다루는 것



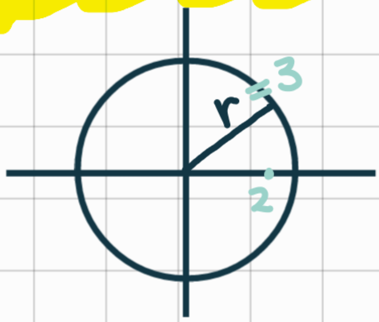




$y = x^2$

$y = \pm 1$ ($+1$, -1)

원의 방정식



$x^2 + y^2 = r^2$

$4 + y^2 = 9$

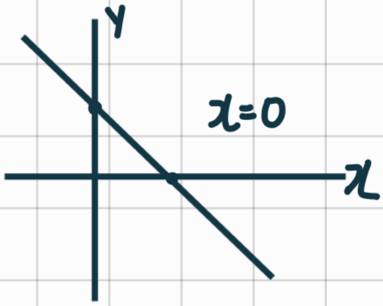
$y^2 = 5$

$y = \sqrt{5}$

$-\sqrt{5}$

직선의 방정식

$2x + y = 5$



$2x + y = 5$

$(0, 5)$

$2x + 0 = 5$

$y = 0 \quad 2x = 5$

$x = \frac{5}{2}$

$\left(\frac{5}{2}, 0\right)$

$y = -2x + 5$

기울기 : -2

y절편 : 5

$(x+1)^2$
$= \underbrace{(x+1)(x+1)}$
$= x^2 + x + x + 1$
$= x^2 + 2x + 1$

=> 완전제곱식

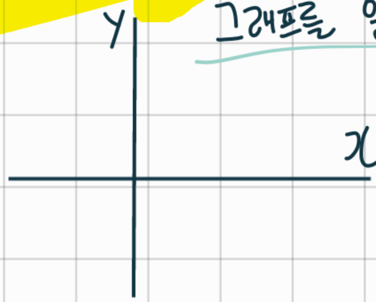
풀었으면
한 번 증명을 했으면
외워야 하는 것이다!

$(a+b)^2$
$= (a+b)(a+b)$
$= a^2 + ab + ba + b^2$
$= a^2 + 2ab + b^2$

$(a-b)^2$
$= (a-b)(a-b)$
$= a^2 - ab - ab + b^2$
$= a^2 - 2ab + b^2$

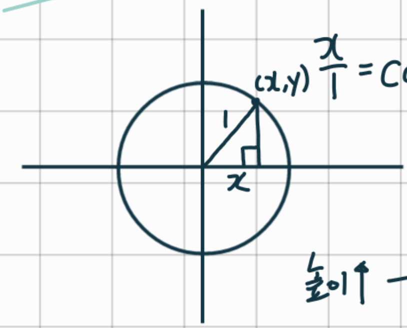
삼각함수

그래프를 알 때



$\cos\theta$
$x = \theta \quad \cos x$

동심원



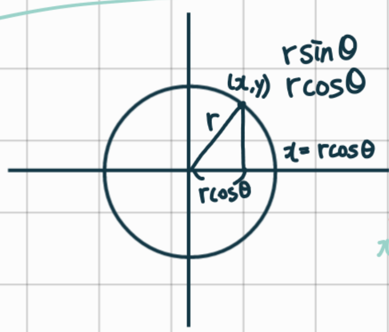
$\frac{x}{1} = \cos\theta$ 밑변 / 빗변

$\frac{y}{1} = \sin\theta$ 높이 / 빗변

높이↑ → 밑변↓

동심원이 아니면

$(ab)^2 = a^2b^2$



$r\sin\theta$
$r\cos\theta$

$x^2 + y^2 = r^2$
$r^2(\cos)^2$
$\underbrace{\cancel{r^2}\cos\theta^2}_{x^2} + \underbrace{\cancel{r^2}\sin\theta^2}_{y^2} = \cancel{r^2}$
$= \cos^2\theta$
$\cos^2\theta + \sin\theta^2 = 1$

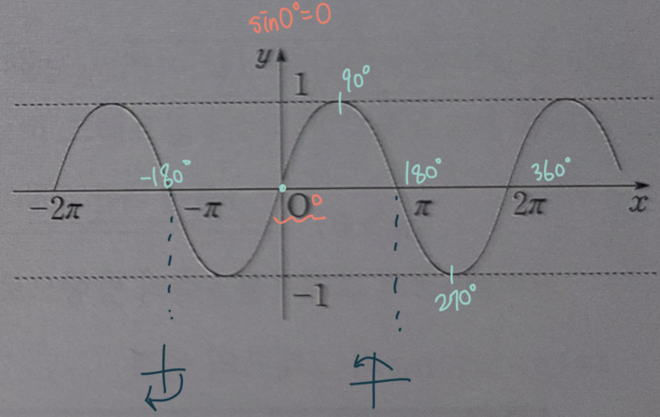
0	$\sin\theta$	$\cos\theta$
0	0	1
30	$\frac{1}{2}$ 그래프	$\square^2$ $\frac{\sqrt{3}}{2}$
45	$\square$ $\frac{1}{\sqrt{2}}$ =	$\square$ $\frac{1}{\sqrt{2}}$
90	1	0

$\left(\frac{1}{2}\right)^2 + \square^2 = 1$
$\frac{1}{4} + \square^2 = 1$
$\square^2 = 1 - \frac{1}{4}$
$\square^2 = \frac{3}{4}$
$\square = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

$\square^2 + \square^2 = 1$
$2\square^2 = 1$
$\square^2 = \frac{1}{2}$
$\square = \frac{1}{\sqrt{2}}$

546-1. $\sin x$의 그래프

① $y=\sin x$의 그래프

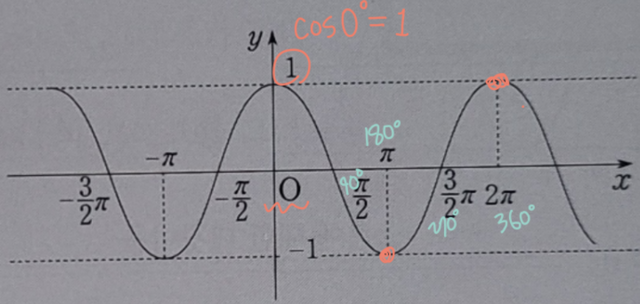


② $y=\sin x$의 그래프의 성질

정의역	모든 실수($\{x \mid x$는 모든 실수$\}$)
치역	$-1 \leqq y \leqq 1$, 최댓값$=1$, 최솟값$=-1$
주기	$2\pi(360^\circ)$
대칭	원점에 대한 대칭(기함수)

547-1. $\cos x$의 그래프

① $y=\cos x$의 그래프

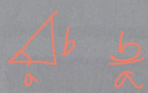


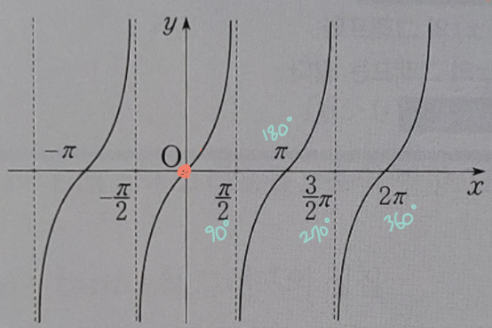
② $y=\cos x$의 그래프의 성질

정의역	모든 실수($\{x \mid x$는 모든 실수$\}$)
치역	$-1 \leqq y \leqq 1$, 최댓값$=1$, 최솟값$=-1$
주기	$2\pi(360^\circ)$
대칭	y축에 대한 대칭(우함수)

548-1. $\tan x$의 그래프

① $y=\tan x$의 그래프





② $y=\tan x$의 그래프의 성질

정의역	$x=n\pi+\frac{\pi}{2}$(n은 정수)를 제외한 모든 실수
치역	모든 실수
주기	$\pi(180^\circ)$
대칭	원점에 대한 대칭(기함수)

로그

$\log ab = \log a + \log b$

$\log_a b^n = n\log_a b$

$\log 1 = 0$

$\log_a a = 1$

$\boxed{\log_a b = x}$

$\Updownarrow$

$\boxed{a^x = b}$

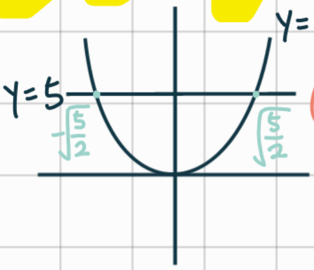
$\log ab = \log a + \log b$

$\log \frac{b}{a} = \log b \times a^{-1}$ $\boxed{\frac{1}{a} = a^{-1}}$ $\frac{1}{2} = 2^{-1}$

$= \log b + \log a^{-1}$

$= \log b - \log a$

미분 전



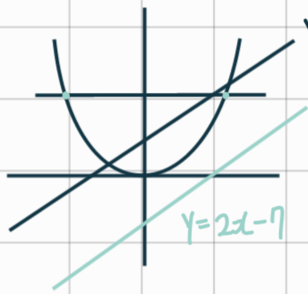
$Y=2x^2$
$Y=5$
의 교점은?

$Y=2x^2=5$

$2x^2=5$

$x^2=\frac{5}{2}$

$x=\pm\sqrt{\frac{5}{2}}$



양수

$y = ax + b$

a가 양수인데
b가 0보다 작으면
만날 수 없어.
b가 양수가 되어야
ex) $Y=3x+2$
면 됨.

$Y=2x^2$
$Y=3x+2$

$2x^2 = 3x+2$

$2x^2-3x-2=0$ → 근의 공식 써야 겠지만 못해도 되고

해가 실근

만날 수 없으면 허근.

$(x+2)(x-2)$

$x^2+2x-2x-4$

x^2-4

$(a+b)(a-b)$

a^2-b^2

$e^a \cdot e^b = e^{a+b}$

$2^3 \cdot 2^5 = 2^8$

$2^3 \times 2^{-2} = 2^{3-2} = 2^1$

$2^3 \times 2^{-4} = 2^{-1} = 2^{\frac{1}{2}}$

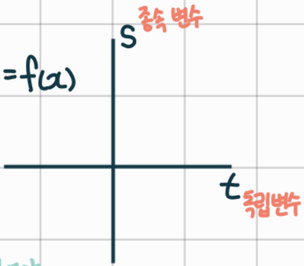
$\log_{10} 7 = \frac{\log_3 7}{\log_3 10}$

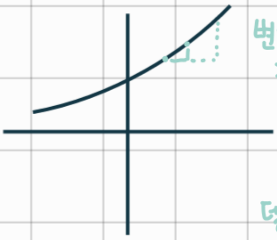
미분의 정의를 알아야 파이썬으로 코딩을 함.

미분 = 기울기 = $\frac{\Delta s}{\Delta t}$

델타(변화량)

$s=f(x)$





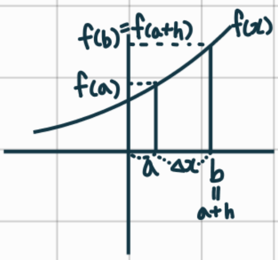
변화량 적어야
근사값 나옴

$\lim_{\Delta x \to 0}$

델타x를 0으로 보낸다는 것은
↳ 이게 0이 아니라는 것임

미분의 정의

$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$ → $\lim_{h \to 0}$



$b = a + \Delta x$ ($\Delta x \to h$)

$= a+h$

$\lim_{b \to a} \frac{f(b)-f(a)}{b-a} = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$

$y=f(x)$

$y'=f'(x)$

$f'(a)$

$(x^n)' = (n \times x^{n-1})$

$3x^7 = 3 \times 7x^6$

$f(x) = 3x^7 \Rightarrow f'(1)$

$= 3 \times 7 \times 1^6$

$= \underline{21}$ x=1에서의 기울기

$y=f(x)$ $\overset{\text{미분}}{\frac{df}{dx}} = \frac{dy}{dx}$

$f(x,y)$ 입력값이 두개라서

- $\frac{\partial f}{\partial x}$ x가 변할 때 y가 어떻게 변하는가
- $\frac{\partial f}{\partial y}$ y가 변할 때 x가 어떻게 변하는가

$y=f(x,t,v)$

- $\frac{\partial y}{\partial x} = ?$
- $\frac{\partial y}{\partial t} = ?$
- $\frac{\partial y}{\partial v} = ?$

라플라

$\nabla = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial t} \\ \frac{\partial}{\partial v} \end{pmatrix}$

$y=f(x^1, x^2, \cdots x^{100})$

$\nabla = \begin{pmatrix} \frac{\partial}{\partial x^1} \\ \frac{\partial}{\partial x_2} \\ \frac{\partial}{\partial x^3} \\ \vdots \\ \frac{\partial}{\partial x^{100}} \end{pmatrix}$

선형 결합 (līner combination)

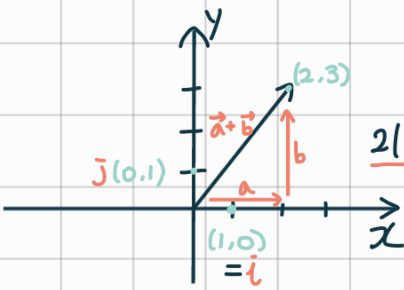
선형대수 (līner algebra)

ㄴ 벡터 . 행렬 . 연산

벡터 (크기, 방향): 공간의 한 점 (2,3)

크기와 방향이 같으면 같은 벡터

스칼라 (크기) (2,0)(0,1)



$2(1,0)+3(0,1)=(2,3)$

수직 관계

$(2,3) = \overset{스칼라}{2}\overset{벡터}{(1,0)} + 3(0,1)$

평행이동함

$=(2,0)+(0,3)$

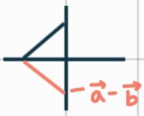
$=2i+3j$

i, j가 벡타

cf. 빼기 (음수는 방향 바꾸는 것)

$\vec{a}-\vec{b} = \vec{a}+(-\vec{b})$

$=(2,-3)$



θ $\vec{c}$ ↗ ↘ $\vec{f}$

$\vec{c}-\vec{f}$ = ↗ ↘

$\vec{c}=(3,2,7)$

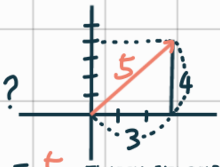
$\vec{f}=(1,0,-3)$

$(3-1, 2-0, 7+3)$

$=(2,2,10)$

$\vec{a}=(3,4)$

$(\vec{a})$의 크기는?



$\sqrt{3^2+4^2}=\sqrt{25}=5$ 피타고라스 정리 3차원판

$\vec{a}=(4,7)$

$(\vec{a})$의 크기는?

$\sqrt{(4-0)^2+(7-0)^2}$

$\sqrt{(5-2)^2+(6-4)^2}$

$=\sqrt{3^2+2^2}=\sqrt{9+4}$

$=\sqrt{13}$

$(a_1, a_2, \cdots, a_n^2)$

$\sqrt{\sum_{j=1}^{n}(a_j)^2} \neq \sum_{j=1}^{n}\sqrt{(a_j)^2}$

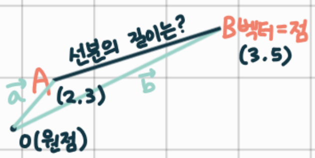
Σ 합=시그마

$=\sqrt{a_1^2}+\sqrt{a_2^2}+\cdots\sqrt{a_n^2}$

$=a_1+a_2+\cdots a_n$

$\vec{a}=(a_1, a_2, \cdots a_n)$

$\vec{b}=(b_1, b_2, \cdots b_n)$



$\overline{AB}=\overline{OB}-\overline{OA}$

$\vec{b}-\vec{a}$

$=\vec{b}+(-\vec{a})$

$(3-2)+(5-3)$

$=(1,2)$

$\sqrt{1^2+2^2}=\sqrt{5}$

Geometry (기하학)

$\vec{a}=(a_1, a_2, \cdots a_n)$

$\vec{b}=(b_1, b_2, \cdots b_n)$

$|\vec{b}-\vec{a}|$

절댓값 쓰는게 벡터의 거리

$(b_1-a_1, b_2-a_2, \cdots, b_n-a_n)$

$=\sqrt{(b_1-a_1)^2+(b_2-a_2)^2, \cdots (b_n-a_n)^2}$

내적: $\overset{점}{\text{dot}}$ $\overset{곱}{\text{product}}$

벡터 $\vec{a}=(1,0)$

$\vec{b}=(0,1)$

$\vec{a}\cdot\vec{b}$

$(1,0)\cdot(0,1)$

$=1\times0+0\times1=\boxed{0}$

내적을 취해서 0이면 수직!

$\vec{a}=(a_1, a_2)$

$\vec{b}=(b_1, b_2)$

$\vec{a}\cdot\vec{b}=(a_1b_1+a_2b_2)$

$\vec{a}=(a_1, a_2, a_3)$

$\vec{c}=(c_1, c_2, c_3)$

$\vec{a}\cdot\vec{c}=a_1c_1+a_2c_2+a_3c_3$

$$\vec{a}\cdot\vec{b} = (a_1, b_1)\cdot(a_2\cdot b_2)$$
$$= a_1 a_2 + b_1 b_2$$
$$= |\vec{a}|\cdot|\vec{b}|\cdot\cos\theta$$

$\vec{a}\cdot\vec{b}$가 이루는 각 θ

$\llcorner$: 수직
$-$: 직선
$\cos\theta\, 90° = 0$

$\vec{a} = (-1, 2)$
$\vec{b} = (2, 1)$

벡데이터
1000차원!
↓
20차원

$\vec{a}\times\vec{b}$
$(1,2)\ (2,3) = 1\times2 + 2\times3$

2가지 [점곱 dot product = 내적 ⇒ 스칼라
cross product = 외적 ⇒ 벡터

$\vec{a} = (1, 2, 3)$
$\vec{b} = (-1, -2, -1)$
$-1 + (-4) + (-3)$ 각각의 요소를 곱하고 더한다.
$\vec{a}\cdot\vec{b} = -8$

$|\vec{a}| = \sqrt{14}$ $\quad |\vec{b}| = \sqrt{6}$

$\vec{a}\cdot\vec{b} = |\vec{a}|\cdot|\vec{b}|\cos\theta$ $\quad \cos\theta = \frac{-8}{\sqrt{14}\sqrt{6}} = \sqrt{2}\sqrt{7}\sqrt{2}\sqrt{3}$
$= 2\sqrt{21}$
$= \frac{-8}{2\sqrt{21}} = \frac{-4}{\sqrt{21}}$

$\theta = \cos^{-1}\left(\frac{-4}{\sqrt{21}}\right)$
ArcCosine

$\cos\theta = 1$
$\cos^{-1} 1 = 0°$
아코사인

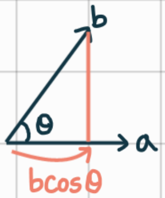
$\vec{a} = (a_1, a_2)$
$\vec{b} = (b_1, b_2)$

$\vec{a}\cdot\vec{b} = (a_1 b_1 + a_2 b_2) =$ 스칼라
$\vec{a}\cdot\vec{b} = |\vec{a}|\cdot|\vec{b}|\cos\theta$ ($\vec{a}\cdot\vec{b}$ 이루는 각)
$= (1\times2 + 2\times(-1) + 3\times2) = 6$

$\vec{a}(1,2,3) = \sqrt{1^2+2^2+3^2}$
$= \sqrt{1+4+9}$
$= \sqrt{14}$
$\vec{b}(2,-1,2) = \sqrt{2^2+(-1)^2+2^2}$
$= \sqrt{4+1+4}$
$= \sqrt{9} = 3$

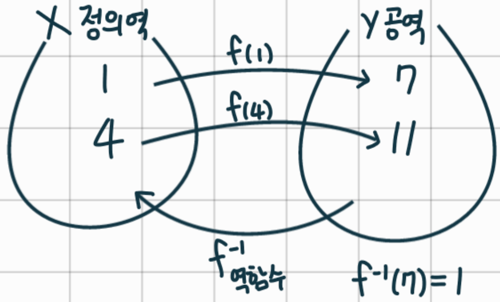
$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|} = \frac{6}{\sqrt{14}\cdot 3} = \frac{2}{\sqrt{14}}$
$= \frac{\sqrt{2}\sqrt{2}}{\sqrt{2}\sqrt{7}} = \frac{\sqrt{2}}{\sqrt{7}} = \sqrt{\frac{2}{7}}$

내적의 물리적 의미
$\vec{a}\cdot\vec{b} = |\vec{a}|\cdot|\vec{b}|\cos\theta = |\vec{a}|$



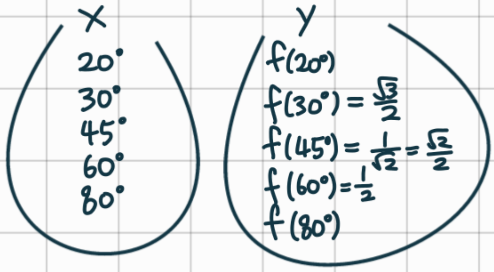
직교 (Orthogona) = 수직

OFDM



대응되는 것만 치역이라 함

$f = \cos\theta$



$f^{-1}\left(\frac{\sqrt{3}}{2}\right) = 30°$
$\cos\theta = \sqrt{\frac{2}{7}}$
$\cos^{-1}\sqrt{\frac{2}{7}} = \theta$ ArcCosine
np.arccos$\left(\sqrt{\frac{2}{7}}\right)$

$$\vec{a}\cdot\vec{b} = |\vec{a}|\cdot|\vec{b}|\cos\theta$$

직선의 방정식

$2x + 3y = 5$ $\times 3$ $\quad 6x + 9y = 15$
$y = \frac{2}{3}x + \frac{5}{3}$
$3x + 2y = 4$ $\times 2$ $\quad 6x + 4y = 8$ $-$
$5y = 7$
$y = \frac{7}{5}$

$2x + 3\times\frac{7}{5} = 5$
$2x + \frac{21}{5} = \frac{25}{5}$
$2x = \frac{4}{5}$
$x = \frac{4}{10}$

교점 $= \left(\frac{4}{10}, \frac{7}{5}\right)$

$$\begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$$

각각 요소끼리 곱하고 더함 (내적)
$2\times x + 3\times y = 5$

행(가로)과 열(세로) 바꿈 (transpos)

$\vec{a} = (2, 3)$
$\vec{x} = (x, y)$
$\vec{a}^T\cdot\vec{x} = \vec{b}$
$x\vec{a}_1 + y\vec{a}_2 = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$
$\frac{4}{10}\begin{pmatrix} 2 \\ 3 \end{pmatrix} + \frac{7}{5}\begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$

선형결합
liner Combination
벡데이터 하는 일
x 찾기

$\alpha\vec{a} + \beta\vec{b} \rightarrow \vec{c}$

벡터 독립이냐 ↑→ 수직이면 독립 O, 독립이면 수직 X
종속이냐 ↗↗ 서로가 배(x)해서 나오면 종속

R^2 real 2차원 평면
독립이어야 표현할 수 있다 (span)
3차원은 독립 3개는 필요.

표준 직교 basis
수직 (span, 독립) ⊕ → 그 집합을 basis 라고 한다.
기저

$\{(1,0)\ (0,1)\}$
basis = 기저

$\{(1,2)(3,2)\}$
얘도 basis 가 무지 많음

$3x+2y+3z=5$
$x+y-z=7$

$$\begin{pmatrix}3&2&3\\1&1&-1\\0&0&0\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}5\\7\\0\end{pmatrix}$$

transpos

$3x+y-z=6$
$2x+y-2z=8$
$x+2y+7z=3$

$$\begin{array}{cc} & \begin{matrix}c_1 & c_2 & c_3\end{matrix} \\ \begin{matrix}a_1\\a_2\\a_3\end{matrix} & \begin{pmatrix}3&1&-1\\2&1&-2\\1&2&7\end{pmatrix}\end{array}\begin{array}{c}x\\ \begin{pmatrix}x\\y\\z\end{pmatrix}\end{array}=\begin{array}{c}b\\ \begin{pmatrix}6\\8\\3\end{pmatrix}\end{array}$$

$a_i^T \cdot x = b$

$$x\begin{pmatrix}3\\2\\1\end{pmatrix}+y\begin{pmatrix}1\\1\\2\end{pmatrix}+z\begin{pmatrix}-1\\1\\7\end{pmatrix}=\begin{pmatrix}6\\8\\3\end{pmatrix}$$

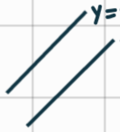
$A^{-1}Ax = bA^{-1}$
행렬 벡터

$$\cos\theta = \frac{(A|\text{벡터} \times B|\text{벡터})}{(A| \times B|)}$$

-단위행렬-

$$\begin{pmatrix}1&3&7\\1&-1&8\\2&4&9\end{pmatrix}\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}=\begin{pmatrix}(0,0)&(0,1)&\\&&\\&&\end{pmatrix}$$

A 3×3 I 3×3 3×3
같아야함



$y=2x$
$y=2x+3$

$2x-y=0$
$2x-y+3=0$

$$\begin{pmatrix}2&-1\\2&-1\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}0\\-3\end{pmatrix}=\frac{X}{ab-bc}$$
A
역행렬 못 구한다는 얘기

$A^{-1}Ax = bA^{-1}x$

np.dot(A,B) $(A)\times(B)$

np.dot(np.linalg.inv(A), b)

$5^{-1}5x = 7\times5^{-1}$
$bA^{-1}\times A^{-1}=b$???

$2x+y=5$ -①
$x-y=3$ -② (x,y)의 값 구하기

①+②
$3x=8$
$x=\frac{8}{3}$ ⇨ $\frac{8}{3}-y=3$
$y=\frac{8}{3}-3=-\frac{1}{3}$

$3^{-1}=\frac{1}{3}$

$x=(-\frac{1}{3}, \frac{8}{3})$ x벡터를 찾은 것!

$$\begin{pmatrix}2&1\\1&-1\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}5\\3\end{pmatrix}$$
행렬 A b 벡터를 곱하고 더하는 것= 내적

$\begin{pmatrix}2\\1\end{pmatrix}\cdot\begin{pmatrix}x\\y\end{pmatrix}=5$

열벡터 $x=\begin{pmatrix}x\\y\end{pmatrix}$ 열벡터 $b=\begin{pmatrix}5\\3\end{pmatrix}$

$AB=I$
$B=A^{-1}$
$AB=BA=I$
$A=B^{-1}$

$\underset{-\frac{1}{3}}{x}\begin{pmatrix}1\\2\end{pmatrix}+\underset{\frac{8}{3}}{y}\begin{pmatrix}1\\-1\end{pmatrix}=\begin{pmatrix}5\\3\end{pmatrix}$

$-\frac{1}{3}\times2+\frac{8}{3}=5$
$-\frac{1}{3}\times1+\frac{8}{3}\times(-1)=3$

$2x+y-2z=3$ -①
$4x+2y-5z=7$ -②
$x-y+6z=4$ -③

$x=\begin{pmatrix}x\\y\\z\end{pmatrix}$

$$\begin{pmatrix}2&1&-2\\4&2&-5\\1&-1&6\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}3\\7\\4\end{pmatrix}$$

+
$7x+2y+z=14$

①-② $4x+2y-4z=6$
$4x+2y-5z=7$
$z=-1$

①+③ $3x+4z=7$
$3x-4=7$
$3x=11$
$x=\frac{11}{3}$

③ $\frac{11}{3}-y-6=4$
$y=\frac{11}{3}-6-4=-10+\frac{11}{3}=-\frac{30}{3}+\frac{11}{3}=-\frac{19}{3}$

$x=\begin{pmatrix}\frac{11}{3}\\-\frac{19}{3}\\-1\end{pmatrix}$

검증) $\begin{pmatrix}2&1&-2\\4&2&-5\\1&-1&6\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}3\\7\\4\end{pmatrix}$

$\overset{\frac{11}{3}}{x}\begin{pmatrix}2\\4\\1\end{pmatrix}+\overset{-\frac{19}{3}}{y}\begin{pmatrix}1\\2\\-1\end{pmatrix}+\overset{-1}{z}\begin{pmatrix}-2\\-5\\-6\end{pmatrix}=\begin{pmatrix}3\\7\\4\end{pmatrix}$ 대입

$\frac{11}{3}\times2-\frac{19}{3}+2=3$
$\frac{22-19+6}{3}=\frac{9}{3}=3$