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System risk assessment with discrete-state components

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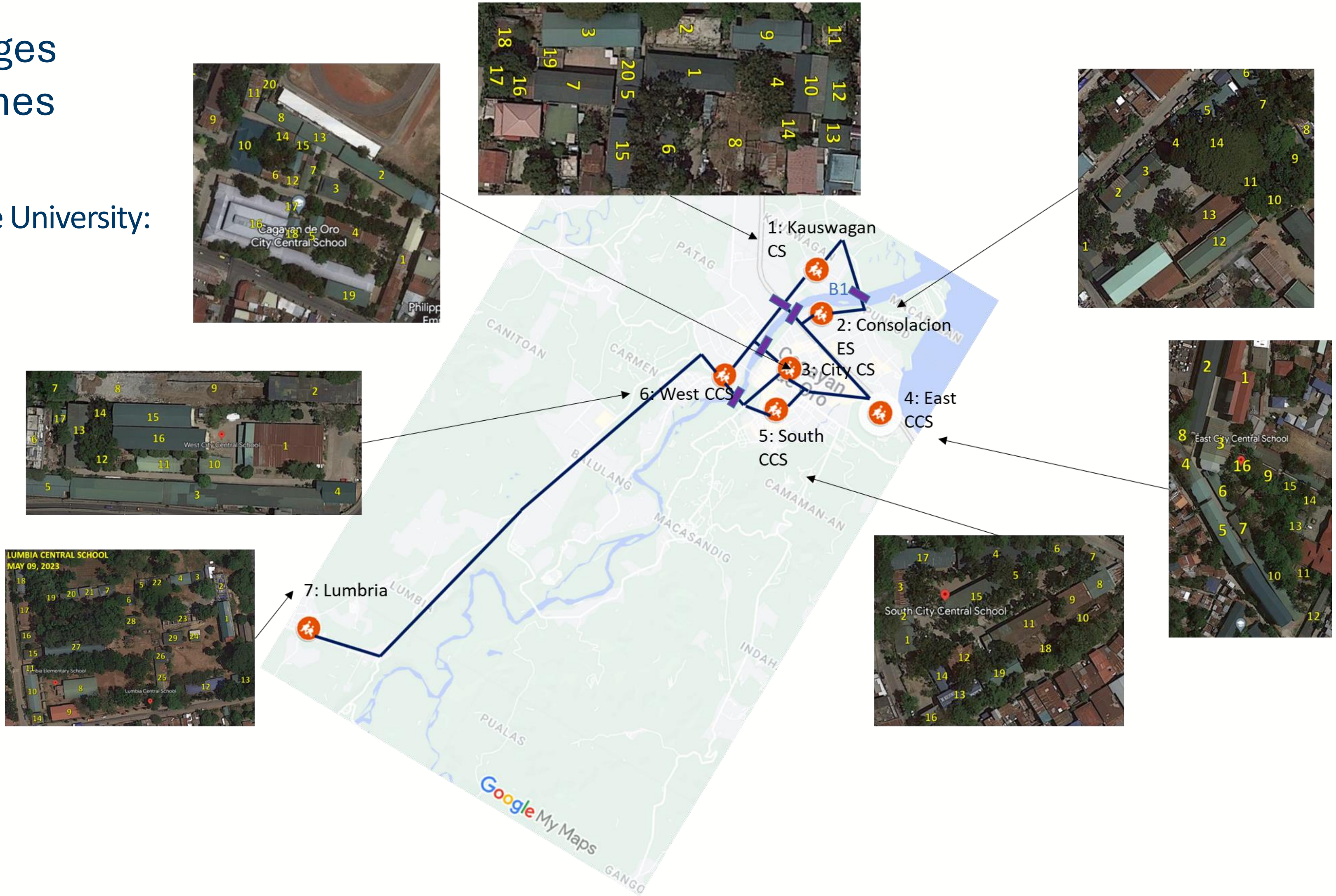
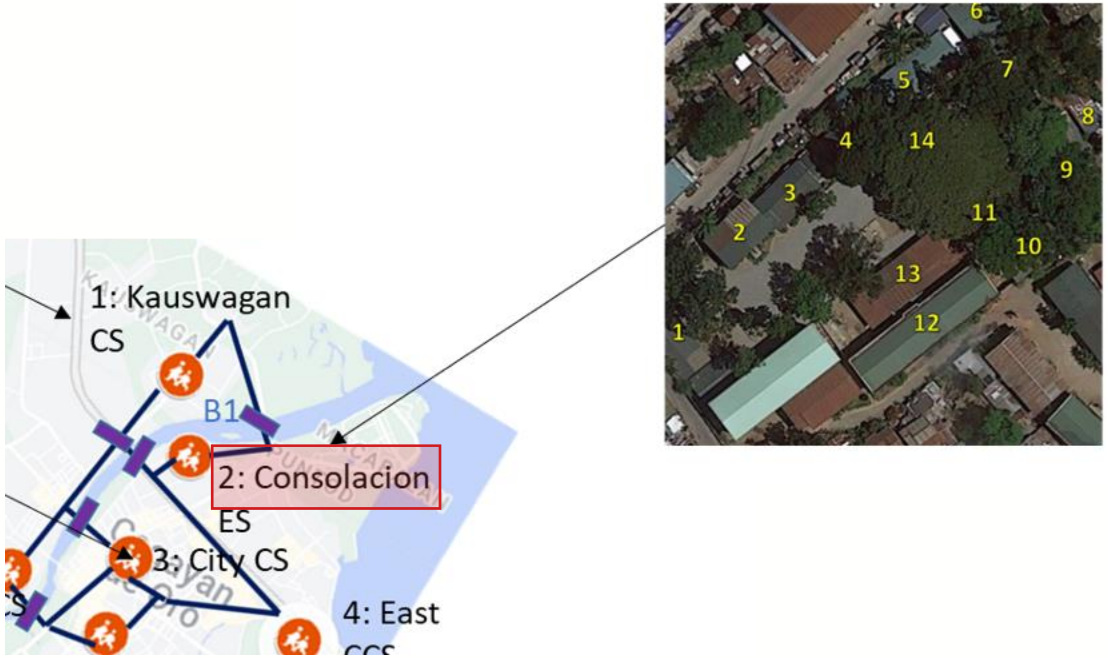
Inherent uncertainties in structural functionalities

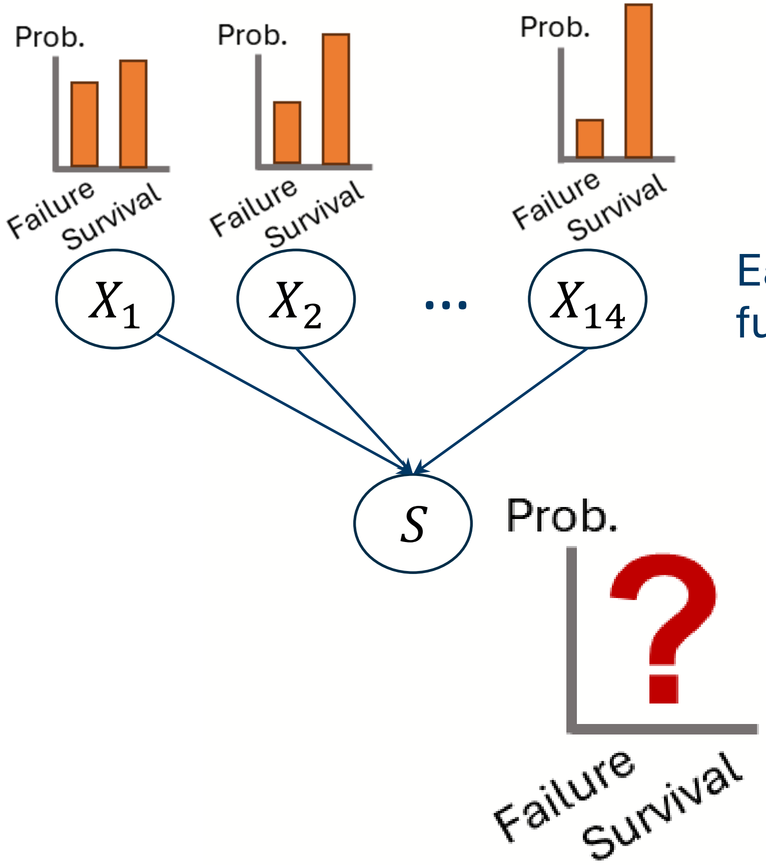
School complexes and bridges
in Cagayan de Oro, Philippines

Data obtained from past projects by
UCL, Xavier University, and De La Salle University:



Inherent uncertainties in structural functionalities





Each building's functionality

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System risk assessment with discrete-state components

Day 1

- Background and definition
- Sampling vs. decomposition
- Introduction to MBNPy

Day 2

- A tutorial on risk assessment of school complexes in Cagayan De Oro, Philippines

At the end of this tutorial, you will

- Understand the concept of system risk assessment and
- Analyse your system risk problems using MBNPy module.

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Challenge in calculating system probability

Exponential number of possible combinations

Component probabilities

$P(X_1)$

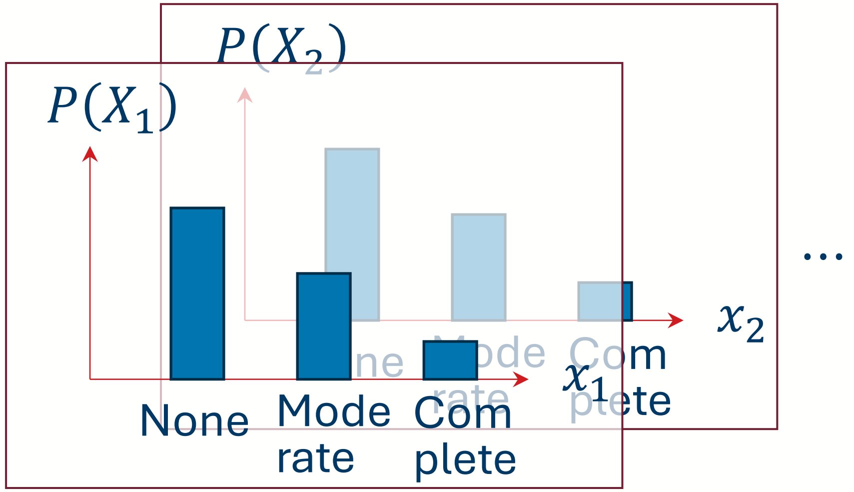
$\vdots$

$P(X_N)$

System function

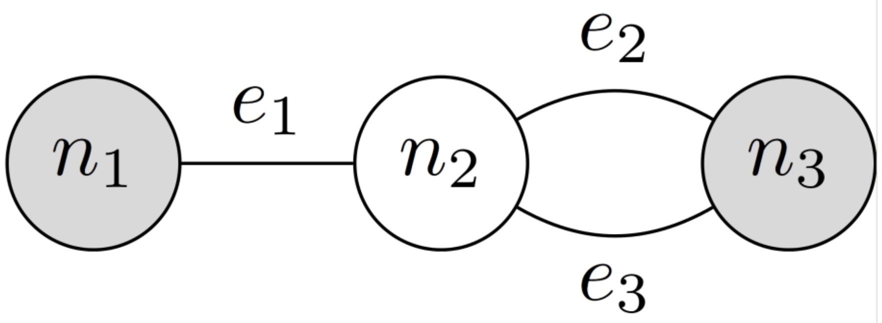
$s = f(x_1, \ldots, x_N)$

$$P(S = \text{fail}) = \sum_{\boxed{x_1,\ldots,x_N}} I[f(x_1,\ldots,x_N) = \text{fail}] \cdot P(x_1,\ldots,x_N)$$



Challenge in calculating system probability
Toy example

e_n's state is represented by random variable X_n.



failure survival

$P(X_1 = 0) = 0.1, P(X_1 = 1) = 0.9$
$P(X_2 = 0) = 0.2, P(X_2 = 1) = 0.8$
$P(X_3 = 0) = 0.3, P(X_3 = 1) = 0.7$

System survival: connection between n_1 and n_3

$(X_n = k) := x_n^k$

$(x_1^1, x_2^1, x_3^1) \to s^1, P(x_1^1, x_2^1, x_3^1) = 0.9 \cdot 0.8 \cdot 0.7 = 0.504$

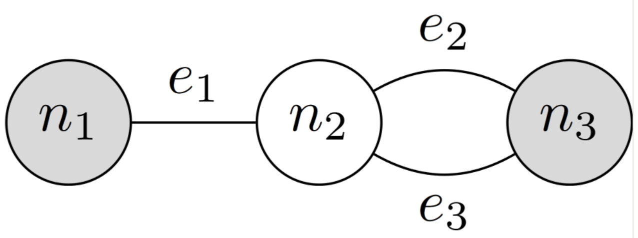
$(x_1^1, x_2^1, x_3^0) \to s^1, P(x_1^1, x_2^1, x_3^0) = 0.9 \cdot 0.8 \cdot 0.3 = 0.216$

$(x_1^1, x_2^0, x_3^1) \to s^1, P(x_1^1, x_2^0, x_3^1) = 0.9 \cdot 0.2 \cdot 0.7 = 0.126$

$(x_1^0, x_2^1, x_3^1) \to s^0, P(x_1^0, x_2^1, x_3^1) = 0.1 \cdot 0.8 \cdot 0.7 = 0.056$

Challenge in calculating system probability

Toy example (cont'd)



System survival: connection between n_1 and n_3

$(x_1^1, x_2^1, x_3^1) \rightarrow s^1, P(x_1^1, x_2^1, x_3^1) = 0.9 \cdot 0.8 \cdot 0.7 = 0.504$

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$(x_1^1, x_2^0, x_3^0) \rightarrow s^0, P(x_1^1, x_2^0, x_3^0) = 0.9 \cdot 0.2 \cdot 0.3 = 0.054$

$(x_1^0, x_2^1, x_3^0) \rightarrow s^0, P(x_1^0, x_2^1, x_3^0) = 0.1 \cdot 0.8 \cdot 0.3 = 0.024$

$(x_1^0, x_2^0, x_3^1) \rightarrow s^0, P(x_1^0, x_2^0, x_3^1) = 0.1 \cdot 0.2 \cdot 0.7 = 0.014$

$(x_1^0, x_2^0, x_3^0) \rightarrow s^0, P(x_1^0, x_2^0, x_3^0) = 0.1 \cdot 0.2 \cdot 0.3 = 0.006$

$+$

$0.056 + 0.054 + 0.024 + 0.014 + 0.006 = \mathbf{0.154}$

$$P(S = \text{fail}) = \sum_{x_1,\dots,x_N} I[f(x_1,\dots,x_N) = \text{fail}] \cdot P(x_1,\dots,x_N)$$

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Challenge in calculating system probability
Toy example (cont'd)

$$\left.\begin{array}{l}
(x_1^1, x_2^1, x_3^1) \rightarrow s^1, P(x_1^1, x_2^1, x_3^1) = 0.9 \cdot 0.8 \cdot 0.7 = 0.504 \\
(x_1^1, x_2^1, x_3^0) \rightarrow s^1, P(x_1^1, x_2^1, x_3^0) = 0.9 \cdot 0.8 \cdot 0.3 = 0.216 \\
(x_1^1, x_2^0, x_3^1) \rightarrow s^1, P(x_1^1, x_2^0, x_3^1) = 0.9 \cdot 0.2 \cdot 0.7 = 0.126 \\
(x_1^0, x_2^1, x_3^1) \rightarrow s^0, P(x_1^0, x_2^1, x_3^1) = 0.1 \cdot 0.8 \cdot 0.7 = 0.056 \\
(x_1^1, x_2^0, x_3^0) \rightarrow s^0, P(x_1^1, x_2^0, x_3^0) = 0.9 \cdot 0.2 \cdot 0.3 = 0.054 \\
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(x_1^0, x_2^0, x_3^1) \rightarrow s^0, P(x_1^0, x_2^0, x_3^1) = 0.1 \cdot 0.2 \cdot 0.7 = 0.014 \\
(x_1^0, x_2^0, x_3^0) \rightarrow s^0, P(x_1^0, x_2^0, x_3^0) = 0.1 \cdot 0.2 \cdot 0.3 = 0.006
\end{array}\right\} (\text{No. of states})^N$$

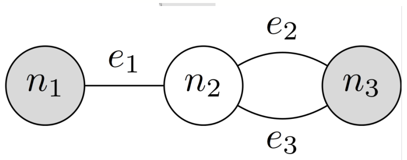
e.g. 100 binary-state components: $2^{100} > 10^{30}$
Simply impossible number!

Any solution?

Common solution: Monte Carlo Simulation

- Brute-force generation of samples by given probabilities
- E.g.

```
prob = [0.1, 0.2, 0.3];
samples = zeros(10, 3);
for i=1:3
    samples(:,i) = randsample([0,1], 10, true, [prob(i), 1-prob(i)]);
end
```



Try 1: $\hat{P}(s^0) = 0.4$

samples =

1	1	0
0	1	1
1	1	1
0	1	0
0	0	0
1	1	1
1	1	1
1	1	1
1	1	1
0	1	0

Try 2 : $\hat{P}(s^0) = 0$

samples =

1	1	1
1	1	1
1	1	1
1	1	1
1	1	1
1	1	0
1	0	1
1	1	1
1	1	1
1	1	1

Try 3 : $\hat{P}(s^0) = 0.4$

samples =

0	0	1
0	1	0
1	1	0
1	1	1
1	0	0
1	1	1
1	1	1
1	1	1
0	1	0
1	1	1

Recall $P(s^0) = 0.154.$

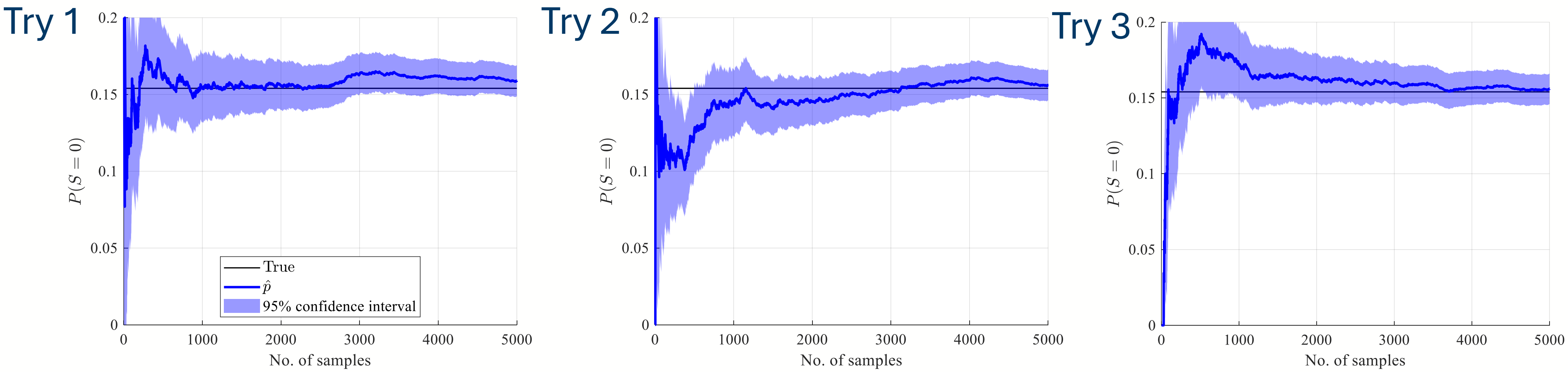
What's wrong?

MCS needs enough samples.

- Measure of confidence: coefficient of variance

$$\hat{\delta} = \frac{\sqrt{\widehat{\mathrm{Var}}[\hat{p}]}}{\hat{p}} = \sqrt{\frac{(1-\hat{p})}{N\hat{p}}} \approx \sqrt{\frac{1}{N\hat{p}}}$$

- Required number of samples for $\hat{\delta}$, $N = 1/\hat{p}\hat{\delta}^2$
  - E.g. we want $\hat{\delta} = 5\%$, we need $N = 1/(0.154 \cdot 0.05^2) = 2{,}597$ samples.



Complexity of MCS

Required number of samples for $\hat{\delta}$, $N = 1/\hat{p}\hat{\delta}^2$

- N increases with a lower $\hat{p}$.
- $N\hat{p} = 1/\hat{\delta}^2 = (\text{No. of failure samples})$
    - For $\hat{\delta} = 0.05$, MCS should see $1/(0.05)^2 = 400$ failure samples.
- E.g. $\hat{p} = 0.001$ and $\hat{\delta} = 0.05$, $N = 400{,}000$.
    - If it takes 1 second for a system simulation, it takes 5 days.

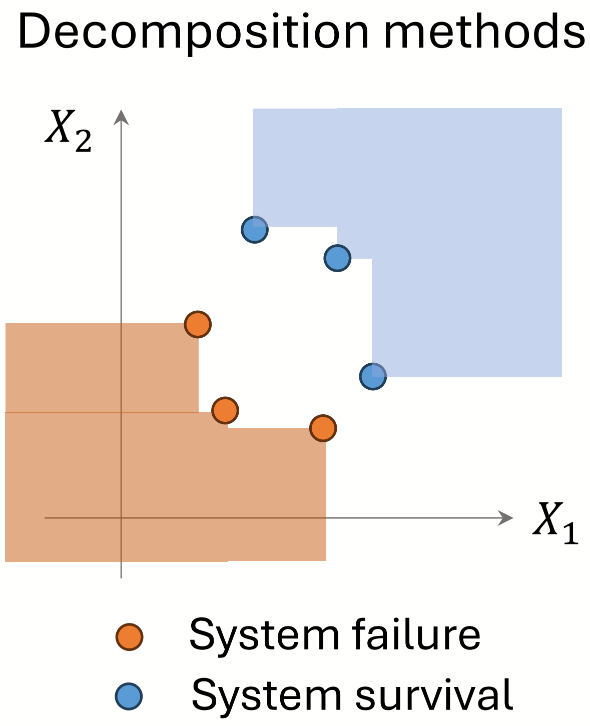
Whenever component probabilities $P(X_n), n = 1, \dots, N$, changes, MCS needs to be done all over again.

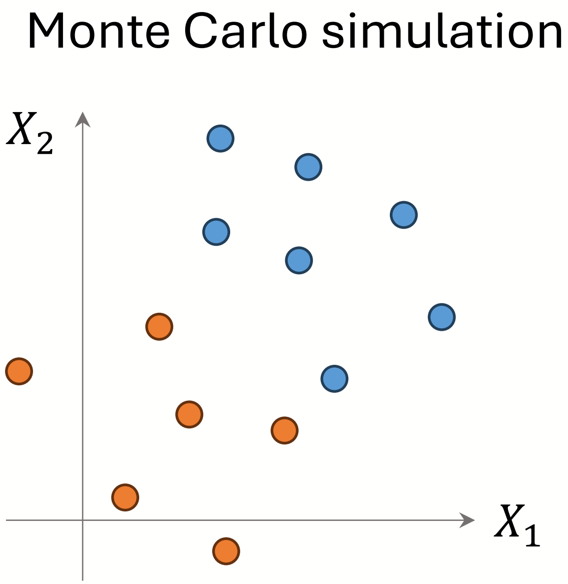
- I.e. previous results cannot be re-used.

Decomposition methods as an alternative solution

Use understandings of a system event for computational efficiency

- Coherency: Better component states do not worsen system state.



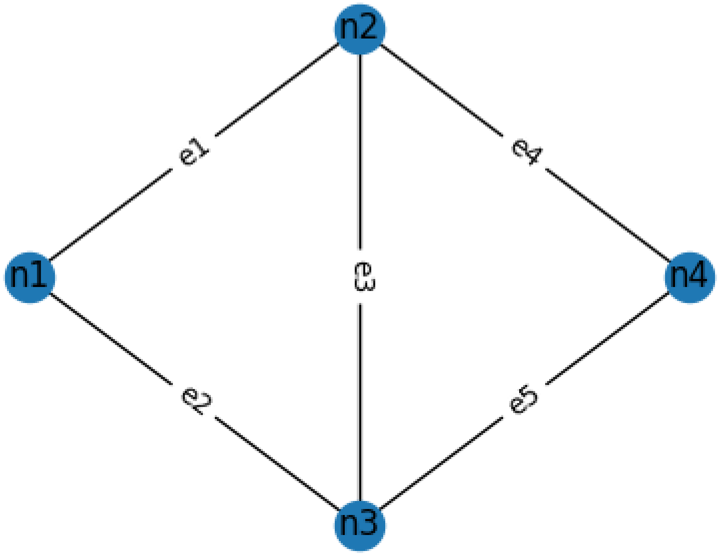


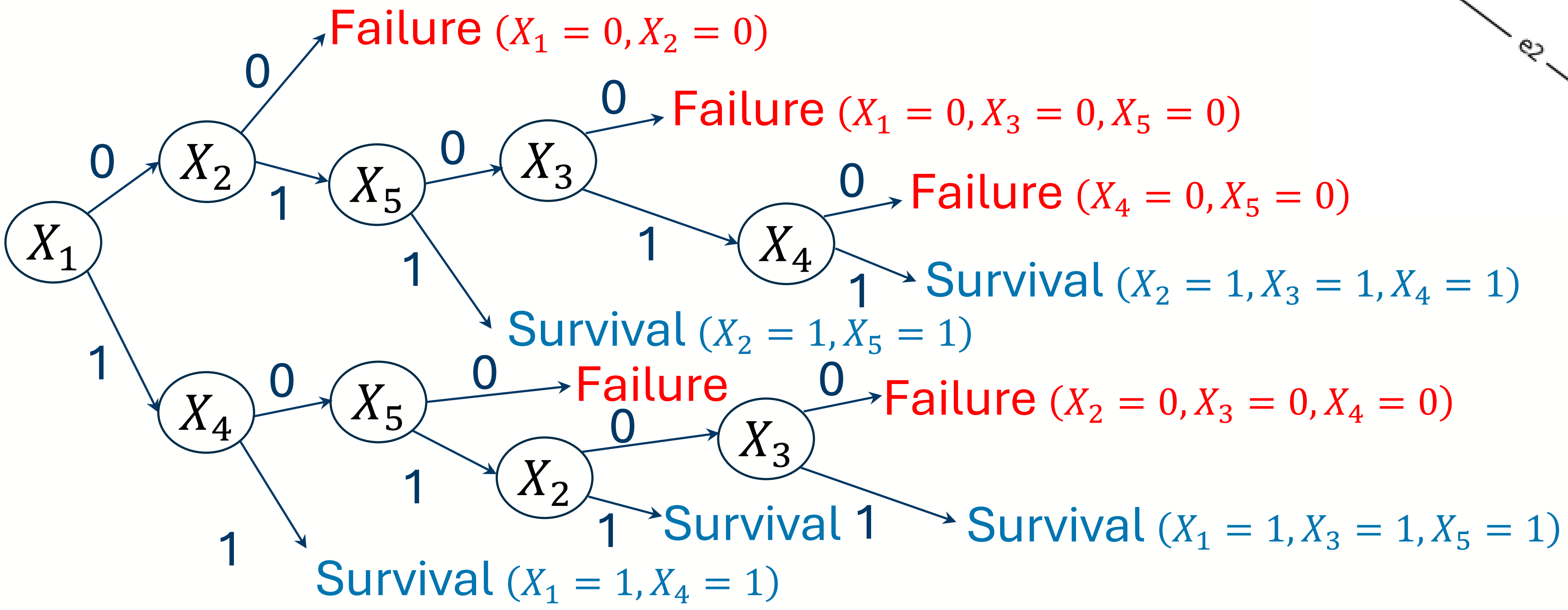
Decomposition methods as an alternative solution

Example: decomposition

Problem

- X_n's state (representing e_n): 0 (failure) or 1 (survival)
- System failure: dysconnectivity between n1 to n4

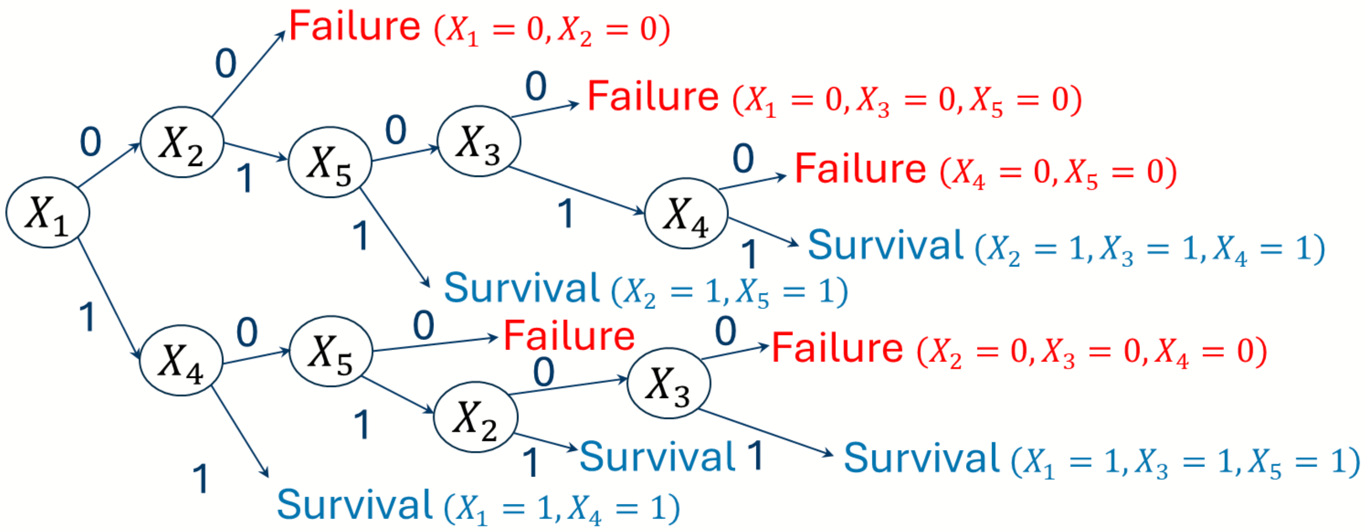




- $2^5 = 32$
  $\rightarrow$10 branches
- 4 survival rules and
  4 failure rules.

Decomposition methods as an alternative solution
Example: computation

- Failure probability = $\sum_{\text{fail branch}} P(\text{fail branch})$



$P(x_n^0) = 0.01, P(x_n^1) = 0.99, n = 1,2$
$P(x_n^0) = 0.05, P(x_n^1) = 0.95, n = 3,4$
$P(x_n^0) = 0.10, P(x_n^1) = 0.90, n = 5$

$\rightarrow$ Failure prob.
$= 0.01 \cdot 0.01 + 0.01 \cdot 0.99 \cdot 0.10 \cdot 0.05 + \cdots$
$= 0.0001 + 0.0000495 + \cdots$

- Probability update: No need for another B&B.

Updated probabilities to
$P(x_n^0) = 0.2, P(x_n^1) = 0.8$ for $n = 1$

$\rightarrow$ Failure prob. $= 0.2 \cdot 0.01 + 0.2 \cdot 0.99 \cdot 0.10 \cdot 0.05 + \cdots$

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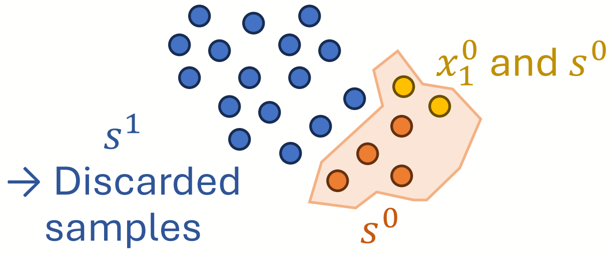
Comparison

MCS

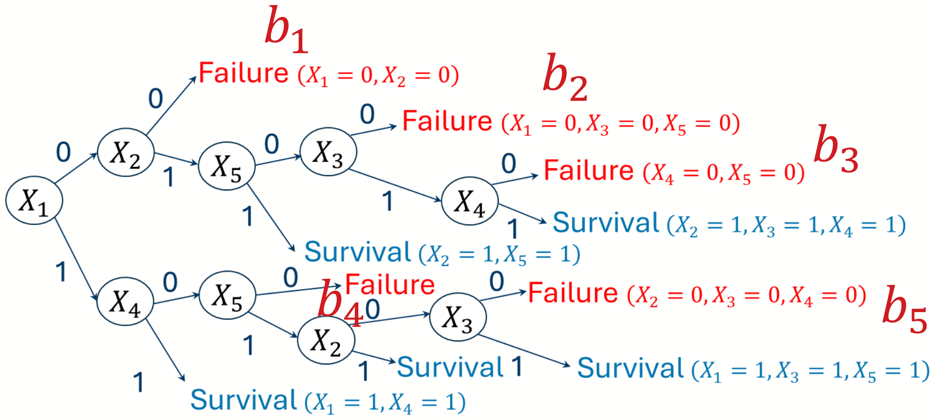
- **Complexity** $O(P(s^0)^{-1})$
- **Probability update** requires a new round.
- **Conditional probability** complexity
$$O(P(s^0)^{-2})$$
  - E.g. component importance measure $P(x_n^0|s^0)$



Effective number of samples
$N \to N \cdot P(s^0)$

Decomposition

- O(# branches), where # branches ↑↑ with # rules ↑ with # components.
- Update adds little computation cost.
- Conditional doesn't increase complexity.

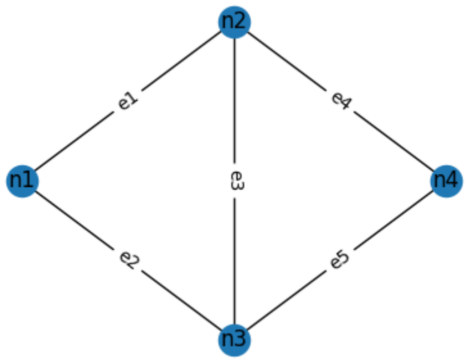


$P(x_n^0|s^0) = \dfrac{P(x_n^0, s^0)}{P(s^0)}$,
where
$P(x_n^0, s^0) = P(b_1) + P(b_2) + P(b_3)$

MBNPy package
Decomposition using BRC

BRC algorithm

- Input 1: System event




```
nodes = {'n1': (0, 0),
         'n2': (1, 1),
         'n3': (1, -1),
         'n4': (2, 0)}

edges = {'e1': ['n1', 'n2'],
         'e2': ['n1', 'n3'],
         'e3': ['n2', 'n3'],
         'e4': ['n2', 'n4'],
         'e5': ['n3', 'n4']}
```

```
def net_conn(comps_st, od_pair, edges, varis): # maximum flow analysis

    G = nx.Graph()
    for k,x in comps_st.items():
        G.add_edge(edges[k][0], edges[k][1]) # we add each edge
        G[edges[k][0]][edges[k][1]]['capacity'] = varis[k].values[x] # define capacity as 0 if state = 0 or 1 if state = 1

    # perform maximum flow analysis between the OD pair
    G.add_edge(od_pair[1], 'new_d', capacity=1) # add a new edge with capacity 1 to ensure we find only ONE path.
    f_val, f_dict = nx.maximum_flow(G, od_pair[0], 'new_d', capacity='capacity', flow_func=shortest_augmenting_path)

    if f_val > 0: # if the flow between the OD pair is greater than 0, the two nodes are connected
        sys_st = 's'

        # We can infer an associated minimum survival rule in case of network connectivity.
        min_comps_st = {}
        for k, x in comps_st.items():
            k_flow = max([f_dict[edges[k][0]][edges[k][1]], f_dict[edges[k][1]][edges[k][0]]])
            if k_flow > 0: # the edges with flows > 0 consitute a minimum survival rule.
                min_comps_st[k] = 1

    else:
        sys_st = 'f'

        # In case of system failur        nimum                          rd.
        min_comps_st = None

    return f_val, sys_st, min_comps_st
```

Either
‘s’ or ‘f’

E.g. {‘x1’: 1, ‘x4’: 1}
If unknown, return
None

MBNPy package
Decomposition using BRC

BRC algorithm

- Input 2: Probability

```
probs = {'e1': {0: 0.01, 1: 0.99}, 'e2': {0: 0.01, 1: 0.99}, 'e3': {0: 0.05, 1: 0.95},
         'e4': {0: 0.05, 1: 0.95}, 'e5': {0: 0.10, 1: 0.90}}
```

MBNPy package
Decomposition using BRC

BRC algorithm

- Run

```
sys_fun = lambda comps_st : net_conn(comps_st, od_pair, edges, varis)
```

```
brs, rules, sys_res, monitor = brc.run(probs, sys_fun)
```

✓ 0.0s

```
*Final decomposition is completed with 11 branches (originally 13 branches).

***Analysis completed with f_sys runs 8: out_flag = complete***
The # of found non-dominated rules (f, s): 8 (4, 4)
Probability of branchs (f, s, u): (5.1688e-03, 9.95e-01, 0.0000e+00)
The # of branches (f, s, u), (min, avg) len of rf: 11 (5, 6, 0), (2, 2.50)
Elapsed seconds (average per round): 1.15e-02 (1.28e-03)
```

MBNPy package
Decomposition using BRC

BRC algorithm

- Outcome 1: Identified rules

```
print(rules['s'])
print(rules['f'])
```

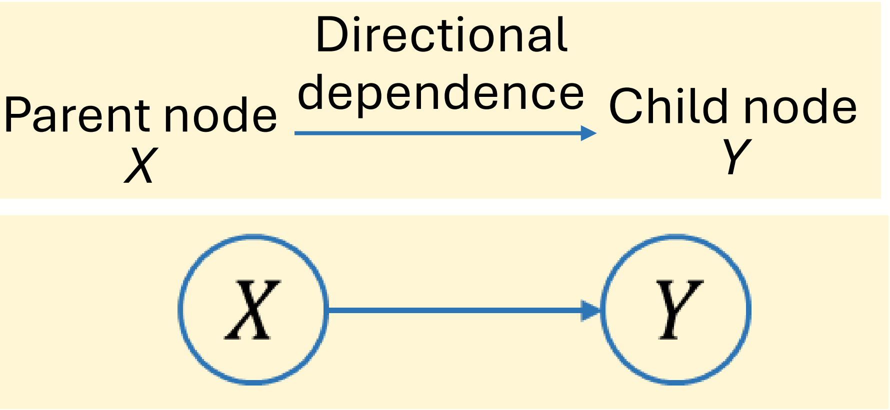
✓ 0.0s

```
[{'e1': 1, 'e4': 1}, {'e2': 1, 'e5': 1}, {'e2': 1, 'e3': 1, 'e4': 1}, {'e1': 1, 'e3': 1, 'e5': 1}]
[{'e4': 0, 'e5': 0}, {'e1': 0, 'e2': 0}, {'e1': 0, 'e3': 0, 'e5': 0}, {'e2': 0, 'e3': 0, 'e4': 0}]
```

MBNPy package
Inference by Matrix-based Bayesian network

Bayesian network as probabilistic graphical models

- Random variables – circular nodes
- Directional dependence – directed edges
    - ∋ Causal dependence
- Directed acyclic graph
- Each node is quantified by a distribution conditional on parent nodes
- Joint distribution = product of all distributions

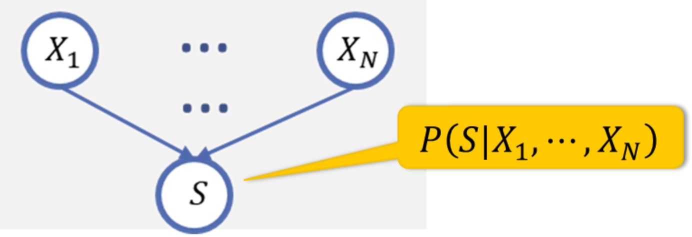


$$P(Y|X)$$

$$P(X,Y) = P(X) \cdot P(Y|X)$$

MBNPy package
Inference by Matrix-based Bayesian network

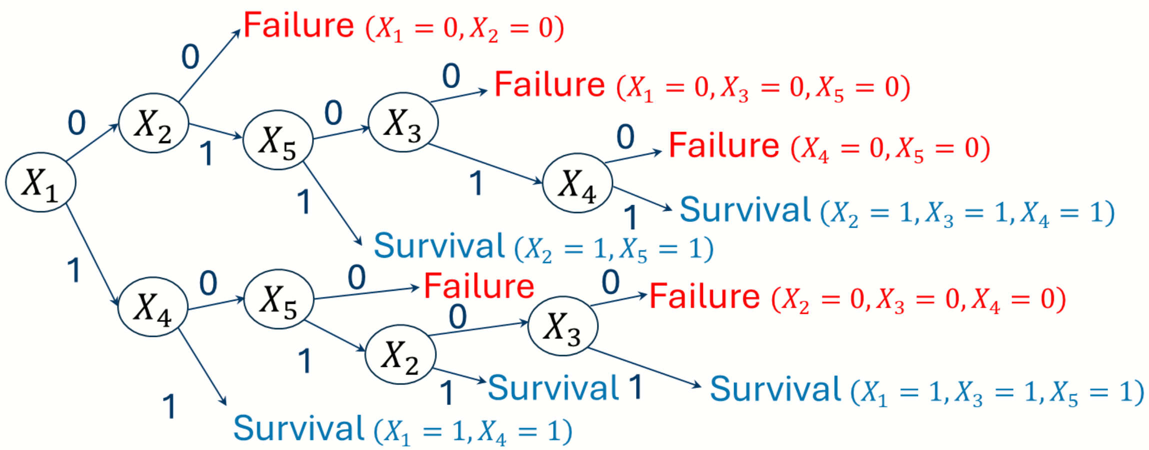
MBN: Alternative BN data structure for system events



$$\left.\begin{aligned}
&P(S=0\,|\,X_1=0, X_2=0, \cdots, X_N=0)\\
&P(S=1\,|\,X_1=0, X_2=0, \cdots, X_N=0)\\
&P(S=0\,|\,X_1=1, X_2=0, \cdots, X_N=0)\\
&\qquad\vdots\\
&P(S=1\,|\,X_1=1, X_2=1, \cdots, X_N=1)
\end{aligned}\right\} 2^{N+1}$$

$x_n^2 := x_n^0$ or x_n^1

- Idea: composite state representing multiple states



$$\begin{array}{cccccc}
S\,| & X_1 & X_2 & X_3 & X_4 & X_5
\end{array}$$

$$\mathbf{C} = \begin{bmatrix}
0 & 0 & 0 & 2 & 2 & 2\\
0 & 0 & 1 & 0 & 2 & 0\\
0 & 0 & 1 & 1 & 0 & 0\\
1 & 0 & 1 & 1 & 1 & 0\\
1 & 0 & 1 & 2 & 2 & 1\\
0 & 1 & 2 & 2 & 0 & 0\\
0 & 1 & 0 & 0 & 0 & 1\\
1 & 1 & 0 & 1 & 0 & 1\\
1 & 1 & 1 & 2 & 0 & 1\\
1 & 1 & 2 & 2 & 1 & 2
\end{bmatrix} \quad \mathbf{p} = \begin{bmatrix}
1.0\\ 1.0\\ 1.0\\ 1.0\\ 1.0\\ 1.0\\ 1.0\\ 1.0\\ 1.0\\ 1.0
\end{bmatrix}$$

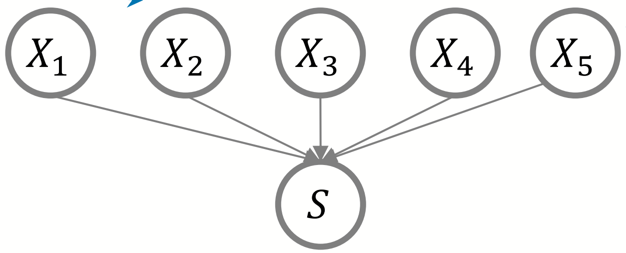
MBNPy package
Inference by Matrix-based Bayesian network

Variable elimination

$\mathbf{C} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $\mathbf{p} = \begin{bmatrix} 0.05 \\ 0.95 \end{bmatrix}$

$\mathbf{C} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

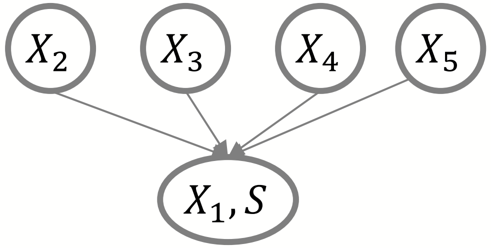
$\mathbf{p} = \begin{bmatrix} 0.01 \\ 0.99 \end{bmatrix}$



$\mathbf{C} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $\mathbf{p} = \begin{bmatrix} 0.10 \\ 0.90 \end{bmatrix}$

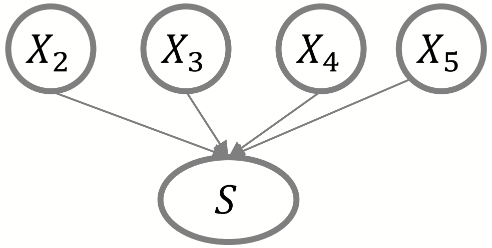
Given elimination order $X_1 \rightarrow X_2 \rightarrow X_3 \rightarrow X_4 \rightarrow X_5$,

Product



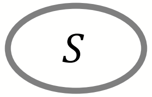
$$P(S|X_1, X_2, \dots, X_5) \cdot P(X_1) = P(S, X_1|X_2, \dots, X_5)$$

Sum



$$\sum_{X_1} P(S, X_1|X_2, \dots, X_5) = P(S|X_2, \cdots, X_5)$$

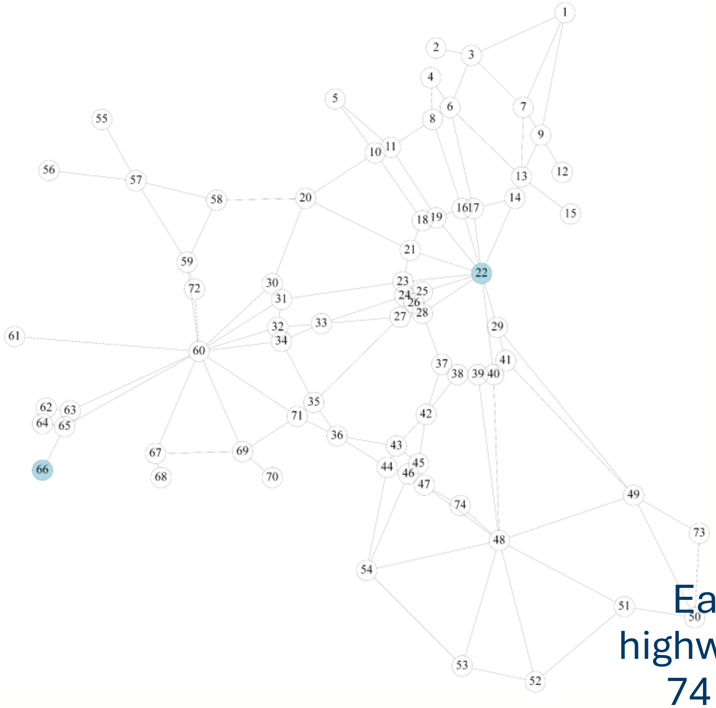
...



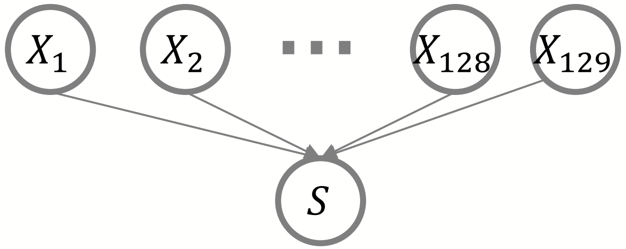
$P(S)$ $\mathbf{C} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $\mathbf{p} = \begin{bmatrix} 5.17 \cdot 10^{-3} \\ 9.95 \cdot 10^{-1} \end{bmatrix}$

EMA benchmark highway network

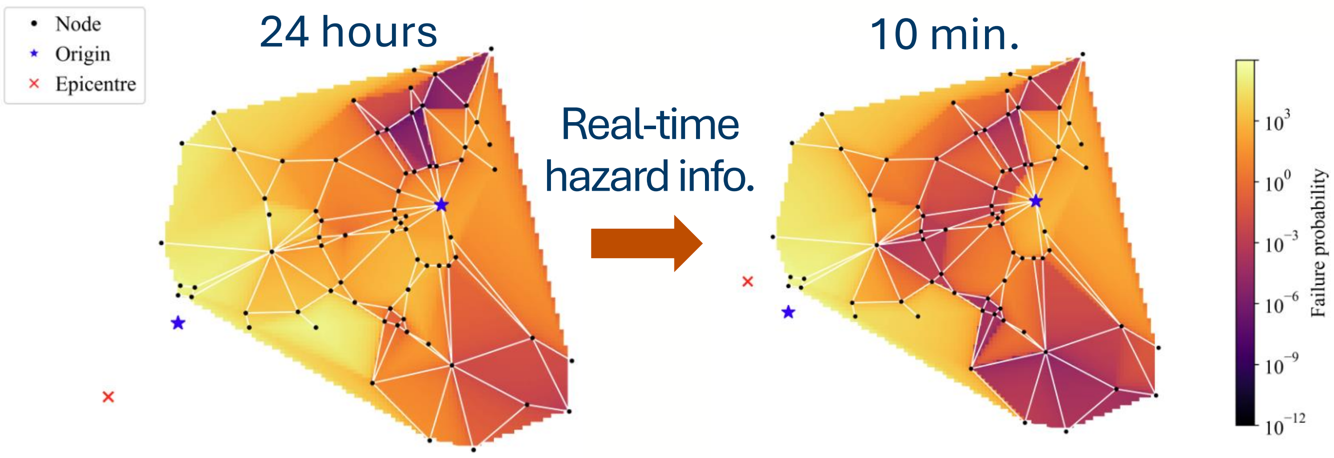
- System failure: more than twice the usual time to airports (blue nodes) → 72 system events
- Termination: 5% bound width w.r.t. lower bound or more than 50,000 branches → MCS on unspecified branches.

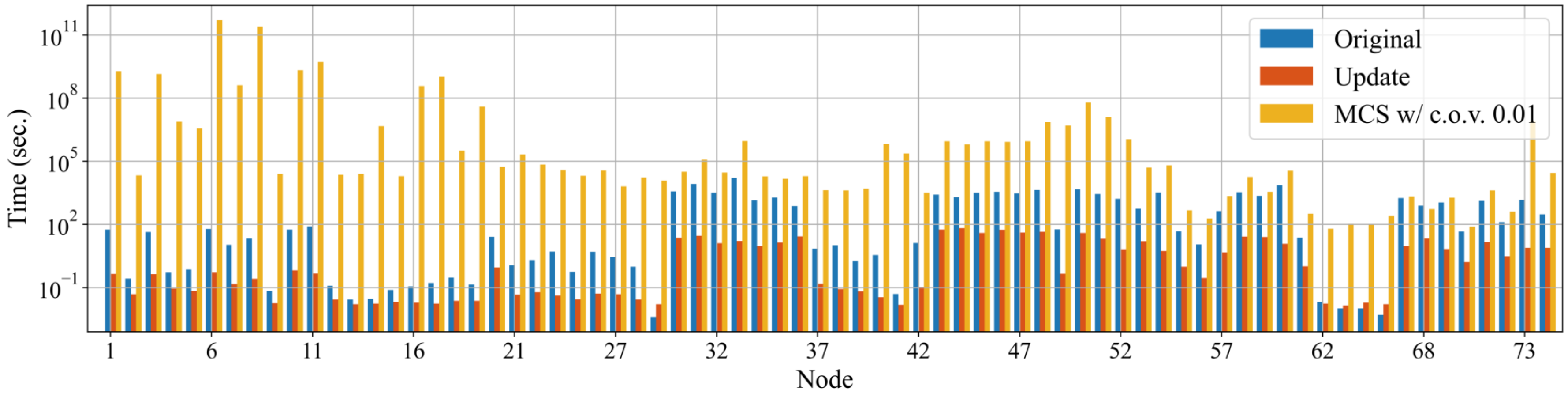


Eastern Massachusetts highway benchmark network: 74 nodes and 129 edges



EMA benchmark highway network

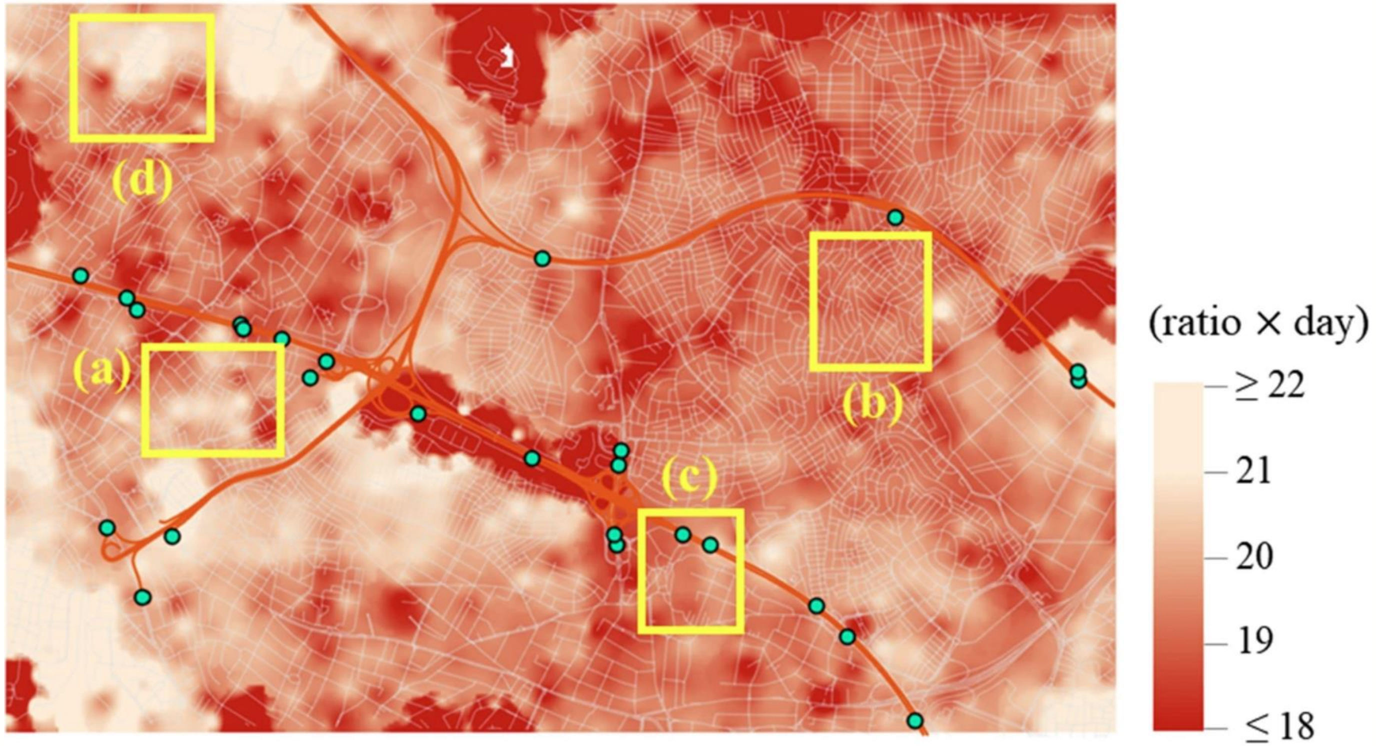




Computation time for original and update scenarios and MCS

Full-scale community-level risk analysis

Map Image from Google, © 2021 Istanbul, Turkey



Reference: Byun, J. E. and D'Ayala, D. (2022). Urban seismic resilience mapping: a transportation network in Istanbul, Turkey. *Scientific reports*, *12*(1), 8188.

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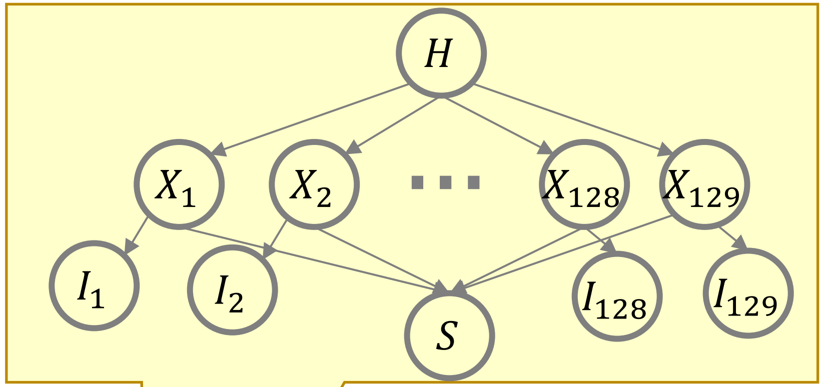
Conclusion



- Summary of Day 1
  - Challenge in system risk assessment – exponential cost w.r.t. # components
  - Introduced solution: decomposition method
  - Implementation using MBNPy package
  - Large-scale benchmark application
  - NB More detailed analyses are possible by adding more nodes.
- Between Day 1 and Day 2:
  https://github.com/jieunbyun/mbnpy_tutorial
  - 01_brc tutorial.ipynb – application of the tutorial example (est. 30 minutes)
  - 02_school reliability.ipynb – application to school complex reliability (est. 1 hour)
- Day 2

If you have any questions while trying,
drop me an email: ji-eun.byun@glasgow.ac.uk

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