Judges' Commentary: Hot Bath Problem

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Introduction

The teams who chose the Hot Bath Problem in the 2016 MCM were asked to find an optimal strategy for taking a hot bath in a traditional tub, that is, one with no circulating jets. This was a deceptively difficult problem to address. Of course, there are many established models for heat transfer and fluid flow, and we saw a number of papers using the heat equation and Navier-Stokes equations; but also, as one would expect in an undergraduate competition, we saw papers that simply used Newton's law of cooling. Many undergraduates do not see partial differential equations in the course of their study, and most of the judges took this into account when evaluating the papers.

The judging process itself should be of interest to teams and advisors; but since it has been described in detail in previous judges' commentaries (see Black [2009; 2011; 2013]), I do not provide an overview here. In particular, the process sheds light on the importance of various components in solution papers. In this commentary, I will focus primarily on specifics as they pertain to this problem and on general advice to teams.

Graphics, Simulations, and Models

We saw a number of simulations and graphical representations of heat flow. The better papers explained well what the graphics were showing and how they related to the model and the team's recommendations. These papers also gave a good description of the algorithm used to create the graphics. All too many papers present graphics, and sometimes include code in an appendix, without giving the judges sufficient information to

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evaluate them. Judges have neither the time nor the interest in reading code in order to discern the algorithm. Descriptive captions, along with clear exposition of how the graphics were created, are a must.

The higher-ranked papers included mathematics that was appropriate and justified. If there was a simulation, it was clear how the simulation was created and there was a clear justification of the model used. Better papers limited the number of models that they used in order to flesh out fully the details on the ones they did use.

Communication

The value of clear communication in this competition cannot be overly stressed. Judges have limited time to read the paper, and the most brilliant modeling will fall by the wayside if the details are not communicated clearly and efficiently.

In particular, the project summary should outline the model used and the results. For this problem, it should have indicated the best strategy for keeping the water hot, what model was used, and how the model led to this strategy.

When data, models, equations, or graphics are taken from other sources, appropriate documentation is required! But teams also need to discuss why this model, equation, data, etc., is relevant and exactly how they adapted it to fit this problem.

The Non-Technical Explanation for Users

The non-technical explanations that are frequently asked for in modeling competitions are important and often not given enough attention by participants. The best mathematics is not going to be helpful in the "real world" unless you can convince a non-technical audience that the results are valid. And the results will be totally useless if they cannot be translated into indications of what the "client" should actually do with them.

In this case, what should the bather do to optimize enjoyment of the bath? There was some room for originality and creativity here. Many of the best papers specifically recognized that you somehow need to stir the water, and that adding hot water to the top of the tub while the overflow valve is also at the top is going to present problems. Some papers recommended draining part of the cooler water before adding new hot water, and others devised mechanisms for moving the hot water to the bottom of the tub.

Bathtubs

Teams that looked at different sizes and shapes of tubs were more highly regarded than those that didn't. It was also expected that the tub would be modeled in three dimensions, not just as a series of two-dimensional slices. The best papers had three-dimensional models and discussed the effect of the shape and size of the tub had on their models.

Modeling vs. Applying Models Correctly

One of the more interesting tensions in this year's judging was between developing and/or adapting models as opposed to finding and appropriately using existing models.

As mentioned previously, there are many accepted models and differential equations available to describe heat transfer and fluid flow. Among papers that used these, the better ones were explicit in stating where the models came from, how and why they were applied, and what results were obtained from their use.

One of the impressive things that we saw in one of the Outstanding papers was a creative solution to the need for appropriate parameters for these models—in particular the proportionality constant k in Newton's law of cooling. The team found sources online that gave the values for k for water in beakers; but they needed to model water in a variety of shapes of bathtubs. They performed experiments to measure the proportionality constant for a particular bathtub, then used a variety of curve-fitting techniques to estimate the parameter value for tubs of other sizes. (Experiments in general, when appropriate, are always a good idea and provide a welcome respite for the judges from textbook approaches.)

The curve-fitting that the team used was informed by analyzing the dimensions involved, making assumptions about which dimensions would impact the value of k, and making sure that the units would cancel out in the curves that they applied. Unfortunately, this team was not perfect. In fact, they had Newton's law of cooling wrong! Because of this, their initial model's prediction was contrary to common sense—the temperature of the bath-water went to freezing instead of to room temperature. Of course, the team should have realized that this was because they had the rate of change of temperature proportional to the temperature of the water rather than proportional to the difference between the temperature of the water and the temperature of the room. They did realize that the predictions were off and adjusted the output of the model appropriately, arriving at the solution to the differential equation that they should have been solving in the first place.

None of the papers that we see are ever perfect. Teams, after all, have only a weekend to come up with a model, solve it, enhance it, and write a clear and coherent paper presenting their results. Anyone who has ever written a textbook will tell you about still finding mistakes (with hope that they are not substantive!) after years of use.

So one of the most difficult, and sometimes contentious, decisions that the judges have to make each year is what constitutes a "fatal" flaw in a paper. What error is so grave that, in and of itself, it would eliminate an otherwise Outstanding paper from receiving the Outstanding designation? This year it was the error noted above in Newton's law of cooling that led to the most heated debate. Some judges felt that the team had redeemed themselves sufficiently, in how they handled the break from reality and the other modeling that they did, to allow us to overlook this flaw; while others disagreed, believing that the team should have recognized that the differential equations model used did not make good sense. This was a rare case where, after lengthy discussion, a consensus was not reached, and the paper was awarded the Outstanding designation by majority vote. However, judges on both sides of the controversy recognized the validity of the argument for the other side.

The take-away from this for teams in future competitions is to double- or triple-check your established models. In particular, when an established model seems to predict something that you know is wrong, question whether you have made a mistake in the model. This small error almost cost this team the Outstanding designation.

Conclusion

The Hot Bath Problem allowed for a variety of approaches and a fair amount of creativity, even though most papers used established models for heat and fluid flow. Teams who thought about the actual situation of taking a bath rose above those who merely took the established models and presented results. As always, communication was key in determining the top papers.

Overall, there has been an improvement in the quality of the papers that we see in final judging. Some of this is certainly due to the increased popularity of the contest and the smaller percentage of papers that make it this far. However, it also seems that teams are making better use of the advice and materials available to them.

In particular, we are seeing more teams perform meaningful sensitivity analysis. We are also seeing better use of assumptions in the modeling process. More are making assumptions that simplify their models and later testing the sensitivity of their results to those assumptions.

All of the teams who successfully participated in this year's competition should be proud of their accomplishment. For those who wish to improve for the future, paying attention to the advice in this and other judges' commentaries would be a good step.

References

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About the Author

Kathleen Shannon is Professor of Mathematics at Salisbury University in Maryland. She earned her bachelor's degree from The College of the Holy Cross with majors in both Mathematics and Physics. She received Master's and Ph.D. degrees in Applied Mathematics from Brown University. Her primary focus is in teaching undergraduate mathematics, from liberal arts courses through calculus and discrete mathematics to numerical analysis and real analysis. She is active in the mathematical modeling community as a regular judge of both the MCM and Moody's Mega Math Challenge. She is a member of the Mathematical Association of America and of the Society for Industrial and Applied Mathematics.