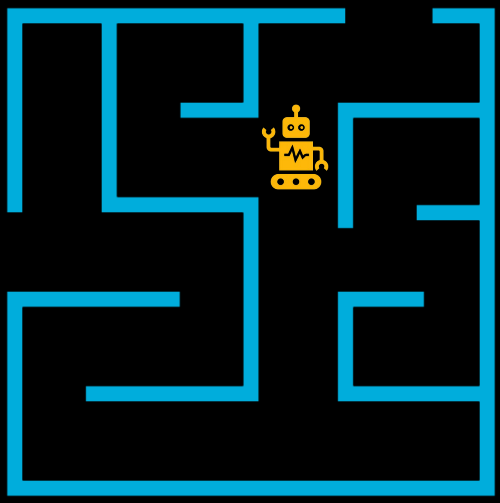
Week 1 Review

How to solve a (simple) maze?

- Hold to the left wall!
- Wall to the left and front is clear → go forward.
- Wall in front but not on left → turn left 90 degrees.
- Wall on the left and wall in front → turn right 90 degrees.
- No wall on the left or front → turn left 90 degrees

Wall Following Robot

- Implement a circuit for a robot that solves a maze.
- 2 inputs (sensors): Front and Left
  - 1 if there is a wall, 0 if not.
- 2-bit output tells robot what to do:
  - 00 means go straight
  - 01 means turn left
  - 10 means turn right
  - 11 means do a u-turn



Make Truth Table!

- Wall to the left and front is clear → go forward.
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- No wall on the left or front → turn left

- 2 wall sensors: Front and Left
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  - 10 means turn right
  - 11 means do a u-turn

F	L	Y_1	Y_0
0	0		
0	1		
1	0		
1	1		

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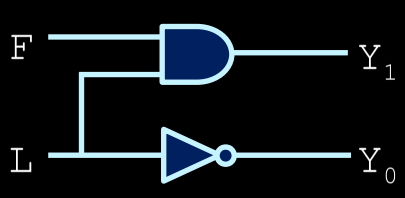
0	0	0	1
0	1	0	0
1	0	0	1
1	1	1	0

Robot Controller Circuit

F	L	Y_1	Y_0
0	0	0	1
0	1	0	0
1	0	0	1
1	1	1	0

$$Y_1 = \overline{F} \cdot L$$
$$Y_0 = L$$



let's form an AI startup!

Question 1: Counting Bits

- We have 4 inputs: A, B, C, D
- The output Y should the number of high bits that we see in the input.

- How many output bits do we need for Y?
  - Y can be 0 to 4, so we need 3 bits
    000 (zero), 001 (one), 010 (two), 011 (three)
    but also 100 (four)

Question 1: Counting Bits

- Make a truth table.
- How many rows?
  - We have 4 input bits,
    so 2^4=16 rows

Question 1: Counting Bits

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- How many rows?
  - We have 4 input bits, so 2^4=16 rows
- Fill in inputs
  - Use standard binary number order: 0000, 0001, 0010 and so on

A	B	C	D	Y_2 Y_1 Y_0

Question 1: Counting Bits

- Make a truth table.
- How many rows?
  - We have 4 input bits, so 2^4=16 rows
- Fill in inputs
  - Use standard binary number order: 0000, 0001, 0010 and so on
- Complete outputs

A	B	C	D	Y_2 Y_1 Y_0
0	0	0	0	
0	0	0	1	
0	0	1	0	
0	0	1	1	
0	1	0	0	
0	1	0	1	
0	1	1	0	
0	1	1	1	
1	0	0	0	
1	0	0	1	
1	0	1	0	
1	0	1	1	
1	1	0	0	
1	1	0	1	
1	1	1	0	
1	1	1	1	

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  - Use standard binary number order: 0000, 0001, 0010 and so on
- Complete outputs

A	B	C	D	Y_2	Y_1	Y_0	
0	0	0	0	0	0	0	0
0	0	0	1	0	0	1	1
0	0	1	0	0	0	1	1
0	0	1	1	0	1	0	2
0	1	0	0	0	0	1	1
0	1	0	1	0	1	0	2
0	1	1	0	0	1	0	2
0	1	1	1	0	1	1	3
1	0	0	0	0	0	1	1
1	0	0	1	0	1	0	2
1	0	1	0	0	1	0	2
1	0	1	1	0	1	1	3
1	1	0	0	0	1	0	2
1	1	0	1	0	1	1	3
1	1	1	0	0	1	1	3
1	1	1	1	1	0	0	4

Next step

- Convert truth table to expression!
- Starting with minterms and maxterms

Minterm expressions

- AND of all inputs or inverted inputs
  - A'BC ABC AB'C but not AC or AB+C
- m_n is 1 for row n and 0 elsewhere.
  - m_3 = A'BC since 3 is 011
    (A=0, B=1, C=1 will cause m_3 to be 1)
- Rows where output is high.

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

Minterm	Y
m_0	0
m_1	1
m_2	1
m_3	1
m_4	1
m_5	0
m_6	1
m_7	0

SOM: Sum of Minterms

- Expression for truth table:
  OR of all minterms for which is it high
- $Y = m_1 + m_2 + m_3 + m_4 + m_6$

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

Minterm	Y
m_0	0
m_1	1
m_2	1
m_3	1
m_4	1
m_5	0
m_6	1
m_7	0

Maxterms

- OR of all inputs or inverted inputs
  - A'+B+C A+B+C A+B'+C but not A+C or A+BC
- M_n is 0 for row n , 1 elsewhere.
  - M_3 = A+B'+C' since 3 is 011
    (only A=0, B=1, C=1 will cause m_3 to be 0)
- Rows where output is low.

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

Maxterm	Y
M_0	0
M_1	1
M_2	1
M_3	1
M_4	1
M_5	0
M_6	1
M_7	0

POM: Product of Maxterms

- Expression for truth table:
  AND of all maxterms for which is it low
- $Y = M_0 \cdot M_5 \cdot M_7$

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

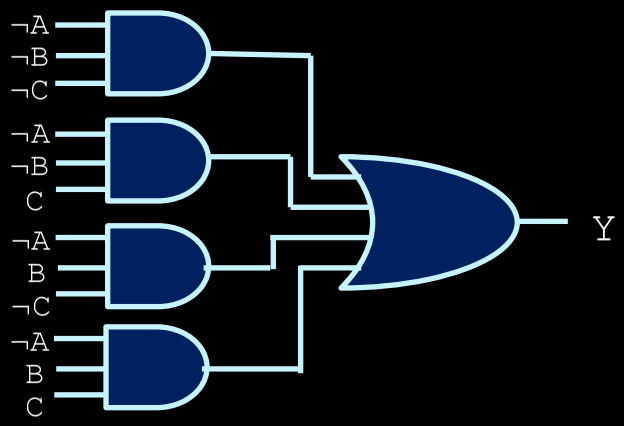
Maxterm	Y
M_0	0
M_1	1
M_2	1
M_3	1
M_4	1
M_5	0
M_6	1
M_7	0

Converting SOM to gates

- Once you have a Sum-of-Minterms expression, it is easy to convert this to the equivalent combination of gates:

$$m_0 + m_1 + m_2 + m_3$$

$$\bar{A}\cdot\bar{B}\cdot\bar{C} + \bar{A}\cdot\bar{B}\cdot C + \bar{A}\cdot B\cdot\bar{C} + \bar{A}\cdot B\cdot C$$



Question 2

- Write Y in SOM (Sum Of Minterms) form.

$$Y = \bar{A}\cdot\bar{B}\cdot\bar{C} + \bar{A}\cdot B\cdot\bar{C} + A\cdot B\cdot\bar{C} + A\cdot B\cdot C$$

$$Y = m_1 + m_2 + m_4 + m_7$$

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Question 3

- Given the minterms below, can you fill in the truth table on the right?

$$Y = m_2 + m_3 + m_7 + m_9 + m_{12} + m_{14}$$

A	B	C	D	Y
0	0	0	0	
0	0	0	1	
0	0	1	0	
0	0	1	1	
0	1	0	0	
0	1	0	1	
0	1	1	0	
0	1	1	1	
1	0	0	0	
1	0	0	1	
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0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	1
1	1	1	1	0

Question 4

- What is the POM form of the function from the truth table on the right?

$$Y = M_3 M_4 M_6 M_{11} M_{12} M_{14}$$

A	B	C	D	Y
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	1
1	1	1	0	0
1	1	1	1	1