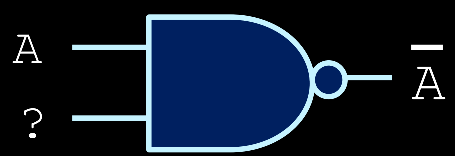
Week 2 Review

Q1

- How can you implement a NOT gate from a 2-input NAND gate?



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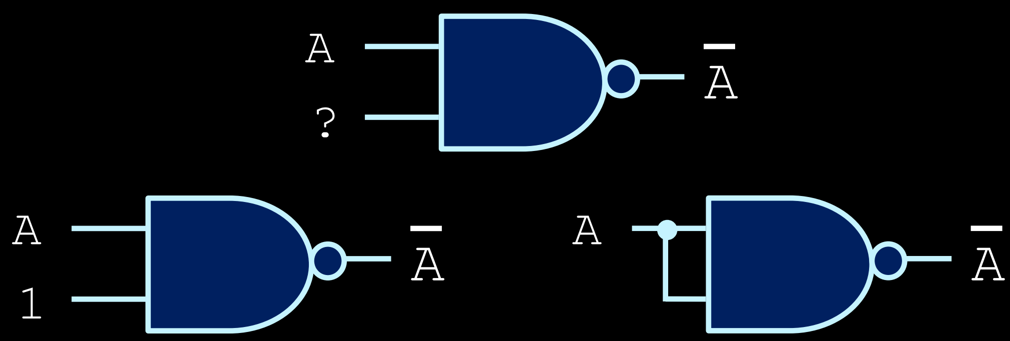


- What about NOT from NOR?

Boolean Algebra

- Use algebraic identities to reduce circuits

$\overline{x}\cdot\overline{y} = \overline{x+y}$

$x+y = x\cdot y$

$x\cdot(x+y) = x \qquad x+(x\cdot y) = x$

$x\cdot(y+z) = x\cdot y + x\cdot z$

$x+(y\cdot z) = (x+y)\cdot(x+z)$

$x+(\overline{x}\cdot y) = x+y \qquad x\cdot(\overline{x}+y) = x\cdot y$

$x\cdot y + \overline{x}\cdot z + y\cdot z = x\cdot y + \overline{x}\cdot z$

Reducing Boolean expressions

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

- Using SOM:

$$Y = \overline{A}\cdot B\cdot C + A\cdot \overline{B}\cdot \overline{C} + A\cdot B\cdot \overline{C} + A\cdot B\cdot C$$

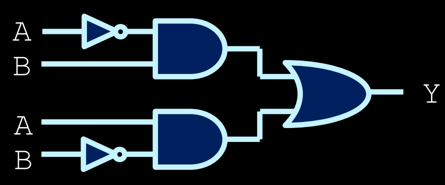
- Now start combining terms, like the last two:

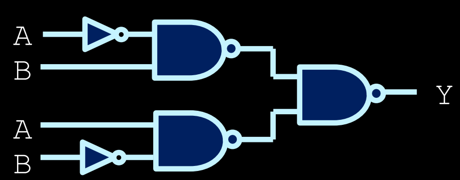
$$Y = \overline{A}\cdot B\cdot C + A\cdot \overline{B}\cdot \overline{C} + \mathbf{A\cdot B}$$

Question 2

- Implement a two-input XOR gate using only NAND and NOT gates
  - Draw the circuit using only these two logic gates.

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0





- Remember De Morgan's!
  - (W + Z) = (W Z)

Karnaugh map review

	$\overline{B}\cdot\overline{C}$	$\overline{B}\cdot C$	$B\cdot C$	$B\cdot\overline{C}$
$\overline{A}$	0	0	1	0
A	1	0	1	1

- K-maps provide an illustration of a circuit's minterms (or maxterms), and a guide to how neighbouring terms may be combined.

$$Y = \overline{A}\cdot B\cdot C + A\cdot\overline{B}\cdot\overline{C} + A\cdot B\cdot\overline{C} + A\cdot B\cdot C$$
$$= B\cdot C + A\cdot\overline{C}$$

Karnaugh map example

- Create a circuit with four inputs (A, B, C, D), and two outputs (X, Y):
  - The output X is high whenever two or more of the inputs are high.
  - The output Y is high when three or more of the inputs are high.

A	B	C	D	X	Y
0	0	0	0		
0	0	0	1		
0	0	1	0		
0	0	1	1		
0	1	0	0		
0	1	0	1		
0	1	1	0		
0	1	1	1		
1	0	0	0		
1	0	0	1		
1	0	1	0		
1	0	1	1		
1	1	0	0		
1	1	0	1		
1	1	1	0		
1	1	1	1		

Karnaugh map example

X:

	$\bar{C}\cdot\bar{D}$	$\bar{C}\cdot D$	$C\cdot D$	$C\cdot\bar{D}$
$\bar{A}\cdot\bar{B}$	0	0	1	0
$\bar{A}\cdot B$	0	1	1	1
$A\cdot B$	1	1	1	1
$A\cdot\bar{B}$	0	1	1	1

$$X = A\cdot B + C\cdot D + B\cdot D + B\cdot C + A\cdot D + A\cdot C$$

Alternative for X: Maxterms

X:

	$\bar{C}\cdot\bar{D}$	$\bar{C}\cdot D$	$C\cdot D$	$C\cdot\bar{D}$
$\bar{A}\cdot\bar{B}$	0	0	1	0
$\bar{A}\cdot B$	0	1	1	1
$A\cdot B$	1	1	1	1
$A\cdot\bar{B}$	0	1	1	1

Alternative for X: Maxterms

X:

	$C+D$	$C+\overline{D}$	$\overline{C}+\overline{D}$	$\overline{C}+D$
$A+B$	0	0	1	0
$A+\overline{B}$	0	1	1	1
$\overline{A}+\overline{B}$	1	1	1	1
$\overline{A}+B$	0	1	1	1

$X = (A+C+D) \cdot (B+C+D) \cdot (A+B+C) \cdot (A+B+D)$

Karnaugh map example

Y:

	$\overline{C} \cdot \overline{D}$	$\overline{C} \cdot D$	$C \cdot D$	$C \cdot \overline{D}$
$\overline{A} \cdot \overline{B}$	0	0	0	0
$\overline{A} \cdot B$	0	0	1	0
$A \cdot B$	0	1	1	1
$A \cdot \overline{B}$	0	0	1	0

$Y = A \cdot B \cdot D + B \cdot C \cdot D + A \cdot B \cdot C + A \cdot C \cdot D$

Question 3

- What is the most reduced sum of products form of the function from the truth table on the right?

$$Y = m_0 + m_1 + m_2 + m_5 + m_7 + m_8 + m_9 + m_{10} + m_{13} + m_{15}$$

A	B	C	D	Y
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	1
1	1	1	0	0
1	1	1	1	1

Question 3 (cont'd)

$$Y = m_0 + m_1 + m_2 + m_5 + m_7 + m_8 + m_9 + m_{10} + m_{13} + m_{15}$$

	$\bar{C}\cdot\bar{D}$	$\bar{C}\cdot D$	$C\cdot D$	$C\cdot\bar{D}$
$\bar{A}\cdot\bar{B}$	m_0	m_1	m_3	m_2
$\bar{A}\cdot B$	m_4	m_5	m_7	m_6
$A\cdot B$	m_{12}	m_{13}	m_{15}	m_{14}
$A\cdot\bar{B}$	m_8	m_9	m_{11}	m_{10}

Question 3 (cont'd)

$$Y = m_0 + m_1 + m_2 + m_5 + m_7 + m_8 + m_9 + m_{10} + m_{13} + m_{15}$$

	$\bar{C}\cdot\bar{D}$	$\bar{C}\cdot D$	$C\cdot D$	$C\cdot\bar{D}$
$\bar{A}\cdot\bar{B}$	1	1	0	1
$\bar{A}\cdot B$	0	1	1	0
$A\cdot B$	0	1	1	0
$A\cdot\bar{B}$	1	1	0	1

Question 3 (cont'd)

$$Y = m_0 + m_1 + m_2 + m_5 + m_7 + m_8 + m_9 + m_{10} + m_{13} + m_{15}$$

	$\bar{C}\cdot\bar{D}$	$\bar{C}\cdot D$	$C\cdot D$	$C\cdot\bar{D}$
$\bar{A}\cdot\bar{B}$	1	1	0	1
$\bar{A}\cdot B$	0	1	1	0
$A\cdot B$	0	1	1	0
$A\cdot\bar{B}$	1	1	0	1

$$Y = \bar{C}\cdot D + B\cdot D + \bar{B}\cdot\bar{D}$$

Question 3 (alternative)

- An alternative grouping:

	$\bar{C}\cdot\bar{D}$	$\bar{C}\cdot D$	$C\cdot D$	$C\cdot\bar{D}$
$\bar{A}\cdot\bar{B}$	1	1	0	1
$\bar{A}\cdot B$	0	1	1	0
$A\cdot B$	0	1	1	0
$A\cdot\bar{B}$	1	1	0	1

$$Y = \bar{B}\cdot\bar{C} + B\cdot D + \bar{B}\cdot\bar{D}$$