Var 计算公式.

Formula Sheet

$E(X^2) = V(X) + E(X)^2$

$\Rightarrow V(X) = E(X^2) - [E(X)]^2$

1. Distributions:

Distribution	pdf or pmf	mean	variance	MGF
Bernoulli(θ)	$\theta^x(1-\theta)^{1-x}$	θ	$\theta(1-\theta)$	$(1-\theta)+\theta e^t$
Binomial (m,θ)	$\binom{m}{x}\theta^x(1-\theta)^{m-x}$	$m\theta$	$m\theta(1-\theta)$	$[(1-\theta)+\theta e^t]^m$
Poisson(λ)	$\frac{e^{-\lambda}\lambda^x}{x!}$	λ	λ	$exp[\lambda(e^t-1)]$
Uniform$[a,b]$	$1/(b-a)$	$(a+b)/2$	$(b-a)^2/12$	$\begin{cases}(e^{tb}-e^{ta})/t(b-a) & ,t\neq 0\\ 1 & ,t=0\end{cases}$
Normal (μ,σ^2)	$(2\pi\sigma^2)^{-1/2}exp[-\frac{1}{2\sigma^2}(x-\mu)^2]$	μ	σ^2	$exp[\mu t+\sigma^2t^2/2]$
Exponential(β)	$\beta e^{-\beta x}$	$1/\beta$	$1/\beta^2$	$(1-t/\beta)^{-1}$
Exponential(θ)	$\frac{1}{\theta}e^{-\frac{x}{\theta}}$	θ	θ^2	$(1-t\theta)^{-1}$
Gamma(α,β)	$\frac{\beta^\alpha}{\Gamma(\alpha)}x^{\alpha-1}e^{-\beta x}$	α/β	α/β^2	$(1-t/\beta)^{-\alpha}, t<\beta$
Beta(a,b)	$\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}x^{a-1}(1-x)^{b-1}$	$a/(a+b)$	$\frac{ab}{(a+b)^2(a+b+1)}$	$1+\sum_{k=1}^{\infty}\left(\prod_{r=0}^{k-1}\frac{\alpha+r}{\alpha+\beta+r}\right)\frac{t^k}{k!}$

2. LLN: $\frac{1}{n}\sum_i X_i \xrightarrow{P} E[X_i]$; CLT: $\bar{X}_n \xrightarrow{D} N(\mu,\frac{\sigma^2}{n})$; $\frac{Z}{\sqrt{\chi^2_{(m)}/m}} \sim t_{(m)}$; $\frac{\chi^2_{(m)}/m}{\chi^2_{(n)}/n} \sim F_{(m,n)}$; $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{(n-1)}$

3. Method of Moments: $\frac{1}{n}\sum_i X_i^k$ is an estimator of $E[X^k]$

$T = \frac{\bar{X}-\mu}{S/\sqrt{n}} \sim t_{n-1}$

4. Maximum Likelihood Estimation:

$E(\chi^2\ df=m) = m$

$V(\chi^2\ df=m) = 2m$

- Log-likelihood, $l(\theta) = \log\prod_i f_\theta(x_i) = \sum_i \log f_\theta(x_i)$
- Score, $l'(\theta) = \frac{\partial}{\partial\theta}l(\theta)$
- At mle $(\hat{\theta})$, $l'(\theta) = 0$
- Fisher Info for single obs, $I(\theta_0) = E[\{\frac{\partial}{\partial\theta}\log f(X|\theta)\big|_{\theta=\theta_0}\}^2] = -E[\frac{\partial^2}{\partial\theta^2}\log f(X|\theta)\big|_{\theta=\theta_0}]$ $= -E(l''(\theta_0))$
- Fisher info for n obs, $nI(\theta_0) = E[\{\frac{\partial}{\partial\theta}\log f(X_1,X_2,...,X_n|\theta)\big|_{\theta=\theta_0}\}^2] = -E[\frac{\partial^2}{\partial\theta^2}\log f(X_1,X_2,...,X_n|\theta)\big|_{\theta=\theta_0}]$
- $l'(\theta_0) \xrightarrow{D} N(0,nI(\theta_0))$; $\frac{1}{n}l''(\theta_0) \xrightarrow{P} -I(\theta_0)$; $\hat{\theta} \xrightarrow{D} N(\theta_0,\frac{1}{nI(\theta_0)})$ $V(\hat{\theta}_{MLE}) = \frac{1}{-l''(\hat{\theta}_{MLE})}$
- Plug-in est. of Fisher Info, $nI(\hat{\theta}) = -E[\frac{\partial^2}{\partial\theta^2}\log f(X_1,X_2,...,X_n|\theta)]\big|_{\theta=\hat{\theta}}$

5. $Bias[T] = E[T]-\theta$; $MSE[T] = Var[T]+(Bias[T])^2$; $eff[T_1,T_2] = \frac{V[T_2]}{V[T_1]}$; C-R Inequality: $V[T] \geq \frac{1}{nI(\theta_0)}$

6. Consistency in probability, $T_n \xrightarrow{P} \theta$ as $n\to\infty$ invariance property.

7. Factorization theorem: $T(X_1,X_2,...X_n)$ is said to be sufficient for θ if the joint probability function factors in the form, $f(x_1,x_2,...x_n|\theta) = g[T(x_1,x_2,...x_n),\theta]*h(x_1,x_2,...x_n)$

$\bar{X} \sim N(0,1)$

8. Formula related to confidence intervals: (σ^2 unknown) $\to \sigma^2$

$\bar{X} \pm z_{\frac{(1+\gamma)}{2}}\frac{\sigma}{\sqrt{n}}$; $\bar{X} \pm t_{\frac{(1+\gamma)}{2}(df=n-1)} * \frac{S}{\sqrt{n}}$; $\left(\frac{(n-1)S^2}{\chi^2_{\frac{1+\gamma}{2}(n-1)}}, \frac{(n-1)S^2}{\chi^2_{\frac{1-\gamma}{2}(n-1)}}\right)$; $\hat{\theta} \pm z_{\frac{(1+\gamma)}{2}}\sqrt{\frac{1}{nI(\hat{\theta})}}$;

$\bar{X} \sim N(\bar{X}, \frac{\bar{X}(1-\bar{X})}{n})$ $\bar{X} \pm z_{\frac{(1+\gamma)}{2}}\sqrt{\frac{\bar{X}(1-\bar{X})}{n}}$ P μ $S \to$ estimate s

9. Margin of Error, $ME = z_{\frac{(1+\gamma)}{2}}\frac{\sigma}{\sqrt{n}}$; $width = 2*ME$; for proportion, $ME = z_{\frac{(1+\gamma)}{2}}\sqrt{\frac{\bar{X}(1-\bar{X})}{n}}$

10. Definitions related to test of hypothesis:

- P[Type-1 error]$= \alpha = P$[reject $H_0|H_0$ true]
- P[Type-2 error]$= \beta = P$[fail to reject $H_0|H_0$ false]
- Power of a test$= 1-\beta = P$[reject $H_0|H_0$ false]

for μ.

11. Some p-value formula:

- for z-test $2\left[1-\Phi\left(\left|\frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}\right|\right)\right]$ where Φ is the CDF of a standard normal distribution. → 已知 σ
- for t-test $2\left[1-G\left(\left|\frac{\bar{x}-\mu_0}{s/\sqrt{n}}\right|\right)\right]$ where G is the CDF of a $t_{(n-1)}$ distribution. → 未知用 $s \xrightarrow{\text{estimate.}} \sigma$
- One sided test:
  - When testing $H_0: \mu=\mu_0$ against $H_1: \mu>\mu_0$, $p-value = 1-\Phi\left(\frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}\right)$
  - When testing $H_0: \mu=\mu_0$ against $H_1: \mu<\mu_0$, $p-value = \Phi\left(\frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}}\right)$
  - Similar idea for t, χ^2 or other tests.

$\sigma^2=\sigma_0^2$

$\frac{(n-1)S^2}{\sigma_0^2}$

$\sim \chi^2_{df=(n-1)}$

12. Comparing two populations: (test stats and distributions)

for two variance

a) $\frac{S_x^2/\sigma_x^2}{S_y^2/\sigma_y^2} \sim F_{n-1,m-1}$; ?

b) $\frac{(\bar{X}-\bar{Y})-(\mu_x-\mu_y)}{\sqrt{\frac{\sigma_x^2}{n}+\frac{\sigma_y^2}{m}}} \sim N(0,1)$ σ_x^2, σ_y^2 known

c) $\frac{(\bar{X}-\bar{Y})-(\mu_x-\mu_y)}{S_p\sqrt{(\frac{1}{n}+\frac{1}{m})}} \sim t_{(n+m-2)}$ where, $S_p^2=\frac{(n-1)S_x^2+(m-1)S_y^2}{n+m-2}$ $= s \to \sigma.$

$\sigma_x^2=\sigma_y^2=\sigma^2$ unknow

d) $\frac{(\bar{X}-\bar{Y})-(\mu_x-\mu_y)}{\sqrt{\frac{S_x^2}{n}+\frac{S_y^2}{m}}} \to t_{(df=v)}$ where, $v=\frac{(\frac{S_x^2}{n}+\frac{S_y^2}{m})^2}{\frac{(S_x^2/n)^2}{n-1}+\frac{(S_y^2/m)^2}{m-1}}$

$\sigma_x^2 \neq \sigma_y^2$ unknow

13. Likelihood ratio test, $-2ln\Lambda \xrightarrow{D} \chi^2_{(df=p-d)}$, where $\Lambda=\frac{max_{\theta\in\Omega_0}[L(\theta)]}{max_{\theta\in\Omega}[L(\theta)]}=\frac{L(\theta_0)}{L(\hat{\theta})}$

14. Wald test, $\frac{\hat{\theta}-\theta_0}{\sqrt{\frac{1}{-l''(\hat{\theta})}}} \xrightarrow{D} N(0,1)$; Score test, $\frac{S(\theta_0)}{\sqrt{nI(\theta_0)}} \xrightarrow{D} N(0,1)$

15. Goodness of fit test (known p_i), $X^2=\sum_{i=1}^k \frac{(X_i-np_i)^2}{np_i} \xrightarrow{D} \chi^2_{(df=k-1)}$

16. Goodness of fit test (unknown p_i), $X^2=\sum_{i=1}^k \frac{(X_i-np_i(\hat{\theta}))^2}{np_i(\hat{\theta})} \xrightarrow{D} \chi^2_{(df=k-1-dim(\Omega))}$

17. Posterior $\propto$ Likelihood * Prior $\implies \pi(\theta|s) \propto \prod f(x_i|\theta) * \pi(\theta)$

18. Regression:

$Y_i \sim \beta_1+\beta_2 x+\varepsilon_i$.

- $b_1=\bar{y}-b_2\bar{x}$ *and* $b_2=\frac{\sum_{i=1}^n(x_i-\bar{x})(y_i-\bar{y})}{\sum_{i=1}^n(x_i-\bar{x})^2}$
- $\frac{B_2-\beta_2}{\sqrt{\frac{S^2}{\sum_{i=1}^n(x_i-\bar{x})^2}}} \sim t_{(n-2)}$ where, $S^2=\frac{1}{n-2}\sum_{i=1}^n(Y_i-b_1-b_2x_i)^2$ → 用作 t test B_2

  if $\beta_2=0$ (X, Y no relationship)
- CI for β_2: $B_2 \pm t_{\frac{(1+\gamma)}{2}(df=n-2)} * \sqrt{\frac{S^2}{\sum_{i=1}^n(x_i-\bar{x})^2}}$
- Total SS $=\sum_{i=1}^n(y_i-\bar{y})^2$; Reg. SS $=b_2^2\sum_{i=1}^n(x_i-\bar{x})^2$; Error SS $=\sum_{i=1}^n(y_i-b_1-b_2x_i)^2$
- Total SS = Reg. SS + Error SS
- $R^2=\frac{Reg.\ SS}{Total\ SS}$

19. Some quantiles from z, t and χ^2 table:

$z_{0.05}=-1.644854$	$z_{0.975}=1.959964$	$z_{0.1}=-1.281552$	$\chi^2_{0.95(1)}=3.841459$
$\chi^2_{0.95(2)}=5.991465$	$\chi^2_{0.95(3)}=7.814728$	$\chi^2_{0.05(4)}=0.710723$	$\chi^2_{0.9(1)}=2.705543$
$t_{0.05(2)}=-2.919986$	$t_{0.975(2)}=4.302653$	$t_{0.95(4)}=2.131847$	$t_{0.95(36)}=1.688298$
$F_{0.05(19,17)}=0.4550151$	$F_{0.95(19,17)}=2.242891$		