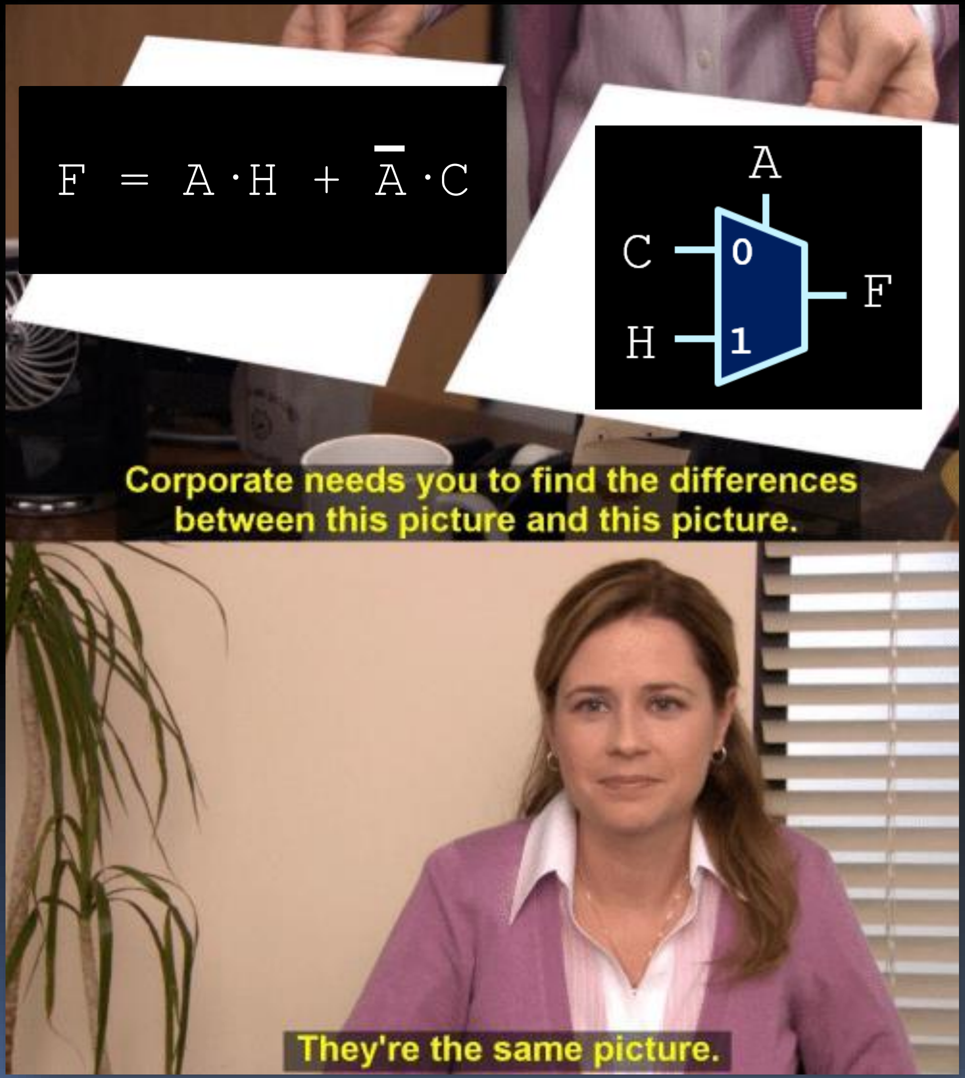
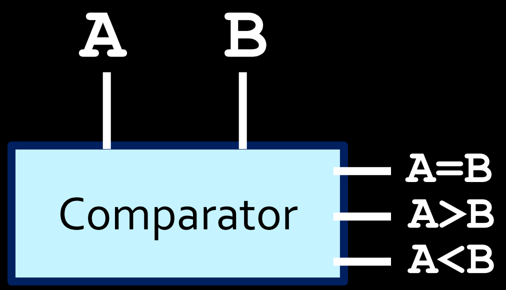
Week 3 Part E: Comparators



Comparators

- A circuit that takes in two input vectors, and determines if the first is greater than, less than or equal to the second.
- How does one make such a circuit?

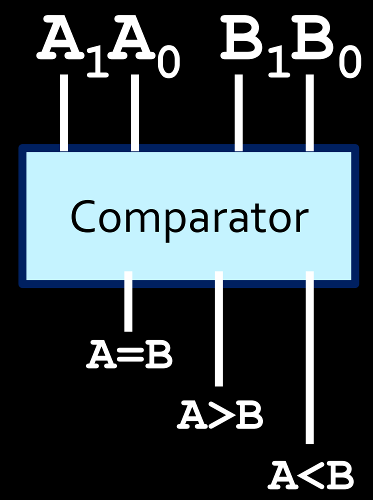
Basic Comparators



- Consider two one-but binary numbers A and B.
- The circuits for this would be:
  - A==B: $A \cdot B + \overline{A} \cdot \overline{B}$
  - A>B: $A \cdot \overline{B}$
  - A<B: $\overline{A} \cdot B$

A	B	
0	0	==
0	1	<
1	0	>
1	1	==

Basic Comparators

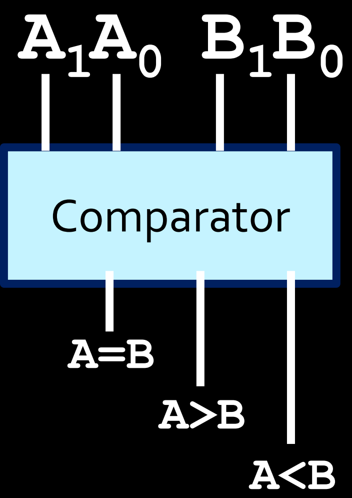


- What if A and B are two bits long?
- The terms for this circuit for have to expand to reflect the additional bit.
- For example:
  - A==B: $\boxed{(A_1 \cdot B_1 + \overline{A}_1 \cdot \overline{B}_1) \cdot (A_0 \cdot B_0 + \overline{A}_0 \cdot \overline{B}_0)}$

Make sure that the values of bit 1 are the same

Make sure that the values of bit 0 are the same

Basic Comparators



- What about checking if A is greater or less than B?
  - A>B: $$A_1 \cdot \overline{B}_1 + (A_1 \cdot B_1 + \overline{A}_1 \cdot \overline{B}_1) \cdot (A_0 \cdot \overline{B}_0)$$

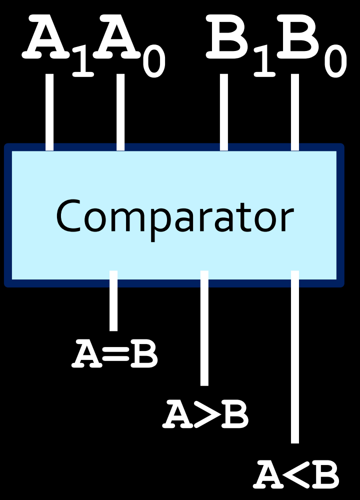
Check if first bit satisfies condition

If not, check that the first bits are equal…

…and then do the 1-bit comparison

  - A<B: $$\overline{A}_1 \cdot B_1 + (A_1 \cdot B_1 + \overline{A}_1 \cdot \overline{B}_1) \cdot (\overline{A}_0 \cdot B_0)$$

Basic Comparators



- The final circuit equations for two-input comparators are shown below.
  - Note the sections they have in common!
  - A==B: $\boxed{(A_1 \cdot B_1 + \overline{A}_1 \cdot \overline{B}_1)} \cdot \boxed{(A_0 \cdot B_0 + \overline{A}_0 \cdot \overline{B}_0)}$
  - A>B: $A_1 \cdot \overline{B}_1 + \boxed{(A_1 \cdot B_1 + \overline{A}_1 \cdot \overline{B}_1)} \cdot (A_0 \cdot \overline{B}_0)$
  - A<B: $\overline{A}_1 \cdot B_1 + \boxed{(A_1 \cdot B_1 + \overline{A}_1 \cdot \overline{B}_1)} \cdot (\overline{A}_0 \cdot B_0)$

General Comparators

- The general circuit for comparators requires you to define equations for each case.
- <u>Case #1:</u> Equality
  - If inputs A and B are equal, then all bits must be the same.
  - Define X_i for any digit i:
    - (equality for digit i)

$$X_i = A_i \cdot B_i + \overline{A}_i \cdot \overline{B}_i$$

  - Equality between A and B is defined as:

$$A{=}{=}B \; : \; X_0 \cdot X_1 \cdot \ldots \cdot X_n$$

What about A > B, A < B?

- Suppose we have two 6-bit numbers.
- Suppose $A_5=B_5$ and $A_4=B_4$
  - This means $X_5=1$, $X_4=1$
- If $A_3=1$ and $B_3=0$ then $A > B$
  - Regardless of digits 2,1,0.
- If $A_3=0$ and $B_3=1$ then $A < B$
  - Regardless of digits 2,1,0.
- If $A_3=B_3$ the we need to check digit 2…
  - Note $A_3=B_3$ means $X_3=1$

Generalize!

- <u>Case #2:</u> $A > B$
  - The first non-matching bits occur at bit i, where $A_i = 1$ and $B_i = 0$. All higher bits match.
  - Using the definition for X_i from before:

$$A>B \;=\; A_n \cdot \overline{B}_n \;+\; X_n \cdot A_{n-1} \cdot \overline{B}_{n-1} \;+\; \ldots \;+\; A_0 \cdot \overline{B}_0 \cdot \prod_{k=1}^{n} X_k$$

- <u>Case #3:</u> $A < B$
  - The first non-matching bits occur at bit i, where $A_i = 0$ and $B_i = 1$. Again, all higher bits match.

$$A<B \;=\; \overline{A}_n \cdot B_n \;+\; X_n \cdot \overline{A}_{n-1} \cdot B_{n-1} \;+\; \ldots \;+\; \overline{A}_0 \cdot B_0 \cdot \prod_{k=1}^{n} X_k$$

Example for 4 bits

A=B

$$A{=}B = X_3 \cdot X_2 \cdot X_1 \cdot X_0$$

$$X_i = A_i \cdot B_i + \overline{A}_i \cdot \overline{B}_i$$

A > B

$$A_3 \cdot \overline{B}_3 + X_3 \cdot A_2 \cdot \overline{B}_2 + X_3 \cdot X_2 \cdot A_1 \cdot \overline{B}_1 + X_3 \cdot X_2 \cdot X_1 \cdot A_0 \cdot \overline{B}_0$$

A< B

$$\overline{A}_3 \cdot B_3 + X_3 \cdot \overline{A}_2 \cdot B_2 + X_3 \cdot X_2 \cdot \overline{A}_1 \cdot B_1 + X_3 \cdot X_2 \cdot X_1 \cdot \overline{A}_0 \cdot B_0$$

Comparator truth table

- Given two input vectors of size `n=2`, output of circuit is shown at right.

Inputs				Outputs		
A_1	A_0	B_1	B_0	`A < B`	`A = B`	`A > B`
0	0	0	0	0	1	0
0	0	0	1	1	0	0
0	0	1	0	1	0	0
0	0	1	1	1	0	0
0	1	0	0	0	0	1
0	1	0	1	0	1	0
0	1	1	0	1	0	0
0	1	1	1	1	0	0
1	0	0	0	0	0	1
1	0	0	1	0	0	1
1	0	1	0	0	1	0
1	0	1	1	1	0	0
1	1	0	0	0	0	1
1	1	0	1	0	0	1
1	1	1	0	0	0	1
1	1	1	1	0	1	0

Comparator example (cont'd)

A<B:

	$\bar{B}_0 \cdot \bar{B}_1$	$B_0 \cdot \bar{B}_1$	$B_0 \cdot B_1$	$\bar{B}_0 \cdot B_1$
$\bar{A}_0 \cdot \bar{A}_1$	0	1	1	1
$A_0 \cdot \bar{A}_1$	0	0	1	1
$A_0 \cdot A_1$	0	0	0	0
$\bar{A}_0 \cdot A_1$	0	0	1	0

$$LT = B_1 \cdot \bar{A}_1 + B_0 \cdot B_1 \cdot \bar{A}_0 + B_0 \cdot \bar{A}_0 \cdot \bar{A}_1$$

Comparator example (cont'd)

A=B:

	$\overline{B_0}\cdot\overline{B_1}$	$B_0\cdot\overline{B_1}$	$B_0\cdot B_1$	$\overline{B_0}\cdot B_1$
$\overline{A_0}\cdot\overline{A_1}$	1	0	0	0
$A_0\cdot\overline{A_1}$	0	1	0	0
$A_0\cdot A_1$	0	0	1	0
$\overline{A_0}\cdot A_1$	0	0	0	1

$$EQ = \overline{B_0}\cdot\overline{B_1}\cdot\overline{A_0}\cdot\overline{A_1} + B_0\cdot\overline{B_1}\cdot A_0\cdot\overline{A_1} + B_0\cdot B_1\cdot A_0\cdot A_1 + \overline{B_0}\cdot B_1\cdot\overline{A_0}\cdot A_1$$

Comparator example (cont'd)

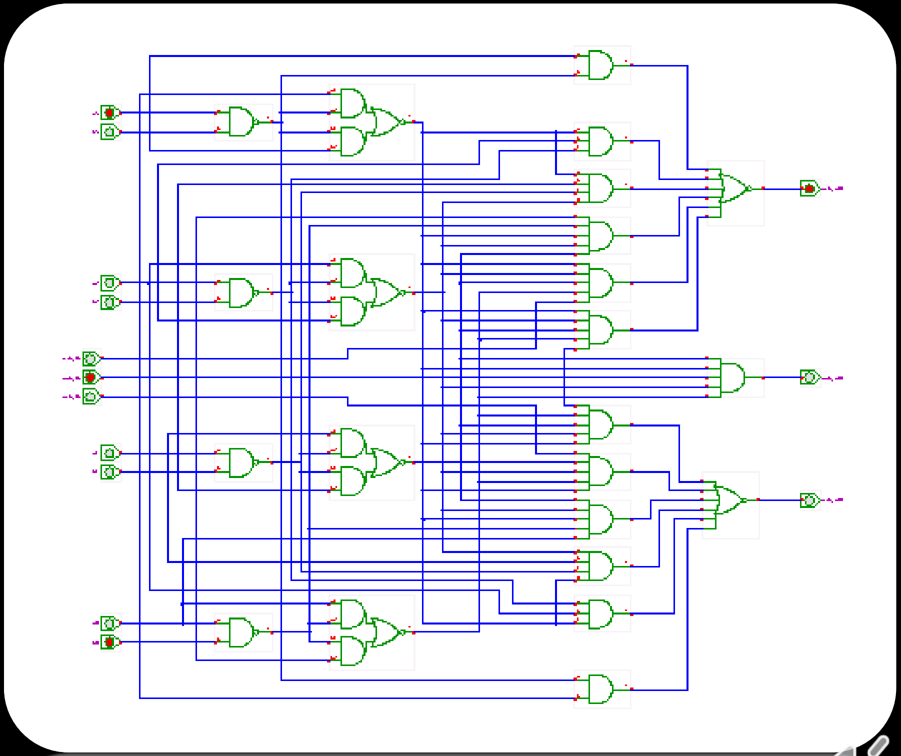
A>B:

	$\overline{B}_0 \cdot \overline{B}_1$	$B_0 \cdot \overline{B}_1$	$B_0 \cdot B_1$	$\overline{B}_0 \cdot B_1$
$\overline{A}_0 \cdot \overline{A}_1$	0	0	0	0
$A_0 \cdot \overline{A}_1$	1	0	0	0
$A_0 \cdot A_1$	1	1	0	1
$\overline{A}_0 \cdot A_1$	1	1	0	0

$$GT = \overline{B}_1 \cdot A_1 + \overline{B}_0 \cdot \overline{B}_1 \cdot A_1 + \overline{B}_0 \cdot A_0 \cdot A_1$$

Comparing larger numbers

- As numbers get larger, the comparator circuit gets more complex.
- For large numbers, it can be easier to simply subtract and check the sign of the result.
  - Easier, less circuitry, just not faster.



A Summary

- We learned several combinatorial devices:
  - Control the flow of a signal: Mux, demux
  - Translate signals, control output: decoder
  - Arithmetic: adder, subtractor, comparator.

- Next week we'll work with sequential circuits: circuits that have memory.