Question-1 [5 points]: Suppose you are testing whether light bulbs produced by two competing companies are equivalent in terms of length of life. You have 10 observations from the first company $(X_1, X_2, \dots, X_{10})$ and 15 observations from the second company $(Y_1, Y_2, \dots, Y_{15})$. You can assume the observations are all independent and drawn from Normal distribution. The table below summarizes the observed samples.

	sample size	mean	sd
X	10	10.48	2.57
Y	15	9.30	4.50

a. [2 points] Test $H_0: \sigma_x^2 = \sigma_y^2$ vs. $H_a: \sigma_x^2 \neq \sigma_y^2$ at $\alpha = 0.05$. What conclusion do you make?

b. [3 points] By assuming the population variances to be equal, test $H_0: \mu_x = \mu_y$ vs. $H_a: \mu_x > \mu_y$ at $\alpha = 0.05$. What conclusion do you make?

Question-2 [5 points]: Suppose $X_1, X_2, \ldots, X_n$ are independently drawn from a Non-normal distribution with mean μ and variance σ^2.

By proving any required identity, show that $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$ is an unbiased estimator of σ^2.

Question-3 [6 points]: Suppose these following observations are independently drawn from a Poisson distribution with an unknown λ

$$3,5,1,3,6,6,10,4,5,4$$

For this sample, $\bar{x} = 4.7$

You are also told that, for Poisson $(\lambda), E[X_i] = \lambda$ and MLE of $\lambda = \bar{X}$.

a.[3 points] Considering the sample large enough, by showing all the necessary steps, Construct a 95% confidence interval for λ.

b. [3 points] Suppose you are testing $H_0: \lambda = 5$ vs $H_a: \lambda = 4$ for sample size, $n = 10$. At level of significance, $\alpha = 0.05$ using $\frac{\bar{X}-\lambda}{\sqrt{\lambda/n}} \xrightarrow{D} N(0,1)$ as the test statistic, calculate the power of your test. Leave your answer in terms of the CDF of standard normal distribution, $\Phi()$ or pnorm().

Question-4 [6 points]: Suppose these following 15 numbers $(Y_1, Y_2, \ldots, Y_{15})$ are randomly drawn from a distribution. The sorting was done after the random draw.

$$0.33, 0.49, 0.51, 0.56, 0.64, 0.66, 0.74, 1.11, 1.24, 1.36, 1.52, 2.21, 2.58, 2.73, 4.40$$

You are also told that $\bar{y} = 1.41$.
By using an appropriate test and converting the range of y to $(0,1)$, $[1,2)$ and $[2, \infty)$, test whether the samples are from an Exponential (θ) distribution.
hint: for Exp (θ), $F(y) = 1 - e^{-\frac{y}{\theta}}$; and MLE of θ is $\bar{Y}$.

Question-5 [6 points]: Suppose $(X_1, X_2, \ldots, X_n)$ are independently distributed as Exp (θ) with pdf

$$f(x \mid \theta) = \frac{1}{\theta} e^{-\frac{x}{\theta}}$$

$\frac{1}{\theta}$ is believed to be from a Gamma (α_0, β_0) distribution with pdf

$$\frac{\beta_0^{\alpha_0}}{\Gamma(\alpha_0)} \frac{1}{\theta^{(\alpha_0 - 1)}} e^{-\frac{\beta_0}{\theta}}$$

a. [3 points] By showing appropriate steps, find out the Posterior distribution of $\frac{1}{\theta}$.

b. [1.5 points] Calculate the posterior mean and the posterior variance of $\frac{1}{\theta}$.

c.[1.5 points] What type of prior have we used here? Propose a non-informative prior.

Question-6 [6 points]: Suppose

$$(X_1, X_2, \dots, X_n) \overset{iid}{\sim} \text{Bernoulli}\,(\theta_x) \text{ and } (Y_1, Y_2, \dots, Y_m) \overset{\text{iid}}{\sim} \text{Bernoulli}\,(\theta_y)$$

You are also told that X and Y are independent.

a. [4 points] Construct a likelihood ratio test for testing the hypothesis, $H_0: \theta_x = \theta_y$ with an alternative hypothesis, $H_a: \theta_x \neq \theta_y$ at significance level α. (This question is asking for three things: the test statistic, corresponding distribution and the rejection region). Simplify your test statistic as much as possible.

b. [2 points] Suppose these following two set of samples are believed to be from a Bernoulli (θ_x) and a Bernoulli (θ_y) distributions.

X	1,1,1,1,0,1,0,1,1,1	$\sum_{i=1}^{10} x_i = 8$
Y	0,1,1,0,0,1,0,1,0,1	$\sum_{i=1}^{10} y_i = 5$

Test $H_0: \theta_x = \theta_y$ vs. $H_a: \theta_x \neq \theta_y$ at $\alpha = 0.05$

Question-7 [6 points]: Suppose you are given a data set with 5 observations. The first two columns of this following table represent the given data.

x_i	y_i	$(x_i - \bar{x})^2$	$(y_i - \bar{y})^2$	$(x_i - \bar{x})(y_i - \bar{y})$
-2	6	4	12.96	7.2
-1	8	1	2.56	1.6
0	11	0	1.96	0.0
1	10	1	0.16	0.4
2	13	4	11.56	6.8
$Sum:0$	48	10	29.20	16.0

Suppose we are fitting this following regression model

$$(Y \mid X = x) \sim N(\beta_1 + \beta_2 x, \sigma^2)$$

a. [1.5 points] Calculate the maximum likelihood estimate of β_1 and β_2 and interpret there values. (you do not need to maximize the likelihood function)

b.[1.5 points] Calculate the value of the coefficient of determination and interpret the value.

c. [1.5 points] Test (at $\alpha = 0.05$) the hypothesis, $H_0: \beta_2 = 1$ vs $H_a: \beta_2 \neq 1$. Write your complete conclusion.

d.[1.5 points] Construct a 95% confidence interval for β_2 and interpret it.