

Q1. $MSE(T) = Bias^2(T) + Var(T)$

$$T_1 = \frac{x_1 + x_2 + x_3}{3} \sim N(\mu, \frac{\sigma^2}{3}).$$

$$Bias(T_1) = Bias\left(\frac{x_1 + x_2 + x_3}{3}\right)$$

$$= E\left(\frac{x_1 + x_2 + x_3}{3}\right) - \mu$$

$$= \frac{1}{3}E[x_1] + \frac{1}{3}E[x_2] + \frac{1}{3}E[x_3] - \mu..$$

$$= \mu - \mu = 0.$$

$$V[T_1] = V\left[\frac{x_1 + x_2 + x_3}{3}\right]$$

$$= \frac{3\sigma^2}{9} = \frac{\sigma^2}{3}$$

$$Var(T) = \frac{\sigma^2}{3}$$

$$\bar{x}_n \sim N(\mu, \frac{\sigma^2}{n})$$

$$\Rightarrow MSE(T_1) = \frac{\sigma^2}{3}$$

$$V[T_2] = V\left[\frac{1}{2}x_1 + \frac{1}{4}x_2 + \frac{1}{4}x_3\right]$$

$$= \frac{V[x_1]}{4} + \frac{V[x_2]}{16} + \frac{V[x_3]}{16}$$

$$= \frac{4\sigma^2 + \sigma^2 + \sigma^2}{16} = \frac{6\sigma^2}{16} = \frac{3\sigma^2}{8}$$

$$\Rightarrow MSE(T_2) = \frac{3\sigma^2}{8}$$

Q2.