TUT

TUT Exercise W1.

a). ① W.T.S: $\exists 1 \in C$

$a = 1 \cdot a$

$b = 1 \cdot b$

$\therefore$ by defn. 1 is the common divisor

$\therefore$ C is nonempty.

② finite.

Let A and B be the sets of divisors of a and b respectly.

$C \subseteq A$ and $C \subseteq B$

A is finite b/c there's finite divisors of a.

B is finite b/c there's finite divisors of b

$\therefore$ C is finite. b/c C is the subset of A.

b). d is a common divisor of a & b iff

d is a common divisor of a & (b-a).

($\Rightarrow$). W.t.s: $\exists z \in \mathbb{Z}$ s.t. $b-a = zd$.

let $m \in \mathbb{Z}$ s.t. $a = md$ by hypothesis.

and let $k \in \mathbb{Z}$ s.t. $b = kd$

$b - a = md - kd$

$= (m-k)d$

$m-k \in \mathbb{Z}$ b/c $m \in \mathbb{Z}, k \in \mathbb{Z}$.

thus, $\exists z = m-k \in \mathbb{Z}$ s.t. $b-a = zd$. as wanted

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$(\Leftarrow)$. w.t.s. $\exists\, z \in \mathbb{Z}$ s.t. $b = zd$

let $m \in \mathbb{Z}$ s.t. $a = md$.

let $k \in \mathbb{Z}$ s.t. $b - a = kd$.

$$
\begin{aligned}
& b - a = kd \\
\Leftrightarrow\ & b = a + kd \\
\Leftrightarrow\ & b = md + kd \\
\Leftrightarrow\ & b = (m+k)d
\end{aligned}
$$

Since $m, k \in \mathbb{Z}$, $m + k \in \mathbb{Z}$.

$\therefore \exists\, z = m + k \in \mathbb{Z}$ s.t. $b = zd$. as wanted.

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