

Kruskal's Algorithm.

prove $\left\{ \begin{array}{l} \text{first conditions} \\ \text{invariance.} \end{array} \right.$ from B36

- 1. each cluster is a tree
- 2. $T \subseteq T_{\min}$ for some MST T_{\min} .

In vertices in cluster is connected & no cycle
 \rightarrow reachable from the other

why important?

maintain the invariant for each loop.

$T \subseteq T_{\min}$ for some minimum spanning tree.

keep adding edges \rightarrow MST \nearrow .

line 4-7

assume (1), (2).

WIP = (1), (2). after line 7.

merging u . cluster & v . cluster

\downarrow

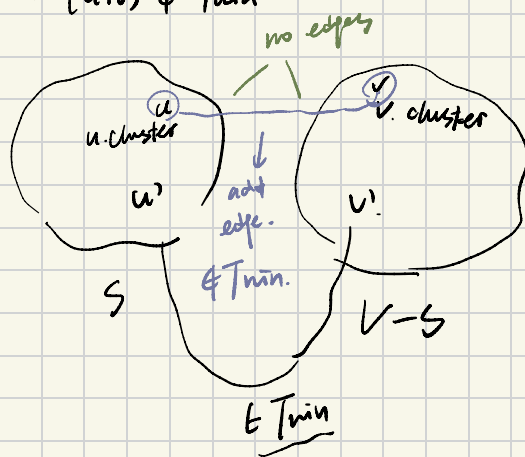
connect with no cycle & connected

\downarrow

a bigger cluster. \Leftarrow (1) holds.

1. $T \in T_{min}$ at line 4.

Case: sth change.
 $(u, v) \rightarrow (u, v) \in T_{min} \checkmark$
 $(u, v) \rightarrow (u, v) \notin T_{min}$



claim
there is a edge.

(u', v')

\forall all MST connect

\Downarrow
there must a way go from
 u to v

• in T_{min} , \exists ! simple path
from u to v .

• take out $(u', v') \rightarrow$ disconnected

• add $(u, v) \rightarrow$ reconnected.
 \Downarrow
get tree again.

• T_{min}' is minimum?

$weight(u, v) \leq$

$weight(u', v')$

\forall the way we store
edges.

