$$\mathcal{L}(p) = \prod_{i=1}^{n} (1-p)^{x_i} p$$

$$= (1-p)^{\sum x_i} p^n$$

$$nI(p) = \mathcal{I}_n(p)$$

$$\ell(p) = \log \mathcal{L}(p) = \log (1-p)^{\sum x_i} + n \log(p)$$

$$S(p) = \frac{\partial \ell(p)}{\partial p} = \frac{\sum x_i}{p-1} + \frac{n}{p}$$

$$\mathrm{Var}(S(p)) = \mathrm{Var}\left(\frac{\sum x_i}{p-1} + \frac{n}{p} \right)$$

$$= \frac{n \mathrm{Var}(x_i)}{(p-1)^2} = \frac{n \cdot \frac{1-p}{p^2}}{(1-p)^2} = \frac{n}{(1-p)p^2}$$

$$\| \quad nI(p)$$

Give an expression for CRLB

CRLB

$$\mathrm{Var}(T) \geq \frac{1}{nI(\theta)} = CRLB$$

(c). $\hat{p}_{MLE} \xrightarrow{d} N\left(p, \frac{1}{nI(p)}\right)$

$$\sqrt{n}\left(\hat{p}_{MLE} - p\right) \xrightarrow{d} N\left(0, \frac{1}{I(p)}\right)$$

$$\hat{p}_{MLE} \pm z_{0.975} \cdot \sqrt{\frac{1}{nI(\hat{p}_{MLE})}}$$

$$\hat{\theta}_{MLE} \xrightarrow{P} \theta$$

$$n \to \infty$$

$$Var(\hat{\theta}_{MLE}) = \frac{1}{nI(\theta)}$$