θ_i & λ_i

We can add weight factors before each (a_i, b_i) pair.
to do the regularization:

$$\Rightarrow d(\vec{a}, \vec{b}) = \sqrt{\sum_{i=1}^{p} (\theta_i a_i - \lambda_i b_i)^2}$$

$$= \left((w_1 \vec{a} - w_2 \vec{b})(w_1 \vec{a} - w_2 \vec{b}) \right)^{1/2}$$

$$= \left([\vec{a} \ \vec{b}] \begin{bmatrix} w_1 \\ -w_2 \end{bmatrix} [w_1 \ -w_2] \begin{bmatrix} \vec{a}^T \\ \vec{b}^T \end{bmatrix} \right)^{1/2}$$

$$= \underbrace{\left([\vec{a} \ \vec{b}] \begin{bmatrix} w_1^2 & -w_1 w_2 \\ -w_1 w_2 & w_2^2 \end{bmatrix} \begin{bmatrix} \vec{a}^T \\ \vec{b}^T \end{bmatrix} \right)^{1/2}}$$

$$w_1 \vec{a} - w_2 \vec{b}$$

$$= [w_1 \ -w_2] \begin{bmatrix} \vec{a}^T \\ \vec{b}^T \end{bmatrix}$$

$$= [\vec{a} \ \vec{b}] \begin{bmatrix} w_1 \\ -w_2 \end{bmatrix}$$

Consider if our input features $\vec{a}$ & $\vec{b}$ with different measurement.
we can use weight factors w_1 & w_2 to adjust the measurement to the same level.
For example, if $\vec{a}$ measured in centimeter and $\vec{b}$ measured in meter, we can take $w_1 = 1$ & $w_2 = 100$. thus the measurement of $\vec{b}$ (meter) will be transfer as the measurement of $\vec{a}$ (become centimeter)

$$d(a, b) = \sqrt{\sum (\theta_i a_i - \lambda_i b_i)^2}$$

$$\left([\theta_i \ -\lambda_i] \begin{bmatrix} a_i \\ b_i \end{bmatrix} \right)^2 \quad \text{where } \vec{c} = \begin{bmatrix} \theta_i \\ \lambda_i \end{bmatrix}$$

$$\left([\vec{a} \ \vec{b}] \begin{bmatrix} 1 \\ -1 \end{bmatrix} [1 \ -1] \begin{bmatrix} a^T \\ \vec{b}^T \end{bmatrix} \right)^{1/2}$$

$$= \left([\vec{a} \ \vec{b}] \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} \vec{a}^T \\ \vec{b}^T \end{bmatrix} \right)^{1/2}$$

$$[a_i \ b_i] \begin{bmatrix} \theta_i \\ -\lambda_i \end{bmatrix} [\theta_i \ -\lambda_i] \begin{bmatrix} a_i \\ b_i \end{bmatrix}$$

$$\sum_{i}^{p} \left([a_i \ b_i] \begin{bmatrix} \theta_i^2 & -\theta_i \lambda_i \\ -\theta_i \lambda_i & \lambda_i^2 \end{bmatrix} \begin{bmatrix} a_i \\ b_i \end{bmatrix} \right)^2$$