1-D Gaussian: pdf

$$X \sim N(\mu, \sigma^2)$$

D-Dim Gaussian: $X \sim N(\mu, \Sigma)$. $X \in \mathbb{R}^D$, $N \in \mathbb{R}^D$

$\Sigma \in \mathbb{R}^{D \times D}$

$$\mu = \begin{bmatrix} E(X_1) \\ \vdots \\ E(X_D) \end{bmatrix} \qquad \Sigma = E[(x-\mu)(x-\mu)^T)] \leftarrow D=2$$

$$= \begin{bmatrix} var(X_1) & cov(X_1, X_2) \\ cov(X_2, X_1) & var(X_2) \end{bmatrix}$$

$$p(x|\mu, \Sigma) = \frac{1}{(2\pi)^{d/2} |\Sigma|^{1/2}} \cdot exp\left(-\frac{1}{2}(x-\mu)^T \Sigma^{-1} (x-\mu)\right)$$

$$= G(X; N, \Sigma)$$

$$G(X; \mu, \Sigma) = \alpha$$

$$-\ln(G(X; \mu, \Sigma)) = -\ln(\alpha)$$

$$\Rightarrow = \frac{1}{2}(x-\mu)^T \Sigma^{-1} (x-\mu) = \underline{constant}.$$

$$\Sigma = USU^{-1}, \quad U = [v_1, \cdots, v_d], \quad S = \begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_D \end{bmatrix} \quad \Rightarrow \Sigma v_i = \lambda_i v_i$$

$$\frac{1}{(2\pi)^{d/2} (USU^T)} exp\left(-\frac{1}{2}(x-\mu)^T (USU^T)^{-1} (x-\mu)\right).$$

$|USU^T| = |U||S||U^T|. = \underline{|S|}$

prop: $\underline{orthonormal}$

$|A^{-1}| = \frac{1}{|A|}$

$U^{-1} = U^T$.

$(USU^T)^{-1} = US^{-1}U^T$.

$$\frac{1}{(2\pi)^{d/2} |S|} exp\left[-\frac{1}{2}[(x-\mu)^T U] S^{-1} [U^T (x-\mu)]\right])$$

$$((x-\mu)^T U)^T = U^T(x-\mu)$$

$$y = U^T(x-\mu)$$
$$= U^T x - U^T \mu$$
$$= Ax + b$$

$$\frac{1}{(2\pi)^{d/2}|S|} \exp\left(-\frac{1}{2} y^T S^{-1} y\right) = G(y; 0, S)$$

$$= \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi\lambda_i}} \exp\left(-\frac{1}{2} \cdot \frac{y^2}{\lambda_1}\right)$$

$$y = U^T(x-\mu).$$