Product Pricing with Consumer Learning

Jieyu Gao*
Department of Economics, University of California, Irvine

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Abstract

We analyze a price signaling game incorporating consumer learning. Initially, buyers are uninformed about the quality of the seller's product. We introduce a probability factor determining the likelihood of the seller's type being fully disclosed before trading. Optimal strategies for sellers are outlined. We employ the undefeated equilibrium refinement to determine optimal choices. Furthermore, our analysis includes comparative statics, investigating the impact of variations in the probability of type revelation, initial customer review scores, and the quality of both high and low-quality seller products on the expected return for sellers.

1 Introduction

Consumers frequently encounter challenges when trying to distinguish between the quality discrepancies among various brands. As a result, they tend to associate higher prices with superior quality. Nonetheless, the proliferation of the internet and online shopping platforms has empowered consumers to leverage customer reviews as a valuable resource for assessing product quality. In this study, we propose a model that delves into optimal pricing strategies within the context of consumer reviews, wherein customers interpret pricing as a signaling mechanism.

Many researchers found evidence supporting the positive correlation between price and product quality (Mastrobuoni, Peracchi and Tetenov, 2014). Bagwell and Riordan (1991) found that high and declining prices signal high-quality products within a dynamic game

*Email addresses: jieyug1@uci.edu

framework. Milgrom and Roberts (1986) presents a signaling game using both price and uninformative advertisement as signals. Wolinsky (1983) and Delacroix and Shi (2013) studied the price signaling game with endogenously chosen price and signal pairs. Our model extends this by incorporating the consumer's learning process from previous production outcomes. Additionally, in their model, the cost of production is higher if the product quality is higher. In our model, the cost of production is the same for both the high-quality and low-quality productions to eliminate this factor. In contrast to Bose et al. (2006) exploration of dynamic monopoly pricing where buyers learn from each other's purchase decisions, our model assumes buyers learn from the review scores. Bar-Isaac (2003) examined the influence of reputation on the longevity of sellers. Their model featured a mechanism where consumers determined the price of the product, with the seller having the option to either accept or decline the offer. In contrast, our model employs pricing as a signal of product quality. This model is more relevant to many real-world markets where sellers typically set prices prior to purchase.

In our model, a seller interacts with multiple buyers within a monopolistic setting. The seller determines the price of the product, with the constraint that only a single unit can be sold. Buyers possess limited information regarding the product's quality from this seller and rely on consumer reviews and posted prices to update their beliefs about the seller's quality.

Following a similar framework to the model outlined in Delacroix and Shi (2013), our model incorporates the possibility that, with a certain probability, consumers may acquire complete information about the seller's type during the second stage after the seller posts the price. This scenario mirrors real-world situations where the true quality of a product may become publicly known after its release, perhaps through product quality investigations or disclosures.

For the sake of simplicity and ease of analysis, we adopt a single time period setting within stage two of our model. While we acknowledge that this may not perfectly capture all real-world dynamics, it still offers valuable insights. We anticipate that future extensions of our model could relax this constraint by incorporating a continuous time setting, allowing for a more nuanced exploration of the dynamics over time.

We characterize our model within the context of asymmetric information, wherein the seller possesses perfect information regarding its own type, while buyers lack full information about the seller's type.

We adopt the lexicographically maximum sequential equilibrium (LMSE) or undefeated equilibrium to refine the multiple equilibria (Mailath, Okuno-Fujiwara and Postlewaite, 1993), which is widely used in the signaling games (see Taylor, 1999; Fishman and Hagerty, 2003; Jiang et al., 2016; Bajaj, 2018; Jiang and Yang, 2019; Wu, Zhang and Xie, 2020; Li, Tian and Zheng, 2021). This refinement allows us to select the most profitable equilibrium from the seller's perspective among all the equilibria.

This model finds application in numerous scenarios where buyers cannot observe the product's quality perfectly prior to purchase, and they rely on the price set by the seller as an indicator of quality. For example, with the widespread availability of the Internet, numerous review websites have emerged, offering evaluations and rankings for various products. Consumers can readily access consumer review scores for specific products and infer product quality by considering both the listed price and the scores they encounter.

The rest of the paper is organized as follows. In Section 2 we introduce our model settings and the equilibria and the refinement. In Section 3 we conduct a comparative analysis to examine the characteristics of the equilibrium. Section 4 offers concluding remarks.

2 Model

Consider a market comprising a single seller and multiple buyers. The seller is assumed to be risk-neutral and can fall into one of two categories: high-quality (denoted as H-type) or low-quality (denoted as L-type). We let S denote the seller. The marginal cost of producing the good is the same for both types of sellers, which is normalized to 0. The H-type seller can make a successful production with probability g and the L-type seller can make a successful production with probability b, where $1 > g > b > c > 0$.

Buyers' valuation of a success is 1 and of a failure is 0. Each buyer can consume at most one unit of the product. Each buyer is capable of consuming at most one unit of the product, and they are considered homogeneous and price-takers. We assume a single consumer in each period, with all buyers being rational. We use B to denote the buyers. The success or failure of the product can be determined after it has been produced and the trade is realized.

In a one-period market, the seller S is born with an initial common belief λ representing the probability that the seller is of the H-type. S has perfect information about their type, and subsequently set a price p for its product. The seller's type is fully revealed to the public with probability ϕ. The buyer B enters the market and observes the price set by S. Upon observing this price and other available information, B updates their belief regarding the seller's type denoted as μ. Subsequently, B decides to make a purchase if the observed price p is less than or equal to the expected value of the product, calculated as $\mu g + (1 - \mu)b$. If B chooses to proceed with the purchase, S produces one unit of the product, and the transaction is completed. Following the consumption of the good by B, the outcome of the production is revealed to the public.

We assume that if S sets a price p different from $p^*(\lambda)$—where $p^*(\lambda)$ represents the equilibrium price determined by the H-type seller—the resulting posterior belief μ is set to zero. Conversely, if the posted price matches the equilibrium price, S is then identified as an H-type seller with a non-zero probability, in other words, $\mu > 0$. It's evident that S would select a price $p \in [b, g]$.

The expected payoff function for the L-type seller is given by

$$V^L(p, \mu) = (1 - \phi)\, p\mathbb{1}\left\{p \leq \mu g + (1 - \mu)b\right\} + \phi b\mathbb{1}\left\{p = b\right\},$$

where with probability ϕ, the trade only occurs if the price posted is b. Similarly, the expected payoff function for the H-type seller is expressed as

$$V^H(p, \mu) = (1 - \phi)\, p\mathbb{1}\left\{p \leq \mu^s g + (1 - \mu)b\right\} + \phi p, \tag{1}$$

where with probability ϕ, the trade occurs as long as the posted price p is less than or equal to g.

We employ backward induction to determine the equilibrium in this model. Initially, we solve for the set of perfect Bayesian equilibria, after which we apply the undefeated equilibrium refinement to further refine the equilibrium set.

2.1 Perfect Bayesian Equilibrium

In this section, we use backward induction to solve for the equilibrium for this model. We discuss three potential types of equilibrium, including separating equilibrium, semi-pooling equilibrium, and pooling equilibrium.

2.1.1 Separating Equilibrium

To attain the separating equilibrium, the L-type opts to set a price b with $\mu = 0$, while the H-type seller selects a price $p^*(\lambda) \neq b$ with $\mu = 1$. Establishing this equilibrium necessitates making it prohibitively expensive for the L-type seller to imitate the pricing strategy of the H-type seller. In essence, this requires that $b \geq (1-\phi)p^*(\lambda)$. This implies that $p^*(\lambda) \leq \left\{\frac{b}{1-\phi}, g\right\}$. If $\phi > \frac{g-b}{g}$, $p^*(\lambda) \leq g$; otherwise, $p^*(\lambda) \leq \frac{b}{1-\phi}$.

This result intuitively aligns with expectations, as when the probability of type revelation is significantly high, it becomes more costly for the L-type seller to mimic the pricing strategy of the H-type seller. This is due to the increased likelihood that the trade will not occur, making it less profitable for the L-type seller to deviate from their own pricing strategy.

2.1.2 Semi-pooling Equilibrium

To achieve the semi-pooling equilibrium, the L-type seller faces a situation of indifference between setting the price b and mimicking the pricing strategy of the H-type seller. We let $d(\lambda)$ denote the probability that the L-type seller mimics the H-type seller. This implies that $b = (1-\phi)p^*(\lambda)$, or $p^*(\lambda) = \frac{b}{1-\phi}$. This equilibrium exists only under conditions where $\phi < \frac{g-b}{g}$ and $\mu \geq \frac{\phi b}{(1-\phi)(g-b)}$, where $\mu = \frac{\lambda_2}{\lambda_2+(1-\lambda_2)d(\lambda_2)}$. This condition implies that $d(\lambda) \leq \frac{\lambda g(1-\phi)-\lambda b}{(1-\lambda)b\phi}$. Therefore, in the semi-pooling equilibrium, when $\phi \leq \frac{g-b}{g}$, the H-type seller selects the price $p^*(\lambda) = \frac{b}{1-\phi}$, while the L-type seller posts a price of $\frac{b}{1-\phi}$ with probability $d(\lambda) \leq \frac{\lambda g(1-\phi)-\lambda b}{(1-\lambda)b\phi}$, and chooses the price b with probability $1 - d(\lambda)$.

2.1.3 Pooling Equilibrium

The pooling equilibrium exists when both types of sellers set the same price. In this scenario, the updated belief μ equals λ, as both types of sellers set the same price with certainty, leading to no further revelation of information of the seller's type. Within this equilibrium, the L-type seller benefits by emulating the pricing strategy of the H-type seller. This suggests that $b \leq (1-\phi)p^*(\lambda)$, or equivalently, $p^*(\lambda) \geq \frac{b}{1-\phi}$, which also infers that $\phi \leq \frac{g-b}{g}$. This condition occurs because when the probability of revelation is sufficiently low enough, it's more advantageous for the L-type to mimic the H-type seller. Moreover, the buyer's willingness to pay is $\mu g + (1-\mu)b$, or equivalently, $\lambda g + (1-\lambda)b$. Thus, we derive $p^*(\lambda) \leq \lambda g + (1-\lambda)b$. This also implies that $\lambda \geq \frac{b\phi}{(1-\phi)(g-b)}$ In summary, when $\phi \leq \frac{g-b}{g}$ and $\lambda \geq \frac{b\phi}{(1-\phi)(g-b)}$, the pooling equilibrium price $p^*(\lambda) \in [\frac{b}{1-\phi}, \lambda g + (1-\lambda)b]$.

2.2 Refinement

In this section, we leverage the undefeated equilibrium (Mailath, Okuno-Fujiwara and Postlewaite, 1993) to refine the perfect Bayesian Equilibria discussed in Section 2.1. An equilibrium is undefeated if and only if no alternative equilibrium exists wherein either the H-type seller finds it more beneficial to deviate to a separating equilibrium, or both types of sellers have incentives to deviate to a pooling equilibrium. The undefeated equilibrium serves as a commonly employed refinement tool in signaling games. It enables us to select the equilibrium that maximizes the profit of the H-type seller—an important consideration in many market scenarios.

If $\phi > \frac{g-b}{g}$, the separating equilibrium price for the H-type seller lies in the interval $(b, g]$, while the L-type seller consistently sets the price at b. Among the set of separating perfect Bayesian equilibria, the optimal choice is made based on maximizing the profit of the H-type seller.

In this scenario, imagine the H-type seller initially setting $p^(\lambda)$ within the range (b, g), while the L-type seller sets the price to be b. In this situation, the H-type seller will deviate to an equilibrium where $p^(\lambda) = g$. Consequently, all other separating equilibria, except the one where $p^*(\lambda) = g$, are defeated. Since only the separating equilibria can exist in this scenario, the undefeated equilibrium is that the H-type seller would set a price $p^*(\lambda) = \frac{b}{1-\phi}$, and the L-type seller would set a price b.

If $\phi \leq \frac{g-b}{g}$, all three types of equilibria become feasible. The separating equilibrium price for the H-type seller is within the range of $(b, \frac{b}{1-\phi}]$. All the separating equilibria where the H-type seller's price $p^*(\lambda) < \frac{b}{1-\phi}$ are defeated by the equilibrium where the H-type seller sets the price $\frac{b}{1-\phi}$ and the L-type seller posts the price b. Thus, the separating equilibrium price for the H-type seller is $\frac{b}{1-\phi}$.

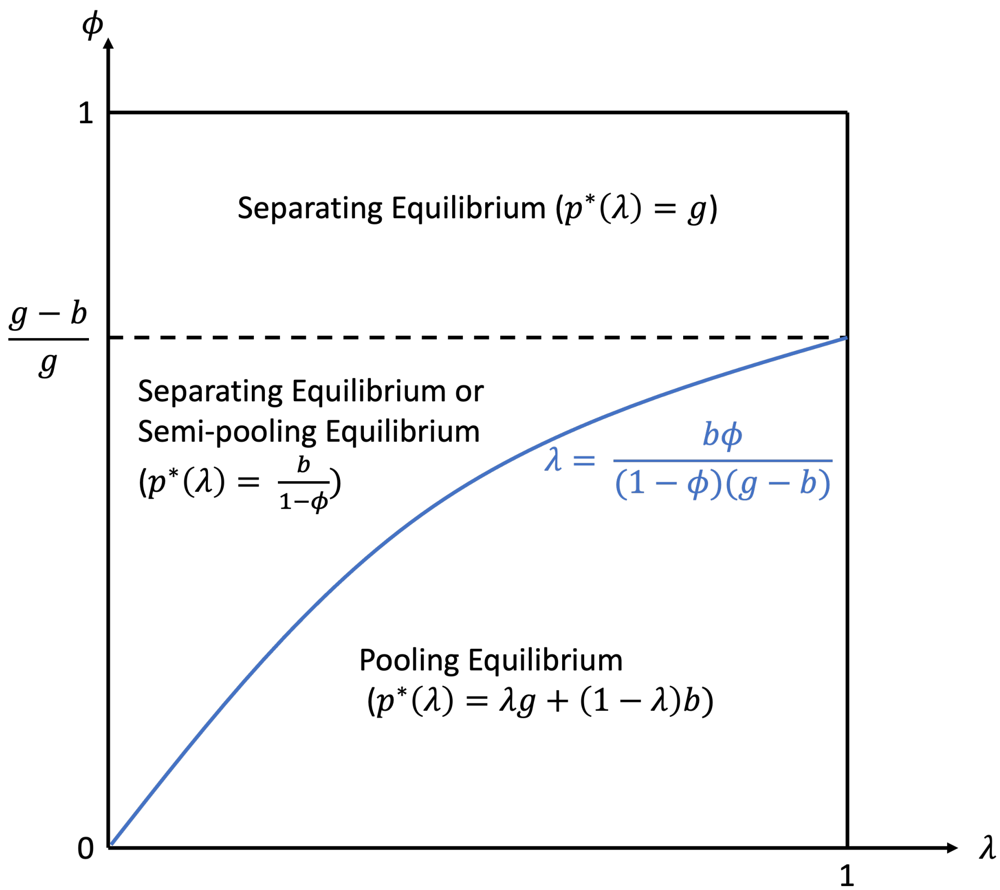
Among the semi-pooling equilibria, we argue that all the other equilibria are defeated by the equilibrium where the H-type seller sets the price $p^*(\lambda) = \frac{b}{1-\phi}$. Suppose that the H-type seller deviates to another semi-pooling equilibrium where $p^*(\lambda) < \frac{b}{1-\phi}$; in such a case, they would be strictly worsen off since the optimal profit is achieved by setting the price at $\frac{b}{1-\phi}$.

Similarly, all the other pooling equilibrium is defeated by the one where the H-type seller sets the price $\lambda g + (1 - \lambda)b$. Suppose the seller sets a price $p^*(\lambda) < \lambda g + (1 - \lambda)b$, both types of the seller would benefit from deviating to the pooling equilibrium where they set the price $p^*(\lambda) = \lambda g + (1 - \lambda)b$.

When $\lambda < \frac{b\phi}{(1-\phi)(g-b)}$, only the semi-pooling equilibrium and the separating equilibrium are feasible. In this scenario, both the H-type and L-type seller finds themselves indifferent between the two equilibria. When $\lambda \geq \frac{b\phi}{(1-\phi)(g-b)}$, the pooling equilibrium becomes feasible. In this scenario, the pooling equilibrium defeats both the separating and the semi-pooling equilibria. This is because the price in the pooling equilibrium, $\lambda g + (1 - \lambda)b$, is weakly greater than the price in the separating and semi-pooling equilibrium, $\frac{b}{1-\phi}$. Consequently, the H-type seller would be better off staying in the pooling equilibrium.

To summarize, in scenarios where $\phi > \frac{g-b}{g}$, the separating equilibrium with $p^(\lambda) = g$ stands as the undefeated equilibrium. In cases where $\phi \leq \frac{g-b}{g}$ and $\lambda < \frac{b\phi}{(1-\phi)(g-b)}$, both the separating and semi-pooling equilibria with $p^(\lambda) = \frac{b}{1-\phi}$ emerge as undefeated equilibria. Conversely, in scenarios where $\phi \leq \frac{g-b}{g}$ and $\lambda \geq \frac{b\phi}{(1-\phi)(g-b)}$, the pooling equilibrium with $p^*(\lambda) = \lambda g + (1 - \lambda)b$ prevails as the undefeated equilibrium. These results are summarized in Figure 1.

Figure 1: Undefeated Equilibrium



3 Comparative Statics

In the preceding section, we demonstrated that equilibrium prices are contingent on variables λ, ϕ, g, and b. In this section, we delve into the comparative statics analysis of these equilibria.

Proposition 1

We denote the expected payoffs at equilibrium as (π^H, π^L), where π^H represents the equilibrium expected return for the H-type seller, and π^L signifies the equilibrium expected return for the L-type seller. We have

1. $\frac{\partial \pi^H}{\partial \phi} \geq 0$, and $\frac{\partial \pi^L}{\partial \phi} \leq 0$;

2. $\frac{\partial \pi^H}{\partial \lambda} \geq 0$, and $\frac{\partial \pi^L}{\partial \lambda} \geq 0$;

3. $\frac{\partial \pi^H}{\partial g} \geq 0$, and $\frac{\partial \pi^L}{\partial g} \geq 0$;

4. $\frac{\partial \pi^H}{\partial b} \geq 0$, and $\frac{\partial \pi^L}{\partial b} \geq 0$.

The first part of the condition indicates that the equilibrium expected return for both seller types exhibits a weak increase as the probability of type disclosure rises, while the equilibrium expected return for the L-type seller decreases in response to the probability of disclosure. Since we normalized the cost of production to be 0, the expected return for the H-type seller is just the price they set. We illustrate the changes using Figure 2.

Based on the figure, consider a seller, denoted as a, is born in the region $\mathcal{C}$ where the pooling equilibrium prevails as the undefeated equilibrium. The equilibrium expected return for the H-type seller is $\lambda g + (1-\lambda)b$, and for the L-type seller is $\pi^L = (1-\phi)\left(\lambda g + (1-\lambda)\, b)\right) > b$. If we increase the value of ϕ while the seller remains in region $\mathcal{C}$, we find $\frac{\partial \pi^H}{\partial \phi} = 0$ and $\frac{\partial \pi^L}{\partial \phi} = -\left(\lambda g + (1-\lambda)\, b\right) < 0$.

Suppose we increase ϕ by δ, leading the seller to transition to point a' in region $\mathcal{B}$. In this region, either the semi-pooling or the separating equilibrium prevails as the undefeated equilibrium, with the expected return being b for the L-type seller and $\frac{b}{1-\phi}$ for the H-type seller. Additionally, in region $\mathcal{B}$, we observe that $\lambda < \frac{b\phi}{(1-\phi)(g-b)}$ and $\phi < \frac{g-b}{g}$, implying $g > \frac{b}{1-\phi} > \lambda g + (1-\lambda)b$. Hence, transitioning from region $\mathcal{C}$ to region $\mathcal{B}$ leads to an increase in the expected return of the H-type seller and a decrease in that of the L-type seller.

Furthermore, consider the scenario where we increase the value of ϕ while the seller remains in region $\mathcal{B}$. In this case, the expected return for the H-type seller would increase because $\frac{\partial \frac{b}{1-\phi}}{\partial \phi} > 0$, while the expected return for the L-type seller would remain unchanged.

Considering the further increase of ϕ by δ', this transition moves the seller to point a'' in region $\mathcal{A}$. In this region, the separating equilibrium is the undefeated equilibrium, characterized by the H-type seller setting the price at g and the L-type seller setting it at b. In region $\mathcal{A}$, the expected return for the H-type seller is g, representing the highest possible expected return for this type of seller, while the L-type seller's expected return remains at b. Consequently, transitioning from region $\mathcal{B}$ to $\mathcal{A}$ results in an increase in the expected return for the H-type seller, with no change in the return for the L-type seller.

In summary, the expected return for the H-type seller experiences a weak increase as ϕ rises, whereas the expected return for the L-type seller decreases with an increase in ϕ. This intuition aligns with real-life scenarios, where a higher likelihood of type disclosure before a purchase benefits the H-type seller due to reduced signaling efforts, while simultaneously disadvantaging the L-type seller.

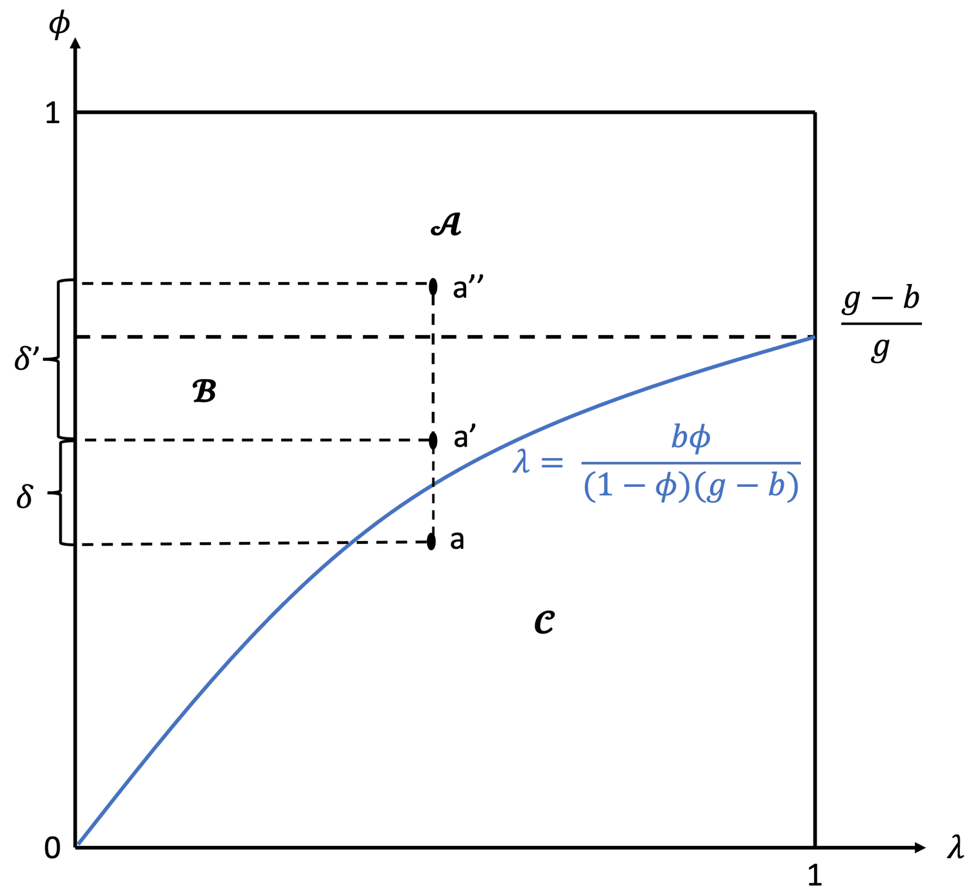


Figure 2: Illustration: ϕ increases

The second part of Proposition 1 indicates that both types of sellers would experience a weak benefit from an increase in the initial reputation parameter λ. We illustrate the changes in Figure 3.

Referring to the figure, We consider a seller, labeled as a, born in region $\mathcal{B}$. If we increase the value of λ while seller a remains in the same region, the expected return for both the H-type seller and the L-type seller would remain unchanged, at $\frac{b}{1-\phi}$ and b respectively.

If we increase the value of λ by δ, the seller a moves to position a' within region $\mathcal{C}$. In this scenario, the expected return for both the H-type seller and the L-type seller increases to $\lambda g + (1-\lambda)b$ and $(1-\phi)(\lambda g + (1-\lambda)b)$ respectively. As discussed in Section 2.1, we know that in this region, $\lambda g + (1-\lambda)b > \frac{b}{1-\phi}$, and $(1-\phi)(\lambda g + (1-\lambda)b) > b$. Therefore, the expected return for both types of sellers increases.

If we increase ϕ while the seller a remains in the region $\mathcal{C}$, the expected return for both types of sellers increases. This is because $\frac{\partial \lambda g+(1-\lambda)b}{\partial \lambda} = g - b > 0$, and $\frac{\partial(1-\phi)(\lambda g+(1-\lambda)b)}{\partial \lambda} = (1-\phi)(g-b) > 0$.

Similarly, if a seller is situated in the region $\mathcal{A}$, the expected return for the H-type seller remains at g and for the L-type seller stays at b as λ increases.

Therefore, the expected return for both types of sellers experiences a weakly increase as λ increases. This aligns with the intuition that both types of sellers would benefit from higher review scores.

The third part of Proposition 1 suggests that both types of sellers benefit from quality improvements in the H-type seller's product. We illustrate these changes using Figure 4. Suppose we increase the value of g to g', where $g' > g$. As a result, the region Δ_1, previously within region $\mathcal{A}$, now belongs to part of region $\mathcal{B}$. Simultaneously, the region Δ_2 and Δ_3, previously within region $\mathcal{B}$ and $\mathcal{A}$ respectively, become part of region $\mathcal{C}$. We define the regions as $\mathcal{A}' = \mathcal{A} \setminus (\Delta_1 \cup \Delta_3)$, $\mathcal{B}' = (\mathcal{B} \setminus \Delta_2) \cup \Delta_1$, and $\mathcal{C}' = \mathcal{C} \cup \Delta_2 \cup \Delta_3$.

Suppose a seller a is born in region Δ_1 with the expected return for the H-type seller being g, and for the L-type seller being b. When we increase g to g', the expected return for the H-type seller becomes $\frac{b}{1-\phi}$, where $g < \frac{b}{1-\phi} < g'$. Meanwhile, the expected return for the L-type seller remains at b. Thus, during this transition, as g increases, the expected return for the H-type seller increases while that for the L-type seller remains the same.

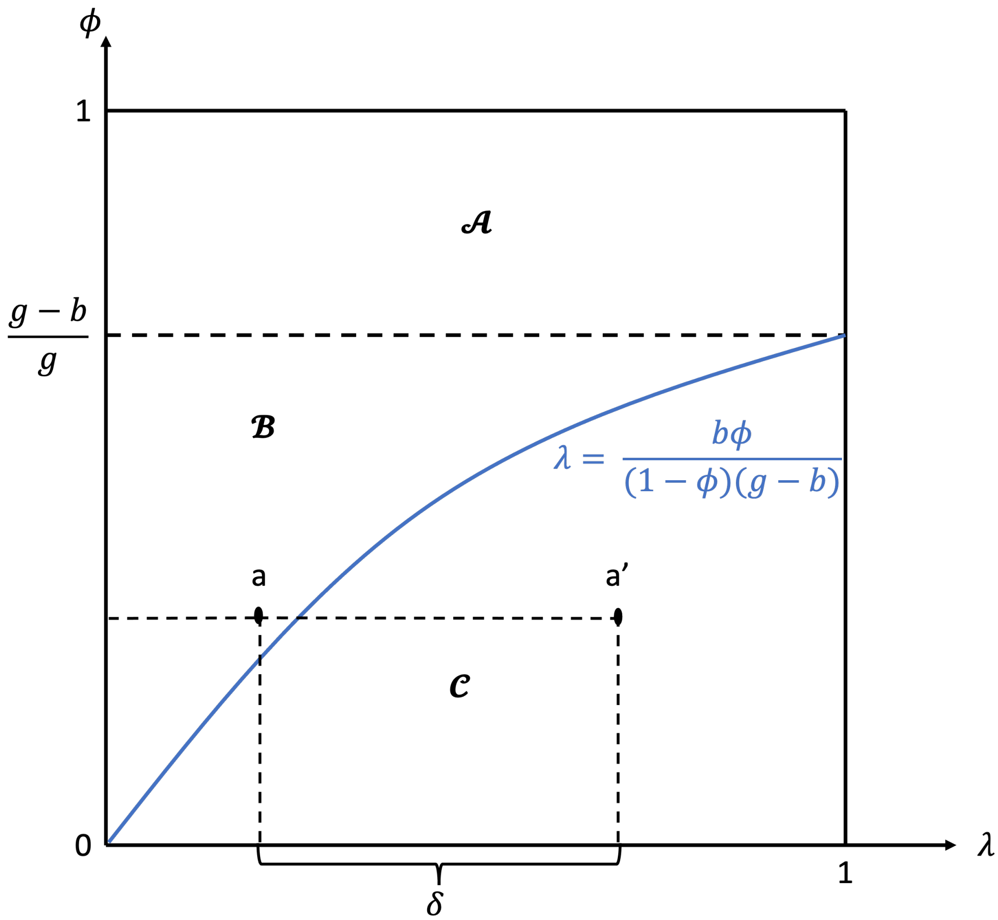


Figure 3: Illustration: λ increases

We then consider a seller a born in region Δ_2, with the expected return for the H-type seller being $\frac{b}{1-\phi}$ and for the L-type seller being b. If we increase g to g', the updated expected return at the undefeated pooling equilibrium for the H-type seller becomes $\lambda g' + (1-\lambda)b > \frac{b}{1-\phi}$, and that of the L-type seller becomes $(1-\phi)(\lambda g' + (1-\lambda)b) > b$. Thus, as g increases, both types of sellers in region Δ_2 would benefit.

Similarly, consider a seller a born in region Δ_3, with the expected return for the H-type seller being g and for the L-type seller being b. If we increase the value of g to g', the updated expected return at the undefeated pooling equilibrium becomes $\lambda g' + (1-\lambda)b > \frac{b}{1-\phi} > g$, while that of the L-type seller becomes $(1-\phi)(\lambda g' + (1-\lambda)b) > b$. Thus, as g increases, both types of sellers in region Δ_3 would be better off.

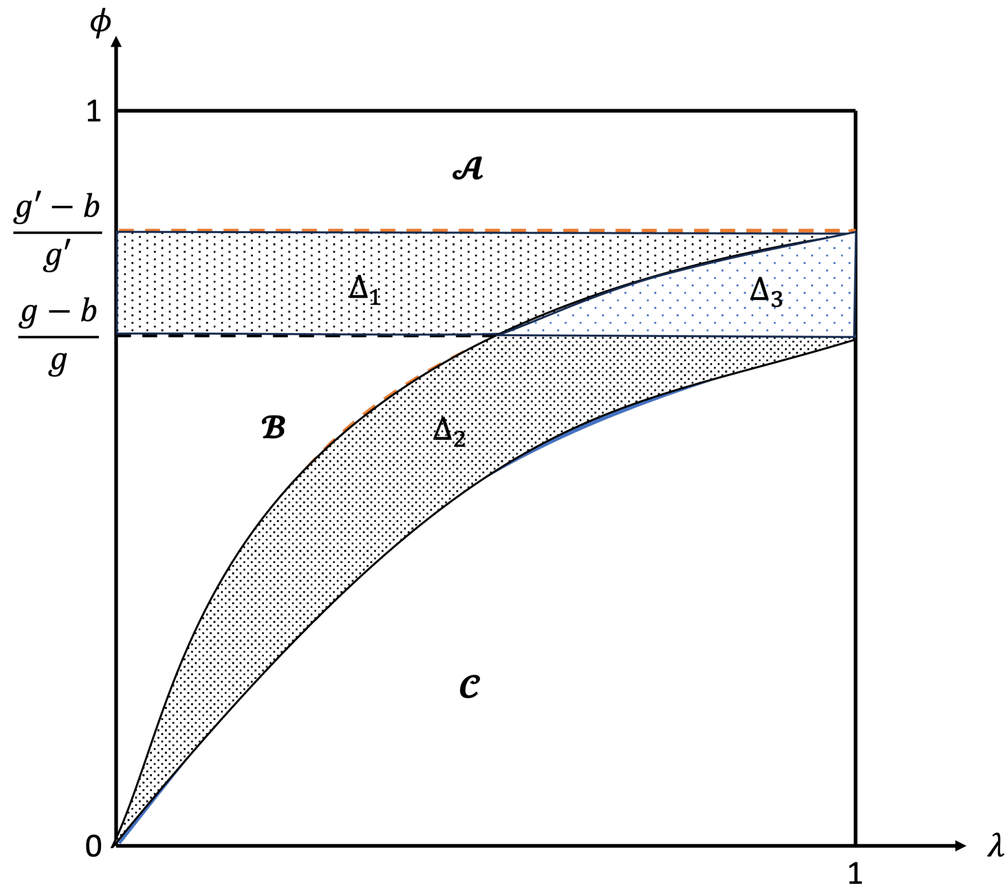


Figure 4: Illustration: g increases

Suppose we increase g to g'. For a seller a remaining in region $\mathcal{A}'$, the expected return for the H-type seller becomes $g' > g$, while for the L-type seller, it remains b. For a seller a staying in region $\mathcal{C}'$, the expected return for both types of sellers increases. This is because $\frac{\partial \lambda g+(1-\lambda)b}{\partial g} = \lambda > 0$, and $\frac{\partial (1-\phi)(\lambda g+(1-\lambda)b)}{\partial g} = (1-\phi)\lambda > 0$. For a seller a remaining in region $\mathcal{B}'$, the expected return for both types of sellers remains unchanged.

In conclusion, as g increases, the expected return for both types of sellers experiences a weak increase.

The final part of Proposition 1 indicates that both types of sellers would benefit from an increase in the product quality of the L-type seller. We illustrate these changes using Figure 5. Suppose we increase the value of b to b', where $b' > b$. Consequently, the regions Δ_1 and Δ_3, previously within regions $\mathcal{B}$ and $\mathcal{C}$ respectively, become part of region $\mathcal{A}$. Simultaneously, the region Δ_2, previously within region $\mathcal{C}$, becomes part of region $\mathcal{B}$. We define the updated regions as $\mathcal{A}' = \mathcal{A} \cup \Delta_1 \cup \Delta_3$, $\mathcal{B}' = (\mathcal{B} \setminus \Delta_1) \cup \Delta_2$, and $\mathcal{C}' = \mathcal{C} \setminus (\Delta_2 \cup \Delta_3)$.

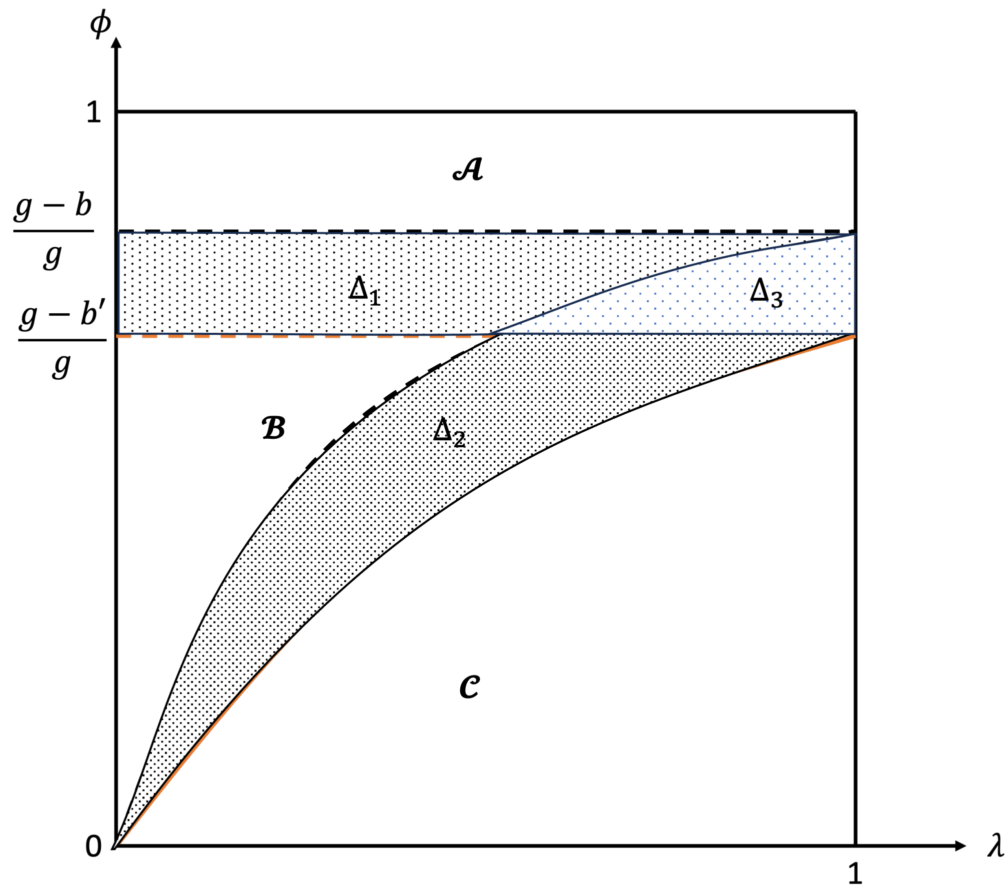


Figure 5: Illustration: b increases

Suppose a seller a is born in region Δ_1. If the seller is of the H-type, their expected return at the undefeated equilibrium is $\frac{b}{1-\phi}$, while for the L-type seller, it remains b. If we increase the value of b to b', the updated expected return at the undefeated equilibrium for the H-type seller becomes $g > \frac{b}{1-\phi}$, and for the L-type seller, it becomes $b' > b$. Thus, both types of sellers experience a benefit from the increase in b.

We consider a seller a born in region Δ_3, with the expected return for the H-type seller being $\lambda g + (1-\lambda)b$ and for the L-type seller being $(1-\phi)(\lambda g + (1-\lambda)b)$. If we increase b to

b', the expected return for the H-type seller becomes $g \geq \lambda g + (1-\lambda)b$, and for the L-type seller it becomes b'. Based on the discussion in Section 2.1, one can deduce that in this region $\frac{b'}{1-\phi} > \lambda g + (1-\lambda)b' > \lambda g + (1-\lambda)b$, which also implies $b' > (1-\phi)(\lambda g + (1-\lambda)b)$. Thus, both types of sellers benefit from the increase in b.

We now consider a seller a born in region Δ_2, where the expected return for the H-type is $\lambda g + (1-\lambda)b$, and for the L-type seller, it's $(1-\phi)(\lambda g + (1-\lambda)b)$. Suppose we increase the value of b to b'. The new equilibrium expected return for the H-type seller becomes $\frac{b'}{1-\phi}$, and for the L-type seller, it's b'. As discussed previously, we have $b' > (1-\phi)(\lambda g + (1-\lambda)b)$, indicating the L-type seller benefits from the increase. Also, based on the previous discussion in Section 2.1, we know that in this region, $\frac{b'}{1-\phi} > \lambda g + (1-\lambda)b'$, implying $\frac{b'}{1-\phi} > \lambda g + (1-\lambda)b$. Thus, both types of sellers benefit from the increase.

Suppose we increase b to b'. For a seller a in region $\mathcal{A}'$, the expected return for the H-type seller remains g, while for the L-type seller it increases to $b' > b$. In region $\mathcal{C}'$, the expected return for both types of sellers increases. This is because $\frac{\partial \lambda g + (1-\lambda)b}{\partial b} = 1 - \lambda \geq 0$, and $\frac{\partial (1-\phi)(\lambda g + (1-\lambda)b)}{\partial b} = (1-\phi)(1-\lambda) \geq 0$. Similarly, in region $\mathcal{B}'$, the expected return for both types of sellers increases since $\frac{\partial \frac{b}{1-\phi}}{\partial b} = \frac{1}{1-\phi} \geq 0$, and $\frac{\partial b}{\partial b} = 1 > 0$.

Hence, as b increases, both the H-type and L-type sellers are better off.

4 Conclusion

Our model extends the signal game by incorporating the review score, a significant aspect in online markets where customers rely on both price and customer scores to gauge seller quality. Additionally, we introduce a probability factor representing the likelihood of the seller's quality being fully disclosed to the public. This aspect becomes particularly pertinent in scenarios where government interventions may occur to investigate and reveal product quality information.

We discovered that when the probability of quality revelation is low, the pooling equilibrium emerges as the undefeated equilibrium under conditions of sufficiently high customer reviews. Conversely, when customer reviews are not favorable, the separating or semi-pooling equilibrium prevails. However, in instances where the probability of type revelation is high, the separating equilibrium becomes the undefeated equilibrium. This is due to the significant cost incurred by low-quality sellers in attempting to mimic high-quality sellers.

We showed the comparative statics of a seller's expected payoff concerning the probability of type revelation, the initial customer review rate, the quality of the high-quality seller's product, and the quality of the low-quality seller's product. We observed that the H-type seller benefits from an increase in the probability of type revelation, whereas the L-type seller experiences a loss. We also demonstrated that both types of sellers benefit from higher customer reviews, as well as from an increase in the quality of both high-quality and low-quality seller's products.

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