# Stability and Fairness of AP Selection Games in IEEE 802.11 Access Networks

Li-Hsing Yen, *Member, IEEE*, Jia-Jun Li, and Che-Ming Lin

***Abstract*—Wireless stations (WSs) in an IEEE 802.11 access network compete with each other for collective bandwidth offered by access points (APs). The competition involves selecting an AP with the consideration of potential link rate and workload status. From the perspective of system, a good AP selection policy should be stable, increase overall system throughput, and maintain bandwidth fairness among WSs. This paper models AP selections under the framework of game theory, where each WS's sole goal is to maximize its achievable throughput. The achievable throughput depends on not only the number of WSs that associate with the same AP but the set of link rates these WSs use as well. It is not a monotonically decreasing function of WS population when considering the effect of performance anomaly. We have proven the stability of this game (Nash equilibrium) and shown that selfish behavior of individual WSs in fact improves overall bandwidth fairness among WSs. Thorough simulations were conducted to demonstrate the validity of the analytical results and compare the performance of the proposed game with that of counterparts.**

***Index Terms*—Quality of service (QoS), wireless local area network (WLAN).**

## I. Introduction

IEEE 802.11 wireless local area networks have been widely deployed as wireless infrastructures, providing data access services in home, corporate, and public environments. In such environments, a wireless station (WS) with an IEEE 802.11 interface sends and receives frames via an access point (AP) to the network infrastructure, and all APs in service constitute an access network. However, traffic load in an access network may not be fairly shared by all serving APs due to the uncoordinated nature of AP selections among WSs. More specifically, WSs typically select and associate with an AP with the highest received signal strength (RSS). This problem motivates many load-balancing schemes for IEEE 802.11 networks [1] with a design goal to make WS–AP associations load aware, preventing WSs from making associations with congested APs. The ultimate goal is to either increase the overall system throughput or maintain bandwidth fairness among WSs.

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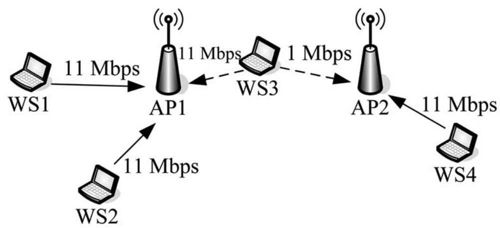
In this paper, we analyze the problem of AP selections under the framework of game theory. Game theory provides a mathematical modeling for the study of competition strategies in a game where players have conflicting benefits or goals. For the last decade, game theory has been used to analyze duty/resource-sharing problems in wireless networks [2]. In these games, selfish players usually bring in undesired results (uneven load distribution or unfair resource share), and researchers have to introduce incentive or punitive mechanisms to force cooperation among players. For example, a commonly adopted mechanism is to design a synthetic utility function for players that penalizes selfish behaviors. The goal is to let games naturally fall into stable states called Nash equilibria, where the system's interest could potentially benefit.

Our framework differs from previous frameworks in that WSs select and reselect APs merely for their own interest (specifically, achievable throughput that a WS may receive from a selection). No other external incentive/punitive mechanisms are introduced to ensure stability or fairness. The purpose of this research is to study the properties of the proposed AP selection game. We shall determine whether a Nash equilibrium exists, even in this context, which eliminates the possibility of unstable association transitions (change of AP selections). Furthermore, we shall explore if the selfish yet rational behaviors of WSs under the proposed framework could improve bandwidth fairness. We shall also present simulation results for numerical analyses on the properties of the proposed game model and other alternatives.

The remainder of this paper is organized as follows: Background information and related work are presented in Section II. Section III analyzes several properties of the proposed game, including stability and fairness. In Section IV, simulation results of the proposed game are discussed and compared with other alternatives. Section V concludes this paper.

## II. Background and Related Work

WSs in an IEEE 802.11 access network essentially compete for bandwidth offered by APs. Clearly, a WS's utility depends on not only its own association choice but on other WS's as well. This is why game theory becomes a useful tool to apply here. Intuitively, WSs should select an AP that is the least crowded to maximize its achievable throughput. Games with player's objective defined to minimize the number of other users that share the same selection are known to be *crowding games* [3]. In the literature, crowding games have been used to model network selections by mobile users [4], [5]. However,




| WS3's choice | Achievable throughput (Mbps) | | | | Total (Mbps) |
|---|---|---|---|---|---|
| | WS1 | WS2 | WS3 | WS4 | |
| AP1 | 2.67 (2.21) | 2.67 (2.23) | 2.67 (2.23) | 8.01 (6.57) | 16.02 (13.24) |
| AP2 | 4.05 (3.34) | 4.05 (3.38) | 0.83 (0.75) | 0.83 (0.84) | 9.76 (8.31) |

Fig. 1. Scenario illustrating performance anomaly. Achievable throughputs are based on the analysis of [6], whereas those in parentheses were obtained through simulation with ns2.

this framework does not well apply to IEEE 802.11 networks, because the achievable throughput of WSs in an AP is not necessarily a monotonically decreasing function of WS population there. The irregularity comes from two design features of IEEE 802.11. One is its nondeterministic medium access control (MAC) scheme, which does not guarantee any bandwidth share to participants. The other is the provision of multiple link rates in IEEE 802.11 a/b/g networks, which may give rise to an undesirable phenomenon called *performance anomaly* [6]. Performance anomaly refers to the effect that, when links operating at different rates coexist within an AP, throughputs of high-rate links will all degrade to the level of the lowest rate link. Performance anomaly not only impairs achievable throughputs of WSs but makes the AP's actual capacity variable as well. Consider the example of Fig. 1, where two IEEE 802.11b APs are serving four WSs. WS3 there could choose to associate with either AP1 or AP2. We can see that selecting AP1 yields a better result, although AP1 is more crowded than AP2. AP1 is also a better choice from the perspective of system's benefit, as selecting AP1 has a higher total achievable throughput than selecting the counterpart. Perception of performance anomaly can perform better. However, this cannot be characterized in crowding games.

The AP selection problem under consideration is modeled as a *noncooperative dynamic* game. In a noncooperative game, players do not cooperate with each other to seek system's benefit. A noncooperative game is dynamic if players take turns to make their decisions, knowing what decisions have already been made. In our model, an associated WS will reassociate with another AP if that reassociation improves its achievable throughput. The achievable throughput in recognition of the effect of performance anomaly can be computed with analytical results from prior work in [6], [7]. Here, we assume WSs pursuing its own throughput improvement rather than the balance of workloads among APs (e.g., [8]). Although a lightly loaded AP, in principle, offers a high achievable throughput and selecting an AP with the least load helps load balancing among APs, we argue that, from WS's perspective, AP selections based on achievable throughput are more straightforward and "natural" than AP selections based on load balancing. Several other approaches also proposed AP selections based on achievable throughput (potential bandwidth) [9]–[11]. Another issue of load-based AP selections comes from the fact that the notion of AP's load is not well defined in IEEE 802.11 networks. It could be the number of WSs associating with an AP, the frame drop rate of the AP's transmission queue during real-time sessions [12], or the total time that an AP takes to provide each WS one unit of traffic [8], [13].

Although there have been many approaches proposed for AP selections, only a few of them treat the problem under the framework of game theory. Mittal *et al.* [14] introduced an AP selection game, which differs from our setting in that WSs may need to travel some distance to reach an AP. The cost of an AP selection is measured by the AP's load and the traveling distance required by that selection. With this cost model, Mittal *et al.* proposed a simple greedy algorithm that brings the game to a Nash equilibrium under the condition of even WS distribution and the absence of dynamic WS arrivals and departures. However, the ability to measure physical distance between WSs and APs, as required by this model, is not yet a primitive feature in today's wireless networks. Shakkottai *et al.* [15] studied the problem of a WS associating with multiple APs and splitting its traffic among these APs (*link-layer multihoming*). They used the model of *population game* [16], which implies that the impact of individual WS's selection on other WS's utilities is infinitesimal. Although link-layer multihoming is possible for WSs using a single wireless interface card [17], this technique is not yet mature and has not been widely adopted. The population game model also does not generally apply to IEEE 802.11 networks. Jiang *et al.* [18] considered base station (BS) selections by mobile users, where each user selfishly chooses a BS that gives her the highest achievable throughput. This work assumes that the throughput each user can receive is controlled by the BS and that the number of users is enormous to apply the population game model. The ability to control user's throughput share by the BS is untenable in native IEEE 802.11 networks. The assumption of numerous users may not hold.

In addition to throughput, *fairness* is also a typical criterion for AP selection problems. In the context of bandwidth sharing, *max-min* [19] is a commonly adopted metric for fairness, particularly when bandwidth requestors have different bandwidth demands. With an objective to maximize the minimum share of a requestor whose demand is not fully satisfied, basic principles of max-min fairness are to allocate bandwidth to requestors in increasing demands, to ensure no requestor receives bandwidth more than its demand, and to equally split the remaining bandwidth to requestors with unsatisfied demands. If we use a tuple to denote the set of allocated bandwidth of every requestor sorted in a nondecreasing order, then a bandwidth allocation is max-min fair when the corresponding tuple has the highest lexicographical value[1] among all of them.

A similar notion, i.e., *min-max* fairness, can be defined for the sharing of workloads among APs. A distribution of workloads

[1]For any two $n$-tuples of numbers $T = (t_1, t_2, \ldots, t_n)$ and $T' = (t'_1, t'_2, \ldots, t'_n)$, $T$ has a higher lexicographical value than $T'$ if $\exists k \in \{1, \ldots, n\} : t_k > t'_k$ and, if $k > 1, \forall i : 1 \leq i < k :: t_i = t'_i$.

is min-max fair if the tuple denoting the set of workloads of every AP sorted in a nonincreasing order has the lowest lexicographical value among all possibilities. Bejerano *et al.* [13] studied AP selections that achieve the min-max fairness of AP workloads. They have proven that, unless link-layer multihoming is allowed, a min-max load balanced association does not imply a max-min fair bandwidth allocation, and *vice versa*.

Max-min fairness well applies to cases where resource requestors have limited demands. In our problem setting, however, every WS has an unlimited bandwidth demand; it could actually consume all bandwidth available to it. For this kind of bandwidth sharing, *balance index* [20] can be used to quantify the fairness of bandwidth share among all competitors. For a bandwidth allocation consisting of $n$ portions numbered 1 to $n$, let $B_i$, $1 \leq i \leq n$ denote the amount of bandwidth allocated to the $i$th portion. The balance index $\beta$ is defined as

$$\beta = \frac{(\sum B_i)^2}{n \times \sum B_i^2}. \quad (1)$$

The value of $\beta$ becomes 1 when all requestors get an equal share, and it approaches $1/n$ in case of extremely unbalanced allocations. Balance index can be related to max-min fairness in the sense that, when WSs all have unlimited bandwidth demands, a bandwidth allocation with $\beta = 1$ is also max-min fair. However, the converse does not generally hold.

## III. Access Point Selection Game

We consider an IEEE 802.11 network consisting of $n$ WSs and $m$ APs. Neighboring APs are assumed to operate at different (nonoverlapping) frequency channels such that there is no interference among APs. Let $A = (a_1, a_2, \ldots, a_m)$ and $W = (w_1, w_2, \ldots, w_n)$ be the tuples of all APs and WSs, respectively. We assume that each WS can access at least one AP and denote the set of APs that $w_i$ can access (i.e., the strategy set of $w_i$) by $A_i$, where $1 \leq i \leq n$. For a possible AP–WS association, the WS's *utility* is defined to be the achievable throughput of the WS that resulted from that association.

We define a *configuration* (a strategy profile) to be an $n$-tuple $C = (c_1, c_2, \ldots, c_n)$, where $c_i \in A_i$ represents $w_i$'s association choice. For a specific $w_i$, we may sometimes express $C$ as $C = (c_i, C_{-i})$, where $C_{-i} = (c_1, c_2, \ldots, c_{i-1}, c_{i+1}, \ldots, c_n)$ denotes all other WS's associations other than $w_i$'s. Function $u_i(C)$ gives $w_i$'s utility with respect to configuration $C$. We shall explore how to evaluate $u_i(C)$ in the next section. The AP selection game $\Gamma = [W; A; \{u_i\}_{i=1}^n]$ can be formally defined by $\max_{c_i \in A_i} u_i(c_i, C_{-i})$ for all $i = 1, 2, \ldots, n$. For the ensuing discussion, most of the symbols used are summarized in Table I.

A WS may seek an optimal association decision if the decision is made by considering all possible decisions that other WSs could make and what rules other WSs would follow to make their decisions. An equivalent approach is to examine all possible configurations to find the best strategy for a particular WS. The proposed game takes a simpler model instead, where WSs do not predict or analyze other WS's intentions or best strategies: They only *respond* to other WS's actions. With the

TABLE I
Partial List of Notations

| Notation | Meaning |
|---|---|
| $m$ | Number of APs |
| $n$ | Number of WSs |
| $A$ | The tuple of all APs; $A = (a_1, a_2, \cdots, a_m)$ |
| $W$ | The tuple of all WSs; $W = (w_1, w_2, \cdots, w_n)$ |
| $A_i$ | The set of APs that $w_i$ can associate with |
| $W_j$ | The set of WSs that associate with $a_j$ |
| $c_i, c_i^*, c_i'$ | The AP that $w_i$ associates with |
| $u_i(C)$ | $w_i$'s utility with respect to configuration $C$ |
| $\Sigma$ | The configuration space (set of all configurations) |
| $t(a, C)$ | The throughput of any WS residing in AP $a$ with respect to configuration $C$. |

knowledge of all other WS's choices, a WS conducts an association change only if that change maximizes the net increase in its utility among all possible reassociation choices under the assumption that all other WSs stay unchanged. Formally, the *best response function* for WS $w_i$ is $b_i(C_{-i}) = \{c_i \in A_i | \forall c_i' \in A_i : u_i(c_i, C_{-i}) \geq u_i(c_i', C_{-i})\}$. This reassociation rule may be shortsighted in the sense that the increase in utility is evaluated without considering possible responses from other WSs. Theoretically, the decision of association change may turn out to be a loss when later other WSs respond with their association changes. Fortunately, this is not a one-shot game; the original WS may recover its loss by making another reassociation. Our major concern is whether such interactions result in nonstop chain reactions and how the game evolves in terms of bandwidth fairness and overall system throughput.

In our game model, a transition from one configuration to another occurs when some WS conducts an association change. For simplicity, we assume that only one association change is conducted at a time; simultaneous transitions are serialized in some arbitrary order. Denote the transition relation by "$\rightsquigarrow$." Formally, for any two configurations $C_i$ and $C_j$, $C_i \rightsquigarrow C_j$ if $u_r(C_i) < u_r(C_j)$, where $w_r$ is the only WS that has different association choices between $C_i$ and $C_j$.

### A. Utility: Achievable Throughput

For each WS $w_i$ in the proposed game, its utility function $u_i(C)$ is defined to be the achievable throughput of $w_i$ in configuration $C$. Heusse *et al.* [6] analyzed achievable throughputs of WSs under IEEE 802.11 multirate environment. Their analysis assumes no interference among neighboring APs and can be summarized as follows: For any WS $w_i$ operating at link rate $r_i$, its MAC-layer throughput can be expressed as

$$X_i = U_i \times \frac{s_d}{r_i T_i} \times r_i \quad (2)$$

where $U_i$ is the fraction of time that $w_i$ is able to access the medium; $T_i$ is the overall transmission time (counting protocol overhead, transmission time, and the time spent in contention procedure) for a single frame sent by $w_i$; and $s_d$ is the size (in bits) of the frame. By definition, $U_i = T_i / I_i$, where $I_i$ is the average time between two consecutive transmissions of $w_i$. Therefore, (2) can be simplified as

$$X_i = \frac{s_d}{I_i}. \quad (3)$$

Let $W_j$ denote the set of all WSs that associate with AP $a_j$. One property of 802.11 MAC scheme is that all WSs in $W_j$ have equal long-term channel access probability, regardless of their link rates. In case of saturated traffic (i.e., every WS always has packets to transmit), this means that each $w_i \in W_j$ is expected to have an $I_i$ value that comprises $T_k$ for all $w_k \in W_j$ and the expected time spent in all possible collisions among WSs in $W_j$ during $I_i$. Formally

$$\forall w_i \in W_j : I_i = \left( \sum_{w_k \in W_j} T_k \right) + \delta(W_j) \tag{4}$$

where $\delta(W_j)$ is the expected time spent in all possible collisions among WSs in $W_j$ during $I_i$. $T_k$ consists of a rate-independent part (corresponding to protocol overhead and the time spent in contention procedure) and a variable-length part (transmission time) that depends on frame length $s_d$ and link rate $r_k$. The duration of a collision is also dominated by the lowest rate of WSs involved in the collision. A WS obtains its maximal throughput when it always has packets to transmit and each frame is of the maximal frame size. Therefore, all WSs that associate with the same AP receive an equal amount of achievable throughput that is determined by the mixture of their link rates but dominated by low-rate links. Consequently, the performance of high-rate links is effectively dragged down by low-rate links. The analysis presented in [7] shows similar conclusions.

Let $t(a, C)$ be the achievable throughput of any WS residing in AP $a$ with respect to configuration $C$. By (3) and (4), we know that

$$t(a, C) = \frac{s_d}{\left(\sum_{c_k=a} T_k\right) + \delta\left(\{c_k = a\}\right)}. \tag{5}$$

In the proposed AP selection game, $u_i(c_i, C_{-i}) = t(c_i, C)$, and the game can thus be defined by

$$\max_{c_i \in A_i} u_i(c_i, C_{-i}) = \max_{c_i \in A_i} \frac{s_d}{\left(\sum_{c_k=c_i} T_k\right) + \delta(\{c_k = c_i\})} \tag{6}$$

for all $i = 1, 2, \ldots, n$.

Without loss of generality, assume that $C_i \rightsquigarrow C_j$ because some WS $w_r$ changes its AP from $a_k$ to $a_l$. Since $C_i \rightsquigarrow C_j$ implies $u_r(C_i) < u_r(C_j)$, we have

$$t(a_k, C_i) < t(a_l, C_j). \tag{7}$$

Note that all and only all WSs associating with either $a_k$ or $a_l$ have their achievable throughputs changed by $C_i \rightsquigarrow C_j$. Therefore, if we are concerned with the total achievable throughput in the system, the net increase due to the transition is

$$\begin{aligned}(d-1) \times t(a_k, C_j) - d \times t(a_k, C_i)& \\ +e \times t(a_l, C_j) - (e-1) \times t(a_l, C_i)&\end{aligned} \tag{8}$$

where $d$ is the number of WSs associating with $a_k$ in $C_i$, and $e$ is the number of WSs (including $w_r$) associating with $a_l$ in $C_j$. Clearly, (7) does not guarantee a positive net increase. In fact, an association change in the proposed AP selection game may lead to degradation of the total achievable throughput in the system.

For comparison purpose, we also define a *public-interest first* (PIF) reassociation model, where a WS makes an association change only if that association results in an increase in the total achievable throughput in the system. In the case of multiple candidates, the WS chooses that which results in the maximal net increase. Although the WS's own benefit may be sacrificed in this model, the system is always benefited from reassociations.

## *B. Stability*

*Definition 1:* Nash equilibrium: Given a game $\Gamma = [W; A; \{u_i\}_{i=1}^n]$, a configuration $C^* = (c_1^*, c_2^*, \ldots, c_n^*)$ is a Nash equilibrium if $\forall i \in \{1, \ldots, n\} : \forall c_i \in A_i :: u_i(c_i^*, C_{-i}^*) \geq u_i(c_i, C_{-i}^*)$.

In other words, Nash equilibrium is a configuration where no WS can further increase its own utility by unilaterally changing its choice. Nash equilibrium is not necessarily a Pareto optimal strategy. A configuration $C = (c_1, c_2, \ldots, c_n)$ is Pareto optimal if and only if there exists no other configuration $C' = (c_1', c_2', \ldots, c_n')$ such that $\forall i \in \{1, \ldots, n\} : u_i(C') \geq u_i(C)$ and $\exists j \in \{1, \ldots, n\} : u_j(C') > u_j(C)$.

Recall that, in our model, an associated WS can reassociate with another AP if that reassociation improves its achievable throughput. The reassociation action may trigger another WS's reassociation, and so on. If Nash equilibria do not exist in this game, reassociation activities will last, and the system cannot enter a stable state. By contrast, stability in the PIF reassociation model is always guaranteed as it is impossible to unlimitedly increase the total achievable throughput of the system. We shall now show the existence of Nash equilibria in the proposed AP selection game.

Let $\Sigma = A_1 \times A_2 \times \cdots \times A_n$ be the configuration space, i.e., the set of all possible configurations. If there exists no Nash equilibrium, then, for any configuration $C_i \in \Sigma$, there must exist another configuration $C_j \in \Sigma$ such that $C_i \rightsquigarrow C_j$. Since the strategy space is finite, nonexistence of Nash equilibrium implies that there must be a series of configurations $C_1', C_2', \ldots, C_p'$, where $p \leq k$, such that $C_1' \rightsquigarrow C_2'$, $C_2' \rightsquigarrow C_3', \ldots, C_p' \rightsquigarrow C_1'$. We shall prove the existence of Nash equilibrium by showing that such series does not exist.

For each configuration $C_i \in \Sigma$, let $T(C_i) = (\alpha_i^1, \alpha_i^2, \ldots, \alpha_i^m)$ be an $m$-tuple of APs, where $\{\alpha_i^1, \alpha_i^2, \ldots, \alpha_i^m\}$ is an ordered set of all APs such that $t(\alpha_i^1, C_i) \leq t(\alpha_i^2, C_i) \leq \cdots \leq t(\alpha_i^m, C_i)$[2]. Let $\Theta = \{T(C) | C \in \Sigma)\}$. We also define a binary relation $\prec$ on $\Theta$ as follows: For $T(C_i)$, $T(C_j) \in \Theta$, we have $T(C_i) \prec T(C_j)$ if $\exists k \in \{1, \ldots, m\} : t(\alpha_i^k, C_i) < t(\alpha_j^k, C_j)$, and if $k > 1$, $\forall l : 1 \leq l < k :: t(\alpha_i^l, C_i) = t(\alpha_j^l, C_j)$. It is not hard to see that "$\prec$" is a precedence relation [21], i.e., it is antisymmetric and transitive.

*Theorem 1:* $\forall C_i, C_j \in \Sigma : C_i \rightsquigarrow C_j \Rightarrow T(C_i) \prec T(C_j)$.

[2]For any AP $\alpha_i^j$ that draws no WS in $C_i$, we define $t(\alpha_i^j, C_i)$ to be the nominal capacity of one AP (11 Mb/s in case of IEEE 802.11b) so that such APs are always ranked after any AP with one or more WS associations.

*Proof:* Without loss of generality, assume that $C_i \rightsquigarrow C_j$, because some WS $w_r$ changes its AP from $a_k$ to $a_l$. Let $a_k$ be the $p$th and $q$th elements in $T(C_i)$ and $T(C_j)$, respectively. In other words, $a_k = \alpha_i^p = \alpha_j^q$. Similarly, let $a_l = \alpha_i^x = \alpha_j^y$. $C_i \rightsquigarrow C_j$ implies that $u_r(C_i) < u_r(C_j)$, which, in turn, implies that

$$t\left(\alpha_i^p, C_i\right) < t\left(\alpha_j^y, C_j\right). \tag{9}$$

If there exists at least one WS associating with $a_k$ after the association migration of $w_r$, then $t(a_k)$ will be increased, meaning that

$$t\left(\alpha_i^p, C_i\right) < t\left(\alpha_j^q, C_j\right). \tag{10}$$

If there is no other WS associating with $a_k$ after $w_r$'s leave, then $t(\alpha_j^q, C_j)$ equals to the nominal capacity of one AP as defined, and (10) still holds. By (9), (10), and the fact that $a_k$ and $a_l$ are the only two APs whose throughput is changed by $C_i \rightsquigarrow C_j$, the first $p-1$ elements in $T(C_i)$ hold their ranks in $T(C_j)$. Thus, we have

$$\forall s : 1 \le s \le p-1 :: t(\alpha_i^s, C_i) = t(\alpha_j^s, C_j). \tag{11}$$

Now consider the relation between $v = \min\{q, y\}$ and $p$. By (9)–(11), it is impossible that $v < p$. If $v = p$, then we have the proof by (11) and either (9) or (10). If $v > p$, then $\alpha_i^p$ must change its rank from the $p$th element in $T(C_i)$ to at least the $v$th element in $T(C_j)$, and all APs in between change their ranks accordingly (see Fig. 2). That is

$$\forall s : p \le s \le v-1 :: t\left(\alpha_j^s, C_j\right) = t\left(\alpha_i^{s+1}, C_i\right). \tag{12}$$

Since $\forall s : p \le s \le v-1 :: t(\alpha_i^{s+1}, C_i) \ge t(\alpha_i^s, C_i)$, (12) implies that

$$\forall s : p \le s \le v-1 :: t\left(\alpha_i^s, C_i\right) \le t\left(\alpha_j^s, C_j\right). \tag{13}$$

Moreover, from (12), we know that $t(\alpha_j^{v-1}, C_j) = t(\alpha_i^v, C_i)$. This, together with the fact $t(\alpha_j^{v-1}, C_j) \le t(\alpha_j^v, C_j)$, implies that

$$t\left(\alpha_i^v, C_i\right) \le t\left(\alpha_j^v, C_j\right). \tag{14}$$

Equations (13) and (14) can be merged into

$$\forall s : p \le s \le v :: t\left(\alpha_i^s, C_i\right) \le t\left(\alpha_j^s, C_j\right) \tag{15}$$

which implies that either

$$\forall s : p \le s \le v :: t\left(\alpha_i^s, C_i\right) = t\left(\alpha_j^s, C_j\right) \tag{16}$$

or

$$\exists s : p \le s \le v :: t\left(\alpha_i^s, C_i\right) < t\left(\alpha_j^s, C_j\right) \tag{17}$$

holds. If (16) holds, then we have $t(\alpha_i^s, C_i) = t(\alpha_i^{s+1}, C_i)$ for all $s$, $p \le s \le v-1$ by (12), which, in turn, implies that $t(\alpha_i^p, C_i) = t(\alpha_j^v, C_j)$. The derived result contradicts with either (9) (when $v = y$) or (10) (when $v = q$). Therefore, only (17) holds. The theorem is thus proven by (11), (15), and (17). ■

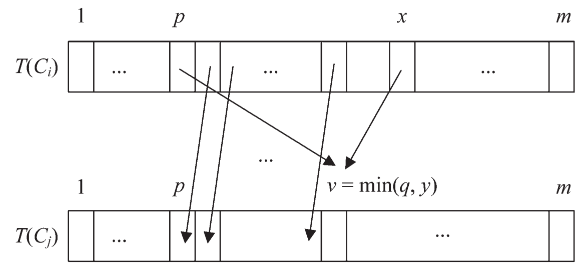



Fig. 2. Rank mapping from $T(C_i)$ to $T(C_j)$ when $v = \min\{q, y\} > p$.

Half of the proof deals with the case that different APs may provide identical throughputs for WSs associating with them. If this case were not considered, the equality in (13) would not hold, and all the subsequent arguments would not be needed.

If there is any series of configuration transitions $C'_1, C'_2, \ldots, C'_p$, where $p \le k$ in the proposed game such that $C'_1 \rightsquigarrow C'_2, C'_2 \rightsquigarrow C'_3, \ldots, C'_p \rightsquigarrow C'_1$, then, by Theorem 1, we have $T(C'_1) \prec T(C'_2), T(C'_2) \prec T(C'_3), \ldots, T(C'_p) \prec T(C'_1)$. It follows that $T(C'_2) \prec T(C'_1)$ as $\prec$ is transitive, which leads to a contradiction since $\prec$ is also antisymmetric. Therefore, Theorem 1 implies that any loop of configuration transitions is impossible and suffices to be a proof for the existence of Nash equilibria in the proposed game. More specifically, starting from any configuration, reassociation activities made by WSs always end up with a configuration where no WS can further increase its own utility by unilaterally changing its choice.

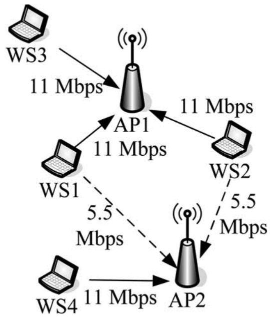
Note that different initial configurations may end up with different Nash equilibria. Even with the same initial configuration, different reassociation orders may lead to different stable configurations. Fig. 3 gives an example, where both WS1 and WS2 have the motivation to reassociate with AP2. Depending on which one moves first, two possible Nash equilibria can be reached.

### C. Fairness

We shall now address the fairness issue of the game. The definition of max-min fairness refers to only one configuration. To quantify the relative degree of fairness for *every* feasible configuration, we propose measuring the lexicographical value of the corresponding utility tuple. More precisely, each configuration $C_i \in \Sigma$ corresponds to a *utility tuple* $U_i = (\mu_i^1, \mu_i^2, \ldots, \mu_i^n)$, which is obtained by sorting $\{u_j(C_i)\}_{j=1}^n$ in a nondecreasing order.

*Definition 2:* Given two configurations $C_i$ and $C_j$ with respective utility tuples $U_i = (\mu_i^1, \mu_i^2, \ldots, \mu_i^n)$ and $U_j = (\mu_j^1, \mu_j^2, \ldots, \mu_j^n)$, we say that $C_i$ is *lexicographically fairer* than $C_j$ if $U_i$ has a higher lexicographical value than $U_j$, i.e., $\exists k \in \{1, \ldots, n\} : \mu_i^k > \mu_j^k$ and, if $k > 1$, $\forall l : 1 \le l < k :: \mu_i^l = \mu_j^l$.

Intuitively, $C_i$ is lexicographically fairer than $C_j$ if the lowest utility in $C_i$ is larger than that in $C_j$, or the lowest utility in $C_i$ is equal to that in $C_j$, but the second lowest utility in $C_i$ is larger than that in $C_j$, and so on. This definition is consistent with max-min fairness in the sense that a configuration is max-min fair if and only if it is lexicographically fairer than any others.




| Configuration | Achievable throughput (Mbps) | | | | |
|---|---|---|---|---|---|
| | WS1 | WS2 | WS3 | WS4 | |
| Initial | 2.67 | 2.67 | 2.67 | 8.01 | |
| WS1 chooses AP2 first | 2.93 | 4.05 | 4.05 | 2.93 | (Nash Equilibrium) |
| WS2 chooses AP2 first | 4.05 | 2.93 | 4.05 | 2.93 | (Nash Equilibrium) |

Fig. 3. Example illustrating the effect of reassociation order. Achievable throughputs are based on the analysis of [6].

We can derive $U_i$ from $T(C_i)$ by seeing that all WSs associating with the same AP receive equal throughput. Let $w(\alpha_i^k)$ be the number of WSs associating with AP $\alpha_i^k \in T(C_i)$, where $1 \le k \le m$. Given $T(C_i)$, we let each AP $\alpha_i^k$ map to $w(\alpha_i^k)$ consecutive elements in $U_i$. Specifically, the following function returns the position of the first element in $U_i$ that corresponds to a WS associating with $\alpha_i^k$ (if there is any WS associating with $\alpha_i^k$):

$$\rho\left(\alpha_i^k\right) = \begin{cases} 1, & k = 1 \\ 1 + \sum_{l=1}^{k-1} w\left(\alpha_i^l\right), & 2 \le k \le m. \end{cases} \tag{18}$$

If $w(\alpha_i^k) = 0$, $\alpha_i^k$ maps to no element in $U_i$. Otherwise, all elements in $U_i$ with ordinal numbers ranging from $\rho(\alpha_i^k)$ to $\rho(\alpha_i^{k+1}) - 1$, $1 \le k \le m-1$ have identical values $t(\alpha_i^k, C_i)$.

With the way to derive $U_i$ from a given $T(C_i)$, we shall further prove that, if $C_i \rightsquigarrow C_j$, then $U_j$ also has a higher lexicographical value than $U_i$, meaning that $C_j$ is lexicographically fairer than $C_i$. Consequently, configuration transitions in the proposed AP selection game always improve utility fairness.

*Theorem 2:* $\forall C_i, C_j \in \Sigma : C_i \rightsquigarrow C_j \Rightarrow U_i \prec U_j$.

*Proof:* $U_i$ and $U_j$ can be derived from $T(C_i)$ and $T(C_j)$, respectively, as previously stated. We assume the same definitions of $p$, $q$, and $y$ as in the proof of Theorem 1. Since the first $p-1$ APs in $T(C_i)$ hold their ranks in $T(C_j)$, all the first $\rho(\alpha_i^p) - 1$ elements in $U_i$ are identical to the corresponding elements in $U_j$. That is

$$\forall k : 1 \le k \le \rho\left(\alpha_i^p\right) - 1 : \mu_j^k = \mu_i^k. \tag{19}$$

Now consider $v = \min\{q, y\}$. If $v = p$, then the $\rho(\alpha_i^p)$th element in $U_i$ is smaller than the corresponding element in $U_j$ by either (9) or (10), and the proof is done. If $v > p$, then the position of either $\alpha_j^q$ or $\alpha_j^y$ is at least the $v$th in $T(C_j)$, which means all the $w(\alpha_i^p)$ elements associating with $\alpha_i^p$ are placed at least in the $\rho(\alpha_j^v)$th position in $U_j$. It follows that

$$\forall k : p \le k \le v-1 :: \rho\left(\alpha_j^k\right) = \rho\left(\alpha_i^{k+1}\right) - w\left(\alpha_i^p\right) \tag{20}$$

$$\forall k : p \le k \le v-1 :: w\left(\alpha_j^k\right) = w\left(\alpha_i^{k+1}\right) \tag{21}$$

$$\forall k : \rho\left(\alpha_j^p\right) \le k \le \rho\left(\alpha_j^v\right) - 1 :: \mu_j^k = \mu_i^{k+w(\alpha_i^p)} \ge \mu_i^k. \tag{22}$$

See Fig. 4 for the rank mapping from $U_i$ to $U_j$ indicated by (20). Equation (22) implies that either

$$\exists s : \rho\left(\alpha_j^p\right) \le s \le \rho\left(\alpha_j^v\right) - 1 :: \mu_j^s > \mu_i^s \tag{23}$$

or

$$\forall k : \rho\left(\alpha_j^p\right) \le k \le \rho\left(\alpha_j^v\right) - 1 :: \mu_j^k = \mu_i^{k+W\left(\alpha_i^p\right)} = \mu_i^k. \tag{24}$$

If (23) holds, the theorem is proven by (19), (22), and (23). If (24) holds, by (20), we have

$$\forall k : \rho\left(\alpha_i^p\right) \le k \le \rho\left(\alpha_i^{v+1}\right) - w(\alpha_i^p) - 1 :: \mu_i^k = \mu_i^{k+w(\alpha_i^p)}$$
$$\Rightarrow \mu_i^{\rho(\alpha_i^p)} = \mu_i^{\rho(\alpha_i^p)+1} = \cdots = \mu_i^{\rho(\alpha_i^{v+1})-1}. \tag{25}$$

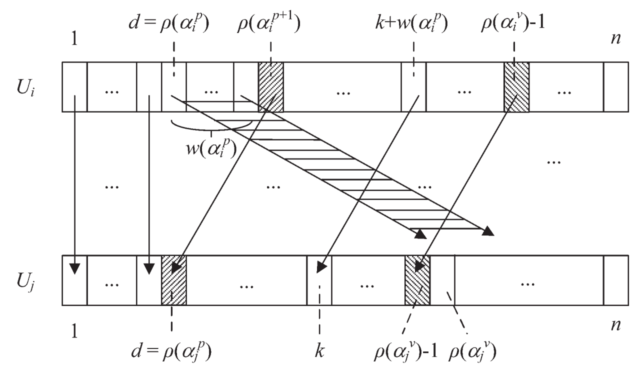
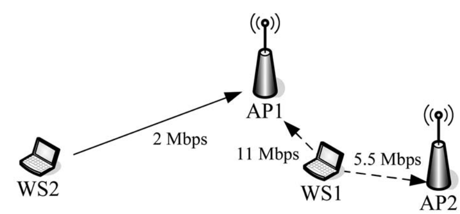



Fig. 4. Rank mapping from $U_i$ to $U_j$ when $v = \min\{q, y\} > p$.




| WS1's choice | Achievable throughput (Mbps) | | Utility tuple | Balance index |
|---|---|---|---|---|
| | WS1 | WS2 | | |
| AP1 | 1.48 | 1.48 | ⟨1.48, 1.48⟩ | 1 |
| AP2 | 4.61 | 1.85 | ⟨1.85, 4.61⟩ | 0.846 |

Fig. 5. Scenario illustrating the difference between lexicographical fairness and balance index.

TABLE II
CONVERSION OF DISTANCE TO LINK RATE

| Range of distance $d$ (m) | Link rate (Mbps) |
|---|---|
| $0 \le d < 50$ | 11 |
| $50 \le d < 80$ | 5.5 |
| $80 \le d < 120$ | 2 |
| $120 \le d < 150$ | 1 |
| $d \ge 150$ | 0 |

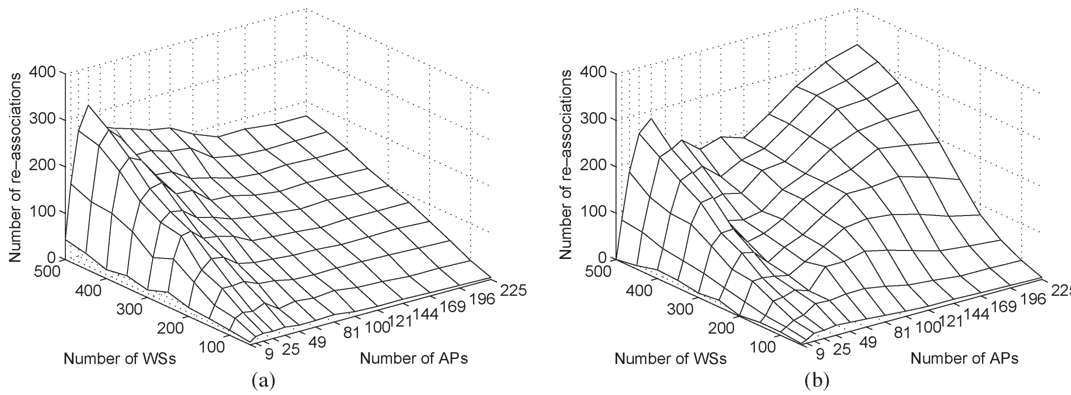



Fig. 6. (a) Number of reassociations in the proposed game before Nash equilibrium. (b) Number of reassociations in the PIF model.

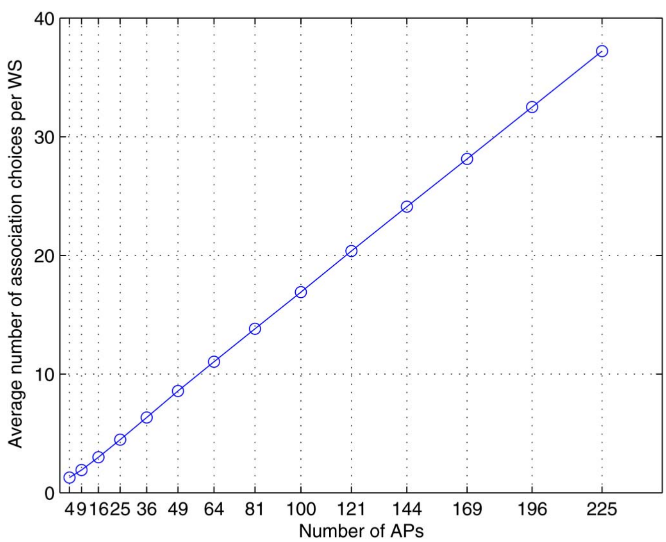



Fig. 7. Average number of association choices per WS.

Let $d = \rho(\alpha_i^p)$ and $e = \rho(\alpha_j^v)$. The derivation of (26), shown below, is based on (18), (20) and (21), i.e.,

$$\begin{aligned} & e = \rho\left(\alpha_j^{v-1}\right) + w\left(\alpha_j^{v-1}\right) \\ \Rightarrow & e = \rho\left(\alpha_i^v\right) - w\left(\alpha_i^p\right) + w(\alpha_j^{v-1}) \\ \Rightarrow & e = \rho\left(\alpha_i^v\right) - w\left(\alpha_i^p\right) + w\left(\alpha_i^v\right) \\ \Rightarrow & e = \rho\left(\alpha_i^{v+1}\right) - w(\alpha_i^p) \leq \rho\left(\alpha_i^{v+1}\right) - 1. \end{aligned} \tag{26}$$

By (25) and (26), we have $\mu_j^e = \mu_i^d$. Furthermore, $\mu_j^e > \mu_i^d$ by (9) and (10). Therefore, we have $\mu_j^e > \mu_i^e$. The theorem is thus proven. ■

This proof is similar to that of Theorem 1 in the sense that much efforts are devoted to deal with the case of identical utility values among adjacent elements in a utility tuple. The key point is, despite the existence of a continuous series of identical utility values, we can always find a break point in the corresponding utility tuple that exhibits a difference between $U_i$ and $U_j$.

It should be noted that the definition of lexicographical fairness does not always comply with the definition of balance index. Refer to the scenario shown in Fig. 5, where WS1 could select to associate with either AP1 or AP2. The former selection results in a higher balance index than the latter, whereas the latter is lexicographically fairer than the former. Despite the existence of such counterexample in a synthesized setting, in the next section, we shall show through simulations that configuration transitions in the proposed AP selection game generally improve bandwidth fairness in terms of balance index.

## IV. Numerical Results

We conducted extended simulations to study the properties of the proposed game. The simulation setting is given as follows: APs form a square grid in a $600 \times 600$ $(\mathrm{m}^2)$ area with the dimension of the sides of the grid squares set to 2 to 15. Neighboring APs (also a border AP and the border of the area) are separated with equal distance. WSs are randomly uniformly distributed over the same region with the number of WSs varied from 50 to 500 in increments of 50. The link rate between a WS and an AP is based on IEEE 802.11b and determined by their in-between distance (see Table II). We also preclude unconnected WSs by randomly relocating such WSs. For each setting, 1000 trials were made for an average result.

We let each WS select an AP based on RSS initially. Here, all APs are assumed identical transmitting power, and a simple path-loss model is adopted, where RSS decreases with the square of the traveling distance of the signal. After its initial association, a WS selects an AP to reassociate with following either the proposed game model or the PIF model. When multiple WSs are eligible to make an association change, we randomly select one WS at a time to do so. The achievable throughput of each WS is calculated based on the analysis of Heusse *et al.* [6].

### A. *Number of Reassociations*

Fig. 6 shows the total number of reassociations in the proposed game and in the PIF model for each possible setting. The reassociation activities in the proposed game all stop after a limited number of times, which validates our analysis. In general, WSs in the PIF model experience more reassociations than those in the proposed game model. The average number of reassociations per WS in the proposed game is 0.367 with a standard deviation of 0.131. For the PIF model, the average value and standard deviation are 0.393 and 0.185, respectively.

Three key factors govern the number of reassociations: 1) the nature of the re-association policy; 2) the number of association

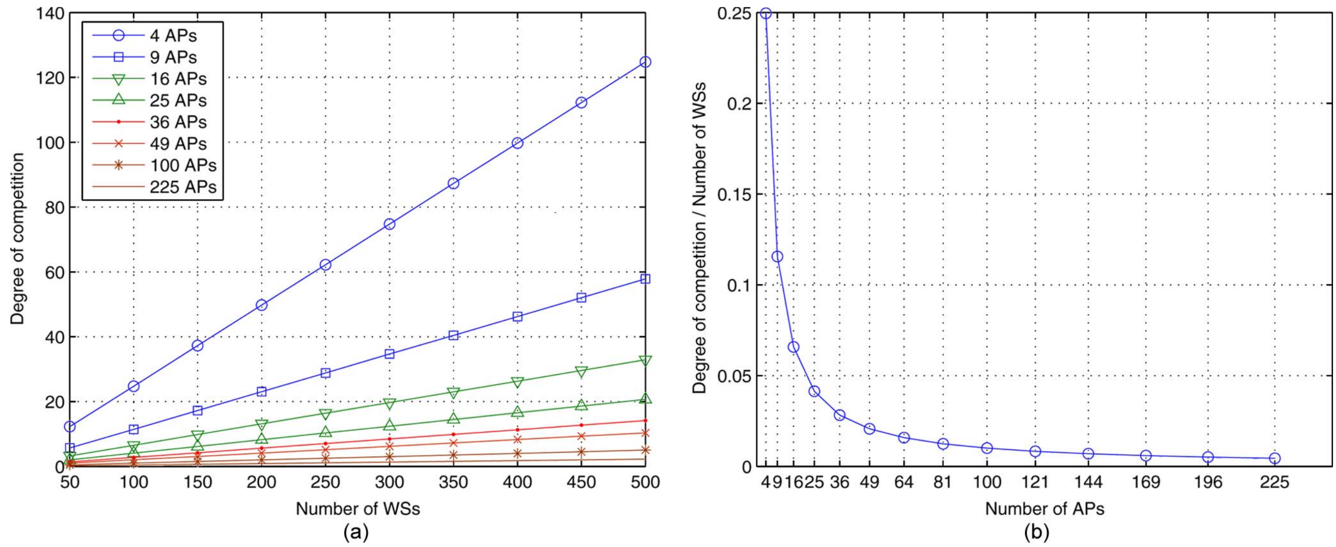



Fig. 8. (a) Expected degree of competition. (b) Expected degree of competition divided by the number of WSs versus the number of APs.

choices; and 3) the degree of association competition. The first factor is model dependent and explains the difference between Fig. 6(a) and (b). The number of reassociations, as a result of competitions, generally increases with the number of association choices and the degree of association competition. Given a certain experiment setting, the expected number of association choices owned by an WS can be measured by the average number of APs accessible to an WS. It is irrelevant to the total number of WSs deployed. Fig. 7 displays how the average number of association choices changes with the number of APs. The degree of competition counts the number of competitors that each WS is expected to face for a particular WS–AP association. For WS $w_i$ to associate with AP $a_j$, the degree of competition depends on not only the number of other WSs that can also associate with $a_j$ but the likelihood that these potential competitors actually do it as well. Without resorting to the knowledge of a specific reassociation model, let us assume that every AP in $A_i$ will be chosen by $w_i$ with equal preference, and this holds for every WS $w_i$. It follows that the expected number of competitors that $w_i$ has to face is

$$\sum_{a_j \in A_i} \left( \frac{1}{|A_i|} \sum_{w_k \in P_j, k \neq i} \frac{1}{|A_k|} \right) \tag{27}$$

where $P_j = \{w_k | a_j \in A_k\}$ is the set of WSs that can associate with $a_j$. We take the average value of (27) over all WSs as the expected degree of competition. Fig. 8(a) shows the measured results. Clearly, the average degree of competition increases with the number of WSs but decreases with the number of APs. Since the average degree of competition has a linear relationship with the number of WSs, we divide the former by the latter and get the result of Fig. 8(b). In the following, we explain the results of Fig. 6 with the help of Figs. 7 and 8.

1) For a fixed number of WSs, the expected number of choices is in proportion to the number of APs (see Fig. 7). However, in Fig. 6, we observe a rather high reassociation count when only four to 25 or 36 APs are deployed. This must be contributed by the extremely high degree of competition in that range [see Fig. 8(b)]. When the number of APs is further increased, the total reassociation count does not rise further but rather declines slightly. This can be justified as the extremely low degree of competition cancels out the trend of increasing reassociations due to the increase in the expected number of choices. Consequently, WSs experience even fewer reassociations to reach Nash equilibria.

2) For a fixed number of APs, the expected number of choices is fixed, whereas the expected degree of competition is proportional to the population of WSs [see Fig. 8(a)]. When the number of WSs is small, modest competitions and few reassociations are observed. As more WSs are involved, competitions among WSs become intense, giving rise to more interactive reassociations. Consequently, total reassociation count roughly increases with the number of WSs. The increasing rate with the PIF model, however, is generally higher than that with our game model, as Fig. 9 indicates.

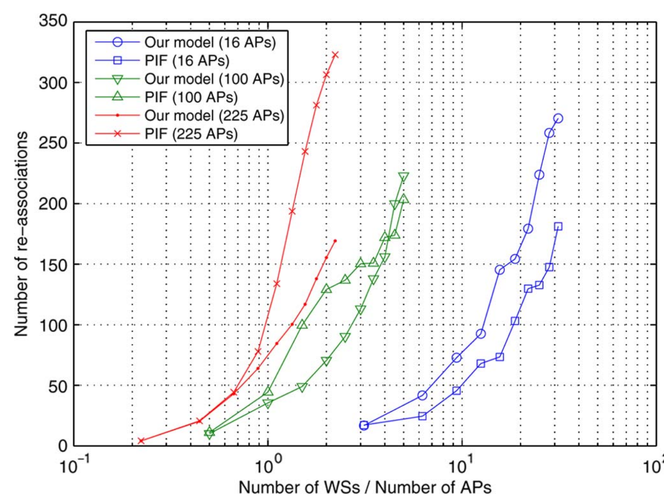



Fig. 9. Comparison of reassociation increase rate between the proposed game and the PIF models.

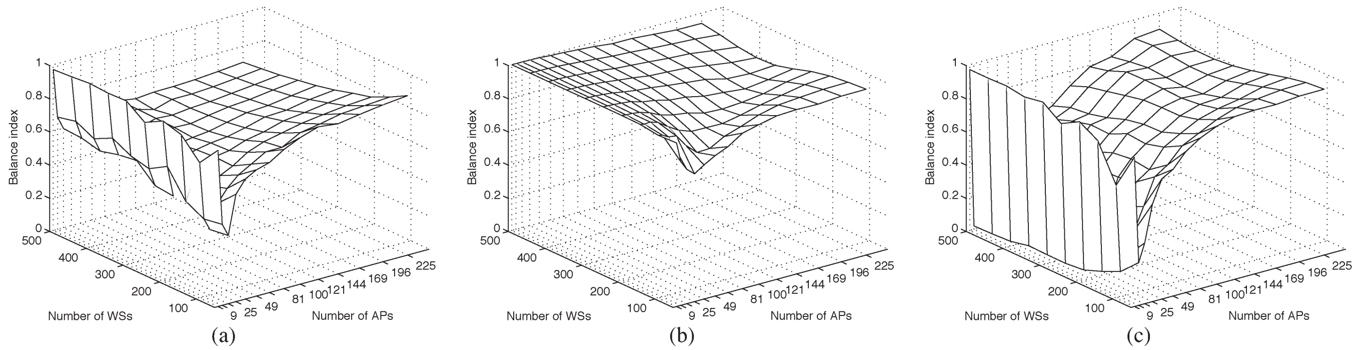



Fig. 10. Balance index (a) after the initial RSS-based associations and after the stop of reassociation activities in (b) the proposed game and (c) the PIF models.

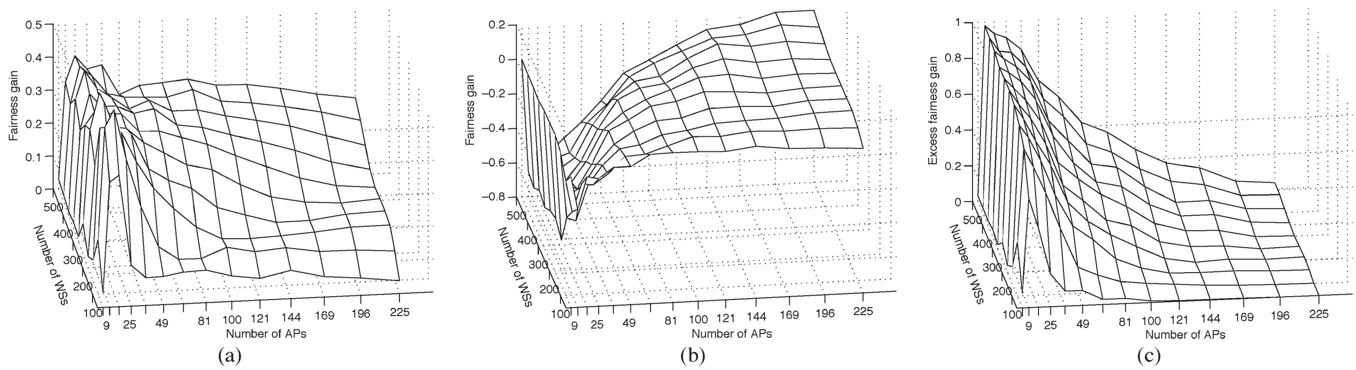



Fig. 11. Fairness gains by reassociations in (a) the proposed game model and (b) the PIF model. (c) Excess fairness gain by the proposed game over PIF.

## B. Balance Index

We also measured balance indices for both AP selection models. For each trial, the balance index was measured after the initial RSS-based associations as well as after all reassociation activities stop. Fig. 10(a) displays the balance indices measured after the initial RSS-based associations. We can see that, when only four APs are deployed, the RSS-based association policy results in rather high balance indices. This can be understood as, when only few APs are accessible to numbers of WSs, all APs are overpopulated and offer a similar amount of bandwidth share (which is extreme low) to each WS. When more APs are deployed, the workloads of APs become diverse due to the fact that RSS-based association policy is not load aware. Different WSs therefore receive different amounts of achievable throughput, which explains the sharp drop of the balance index when the number of APs is increased from four to 16. When the number of APs is further increased or when the number of WSs is decreased, the expected degree of competition and, thus, the impact of performance anomaly both lessen. It turns out that the difference of achievable throughputs among WSs diminishes. This justifies both the rise of balance indices with the number of APs and the descent of balance indices with the number of WSs in the right half of Fig. 10(a).

Fig. 10(b) shows balance indices measured after the stop of the proposed game model, whereas Fig. 10(c) shows the same results for the PIF model. Observe that the game model raises the balance indices for all experimental settings, but the PIF model does not. Although we only prove that reassociations improve lexicographical fairness, simulation results reveal that fairness in terms of balance index also benefits from reassociation activities in the proposed game.

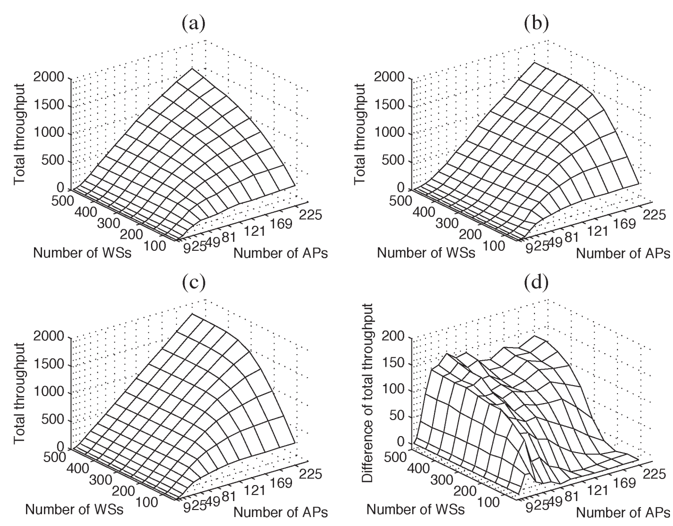



Fig. 12. (a) Aggregated throughputs after the initial RSS-based associations. (b) Aggregated throughputs after the stop of the proposed game model. (c) Aggregated throughputs after the stop of the PIF model. (d) Aggregated throughput of the PIF model minus that of the proposed game model.

For each trial, the difference of the balance indices between the initial and the final association configurations can be viewed as the gain of fairness by reassociations in the trial. Fig. 11(a) displays the average result for the proposed game model. We observe all nonnegative gains, with the maximum, average value, and standard deviation of 0.558, 0.240, and 0.126, respectively. The gain generally increases with the number of WSs, particularly when adequate APs are provided. The result of the PIF model is shown in Fig. 11(b), where we observe negative to positive fairness gains. The minimal value, average

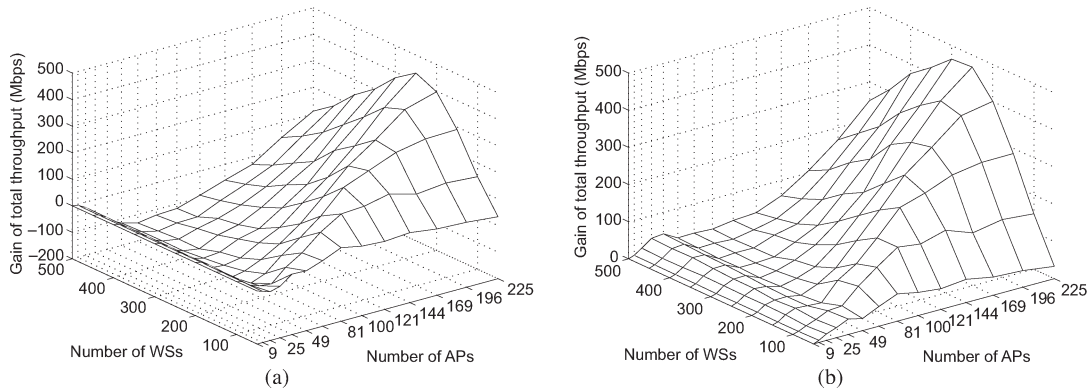



Fig. 13. Gain of aggregated throughput due to reassociations in (a) the proposed game model and (b) the PIF model.

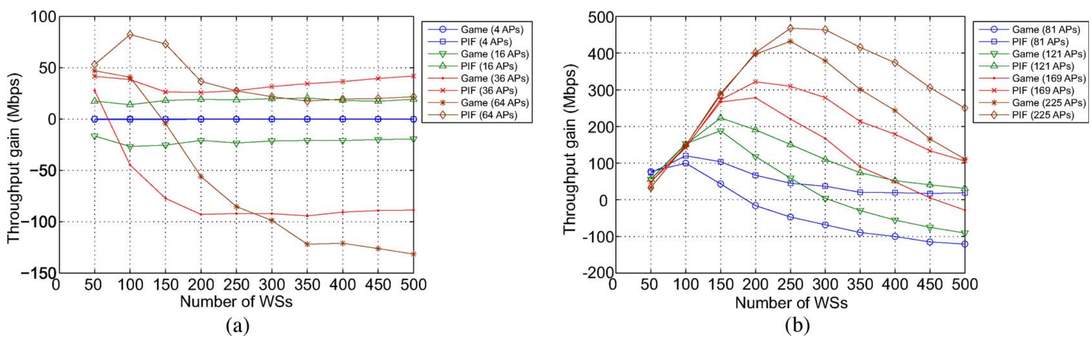



Fig. 14. Comparison of throughput gains between the proposed game and the PIF models.

value, and standard deviation of the gain are $-0.679$, $-0.085$, and 0.238, respectively. Fig. 11(c) shows excess fairness gains by the proposed game over the PIF model. We found that the superiority of the proposed game over the PIF becomes more significant when fewer APs are introduced. This trend is generally consistent with the behavior exhibited by the expected degree of competition (see Fig. 8).

### *C. Aggregated Throughput*

Aggregated throughput (counting all WSs) was also investigated. The results of the RSS-based association policy, the proposed game model, and the PIF model are shown in Fig. 12(a)–(c). Clearly, the PIF model outperforms the others. If a configuration is Pareto optimal, then it must have the highest aggregated throughput among all of them. Therefore, the superiority of the PIF model over our game model further confirms that Nash equilibria in the proposed game are typically not Pareto optimal. Fig. 12(d) shows the excess of the PIF model over the proposed game model in terms of aggregated throughput.

For each trial, the difference of the aggregated throughputs between the initial and the final association configurations is viewed as the gain of aggregated throughput by reassociations in the trial. Fig. 13 shows the gains of aggregated throughput due to reassociations in the proposed game model and in the PIF model. We can see that the PIF model yields all-positive gains, whereas the proposed game model does not necessarily improve aggregated throughput. Fig. 14 compares throughput gains between the proposed game and the PIF models. When few APs are deployed, the difference of the gain between these two models is either negligible (four APs) or nearly a constant (nine or 16 APs). When more APs are deployed, the throughput gains in both models depend on the number of WSs. For a specific number of APs, there is an optimal number of WSs for which the gain of aggregated throughput due to reassociations is maximized. Deviation from this value diminishes the gain and might even degrade the aggregated throughput (in case of the game model). The optimal number of WSs for 81, 121, 169, and 225 APs are 100, 150, 200, and 250, respectively. For 49 or fewer APs, the optimal number of WSs is smaller than 50. Thus, the results only exhibit a decrease in throughput gain with the number of WSs. This phenomenon can be explained as, when not too many WSs are engaged in a bandwidth competition, the RSS-based association policy fails to fully exploit potential bandwidth collectively offered by all APs, leaving much space for both reassociation models to improve. When many WSs are introduced to the access network such that few APs are lightly loaded, the RSS-based association policy leaves little space for reassociations to improve. Thus, the throughput gains decline. In particular, a WS in the proposed game is likely to increase its throughput through reassociations at the price of decreasing other WS's throughput. Overall, throughput therefore may suffer from such reassociations.

In both models, the maximal gain that can be obtained roughly increases with the number of APs. This is reasonable as more APs provide more potential bandwidth. With 81 or more APs, the proposed game behaves like the PIF model if not too many WSs are involved. Their difference emerges when more WSs are added and increases with the number of WSs. The PIF model performs better than the game model in finding potential bandwidth the whole access network can provide.

## V. Conclusion

This paper has proposed and analyzed an AP selection game where WSs select APs merely to maximize their achievable throughput. We have proven that Nash equilibria exist in such games with the effect of performance anomaly on achievable throughput considered, which guarantees the convergence of configuration transitions. Furthermore, we have shown that association transitions triggered by selfish WSs in fact improve fairness of bandwidth share, which was not previously expected. We have conducted extended simulations to study the properties of the proposed game and compared the results with those of the PIF reassociation model. The results confirm that the number of association transitions in the proposed game is always limited and generally smaller than that in the PIF model. The proposed game also results in higher bandwidth fairness (in terms of balance index) than the PIF model in all settings. Concerning aggregated throughput, the PIF model always improves the results of the RSS-based association policy, whereas the proposed game model does not always yield positive improvements.

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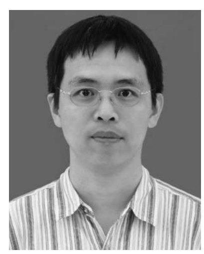

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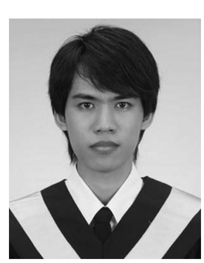

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