CSE 150. Assignment 1

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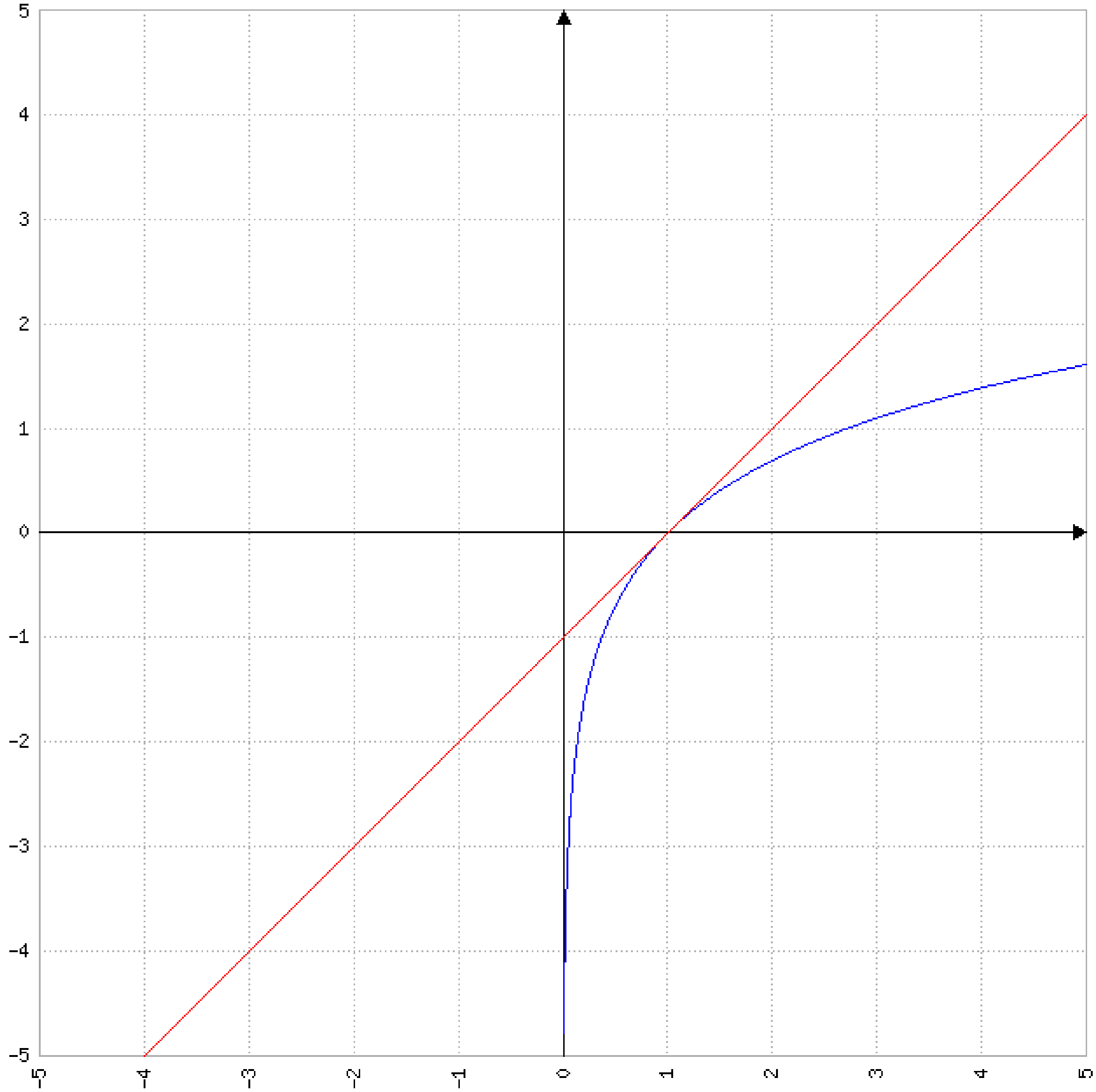
1 Kullback-Leibler distance



1. We can see with this graph that the function than $log(x)$ <= $x - 1$.

   Differentiation of $log(x) - (x - 1)$:

   $f(x) = log(x) - (x - 1)$

   $f'(x) = 1/x * ln(e) - (1 - 0)$
   $f'(x) = (1/x) - 1$

x	0		1		$+\infty$
$f'(x)$		$+$	0	$-$	
$f(x)$	-∞	$\nearrow$	0	$\searrow$	-∞

   We then can conclude that $log(x) <= x - 1$

2. If $p_i = q_i$ then :

   $KL(p, p) = \sum_i p_i log(p_i/p_i)$
   $KL(p, p) = \sum_i p_i log(1)$
   $KL(p, p) = \sum_i p_i * 0$
   $KL(p, p) = 0$

   Then $KL(p, q) >= 0$ if $p_i = q_i$

3.

2 Conditional independence

Let :

$P(X, Y|Z) = \frac{P(X,Y,Z)}{P(Z)}$

$P(X, Y|Z) = \frac{P(X,Y,Z)}{P(Y,Z)} * \frac{P(Y,Z)}{P(Z)}$

$P(X, Y|Z) = P(X|Y, Z) * \frac{P(Y,Z)}{P(Z)}$

$P(X, Y|Z) = P(X|Y, Z) * P(Y, Z)$

As $P(X|Y, Z) = P(X|Z)$ then :

$P(X, Y|Z) = P(X|Z) * P(Y, Z)$

The three statements are equivalent.

3 Creative writing

1. Lets choose the random variables as follow :

   X = Burglary ?
   Y = Yann calls ?
   Z = Alarm ?

   So that
   $P(X=1|Y=1) > P(X=1)$,
   $P(X=1|Y=1,Z=1) < P(X=1|Y=1)$

2. Lets choose the random variables as follow :

   X = Alarm ?
   Y = Yann calls ?
   Z = Zoe calls ?

   So that
   $P(X=1) < P(X=1|Y=1) < P(X=1|Y=1,Z=1)$

3. Lets choose the random variables as follow :

   X = Alarm ?
   Y = Earthquake ?
   Z = Zoe calls ?

   So that
   $P(X=1,Y=1) = P(X=1)P(Y=1)$
   $P(X=1,Y=1|Z=1) = P(X=1|Z=1)P(Y=1|Z=1)$

4 Probabilistic inference

Conditional independence assumptions :
$P(B|E) = P(B)$
$P(J|A) = P(J|A, B, E)$
$P(M|A) = P(M|A, B, E, J)$

a)

$$P(B = 1|M = 1) = \frac{P(M = 1|B = 1)P(B = 1)}{P(M = 1)}$$

$$\begin{aligned}
P(M = 1) &= \sum_{a,b,e,j} P(M = 1, A = a, B = b, E = e, J = j) \\
&= \sum_{a} P(M = 1, A = a) \\
&= \sum_{a} P(M = 1|A = a)P(A = a) \\
&= P(M = 1|A = 0)P(A = 0) + P(M = 1|A = 1)P(A = 1) \\
&= 0,01 * P(A = 0) + 0,70 * P(A = 1)
\end{aligned}$$

$$\begin{aligned}
P(A = 1) &= \sum_{b,e} P(E = e, B = b, A = 1) \\
&= \sum_{b,e} P(E = e)P(B = b|E = e)P(A = 1|E = e, B = b) \\
&= \sum_{b,e} P(E = e)P(B = b)P(A = 1|E = e, B = b) \\
&= P(E = 0)P(B = 0|E = 0)P(A = 1|E = 0, B = 0) + P(E = 1)P(B = 0|E = 1)P(A = 1|E = 1, B = 0)+ \\
&P(E = 0)P(B = 1|E = 0)P(A = 1|E = 0, B = 1) + P(E = 1)P(B = 1|E = 1)P(A = 1|E = 1, B = 1) \\
&= (1 - 0,002)(1 - 0,001)(0,001) + (0,002)(1 - 0,001)(0,29)+ \\
&(1 - 0,002)(0,001)(0,94) + (0,002)(0,001)(0,95) \\
&= 0,00252
\end{aligned}$$

$$\begin{aligned}
P(M = 1) &= 0,01 * P(A = 0) + 0,70 * P(A = 1) \\
&= 0,01 * (1 - 0,00252) + 0,70 * 0,00252 \\
&= 0,01174
\end{aligned}$$

$$
\begin{aligned}
P(M=1|B=1) &= \sum_a P(M=1, A=a|B=1) \\
&= \sum_a P(M=1|A=a, B=1)P(A=a|B=1) \\
&= \sum_a P(M=1|A=a)P(A=a|B=1)
\end{aligned}
$$

$$
\begin{aligned}
P(A=1|B=1) &= \sum_e P(A=1, E=e|B=1) \\
&= \sum_e P(A=1|E=e, B=1)P(E=e, B=1) \\
&= \sum_e P(A=1|E=e, B=1)P(E=e) \\
&= P(A=1|E=0, B=1)P(E=0) + P(A=1|E=1, B=1)P(E=1) \\
&= (0,94)(1-0,002) + (0,95)(0,002) \\
&= 0,94002
\end{aligned}
$$

$$
\begin{aligned}
P(M=1|B=1) &= \sum_a P(M=1|A=a)P(A=a|B=1) \\
&= P(M=1|A=0)P(A=0|B=1) + P(M=1|A=1)P(A=1|B=1) \\
&= (0,01)(1-0,94002) + (0,70)(0,94002) \\
&= 0,65861
\end{aligned}
$$

$$
\begin{aligned}
P(B=1|M=1) &= \frac{P(M=1|B=1)P(B=1)}{P(M=1)} \\
&= \frac{0,65861 * 0,001}{0,01174} \\
&= 0,0561
\end{aligned}
$$

b)

$$
\begin{aligned}
P(B=1|M=1,E=1) &= \frac{P(M=1|B=1,E=1)P(B=1|E=1)}{P(M=1|E=1)} \\
&= \frac{P(M=1|B=1,E=1)P(B=1)}{P(M=1|E=1)}
\end{aligned}
$$

$$
\begin{aligned}
P(M=1|E=1) &= \sum_a P(M=1,A=a|E=1) \\
&= \sum_a P(M=1|A=a|E=1)P(A=a|E=1) \\
&= \sum_a P(M=1|A=a|E=1)P(A=a) \\
&= P(M=1|A=0)P(A=0) + P(M=1|A=1)P(A=1) \\
&= (0,01)(1-0,00252) + (0,70)(0,00252) \\
&= 0,01174
\end{aligned}
$$

$$
\begin{aligned}
P(M=1|B=1,E=1) &= P(M=1|B=1) \\
&= 0,65861
\end{aligned}
$$

$$
\begin{aligned}
P(B=1|M=1,E=1) &= \frac{P(M=1|B=1,E=1)P(B=1)}{P(M=1|E=1)} \\
&= \frac{0,65861 * 0,001}{0,01174} \\
&= 0,0561
\end{aligned}
$$

$P(B=1|M=1) = P(B=1|M=1,E=1)$

c)

$$P(A=1|J=0)=\frac{P(J=0|A=1)P(A=1)}{P(J=0)}$$

$$\begin{aligned}
P(J=0) &= \sum_a P(J=0,A=a) \\
&= \sum_a P(J=0|A=a)P(A=a) \\
&= P(J=0|A=0)P(A=0)+P(J=0|A=1)P(A=1) \\
&= (1-0,05)(1-0,00252)+(1-0,90)(0,00252) \\
&= 0,94786
\end{aligned}$$

$$\begin{aligned}
P(A=1|J=0) &= \frac{P(J=0|A=1)P(A=1)}{P(J=0)} \\
&= \frac{(1-0,90)(0,00252)}{0,94786} \\
&= 0,00027
\end{aligned}$$

d)

$$\begin{aligned}
P(A=1|J=0,M=1) &= \frac{P(J=0,M=1|A=1)P(A=1)}{P(J=0,M=1)}\\
&= \frac{P(J=0|A=1)P(M=1|A=1,J=0)P(A=1)}{P(J=0,M=1)}\\
&= \frac{P(J=0|A=1)P(M=1|A=1)P(A=1)}{P(J=0,M=1)}\\
&= \frac{(1-0,90)(0,70)(0,00252)}{P(J=0,M=1)}
\end{aligned}$$

$$\begin{aligned}
P(J=0,M=1) &= \sum_a P(A=a,J=0,M=1)\\
&= \sum_a P(A=a)P(J=0|A=a)P(M=1|A=a,J=0)\\
&= \sum_a P(A=a)P(J=0|A=a)P(M=1|A=a)\\
&= P(A=0)P(J=0|A=0)P(M=1|A=0)+\\
&\quad P(A=1)P(J=0|A=1)P(M=1|A=1)\\
&= (1-0,00252)(1-0,05)(0,01)+(0,00252)(1-0,90)(0,70)\\
&= 0,00964
\end{aligned}$$

$$\begin{aligned}
P(A=1|J=0,M=1) &= \frac{(1-0,90)(0,70)(0,00252)}{P(J=0,M=1)}\\
&= \frac{(1-0,90)(0,70)(0,00252)}{0,00964}\\
&= 0,0183
\end{aligned}$$

$P(A=1|J=0) > P(A=1|J=0,M=1)$

e)

$$\begin{aligned}
P(A=1|B=0) &= \sum_e P(A=1,E=e|B=0)\\
&= \sum_e P(A=1|E=e,B=0)P(E=e,B=0)\\
&= \sum_e P(A=1|E=e,B=0)P(E=e)\\
&= P(A=1|E=0,B=0)P(E=0)+P(A=1|E=1,B=0)P(E=1)\\
&= (0,001)(1-0,002)+(0,29)(0,002)\\
&= 0,001578
\end{aligned}$$

f)

$$P(A=1|B=0,M=1) = \frac{P(M=1|A=1,B=0)P(A=1|B=0)}{P(M=1|B=0)}$$

$$\begin{aligned}
P(M=1|B=0) &= \sum_a P(M=1,A=a|B=0) \\
&= \sum_a P(A=a|B=0)P(M=1|A=a,B=0) \\
&= P(A=0|B=0)P(M=1|A=0,B=0)+ \\
&P(A=1|B=0)P(M=1|A=1,B=0) \\
&= P(A=0|B=0)(0,01)+(0,001578)(0,70)
\end{aligned}$$

$$\begin{aligned}
P(A=0|B=0) &= \sum_e P(A=0,E=e|B=0) \\
&= \sum_e P(A=0|E=e,B=0)P(E=e,B=0) \\
&= \sum_e P(A=0|E=e,B=0)P(E=e) \\
&= P(A=0|E=0,B=0)P(E=0)+P(A=0|E=1,B=0)P(E=1) \\
&= (1-0,001)(1-0,002)+(1-0,29)(0,002) \\
&= 0,998422
\end{aligned}$$

$$\begin{aligned}
P(M=1|B=0) &= P(A=0|B=0)(0,01)+(0,001578)(0,70) \\
&= (0,998422)(0,01)+(0,001578)(0,70) \\
&= 0,01109
\end{aligned}$$

$$\begin{aligned}
P(A=1|B=0,M=1) &= \frac{P(M=1|A=1,B=0)P(A=1|B=0)}{P(M=1|B=0)} \\
&= \frac{0,70 * 0,001578}{0,01109} \\
&= 0,09960
\end{aligned}$$

$P(A=1|B=0,M=1) > P(A=1|B=0)$

All the results above seems consistent with commonsense patterns or reasoning.