CSE 150. Assignment 2

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1 Probabilistic reasoning

$$
\begin{aligned}
r_k &= \frac{P(X=1|Y_1=1,Y_2=1,...,Y_k=1)}{P(X=0|Y_1=1,Y_2=1,...,Y_k=1)} \\
&= \frac{\frac{P(X=1)*\prod_{i=2}^{k} P(Y_i=1|X=1)}{P(Y_1=1,Y_2=1,...,Y_k=1)}}{\frac{P(X=0)*\prod_{i=1}^{k} P(Y_i=1|X=0)}{P(Y_1=1,Y_2=1,...,Y_k=1)}} \\
&= \frac{\prod_{i=2}^{k} P(Y_i=1|X=1)}{\prod_{i=1}^{k} P(Y_i=1|X=0)} \\
&= \frac{\prod_{i=2}^{k} \frac{2^{i-1}+(-1)^{i-1}}{2^i+(-1)^i}}{\prod_{i=1}^{k} 1-(1/2)}
\end{aligned}
$$

Here are some values of x computed with $f(x)$:

x	$f(x)$
1	1.000
2	0.8
3	1.142
4	0.940
5	1.030
...	...
31	1.000

It depends on the day of the month due to the product for $i = 2$ and $i = 1$ on both side of the division, to k.
The diagnosis seems to become less certain while k becomes bigger. As a matter of fact, the more symptoms you have, the more uncertain it is to know what is the disease. Assuming all the symptoms match the two forms of the disease.

2 Noisy-OR

1. $P(Z=1|X=0,Y=1) > P(Z=1|X=0,Y=0)$

$$\begin{aligned} P(Z=1|X=0,Y=1) &= 1-(1-Px)^0 * (1-Py)^1 \\ &= 1-(1)(1-Py) \\ &= 1-(1-Py) \\ &= 1-1+Py \\ &= Py \end{aligned}$$

$$\begin{aligned} P(Z=1|X=0,Y=0) &= 1-(1-Px)^0 * (1-Py)^0 \\ &= 1-(1)*(1) \\ &= 0 \end{aligned}$$

2. $P(Z=1|X=1,Y=0) < P(Z=1|X=0,Y=1)$

$$\begin{aligned} P(Z=1|X=1,Y=0) &= 1-(1-Px)^1 * (1-Py)^0 \\ &= 1-(1-Px)(1) \\ &= Px \end{aligned}$$

$$P(Z=1|X=0,Y=1) = Py$$

3. $P(Z=1|X=1,Y=1) > P(Z=1|X=1,Y=0)$

$$\begin{aligned} P(Z=1|X=1,Y=1) &= 1-(1-Px)^1 * (1-Py)^1 \\ &= 1-(1-Px)(1-Py) \\ &= 1-(1-Py-Px+(Px*Py)) \\ &= Py+Px-(Px*Py) \end{aligned}$$

$$P(Z=1|X=1,Y=0) = Px$$

3 Conditional independence

1. True
2. False
3. True
4. True
5. True
6. False
7. True
8. False
9. False
10. False

4 Subsets

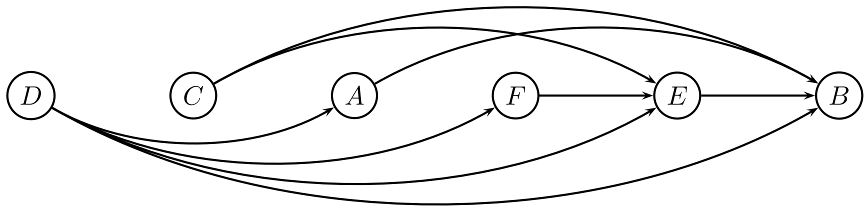
1. $P(A) = P(A|E, C, F)$
2. $P(A|B) = P(A|B, E, C, F)$
3. $P(A|B, D) = P(A|B, D, E, C, F)$
4. $P(B) = P(B|F, D)$
5. $P(B|A, E) = P(B|A, E, F, D)$
6. $P(B|A, C, E) = P(B|A, C, E, F, D)$
7. $P(C) = P(C|A)$
8. $P(C|E, F) = P(C|E, F, A)$
9. $P(C|B, D, E, F) = P(C|B, D, E, F, A)$
10. $P(E) = P(E|A, D, F)$

5 Node ordering

1. $P(D,C,A,F,E,B) = P(D)P(C|D)P(A|C,D)P(F|A,C,D)P(E|F,A,C,D)P(B|E,F,A,C,D)$

   $P(B|E,F,A,C,D) = P(B|E,A,C,D)$
   $P(E|F,A,C,D) = P(E|F,C,D)$
   $P(F|A,C,D) = P(F|D)$
   $P(A|C,D) = P(A|D)$
   $P(C|D) = P(C)$

   $P(D,C,A,F,E,B) = P(D)P(C)P(A|D)P(F|D)P(E|F,C,D)P(B|E,A,C,D)$

2. $P(C,A,F,B,D,E) = P(C)P(A|C)P(F|A,C)P(B|F,A,C)P(D|B,F,A,C)P(E|D,B,F,A,C)$

   $P(E|D,B,F,A,C) = P(E|D,B,F,C)$
   $P(D|B,F,A,C) = P(D|B,F,A)$
   $P(B|F,A,C) = P(B|A,C)$
   $P(F|A,C) = P(F)$
   $P(A|C) = P(A)$

   $P(C,A,F,B,D,E) = P(C)P(A)P(F)P(B|A,C)P(D|B,F,A)P(E|D,B,F,C)$