❖ Transpose of Matrix :

- The transpose of an $m \times n$ matrix $A = [a_{ij}]$ is defined as the $n \times m$ matrix $B = [b_{ij}]$, with $b_{ij} = a_{ij}$ for $1 \le i \le m$ and $1 \le j \le n$. The transpose of A is denoted by A^t.

- That is, by the transpose of an $m \times n$ matrix A, we mean a matrix of order $n \times m$ having the rows of A as its columns and the columns of A as its rows.

For example, if $A = \begin{bmatrix} 1 & 4 & 5 \\ 0 & 1 & 2 \end{bmatrix}$ then $A^t = \begin{bmatrix} 1 & 0 \\ 4 & 1 \\ 5 & 2 \end{bmatrix}$.

Thus, the transpose of a row vector is a column vector and vice-versa.

Theorem 1.2.2 For any matrix A, we have $(A^t)^t = A$.

PROOF. Let $A = [a_{ij}]$, $A^t = [b_{ij}]$ and $(A^t)^t = [c_{ij}]$. Then, the definition of transpose gives

$$c_{ij} = b_{ji} = a_{ij} \quad \text{for all} \quad i, j$$

and the result follows.