# MATRIX

## ❖ MULTIPLY BY A CONSTANT

- We can multiply a matrix by a **constant** *(the value 2 in this case)*:

2x4=8

$$2 \times \begin{bmatrix} 4 & 0 \\ 1 & -9 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 2 & -18 \end{bmatrix}$$

These are the calculations:

$2\times4=8 \quad 2\times0=0$

$2\times1=2 \quad 2\times-9=-18$

- We call the constant a **scalar**, so officially this is called "scalar multiplication".

## ❖ MULTIPLYING A MATRIX BY ANOTHER MATRIX

➢ But to multiply a matrix **by another matrix** we need to do the "**dot product**" of rows and columns ... what does that mean? Let us see with an example:

➢ To work out the answer for the **1st row** and **1st column**:

"Dot Product"

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} 58 & \\ & \end{bmatrix}$$

➢ The "Dot Product" is where we **multiply matching members**, then sum up:

- $(1, 2, 3) \cdot (7, 9, 11) = 1\times7 + 2\times9 + 3\times11 = 58$

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- We match the 1st members (1 and 7), multiply them, likewise for the 2nd members (2 and 9) and the 3rd members (3 and 11), and finally sum them up.

- Want to see another example? Here it is for the 1st row and **2nd column**:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} 58 & 64 \\ & \end{bmatrix}$$

  - (1, 2, 3) · (8, 10, 12) = 1×8 + 2×10 + 3×12 = 64

- We can do the same thing for the **2nd row** and **1st column**:

  (4, 5, 6) · (7, 9, 11) = 4×7 + 5×9 + 6×11 = 139

- And for the **2nd row** and **2nd column**:

  (4, 5, 6) · (8, 10, 12) = 4×8 + 5×10 + 6×12 = 154

- And we get:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} 58 & 64 \\ 139 & 154 \end{bmatrix}$$

DONE!

## ❖ Why Do It This Way?

- This may seem an odd and complicated way of multiplying, but it is necessary!

- I can give you a real-life example to illustrate why we multiply matrices in this way.

- Example: The local shop sells 3 types of pies.

  - Apple pies cost **$3** each
  - Cherry pies cost **$4** each
  - Blueberry pies cost **$2** each

- And this is how many they sold in 4 days:

| | Mon | Tue | Wed | Thu |
|---|---|---|---|---|
| *Apple* | 13 | 9 | 7 | 15 |
| *Cherry* | 8 | 7 | 4 | 6 |
| *Blueberry* | 6 | 4 | 0 | 3 |

- Now think about this ... the **value of sales** for Monday is calculated this way:

- Apple pie value + Cherry pie value + Blueberry pie value

  $3 × 13 + $4 × 8 + $2 × 6 = $83

- So it is, in fact, the "dot product" of prices and how many were sold:

($3, $4, $2) • (13, 8, 6) = $3×13 + $4×8 + $2×6 = $83

- We **match** the price to how many sold, **multiply** each, and then **sum** the result.

- In other words:

  - The sales for Monday were: Apple pies: **$3×13=$39**, Cherry pies: **$4×8=$32**, and Blueberry pies: **$2×6=$12**. Together that is $39 + $32 + $12 = **$83**
  - And for Tuesday: **$3×9 + $4×7 + $2×4 = $63**
  - And for Wednesday: **$3×7 + $4×4 + $2×0 = $37**
  - And for Thursday: **$3×15 + $4×6 + $2×3 = $75**

- So it is important to match each price to each quantity.

- Now you know why we use the "dot product".

- And here is the full result in Matrix form:

$$\begin{bmatrix}\$3 & \$4 & \$2\end{bmatrix} \times \begin{bmatrix}13 & 9 & 7 & 15\\ 8 & 7 & 4 & 6\\ 6 & 4 & 0 & 3\end{bmatrix} = \begin{bmatrix}\$83 & \$63 & \$37 & \$75\end{bmatrix}$$

$$\$3{\times}13 + \$4{\times}8 + \$2{\times}6$$

- They sold **$83** worth of pies on Monday, **$63** on Tuesday, etc. (You can put those values into the **Matrix Calculator** to see if they work.)

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## ❖ Rows and Columns

- To show how many rows and columns a matrix has we often write **rows×columns**.

- Example: This matrix is **2×3** (2 rows by 3 columns):

$$\begin{bmatrix} 6 & 4 & 24 \\ 1 & -9 & 8 \end{bmatrix}$$

- When we do multiplication:

- The number of **columns of the 1st matrix** must equal the number of **rows of the 2nd matrix**.

- And the result will have the same number of **rows as the 1st matrix**, and the same number of **columns as the 2nd matrix**.

- Example:

$$\begin{bmatrix} \$3 & \$4 & \$2 \end{bmatrix} \times \begin{bmatrix} 13 & 9 & 7 & 15 \\ 8 & 7 & 4 & 6 \\ 6 & 4 & 0 & 3 \end{bmatrix} = \begin{bmatrix} \$83 & \$63 & \$37 & \$75 \end{bmatrix}$$

$$\$3\text{x}13 + \$4\text{x}8 + \$2\text{x}6$$

- In that example we multiplied a 1×3 matrix by a 3×4 matrix (note the 3s are the same), and the result was a 1×4 matrix.

- *In General:*

To multiply an **m×n** matrix by an **n×p** matrix, the **n**s must be the same, and the result is an **m×p** matrix.

$$m \times n \; \times \; n \times p \; \rightarrow \; m \times p$$

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➢ So ... multiplying a **1×3** by a **3×1** gets a **1×1** result:

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 1\times4+2\times5+3\times6 \end{bmatrix} = \begin{bmatrix} 32 \end{bmatrix}$$

➢ But multiplying a **3×1** by a **1×3** gets a **3×3** result:

$$\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 4\times1 & 4\times2 & 4\times3 \\ 5\times1 & 5\times2 & 5\times3 \\ 6\times1 & 6\times2 & 6\times3 \end{bmatrix} = \begin{bmatrix} 4 & 8 & 12 \\ 5 & 10 & 15 \\ 6 & 12 & 18 \end{bmatrix}$$

## ❖ IDENTITY MATRIX

➢ The "Identity Matrix" is the matrix equivalent of the number "1":

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

➢ A 3×3 Identity Matrix

- It is "square" (has same number of rows as columns)
- It can be large or small (2×2, 100×100, ... whatever)
- It has **1**s on the main diagonal and **0**s everywhere else
- Its symbol is the capital letter **I**

➢ It is a **special matrix**, because when we multiply by it, the original is unchanged:

$$A \times I = A$$

$$I \times A = A$$

## ❖ Order of Multiplication

- In arithmetic we are used to:

$3 \times 5 = 5 \times 3$
(The Commutative Law of Multiplication)

- But this is **not** generally true for matrices (matrix multiplication is **not commutative**):

$AB \neq BA$

- When we change the order of multiplication, the answer is (usually) **different**.

- Example:
    - See how changing the order affects this multiplication:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1\times2+2\times1 & 1\times0+2\times2 \\ 3\times2+4\times1 & 3\times0+4\times2 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 10 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2\times1+0\times3 & 2\times2+0\times4 \\ 1\times1+2\times3 & 1\times2+2\times4 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 7 & 10 \end{bmatrix}$$

- The answers are different!

- It can have the same result (such as when one matrix is the Identity Matrix) but not usually.