SET THEORY



Power Set:

- → Given the set $S = \{0, 1\}$. What are all the possible subsets of S? They are: \emptyset (as it is a subset of all sets), $\{0\}$, $\{1\}$, and $\{0, 1\}$.
- The power set of S written as P(S) is the set of all the subsets of S $P(S) = \{\emptyset, \{0\}, \{1\}, \{0,1\}\}.$
 - Note that |S| = 2 and |P(S)| = 4.
- → The *power set* P(S) of a set S is the set of all subsets of S. $P(S) = \{x \mid x \subseteq S\}$.
- → E.g. $P(\{a, b\}) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}.$
- \rightarrow Sometimes P(S) is written 2^{s} .
 - Note that for finite S, $|P(S)| = 2^{|S|}$.
- → It turns out that |P(N)| > |N|.

 There are different sizes of infinite sets!
- → Let T = {0, 1, 2}. The P(T) = {∅, {0}, {1}, {2}, {0,1}, {0,2}, {1,2}, {0,1,2}}.
 - Note that |T| = 3 and |P(T)| = 8
- \rightarrow P(\varnothing) = { \varnothing }
 - Note that $|\emptyset| = 0$ and $|P(\emptyset)| = 1$
- \rightarrow If a set has n elements, then the power set will have 2^n elements.