



## ❖ Union of Sets :

- ➔ The **union** of two or more sets contains all the elements in all the sets.
- ➔ The union of sets **A** and **B** is the elements in either set **A** or **B** or **both**.
- ➔ Unions of sets are denoted by the symbol  $\cup$ .
- What is the union of set **A**, the outcomes of rolling a dice, and set **B**, all positive integers between 7 and 11?
  - set **A** = {1, 2, 3, 4, 5, 6}
  - set **B** = {7, 8, 9, 10, 11}
  - $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$
- ➔ For sets **A**, **B**, their **union**  $A \cup B$  is the set containing all elements that are either in **A**, or in **B** (or, of course, in both).
- ➔ Formally,  $\forall A, B: A \cup B = \{x \mid x \in A \vee x \in B\}$ .
- ➔ Note that  $A \cup B$  contains all the elements of **A** and it contains all the elements of **B**:  
 $\forall A, B: (A \cup B \supseteq A) \wedge (A \cup B \supseteq B)$ 
  - $\{a, b, c\} \cup \{2, 3\} = \{a, b, c, 2, 3\}$
  - $\{2, 3, 5\} \cup \{3, 5, 7\} = \{2, 3, 5, 3, 5, 7\} = \{2, 3, 5, 7\}$
- ➔ Formal definition for the union of two sets:  
 $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$
- ➔ Further Examples
  - $\{1, 2, 3\} \cup \{3, 4, 5\} = \{1, 2, 3, 4, 5\}$
  - $\{\text{New York, Washington}\} \cup \{3, 4\} = \{\text{New York, Washington, 3, 4}\}$
  - $\{1, 2\} \cup \emptyset = \{1, 2\}$

### ● Properties of the union operation

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|---|-----------------|
| ▪ $A \cup \emptyset = A$                  | Identity law    |
| ▪ $A \cup U = U$                          | Domination law  |
| ▪ $A \cup A = A$                          | Idempotent law  |
| ▪ $A \cup B = B \cup A$                   | Commutative law |
| ▪ $A \cup (B \cup C) = (A \cup B) \cup C$ | Associative law |