❖ INVERSE OF A MATRIX USING MINORS, COFACTORS AND ADJUGATE

➢ We can calculate the Inverse of matrix by:

- Step 1: calculating the Matrix of Minors,
- Step 2: then turn that into the Matrix of Cofactors,
- Step 3: then the Adjugate, and
- Step 4: multiply that by 1/Determinant.

But it is best explained by working through an example!

➢ **Example: find the Inverse of A:**

$$A = \begin{bmatrix} 3 & 0 & 2 \\ 2 & 0 & -2 \\ 0 & 1 & 1 \end{bmatrix}$$

It needs 4 steps. It is all simple arithmetic but there is a lot of it, so try not to make a mistake!

➢ **Step 1: Matrix of Minors**

The first step is to create a "Matrix of Minors". This step has the most calculations.

For each element of the matrix:

- Ignore the values on the current row and column
- Calculate the determinant of the remaining values

Put those determinants into a matrix (the "Matrix of Minors")

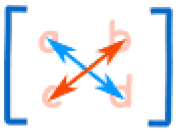
MATRIX

❖ DETERMINANT

➢ For a 2×2 matrix (2 rows and 2 columns) the determinant is easy: **ad-bc**

Think of a cross:

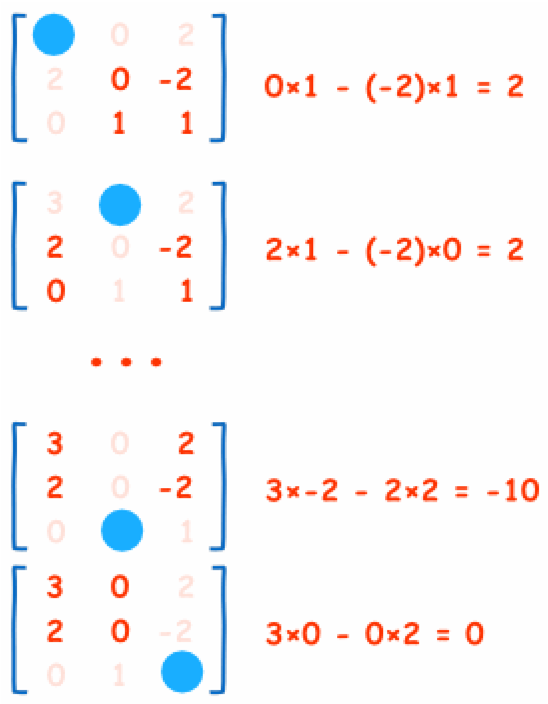
- Blue means positive (+ad),
- Red means negative (-bc)



(It gets harder for a 3×3 matrix, etc)

➢ The Calculations

Here are the first two, and last two, calculations of the "**Matrix of Minors**" (notice how I ignore the values in the current row and columns, and calculate the determinant using the remaining values):



And here is the calculation for the whole matrix:

- **Step 2: Matrix of Cofactors**

  This is easy! Just apply a "checkerboard" of minuses to the "Matrix of Minors". In other words, we need to change the sign of alternate cells.

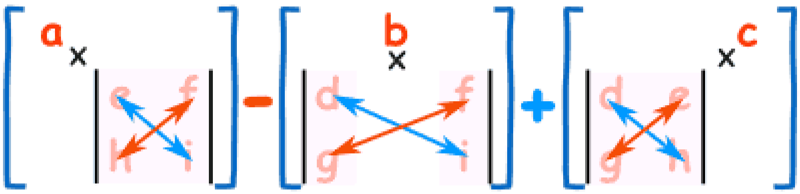
- **Step 3: Adjugate (also called Adjoint)**

  Now "Transpose" all elements of the previous matrix... in other words swap their positions over the diagonal (the diagonal stays the same):

$$\begin{bmatrix} 2 & 2 & 0 \\ -2 & 3 & 10 \\ 2 & -3 & 0 \end{bmatrix}$$

- **Step 4: Multiply by 1/Determinant**

  Now find the determinant of the original matrix. This isn't too hard, because we already calculated the determinants of the smaller parts when we did "Matrix of Minors".



- In practice we can just multiply each of the top row elements by the cofactor for the same location:

  Elements of top row: 3, 0, 2
  Cofactors for top row: 2, −2, 2

  Determinant = 3×2 + 0×(−2) + 2×2 = **10**

  (Just for fun: try this for any other row or column, they should also get 10.)

  And now multiply the Adjoint by 1/Determinant:

$$A^{-1} = \frac{1}{10}\underbrace{\begin{bmatrix} 2 & 2 & 0 \\ -2 & 3 & 10 \\ 2 & -3 & 0 \end{bmatrix}}_{\text{Adjugate}} = \underbrace{\begin{bmatrix} 0.2 & 0.2 & 0 \\ -0.2 & 0.3 & 1 \\ 0.2 & -0.3 & 0 \end{bmatrix}}_{\text{Inverse}}$$

And we are done!