MATRIX

❖ SOME SPECIAL TYPES OF MATRIX :

- **Equal Matrix :**

  - Two matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ having the same order $m \times n$ are equal if $a_{ij} = b_{ij}$ for each $i = 1, 2, . . . , m$ and $j = 1, 2, . . . , n$.

  - In other words, two matrices are said to be equal if they have the same order and their corresponding entries are equal.

- **Row Matrix and Column Matrix:**

  - A matrix consisting of a single row is called a row matrix or a row vector, whereas a matrix having single column is called a column matrix or a column vector.

- **Null or Zero Matrix:**

  - A matrix in which each element is "0" is called a Null or Zero matrix. Zero matrices are generally denoted by the symbol O. This distinguishes zero matrix from the real number 0.

  For example $O = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ is a zero matrix of order 2 x 4.

  - The matrix O_{mxn} has the property that for every matrix A_{mxn}, $A + O = O + A = A$

- **Square matrix:**

  - A matrix A having same numbers of rows and columns is called a square matrix. A matrix A of order m x n can be written as A_{mxn}. If $m = n$, then the matrix is said to be a square matrix. A square matrix of order n x n, is simply written as A_n.

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Thus $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$ and $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ are square matrix of order 2 and 3

- **Main or Principal (leading) Diagonal:**

  - The principal diagonal of a square matrix is the ordered set of elements a_{ij}, where $i = j$, extending from the upper left-hand corner to the lower right-hand corner of the matrix. Thus, the principal diagonal contains elements a_{11}, a_{22}, a_{33} etc.

  - For example, the principal diagonal of

$$\begin{bmatrix} 1 & 3 & -1 \\ 5 & 2 & 3 \\ 6 & 4 & 0 \end{bmatrix}$$

  - consists of elements 1, 2 and 0, in that order.

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- **Diagonal matrix:**

  - A square matrix in which all elements are zero except those in the main or principal diagonal is called a diagonal matrix. Some elements of the principal diagonal may be zero but not all.

For example $\begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

are diagonal matrices.

In general $$A = \begin{bmatrix} a_{11} & a_{12} & --- & a_{1n} \\ a_{21} & a_{22} & --- & a_{2n} \\ --- & --- & --- & --- \\ --- & --- & --- & --- \\ a_{n1} & a_{n2} & --- & a_{nn} \end{bmatrix} = (a_{ij})_{nxn}$$

is a diagonal matrix if and only if

$a_{ij} = 0$ for $i \neq j$

$a_{ij} \neq 0$ for at least one $i = j$

- **Scalar matrix:**

  - A diagonal matrix in which all the diagonal elements are same, is called a scalar matrix i.e.

Thus

$\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ and $\begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix}$ are scalar matrices ,

- **Identity or Unit matrix:**

  - A scalar matrix in which each diagonal element is 1(unity) is called a unit matrix. An identity matrix of order n is denoted by I_n.

Thus $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

are the identity matrices of order 2 and 3 .

In general, $A = \begin{bmatrix} a_{11} & a_{12} & ----a_{1n} \\ a_{21} & a_{22} & ----a_{2n} \\ --- & ---- & ------- \\ --- & ---- & ------- \\ a_{m1} & a_{m2} & ----a_{mn} \end{bmatrix} = [a_{ij}]_{mxn}$

is an identity matrix if and only if

$a_{ij} = 0$ for $i \neq j$ and $a_{ij} = 1$ for $i = j$

  - Note: If a matrix A and identity matrix I are comfortable for multiplication, then I has the property that AI = IA = A i.e., I is the identity matrix for multiplication.

- **Triangular (Lower / Upper) matrix:**

  - Triangular matrices: A square matrix with elements $s_{ij} = 0$ for $j < i$ is termed upper triangular matrix. In other words, a square matrix is upper triangular if all its entries below the main diagonal are zero.

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- Example of a 2 × 2 upper triangular matrix:
  $$A = \begin{bmatrix} u_{11} & u_{12} \\ 0 & u_{22} \end{bmatrix}$$

- A square matrix with elements $s_{ij} = 0$ for $j > i$ is termed lower triangular matrix. In other words, a square matrix is lower triangular if all its entries above the main diagonal are zero.

- Diagonal matrices are both upper and lower triangular since they have zeroes above and below the main diagonal.

- The inverse of a lower triangular matrix is also lower triangular.

- The product of two or more lower triangular matrices is also lower triangular.

- The transpose of a lower triangular matrix is upper triangular.

- The inverse of an upper triangular matrix is also upper triangular.

- The product of two or more upper triangular matrices is also upper triangular.

- The transpose of an upper triangular matrix is lower triangular.