Determinant of a Matrix

- The determinant of a matrix is a **special number** that can be calculated from a square matrix.

- A matrix is an array of numbers:

$$\begin{bmatrix} 3 & 8 \\ 4 & 6 \end{bmatrix}$$

A Matrix
(This one has 2 Rows and 2 Columns)

- The determinant of that matrix is (calculations are explained later):

$3 \times 6 - 8 \times 4 = 18 - 32 = \mathbf{-14}$

What is it for?

- The determinant helps us find the inverse of matrix, tells us things about the matrix that are useful in systems of linear equations, calculus and more.

Symbol

- The **symbol** for determinant is two vertical lines either side.
- Example:

|A| means the determinant of the matrix **A**

(Exactly the same symbol as absolute value.)

Calculating the Determinant

- First of all the matrix must be **square** (i.e. have the same number of rows as columns). Then it is just basic arithmetic. Here is how:

MATRIX

For a 2×2 Matrix

- For a 2×2 matrix (2 rows and 2 columns):

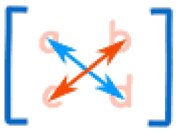
$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

- The determinant is:

$$|A| = ad - bc$$

"The determinant of A equals a times d minus b times c"

It is easy to remember when you think of a cross:



- Blue is positive (+ad),
- Red is negative (−bc)

- Example:

$$B = \begin{bmatrix} 4 & 6 \\ 3 & 8 \end{bmatrix}$$

$$\begin{aligned} |B| &= 4\times8 - 6\times3 \\ &= 32 - 18 \\ &= 14 \end{aligned}$$

MATRIX

For a 3×3 Matrix

- For a 3×3 matrix (3 rows and 3 columns):

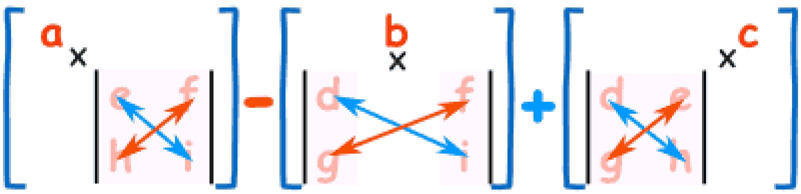
$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

- The determinant is:

$$|A| = a(ei - fh) - b(di - fg) + c(dh - eg)$$

"The determinant of A equals ... etc"

- It may look complicated, but **there is a pattern**:



- To work out the determinant of a **3×3** matrix:
  - Multiply **a** by the **determinant of the 2×2 matrix** that is **not in a**'s row or column.
  - Likewise for **b**, and for **c**
  - Sum them up, but remember the minus in front of the **b**

- As a formula *(remember the vertical bars || mean "determinant of")*:

$$|A| = a \cdot \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \cdot \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \cdot \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

"The determinant of A equals a times the determinant of ... etc"

- Example:

$$C = \begin{bmatrix} 6 & 1 & 1 \\ 4 & -2 & 5 \\ 2 & 8 & 7 \end{bmatrix}$$

$$|C| = 6\times(-2\times7 - 5\times8) - 1\times(4\times7 - 5\times2) + 1\times(4\times8 - (-2\times2))$$
$$= 6\times(-54) - 1\times(18) + 1\times(36)$$
$$= \mathbf{-306}$$

Not The Only Way

- This method of calculation is called the "Laplace expansion" and I like it because the pattern is easy to remember. But there are other methods (just so you know).

Summary

- For a 2×2 matrix the determinant is **ad - bc**
- For a 3×3 matrix multiply **a** by the **determinant of the 2×2 matrix** that is **not** in **a**'s row or column, likewise for **b** and **c**, but remember that **b** has a negative sign!
- The pattern continues for larger matrices: multiply **a** by the **determinant of the matrix** that is **not** in **a**'s row or column, continue like this across the whole row, but remember the + − + − pattern.