❖ Cartesian product:

- A Cartesian product is a set of all ordered 2-tuples where each "part" is from a given set Denoted by A x B, and uses parenthesis (not curly brackets).

- For example, 2-D Cartesian coordinates are the set of all ordered pairs **Z** x **Z**
  Recall **Z** is the set of all integers.

- This is all the possible coordinates in 2-D space,
  Example: Given **A** = **{a, b}** and **B** = **{0, 1}**, what is their Cartiesian product?
  C = A x B = { (a,0), (a,1), (b,0), (b,1) }.

- Note that Cartesian products have only 2 parts in these examples (later examples have more parts)

- Formal definition of a Cartesian product:
  A x B = { (*a*, *b*) | *a* ∈ A and *b* ∈ B }.

- All the possible grades in this class will be a Cartesian product of the set S of all the students in this class and the set G of all possible grades.
  Let S = { Alice, Bob, Chris } and G = { A, B, C },
  D = {(Alice, A), (Alice, B), (Alice, C), (Bob, A), (Bob, B), (Bob, C), (Chris, A), (Chris, B), (Chris, C) }

- There can be Cartesian products on more than two sets.

- A 3-D coordinate is an element from the Cartesian product of **Z** x **Z** x **Z**

- For sets *A*, *B*, their ***Cartesian product***
  $A \times B :\equiv \{(a, b) \mid a \in A \wedge b \in B\}$.

- ***E.g.*** **{a, b } x {1,2} = {(a,1), (a,2), (b,1), (b,2)}**

- Note that for finite *A*, *B*, $|A \times B| = |A||B|$.

- Note that the Cartesian product is ***not*** commutative: $\forall AB$: $A \times B = B \times A$.

- Extends to $A_1 \times A_2 \times \ldots \times A_n$...