Fitting Univariate and Multivariate distributions, and Time-Series Models to Equities, Housing units and Currency Exchange Rate data

Final Project Report

Financial Econometrics (ECON 8780)

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Jigar Mehta (Id: 002-22-2932) **jmehta3@student.gsu.edu**

Introduction

The project report is divided into five sections:

1. Fitting univariate distribution to daily log returns of Apple stock price data
2. Fitting multivariate distribution to daily log returns of 3 correlated stocks
3. Fitting VAR Model to Deutsche bank and Credit Suisse stock returns
4. Fitting Multiplicative ARIMA model to monthly new one family houses sold in US
5. Fitting AR/GARCH model to US dollar to INR exchange rate data

Data Description

Section 1: Fitting Univariate distribution to daily log returns of Apple stock

The daily adjusted close price of Apple Inc. stock (listed on NASDAQ: AAPL) from Apr 1st, 2010 to Oct 30th, 2015. The data consists of 1468 active trading days of stock price data.
(**Source**: yahoo finance)

Section 2: Fitting Multivariate distribution to a portfolio of 3 assets

For Credit Suisse, Deutsche bank and HSBC bank, the daily stock adjusted close price (listed on NASDAQ: AAPL) from Apr 1st, 2010 to Oct 30th, 2015 was used. The multivariate model was fit on the daily log returns of three assets. The data consists of 1468 active days of daily stock price.
(**Source**: yahoo finance)

Section 3: Fitting VAR Model to Deutsche bank and Credit Suisse stock returns

A VAR model is fitted to the daily log returns of Credit Suisse and Deutsche bank stocks. The data is same as in Section 2.

Section 4: Fitting Multiplicative ARIMA Model to housing units sold data

The monthly new one family housing units (in thousands) sold across US from Jan 1st, 1963 to Sep 1st, 2015.
(**Source**: Economic Research: Federal Reserve bank of St. Loius
https://research.stlouisfed.org/fred2/series/HSN1FNSA#)

Section 5: Fitting AR - GARCH Model to US dollar to IND exchange rate data

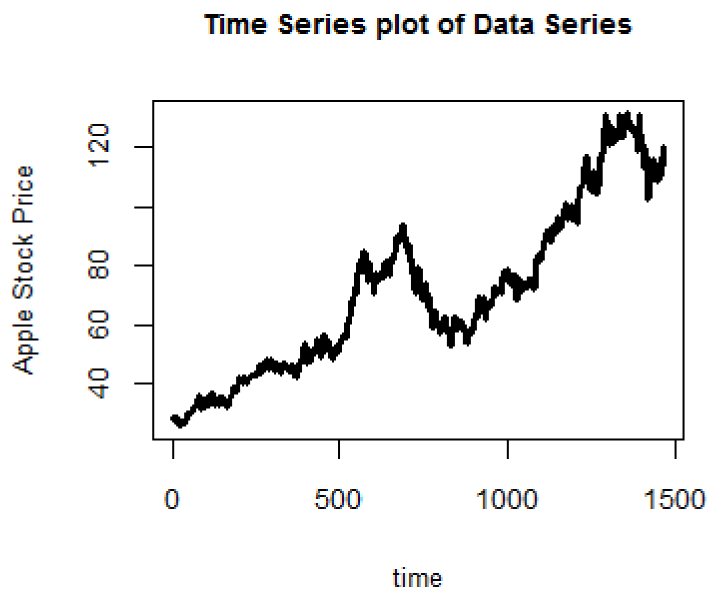
The daily exchange rate of USD to INR from Nov 1st, 2005 to Oct 30th, 2015. The data consists of 2512 active days of exchange rate data.
(**Source:** Economic Research: Federal Reserve bank of St. Loius
https://research.stlouisfed.org/fred2/series/DEXINUS)

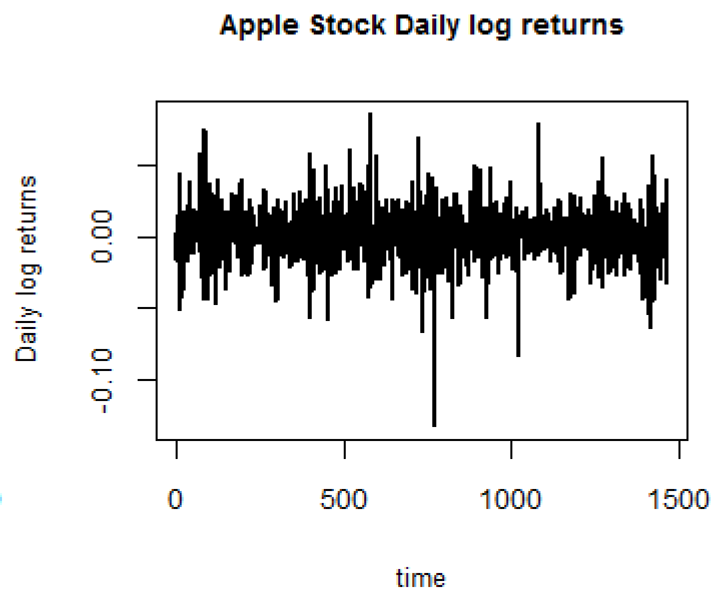
Note: For all stocks, the daily adjusted close price is considered because it takes care of all corporate actions such as stock splits, dividends, rights offerings and effect of M&As by the company.

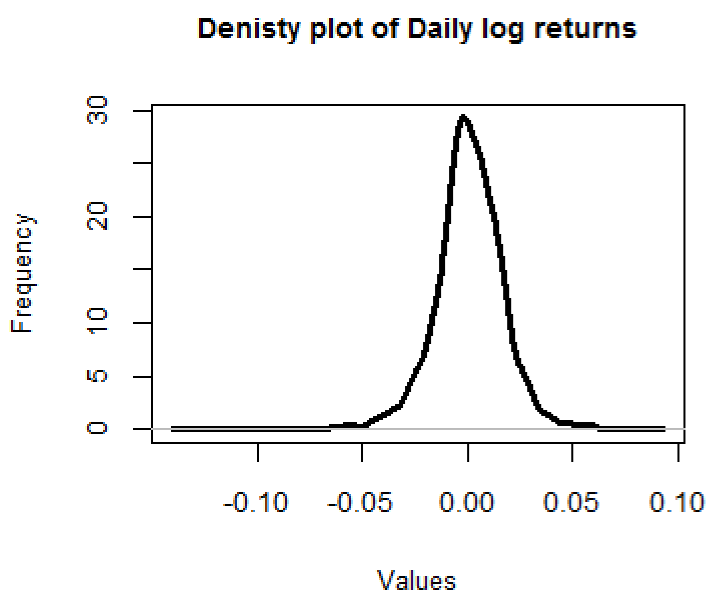
Section1: Fitting Univariate distribution to daily log returns of Apple stock

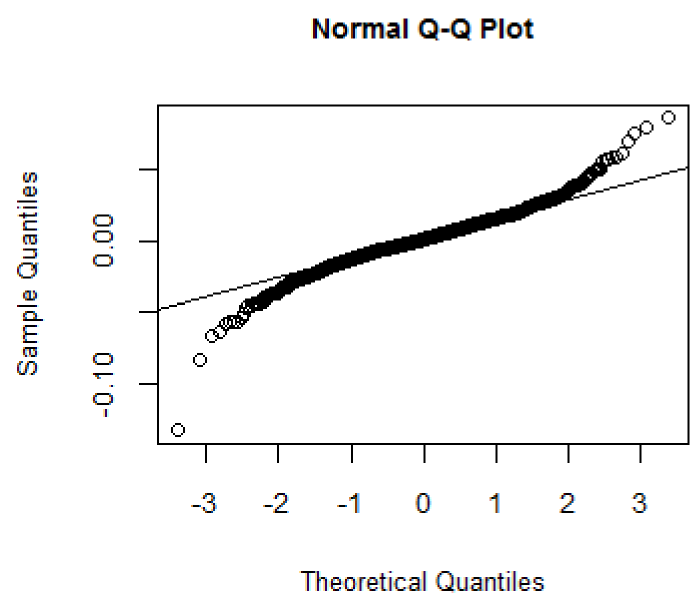
In this section, the daily log returns of Apple stock adjusted closing price is used as time series to fit and compare various univariate distributions. The models are fitted and parameters are estimated using Maximum Likelihood Estimation (MLE) technique. Finally, the best fit parsimonious model is selected from the quality parameters like maximum likelihood values, AIC and BIC.

As we can see form the time series plot of the data, the stock price has a growing trend, non-constant mean and variance, and volatility. We use the differenced (daily log returns) time series to obtain daily returns of the stock price.









The density plot and Normal Q-Q plot of the time sereis data clearly show that the data is almost symmnetrically distributied in the centre and shoulder region but with heavier tails. The Q-Q line shows the deviation from normality in the tails region.

The student t, skewed t, ged and skewed ged were the different distributions fitted to the daily log returns data using the stdFit, sstdFit, gedFit and sgetFit functions, repsectively, from the fGarch package in R. Here is the output from the t-distribution model:

t-distribution R output:

```
$par
     mean          sd            nu
0.001132593   0.016937859   4.416574524

$objective
[1] -3995.551
$convergence
[1] 0

$iterations
[1] 16
```

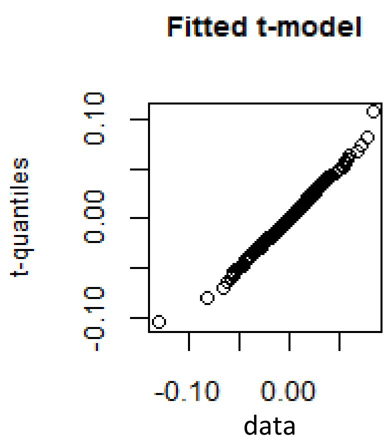
The performance of the model

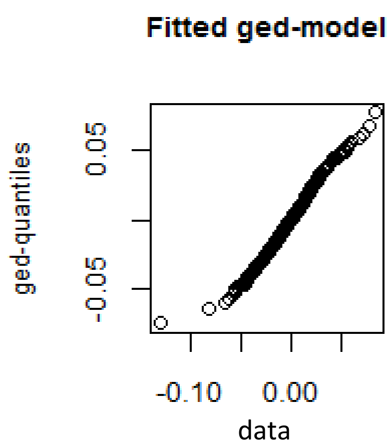
Distribution	Param	AIC	BIC
t	3	-7985.1	-7969.2
Skewed t	4	-7983.3	-7962.1
ged	3	-7961.7	-7945.7
Skewed ged	4	-7959.7	-7938.5

AIC = -2*Log_Likelihood value + 2*p

BIC = -2*Log_Likelihood value + log(n)*p

The sample-size, n = 1467





Inferences and Conclusion:

a) The estimated shape parameter value for a skewed-t distribution is xi = 0.983, nearly 1, which suggest tha a skewed model is not required for this data. Also, the estimated log-likelihood, AIC and BIC values are better for t-distribution. The # parameters required to be estimated for skewed-t are 1 less than t-distribtion. All these reasons imply that symmetric t-distribution provides a better fit to the data.

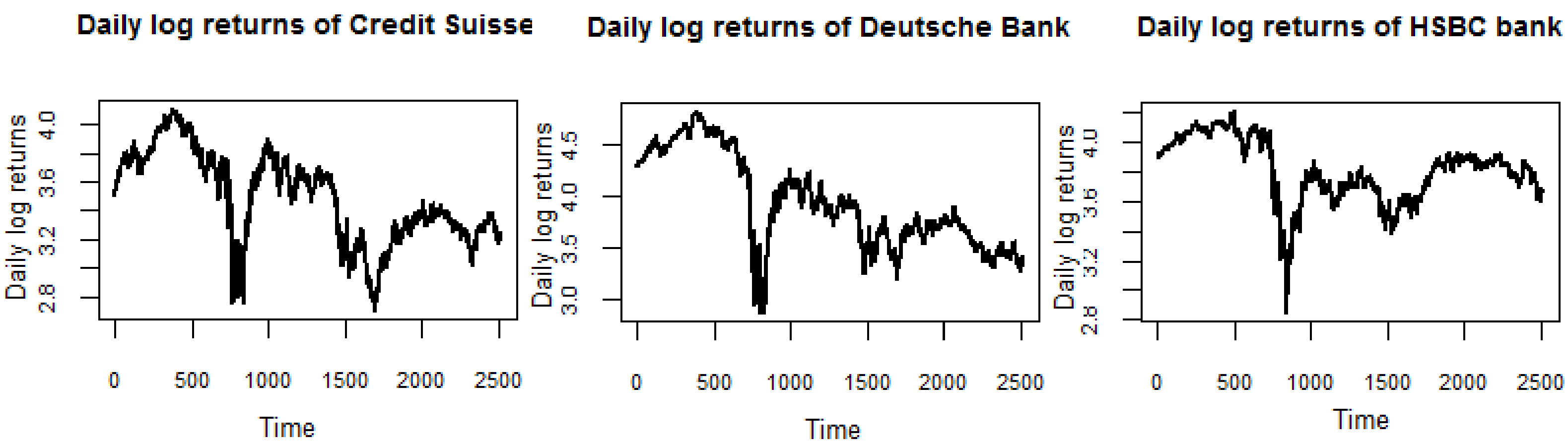
b) The maximum value of the likelihood for ged model is much smaller than the value -3995, obtained using the t-distribution. As seen from QQplots, above t-distributions fits better than ged model. The reason for that is that like the t-distributions, the density of this data seems to be rounded near the median.

c) The performance of all the models is summarized above in the table and it is seen that symmetric t-distribution has a higher performance out of all the distributions.

Section 2: Fitting multivariate distribution to daily returns of Deutsche bank, Credit Suisse and HSBC stock prices

Many a times, we are not interested merely in a single random variable but rather in the joint behavior of several random variables, for example, returns on several assets. Multivariate distributions describe such joint behavior.

In this section, we analyze three correlated stocks of Deutsche bank, Credit Suisse and HSBC bank. We use daily log returns of three assets and fit a multivariate distribution to understand the joint behavior of all three. The adjusted close price of three stocks from Apr 1st, 2010 to Oct 30th, 2015 are considered which includes dividends. The data consists of 1468 active days of daily stock price for each asset. The figure below depicts the univariate distribution of daily log returns v/s. time for each asset.



The relationship between two random variables depends on their variances as well as the strength of the linear relationship between them. It is very important to study both covariance and correlation values. Covariance matrix and Correlation matrix are extremely important as input to, for example, a portfolio analysis, to understand the relationship between variables. The corresponding matrices for three assets are:

Covariance matrix

	cred_suis	deut_bk	Hsbc
cred_suis	2	0.983	0.741
deut_bk	0.983	0.763	0.433
Hsbc	0.741	0.433	0.553

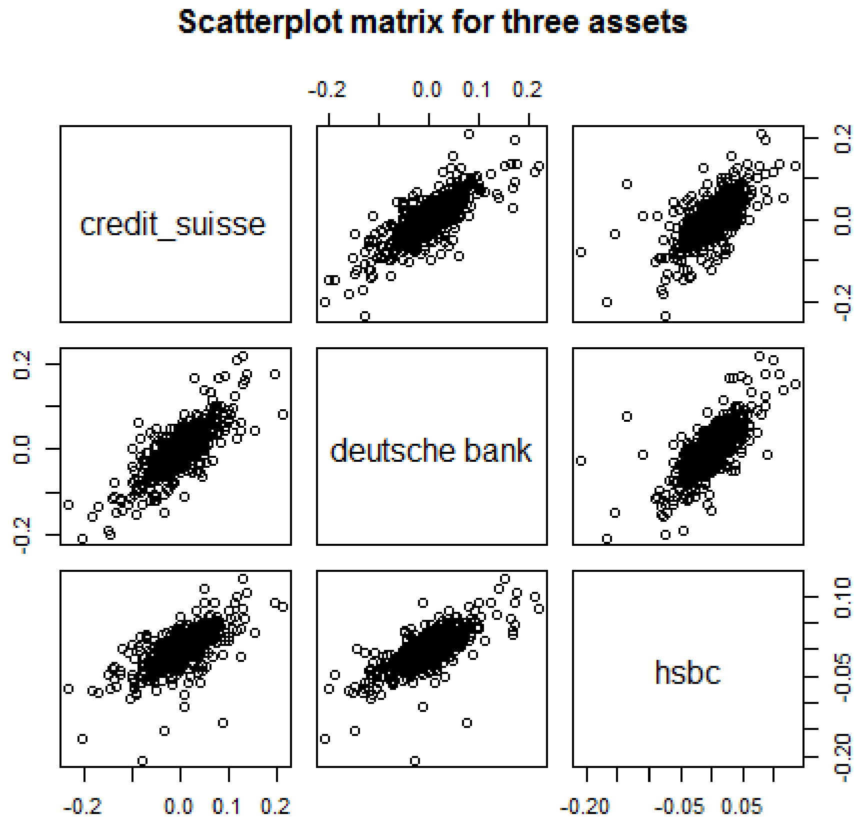
Correlation matrix

	cred_suis	deut_bk	hsbc
cred_suis	1	0.796	0.704
deut_bk	0.796	1	0.667
hsbc	0.704	0.667	1

We can see that all correlations are positive suggesting that all the stocks vary in the same direction in a given time window. The largest correlation of ~0.8 is between deutsche bank and credit Suisse which is strong correlation. The correlations between other stocks, as we can see, are also considered strong.

A correlation coefficient is only a summary of the linear relationship between variables. Interesting features, such as nonlinearity or the joint behavior of extreme values, remain hidden when only correlations are examined. A solution to this problem is the so-called scatterplot matrix, which is a matrix of scatterplots, one for each pair of variables.

Scatter Plot matrix:

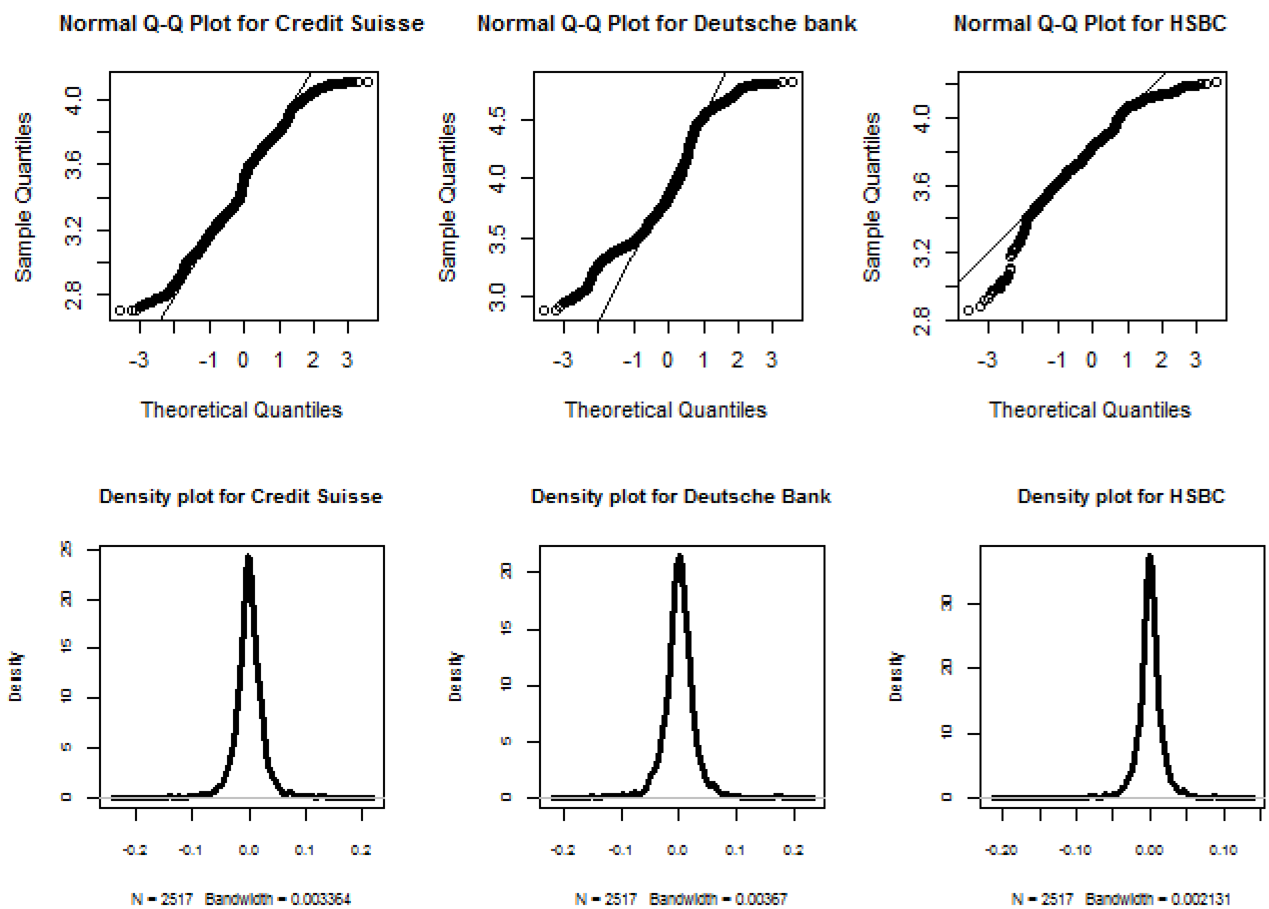


We can look at scatterplots to investigate whether extreme values cluster together. A portfolio is riskier if large negative returns on its assets tend to occur together on the same days. In the scatterplot for Deutsche bank and Credit Suisse, extreme returns for one stock do tend to occur on the same days as extreme returns on the other stock. This can be seen by noticing that the outliers are more likely to occur together when they tend to be in the upper-right and lower-left corners, rather than being concentrated along the axes. The extreme-value behavior for HSBC and Credit Suisse is a bit different as the outliers tend to fall along the *x*- and *y*-axes. The scatterplot for Deutsche bank and Credit Suisse shows tail dependence, and HSBC and Credit Suisse scatterplot shows tail independence (compared to other, but not very convincing).

A multivariate t-distribution as a simple and powerful tool for calculating expected returns on assets due to the fat tails of returns. It is the most efficient estimator of the expected return of an asset and is drastically different from the sample average return.

Fitting the Multivariate skewed-t Distribution by Maximum Likelihood:

The next step is to fit a multivariate distribution model to the dataset of three assets. The density plots and normal Q-Q plots helps in deciding which distribution to fit to the data. The corresponding plots for three assets are as follows:



We can see from the normal plots and density plots that the three assets are symmetrically distributed at the center but with heavier tails. Therefore, it is desirable to fit a model for vectors of returns such that the univariate marginals are *t*-distributed. The multivariate *t*-distribution has this property. The random vector *Y* has a multivariate t_ν (μ, A) distribution if :

$$Y = \mu + \sqrt{\frac{\nu}{W}} Z$$

where W is chi-squared distributed with ν degrees of freedom, Z is N_d (0,A) distributed, and W and Z are independent. When the data are t-distributed, the maximum likelihood estimates are superior to the sample mean and covariance matrix in several respects – the MLE is more accurate and it is less sensitive to outliers. So, I fit the multivariate skewed-t distribution using the mst.mple function.

In the multivariate skewed-t distribution model, in addition to the shape parameter v determining tail weight, the skewed t-distribution has a vector $\alpha = (\alpha 1, \ldots, \alpha d)^T$ of shape parameters determining the amounts of skewness in the components of the distribution. If Y has a skewed t-distribution, then Yi is left-skewed, symmetric, or right-skewed depending on whether $\alpha i < 0$, $\alpha i = 0$, or $\alpha i > 0$.

The function mst.mple is in R's sn package and it is called to fix the multivariate skewed-t distribution model. This function maximizes the likelihood over all parameters, so there is no need to use the profile likelihood.

R – Output:

```
$opt.method$objective
[1] 14472.61

$opt.method$convergence
[1] 0

$opt.method$iterations
[1] 115

$opt.method$evaluations
function gradient
    139      116

$opt.method$method
[1] "nlminb"
```

The estimates are as follows:

```
> fit2$dp
$beta
           [,1]       [,2]       [,3]
[1,] 0.06105093 0.05537459 0.03629728

$Omega
          [,1]      [,2]      [,3]
[1,] 0.8492661 0.3947493 0.2977115
[2,] 0.3947493 0.2890238 0.1657745
[3,] 0.2977115 0.1657745 0.2250193

$alpha
[1]  0.02407278 -0.10679703 -0.01452983

$nu
[1] 3.129129
```

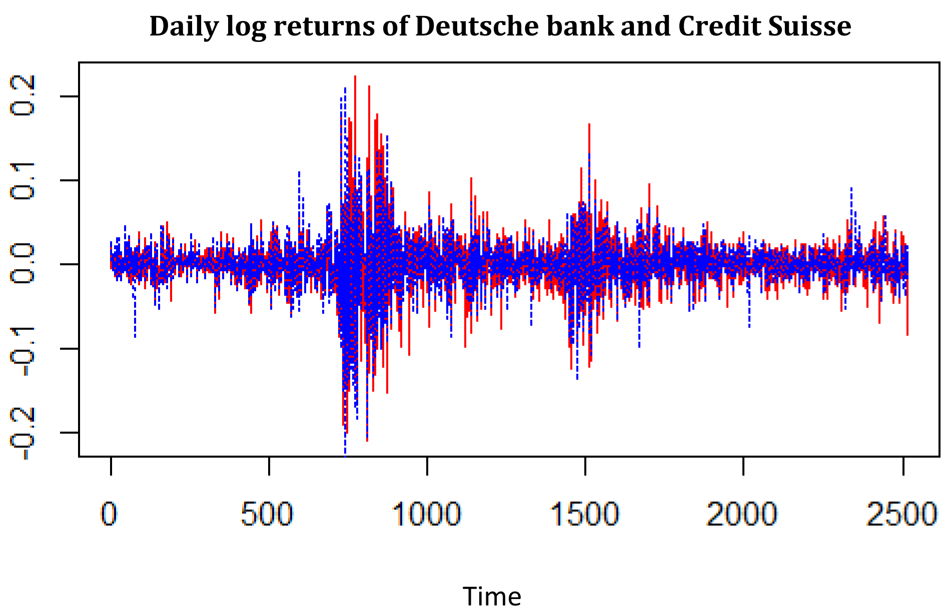
Here fit2$beta is the estimate of μ, fit2$Omega is the estimate of Σ, fit2$alpha is the estimate of α, and fit2$nu is the estimate of ν. It is clear that the estimates of all components of α are small but not zero, which suggests that there is little skewness in the data. In summary, the three assets are well fit by a skewed t-distribution.

Inferences and Conclusion:

- Fitting a multivariate distribution to a set of correlated daily returns of stock prices helps in modeling the future returns of these stocks.
- The scatterplot for Deutsche bank and Credit Suisse shows tail dependence, and HSBC and Credit Suisse scatterplot shows tail independence
- Using normal Q-Q plots and density plots, we found that distribution is skewed accordingly, selected the skewed-t distribution.
- Next, we fit the multivariate skewed-t distribution on the three stocks using R using the mst.mple function in the sn package. The model parameters helps in understanding the model fit. The significant values for alpha parameters further supports the need for skewed-t model.

Section 3: Fitting VAR Model to Deutsche bank and Credit Suisse stock returns

In this section we try to discover any influence that may exist among the daily log returns of Deutsche bank and Credit Suisse stocks. Rationale is that, if there is a strong correlation between the two time series, there may be a good change that the movement of one stock price influences the movement of others. As we saw in the previous section, the daily log returns of both the series are not stationary and hence, I use the differencing. To make it more visible, I transform the data to indices and plot them on the same axis.



The two time series look stochastic process with clustering volatility, and there are many points in time where they go along the same trend and sometimes trend with lags. Assuming the autocorrelation among both the time series and possibility of cross-correlation, we try to fit multivariate ARMA (p,q) model with q=0 i.e. Vector Autoregressive (VAR) model.

Multivariate Time Series:

Suppose that for each t, Yt = (Y1;t;. . . . ; Yd;t) is a d-dimensional random vector representing quantities that were measured at time t, e.g., returns on d equities. Then Y1; Y2. . . is called a d-dimensional multivariate time series. A multivariate time series said to be stationary if for every n and m, Y1; : : : ;Yn and Y1+m; : : : ; Yn+m have the same distributions.

The cross-correlation function:

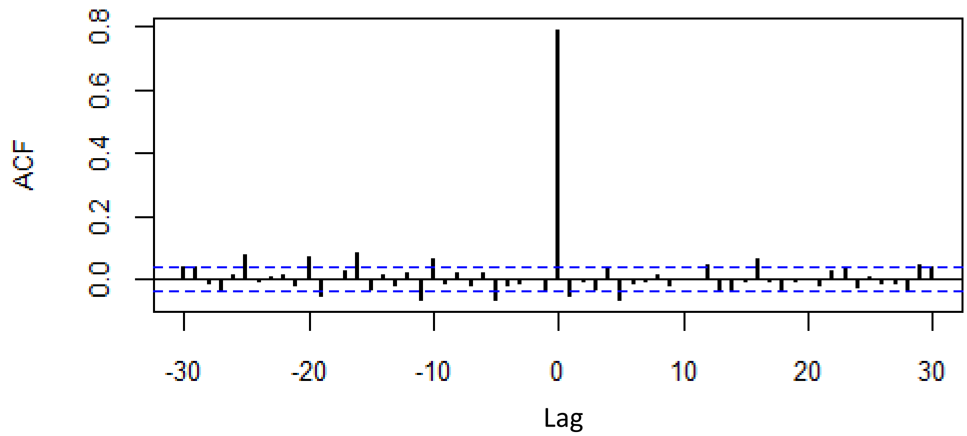
Suppose that *Yj* and *Yj0* are the two component series of a stationary multi-variate time series. The *cross-correlation function* (CCF) between *Yj* and *Yj0* is defined as:

$$\rho_{Y_j,Y_{j'}}(k) = \mathrm{Corr}\{Y_j(t), Y_{j'}(t-k)\}$$

and is the correlation between *Yj* at a time *t* and *Yj0* at *k* time units earlier. As with autocorrelation, *k* is called the *lag*. However, unlike the ACF, the CCF is not symmetric in the lag variable *k*.

Cross-correlations can suggest how the component series might be influencing each other or might be influenced by a common factor. Like all correlations, cross-correlations only show statistical association, not causation, but causal relationship might be deduced from other knowledge. The cross-correlation function between two time series is plotted.

CCF Plot for Returns on Deutsche bank and Credit Suisse



The result, showing the lag (h) in $xt+h$ and the correlation with yt is:

```
Autocorrelations of series 'X', by lag

   -30    -29    -28    -27    -26    -25    -24    -23    -22    -21    -20    -19    -18
 0.041  0.041 -0.010 -0.037  0.015  0.079 -0.005  0.005  0.017 -0.018  0.073 -0.051  0.004
   -17    -16    -15    -14    -13    -12    -11    -10     -9     -8     -7     -6     -5
 0.027  0.085 -0.027  0.015 -0.017  0.020 -0.062  0.063 -0.009  0.021 -0.014  0.020 -0.063
    -4     -3     -2     -1      0      1      2      3      4      5      6      7      8
-0.018 -0.013  0.004 -0.037  0.792 -0.049 -0.007 -0.030  0.042 -0.063 -0.008 -0.006  0.014
     9     10     11     12     13     14     15     16     17     18     19     20     21
-0.020  0.004  0.001  0.047 -0.029 -0.035 -0.006  0.064 -0.005 -0.032 -0.006  0.005 -0.017
    22     23     24     25     26     27     28     29     30
 0.027  0.036 -0.021  0.005 -0.008 -0.011 -0.033  0.048  0.039
```

The most dominant cross correlations actually appears at lag=0, which is exactly the correlation of daily log returns of two stocks. CCF=0.8. After that, there are small significant lag values but not powerful. In general, the cross correlation decays quickly as the lag's absolute value increases and the dominant correlation does not include any lags. This implies that, even though the two stocks are highly correlated, the future changes in Deutsche bank stock prices doesn't have any significant influence on the future changes of Credit Suisse stock price. This makes sense as correlation doesn't mean causation.

In this case, we will verify the poor cross-correlation result with a VAR model. If the model returns small and insignificant values for lagged estimators, it will further strengthen the case that VAR model is not suitable for this two series. So, the next step is to fit a VAR model in R using 'VAR' function in the R's vars packages.

Model Identification:

The VAR s an n-equation, n-variable linear model in which each variable is in turn explained by its own lagged values, plus the past values of the remaining n-1 variables. The model is a d-dimensional multivariate time series $\boldsymbol{Y_t}$ that has a multivariate AR(p) process with mean μ if for p by p matrix $\emptyset_1, \ldots \emptyset_p$.

$$\boldsymbol{Y_t} - \boldsymbol{\mu} = \emptyset_1(\boldsymbol{Y_{t-1}} - \boldsymbol{\mu}) + \cdots + \emptyset_p(\boldsymbol{Y_{t-p}} - \boldsymbol{\mu}) + \epsilon_t$$

where $\epsilon_1 \ldots \epsilon_n$ is a multivariate $\mathrm{WN}(0, \boldsymbol{\Sigma})$ process satisfying

1. $\mathbf{E}(\epsilon_t) = \mathbf{0}$ – Every error term has mean zero
2. $\mathbf{E}(\epsilon_t \epsilon_t{}')= \boldsymbol{\Sigma}$ -the covariance matrix of error term
3. $\mathbf{E}(\epsilon_t \epsilon_{t-k}{}')=0$ for any non-zero k. There is no serial correlation across individual error terms.

In matrix $\emptyset^k$, the component $\emptyset^k_{i,j}$ (the i,j th component of $\emptyset^k$) is the influcence of $\boldsymbol{Y_{j,t-k}}$ on $\boldsymbol{Y_{i,t}}$, k=1,2,… p.

The number of lagged values to include in each equation can be determined by a number of different methods such as AIC and SIC. Here we use AIC method. Just like the univariate AR, the error terms in these regressions are the "surprise" movements in the variables after taking its past values into account. When lag=10, the model achieves smallest AIC.

AIC for the fitting Model of VAR from lag=1 to lag=10

lag	1	2	3	4	5	6	7	8	9	10
AIC(n)	-15.1043	-15.1077	-15.1063	-15.1103	-15.1151	-15.1136	-15.1194	-15.1225	-15.1206	-15.1319

R – Output: Fitting the model we get the output as follows: (Only significant parameters shown)

VAR(y = y, p = lag_m, type = "const")

VAR Estimation Results:
=========================
Endogenous variables: db, cr
Deterministic variables: const
Sample size: 2507
Log Likelihood: 11895.31

Estimation results for equation Deutsche bank:

```
        Estimate  Std. Error t value Pr(>|t|)
cr.l5  -0.0961207  0.0358659  -2.680  0.00741 **
db.l10  0.1871375  0.0329276   5.683 1.48e-08 ***
cr.l10 -0.1657341  0.0357109  -4.641 3.65e-06 ***
const  -0.0003439  0.0006041  -0.569  0.56924
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.03021 on 2486 degrees of freedom
Multiple R-Squared: 0.0251,    Adjusted R-squared: 0.01726
F-statistic: 3.201 on 20 and 2486 DF,  p-value: 2.071e-06
```

Estimation results for equation Credit Suisse:

```
          Estimate Std. Error t value Pr(>|t|)    
db.l1    6.814e-02  3.043e-02   2.239 0.025237 *  
cr.l1   -1.366e-01  3.299e-02  -4.142 3.56e-05 ***
db.l2    6.575e-02  3.060e-02   2.148 0.031774 *  
cr.l2   -9.678e-02  3.320e-02  -2.915 0.003593 ** 
cr.l3   -7.482e-02  3.323e-02  -2.252 0.024411 *  
cr.l5   -7.331e-02  3.320e-02  -2.208 0.027348 *  
db.l7    6.894e-02  3.051e-02   2.259 0.023947 *  
cr.l7   -1.127e-01  3.314e-02  -3.401 0.000683 ***
cr.l8    6.563e-02  3.323e-02   1.975 0.048370 *  
db.l10   1.143e-01  3.048e-02   3.751 0.000180 ***
cr.l10  -8.322e-02  3.306e-02  -2.517 0.011898 *  
const   -9.546e-05  5.593e-04  -0.171 0.864494    

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.02797 on 2486 degrees of freedom
Multiple R-Squared: 0.03189,   Adjusted R-squared: 0.0241 
F-statistic: 4.095 on 20 and 2486 DF,  p-value: 2.802e-09


Covariance matrix of residuals:
          db        cr
db 0.0009129 0.0006713
cr 0.0006713 0.0007825

Correlation matrix of residuals:
       db     cr
db 1.0000 0.7943
cr 0.7943 1.0000
```

The covariance matrix of $\boldsymbol{\epsilon n}$ is $\begin{pmatrix} 0.0009129 & 0.0006713 \\ 0.0006713 & 0.0007825 \end{pmatrix}$

Inferences and Conclusion:

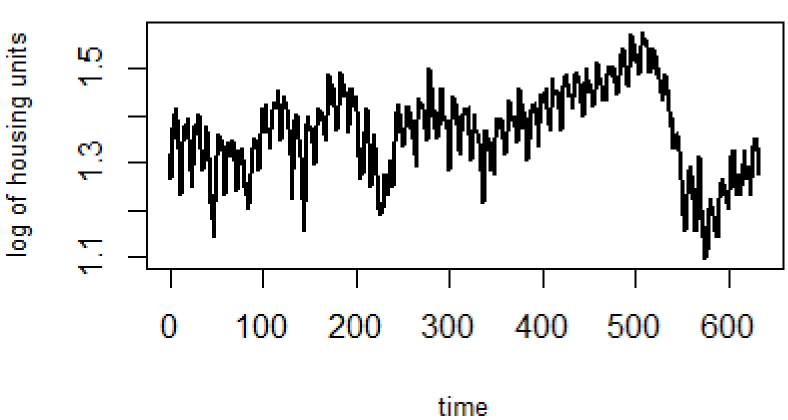
Notice that the $\emptyset 1{,}2k$ are significant in some lags, meaning that lagged Deuts Bank has some influence on Crd Suis. While at the same time all $\emptyset 2{,}1k$ are not significant, which means that lagged Crd Suis does not have an influence on Deuts Bank. But even though some $\emptyset 1{,}2k$ are significant, all the $\emptyset 1{,}2k$ are very small, indicating that the change in cross stock price difference has a very small influence on the difference of exchange rate. But it is still significant here due to the relatively small standard error. On the other hand, the absolute value for $\emptyset 2{,}2k$is relatively larger and decays slowly as k increases. The decreasing $\emptyset 2{,}2k$ is an indication of AR process since price differences depend only on its historical values.

In sum, the VAR model does not fit well on the data. The evidence include the small value of lagged estimators, insignificant lagged estimators and poor cross correlation.

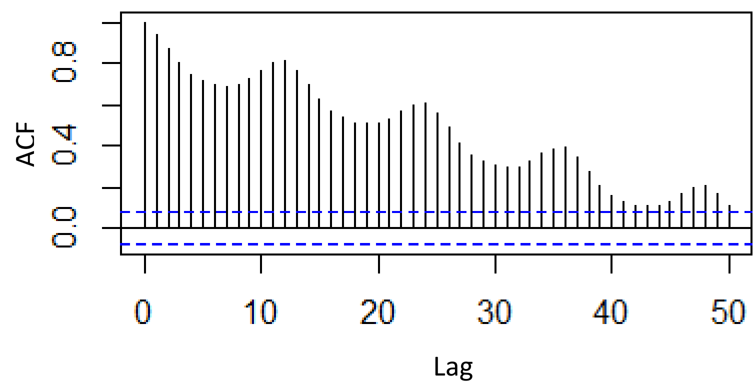
Section 4: Fitting Seasonal ARIMA Model to monthly housing units sold data

Economic time series are often exhibit seasonal variations. In this section, we analyze the log of monthly new single family houses sold across US from Jan 1963 to Sept 2015. Taking the log transformation of the time series stabilizes the increasing oscillations.

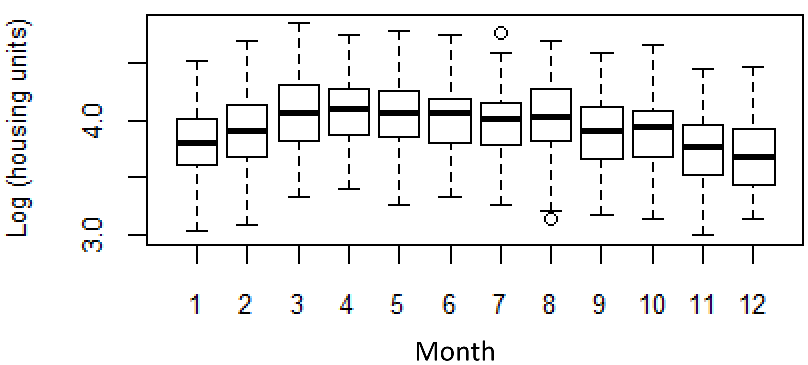
Time series plot of log of Housing units



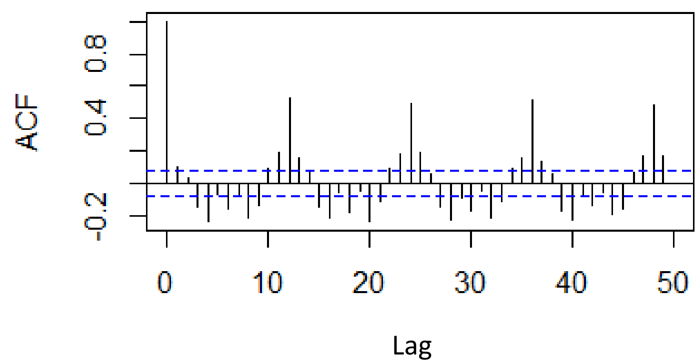
ACF plot of log of housing data



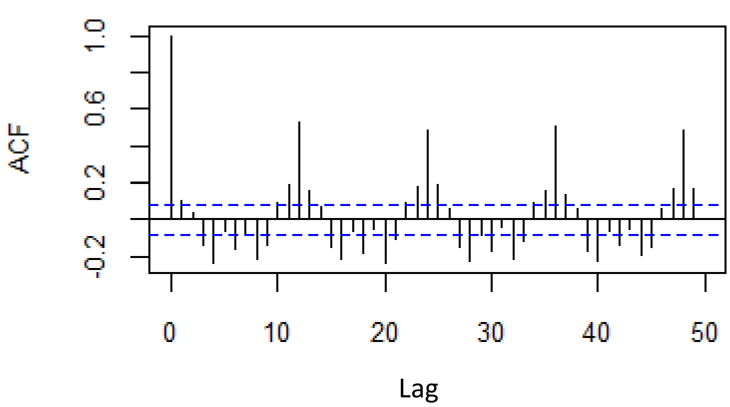
BoxPlot of housing data



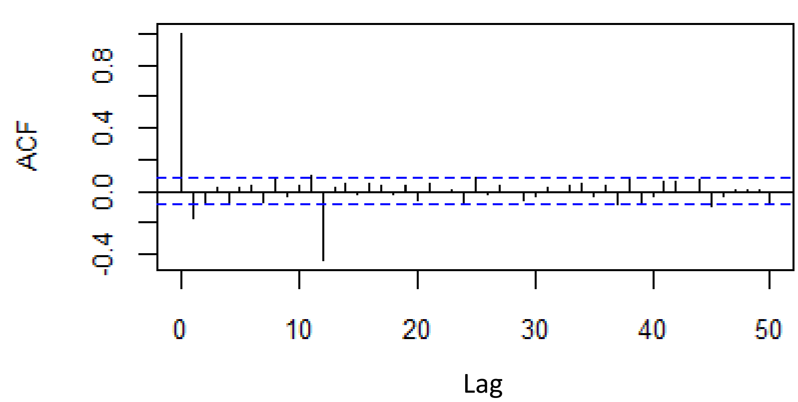
ACF plot of only non-seasonal differencing



ACF plot – only seasonal differencing



ACF plot - seasonal and non-seasonal differencing



$\Delta_s = 1 - B^s$ is the first-order seasonal differencing operator

$\Delta_s = (1 - B)^s$ is the sth order nonseasonal differencing operator.

As seen from the time series plot, there are two types of non-stationarity: 1) there is strong seasonality and 2) whether the seasonal subseries revert to a fixed mean is unclear and, if not, then this is a second type of non-stationarity because the process is integrated. Also, ACF plot shows at lags of multiples of 12, the autocorrelations are large, and decay slowly to zero. At other lags, the autocorrelations are smaller but also decay somewhat slowly. The boxplots in the third figure below shows the seasonal effects clearly. Housing units sold in the start and last four months are much lower compared to mid of the year.

To remove seasonal nonstationary, one uses seasonal differencing. For the housing units sold data, the seasonally differenced and non-seasonal differencing series appears stationary and might be preferred since it might result in a parsimonious model.

Fitting Multiplicative ARIMA Model to the log housing data:

The ARIMA $\{(p, d, q) \times (p_s, d_s, q_s)_s\}$ process is :

$$(1-\phi_1 B-\cdots-\phi_p B^p)\{1-\phi_1^* B^s-\cdots-\phi_{p_s}^*(B^s)^{p_s}\}\{\Delta^d(\Delta_s^{d_s} Y_t)-\mu\}$$
$$=(1+\theta_1 B+\ldots+\theta_q B^q)\{1+\theta_1^* B^s+\ldots+\theta_{q_s}(B^s)^{q_s}\}\epsilon_t.$$

This process multiplies together the AR components, MA components and the differencing components of two processes: the nonseasonal ARIMA *(p, d, q)* process:

$$(1-\phi_1 B-\cdots-\phi_p B^p)\{(\Delta^d Y_t)-\mu\}=(1+\theta_1 B+\ldots+\theta_q B^q)\epsilon_t$$

and the seasonal ARIMA $(p_s, d_s, q_s)_s$ process

$$\{1-\phi_1^* B^s-\cdots-\phi_{p_s}^*(B^s)^{p_s}\}\{(\Delta_s^{d_s} Y_t)-\mu\}=\{1+\theta_1^* B^s+\ldots+\theta_{q_s}(B^s)^{q_s}\}\epsilon_t$$

Out of the many models tried with both seasonal and non-seasonal differencing, I selected ARIMA (1,1,1) x (1,1,1) [12].

R Output:

ARIMA (1,1,1)(1,1,1)[12]

Coefficients:

	ar1	ma1	sar1	sma1
	0.3267	-0.5254	0.0472	-0.911
s.e.	0.1574	0.1413	0.0456	0.022

sigma^2 estimated as 0.006808: log likelihood=656.91
AIC=-1303.81 AICc=-1303.72 BIC=-1281.67

Thus, the fitted model is:

$(1 - 0.3267B)*(1- 0.0472B_{12})Y_t* = (1 + 0.5253B)*(1+0.911B_{12})\epsilon_t$

where, $Y_t^* = \Delta(\Delta_{12} Y_t)$ and ϵ_t is white noise.

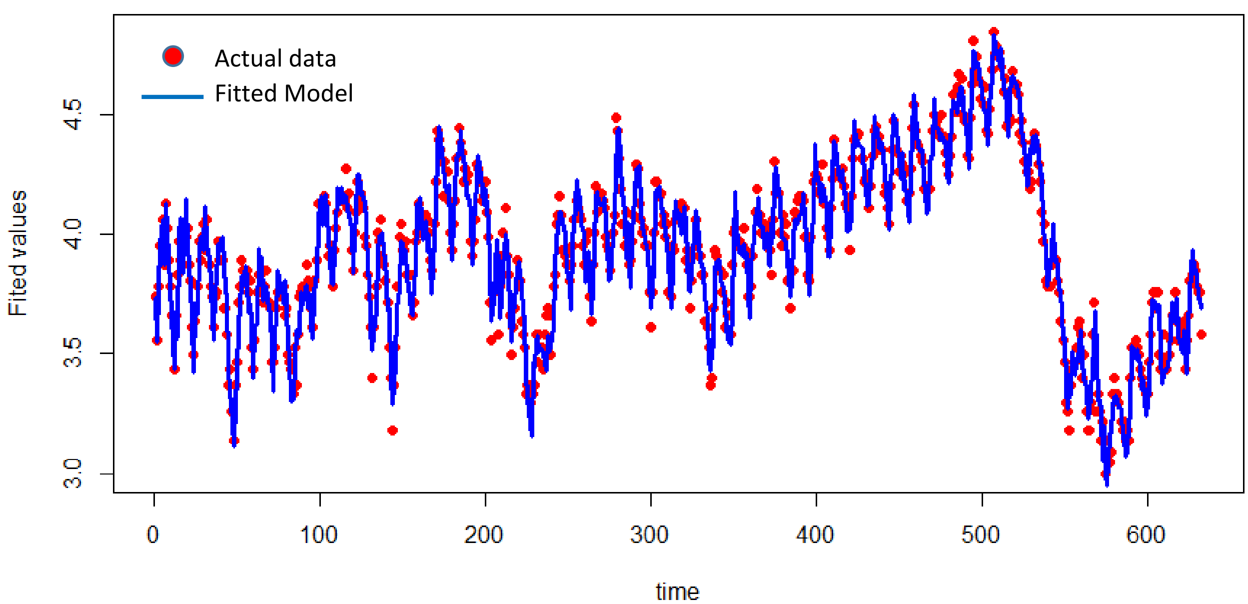
The coefficients for ar1, ma1 and sma1 are significant and different from zero, which suggests that the model is a good fit. However, sar1 coefficient is almost 0, which says that the ARIMA 1,1,1 x 0,1,1 could be a good fit. But, with the latter model the AIC is very large which implies the current model is required and a good fit.

The criterion we use for model selection is Akaike's Information Criterion method. AIC= -2*ln (L) +2*k, where k is the number of parameters in the model and L is the maximized value of the log likelihood function. With this method, the model with the lowest AIC is selected

Actual vs fitted Plot:

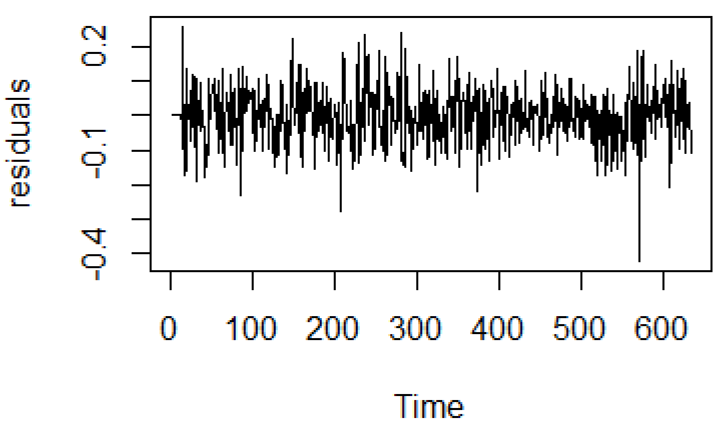
Here, the Actual vs fitted model graph depicts how accurately the model fits my actual data. Some of the red points are not connected. We can use fractional differencing and a multip -licative FARIMA *(p,d,q)* model for obtaining a better fit to the data.

Actual vs Fitted Plot

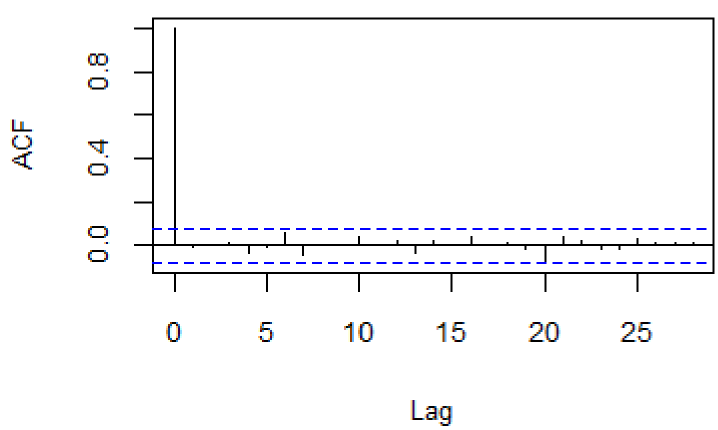


Diagnostic Checking: Residual Analysis

Residuals - ARIMA{(1,1,1) x (1,1,1)p=12} model



ACF of Residuals-ARIMA { (1,1,1)x(1,1,1)12 } model



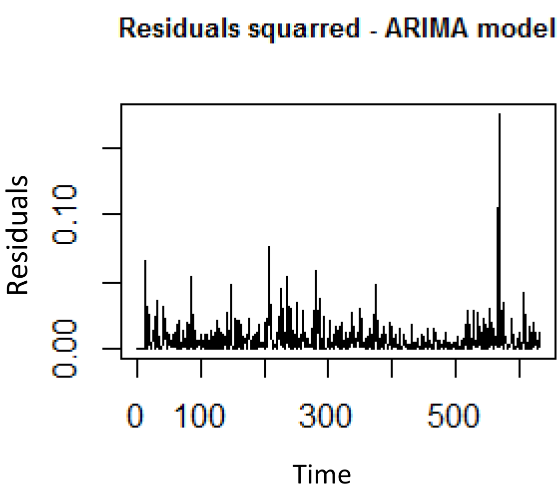
The residuals of the ARIMA model, as seen from above figure, are random in nature and almost normally distributed. The ACF plot shows no significant value at lags up to 30, which proves that it is a white noise. Besides, the Ljung-Box Test below confirms that residuals are a white noise up to lag 30.

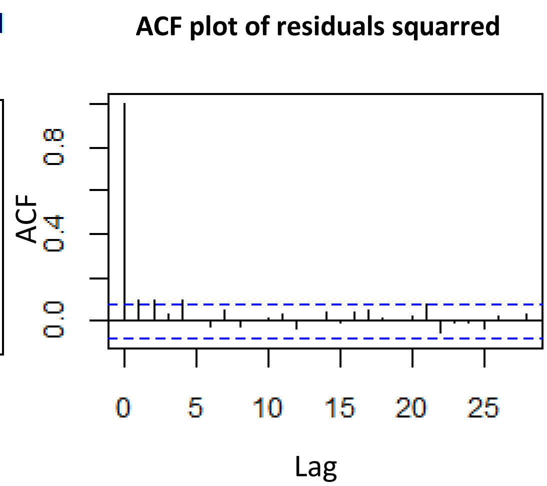
```
Box-Ljung test
data:  res
X-squared = 16.3664, df = 30, p-value = 0.9794
```

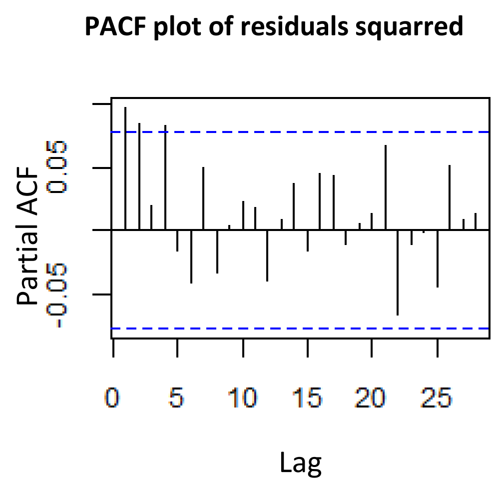
Box test tests the null hypothesis that the first n autocorrelations are jointly zero. If we fail to reject the null hypothesis that the autocorrelation is zero, then the lagged values are jointly independent. Here the p-value > 0.05 including lags as large as 30, hence there is no serial correlation in the residuals and the ARIMA {(1,1,1)x(1,1,1)12} model performs well.

Detection for Volatility Clustering presence in the residuals squared:

The residuals of the ARIMA model does not show any significant visble clustering volatility. To confirm, check the plot of squared residuals as well as the ACF and PACF of squared residuals. The ACF for of first few lags are almost near threshold and hence, residuals squarred can be considered as white noise. The PACF of residuals squarred shows threshold breach at lag 1 and does die down, which suggests that its not that severe. The Ljung-Box test below also supports that residuals squarred is white noise upto lag = 30.





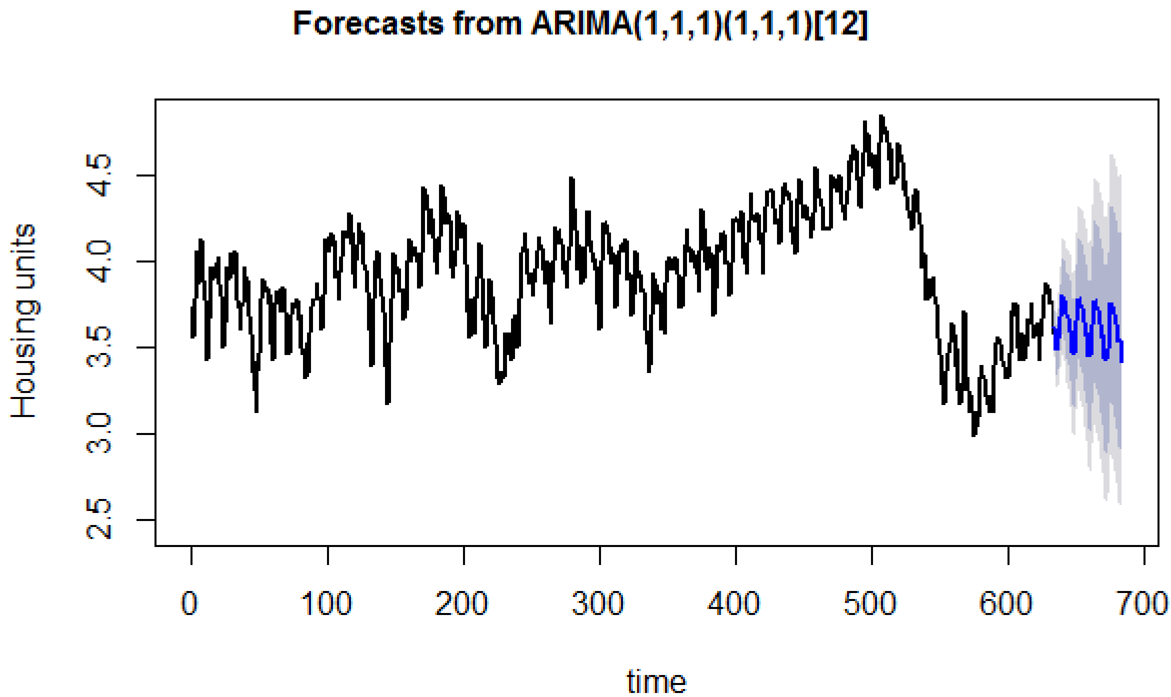


Ljung-Box test for checking white noise for residuals squared:

```
Box-Ljung test
data:  res^2
X-squared = 36.1511, df = 30, p-value = 0.2032
```

Once, we have checked the diagnostics and ensured that ARIMA {(1,1,1) x (1,1,1)12} is good fit to the data, next we use this model to forecast n data points ahead in time.

Forecasting values ahead in time (for n=50):



The above figure shows the forecasts of monthy log housing units sold for 50 months ahead in time. The darke blue line indiates the forecast value and the bands above and top shows +- 95% and 90% forecast limits.

Inferences and Conclusion:

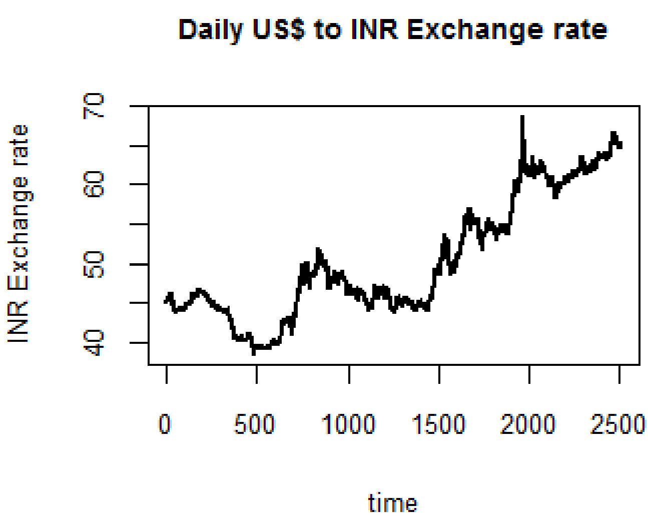
- If the data is seasonal in nature, then we use both non-seasonal and seasonal differencing to bring the series to stationarity. The acf and pacf plots helps in deciding the differencing type and number of differencing required. For this series we used both types of differencing to achieve stationarity.
- Sometimes, there arises a problem called overdifferecning where we see a strong negative coorelation at lag 1. This is due to the fact that diff funtion in R allows only interger differencing values. Non-seasonal differecning paaramter, d of 1 in this example causes overdiffercing.
- A solution to overdifferecing is to use Fractional differecing technique, where the differecning estimate can be specified in decimals. This is achieved using fracdiff function in R. The value of d = 0.7 was accurate for overcoming overdifferecing for this time series.
- We used a multiplicative ARIMA(1,1,1)x (1,1,1) model to fit the seasonal data. The residuals are white noise and resdiuals squarred doesn't show volatility clustering problem of the fitted model. Hence, the model fits well to the time series.
- We used the model to forecast the 50 months values ahead in time. Knowing the values ahead in time helps to take informed business decisions and minimize risks and loses.

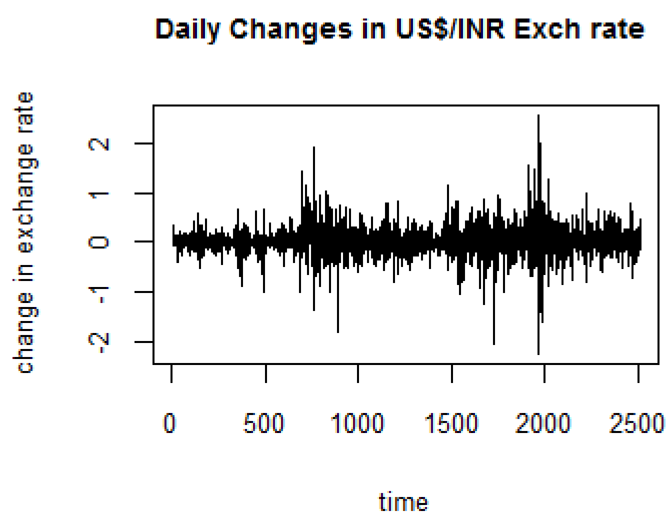
Section 5: Fitting AR - GARCH model to US to INR dollar exchange rate

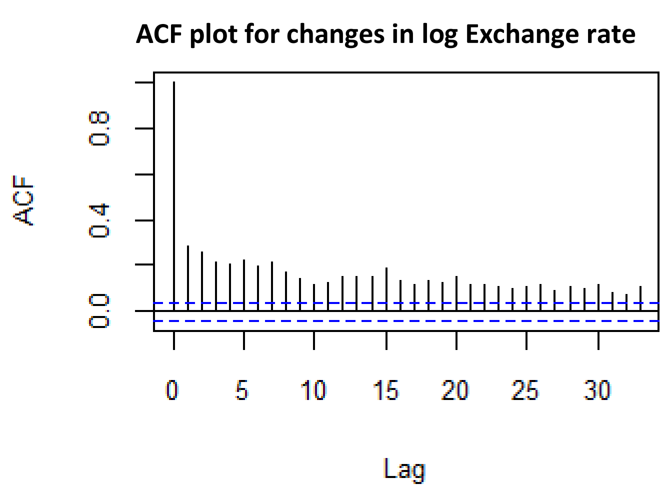
The exchange rate between US and India is dependent upon many factors and is highly volatile. Modelling such a distribution is very important considering that the exchange rate next day tend to lie closely to the previous day's data. It is reasonable to assume that the exchange rate would be an auto-correlated time series data. In this section, we use the US$ to INR exchange rate from Nov 1st, 2005 to Oct 30th, 2015 data to fit two models:

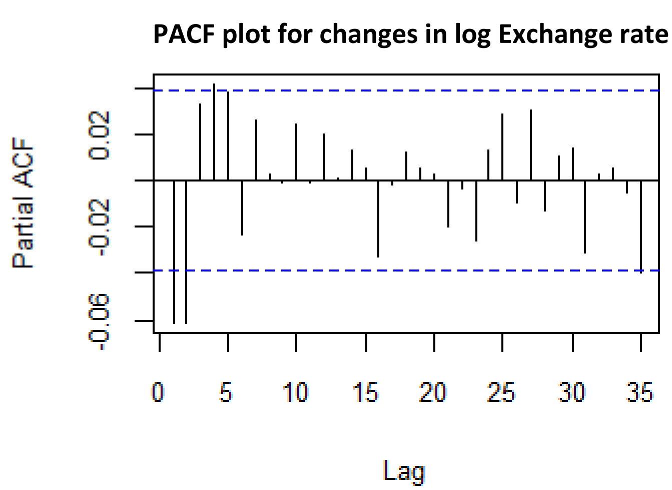
- First, we fit ARIMA model to the rate data. The residuals squared analysis reveals the volatility clustering of residuals and hence,
- Second, we fit a ARIMA-GARCH model

From the time series plot, we can observe several characters. First, historical data is correlated with itself. Exchange rate is highly persistent, which also indicates a possibility of non-stationarity. Second, there seems to be high volatility clustering at points 700-900 and 1900-2100 and at 1200-1400 the volatility is significantly smaller.









The ACF graph significant lags and typical one-sided dumping pattern which indicates strong correlation of y with its lagged values. The data is not stationary. The PACF graph also supports the evidence of strong negative correlation between Yt and Yt-1 and Yt and Yt-2, since the partial correlation dies down immediately when lag is larger than two.

Fitting ARIMA Model to log exchange rate:

A time series Yt is said to be an Autoregressive Integrated Moving Average (p,d,q) process if $\Delta^d Yt$ is ARMA(p,q). Same to ARMA, it has moving average parameters and autoregressive parameters. d is the number of differencing. An ARIMA (p,d,q) is stationary if d=0. Otherwise, only its differences of order d or above are stationary. Often the first or second differences of non-stationary time series are stationary.

R output :

```
> fit_w<-auto.arima(w,max.p=10,max.d=2,max.q=5,max.P=0,max.D=0,max.Q=0,ic='aic')
```

```
> summary(fit_w)
Series: w
ARIMA(3,1,1)

Coefficients:
         ar1      ar2     ar3      ma1
      0.6996  -0.0089  0.0856  -0.7641
s.e.  0.1427   0.0257  0.0199   0.1426

sigma^2 estimated as 3.129e-05:  log likelihood=9459.37
AIC=-18908.75   AICc=-18908.72   BIC=-18879.6

Training set error measures:
                       ME        RMSE         MAE         MPE
Training set 0.0001427246 0.005593084 0.003819633 0.003481955
```

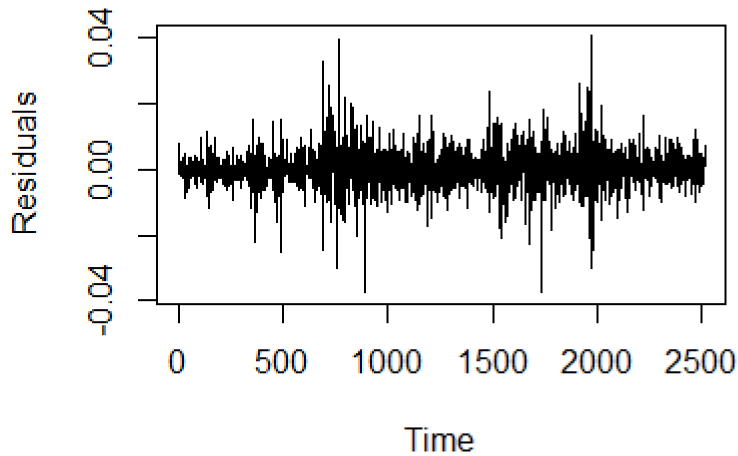
As we see from the R-output, the coeffients for ar2 and ar3 arent significant as they are closee to 0. So, we fit arima (1,1,1) and compare the AIC values. The R-model for ARIMA (1,1,1) is :

```
Call: arima(x = w, order = c(1, 1, 1))

Coefficients:
     ar1     ma1
  0.2383  -0.3068
s.e.  0.1475   0.1435

sigma^2 estimated as 3.148e-05:  log likelihood = 9451.73,  aic = -19897.45

Training set error measures:
                    ME        RMSE         MAE         MPE       MAPE      MASE        ACF1
Training set 0.0001647287 0.005610161 0.003826256 0.004018516 0.09772825 0.9951696 0.004353773
```

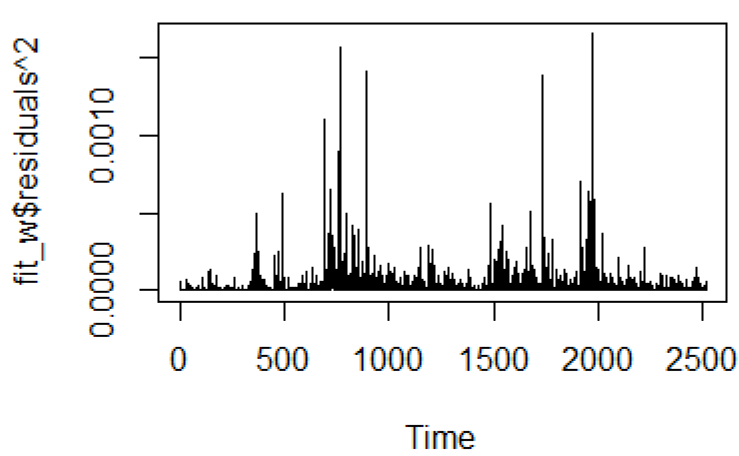
So, we observe that ARIMA (1,1,1) has much greater AIC values, which suggets it is not a better fit. Hence, we select the former ARIMA model (3,1,1). Next, step is to check residuals.

Diagnostic Checking : Resdidual Analysis:

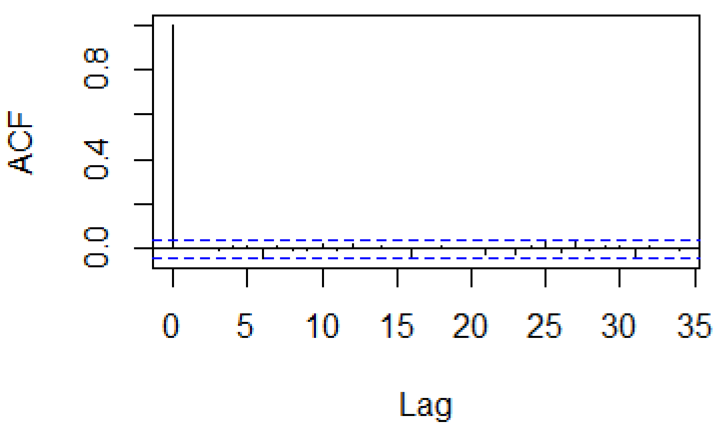
Residuals of ARIMA (3,1,1) fitted model



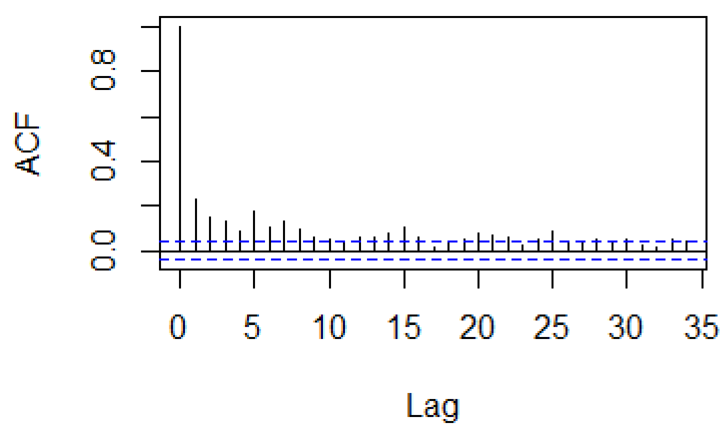
Residuals^2 of ARIMA (3,1,1) fitted model



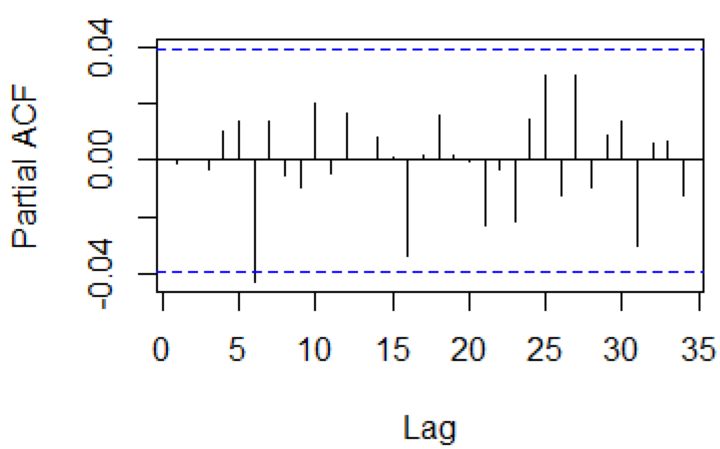
ACF of Residuals of ARIMA (3,1,1) model



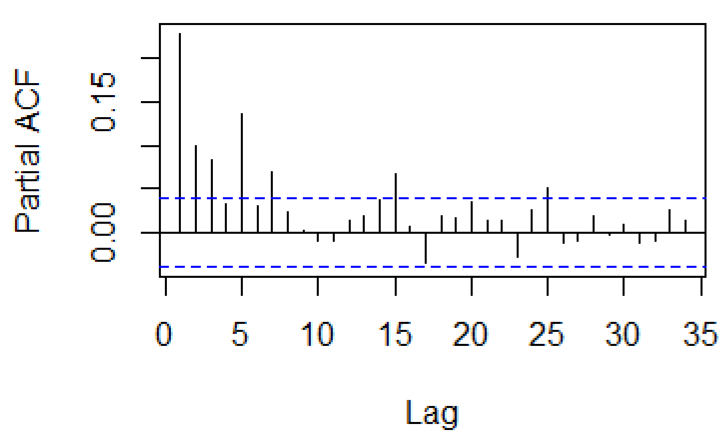
ACF of Residuals^2 of ARIMA (3,1,1) model



PACF of Residuals of ARIMA (3,1,1) model



PACF of Residuals^2 of ARIMA (3,1,1) model




```
 Box-Ljung test

data:  fit_w$residuals
X-squared = 21.3478, df = 30, p-value = 0.8767
```

```
 Box-Ljung test

data:  fit_w$residuals^2
X-squared = 594.9973, df = 30, p-value < 2.2e-16
```

Above on the left side, the plot of the residuals does not observe a significant trend, and the ACF/PACF do not have any significant lags, indicating that ARIMA (3, 1, 1) is a good fit. Besides, the Ljung-Box test also gives the result that residuals is a white noise.

Fitting AR-GARCH Model:

Above on the right side, the residuals squared plots and the ACF/PACF plots of reveal that that the residuals of the ARIMA model has clustering volatility. Also, the Ljung-Box test gives the result that residuals squared is not a white noise.

Three visible evidence supports the volatility clustering:

1. At some points of the plot, volatility seems to be persistent and clustering.
2. The ACF for all the lags are significantly large, indicating that error term $\varepsilon^2{}_t$ is correlated with past $\varepsilon^2{}_{t-k}$, meaning large changes in returns tend to cluster together and small changes tend to cluster together. That leads to conditionally heteroscedasticity.
3. PACF does not die down and some lags are significantly large.

To allow for changing volatility we apply ARCH/GARCH to model the conditional variance. The GARCH(p,q) model is

$$a_t = \epsilon_t \sigma_t \text{ where } \sigma_t = \sqrt{\alpha_0 + \sum_{i=1}^{p} \alpha_i a_{t-i}^2 + \sum_{i=1}^{q} \beta_i \sigma_{t-i}^2} \text{ (suppose for now } \epsilon_t \text{be a white noise)}$$

Note that for the process to be strictly stationary with finite variance, $\sum_{i=1}^{p} \alpha_i + \sum_{i=1}^{q} \beta_i < 1$. If equal to one, the solution will have infinite variance and is an IGARCH(p,q) model. The conditional variance depends on past observations a_{t-i}^2 and past conditional variances σ_{t-i}^2. Compared to AR model which has constant conditional variance and inconstant conditional expectation, an ARCH model had a constant conditional expectation and varying conditional variance. Nevertheless the GARCH process is still stationary because it has constant expectation, constant unconditional variance and correlation $\sigma_t(h) = 0$.

The following models were tried and the corresponding AIC values are:

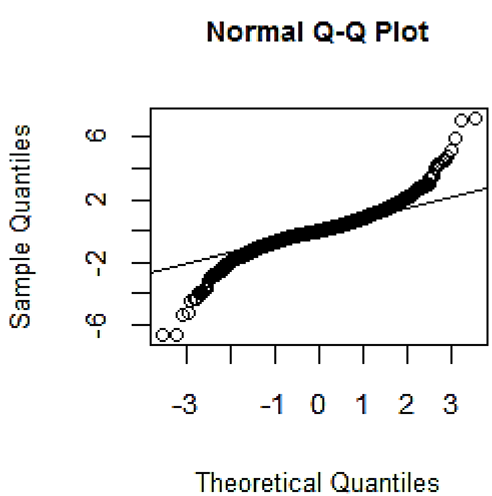
Model Specification	AIC normalized
AR(1)/GARCH(1,1)	-7.852
ARMA(1,1)/GARCH(1,1)	-7.86
AR(2)/GARCH(1,1)	-7.8603
ARMA(2,1)/GARCH(1,1)	-7.8604
AR(1)/GARCH(2,1)	-7.852
ARMA(2,1)/GARCH(1,1)	-7.859

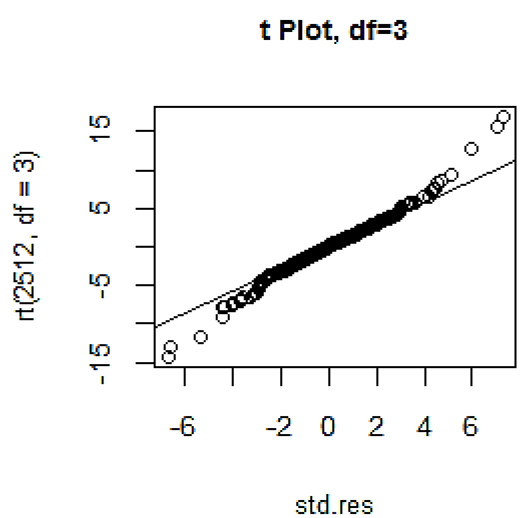
Now, initially we used normalized errors in Garch model and we got following R-output for various tests: (only partial output is shown, the model coefficeints are not shown here)

```
Standardised Residuals Tests:
                                 Statistic p-Value
 Jarque-Bera Test   R    Chi^2  2618.065   0
 Shapiro-Wilk Test  R    W      0.9587991 0
 Ljung-Box Test     R    Q(10)  19.33136   0.03625026
 Ljung-Box Test     R    Q(15)  24.97965   0.0502174
 Ljung-Box Test     R    Q(20)  26.65397   0.145283
 Ljung-Box Test     R^2  Q(10)  7.402632   0.6869652
 Ljung-Box Test     R^2  Q(15)  8.773419   0.8890635
 Ljung-Box Test     R^2  Q(20)  11.39551   0.935314
 LM Arch Test       R    TR^2   7.81721    0.7992467

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-7.735918 -7.724316 -7.735926 -7.731707
```

The Jarque-Bera test of normality strongly rejects the null hypothesis that the white noise innovation process ∊t is Gaussian. The Ljung-Box test shows no joint correlation of the lagged values, for both residuals and squared residuals. The GARCH(1,1) fits the residuals well, except for the nonnormality of the ∊t noted. Now, garchFit function allows the white noise to have a non Gaussian distribution.

A t-distribution was fit to std. residuals by maximum likelihood estimation using R's *fitdistr* function. The MLE of the degrees-of-freedom parameter was 3. This confirms the good fit by this distribution as seen below in Q-Q plots.





It can be confirmed that the latter plot is nearly a straight line except for small number of outliers in the tails. Here, the sample size of the data is 2512. so the outliers are a very small fraction of the data. Thus, it seems like a t-model would be suitable for the white noise.

Hence, <u>the garchFit function was used to refit different models assuming t-distributed errors</u>. As per the lowest AIC values and least numbers of parameters required to estimate, we select AR(1)/GARCH(1,1) model as our final model. To select a best fit model, AIC was used as a quality parameter to select amongst the many models.

The R-Output for the final model - AR(1)/GARCH(1,1) is shown here:

```
Title:
 GARCH Modelling

Call:
 garchFit(formula = ~arma(1, 0) + garch(1, 1), data = w, cond.dist = "std")

Mean and Variance Equation:
 data ~ arma(1, 0) + garch(1, 1)
<environment: 0x29f69a68>
 [data = w]

Conditional Distribution:
 std

Coefficient(s):
         mu          ar1         omega        alpha1         beta1         shape
-2.0270e-05   1.0000e+00   5.4250e-07   1.0721e-01   8.8669e-01   4.3073e+00

Std. Errors:
 based on Hessian

Error Analysis:
         Estimate  Std. Error   t value Pr(>|t|)
mu      -2.027e-05   1.202e-04    -0.169    0.866
ar1      1.000e+00   4.926e-05 20300.879  < 2e-16 ***
omega    5.425e-07   7.315e-07     0.742    0.458
alpha1   1.072e-01   1.735e-02     6.179 6.45e-10 ***
beta1    8.867e-01   1.254e-02    70.689  < 2e-16 ***
shape    4.307e+00   3.870e-01    11.130  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Log Likelihood:
 9869.096    normalized:  3.92878

Description:
 Sat Nov 21 02:03:19 2015 by user: Jigar Mehta
```

Diagnostic Checking

The output also included the following tests applied to the standardized residuals and squared residuals.

```
Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  3345.776  0
 Shapiro-Wilk Test  R    W      0.9556686 0
 Ljung-Box Test     R    Q(10)  17.45075  0.06496634
 Ljung-Box Test     R    Q(15)  23.43437  0.0753428
 Ljung-Box Test     R    Q(20)  24.8551   0.2070309
 Ljung-Box Test     R^2  Q(10)  5.144371  0.8813274
 Ljung-Box Test     R^2  Q(15)  7.240103  0.9506412
 Ljung-Box Test     R^2  Q(20)  9.715517  0.9730419
 LM Arch Test       R    TR^2   6.228566  0.9041243

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-7.852783 -7.838861 -7.852794 -7.847730
```

The p-values of alpha1 beta1 is smaller than 0.05, indicating the parameters are significantly different from zero. In the output, ϕ is denoted by ar1, the mean is mean, and w is called omega. Note that ϕ = 0.0986 and is statistically significant, implying that this is a small amount of positive autocorrelation. Both $\alpha 1$ and β1 are highly significant and β1 = 0.859, which implies rather persistent volatility clustering.

In the output from garchFit, the normalized log-likelihood is the log-likelihood divided by n. The AIC and BIC values have also been normalized by dividing by n, so these values should be multiplied by n = 2518 to have their usual values.

The Ljung-Box tests for residuals R and squared residuals are larger than 0.05 up to lag=20 which implies there are no auto-correlations. This confirms that R and R^2 are white noise and the AR(1)/GARCH(1,1) model fits the data well. The nonsignificant LM Arch Test indicates the same.

Inferences and Conclusion:

- ARIMA(3,1,1) was fitted to the original US to INR daily log returns time series. The residuals of the fitted model were normal and white noise.

- However, the standardized residuals squared of fitted ARIMA model of US to INR exchange rate time series shows periods of high volatility and periods of low volatility i.e. volatility clustering, which suggest ARIMA model is not sufficient for such time series.

- ARMA models are used to model the conditional expectation of a process given the past, but in an ARMA model the conditional variance given the past is constant.

- For accurate modeling of time-varying volatility is needed, AR-GARCH time series model is fitted to the original series. The GARCH model helps in modeling both the conditional heteroscedasticity and the heavy-tailed distribution

- The capability of the GARCH(1,1) model to fit the lag-1 autocorrelation and the subsequent rate of decay separately is the main reason that the GARCH(1,1) model fits our given financial time series.

R Codes:

Section1: Fitting Univariate distribution to daily log returns of Apple stock

```
library("fGarch")
library("tseries")
library("sn")
library("LambertW")
library("forecast")

cr<-read.csv("C:\\Users\\Jigar Mehta\\Desktop\\GSU\\FALL 15 Live courses\\ECON 8780 - Fin econo\\Final
Project\\apple.csv")
View(cr)
cr$year=year(as.POSIXlt(cr$Date, format="%m/%d/%Y"))
cs<-cr$Adj.Close
plot(cr$Date,cr$Adj.Close,type='l',lwd=3,ylab='Apple Stock Price',xlab="time",main="Time Series plot of Data
Series",cex.main=0.9,cex.axis=0.9,cex.lab=0.8)
css<-diff(log(cs))
plot(css,type='l',lwd=2,main="Apple Stock Daily log returns",ylab='Daily log
returns',xlab="time",cex.main=0.9,cex.axis=0.9,cex.lab=0.8)
plot(density(css),type='l',lwd=3,ylab='Frequency',cex.main=0.9,cex.axis=0.9,cex.lab=0.8,xlab='Values',main='Den
isty plot of Daily log returns')

qqnorm(css,cex.main=0.9,cex.axis=0.9,cex.lab=0.8)
qqline(css)

r1<-stdFit(css)
aic_r1<- 2*(r1$objective)+2*3
bic_r1<- 2*(r1$objective)+log(1467)*3

r2<-sstdFit(css)
aic_r2<- 2*(r2$minimum)+2*4
bic_r2<- 2*(r2$minimum)+log(1467)*4

r3<-gedFit(css)
aic_r3<- 2*(r3$objective)+2*3
bic_r3<- 2*(r3$objective)+log(1467)*3

r4<-sgedFit(css)
aic_r4<- 2*(r4$objective)+2*4
bic_r4<- 2*(r4$objective)+log(1467)*4

s<-qged(ppoints(length(css)),mean=r1$par[1],sd=r1$par[2],nu=r1$par[3])
g<-qged(ppoints(length(css)),mean=r3$par[1],sd=r3$par[2],nu=r3$par[3])
qqplot(css,s,main="Fitted t-model",cex.main=1,cex.lab=1,xlab="data",ylab="t-quantiles")
qqplot(css,g,main="Fitted ged-model",cex.main=1,cex.lab=1,xlab="data",ylab="ged-quantiles")
```

Section 2: Fitting multivariate distribution to daily returns of Deutsche bank, Credit Suisse and HSBC stock prices

```
par(mfrow=c(1,1))
library("tseries")
library("forecast")
library("MASS")
library("sn")

db <- read.csv("C:/Users/Jigar Mehta/Desktop/GSU/FALL 15 Live courses/ECON 8780 - Fin econo/Final Project/DB .csv")
View(db)
attach(db)
db<-db[order(Date),]
db<-db$Adj.Close
db<-log(db)
detach(db)

cr <- read.csv("C:/Users/Jigar Mehta/Desktop/GSU/FALL 15 Live courses/ECON 8780 - Fin econo/Final Project/credit suise.csv")
View(cr)
attach(cr)
cr<-cr[order(Date),]
cr<-cr$Adj.Close
cr<-log(cr)
View(cr)
detach(cr)

cit <- read.csv("C:/Users/Jigar Mehta/Desktop/GSU/FALL 15 Live courses/ECON 8780 - Fin econo/Final Project/hsbc.csv")
View(cit)
attach(cit)
cit<-cit[order(Date),]
cit<-cit$Adj.Close
cit<-log(cit)
View(cit)
detach(cit)

plot(db,type='l',lwd=2,cex.main=0.9,cex.axis=0.7,cex.lab=0.7,main="Daily log returns of Deutsche Bank",ylab='Daily log returns',xlab='Time')
plot(cr,type='l',lwd=2,cex.main=0.9,cex.axis=0.7,cex.lab=0.7,main="Daily log returns of Credit Suisse",ylab='Daily log returns',xlab='Time')
plot(cit,type='l',lwd=2,cex.main=0.9,cex.axis=0.7,cex.lab=0.7,main="Daily log returns of HSBC bank",ylab='Daily log returns',xlab='Time')

par(mfrow=c(1,3))
```

```
qqnorm(cr,main="Normal Q-Q Plot for Credit Suisse",cex.main=1);qqline(cr)
qqnorm(db,,main="Normal Q-Q Plot for Deutsche bank",cex.main=1);qqline(db)
qqnorm(cit,,main="Normal Q-Q Plot for HSBC",cex.main=01);qqline(cit)

par(mfrow=c(1,3))

plot(density(cr),type='l',lwd=3,cex.main=0.9,cex.axis=0.7,cex.lab=0.8,main='Density plot for Credit Suisse')
plot(density(db),type='l',lwd=3,cex.main=0.9,cex.axis=0.7,cex.lab=0.8,main='Density plot for Deutsche Bank')
plot(density(cit),type='l',lwd=3,cex.main=0.9,cex.axis=0.7,cex.lab=0.8,main='Density plot for HSBC')

cit<-diff(cit)
cr<-diff(cr)
db<-diff(db)

dat<-cbind(cr,db,cit)
cor(dat)
cov(dat)
pairs(dat,main='Scatterplot matrix for three assets',cex.main=1.0)

fit1=selm(dat[,1:3]~1,family="st")
print(fit1)
summary(fit1,param.type="DP")
summary(fit1,param.type="DP")

fit2=mst.mple(y=dat[,1:3],penalty=NULL)
print(fit2)
fit2$dp
```

Section 3: Fitting VAR Model to Deutsche bank and Credit Suisse stock returns

```
library("vars")
y=cbind(db,cr)
ts.plot(db,col="red")
lines(cr,col="blue",lty=3)
View(y)
lag=VARselect(y, lag.max = 10, type = "const")
lag_m=max(lag$selection)
var_mod=VAR(y, p = lag_m, type = "const")
summary(var_mod)
```

Section 4: Fitting Seasonal ARIMA Model to monthly housing units sold data

```
library("xlsx")
library("tseries")
library("forecast")
library("fracdiff")
```

```
library("lubridate")

tr<-read.xlsx("C:\\Users\\Jigar Mehta\\Desktop\\GSU\\FALL 15 Live courses\\ECON 8780 - Fin econo\\Final Project\\hou.xlsx",sheetName="FRED Graph")
View(tr)
ts<-(tr$units)
ts<-log(ts)
plot(ts,type='l',lwd=2,ylab="log of housing units",xlab='time',cex.lab=0.8,cex.main=0.8,main="Time series plot of log of Housing units")

boxplot(log(tr$units)~tr$mon,main="BoxPlot of housing data",cex.lab=0.8,cex.axis=0.8,cex.main=0.9,xlab="Months",ylab="Log (housing units)")

acf(ts,lag=50,main="")
acf(diff(ts,1),lag=50,main='',cex.lab=0.8,cex.axis=0.8)
acf(diff(ts,12),lag=50,main='')
acf(diff(diff(ts,1),12),lag=50,main='',cex.lab=0.8,cex.axis=0.8)

fit_w<-Arima(x=ts,order=c(1,1,1),seasonal=list(order=c(1,1,1),period=12))
fit_w
res<-fit_w$residuals
plot(res,ylab='residuals',main='Residuals - ARIMA{(1,1,1) x (1,1,1)p=12} model',cex.main=0.9,cex.axis=0.9,cex.lab=0.8)
acf(res,main="",cex.main=0.9,cex.axis=0.9,cex.lab=0.8)
plot(density(res^2))
Box.test(res,lag=30,'Ljung-Box')
plot(res^2,ylab='residuals',main='Residuals squarred - ARIMA model',cex.main=0.8,cex.axis=0.9,cex.lab=0.8)
acf(res^2,main="",cex.main=0.8,cex.axis=0.9,cex.lab=0.8)
pacf(res^2,main="",cex.main=0.8,cex.axis=0.9,cex.lab=0.8)
plot(res^2)
Box.test(res^2,lag=30,'Ljung-Box')

plot(fit_w$x,col='red',pch=20)
lines(fitted(fit_w),lwd=1.5,col='blue')

plot(forecast(fit_w,h=50),lwd=2,xlab="time",ylab='Housing units',cex.main=1,cex.axis=0.9,cex.lab=0.8)
```

Section 5: Fitting AR- GARCH model to US to INR dollar exchange rate

```
par(mfrow=c(1,1))
library("tseries")
library("forecast")
library("fGarch")
inr <- read.csv("C:/Users/Jigar Mehta/Desktop/GSU/FALL 15 Live courses/ECON 8780 - Fin econo/Final Project/inr.csv")
View(inr)
```

```
w1<-(inr$DEXINUS)
plot(w,type='l',lwd=2,cex.main=0.9,cex.axis=0.8,cex.lab=0.8,xlab="time",ylab="INR Exchange rate",main="Daily
US$ to INR Exchange rate")

w<-diff(log(w1))
w<-log(w1)
plot(w,type='l',lwd=1,cex.main=0.9,cex.axis=0.8,cex.lab=0.8,xlab="time",ylab="change exchange
rate",main="Daily Changes in US$/INR Exch rate ")
acf(w,main="",cex.main=0.9,cex.axis=0.8,cex.lab=0.8)
pacf(w,lag=35,main="",cex.main=0.9,cex.axis=0.8,cex.lab=0.8,)
Box.test(w,lag=10,'Ljung-Box')

plot(density(w))
qqnorm(w)
qqline(w)
dif_w<-diff(w1)
acf(dif_w)
Box.test(dif_w,lag=5,'Ljung-Box')
adf.test(dif_w)
pp.test(dif_w)
kpss.test(dif_w)

fit_w<-auto.arima(w,max.p=10,max.d=2,max.q=5,max.P=0,max.D=0,max.Q=0,ic='aic')
fit_w

plot(fit_w$residuals,type='l',xlab='Time',ylab='Residuals',main='Residuals of ARIMA (3,1,1) fitted
model',cex.main=0.8,cex.axis=0.6,cex.lab=0.6)

acf(fit_w$residuals,main='')
pacf(fit_w$residuals,main='')

plot(fit_w$residuals^2,main='Residuals^2 of ARIMA (3,1,1) fitted model',cex.main=0.8)
acf(fit_w$residuals^2,main='',cex.main=0.8,cdex.lab=0.7)
pacf(fit_w$residuals^2,main='')
Box.test(fit_w$residuals,lag=30,'Ljung-Box')
Box.test(fit_w$residuals^2,lag=30,'Ljung-Box')

nrow(data.frame(w))
res.arima<-fit_w$residuals
sd(res.arima)
std.res=res.arima/sd(res.arima)
qqnorm(std.res,cex.main=0.9,cex.axis=0.9,cex.lab=0.8)
qqline(std.res)
qqplot(std.res,rt(2512,df=3),main="t Plot, df=3",cex.main=0.9,cex.axis=0.9,cex.lab=0.8)
qqline(std.res,rt(2512,df=3),main="t Plot,df=3")

gmo<-garchFit(~ arma(1,0)+garch(1, 1), data = w,cond.dist = "std")
summary(gmo)
```