1 Prove or Disprove — Choosing a technique

(a) $(\forall n \in \mathbb{N})$ if n is odd then $n^2 + 4n$ is odd.

(b) $(\forall a, b \in \mathbb{R})$ if $a + b \leq 15$ then $a \leq 11$ or $b \leq 4$.

(c) $(\forall r \in \mathbb{R})$ if r^2 is irrational, then r is irrational.

(d) $(\forall n \in \mathbb{Z}^+)$ $5n^3 > n!$. (Note: $\mathbb{Z}^+$ is the set of positive integers)

① Write the stmt as a prop
② Try to prove it
③ Get stuck?

2 Pigeonhole Principle

Prove the following statement: If you put $n+1$ balls into n bins, however you want, then at least one bin must contain at least two balls. This is known as the *pigeonhole principle*.

Assume every bin has ≤ 1 ball.
Max n balls.
But, we have $n+1$!!!

3 Numbers of Friends Just a proof

Prove that if there are $n \geq 2$ people at a party, then at least 2 of them have the same number of friends at the party. Assume that friendships are always reciprocated: that is, if Alice is friends with Bob, then Bob is also friends with Alice.
(Hint: The Pigeonhole Principle states that if n items are placed in m containers, where $n > m$, at least one container must contain more than one item. You may use this without proof.)

$\overset{P}{(n \geq 2 \text{ ppl @ party})} \Rightarrow \overset{Q}{\underbrace{\text{at least two ppl have same \# frnds}}_{\text{assume not true}}}$

$P \Rightarrow Q$

$\neg Q \Rightarrow R$

3) Assume everyone has diff # friends.

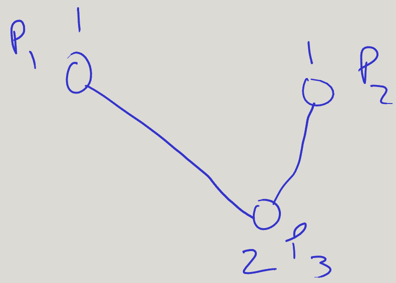
There are n ppl @ party.

$\# P_1 = 0$

$\# P_2 = 1$

$\vdots \; P_i \quad \vdots \; i-1$

$\# P_n = n-1$



1. (a) Let $n = 2k+1$

$$n^2 + 4n$$

$$= (2k+1)^2 + 8k + 4$$

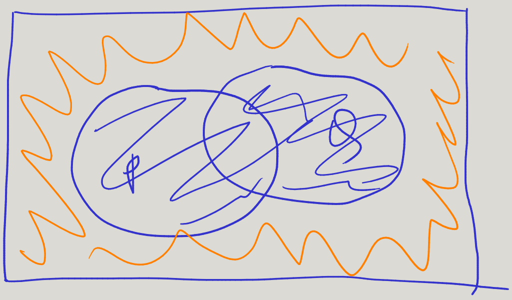
$$= 4k^2 + 4k + 1 + 8k + 4$$

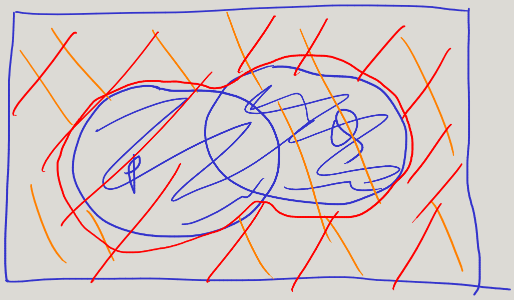
$$= 2\underbrace{(2k^2 + 6k + 2)}_{\ell \in \mathbb{N}} + 1$$

1. (b) $(a+b \leq 15) \Rightarrow (a \leq 11 \vee b \leq 4)$ True

$(a > 11 \wedge b > 4) \Rightarrow a+b > 15$

$$\neg(P \vee Q) \equiv \neg P \wedge \neg Q$$





1. (c) $r = \dfrac{p}{q}$

$$r^2 = \frac{p^2}{q^2}$$

1. (d) $7! = 5040$, $5 \cdot 7^3 = 1715$