$GCD(P(x), Q(x)) = 1$

$gcd(p,q) = 1$

$\underline{A(x)}P(x) + \underline{\underline{B(x)}}Q(x) = 1 \quad \text{mod } Q(x)$

$\frac{p}{q}$ rem 1

$\Rightarrow A(x)P(x) \equiv 1 \pmod{Q(x)}$

$p = kq + 1$

$\Rightarrow A(x) \equiv P^{-1}(x) \pmod{Q(x)}$

$ap + b\overset{-k}{q} = 1$ iff $gcd(p,q) = 1$

$ap \equiv 1 \pmod{q}$

$\deg P(x) \geq \deg Q(x)$

$x^3 + x \qquad x^2 + 1$

$\deg P(x) \leq \deg Q(x)$

$P(x) = B(x)Q(x) + 1$

$Q(x) = A(x)P(x) + 1$

$\Rightarrow 1 \cdot P(x) - B(x)Q(x) = 1$

$-A(x)P(x) + 1 \cdot Q(x) = 1$

$\underbrace{(x^2+1)} - (\underbrace{(x^3+x^2+1)} - \underbrace{(x^2+1)}(x+1))(-x)$

$= (x^2+1)(\quad) + \underline{x}(x^3+x^2+1)$

X # of heads in 5 tosses

$\underline{H}\ \underline{H}\ \underline{T}\ \underline{H}\ \underline{T}$

$X = \sum_{i=1}^{5} X_i$

$X_1 = 1,\ X_2 = 1,\ X_3 = 0,\ X_4 = 1,\ X_5 = 0$

$X = 3$

$E[X] = E\left[\sum_{i=1}^{5} X_i\right]$

$X_i = \begin{cases} 1 & \text{if toss } i \text{ is } H \\ 0 & \text{o/w} \end{cases}$

$X_1 \sim \text{Bernoulli}(p)$

$X_2 \sim \text{Bernoulli}(\underline{\underline{p}})$

$= E[X_1 + X_2 + X_3 + X_4 + X_5]$

$= E[X_1] + E[X_2] + E[X_3] + E[X_4] + E[X_5]$

$= 5E[X_1] = 5\underline{\underline{Pr[X_1 = 1]}} = 5\underline{\underline{p}}$

$$\mathrm{Var}(X) = \cancel{\mathrm{Var}\left(\sum_{i=1}^{n} X_i\right)}$$

$$= \mathbb{E}[X^2] - (\mathbb{E}[X])^2$$

$X \sim \mathrm{Bern}(p)$

$\Downarrow$

$X^2 \sim \mathrm{Bern}(p)$

$0, \quad 1$

$(1-p) \quad p$

$\forall i \quad \Pr[X_i = 1] = p$

$\cancel{\Rightarrow} \; X_i \perp X_j \quad \forall i, j$

$$\mathbb{E}[X^2] = \mathbb{E}\left[\left(\sum_{i=1}^{n} X_i\right)^2\right] = \mathbb{E}\left[\sum_{i=1}^{n} X_i^2 + \sum_{1 \le i \ne j \le n} X_i X_j\right]$$

$$= \sum_{i=1}^{n} \mathbb{E}[X_i^2] + \sum_{1 \le i \ne j \le n} \mathbb{E}[X_i X_j]$$

$$= n\Pr[X_i = 1] + (n(n-1)) \Pr[X_i = 1 \cap X_j = 1]$$

$X_i X_j \sim \mathrm{Bern}(?)$

$\Pr[X_i X_j = 1]$

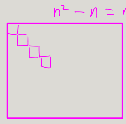
$= \Pr[X_i = 1 \cap X_j = 1]$

Throw 6 balls into n bins.

X # of empty bins.

$$X_i = \begin{cases} 1 & \text{if bin } i \text{ is empty} \\ 0 & \text{o/w} \end{cases}$$

$n^2 - n = n(n-1)$



$$\begin{array}{l|cccc} X_i & X_1 & X_2 & X_3 \cdots & X_n \\ X_j & X_2 & X_1 & & \\ & X_3 & X_3 & & \\ & \vdots & \vdots & & \\ & X_n & X_n & & \end{array}$$

n, $n-1$

$$\Pr[X_3 = 1 \mid X_1 = 1] \underset{\ne}{<} \Pr[X_3 = 1]$$

$$\left(\frac{n-2}{n-1}\right)^6 \qquad \left(\frac{n-1}{n}\right)^6$$

$X \sim \mathrm{Bern}(0.4)$

$Y \sim \mathrm{Bern}(0.4)$

$Z \sim \mathrm{Poisson}(0.4)$

$W \sim \mathrm{Binomial}(2, 0.3)$

$V \sim \mathrm{Binomial}(2, 0.4)$

$$\sum_{x \in X} \Pr[X = x] = 1$$

$$\binom{n}{k} p^k (1-p)^{n-k}$$

$$\binom{2}{2}(0.4)^2 (0.6)^0 = 0.16$$

X	$\Pr[X]$	Y	$\Pr[Y]$
0	$1-0.4$	0	$1-0.4$
1	0.4	1	0.4

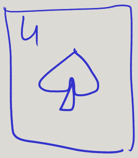
$$X_i = \begin{cases} 1 & A + B \text{ get } i^{th} \text{ card} \\ 0 & \text{o/w} \end{cases}$$

$$\Pr[X_i = 1] = \left(1 - \left(\frac{51}{52}\right)^k\right)^2$$

$\Pr[X = k] = \Pr[Y = k] \quad \forall k$

$\Pr[X = 2] = 0 \ne \Pr[V = 2] = 0.16$

Deck of 52 cards

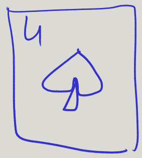
w/o repl. 

$\Pr[4\spadesuit \mid 4\spadesuit] = 0$

$\Pr[X_4 \spadesuit] = \frac{1}{52}$

$\Pr[X_3 \diamondsuit] = \frac{1}{52}$

identically distr.

w/ repl. 

$\Pr[4\spadesuit \mid 4\spadesuit] = \frac{1}{52}$

$\Pr[X_4 \spadesuit] = \frac{1}{52}$

$\Pr[X_3 \diamondsuit] = \frac{1}{52}$

$\Pr[\underset{X}{X_{4\spadesuit}} \mid \underset{Y}{X_{3\diamondsuit}}]$

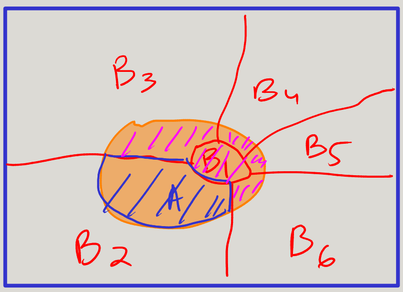
$$A = \{ (\underbrace{n, 0, 0, 0, \cdots, 0}_{k}), (\underbrace{0, n, 0, 0, \cdots, 0}_{k}) \cdots \}$$

$$\Omega \setminus A = \{ (n-1, 1, 0, \cdots), (1, 1, n-2, \cdots) \}$$

Toss a die. A = get an odd # $= \{1, 3, 5\}$ $\quad \Omega = \{1, 2, 3, 4, 5, 6\}$

B = get a prime # $= \{2, 3, 5\}$

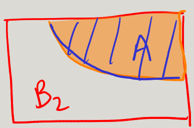
$P(\underset{\text{ask}}{A} \mid \underset{\text{tell}}{B}) = \frac{2}{3}$ $\qquad$ $P(\underset{\text{ask}}{A} \cap \underset{\text{ask}}{B}) = \frac{2}{6} = \frac{1}{3}$



$\Pr[A \mid B_k]$

$\Pr[A \cap B_k]$

$= \Pr[A \mid B_k] \Pr[B_k]$



$$= \frac{1}{k} \binom{n-1}{k-1} \left(\frac{1}{n}\right)^{k-1} \left(1 - \frac{1}{n}\right)^{n-k}$$

$n-k+k-1 = n-1$

$X \sim \mathrm{Binom}(n-1, 1/n)$ $\quad \Pr[X = k-1]$