Book: Introduction to Algorithms 3e

Chap: 17.1 Aggregate Aanlysis

Name: JHD

17.1-1 理想栈没有上限，不为$O(1)$，而是$O(k)$

17.1-2 $0111 \underset{-1}{\overset{+1}{\rightleftarrows}} 1000$ 不停抖动为$\Theta(nk)$

附加1段推导.

$$T_n = \sum_{i=0}^{k-1} \lfloor \frac{n}{2^i} \rfloor$$

$$= \sum_{i=0}^{k-1} \left(\frac{n}{2^i} - \{\frac{n}{2^i}\} \right)$$

$$= n\sum_{i=0}^{k-1} \frac{1}{2^x} \Delta x - \sum_{i=0}^{k-1} \{\frac{n}{2^i}\}$$

$$= n\cdot(2 - \frac{1}{2^{k-1}}) - \underbrace{\sum_{i=0}^{k-1} \{\frac{n}{2^i}\}}_{\in [0,\, k)}$$

$$\frac{T_n}{n} < 2 - \frac{1}{2^{k-1}} = O(1)$$

17.1-3

$$T_n = \sum_{\substack{1 \le i \le n \\ m}} [i = 2^m](2^m - 1) + n$$ (使用艾弗森约定)

$$= \sum_{m} [0 \le m \le \lfloor \lg n \rfloor](2^m - 1) + n$$

$$= \sum_{0}^{\lceil \lg n \rceil} (2^x - 1)\Delta x + n$$ (使用《Concrete Mathematics》的定和式技巧)

$$= (2^x - x)\Big|_0^{\lceil \lg n \rceil} + n$$

$$= 2^{\lceil \lg n \rceil} - \lceil \lg n \rceil - 1 + n$$

$$T_n/n = (2^{\lceil \lg n \rceil} - \lceil \lg n \rceil - 1 + n)/n$$

$$\le (2n - \lceil \lg n \rceil)/n = \Theta(1)$$