COURSE: Concrete Mathematics 2e
THEME: chap2 sums — warmups
NAME: JHD

2.1 no defination. (不是不定和式)

2.2 $x([x>0]-[x<0]) = |x|$

2.3 $\sum_{0\le k\le 5} a_k = a_0+a_1+a_2+a_3+a_4+a_5$ $\sum_{0\le k^2\le 5} a_{k^2} = a_4+a_1+a_0+a_1+a_4$

2.4 (a) $\sum_{1\le i<j<k\le 4} a_{ijk} = \sum_{1\le i\le 4}\sum_{i<j\le 4}\sum_{j<k\le 4} a_{ijk} = \sum_{i=1}^{4}\sum_{j=i+1}^{4}\sum_{k=j+1}^{4} a_{ijk}$

对k求和,将同ij结合;
对j求和,将同i结合

$((a_{123}+a_{124})+(a_{134}))+((a_{234}))$

$\sum_{1\le i<j<k\le 4} a_{ijk} = \sum_{1\le k\le 4}\sum_{1\le j<k}\sum_{1\le i<j} a_{ijk} = \sum_{k=1}^{4}\sum_{j=1}^{k-1}\sum_{i=1}^{j-1} a_{ijk}$

$((a_{123}))+((a_{124})+(a_{134}+a_{234}))$

2.5 stupid mistake. 怎能用k换j. □ 强行变成 \

2.6 as function of j and n ≡ 对k求和

$\sum_k [1\le j\le k\le n] = \sum_k [1\le j\le n][j\le k\le n] = [1\le j\le n]\sum_{k=j}^{n} = [1\le j\le n](n-j+1)$

2.7 $\nabla(x^{\overline{m}}) = \nabla(x(x+1)\cdots(x+m-1))$

$= x(x+1)\cdots(x-m-2)(x+m-1)$
$-(x-1)x(x+1)\cdots(x-m-2)$
$= m\cdot x(x+1)\cdots(x-m-2) = m\cdot x^{\overline{m-1}}$

与 $\Delta(x^{\underline{m}}) = m x^{\underline{m-1}}$ 对应

2.8 $0^{\overline{m}} = 0\cdot\ldots = 0$

2.9 $x^{\overline{m+n}} = \underbrace{x(x+1)\cdots(x+m-1)}_{m\text{ terms}}\underbrace{(x+m)\cdots(x+m+n-1)}_{n\text{ terms}} = x^{\overline{m}}(x+m)^{\overline{n}}$

与 $x^{\underline{m+n}} = x^{\underline{m}}(x-m)^{\underline{n}}$ 对应

$x^{\overline{0}} = x^{\overline{-n+n}} = x^{\overline{-n}}(x-n)^{\overline{n}} = 1$, $x^{\overline{-n}} = \frac{1}{(x-n)^{\overline{n}}} = \frac{1}{(x-1)\cdots(x-n)}$

2.10 $\Delta(uv) = u\cdot\Delta v + Ev\Delta u$ $\Delta(vu) = v\cdot\Delta u + Eu\cdot\Delta v$