COURSE: Concrete Mathematics

THEME: chap3 integer functions

NAME: JHD

3.1 $m=\lfloor \lg n\rfloor \qquad l = n - 2^{\lfloor \lg n\rfloor}$

3.2 $\lfloor x\rfloor[x\le \frac{\lfloor x\rfloor+\lceil x\rceil}{2}]+\lceil x\rceil[x>\frac{\lfloor x\rfloor+\lceil x\rceil}{2}]$

$=\lfloor x\rfloor[x\le \frac{\lfloor x\rfloor+\lceil x\rceil}{2}]+\lceil x\rceil(1-[x\le \frac{\lfloor x\rfloor+\lceil x\rceil}{2}])$

$=\lceil x\rceil-(\lceil x\rceil-\lfloor x\rfloor)[x\le \frac{\lfloor x\rfloor+\lceil x\rceil}{2}]$

$=\lceil x\rceil-$ [x不为整数] $[x\le \frac{\lfloor x\rfloor+\lceil x\rceil}{2}]$

(a) $\lfloor x+0.5\rfloor$

(b) $\lceil x-0.5\rceil$

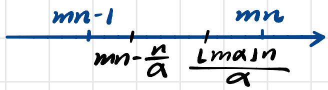
3.3 $\because m\alpha-1<\lfloor m\alpha\rfloor < m\alpha$ 无理数α

$\therefore mn-\frac{n}{\alpha}<\frac{\lfloor m\alpha\rfloor n}{\alpha}<mn$

$\because \alpha>n,\ \frac{n}{\alpha}<1$

$\therefore \lfloor mn-\frac{n}{\alpha}\rfloor = mn-1$

$\lfloor \frac{\lfloor m\alpha\rfloor n}{\alpha}\rfloor = mn-1$



3.4 什么鬼？连证明都没有的问题？

3.5 $n\in N^+$: $\lfloor nx\rfloor = n\lfloor x\rfloor$

$\Leftrightarrow \lfloor n\lfloor x\rfloor+n\{x\}\rfloor = n\lfloor x\rfloor$

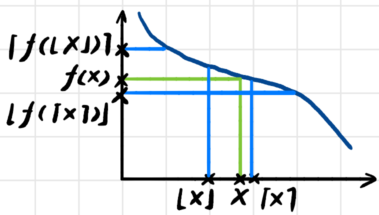
$\Leftrightarrow \lfloor n\{x\}\rfloor=0$

$\Leftrightarrow 0\le n\{x\}<1$

$\therefore 0\le\{x\}<\frac{1}{n}$

3.6 $f(x)\in Z \to x\in Z$

如图，有 $\begin{cases}\lfloor f(x)\rfloor = \lfloor f(\lceil x\rceil)\rfloor\\ \lceil f(x)\rceil = \lceil f(\lfloor x\rfloor)\rceil\end{cases}$



3.7 $X_n = X_{n-m}+1 = X_{n-2m}+2=\cdots$

$=X_{n \bmod m}+\lfloor\frac{n}{m}\rfloor$

$\therefore X_n = n \bmod m+\lfloor\frac{n}{m}\rfloor$

3.8 (离散学过)

3.9 (需证明极限逼近1)

$\frac{m}{n}=\frac{1}{q}+\frac{q-\frac{n}{m}}{\frac{n}{m}\cdot q}=\frac{1}{q}+\frac{n \text{ mumble } m}{\frac{n}{m}\cdot q}=\frac{1}{q}+\frac{m}{nq/n \text{ mumble } m}$

$\begin{cases} m\leftarrow m\\ n\leftarrow n\cdot q/n \text{ mumble } m\end{cases}$ $(\langle q=\lceil\frac{n}{m}\rceil,\ n \text{ mumble } m = \lceil\frac{n}{m}\rceil - m,\ n\nearrow)$

$\therefore$ 可以逼近$\frac{n}{m}$