COURSE: Concrete Mathematics 2e
THEME: chap2 sums - homework
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2.21

$$\sum_{k=0}^{n}(-1)^{n-k}+(-1)^{-1}=\sum_{k=-1}^{n}(-1)^{n-(k+1)}$$
$$=\sum_{k=0}^{n}(-1)^{n-(k+1)}+(-1)^{n}$$
$$=-\sum_{k=0}^{n}(-1)^{n-k}+(-1)^{n}$$

$$\therefore S_n=\frac{(-1)^n+1}{2}$$

$$\sum_{k=0}^{n}(-1)^{n-k}k+(-1)^{-1}\cdot(n+1)=\sum_{k=-1}^{n}(-1)^{n-(k+1)}(k+1)$$
$$=\sum_{k=0}^{n}(-1)^{n-(k+1)}(k+1)+0$$
$$=\sum_{k=0}^{n}(-1)^{n-(k+1)}k+\sum_{k=0}^{n}(-1)^{n-(k+1)}$$
$$=-\sum_{k=0}^{n}(-1)^{n-k}\cdot k-\sum_{k=0}^{n}(-1)^{n-k}$$

$$\therefore T_n=\frac{n+1-S_n}{2}=\frac{n+1-\frac{(-1)^n+1}{2}}{2}=\frac{2n+1-(-1)^n}{4}$$

$$\sum_{k=0}^{n}(-1)^{n-k}k^2+(-1)^{-1}(n+1)^2=\sum_{k=-1}^{n}(-1)^{n-(k+1)}(k+1)^2$$
$$=\sum_{k=0}^{n}(-1)^{n-(k+1)}(k+1)^2+0$$
$$=-\sum_{k=0}^{n}(-1)^{n-k}(k^2+2k+1)$$
$$=-\sum_{k=0}^{n}(-1)^{n-k}k^2-2T_n-S_n$$

$$\therefore U_n=\frac{(n+1)^2-2T_n-S_n}{2}=\frac{n^2+2n+1-(n+1)}{2}$$
$$=\frac{n(n+1)}{2}$$

$$S_n=[n\text{ is even}]$$
$$T_n=\frac{n+[n\text{ is even}]}{2}$$
$$U_n=\frac{n(n+1)}{2}$$