COURSE: Concrete Mathematics
THEME: chap3 integer functions
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3.14 $(x \bmod ny) \bmod y = x \bmod y,\ n \in \mathbb{Z}$?

$\because x = kny + x \bmod ny$

$\therefore (x \bmod ny) \bmod y = (x - kny) \bmod y = x \bmod y$

简单离散结论

3.15 $n = \lfloor \frac{n}{m} \rfloor + \lfloor \frac{n+1}{m} \rfloor + \cdots + \lfloor \frac{n+m-1}{m} \rfloor$

导出3.26

$n = \lceil \frac{n}{m} \rceil + \lceil \frac{n-1}{m} \rceil + \cdots + \lceil \frac{n-m+1}{m} \rceil$

导出 $mx = \lceil x \rceil + \lceil x - \frac{1}{m} \rceil + \cdots + \lceil x - \frac{m-1}{m} \rceil$

3.16 $n \bmod 2 = [n\text{为奇数}] = (1-(-1)^n)/2$

$$\begin{cases} n \bmod 3 = 0: & a + b\cdot\omega^{3k} + c\omega^{6k} = a+b+c = 0 \\ n \bmod 3 = 1: & a + b\cdot\omega^{3k+1} + c\omega^{6k+2} = a + b\omega + c\omega^2 = 1 \\ n \bmod 3 = 2: & a + b\cdot\omega^{3k+2} + c\omega^{6k+4} = a + b\omega^2 + c\omega = 2 \end{cases} \therefore \begin{cases} a = 1 \\ b = \frac{1}{\omega^2 - 1} \\ c = \frac{1}{\omega - 1} \end{cases}$$

3.17 $$\sum_{0 \le k < m} \lfloor x + \frac{k}{m} \rfloor$$

以j换k

$$= \sum_{0 \le k < m} \sum_j [1 \le j \le x + \frac{k}{m}]$$

$$= \sum_j \sum_k [1 \le j \le x + \frac{k}{m}][0 \le k < m]$$

$$= \sum_j m[1 \le j \le x]$$ $x + \frac{k}{m} \in [x, x+1)$, 即仅可取 x

$$= m(x - 1 + 1) = mx$$