COURSE: Concrete Mathematics 2e
THEME: chap2 sums - homework
NAME: JHD

2.24 $$\sum_{0\le k<n} \frac{H_k}{(k+1)(k+2)} = \sum_0^n H_x\, x^{\underline{-2}}\,\delta x$$

$$\sum H_x\, x^{\underline{m}}\,\Delta x = \sum H_x\, \Delta\left(\frac{x^{\underline{m+1}}}{m+1}\right)$$

$$= H_x \cdot \frac{x^{\underline{m+1}}}{m+1} - \sum \frac{(x+1)^{\underline{m+1}}}{m+1}\Delta H_x + C$$

$$= H_x \cdot \frac{x^{\underline{m+1}}}{m+1} - \sum \frac{x^{\underline{m}}}{m+1}\Delta x + C$$

$$= H_x \cdot \frac{x^{\underline{m+1}}}{m+1} - \frac{x^{\underline{m+1}}}{(m+1)^2} + C$$

$$\therefore \sum_0^n H_x x^{\underline{m}}\Delta x = H_x \cdot \frac{x^{\underline{m+1}}}{m+1}\Big|_0^n - \frac{x^{\underline{m+1}}}{(m+1)^2}\Big|_0^n$$

$$= H_n \cdot \frac{n^{\underline{m+1}}}{m+1} - \frac{n^{\underline{m+1}}}{(m+1)^2} + \frac{1}{(m+1)^2}$$

$$= H_n \cdot \frac{n^{\underline{m+1}}}{m+1} + \frac{1}{(m+1)^2}\left(1 - n^{\underline{m+1}}\right)$$

$m=-2$ 때. $$\sum_0^n H_x x^{\underline{-2}}\Delta x = H_n \cdot \frac{n^{\underline{-1}}}{-1} + 1 - n^{\underline{-1}}$$

$$= 1 - \frac{1}{n+1}(H_n + 1)$$

2.25

distributive $$\Big(\prod_{k\in K} a_k\Big)^c = \prod_{k\in K} a_k^c$$

associative $$\prod_{k\in K} a_k \cdot b_k = \prod_{k\in K} a_k \cdot \prod_{k\in K} b_k$$

commutative $$\prod_{k\in K} a_k = \prod_{p(k)\in K} a_{p(k)}$$

2.26 $$\prod_{1\le j\le k\le n} a_j a_k = \prod_{1\le j\le k\le n} a_k a_j$$

$$\prod_{1\le j\le k\le n} a_j a_k = \sqrt{\prod_{1\le j,k\le n} a_j a_k \cdot \prod_{1\le j=k\le n} a_j a_k}$$

$$= \sqrt{\Big(\prod_{1\le k\le n} a_k\Big)^{2n} \cdot \prod_{1\le k\le n} a_k^2} = \Big(\prod_{1\le k\le n} a_k\Big)^{n+1}$$