COURSE: Concrete Mathematics
THEME: chap 1 recurent problem exercises
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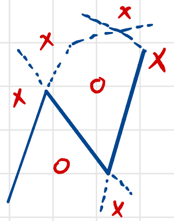
1.13



N线等价于3条/，但是每划3条/就损失5个区域

$T(n) = T(n-1) + (3n-2) + (3n-1) + 3n - 5$

$\quad = T(n-1) + 9n - 8$

$\therefore T(n) = \cancel{T(n-1)} + 9n - 8$

$\cancel{T(n-1)} = \cancel{T(n-2)} + 9(n-1) - 8$

$\cdots \qquad \cdots$

$\cancel{T(2)} = \cancel{T(1)} + 9 \cdot 2 - 8$

$\cancel{T(1)} = \cancel{T(0)} + 9 \cdot 1 - 8$

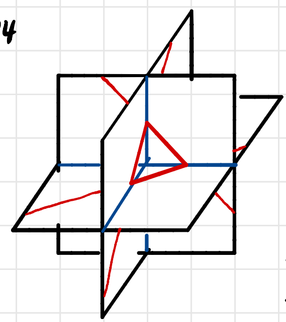
$\cancel{T(0)} = 1$

$\therefore T(n) = 1 + 9 \cdot \frac{(1+n)n}{2} - 8n$

$T(n) = \frac{9n^2 - 7n + 2}{2}$

$T(0) = 1 \quad T(1) = 2 \quad T(2) = 12$

1.14



$\overset{P(0)}{1} \xrightarrow[]{+1\ (L(0))} \overset{P(1)}{2} \xrightarrow[]{+2\ (L(1))} 4 \xrightarrow[]{+4\ (L(2))} 8 \xrightarrow[]{+7\ (L(3))} \underline{15} \xrightarrow[]{+11} 26?$

3点确定1个平面.

新切面被原有面截出的子面数 = 新的子块数

$\because$ 面与面的相交，有且仅有1条直线

$\therefore$ 与原有面数相等的直线在面上形成的子面数 $(L(n-1))$

$\therefore P(n) = P(n-1) + L(n-1)$

$P(0) = 1$

$\because L(n) = \frac{n(n+1)}{2} + 1 = \frac{n^2}{2} + \frac{n}{2} + 1$

$\therefore P(n) = \cancel{P(n-1)} + \frac{(n-1)^2}{2} + \frac{n-1}{2} + 1$

$\cancel{P(n-1)} = \cancel{P(n-2)} + \frac{(n-2)^2}{2} + \frac{n-2}{2} + 1$

$\cdots = \cdots$

$\cancel{P(1)} = \cancel{P(0)} + 0 + 0 + 1$

$\cancel{P(0)} = 1$

$\therefore \begin{cases} P(n) = \sum_{i=0}^{n-1} \frac{i(i+1)}{2}, & n \geq 1 \\ P(0) = 1 \end{cases}$