COURSE: Concrete Mathematics 2e
THEME: chap2 sums — basics
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2.11 $\sum_{0\le k<n}(a_{k+1}-a_k)b_k = a_nb_n - a_0b_0 - \sum_{0\le k<n} a_{k+1}(b_{k+1}-b_k), \quad n\ge 0$

$= \sum_{0\le k<n} a_{k+1}b_k - \sum_{0\le k<n} a_k b_k = \sum_{0\le k<n} a_{k+1}b_k - \sum_{0\le k+1<n} a_{k+1}b_{k+1}$

$= \sum_{0\le k<n} a_{k+1}b_k - \sum_{0\le k<n} a_{k+1}b_{k+1} + a_nb_n - a_0b_0 = a_nb_n - a_0b_0 - \sum_{0\le k<n} a_{k+1}(b_{k+1}-b_k)$

2.12 证明双射. one-to-one 即可. 离散数学

2.13 repetoire method

$\sum_{k=0}^{n}(-1)^k k^2 \Rightarrow \begin{cases} R(0)=0 \\ R(n)=R(n-1)+(-1)^n n^2 \end{cases}$

其 repetoire 为 $\begin{cases} R(0)=0 \\ R(n)=R(n-1)+(-1)^n(\alpha n^2+\beta n+\gamma) \end{cases}$

n	$R(n)$
0	
1	$-\alpha - \beta - \gamma$
2	$+3\alpha + \beta$
3	$-6\alpha - 2\beta - \gamma$
4	$+10\alpha + 2\beta$

设计好的 repetoire

$\therefore R(n)=(-1)^n(A(n)\cdot\alpha + B(n)\beta + C(n)\gamma)$

① $R(n)=(-1)^n\cdot n = (-1)^{n-1}(n-1) + (-1)^n(\alpha n^2+\beta n+\gamma)$

$n = 1-n+\alpha n^2+\beta n+\gamma, \quad \alpha=0,\ \beta=2,\ \gamma=-1$

$\therefore 2B(n)-C(n)=n$

② $R(n)=(-1)^n\cdot n^2 = (-1)^{n-1}(n-1)^2 + (-1)^n(\alpha n^2+\beta n+\gamma)$

$n^2 = -n^2+2n-1+\alpha n^2+\beta n+\gamma, \quad \alpha=2,\ \beta=-2,\ \gamma=1$

$\therefore 2A(n)-2B(n)+C(n)=n^2$

$\therefore A(n)=\frac{n^2+n}{2}$

$\therefore \alpha=1,\ \beta=0,\ \gamma=0 \qquad R(n)=(-1)^n A(n) = (-1)^n\cdot\frac{n^2+n}{2}$

先求得关键的 A(n), 再代入我们问题的参数