

COURSE: Concrete Mathematics

THEME: chap1 recurrent problem exercises

NAME: JHD

18:

n	0	1	2	3	4	5	6
Q_n	α	β	$\frac{1+\beta}{\alpha}$	$\frac{1+\alpha+\beta}{\alpha\beta}$	$\frac{1+\alpha}{\beta}$	α	β

以5为周期重复

19: (a) 代入 $X_n = \frac{X_1 + \dots + X_{n-1}}{n-1}$:

$$P(n): X_1 \dots X_n \leq \left(\frac{X_1 + \dots + X_n}{n} \right)^n = X_n^n$$

$$P(n-1): X_1 \dots X_{n-1} \leq \left(\frac{X_1 + \dots + X_{n-1}}{n-1} \right)^{n-1} = X_{n-1}^{n-1}$$

$$\therefore P(n) \wedge X_n = \frac{X_1 + \dots + X_{n-1}}{n-1} \wedge n > 1 \rightarrow P(n-1)$$

(b) if $P(n)$

then $\forall X_1, \dots, X_{2n} \geq 0$

$$\begin{cases} X_1 \dots X_n \leq \left(\frac{X_1 + \dots + X_n}{n} \right)^n \\ X_{n+1} \dots X_{2n} \leq \left(\frac{X_{n+1} + \dots + X_{2n}}{n} \right)^n \end{cases}$$

$$X_1 \dots X_{2n} \leq \left(\frac{X_1 + \dots + X_n}{n} \right)^n \cdot \left(\frac{X_{n+1} + \dots + X_{2n}}{n} \right)^n$$

$$\text{if } P(2): X_1 X_2 \leq \left(\frac{X_1 + X_2}{2} \right)^2$$

$$\begin{aligned} \text{then } X_1 \dots X_{2n} &\leq \left(\frac{\frac{X_1 + \dots + X_n}{n} + \frac{X_{n+1} + \dots + X_{2n}}{n}}{2} \right)^{2n} \\ &= \left(\frac{X_1 + \dots + X_{2n}}{2n} \right)^{2n}, P(2n) \end{aligned}$$

$$\therefore P(n), P(2) \rightarrow P(2n)$$

(c) $P(2)$



$P(4) \rightarrow P(3)$



$P(8) \rightarrow P(7) \rightarrow P(6)$



...

$\therefore \forall n > 1$

$P(2)$

$P(2), P(n) \rightarrow P(2n)$

$P(n) \rightarrow P(n-1)$

$\therefore \forall n \quad P(n)$