COURSE: Concrete Mathematics
THEME: chap 1 recurent problem exercises
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1.16

$$\begin{cases} g(1) = \alpha \\ g(2n+j) = 3g(n) + \gamma n + \beta_j \end{cases}$$

n	$g(n)$
1	α
2	$3\alpha + \gamma + \beta_0$
3	$3\alpha + \gamma + \quad + \beta_1$
4	$9\alpha + 5\gamma + 4\beta_0$
5	$9\alpha + 5\gamma + 3\beta_0 + \beta_1$
6	$9\alpha + 6\gamma + \beta_0 + 3\beta_1$
7	$9\alpha + 6\gamma \quad + 4\beta_1$
8	$27\alpha + 19\gamma + 13\beta_0$
9	$27\alpha + 19\gamma + 12\beta_0 + \beta_1$
10	$27\alpha + 20\gamma + 10\beta_0 + 3\beta_1$

写了表也没用

for $(\alpha = 1,\ \gamma = \beta_0 = \beta_1 = 1)$ 先正向设参数

$$g(2^m + l) = 3^m = \underline{A(n)}$$

for $(\gamma = 1,\ \alpha = \beta_0 = \beta_1 = 1)$

$$g(2^m + l) = \sum_{i=0}^{m-1} 3^{m-1-i} 2^i$$

$$= 3^{m-1} \cdot \sum_{i=0}^{m-1} \left(\frac{2}{3}\right)^i = \underline{3^m - 2^m = C(n)}$$

for $g(n) = n$, 再反向找特解

$(\alpha = 1,\ \gamma = -1,\ \beta_0 = 0,\ \beta_1 = 1)$

$$\underline{A(n) - C(n)} \ \underline{+ B_1(n) = n}$$

for $g(n) = 1$

$(\alpha = 1,\ \gamma = 0,\ \beta_0 = \beta_1 = -2)$

$$\underline{A(n) - 2B_0(n) - 2B_1(n) = 1}$$

$$g(n) = A(n)\alpha + C(n)\gamma + B_0(n)\beta_0 + B_1(n)\beta_1$$

$n = 2^m + l$, for $0 \le l < 2^m$

$$\begin{cases} A(n) = 3^m \\ C(n) = 3^m - 2^m \\ B_0(n) = \dfrac{3^m - 1}{2} - l \\ B_1(n) = l \end{cases}$$

repertoire method 也就是特殊值法

本题也可以二进制 relax, 但效果不理想