COURSE: Concrete Mathematics 2e

THEME: chap2 sums - homework

NAME: JHD

2.22 $$\sum_{1\le j<k\le n}(a_jb_k-a_kb_j)^2=\sum_{1\le j<k\le n}(a_kb_j-a_jb_k)^2$$

let $S_{\triangledown}=\sum_{1\le j<k\le n}(a_jb_k-a_kb_j)^2$ $\quad S_{\triangle}=\sum_{1\le k\le j\le n}(a_jb_k-a_kb_j)^2$

$$S_{\triangledown}=\frac{S_{\triangledown}+S_{\triangle}}{2}=\frac{1}{2}\Big(\sum_{1\le j,k\le n}(a_jb_k-a_kb_j)^2+\sum_{1\le j=k\le n}(a_jb_k-a_kb_j)^2\Big)$$

$$=\frac{1}{2}\sum_{1\le j,k\le n}(a_j^2b_k^2-2a_ja_kb_jb_k+a_k^2b_j^2)$$

$$=\sum_{1\le j,k\le n}(a_j^2b_k^2-a_ja_kb_jb_k)$$

$$=\sum_{1\le j\le n}a_j^2\sum_{1\le k\le n}b_k^2-\sum_{1\le j\le n}a_jb_j\cdot\sum_{1\le j\le k}a_kb_k$$

$$=\Big(\sum_{k=1}^{n}a_k^2\Big)\Big(\sum_{k=1}^{n}b_k^2\Big)-\Big(\sum_{k=1}^{n}a_kb_k\Big)^2$$

2.23 (a) $$\sum_{k=1}^{n}\frac{2k+1}{k(k+1)}=\sum_{k=1}^{n}(2k+1)\Big(\frac{1}{k}-\frac{1}{k+1}\Big)$$

$$=\sum_{k=1}^{n}2k\cdot\Big(\frac{1}{k}-\frac{1}{k+1}\Big)+\begin{pmatrix}\frac{1}{n}-\frac{1}{n+1}\\ \frac{1}{n-1}-\cancel{\frac{1}{n}}\\ \cdots\\ \frac{1}{1}-\cancel{\frac{1}{2}}\end{pmatrix}=1-\frac{1}{n+1}$$

$$=\sum_{k=1}^{n}\frac{2}{k+1}+1-\frac{1}{n+1}$$

$$=2\sum_{k=2}^{n+1}\frac{1}{k}+1-\frac{1}{n+1}$$

$$=2(H_{n+1}-1)+1-\frac{1}{n+1}$$

$$=H_{n+1}+H_n-1$$

(b) $$\sum_{k=1}^{n}\frac{2k+1}{k(k+1)}=\sum\nolimits_1^{n+1}\frac{2x+1}{x(x+1)}\Delta x$$

$$=\sum\nolimits_1^{n+1}(2x+1)(x-1)^{\underline{-2}}$$

$$=\sum\nolimits_1^{n+1}(2x+1)\Delta\big(-(x-1)^{\underline{-1}}\big)$$

$$=-(2x+1)(x-1)^{\underline{-1}}\Big|_1^{n+1}+\sum\nolimits_1^{n+1}x^{\underline{-1}}\Delta(2x+1)$$

$$=-\frac{2x+1}{x}\Big|_1^{n+1}+2\cdot H_x\Big|_1^{n+1}$$

$$=\Big(2H_x-2-\frac{1}{x}\Big)_1^{n+1}$$

$$=H_{n+1}+H_n-2-(2\cdot1-2-1)$$

$$=H_{n+1}+H_n-1$$