COURSE: Concrete Mathematics 2e
THEME: chap2 sums - basics
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2.14 $$\sum_{k=1}^{n} k2^k = \sum_{1\le j\le k\le n} 2^k = \sum_{1\le j\le n}\ \sum_{j\le k\le n} 2^k = \sum_{j=1}^{n}\sum_{k=j}^{n} 2^k = \sum_{j=1}^{n}\left(2^k\Big|_{j}^{n+1}\right)$$
$$= \sum_{j=1}^{n}(2^{n+1}-2^j) = n\cdot 2^{n+1} - 2^j\Big|_{1}^{n+1} = (n-1)\cdot 2^{n+1}+2$$

2.15 $$\left.\begin{aligned} &\boxdot_n + \square_n = 2\sum_{1\le j\le k\le n} jk \\ &S_{\triangleleft} = \sum_{1\le j\le k\le n} a_j a_k = \frac{1}{2}\Big(\big(\sum_{k=1}^{n} a_k\big)^2 + \sum_{k=1}^{n} a_k^2\Big)\end{aligned}\right\}$$
$$\Rightarrow \quad \boxdot_n + \square_n = \Big(\sum_{k=1}^{n} k\Big)^2 + \sum_{k=1}^{n} k^2$$
$$\boxdot_n = \Big(\frac{n(n+1)}{2}\Big)^2$$

2.16 $$\frac{x^{\underline{m}}}{(x-n)^{\underline{m}}} = \frac{x^{\underline{m-n+n}}}{(x-n)^{\underline{m}}} = \frac{x^{\underline{n}}(x-n)^{\underline{m-n}}}{(x-n)^{\underline{m}}} \quad \text{(分母不为0)}$$
$$= \frac{x^{\underline{n}}(x-n)^{\underline{m-n}}}{(x-n)^{\underline{m-n+n}}} = \frac{x^{\underline{n}}\cancel{(x-n)^{\underline{m-n}}}}{\cancel{(x-n)^{\underline{m-n}}}(x-m)^{\underline{n}}} = \frac{x^{\underline{n}}}{(x-m)^{\underline{n}}}$$

2.17 $$x^{\overline{m}} = x(x+1)\cdots(x+m-1) \qquad = (x+m-1)^{\underline{m}}$$
$$= (-x)(-x-1)\cdots(-x-m+1)\cdot(-1)^m = (-x)^{\underline{m}}\cdot(-1)^m$$
$$\because 1 = (x-1)^{\underline{0}} = (x-1)^{\underline{m-m}} = (x-1)^{\underline{-m}}\cdot(x-1+m)^{\underline{m}}$$
$$\therefore 1/(x-1)^{\underline{-m}} = (x+m-1)^{\underline{m}}$$

2.18 to prove: $\sum_{k\in K} a_k$ 绝对收敛 $\equiv$ ∃常数 B, ∀有限子集 $F\subseteq K$, $\sum_{k\in F}|a_k| \le B$