COURSE: Concrete Mathematics 2e
THEME: chap2 sums - homework
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2.19 $\begin{cases} T_0 = 5 \\ 2T_n = nT_{n-1} + 3n! \end{cases}$

$(a_n = 2,\ b_n = n,\ c_n = 3n!)$

$2S_nT_n = nS_nT_{n-1} + 3S_n \cdot n!$

let $2S_{n-1} = nS_n$, $S_n = \frac{2}{n}S_{n-1}$

$\therefore\ S_n = \frac{a_{n-1}\cdots a_0}{b_n\cdots b_1} = \frac{2^n}{n!}$

$\therefore\ \frac{2^{n+1}}{n!}T_n = \frac{2^n}{(n-1)!}T_{n-1} + 3\cdot 2^n$

let $S_n = \frac{2^{n+1}}{n!}T_n$

$\therefore\ S_n = \cancel{S_{n-1}} + 3\cdot 2^n = 10 + 3\cdot 2\cdot\frac{2^n-1}{2-1} = 3\cdot 2^{n+1} + 4$

$\cancel{S_{n-1}} = \cancel{S_{n-2}} + 3\cdot 2^{n-1}$

$\cancel{\cdots} =$

$\cancel{S_1} = \cancel{S_0} + 3\cdot 2^1$

$\cancel{S_0} = 2T_0 = 10$

$\therefore\ T_n = \frac{n!}{2^{n+1}}S_n = \left(3 + \frac{1}{2^{n-1}}\right)n!$

2.20 $\underline{\sum_{k=0}^{n} kH_k} + (n+1)H_{n+1} = \sum_{k=0}^{n}(k+1)H_{k+1}$

$= \sum_{k=0}^{n} kH_{k+1} + \sum_{k=0}^{n} H_{k+1}$

$= \sum_{k=0}^{n} k\left(H_k + \frac{1}{k+1}\right) + \sum_{k=0}^{n} H_{k+1}$

$= \sum_{k=0}^{n}\left(kH_k + 1 - \frac{1}{k+1}\right) + \sum_{k=0}^{n} H_{k+1}$

$= \sum_{k=0}^{n} kH_k + (n+1) - H_{n+1} + \sum_{k=0}^{n} H_{k+1}$

$= \underline{\sum_{k=0}^{n} kH_k} + (n+1) + \sum_{k=0}^{n} H_k$

$\therefore\ \sum_{k=0}^{n} H_k = (n+1)(H_{n+1} - 1)$