Verified Message-Passing Concurrency in Iris

Separation Logic Meets Session Types

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Tutorial Timeline

Part 1: 14:00 – 15:30

- Introduction (10 min)
- Layered implementation of session channels (10 min)
- Basic concurrent separation logic and one-shot protocols (30 min)
- **Break** (10 min)
- Dependent separation protocols (30 min)

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Part 2: 16:00 – 17:30

- Iris invariants and ghost state (30 min)
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Message Passing Concurrency

Shared-memory message passing concurrency:

- Structured approach to concurrent programming
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Bi-directional session channels:

new()	Create channel and return two endpoints c1 and c2
c.**send**(v)	Send value v over endpoint c
c.**recv**()	Receive and return next inbound value on endpoint c

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Example program:

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\begin{array}{l}
\texttt{let}\,(c_1, c_2) = \texttt{new}\,()\,\texttt{in} \\
\left(\begin{array}{l}
c_1.\texttt{send}(40); \\
\texttt{let}\, y = c_1.\texttt{recv}()\,\texttt{in} \\
\texttt{assert}(y = 42)
\end{array}
\;\middle\|\;
\begin{array}{l}
\texttt{let}\, x = c_2.\texttt{recv}()\,\texttt{in} \\
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\\
\end{array}\right)
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Safety and Functional Correctness

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Goal: Prove crash-freedom (safety) in presence of asserts (functional correctness)

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Actris: Session type-based extension of Iris

- **Session type-based:** Reasoning about message-passing concurrency via dependent separation protocols
- **MiniActris:** Layered minimalistic version of Actris from first principles (ICFP'23 Functional Pearl)

Learning Goals of this Tutorial

After this tutorial you will be able to:

- Design layers of abstractions in concurrent separation logic
- Verify sample programs using these abstractions
- Verify these abstractions using the Iris methodology
- Mechanize these results using the Iris Proof Mode in Coq

Overview of Abstraction Layers

Layer	Reasoning principles / specifications
#1 Iris's HeapLang	Basic concurrent separation logic Iris invariants and ghost state
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Layered implementation of session channels ala. MiniActris

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Layer #1: Iris's Heap Lang

Untyped OCaml-like language with

- Mutable references
- Higher-order recursive functions
- Parallel composition-based concurrency
- Assert statements

$$
\begin{aligned}
v, w \in \mathsf{Val} ::=\ & z \mid \mathbf{true} \mid \mathbf{false} \mid () \mid \ell \mid & (z \in \mathbb{Z}, \ell \in \mathsf{Loc}) \\
& \mathbf{rec}\ f\ x = e \mid (v, w) \mid \mathbf{Some}\, v \mid \mathbf{None} \\
e \in \mathsf{Expr} ::=\ & v \mid x \mid e_1\ e_2 \mid \\
& \mathbf{fst}\, e \mid \mathbf{snd}\, e \mid \\
& (\mathbf{match}\, e\, \mathbf{with}\, \mathbf{Some}\, v \Rightarrow e_1 \mid \mathbf{None} \Rightarrow e_2\ \mathbf{end}) \mid \\
& \mathbf{ref}\, e \mid \mathbf{free}\, e \mid !\, e \mid e_1 \leftarrow e_2 \mid \\
& (e_1 \parallel e_2) \mid \mathbf{assert}(e) \mid \ldots
\end{aligned}
$$

Example Program – Sequential

Simple sequential program:

$$
\begin{array}{l}
\texttt{ref\_prog} \triangleq \\
\quad \textbf{let}\ \ell = \textbf{ref None in} \\
\quad \ell \leftarrow \textbf{Some}\ 42; \\
\quad \textbf{let}\ x = !\,\ell\ \textbf{in} \\
\quad \textbf{free}\ \ell; \\
\quad \textbf{assert}(x = \textbf{Some}\ 42)
\end{array}
$$

The **assert** statement halts the program if the condition does not reduce to **true**

One-shot channel implementation:

$$
\begin{aligned}
\texttt{new1}\,() &\triangleq \texttt{ref None} \\
\texttt{send1}\,c\,v &\triangleq c \leftarrow \texttt{Some}\,v \\
\texttt{recv1}\,c &\triangleq \texttt{let}\,x = {!}\,c\,\texttt{in} \\
&\quad\ \texttt{match}\,x\,\texttt{with} \\
&\qquad \texttt{None} \Rightarrow \texttt{recv1}\,c \\
&\quad\ \mid \texttt{Some}\,v \Rightarrow \texttt{free}\,c;\ v \\
&\quad\ \texttt{end}
\end{aligned}
$$

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&\quad\ \mathtt{match}\,x\,\mathtt{with} \\
&\quad\quad \mathtt{None} \Rightarrow \mathtt{recv1}\,c \\
&\quad\ \mid \mathtt{Some}\,v \Rightarrow \mathtt{free}\,c;\ v \\
&\quad\ \mathtt{end}
\end{aligned}$$

Concurrent program that uses one-shot channels:

$$\begin{aligned}
&\mathtt{oneshot_prog} \triangleq \\
&\quad \mathtt{let}\,c = \mathtt{new1}\,()\,\mathtt{in} \\
&\quad \left(\mathtt{send1}\,c\,42 \ \middle\|\ \begin{array}{l}\mathtt{let}\,x = \mathtt{recv1}\,c\,\mathtt{in} \\ \mathtt{assert}(x = 42)\end{array}\right)
\end{aligned}$$

Example Programs – Reference Passing

Passing references over one-shot channels:

$$
\begin{array}{l}
\texttt{oneshot\_ref\_prog} \triangleq \\
\quad \textbf{let}\ c = \textbf{new1}\,()\ \textbf{in} \\
\quad \left(\begin{array}{l} \textbf{let}\ \ell = \textbf{ref}\ 42\ \textbf{in} \\ \textbf{send1}\ c\ \ell \\ \end{array} \middle\| \begin{array}{l} \textbf{let}\ \ell = \textbf{recv1}\ c\ \textbf{in} \\ \textbf{let}\ x = !\,\ell\ \textbf{in}\ \textbf{free}\ \ell; \\ \textbf{assert}(x = 42) \end{array}\right)
\end{array}
$$

Example Programs – Higher-Order Channels

Passing one-shot channels over one-shot channels:

$$
\begin{array}{l}
\texttt{oneshot\_chan\_prog} \triangleq \\
\quad \mathbf{let}\, c = \mathbf{new1}\,()\,\mathbf{in} \\
\quad \left(\begin{array}{l}
\mathbf{let}\, \ell = \mathbf{ref}\, 40\, \mathbf{in} \\
\mathbf{let}\, c' = \mathbf{new1}\,()\,\mathbf{in}\, \mathbf{send1}\, c\, (\ell, c'); \\
\mathbf{recv1}\, c'; \\
\mathbf{let}\, x = !\,\ell\, \mathbf{in}\, \mathbf{free}\, \ell; \\
\mathbf{assert}(x = 42)
\end{array}
\middle\|
\begin{array}{l}
\mathbf{let}\, (\ell, c') = \mathbf{recv1}\, c\, \mathbf{in} \\
\ell \leftarrow (!\,\ell + 2); \\
\mathbf{send1}\, c'\, () \\
\\
\\
\end{array}\right)
\end{array}
$$

Layer #3: Functional Session Channels

Implementation (inspired by Kobayashi et al., Dardha et al.):

$$\mathbf{new}_{\mathbf{fun}}\,() \triangleq \mathbf{new1}\,()$$

$$\mathbf{send}\,c\,v \triangleq \mathbf{let}\,c' = \mathbf{new1}\,()\,\mathbf{in}\,\mathbf{send1}\,c\,(v, c');\; c' \qquad \mathbf{close}\,c \triangleq \mathbf{send1}\,c\,()$$

$$\mathbf{recv}\,c \triangleq \mathbf{recv1}\,c \qquad \mathbf{wait}\,c \triangleq \mathbf{recv1}\,c$$

Recovering the one-shot channel example:

$$\begin{array}{l}
\mathtt{ses_fun_ref_prog} \triangleq \\
\quad \mathbf{let}\,c = \mathbf{new}_{\mathbf{fun}}\,()\,\mathbf{in} \\
\quad \left(\begin{array}{l}
\mathbf{let}\,\ell = \mathbf{ref}\,40\,\mathbf{in} \\
\mathbf{let}\,c' = \mathbf{send}\,c\,\ell\,\mathbf{in} \\
\mathbf{wait}\,c'; \\
\mathbf{let}\,x = \,!\,\ell\,\mathbf{in}\,\mathbf{free}\,\ell; \\
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\begin{array}{l}
\mathbf{let}\,(\ell, c') = \mathbf{recv}\,c\,\mathbf{in} \\
\ell \leftarrow (!\,\ell + 2); \\
\mathbf{close}\,c' \\
\\
\\
\end{array}\right)
\end{array}$$

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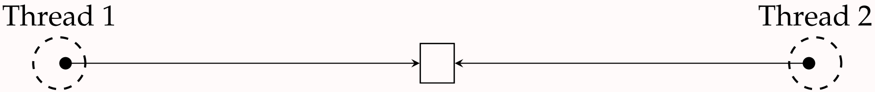
Emerging polarized bi-directional linked list:



$$\mathbf{let}\,c_1 = \mathbf{send}\,c_1\,0\,\mathbf{in} \qquad \mathbf{let}\,(c_2, \_) = \mathbf{recv}\,c_2\,\mathbf{in}$$

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$$\mathbf{let}\,(c_1, \_) = \mathbf{recv}\,c_1\,\mathbf{in} \qquad \mathbf{let}\,c_2 = \mathbf{send}\,c_2\,3\,\mathbf{in}$$

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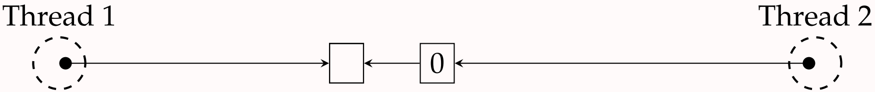

Thread 1	Thread 2
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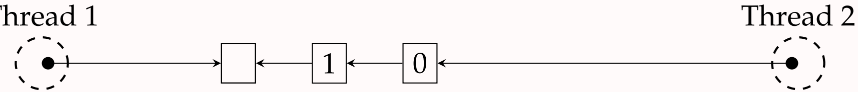

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Emerging polarized bi-directional linked list:

Thread 1	Thread 2
$\mathbf{let}\;c_1 = \mathbf{send}\,c_1\,0\;\mathbf{in}$	$\mathbf{let}\;(c_2, \_) = \mathbf{recv}\,c_2\;\mathbf{in}$
$\mathbf{let}\;c_1 = \mathbf{send}\,c_1\,1\;\mathbf{in}$	$\mathbf{let}\;(c_2, \_) = \mathbf{recv}\,c_2\;\mathbf{in}$
$\mathbf{let}\;c_1 = \mathbf{send}\,c_1\,2\;\mathbf{in}$	$\mathbf{let}\;(c_2, \_) = \mathbf{recv}\,c_2\;\mathbf{in}$
$\mathbf{let}\;(c_1, \_) = \mathbf{recv}\,c_1\;\mathbf{in}$	$\mathbf{let}\;c_2 = \mathbf{send}\,c_2\,3\;\mathbf{in}$

Layer #3: Functional Session Channels – Intuition

Implementation (inspired by Kobayashi et al., Dardha et al.):

$$\mathbf{new_{fun}}\,() \triangleq \mathbf{new1}\,()$$

$$\mathbf{send}\,c\,v \triangleq \mathbf{let}\,c' = \mathbf{new1}\,()\,\mathbf{in}\,\mathbf{send1}\,c\,(v, c');\ c' \qquad \mathbf{close}\,c \triangleq \mathbf{send1}\,c\,()$$

$$\mathbf{recv}\,c \triangleq \mathbf{recv1}\,c \qquad \mathbf{wait}\,c \triangleq \mathbf{recv1}\,c$$

Emerging polarized bi-directional linked list:

$\mathbf{let}\,c_1 = \mathbf{send}\,c_1\,0\,\mathbf{in}$
$\mathbf{let}\,c_1 = \mathbf{send}\,c_1\,1\,\mathbf{in}$
$\mathbf{let}\,c_1 = \mathbf{send}\,c_1\,2\,\mathbf{in}$
$\mathbf{let}\,(c_1, \_) = \mathbf{recv}\,c_1\,\mathbf{in}$

$\mathbf{let}\,(c_2, \_) = \mathbf{recv}\,c_2\,\mathbf{in}$
$\mathbf{let}\,(c_2, \_) = \mathbf{recv}\,c_2\,\mathbf{in}$
$\mathbf{let}\,(c_2, \_) = \mathbf{recv}\,c_2\,\mathbf{in}$
$\mathbf{let}\,c_2 = \mathbf{send}\,c_2\,3\,\mathbf{in}$

Layer #4: Session Channels

Session channel implementation:

$$\mathbf{new}\,() \triangleq \mathbf{let}\,c = \mathbf{new_{fun}}\,()\,\mathbf{in}\,(\mathbf{ref}\,c, \mathbf{ref}\,c)$$

$$c.\mathbf{send}(v) \triangleq c \leftarrow \mathbf{send}\,(!\,c)\,v \qquad c.\mathbf{close}() \triangleq \mathbf{close}\,(!\,c); \mathbf{free}\,c$$

$$c.\mathbf{recv}() \triangleq \mathbf{let}\,(v, c') = \mathbf{recv}\,!\,c\,\mathbf{in}\,c \leftarrow c'; v \qquad c.\mathbf{wait}() \triangleq \mathbf{wait}\,(!\,c); \mathbf{free}\,c$$

Layer #4: Session Channels

Session channel implementation:

$$\mathbf{new}\,() \triangleq \mathbf{let}\, c = \mathbf{new_{fun}}\,()\,\mathbf{in}\,(\mathbf{ref}\,c, \mathbf{ref}\,c)$$

$$c.\mathbf{send}(v) \triangleq c \leftarrow \mathbf{send}\,(!\,c)\,v \qquad c.\mathbf{close}() \triangleq \mathbf{close}\,(!\,c); \mathbf{free}\,c$$

$$c.\mathbf{recv}() \triangleq \mathbf{let}\,(v, c') = \mathbf{recv}\;!\,c\;\mathbf{in}\,c \leftarrow c'; v \qquad c.\mathbf{wait}() \triangleq \mathbf{wait}\,(!\,c); \mathbf{free}\,c$$

Session channel example:

$$\begin{array}{l}
\mathtt{ses_ref_prog} \triangleq \\
\quad \mathbf{let}\,(c_1, c_2) = \mathbf{new}\,()\,\mathbf{in} \\
\quad \left(\begin{array}{l}
\mathbf{let}\,\ell = \mathbf{ref}\,40\,\mathbf{in} \\
c_1.\mathbf{send}(\ell); c_1.\mathbf{wait}(); \\
\mathbf{let}\,x = !\,\ell\,\mathbf{in}\,\mathbf{free}\,\ell; \\
\mathbf{assert}(x = 42)
\end{array}
\;\middle\|\;
\begin{array}{l}
\mathbf{let}\,\ell = c_2.\mathbf{recv}()\,\mathbf{in} \\
\ell \leftarrow (!\,\ell + 2); c_2.\mathbf{close}() \\
\\
\\
\end{array}\right)
\end{array}$$

Layer #4: Session Channels

Session channel implementation:

$$\mathbf{new}\,() \triangleq \mathbf{let}\; c = \mathbf{new_{fun}}\,()\;\mathbf{in}\;(\mathbf{ref}\, c, \mathbf{ref}\, c)$$

$$c.\mathbf{send}(v) \triangleq c \leftarrow \mathbf{send}\,(!\,c)\,v \qquad\qquad c.\mathbf{close}() \triangleq \mathbf{close}\,(!\,c); \mathbf{free}\, c$$

$$c.\mathbf{recv}() \triangleq \mathbf{let}\,(v, c') = \mathbf{recv}\;!\,c\;\mathbf{in}\; c \leftarrow c'; v \qquad c.\mathbf{wait}() \triangleq \mathbf{wait}\,(!\,c); \mathbf{free}\, c$$

Session channel example:

$$\begin{aligned}
&\mathtt{ses_ref_prog} \triangleq \\
&\quad \mathbf{let}\,(c_1, c_2) = \mathbf{new}\,()\;\mathbf{in} \\
&\quad \left(\begin{array}{l} \mathbf{let}\,\ell = \mathbf{ref}\,40\;\mathbf{in} \\ c_1.\mathbf{send}(\ell); c_1.\mathbf{wait}(); \\ \mathbf{let}\, x = !\,\ell\;\mathbf{in}\;\mathbf{free}\,\ell; \\ \mathbf{assert}(x = 42) \end{array} \middle\| \begin{array}{l} \mathbf{let}\,\ell = c_2.\mathbf{recv}()\;\mathbf{in} \\ \ell \leftarrow (!\,\ell + 2); c_2.\mathbf{close}() \\ \\ \\ \end{array}\right)
\end{aligned}$$

Goal: Verify this example and all its dependencies in Iris

Questions?

Basic concurrent separation logic and one-shot protocols

Tutorial Timeline

Part 1: 14:00 – 15:30

- ~~Introduction (10 min)~~
- ~~Layered implementation of session channels (10 min)~~
- Basic concurrent separation logic and one-shot protocols (30 min)
- **Break** (10 min)
- Dependent separation protocols (30 min)

Break (30 min)

Part 2: 16:00 – 17:30

- Iris invariants and ghost state (30 min)
- **Break** (10 min)
- Supervised Coq hacking (50 min)

Overview of Abstraction Layers

Layer	Reasoning principles / specifications
#1 Iris's HeapLang	Basic concurrent separation logic Iris invariants and ghost state
#2 One-shot channels	One-shot protocols
#3 Functional session channels	Dependent separation protocols
#4 Session channels	Dependent separation protocols

Hoare Triples

Hoare triples for partial functional correctness:

$$\{P\}e\{w.\,Q\}$$

Precondition

Postcondition

Binder for return value

If the initial state satisfies P, then:

- **Safety:** e does not crash
- **Postcondition validity:** if e terminates with value v, then the final state satisfies $Q[v/w]$

We often write $\{P\}\,e\,\{Q\} \triangleq \{P\}\,e\,\{w.\,w = () * Q\}$

Separation Logic [O'Hearn, Reynolds, Yang 2001]

Separation logic propositions assert <u>ownership</u> of resources

The points-to connective $\ell \mapsto v$

- Provides the knowledge that location ℓ has value v, and
- Provides exclusive ownership of ℓ

Separating conjunction $P * Q$ captures that the state consists of <u>disjoint parts</u> satisfying P and Q.

Separation Logic [O'Hearn, Reynolds, Yang 2001]

Separation logic propositions assert ownership of resources

The points-to connective $\ell \mapsto v$

- Provides the knowledge that location ℓ has value v, and
- Provides exclusive ownership of ℓ

Separating conjunction $P * Q$ captures that the state consists of disjoint parts satisfying P and Q.

Enables modular reasoning, through disjointness:

$$\textsc{Ht-frame}\quad \frac{\{P\}\, e\, \{w.\, Q\}}{\{P * R\}\, e\, \{w.\, Q * R\}}$$

Sample of Separation Logic Rules

Heap manipulation:

HT-ALLOC
$$\{\mathsf{True}\}\ \mathtt{ref}\, v\ \{w.\, \exists \ell.\, w = \ell * \ell \mapsto v\}$$

HT-LOAD
$$\{\ell \mapsto v\}\ !\,\ell\ \{w.\, w = v * \ell \mapsto v\}$$

HT-STORE
$$\{\ell \mapsto v\}\ \ell \leftarrow w\ \{\ell \mapsto w\}$$

HT-FREE
$$\{\ell \mapsto v\}\ \mathtt{free}\, \ell\ \{\mathsf{True}\}$$

Structural and general rules:

HT-LET
$$\frac{\{P\}\ e_1\ \{w_1.\, Q\} \qquad \forall w_1.\, \{Q\}\ e_2[w_1/x]\ \{w_2.\, R\}}{\{P\}\ \mathtt{let}\, x = e_1\ \mathtt{in}\, e_2\ \{w_2.\, R\}}$$

HT-VAL
$$\{\mathsf{True}\}\ v\ \{w.\, w = v\}$$

HT-SEQ
$$\frac{\{P\}\ e_1\ \{w_1.\, Q\} \qquad \forall w_1.\, \{Q\}\ e_2\ \{w_2.R\}}{\{P\}\ e_1; e_2\ \{w_2.\, R\}}$$

HT-ASSERT
$$\frac{\{P\}\ e\ \{w.\, w = \mathtt{true} * Q\}}{\{P\}\ \mathtt{assert}(e)\ \{Q\}}$$

Simple Verification Example – Sequential Reference Program

$$
\begin{array}{l}
\texttt{let}\ \ell = \texttt{ref}\ \texttt{None}\ \texttt{in} \\
\ell \leftarrow \texttt{Some}\ 42; \\
\texttt{let}\ x = !\,\ell\ \texttt{in} \\
\texttt{free}\ \ell; \\
\texttt{assert}\ (x = \texttt{Some}\ 42)
\end{array}
$$

Simple Verification Example – Sequential Reference Program

$\{\mathsf{True}\}$
$\mathtt{let}\ \ell = \mathtt{ref}\ \mathtt{None}\ \mathtt{in}$
$\ell \leftarrow \mathtt{Some}\ 42;$
$\mathtt{let}\ x = !\,\ell\ \mathtt{in}$
$\mathtt{free}\ \ell;$
$\mathtt{assert}\,(x = \mathtt{Some}\ 42)$

Simple Verification Example – Sequential Reference Program

$\{\mathsf{True}\}$
$\texttt{let}\ \ell = \texttt{ref None in}$ $\quad$ // HT-LET, HT-ALLOC
$\{\ell \mapsto \texttt{None}\}$
$\ell \leftarrow \texttt{Some}\ 42;$
$\texttt{let}\ x = !\,\ell\ \texttt{in}$
$\texttt{free}\ \ell;$
$\texttt{assert}\,(x = \texttt{Some}\ 42)$

Simple Verification Example – Sequential Reference Program

$$
\begin{array}{ll}
\{\mathsf{True}\} & \\
\mathtt{let}\ \ell = \mathtt{ref}\ \mathtt{None}\ \mathtt{in} & //\ \textsc{Ht-let}, \textsc{Ht-alloc} \\
\{\ell \mapsto \mathtt{None}\} & \\
\ell \leftarrow \mathtt{Some}\ 42; & //\ \textsc{Ht-seq}, \textsc{Ht-store} \\
\{\ell \mapsto \mathtt{Some}\ 42\} & \\
\mathtt{let}\ x = !\,\ell\ \mathtt{in} & \\
\mathtt{free}\ \ell; & \\
\mathtt{assert}\,(x = \mathtt{Some}\ 42) &
\end{array}
$$

Simple Verification Example – Sequential Reference Program

$\{\mathsf{True}\}$
$\texttt{let}\ \ell = \texttt{ref None in}$ // HT-LET, HT-ALLOC
$\{\ell \mapsto \texttt{None}\}$
$\ell \leftarrow \texttt{Some}\ 42;$ // HT-SEQ, HT-STORE
$\{\ell \mapsto \texttt{Some}\ 42\}$
$\texttt{let}\ x = !\,\ell\ \texttt{in}$ // HT-LET, HT-LOAD
$\{\ell \mapsto 42 * x = \texttt{Some}\ 42\}$
$\texttt{free}\ \ell;$
$\texttt{assert}\,(x = \texttt{Some}\ 42)$

Simple Verification Example – Sequential Reference Program

$\{\mathsf{True}\}$
$\texttt{let}\ \ell = \texttt{ref None in}$ // HT-LET, HT-ALLOC
$\{\ell \mapsto \texttt{None}\}$
$\ell \leftarrow \texttt{Some}\ 42;$ // HT-SEQ, HT-STORE
$\{\ell \mapsto \texttt{Some}\ 42\}$
$\texttt{let}\ x = !\,\ell\ \texttt{in}$ // HT-LET, HT-LOAD
$\{\ell \mapsto 42 * x = \texttt{Some}\ 42\}$
$\texttt{free}\ \ell;$ // HT-SEQ, HT-FREE
$\{x = \texttt{Some}\ 42\}$
$\texttt{assert}\ (x = \texttt{Some}\ 42)$

Simple Verification Example – Sequential Reference Program

$$
\begin{array}{ll}
\{\mathsf{True}\} & \\
\mathtt{let}\ \ell = \mathtt{ref}\ \mathtt{None}\ \mathtt{in} & // \textsc{Ht-let}, \textsc{Ht-alloc} \\
\{\ell \mapsto \mathtt{None}\} & \\
\ell \leftarrow \mathtt{Some}\ 42; & // \textsc{Ht-seq}, \textsc{Ht-store} \\
\{\ell \mapsto \mathtt{Some}\ 42\} & \\
\mathtt{let}\ x = !\,\ell\ \mathtt{in} & // \textsc{Ht-let}, \textsc{Ht-load} \\
\{\ell \mapsto 42 * x = \mathtt{Some}\ 42\} & \\
\mathtt{free}\ \ell; & // \textsc{Ht-seq}, \textsc{Ht-free} \\
\{x = \mathtt{Some}\ 42\} & \\
\mathtt{assert}\ (x = \mathtt{Some}\ 42) & // \textsc{Ht-assert} \\
\{\mathsf{True}\} &
\end{array}
$$

One-Shot Channel Specifications

Channel ownership $c \rightarrowtail p$

- Provides exclusive permission to use the channel c according to the protocol p

Protocols and duality:

$$\textbf{Protocols:} \quad p ::= (\mathsf{Send}, \Phi) \mid (\mathsf{Recv}, \Phi) \quad \text{where} \quad \Phi : \mathsf{Val} \to \mathsf{Prop}$$

$$\textbf{Duality:} \quad \overline{(\mathsf{Send}, \Phi)} \triangleq (\mathsf{Recv}, \Phi) \qquad \overline{(\mathsf{Recv}, \Phi)} \triangleq (\mathsf{Send}, \Phi)$$

One-Shot Channel Specifications

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$$\textbf{Duality:} \quad \overline{(\mathsf{Send}, \Phi)} \triangleq (\mathsf{Recv}, \Phi) \qquad \overline{(\mathsf{Recv}, \Phi)} \triangleq (\mathsf{Send}, \Phi)$$

One-shot channel specifications:

Ht-new

$$\{\mathsf{True}\}\ \mathtt{new1}\ ()\ \{w.\ \exists c.\ w = c * c \rightarrowtail p * c \rightarrowtail \overline{p}\}$$

Ht-send

$$\{c \rightarrowtail (\mathsf{Send}, \Phi) * \Phi\ v\}\ \mathtt{send1}\ c\ v\ \{\mathsf{True}\}$$

Ht-recv

$$\{c \rightarrowtail (\mathsf{Recv}, \Phi)\}\ \mathtt{recv1}\ c\ \{w.\ \Phi\ w\}$$

Concurrency Specifications

Parallel composition rule:

$$\frac{\{P_1\}\, e_1\, \{w_1.Q_1\} \qquad \{P_2\}\, e_2\, \{w_2.Q_2\}}{\{P_1 * P_2\}\, (e_1 \parallel e_2)\, \{w.\, \exists w_1, w_2.\, w = (w_1, w_2) * Q_1 * Q_2\}}\ \textsc{Ht-par}$$

One-Shot Channel Verification Examples – Basics

$$
\begin{array}{l}
\mathtt{let}\ c = \mathtt{new1}\,()\ \mathtt{in} \\
\left(\begin{array}{l} \mathtt{send1}\ c\ 42 \end{array} \;\middle\|\; \begin{array}{l} \mathtt{let}\ x = \mathtt{recv1}\ c\ \mathtt{in} \\ \mathtt{assert}(x = 42) \end{array}\right)
\end{array}
$$

One-Shot Channel Verification Examples – Basics

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\texttt{let}\, c = \texttt{new1}\,()\,\texttt{in} \\
\left(\begin{array}{l} \texttt{send1}\, c\, 42 \end{array} \middle\| \begin{array}{l} \texttt{let}\, x = \texttt{recv1}\, c\, \texttt{in} \\ \texttt{assert}(x = 42) \end{array}\right)
\end{array}
$$

One-Shot Channel Verification Examples – Basics

$$\{\mathsf{True}\}$$
$$\texttt{let}\, c = \texttt{new1}\,()\,\texttt{in}$$
$$\left(\texttt{send1}\, c\, 42 \;\middle\|\; \begin{array}{l}\texttt{let}\, x = \texttt{recv1}\, c\, \texttt{in} \\ \texttt{assert}(x = 42)\end{array}\right)$$

One-shot protocol:

$$\texttt{prot} \triangleq (\mathsf{Send}, \lambda w.\, w = 42)$$

One-Shot Channel Verification Examples – Basics

$$\{\mathsf{True}\}$$
$$\textbf{let}\ c = \textbf{new1}\,()\ \textbf{in}$$
$$\{c \rightarrowtail \texttt{prot} * c \rightarrowtail \overline{\texttt{prot}}\}$$
$$\left(\textbf{send1}\ c\ 42 \;\middle\|\; \begin{array}{l}\textbf{let}\ x = \textbf{recv1}\ c\ \textbf{in}\\ \textbf{assert}(x = 42)\end{array}\right)$$

One-shot protocol:

$$\texttt{prot} \triangleq (\mathsf{Send}, \lambda w.\, w = 42)$$

One-Shot Channel Verification Examples – Basics

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathtt{let}\; c = \mathtt{new1}\,()\;\mathtt{in} \\
\{c \rightarrowtail \mathtt{prot} * c \rightarrowtail \overline{\mathtt{prot}}\} \\
\left(\begin{array}{l|l}
\{c \rightarrowtail \mathtt{prot}\} & \{c \rightarrowtail \overline{\mathtt{prot}}\} \\
\mathtt{send1}\; c\; 42 & \mathtt{let}\; x = \mathtt{recv1}\; c\;\mathtt{in} \\
 & \mathtt{assert}(x = 42)
\end{array}\right)
\end{array}
$$

One-shot protocol:

$$\mathtt{prot} \triangleq (\mathsf{Send}, \lambda w.\, w = 42)$$

One-Shot Channel Verification Examples – Basics

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathtt{let}\, c = \mathtt{new1}\,()\,\mathtt{in} \\
\{c \rightarrowtail \mathtt{prot} * c \rightarrowtail \overline{\mathtt{prot}}\} \\
\left(\begin{array}{l}
\{c \rightarrowtail \mathtt{prot}\} \\
\mathtt{send1}\, c\, 42 \\
\{\mathsf{True}\}
\end{array}
\middle\|
\begin{array}{l}
\{c \rightarrowtail \overline{\mathtt{prot}}\} \\
\mathtt{let}\, x = \mathtt{recv1}\, c\, \mathtt{in} \\
\mathtt{assert}(x = 42)
\end{array}\right)
\end{array}
$$

One-shot protocol:

$$\mathtt{prot} \triangleq (\mathsf{Send}, \lambda w.\, w = 42)$$

One-Shot Channel Verification Examples – Basics

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\texttt{let}\ c = \texttt{new1}\,()\ \texttt{in} \\
\{c \rightarrowtail \texttt{prot} * c \rightarrowtail \overline{\texttt{prot}}\} \\
\left(\begin{array}{l} \{c \rightarrowtail \texttt{prot}\} \\ \texttt{send1}\ c\ 42 \\ \{\mathsf{True}\} \\ \ \end{array} \middle\| \begin{array}{l} \{c \rightarrowtail \overline{\texttt{prot}}\} \\ \texttt{let}\ x = \texttt{recv1}\ c\ \texttt{in} \\ \{x = 42\} \\ \texttt{assert}(x = 42) \end{array}\right)
\end{array}
$$

One-shot protocol:

$$\texttt{prot} \triangleq (\mathsf{Send}, \lambda w.\, w = 42)$$

One-Shot Channel Verification Examples – Basics

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathtt{let}\ c = \mathtt{new1}\ ()\ \mathtt{in} \\
\{c \rightarrowtail \mathtt{prot} * c \rightarrowtail \overline{\mathtt{prot}}\} \\
\left(\begin{array}{l}
\{c \rightarrowtail \mathtt{prot}\} \\
\mathtt{send1}\ c\ 42 \\
\{\mathsf{True}\} \\
\\
\\
\end{array}
\ \middle\|\
\begin{array}{l}
\{c \rightarrowtail \overline{\mathtt{prot}}\} \\
\mathtt{let}\ x = \mathtt{recv1}\ c\ \mathtt{in} \\
\{x = 42\} \\
\mathtt{assert}(x = 42) \\
\{\mathsf{True}\}
\end{array}\right)
\end{array}
$$

One-shot protocol:

$$\mathtt{prot} \triangleq (\mathsf{Send}, \lambda w.\, w = 42)$$

One-Shot Channel Verification Examples – Basics

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathbf{let}\ c = \mathbf{new1}\,()\ \mathbf{in} \\
\{c \rightarrowtail \mathtt{prot} * c \rightarrowtail \overline{\mathtt{prot}}\} \\
\left(\begin{array}{l}
\{c \rightarrowtail \mathtt{prot}\} \\
\mathbf{send1}\ c\ 42 \\
\{\mathsf{True}\} \\
\\
\\
\end{array}
\;\middle\|\;
\begin{array}{l}
\{c \rightarrowtail \overline{\mathtt{prot}}\} \\
\mathbf{let}\ x = \mathbf{recv1}\ c\ \mathbf{in} \\
\{x = 42\} \\
\mathbf{assert}(x = 42) \\
\{\mathsf{True}\}
\end{array}\right) \\
\{\mathsf{True}\}
\end{array}
$$

One-shot protocol:

$$\mathtt{prot} \triangleq (\mathsf{Send}, \lambda w.\, w = 42)$$

One-Shot Channel Verification Examples – References

$$
\begin{array}{l}
\texttt{let}\, c = \texttt{new1}\,()\,\texttt{in} \\
\left(\begin{array}{l}
\texttt{let}\,\ell = \texttt{ref}\,42\,\texttt{in} \\
\texttt{send1}\,c\,\ell \\

\end{array}
\middle\|
\begin{array}{l}
\texttt{let}\,\ell = \texttt{recv1}\,c\,\texttt{in} \\
\texttt{let}\,x = !\,\ell\,\texttt{in}\ \texttt{free}\,\ell; \\
\texttt{assert}(x = 42)
\end{array}\right)
\end{array}
$$

One-Shot Channel Verification Examples – References

$$\{\mathsf{True}\}$$

$$\begin{array}{l}
\texttt{let}\, c = \texttt{new1}\,()\,\texttt{in} \\
\left(\begin{array}{l}
\texttt{let}\, \ell = \texttt{ref}\, 42\, \texttt{in} \\
\texttt{send1}\, c\, \ell \\
\\
\end{array}
\middle\|
\begin{array}{l}
\texttt{let}\, \ell = \texttt{recv1}\, c\, \texttt{in} \\
\texttt{let}\, x = !\, \ell\, \texttt{in}\ \texttt{free}\, \ell; \\
\texttt{assert}(x = 42)
\end{array}\right)
\end{array}$$

One-Shot Channel Verification Examples – References

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathtt{let}\, c = \mathtt{new1}\,()\,\mathtt{in} \\
\left(\begin{array}{l|l}
\mathtt{let}\,\ell = \mathtt{ref}\,42\,\mathtt{in} & \mathtt{let}\,\ell = \mathtt{recv1}\,c\,\mathtt{in} \\
\mathtt{send1}\,c\,\ell & \mathtt{let}\,x = !\,\ell\,\mathtt{in}\,\mathtt{free}\,\ell; \\
 & \mathtt{assert}(x = 42)
\end{array}\right)
\end{array}
$$

One-shot reference protocol:

$$\mathtt{ref\_prot} \triangleq (\mathsf{Send}, \lambda w.\, \exists(\ell : \mathsf{Loc}).\, w = \ell * \ell \mapsto 42)$$

One-Shot Channel Verification Examples – References

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathtt{let}\, c = \mathtt{new1}\,()\,\mathtt{in} \\
\{c \rightarrowtail \mathtt{ref\_prot} * c \rightarrowtail \overline{\mathtt{ref\_prot}}\} \\
\left(\begin{array}{l} \mathtt{let}\,\ell = \mathtt{ref}\,42\,\mathtt{in} \\ \mathtt{send1}\,c\,\ell \\ \end{array} \middle\| \begin{array}{l} \mathtt{let}\,\ell = \mathtt{recv1}\,c\,\mathtt{in} \\ \mathtt{let}\,x = !\,\ell\,\mathtt{in}\ \mathtt{free}\,\ell; \\ \mathtt{assert}(x = 42) \end{array}\right)
\end{array}
$$

One-shot reference protocol:

$$\mathtt{ref\_prot} \triangleq (\mathsf{Send}, \lambda w.\, \exists (\ell : \mathsf{Loc}).\, w = \ell * \ell \mapsto 42)$$

One-Shot Channel Verification Examples – References

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathbf{let}\; c = \mathbf{new1}\,()\;\mathbf{in} \\
\{c \rightarrowtail \mathtt{ref\_prot} * c \rightarrowtail \overline{\mathtt{ref\_prot}}\} \\
\left(\begin{array}{l}
\{c \rightarrowtail \mathtt{ref\_prot}\} \\
\mathbf{let}\;\ell = \mathbf{ref}\,42\;\mathbf{in} \\
\mathbf{send1}\;c\;\ell \\
\\
\end{array}
\;\middle\|\;
\begin{array}{l}
\{c \rightarrowtail \overline{\mathtt{ref\_prot}}\} \\
\mathbf{let}\;\ell = \mathbf{recv1}\;c\;\mathbf{in} \\
\mathbf{let}\;x = !\,\ell\;\mathbf{in}\;\mathbf{free}\;\ell; \\
\mathbf{assert}(x = 42)
\end{array}\right)
\end{array}
$$

One-shot reference protocol:

$$\mathtt{ref\_prot} \triangleq (\mathsf{Send}, \lambda w.\, \exists(\ell : \mathsf{Loc}).\, w = \ell * \ell \mapsto 42)$$

One-Shot Channel Verification Examples – References

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathbf{let}\, c = \mathbf{new1}\,()\,\mathbf{in} \\
\{c \rightarrowtail \mathtt{ref\_prot} * c \rightarrowtail \overline{\mathtt{ref\_prot}}\} \\
\left(\begin{array}{l|l}
\{c \rightarrowtail \mathtt{ref\_prot}\} & \{c \rightarrowtail \overline{\mathtt{ref\_prot}}\} \\
\mathbf{let}\, \ell = \mathbf{ref}\, 42\, \mathbf{in} & \mathbf{let}\, \ell = \mathbf{recv1}\, c\, \mathbf{in} \\
\{c \rightarrowtail \mathtt{ref\_prot} * \ell \mapsto 42\} & \mathbf{let}\, x = !\, \ell\, \mathbf{in}\ \mathbf{free}\, \ell; \\
\mathbf{send1}\, c\, \ell & \mathbf{assert}(x = 42)
\end{array}\right)
\end{array}
$$

One-shot reference protocol:

$$\mathtt{ref\_prot} \triangleq (\mathsf{Send}, \lambda w.\, \exists (\ell : \mathsf{Loc}).\, w = \ell * \ell \mapsto 42)$$

One-Shot Channel Verification Examples – References

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\textbf{let}\, c = \textbf{new1}\,()\,\textbf{in} \\
\{c \rightarrowtail \texttt{ref\_prot} * c \rightarrowtail \overline{\texttt{ref\_prot}}\} \\
\left(\begin{array}{l|l}
\{c \rightarrowtail \texttt{ref\_prot}\} & \{c \rightarrowtail \overline{\texttt{ref\_prot}}\} \\
\textbf{let}\, \ell = \textbf{ref}\, 42\, \textbf{in} & \textbf{let}\, \ell = \textbf{recv1}\, c\, \textbf{in} \\
\{c \rightarrowtail \texttt{ref\_prot} * \ell \mapsto 42\} & \textbf{let}\, x = !\, \ell\, \textbf{in}\, \textbf{free}\, \ell; \\
\textbf{send1}\, c\, \ell & \textbf{assert}(x = 42) \\
\{\mathsf{True}\} &
\end{array}\right)
\end{array}
$$

One-shot reference protocol:

$$\texttt{ref\_prot} \triangleq (\mathsf{Send}, \lambda w.\, \exists (\ell : \mathsf{Loc}).\, w = \ell * \ell \mapsto 42)$$

One-Shot Channel Verification Examples – References

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathbf{let}\ c = \mathbf{new1}\,()\ \mathbf{in} \\
\{c \rightarrowtail \mathtt{ref\_prot} * c \rightarrowtail \overline{\mathtt{ref\_prot}}\} \\
\left(\begin{array}{l}
\{c \rightarrowtail \mathtt{ref\_prot}\} \\
\mathbf{let}\ \ell = \mathbf{ref}\ 42\ \mathbf{in} \\
\{c \rightarrowtail \mathtt{ref\_prot} * \ell \mapsto 42\} \\
\mathbf{send1}\ c\ \ell \\
\{\mathsf{True}\}
\end{array}
\middle\Vert
\begin{array}{l}
\{c \rightarrowtail \overline{\mathtt{ref\_prot}}\} \\
\mathbf{let}\ \ell = \mathbf{recv1}\ c\ \mathbf{in} \\
\{\ell \mapsto 42\} \\
\mathbf{let}\ x = !\,\ell\ \mathbf{in}\ \mathbf{free}\ \ell; \\
\mathbf{assert}(x = 42)
\end{array}\right)
\end{array}
$$

One-shot reference protocol:

$$
\mathtt{ref\_prot} \triangleq (\mathsf{Send}, \lambda w.\, \exists(\ell : \mathsf{Loc}).\, w = \ell * \ell \mapsto 42)
$$

One-Shot Channel Verification Examples – References

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathbf{let}\ c = \mathbf{new1}\ ()\ \mathbf{in} \\
\{c \rightarrowtail \texttt{ref\_prot} * c \rightarrowtail \overline{\texttt{ref\_prot}}\} \\
\left(\begin{array}{l|l}
\{c \rightarrowtail \texttt{ref\_prot}\} & \{c \rightarrowtail \overline{\texttt{ref\_prot}}\} \\
\mathbf{let}\ \ell = \mathbf{ref}\ 42\ \mathbf{in} & \mathbf{let}\ \ell = \mathbf{recv1}\ c\ \mathbf{in} \\
\{c \rightarrowtail \texttt{ref\_prot} * \ell \mapsto 42\} & \{\ell \mapsto 42\} \\
\mathbf{send1}\ c\ \ell & \mathbf{let}\ x = !\,\ell\ \mathbf{in}\ \mathbf{free}\ \ell; \\
\{\mathsf{True}\} & \{x = 42\} \\
 & \mathbf{assert}(x = 42)
\end{array}\right)
\end{array}
$$

One-shot reference protocol:

$$\texttt{ref\_prot} \triangleq (\mathsf{Send}, \lambda w.\, \exists (\ell : \mathsf{Loc}).\, w = \ell * \ell \mapsto 42)$$

One-Shot Channel Verification Examples – References

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathbf{let}\, c = \mathbf{new1}\,()\, \mathbf{in} \\
\{c \rightarrowtail \mathtt{ref\_prot} * c \rightarrowtail \overline{\mathtt{ref\_prot}}\} \\
\left(\begin{array}{l}
\{c \rightarrowtail \mathtt{ref\_prot}\} \\
\mathbf{let}\, \ell = \mathbf{ref}\, 42\, \mathbf{in} \\
\{c \rightarrowtail \mathtt{ref\_prot} * \ell \mapsto 42\} \\
\mathbf{send1}\, c\, \ell \\
\{\mathsf{True}\} \\
\\
\\
\end{array}
\middle\|
\begin{array}{l}
\{c \rightarrowtail \overline{\mathtt{ref\_prot}}\} \\
\mathbf{let}\, \ell = \mathbf{recv1}\, c\, \mathbf{in} \\
\{\ell \mapsto 42\} \\
\mathbf{let}\, x = !\, \ell\, \mathbf{in}\, \mathbf{free}\, \ell; \\
\{x = 42\} \\
\mathbf{assert}(x = 42) \\
\{\mathsf{True}\}
\end{array}\right)
\end{array}
$$

One-shot reference protocol:

$$\mathtt{ref\_prot} \triangleq (\mathsf{Send}, \lambda w.\, \exists (\ell : \mathsf{Loc}).\, w = \ell * \ell \mapsto 42)$$

One-Shot Channel Verification Examples – References

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathbf{let}\, c = \mathbf{new1}\,()\ \mathbf{in} \\
\{c \rightarrowtail \mathtt{ref\_prot} * c \rightarrowtail \overline{\mathtt{ref\_prot}}\} \\
\left(\begin{array}{l|l}
\{c \rightarrowtail \mathtt{ref\_prot}\} & \{c \rightarrowtail \overline{\mathtt{ref\_prot}}\} \\
\mathbf{let}\, \ell = \mathbf{ref}\, 42\ \mathbf{in} & \mathbf{let}\, \ell = \mathbf{recv1}\, c\ \mathbf{in} \\
\{c \rightarrowtail \mathtt{ref\_prot} * \ell \mapsto 42\} & \{\ell \mapsto 42\} \\
\mathbf{send1}\, c\, \ell & \mathbf{let}\, x = {!}\,\ell\ \mathbf{in}\ \mathbf{free}\, \ell; \\
\{\mathsf{True}\} & \{x = 42\} \\
 & \mathbf{assert}(x = 42) \\
 & \{\mathsf{True}\}
\end{array}\right) \\
\{\mathsf{True}\}
\end{array}
$$

One-shot reference protocol:

$$\mathtt{ref\_prot} \triangleq (\mathsf{Send}, \lambda w.\, \exists(\ell : \mathsf{Loc}).\, w = \ell * \ell \mapsto 42)$$

One-Shot Channel Verification Examples – Higher-Order

$$
\begin{array}{l}
\mathbf{let}\, c = \mathbf{new1}\,()\,\mathbf{in} \\
\left(\begin{array}{l}
\mathbf{let}\, \ell = \mathbf{ref}\, 40\, \mathbf{in} \\
\mathbf{let}\, c' = \mathbf{new1}\,()\,\mathbf{in}\, \mathbf{send1}\, c\, (\ell, c'); \\
\mathbf{recv1}\, c'; \\
\mathbf{let}\, x = !\, \ell\, \mathbf{in}\, \mathbf{free}\, \ell; \\
\mathbf{assert}(x = 42)
\end{array}
\;\middle\|\;
\begin{array}{l}
\mathbf{let}\, (\ell, c') = \mathbf{recv1}\, c\, \mathbf{in} \\
\ell \leftarrow (!\, \ell + 2); \\
\mathbf{send1}\, c'\, () \\
\\
\\
\end{array}\right)
\end{array}
$$

One-Shot Channel Verification Examples – Higher-Order

$$
\begin{array}{l}
\color{purple}{\{\mathsf{True}\}} \\
\mathbf{let}\; c = \mathbf{new1}\,()\;\mathbf{in} \\
\left(\begin{array}{l}
\mathbf{let}\;\ell = \mathbf{ref}\,40\;\mathbf{in} \\
\mathbf{let}\;c' = \mathbf{new1}\,()\;\mathbf{in}\;\mathbf{send1}\,c\,(\ell, c'); \\
\mathbf{recv1}\,c'; \\
\mathbf{let}\;x = !\,\ell\;\mathbf{in}\;\mathbf{free}\,\ell; \\
\mathbf{assert}(x = 42)
\end{array}
\;\middle\|\;
\begin{array}{l}
\mathbf{let}\;(\ell, c') = \mathbf{recv1}\,c\;\mathbf{in} \\
\ell \leftarrow (!\,\ell + 2); \\
\mathbf{send1}\,c'\,() \\
\\
\\
\end{array}\right)
\end{array}
$$

One-Shot Channel Verification Examples – Higher-Order

$\{\mathsf{True}\}$

$$
\begin{array}{l}
\mathtt{let}\, c = \mathtt{new1}\,()\,\mathtt{in} \\
\left(\begin{array}{l}
\mathtt{let}\,\ell = \mathtt{ref}\,40\,\mathtt{in} \\
\mathtt{let}\,c' = \mathtt{new1}\,()\,\mathtt{in}\,\mathtt{send1}\,c\,(\ell, c'); \\
\mathtt{recv1}\,c'; \\
\mathtt{let}\,x = !\,\ell\,\mathtt{in}\,\mathtt{free}\,\ell; \\
\mathtt{assert}(x = 42)
\end{array}
\;\middle\|\;
\begin{array}{l}
\mathtt{let}\,(\ell, c') = \mathtt{recv1}\,c\,\mathtt{in} \\
\ell \leftarrow (!\,\ell + 2); \\
\mathtt{send1}\,c'\,() \\
\\
\\
\end{array}\right)
\end{array}
$$

One-shot channel protocols:

$$
\begin{array}{l}
\mathtt{chan\_prot} \triangleq (\mathsf{Send}, \lambda w.\, \exists (\ell : \mathsf{Loc}), c'.\, w = (\ell, c') * \ell \mapsto 40 * c' \rightarrowtail \overline{\mathtt{chan\_prot}'\ \ell}) \\
\mathtt{chan\_prot}'\ (\ell : \mathsf{Loc}) \triangleq (\mathsf{Recv}, \lambda w.\, w = () * \ell \mapsto 42)
\end{array}
$$

One-Shot Channel Verification Examples – Higher-Order

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathbf{let}\ c = \mathbf{new1}\,()\ \mathbf{in} \\
\{c \rightarrowtail \mathtt{chan\_prot} * c \rightarrowtail \overline{\mathtt{chan\_prot}}\} \\
\left(\begin{array}{l|l}
\mathbf{let}\ \ell = \mathbf{ref}\,40\ \mathbf{in} & \mathbf{let}\,(\ell, c') = \mathbf{recv1}\,c\ \mathbf{in} \\
\mathbf{let}\ c' = \mathbf{new1}\,()\ \mathbf{in}\ \mathbf{send1}\,c\,(\ell, c'); & \ell \leftarrow (!\,\ell + 2); \\
\mathbf{recv1}\,c'; & \mathbf{send1}\,c'\,() \\
\mathbf{let}\ x = !\,\ell\ \mathbf{in}\ \mathbf{free}\,\ell; & \\
\mathbf{assert}(x = 42) &
\end{array}\right)
\end{array}
$$

One-shot channel protocols:

$$
\begin{array}{l}
\mathtt{chan\_prot} \triangleq (\mathsf{Send}, \lambda w.\, \exists (\ell : \mathsf{Loc}), c'.\, w = (\ell, c') * \ell \mapsto 40 * c' \rightarrowtail \overline{\mathtt{chan\_prot}'\ \ell}) \\
\mathtt{chan\_prot}'\ (\ell : \mathsf{Loc}) \triangleq (\mathsf{Recv}, \lambda w.\, w = () * \ell \mapsto 42)
\end{array}
$$

One-Shot Channel Verification Examples – Higher-Order

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathtt{let}\ c = \mathtt{new1}\ ()\ \mathtt{in} \\
\{c \rightarrowtail \mathtt{chan\_prot} * c \rightarrowtail \overline{\mathtt{chan\_prot}}\} \\
\left(\begin{array}{l}
\{c \rightarrowtail \mathtt{chan\_prot}\} \\
\mathtt{let}\ \ell = \mathtt{ref}\ 40\ \mathtt{in} \\
\mathtt{let}\ c' = \mathtt{new1}\ ()\ \mathtt{in}\ \mathtt{send1}\ c\ (\ell, c'); \\
\mathtt{recv1}\ c'; \\
\mathtt{let}\ x = !\,\ell\ \mathtt{in}\ \mathtt{free}\ \ell; \\
\mathtt{assert}(x = 42)
\end{array}
\middle\|
\begin{array}{l}
\{c \rightarrowtail \overline{\mathtt{chan\_prot}}\} \\
\mathtt{let}\ (\ell, c') = \mathtt{recv1}\ c\ \mathtt{in} \\
\ell \leftarrow (!\,\ell + 2); \\
\mathtt{send1}\ c'\ () \\
\\
\\
\end{array}\right)
\end{array}
$$

One-shot channel protocols:

$$
\begin{aligned}
&\mathtt{chan\_prot} \triangleq (\mathsf{Send}, \lambda w.\, \exists (\ell : \mathsf{Loc}), c'.\, w = (\ell, c') * \ell \mapsto 40 * c' \rightarrowtail \overline{\mathtt{chan\_prot}'\ \ell}) \\
&\mathtt{chan\_prot}'\ (\ell : \mathsf{Loc}) \triangleq (\mathsf{Recv}, \lambda w.\, w = () * \ell \mapsto 42)
\end{aligned}
$$

One-Shot Channel Verification Examples – Higher-Order

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathbf{let}\ c = \mathbf{new1}\ ()\ \mathbf{in} \\
\{c \rightarrowtail \mathtt{chan\_prot} * c \rightarrowtail \overline{\mathtt{chan\_prot}}\} \\
\left(\begin{array}{l}
\{c \rightarrowtail \mathtt{chan\_prot}\} \\
\mathbf{let}\ \ell = \mathbf{ref}\ 40\ \mathbf{in} \\
\{c \rightarrowtail \mathtt{chan\_prot} * \ell \mapsto 40\} \\
\mathbf{let}\ c' = \mathbf{new1}\ ()\ \mathbf{in}\ \mathbf{send1}\ c\ (\ell, c'); \\
\mathbf{recv1}\ c'; \\
\mathbf{let}\ x = !\,\ell\ \mathbf{in}\ \mathbf{free}\ \ell; \\
\mathbf{assert}(x = 42)
\end{array}
\;\middle\|\;
\begin{array}{l}
\{c \rightarrowtail \overline{\mathtt{chan\_prot}}\} \\
\mathbf{let}\ (\ell, c') = \mathbf{recv1}\ c\ \mathbf{in} \\
\ell \leftarrow (!\,\ell + 2); \\
\mathbf{send1}\ c'\ () \\
\\ \\ \\
\end{array}\right)
\end{array}
$$

One-shot channel protocols:

$$
\begin{array}{l}
\mathtt{chan\_prot} \triangleq (\mathsf{Send}, \lambda w.\, \exists (\ell : \mathsf{Loc}), c'.\, w = (\ell, c') * \ell \mapsto 40 * c' \rightarrowtail \overline{\mathtt{chan\_prot}'\ \ell}) \\
\mathtt{chan\_prot}'\ (\ell : \mathsf{Loc}) \triangleq (\mathsf{Recv}, \lambda w.\, w = () * \ell \mapsto 42)
\end{array}
$$

One-Shot Channel Verification Examples – Higher-Order

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathbf{let}\ c = \mathbf{new1}\ ()\ \mathbf{in} \\
\{c \rightarrowtail \mathtt{chan\_prot} * c \rightarrowtail \overline{\mathtt{chan\_prot}}\} \\
\left(\begin{array}{l}
\{c \rightarrowtail \mathtt{chan\_prot}\} \\
\mathbf{let}\ \ell = \mathbf{ref}\ 40\ \mathbf{in} \\
\{c \rightarrowtail \mathtt{chan\_prot} * \ell \mapsto 40\} \\
\mathbf{let}\ c' = \mathbf{new1}\ ()\ \mathbf{in}\ \mathbf{send1}\ c\ (\ell, c'); \\
\{c' \rightarrowtail (\mathtt{chan\_prot}'\ \ell\ 40)\} \\
\mathbf{recv1}\ c'; \\
\mathbf{let}\ x = !\,\ell\ \mathbf{in}\ \mathbf{free}\ \ell; \\
\mathbf{assert}(x = 42)
\end{array}
\middle\|
\begin{array}{l}
\{c \rightarrowtail \overline{\mathtt{chan\_prot}}\} \\
\mathbf{let}\ (\ell, c') = \mathbf{recv1}\ c\ \mathbf{in} \\
\ell \leftarrow (!\,\ell + 2); \\
\mathbf{send1}\ c'\ () \\
\\ \\ \\ \\
\end{array}\right)
\end{array}
$$

One-shot channel protocols:

$$
\begin{array}{l}
\mathtt{chan\_prot} \triangleq (\mathsf{Send}, \lambda w.\, \exists (\ell : \mathsf{Loc}), c'.\, w = (\ell, c') * \ell \mapsto 40 * c' \rightarrowtail \overline{\mathtt{chan\_prot}'\ \ell}) \\
\mathtt{chan\_prot}'\ (\ell : \mathsf{Loc}) \triangleq (\mathsf{Recv}, \lambda w.\, w = () * \ell \mapsto 42)
\end{array}
$$

One-Shot Channel Verification Examples – Higher-Order

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathbf{let}\, c = \mathbf{new1}\,()\,\mathbf{in} \\
\{c \rightarrowtail \mathtt{chan\_prot} * c \rightarrowtail \overline{\mathtt{chan\_prot}}\} \\
\left(\begin{array}{l|l}
\{c \rightarrowtail \mathtt{chan\_prot}\} & \{c \rightarrowtail \overline{\mathtt{chan\_prot}}\} \\
\mathbf{let}\, \ell = \mathbf{ref}\, 40\, \mathbf{in} & \mathbf{let}\,(\ell, c') = \mathbf{recv1}\, c\, \mathbf{in} \\
\{c \rightarrowtail \mathtt{chan\_prot} * \ell \mapsto 40\} & \ell \leftarrow (!\,\ell + 2); \\
\mathbf{let}\, c' = \mathbf{new1}\,()\,\mathbf{in}\, \mathbf{send1}\, c\,(\ell, c'); & \mathbf{send1}\, c'\,() \\
\{c' \rightarrowtail (\mathtt{chan\_prot}'\,\ell\, 40)\} & \\
\mathbf{recv1}\, c'; & \\
\{\ell \mapsto 42\} & \\
\mathbf{let}\, x = !\,\ell\, \mathbf{in}\, \mathbf{free}\,\ell; & \\
\mathbf{assert}(x = 42) &
\end{array}\right)
\end{array}
$$

One-shot channel protocols:

$$
\begin{aligned}
\mathtt{chan\_prot} &\triangleq (\mathsf{Send}, \lambda w.\, \exists (\ell : \mathsf{Loc}), c'.\, w = (\ell, c') * \ell \mapsto 40 * c' \rightarrowtail \overline{\mathtt{chan\_prot}'\,\ell}) \\
\mathtt{chan\_prot}'\,(\ell : \mathsf{Loc}) &\triangleq (\mathsf{Recv}, \lambda w.\, w = () * \ell \mapsto 42)
\end{aligned}
$$

One-Shot Channel Verification Examples – Higher-Order

$$
\begin{array}{l}
\{\mathsf{True}\}\\
\mathtt{let}\ c = \mathtt{new1}\,()\ \mathtt{in}\\
\{c \rightarrowtail \mathtt{chan\_prot} * c \rightarrowtail \overline{\mathtt{chan\_prot}}\}\\
\left(\begin{array}{l||l}
\{c \rightarrowtail \mathtt{chan\_prot}\} & \{c \rightarrowtail \overline{\mathtt{chan\_prot}}\}\\
\mathtt{let}\ \ell = \mathtt{ref}\,40\ \mathtt{in} & \mathtt{let}\ (\ell, c') = \mathtt{recv1}\,c\ \mathtt{in}\\
\{c \rightarrowtail \mathtt{chan\_prot} * \ell \mapsto 40\} & \ell \leftarrow (!\,\ell + 2);\\
\mathtt{let}\ c' = \mathtt{new1}\,()\ \mathtt{in}\ \mathtt{send1}\,c\,(\ell, c'); & \mathtt{send1}\,c'\,()\\
\{c' \rightarrowtail (\mathtt{chan\_prot}'\ \ell\ 40)\} & \\
\mathtt{recv1}\,c'; & \\
\{\ell \mapsto 42\} & \\
\mathtt{let}\ x = !\,\ell\ \mathtt{in}\ \mathtt{free}\,\ell; & \\
\{x = 42\} & \\
\mathtt{assert}(x = 42) &
\end{array}\right)
\end{array}
$$

One-shot channel protocols:

$$
\begin{array}{l}
\mathtt{chan\_prot} \triangleq (\mathsf{Send}, \lambda w.\, \exists (\ell : \mathsf{Loc}), c'.\, w = (\ell, c') * \ell \mapsto 40 * c' \rightarrowtail \overline{\mathtt{chan\_prot}'\ \ell})\\
\mathtt{chan\_prot}'\ (\ell : \mathsf{Loc}) \triangleq (\mathsf{Recv}, \lambda w.\, w = () * \ell \mapsto 42)
\end{array}
$$

One-Shot Channel Verification Examples – Higher-Order

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathtt{let}\ c = \mathtt{new1}\ ()\ \mathtt{in} \\
\{c \rightarrowtail \mathtt{chan\_prot} * c \rightarrowtail \overline{\mathtt{chan\_prot}}\} \\
\left(\begin{array}{l|l}
\{c \rightarrowtail \mathtt{chan\_prot}\} & \{c \rightarrowtail \overline{\mathtt{chan\_prot}}\} \\
\mathtt{let}\ \ell = \mathtt{ref}\ 40\ \mathtt{in} & \mathtt{let}\ (\ell, c') = \mathtt{recv1}\ c\ \mathtt{in} \\
\{c \rightarrowtail \mathtt{chan\_prot} * \ell \mapsto 40\} & \ell \leftarrow (!\,\ell + 2); \\
\mathtt{let}\ c' = \mathtt{new1}\ ()\ \mathtt{in}\ \mathtt{send1}\ c\ (\ell, c'); & \mathtt{send1}\ c'\ () \\
\{c' \rightarrowtail (\mathtt{chan\_prot}'\ \ell\ 40)\} & \\
\mathtt{recv1}\ c'; & \\
\{\ell \mapsto 42\} & \\
\mathtt{let}\ x = !\,\ell\ \mathtt{in}\ \mathtt{free}\ \ell; & \\
\{x = 42\} & \\
\mathtt{assert}(x = 42) & \\
\{\mathsf{True}\} &
\end{array}\right)
\end{array}
$$

One-shot channel protocols:

$$\mathtt{chan\_prot} \triangleq (\mathsf{Send}, \lambda w.\, \exists (\ell : \mathsf{Loc}), c'.\, w = (\ell, c') * \ell \mapsto 40 * c' \rightarrowtail \overline{\mathtt{chan\_prot}'\ \ell})$$
$$\mathtt{chan\_prot}'\ (\ell : \mathsf{Loc}) \triangleq (\mathsf{Recv}, \lambda w.\, w = () * \ell \mapsto 42)$$

One-Shot Channel Verification Examples – Higher-Order

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathbf{let}\; c = \mathbf{new1}\,()\;\mathbf{in} \\
\{c \rightarrowtail \mathtt{chan\_prot} * c \rightarrowtail \overline{\mathtt{chan\_prot}}\} \\
\left(\begin{array}{l}
\{c \rightarrowtail \mathtt{chan\_prot}\} \\
\mathbf{let}\; \ell = \mathbf{ref}\, 40\;\mathbf{in} \\
\{c \rightarrowtail \mathtt{chan\_prot} * \ell \mapsto 40\} \\
\mathbf{let}\; c' = \mathbf{new1}\,()\;\mathbf{in}\;\mathbf{send1}\, c\,(\ell, c'); \\
\{c' \rightarrowtail (\mathtt{chan\_prot}'\;\ell\;40)\} \\
\mathbf{recv1}\, c'; \\
\{\ell \mapsto 42\} \\
\mathbf{let}\; x = !\,\ell\;\mathbf{in}\;\mathbf{free}\,\ell; \\
\{x = 42\} \\
\mathbf{assert}(x = 42) \\
\{\mathsf{True}\}
\end{array}
\;\middle\|\;
\begin{array}{l}
\{c \rightarrowtail \overline{\mathtt{chan\_prot}}\} \\
\mathbf{let}\; (\ell, c') = \mathbf{recv1}\, c\;\mathbf{in} \\
\{c' \rightarrowtail \overline{(\mathtt{chan\_prot}'\;\ell)} * \ell \mapsto 40\} \\
\ell \leftarrow (!\,\ell + 2); \\
\mathbf{send1}\, c'\,() \\
\\ \\ \\ \\ \\ \\
\end{array}\right)
\end{array}
$$

One-shot channel protocols:

$$
\begin{array}{l}
\mathtt{chan\_prot} \triangleq (\mathsf{Send}, \lambda w.\, \exists (\ell : \mathsf{Loc}), c'.\, w = (\ell, c') * \ell \mapsto 40 * c' \rightarrowtail \overline{\mathtt{chan\_prot}'\;\ell}) \\
\mathtt{chan\_prot}'\;(\ell : \mathsf{Loc}) \triangleq (\mathsf{Recv}, \lambda w.\, w = () * \ell \mapsto 42)
\end{array}
$$

One-Shot Channel Verification Examples – Higher-Order

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathtt{let}\ c = \mathtt{new1}\ ()\ \mathtt{in} \\
\{c \rightarrowtail \mathtt{chan\_prot} * c \rightarrowtail \overline{\mathtt{chan\_prot}}\} \\
\left(\begin{array}{l|l}
\{c \rightarrowtail \mathtt{chan\_prot}\} & \{c \rightarrowtail \overline{\mathtt{chan\_prot}}\} \\
\mathtt{let}\ \ell = \mathtt{ref}\ 40\ \mathtt{in} & \mathtt{let}\ (\ell, c') = \mathtt{recv1}\ c\ \mathtt{in} \\
\{c \rightarrowtail \mathtt{chan\_prot} * \ell \mapsto 40\} & \{c' \rightarrowtail (\overline{\mathtt{chan\_prot}'\ \ell}) * \ell \mapsto 40\} \\
\mathtt{let}\ c' = \mathtt{new1}\ ()\ \mathtt{in}\ \mathtt{send1}\ c\ (\ell, c'); & \ell \leftarrow (!\,\ell + 2); \\
\{c' \rightarrowtail (\mathtt{chan\_prot}'\ \ell\ 40)\} & \{c' \rightarrowtail (\overline{\mathtt{chan\_prot}'\ \ell}) * \ell \mapsto 42\} \\
\mathtt{recv1}\ c'; & \mathtt{send1}\ c'\ () \\
\{\ell \mapsto 42\} & \\
\mathtt{let}\ x = !\,\ell\ \mathtt{in}\ \mathtt{free}\ \ell; & \\
\{x = 42\} & \\
\mathtt{assert}(x = 42) & \\
\{\mathsf{True}\} &
\end{array}\right)
\end{array}
$$

One-shot channel protocols:

$$
\begin{array}{l}
\mathtt{chan\_prot} \triangleq (\mathsf{Send}, \lambda w.\, \exists (\ell : \mathsf{Loc}), c'.\, w = (\ell, c') * \ell \mapsto 40 * c' \rightarrowtail \overline{\mathtt{chan\_prot}'\ \ell}) \\
\mathtt{chan\_prot}'\ (\ell : \mathsf{Loc}) \triangleq (\mathsf{Recv}, \lambda w.\, w = () * \ell \mapsto 42)
\end{array}
$$

One-Shot Channel Verification Examples – Higher-Order

$$
\begin{array}{l}
{\color{blue}\{\mathsf{True}\}} \\
\mathbf{let}\ c = \mathbf{new1}\,()\ \mathbf{in} \\
{\color{blue}\{c \rightarrowtail \mathtt{chan\_prot} * c \rightarrowtail \overline{\mathtt{chan\_prot}}\}} \\
\left(\begin{array}{l||l}
{\color{blue}\{c \rightarrowtail \mathtt{chan\_prot}\}} & {\color{blue}\{c \rightarrowtail \overline{\mathtt{chan\_prot}}\}} \\
\mathbf{let}\ \ell = \mathbf{ref}\,40\ \mathbf{in} & \mathbf{let}\,(\ell, c') = \mathbf{recv1}\,c\ \mathbf{in} \\
{\color{blue}\{c \rightarrowtail \mathtt{chan\_prot} * \ell \mapsto 40\}} & {\color{blue}\{c' \rightarrowtail (\overline{\mathtt{chan\_prot}'\ \ell}) * \ell \mapsto 40\}} \\
\mathbf{let}\ c' = \mathbf{new1}\,()\ \mathbf{in}\ \mathbf{send1}\,c\,(\ell, c'); & \ell \leftarrow (!\,\ell + 2); \\
{\color{blue}\{c' \rightarrowtail (\mathtt{chan\_prot}'\ \ell\ 40)\}} & {\color{blue}\{c' \rightarrowtail (\overline{\mathtt{chan\_prot}'\ \ell}) * \ell \mapsto 42\}} \\
\mathbf{recv1}\,c'; & \mathbf{send1}\,c'\,() \\
{\color{blue}\{\ell \mapsto 42\}} & {\color{blue}\{\mathsf{True}\}} \\
\mathbf{let}\ x = !\,\ell\ \mathbf{in}\ \mathbf{free}\,\ell; & \\
{\color{blue}\{x = 42\}} & \\
\mathbf{assert}(x = 42) & \\
{\color{blue}\{\mathsf{True}\}} &
\end{array}\right)
\end{array}
$$

One-shot channel protocols:

$$
\begin{aligned}
&\mathtt{chan\_prot} \triangleq (\mathsf{Send}, \lambda w.\, \exists (\ell : \mathsf{Loc}), c'.\, w = (\ell, c') * \ell \mapsto 40 * c' \rightarrowtail \overline{\mathtt{chan\_prot}'\ \ell}) \\
&\mathtt{chan\_prot}'\ (\ell : \mathsf{Loc}) \triangleq (\mathsf{Recv}, \lambda w.\, w = () * \ell \mapsto 42)
\end{aligned}
$$

One-Shot Channel Verification Examples – Higher-Order

$$
\begin{array}{l}
\{\mathsf{True}\}\\
\mathbf{let}\ c = \mathbf{new1}\,()\ \mathbf{in}\\
\{c \rightarrowtail \mathtt{chan\_prot} * c \rightarrowtail \overline{\mathtt{chan\_prot}}\}\\
\left(\begin{array}{l|l}
\{c \rightarrowtail \mathtt{chan\_prot}\} & \{c \rightarrowtail \overline{\mathtt{chan\_prot}}\}\\
\mathbf{let}\ \ell = \mathbf{ref}\,40\ \mathbf{in} & \mathbf{let}\ (\ell, c') = \mathbf{recv1}\,c\ \mathbf{in}\\
\{c \rightarrowtail \mathtt{chan\_prot} * \ell \mapsto 40\} & \{c' \rightarrowtail (\overline{\mathtt{chan\_prot}'\ \ell}) * \ell \mapsto 40\}\\
\mathbf{let}\ c' = \mathbf{new1}\,()\ \mathbf{in}\ \mathbf{send1}\,c\,(\ell, c'); & \ell \leftarrow (!\,\ell + 2);\\
\{c' \rightarrowtail (\mathtt{chan\_prot}'\ \ell\ 40)\} & \{c' \rightarrowtail (\overline{\mathtt{chan\_prot}'\ \ell}) * \ell \mapsto 42\}\\
\mathbf{recv1}\,c'; & \mathbf{send1}\,c'\,()\\
\{\ell \mapsto 42\} & \{\mathsf{True}\}\\
\mathbf{let}\ x = !\,\ell\ \mathbf{in}\ \mathbf{free}\,\ell; & \\
\{x = 42\} & \\
\mathbf{assert}(x = 42) & \\
\{\mathsf{True}\} &
\end{array}\right)\\
\{\mathsf{True}\}
\end{array}
$$

One-shot channel protocols:

$$
\begin{aligned}
\mathtt{chan\_prot} &\triangleq (\mathsf{Send}, \lambda w.\, \exists (\ell : \mathsf{Loc}), c'.\, w = (\ell, c') * \ell \mapsto 40 * c' \rightarrowtail \overline{\mathtt{chan\_prot}'\ \ell})\\
\mathtt{chan\_prot}'\ (\ell : \mathsf{Loc}) &\triangleq (\mathsf{Recv}, \lambda w.\, w = () * \ell \mapsto 42)
\end{aligned}
$$

Questions?

Break (10 min!)

Dependent separation protocols

Tutorial Timeline

Part 1: 14:00 – 15:30

- ~~Introduction (10 min)~~
- ~~Layered implementation of session channels (10 min)~~
- ~~Basic concurrent separation logic and one-shot protocols (30 min)~~
- ~~**Break** (10 min)~~
- Dependent separation protocols (30 min)

Break (30 min)

Part 2: 16:00 – 17:30

- Iris invariants and ghost state (30 min)
- **Break** (10 min)
- Supervised Coq hacking (50 min)

Overview of Abstraction Layers

Layer	Reasoning principles / specifications
#1 Iris's HeapLang	Basic concurrent separation logic Iris invariants and ghost state
#2 One-shot channels	One-shot protocols
#3 Functional session channels	Dependent separation protocols
#4 Session channels	Dependent separation protocols

Functional Session Channels

Implementation (inspired by Kobayashi et al., Dardha et al.):

$$\mathbf{new}_{\mathbf{fun}}\,() \triangleq \mathbf{new1}\,()$$

$$\mathbf{send}\,c\,v \triangleq \mathbf{let}\,c' = \mathbf{new1}\,()\,\mathbf{in}\,\mathbf{send1}\,c\,(v, c');\; c' \qquad \mathbf{close}\,c \triangleq \mathbf{send1}\,c\,()$$

$$\mathbf{recv}\,c \triangleq \mathbf{recv1}\,c \qquad \mathbf{wait}\,c \triangleq \mathbf{recv1}\,c$$

Example program:

$$\begin{array}{l}
\texttt{ses_fun_ref_prog} \triangleq \\
\quad \mathbf{let}\,c = \mathbf{new}_{\mathbf{fun}}\,()\,\mathbf{in} \\
\quad \left(\begin{array}{l|l}
\mathbf{let}\,\ell = \mathbf{ref}\,40\,\mathbf{in} & \mathbf{let}\,(\ell, c') = \mathbf{recv}\,c\,\mathbf{in} \\
\mathbf{let}\,c' = \mathbf{send}\,c\,\ell\,\mathbf{in} & \ell \leftarrow (!\,\ell + 2); \mathbf{close}\,c' \\
\mathbf{wait}\,c'; \mathbf{let}\,x = !\,\ell\,\mathbf{in}\,\mathbf{free}\,\ell; & \\
\mathbf{assert}(x = 42) &
\end{array}\right)
\end{array}$$

Functional Session Channel Specifications?

Implementation (inspired by Kobayashi et al., Dardha et al.):

$$\mathbf{new}_{\mathbf{fun}}\,() \triangleq \mathbf{new1}\,()$$

$$\mathbf{send}\, c\, v \triangleq \mathbf{let}\, c' = \mathbf{new1}\,()\,\mathbf{in}\,\mathbf{send1}\, c\,(v, c');\; c' \qquad \mathbf{close}\, c \triangleq \mathbf{send1}\, c\,()$$

$$\mathbf{recv}\, c \triangleq \mathbf{recv1}\, c \qquad \mathbf{wait}\, c \triangleq \mathbf{recv1}\, c$$

Specifications:

$$\{\mathsf{True}\}\ \mathbf{new}_{\mathbf{fun}}\,()\ \{w.\, \exists c.\, w = c * c \rightarrowtail p * c \rightarrowtail \overline{p}\}$$

$$\{c \rightarrowtail ??? * ???\}\ \mathbf{send}\, c\, v\ \{w.\, ???\}$$

$$\{c \rightarrowtail ???\}\ \mathbf{recv}\, c\ \{w.\, ???\}$$

$$\{c \rightarrowtail ??? * ???\}\ \mathbf{close}\, c\ \{w.\, ???\}$$

$$\{c \rightarrowtail ???\}\ \mathbf{wait}\, c\ \{w.\, ???\}$$

Dependent Separation Protocols

$$\texttt{chan\_prot} \triangleq$$
$$(\mathsf{Send}, \lambda w.\, \exists (\ell : \mathsf{Loc}), c'.\, w = (\ell, c') * \ell \mapsto 40 * c' \rightarrowtail \overline{\texttt{chan\_prot}'\ \ell})$$

Dependent Separation Protocols

$$\texttt{send\_prot} \triangleq$$
$$(\mathsf{Send}, \lambda w.\, \exists(\ell : \mathsf{Loc}), c'.\, w = (\ell, c') * \ell \mapsto 40 * c' \rightarrowtail \overline{\texttt{chan\_prot}'\ \ell})$$

Dependent Separation Protocols

$$\texttt{send\_prot} \triangleq$$
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Dependent Separation Protocols

$$\texttt{send\_prot}\ (p : \mathsf{Loc} \to \mathsf{iProto}) \triangleq$$
$$\quad (\mathsf{Send}, \lambda w.\, \exists (\ell : \mathsf{Loc}), c'.\, w = (\ell, c') * \ell \mapsto 40 * c' \rightarrowtail \overline{p\ \ell})$$

Dependent Separation Protocols

$$\texttt{send\_prot}\ (P : \mathsf{Loc} \to \mathsf{iProp})\ (p : \mathsf{Loc} \to \mathsf{iProto}) \triangleq$$
$$(\mathsf{Send}, \lambda w.\, \exists(\ell : \mathsf{Loc}), c'.\, w = (\ell, c') * P\ \ell * c' \rightarrowtail \overline{p\ \ell})$$

Dependent Separation Protocols

$$\texttt{send\_prot}\ (v : \mathsf{Loc} \to \mathsf{Val})\ (P : \mathsf{Loc} \to \mathsf{iProp})\ (p : \mathsf{Loc} \to \mathsf{iProto}) \triangleq$$
$$(\mathsf{Send}, \lambda w.\, \exists (\ell : \mathsf{Loc}), c'.\, w = (v\ \ell, c') * P\ x * c' \rightarrowtail \overline{p\ x})$$

Dependent Separation Protocols

$$\texttt{send\_prot}\ (\tau : \mathsf{Type})\ (v : \tau \to \mathsf{Val})\ (P : \tau \to \mathsf{iProp})\ (p : \tau \to \mathsf{iProto}) \triangleq$$
$$(\mathsf{Send}, \lambda w.\, \exists (x : \tau), c'.\, w = (v\ x, c') * P\ x * c' \rightarrowtail \overline{p\ x})$$

Dependent Separation Protocols

$$\mathbf{!}\,(x : \tau)\,\langle v \rangle \{P\}.\, p \triangleq$$
$$\quad (\mathsf{Send}, \lambda w.\, \exists (x : \tau), c'.\, w = (v\ x, c') * P\ x * c' \rightarrowtail \overline{p\ x})$$

Dependent Separation Protocols

$$!\,(x : \tau)\,\langle v\rangle\{P\}.\,p \triangleq (\mathsf{Send}, \lambda w.\,\exists (x : \tau), c'.\, w = (v\,x, c') * P\,x * c' \rightarrowtail \overline{p\,x})$$

Dependent Separation Protocols

$$\mathbf{!}\,(x : \tau)\,\langle v\rangle\{P\}.\,p \triangleq (\mathsf{Send}, \lambda w.\, \exists (x:\tau), c'.\, w = (v\;x, c') * P\;x * c' \rightarrowtail \overline{p\;x})$$
$$\mathbf{?}(x : \tau)\,\langle v\rangle\{P\}.\,p \triangleq \overline{\mathbf{!}\,(x : \tau)\,\langle v\rangle\{P\}.\,\overline{p}}$$

Dependent Separation Protocols

$$\mathbf{!}\,(x : \tau)\,\langle v\rangle\{P\}.\,p \triangleq (\mathsf{Send}, \lambda w.\,\exists(x : \tau), c'.\, w = (v\,x, c') * P\,x * c' \rightarrowtail \overline{p\,x})$$
$$\mathbf{?}(x : \tau)\,\langle v\rangle\{P\}.\,p \triangleq (\mathsf{Recv}, \lambda w.\,\exists(x : \tau), c'.\, w = (v\,x, c') * P\,x * c' \rightarrowtail \overline{\overline{p\,x}})$$

Dependent Separation Protocols

$$\mathbf{!}\,(x:\tau)\,\langle v\rangle\{P\}.\,p \triangleq (\mathsf{Send}, \lambda w.\,\exists (x:\tau), c'.\, w = (v\,x, c') * P\,x * c' \rightarrowtail \overline{p\,x})$$
$$\mathbf{?}(x:\tau)\,\langle v\rangle\{P\}.\,p \triangleq (\mathsf{Recv}, \lambda w.\,\exists (x:\tau), c'.\, w = (v\,x, c') * P\,x * c' \rightarrowtail p\,x)$$

Dependent Separation Protocols

$$\mathbf{!}\,(x:\tau)\,\langle v\rangle\{P\}.\,p \triangleq (\mathsf{Send}, \lambda w.\,\exists(x:\tau), c'.\, w = (v\,x, c') * P\,x * c' \rightarrowtail \overline{p\,x})$$

$$\mathbf{?}(x:\tau)\,\langle v\rangle\{P\}.\,p \triangleq (\mathsf{Recv}, \lambda w.\,\exists(x:\tau), c'.\, w = (v\,x, c') * P\,x * c' \rightarrowtail p\,x)$$

$$\texttt{chan\_prot}'\,(\ell : \mathsf{Loc}) \triangleq (\mathsf{Recv}, \lambda w.\, w = () * \ell \mapsto 42)$$

Dependent Separation Protocols

$$\mathbf{!}\,(x : \tau)\,\langle v\rangle\{P\}.\,p \triangleq (\mathsf{Send}, \lambda w.\,\exists (x : \tau), c'.\,w = (v\;x, c') * P\;x * c' \rightarrowtail \overline{p\;x})$$
$$\mathbf{?}(x : \tau)\,\langle v\rangle\{P\}.\,p \triangleq (\mathsf{Recv}, \lambda w.\,\exists (x : \tau), c'.\,w = (v\;x, c') * P\;x * c' \rightarrowtail p\;x)$$

$$\texttt{close\_prot}\;(\ell : \mathsf{Loc}) \triangleq (\mathsf{Recv}, \lambda w.\,w = () * \ell \mapsto 42)$$

Dependent Separation Protocols

$$\mathbf{!}\,(x : \tau)\,\langle v\rangle\{P\}.\,p \triangleq (\mathsf{Send}, \lambda w.\, \exists (x : \tau), c'.\, w = (v\ x, c') * P\ x * c' \rightarrowtail \overline{p\ x})$$

$$\mathbf{?}(x : \tau)\,\langle v\rangle\{P\}.\,p \triangleq (\mathsf{Recv}, \lambda w.\, \exists (x : \tau), c'.\, w = (v\ x, c') * P\ x * c' \rightarrowtail p\ x)$$

$$\texttt{close\_prot}\ (\ell : \mathsf{Loc}) \triangleq (\mathsf{Send}, \lambda w.\, w = () * \ell \mapsto 42)$$

Dependent Separation Protocols

$$\mathbf{!}\,(x:\tau)\,\langle v\rangle\{P\}.\,p \triangleq (\mathsf{Send}, \lambda w.\,\exists (x:\tau), c'.\, w = (v\,x, c') * P\,x * c' \rightarrowtail \overline{p\,x})$$

$$\mathbf{?}(x:\tau)\,\langle v\rangle\{P\}.\,p \triangleq (\mathsf{Recv}, \lambda w.\,\exists (x:\tau), c'.\, w = (v\,x, c') * P\,x * c' \rightarrowtail p\,x)$$

$$\texttt{close\_prot}\,(P : \mathsf{iProp}) \triangleq (\mathsf{Send}, \lambda w.\, w = () * P)$$

Dependent Separation Protocols

$$\mathbf{!}\,(x:\tau)\,\langle v\rangle\{P\}.\,p \triangleq (\mathrm{Send}, \lambda w.\,\exists (x:\tau), c'.\, w = (v\,x, c') * P\,x * c' \rightarrowtail \overline{p\,x})$$
$$\mathbf{?}(x:\tau)\,\langle v\rangle\{P\}.\,p \triangleq (\mathrm{Recv}, \lambda w.\,\exists (x:\tau), c'.\, w = (v\,x, c') * P\,x * c' \rightarrowtail p\,x)$$

$$\mathbf{!}\,\mathbf{end}\{P\} \triangleq (\mathrm{Send}, \lambda w.\, w = () * P)$$

Dependent Separation Protocols

$$\mathbf{!}\,(x:\tau)\,\langle v\rangle\{P\}.\,p \triangleq (\mathsf{Send}, \lambda w.\,\exists (x:\tau), c'.\, w = (v\,x, c') * P\,x * c' \rightarrowtail \overline{p\,x})$$
$$\mathbf{?}(x:\tau)\,\langle v\rangle\{P\}.\,p \triangleq (\mathsf{Recv}, \lambda w.\,\exists (x:\tau), c'.\, w = (v\,x, c') * P\,x * c' \rightarrowtail p\,x)$$

$$\mathbf{!\,end}\{P\} \triangleq (\mathsf{Send}, \lambda w.\, w = () * P)$$
$$\mathbf{?end}\{P\} \triangleq \overline{\mathbf{!\,end}\{P\}}$$

Dependent Separation Protocols

$$\mathbf{!}\,(x : \tau)\,\langle v \rangle \{P\}.\, p \triangleq (\mathsf{Send}, \lambda w.\, \exists (x : \tau), c'.\, w = (v\ x, c') * P\ x * c' \rightarrowtail \overline{p\ x})$$
$$\mathbf{?}(x : \tau)\,\langle v \rangle \{P\}.\, p \triangleq (\mathsf{Recv}, \lambda w.\, \exists (x : \tau), c'.\, w = (v\ x, c') * P\ x * c' \rightarrowtail p\ x)$$

$$\mathbf{!\,end}\{P\} \triangleq (\mathsf{Send}, \lambda w.\, w = () * P)$$
$$\mathbf{?end}\{P\} \triangleq (\mathsf{Recv}, \lambda w.\, w = () * P)$$

Dependent Separation Protocols

$$\mathbf{!}\,(x:\tau)\,\langle v\rangle\{P\}.\,p \triangleq (\mathsf{Send}, \lambda w.\,\exists(x:\tau), c'.\,w = (v\,x, c') * P\,x * c' \rightarrowtail \overline{p\,x})$$
$$\mathbf{?}(x:\tau)\,\langle v\rangle\{P\}.\,p \triangleq (\mathsf{Recv}, \lambda w.\,\exists(x:\tau), c'.\,w = (v\,x, c') * P\,x * c' \rightarrowtail p\,x)$$

$$\mathbf{!\,end}\{P\} \triangleq (\mathsf{Send}, \lambda w.\,w = () * P)$$
$$\mathbf{?end}\{P\} \triangleq (\mathsf{Recv}, \lambda w.\,w = () * P)$$

$$\texttt{chan\_prot} \triangleq \mathbf{!}\,(\ell:\mathsf{Loc})\,\langle\ell\rangle\{\ell \mapsto 40\}.\,\mathbf{?end}\{\ell \mapsto 42\}$$

Functional Session Channels Specifications!

Dependent session protocols:

$$
\begin{aligned}
\mathbf{!}\,(x:\tau)\,\langle v\rangle\{P\}.\,p &\triangleq (\mathsf{Send}, \lambda w.\, \exists (x:\tau), c'.\, w = (v\,x, c') * (P\,x) * c' \rightarrowtail \overline{p\,x}) \\
\mathbf{?}(x:\tau)\,\langle v\rangle\{P\}.\,p &\triangleq (\mathsf{Recv}, \lambda w.\, \exists (x:\tau), c'.\, w = (v\,x, c') * (P\,x) * c' \rightarrowtail p\,x) \\
\mathbf{!\,end}\{P\} &\triangleq (\mathsf{Send}, \lambda w.\, w = () * P) \\
\mathbf{?end}\{P\} &\triangleq (\mathsf{Recv}, \lambda w.\, w = () * P)
\end{aligned}
$$

Functional session channel specifications:

$$
\begin{aligned}
\{\mathsf{True}\}\ &\mathtt{new_{fun}}\,()\ \{w.\, \exists c.\, w = c * c \rightarrowtail p * c \rightarrowtail \overline{p}\} \\
\{c \rightarrowtail (\mathbf{!}\,(x:\tau)\,\langle v\rangle\{P\}.\,p) * P\,t\}\ &\mathtt{send}\,c\,(v\,t)\ \{w.\, \exists c'.\, w = c' * c' \rightarrowtail p\,t\} \\
\{c \rightarrowtail (\mathbf{?}\,(x:\tau)\,\langle v\rangle\{P\}.\,p)\}\ &\mathtt{recv}\,c\ \{w.\, \exists (x:\tau), c'.\, w = (v\,x, c') * P\,x * c' \rightarrowtail p\,x\} \\
\{c \rightarrowtail \mathbf{!\,end}\{P\} * P\}\ &\mathtt{close}\,c\ \{\mathsf{True}\} \\
\{c \rightarrowtail \mathbf{?end}\{P\}\}\ &\mathtt{wait}\,c\ \{P\}
\end{aligned}
$$

Proof of Send Specification

$$
\begin{array}{l}
\mathtt{\textbf{let}}\; c' = \mathtt{\textbf{new1}}\,()\;\mathtt{\textbf{in}} \\
\mathtt{\textbf{send1}}\; c\,(v\; t, c'); \\
c'
\end{array}
$$

Proof of Send Specification

$$\{c \rightarrowtail \mathbf{!}\,(x : \tau)\,\langle v \rangle\{P\}.\,p \quad * \quad P\ t\}$$
$$\texttt{let}\ c' = \texttt{new1}\ ()\ \texttt{in}$$
$$\texttt{send1}\ c\ (v\ t, c');$$
$$c'$$

Proof of Send Specification

$\{c \rightarrowtail \mathbf{!}\,(x : \tau)\,\langle v\rangle\{P\}.\,p \quad * \quad P\,t\}$
$\mathtt{let}\; c' = \mathtt{new1}\,()\;\mathtt{in}$
$\mathtt{send1}\; c\,(v\;t, c');$
c'

Send protocol:

$$\mathbf{!}\,(x : \tau)\,\langle v\rangle\{P\}.\,p \triangleq (\mathsf{Send}, \lambda w.\, \exists (x : \tau), c'.\, w = (v\;x, c') * (P\;x) * c' \rightarrowtail \overline{p\;x})$$

Proof of Send Specification

$\{c \rightarrowtail \mathbf{!}\,(x : \tau)\,\langle v \rangle\{P\}.\,p \quad * \quad P\,t\}$
$\mathtt{let}\ c' = \mathtt{new1}\,()\ \mathtt{in}$
$\{c \rightarrowtail \mathbf{!}\,(x : \tau)\,\langle v \rangle\{P\}.\,p \quad * \quad P\,t \quad * \quad c' \rightarrowtail p\,t \quad * \quad c' \rightarrowtail \overline{p\,t}\}$
$\mathtt{send1}\,c\,(v\,t, c');$
c'

Send protocol:

$$\mathbf{!}\,(x : \tau)\,\langle v \rangle\{P\}.\,p \triangleq (\mathsf{Send}, \lambda w.\,\exists (x : \tau), c'.\,w = (v\,x, c') * (P\,x) * c' \rightarrowtail \overline{p\,x})$$

Proof of Send Specification

$$
\begin{array}{l}
\{c \rightarrowtail \mathbf{!}\,(x:\tau)\,\langle v\rangle\{P\}.\,p \quad * \quad P\;t\} \\
\mathtt{let}\;c' = \mathtt{new1}\,()\;\mathtt{in} \\
\{c \rightarrowtail \mathbf{!}\,(x:\tau)\,\langle v\rangle\{P\}.\,p \quad * \quad P\;t \quad * \quad c' \rightarrowtail p\;t \quad * \quad c' \rightarrowtail \overline{p\;t}\} \\
\mathtt{send1}\,c\,(v\;t, c'); \\
\{c' \rightarrowtail p\;t\} \\
c'
\end{array}
$$

Send protocol:

$$
\mathbf{!}\,(x:\tau)\,\langle v\rangle\{P\}.\,p \triangleq (\mathsf{Send}, \lambda w.\,\exists(x:\tau), c'.\,w = (v\;x, c') * (P\;x) * c' \rightarrowtail \overline{p\;x})
$$

Proof of Send Specification

$$\{c \rightarrowtail \mathbf{!}\,(x : \tau)\,\langle v\rangle\{P\}.\,p \quad * \quad P\ t\}$$
$$\mathbf{let}\ c' = \mathbf{new1}\,()\ \mathbf{in}$$
$$\{c \rightarrowtail \mathbf{!}\,(x : \tau)\,\langle v\rangle\{P\}.\,p \quad * \quad P\ t \quad * \quad c' \rightarrowtail p\ t \quad * \quad c' \rightarrowtail \overline{p\ t}\}$$
$$\mathbf{send1}\,c\,(v\ t, c');$$
$$\{c' \rightarrowtail p\ t\}$$
$$c'$$
$$\{w.\ \exists c'.\ w = c' * c' \rightarrowtail p\ t\}$$

Send protocol:

$$\mathbf{!}\,(x : \tau)\,\langle v\rangle\{P\}.\,p \triangleq (\mathsf{Send}, \lambda w.\ \exists(x : \tau), c'.\ w = (v\ x, c') * (P\ x) * c' \rightarrowtail \overline{p\ x})$$

Functional Session Channel Example

$$\mathbf{let}\; c = \mathbf{new}_{\mathbf{fun}}\,()\;\mathbf{in}$$
$$\left(\begin{array}{l} \mathbf{let}\;\ell = \mathbf{ref}\,40\;\mathbf{in} \\ \mathbf{let}\;c' = \mathbf{send}\,c\,\ell\;\mathbf{in} \\ \mathbf{wait}\;c'; \\ \mathbf{let}\;x = !\,\ell\;\mathbf{in}\;\mathbf{free}\,\ell; \\ \mathbf{assert}(x = 42) \end{array} \;\middle\|\; \begin{array}{l} \mathbf{let}\;(\ell, c') = \mathbf{recv}\,c\;\mathbf{in} \\ \ell \leftarrow (!\,\ell + 2); \\ \mathbf{close}\;c' \\ \\ \\ \end{array}\right)$$

Functional Session Channel Example

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\texttt{let}\, c = \texttt{new}_{\texttt{fun}}\,()\;\texttt{in} \\
\left(\begin{array}{l}
\texttt{let}\,\ell = \texttt{ref}\,40\;\texttt{in} \\
\texttt{let}\,c' = \texttt{send}\,c\,\ell\;\texttt{in} \\
\texttt{wait}\,c'; \\
\texttt{let}\,x = !\,\ell\;\texttt{in}\;\texttt{free}\,\ell; \\
\texttt{assert}(x = 42)
\end{array}
\;\middle\|\;
\begin{array}{l}
\texttt{let}\,(\ell, c') = \texttt{recv}\,c\;\texttt{in} \\
\ell \leftarrow (!\,\ell + 2); \\
\texttt{close}\,c' \\
\\
\\
\end{array}\right)
\end{array}
$$

Functional Session Channel Example

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathtt{let}\, c = \mathtt{new}_{\mathtt{fun}}\,()\,\mathtt{in} \\
\left(\begin{array}{l}
\mathtt{let}\, \ell = \mathtt{ref}\, 40\, \mathtt{in} \\
\mathtt{let}\, c' = \mathtt{send}\, c\, \ell\, \mathtt{in} \\
\mathtt{wait}\, c'; \\
\mathtt{let}\, x = !\, \ell\, \mathtt{in}\ \mathtt{free}\, \ell; \\
\mathtt{assert}(x = 42)
\end{array}
\;\middle\|\;
\begin{array}{l}
\mathtt{let}\, (\ell, c') = \mathtt{recv}\, c\, \mathtt{in} \\
\ell \leftarrow (!\, \ell + 2); \\
\mathtt{close}\, c' \\
\\
\\
\end{array}\right)
\end{array}
$$

Protocol:

$$\mathtt{ses\_prot} \triangleq \mathbf{!}\,(\ell : \mathsf{Loc})\,\langle \ell \rangle \{\ell \mapsto 40\}.\,\mathbf{?end}\{\ell \mapsto 42\}$$

Functional Session Channel Example

$$\{\mathsf{True}\}$$
$$\mathtt{let}\, c = \mathtt{new_{fun}}\,()\,\mathtt{in}$$
$$\{c \rightarrowtail \mathtt{ses\_prot} * c \rightarrowtail \overline{\mathtt{ses\_prot}}\}$$
$$\left(\begin{array}{l} \mathtt{let}\,\ell = \mathtt{ref}\,40\,\mathtt{in} \\ \mathtt{let}\,c' = \mathtt{send}\,c\,\ell\,\mathtt{in} \\ \mathtt{wait}\,c'; \\ \mathtt{let}\,x = !\,\ell\,\mathtt{in}\ \mathtt{free}\,\ell; \\ \mathtt{assert}(x = 42) \end{array} \middle\| \begin{array}{l} \mathtt{let}\,(\ell, c') = \mathtt{recv}\,c\,\mathtt{in} \\ \ell \leftarrow (!\,\ell + 2); \\ \mathtt{close}\,c' \\ \\ \\ \end{array}\right)$$

Protocol:

$$\mathtt{ses\_prot} \triangleq \mathbf{!}\,(\ell : \mathsf{Loc})\,\langle \ell \rangle \{\ell \mapsto 40\}.\,\mathbf{?end}\{\ell \mapsto 42\}$$

Functional Session Channel Example

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\texttt{let}\ c = \texttt{new}_{\mathsf{fun}}\ ()\ \texttt{in} \\
\{c \rightarrowtail \texttt{ses\_prot} * c \rightarrowtail \overline{\texttt{ses\_prot}}\} \\
\left(\begin{array}{l}
\{c \rightarrowtail \texttt{ses\_prot}\} \\
\texttt{let}\ \ell = \texttt{ref}\ 40\ \texttt{in} \\
\texttt{let}\ c' = \texttt{send}\ c\ \ell\ \texttt{in} \\
\texttt{wait}\ c'; \\
\texttt{let}\ x = !\,\ell\ \texttt{in}\ \texttt{free}\ \ell; \\
\texttt{assert}(x = 42)
\end{array}
\middle\|
\begin{array}{l}
\{c \rightarrowtail \overline{\texttt{ses\_prot}}\} \\
\texttt{let}\ (\ell, c') = \texttt{recv}\ c\ \texttt{in} \\
\ell \leftarrow (!\,\ell + 2); \\
\texttt{close}\ c' \\
\\
\\
\end{array}\right)
\end{array}
$$

Protocol:

$$\texttt{ses\_prot} \triangleq \mathbf{!}\,(\ell : \mathsf{Loc})\,\langle \ell \rangle \{\ell \mapsto 40\}.\,\mathbf{?end}\{\ell \mapsto 42\}$$

Functional Session Channel Example

$$
\begin{array}{l}
\{\mathsf{True}\}\\
\mathtt{let}\ c = \mathtt{new_{fun}}\ ()\ \mathtt{in}\\
\{c \rightarrowtail \mathtt{ses\_prot} * c \rightarrowtail \overline{\mathtt{ses\_prot}}\}\\
\left(\begin{array}{l}
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\mathtt{let}\ \ell = \mathtt{ref}\ 40\ \mathtt{in}\\
\{c \rightarrowtail \mathtt{ses\_prot} * \ell \mapsto 40\}\\
\mathtt{let}\ c' = \mathtt{send}\ c\ \ell\ \mathtt{in}\\
\mathtt{wait}\ c';\\
\mathtt{let}\ x = !\,\ell\ \mathtt{in}\ \mathtt{free}\ \ell;\\
\mathtt{assert}(x = 42)
\end{array}
\middle\|
\begin{array}{l}
\{c \rightarrowtail \overline{\mathtt{ses\_prot}}\}\\
\mathtt{let}\ (\ell, c') = \mathtt{recv}\ c\ \mathtt{in}\\
\ell \leftarrow (!\,\ell + 2);\\
\mathtt{close}\ c'\\
\\
\\
\\
\end{array}\right)
\end{array}
$$

Protocol:

$$\mathtt{ses\_prot} \triangleq \mathbf{!}\,(\ell : \mathsf{Loc})\,\langle \ell \rangle \{\ell \mapsto 40\}.\,\mathbf{?end}\{\ell \mapsto 42\}$$

Functional Session Channel Example

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathbf{let}\, c = \mathbf{new}_{\mathbf{fun}}\, ()\, \mathbf{in} \\
\{c \rightarrowtail \mathtt{ses\_prot} * c \rightarrowtail \overline{\mathtt{ses\_prot}}\} \\
\left(\begin{array}{l}
\{c \rightarrowtail \mathtt{ses\_prot}\} \\
\mathbf{let}\, \ell = \mathbf{ref}\, 40\, \mathbf{in} \\
\{c \rightarrowtail \mathtt{ses\_prot} * \ell \mapsto 40\} \\
\mathbf{let}\, c' = \mathbf{send}\, c\, \ell\, \mathbf{in} \\
\{c' \rightarrowtail \mathbf{?end}\{\ell \mapsto 42\}\} \\
\mathbf{wait}\, c'; \\
\mathbf{let}\, x = !\, \ell\, \mathbf{in}\ \mathbf{free}\, \ell; \\
\mathbf{assert}(x = 42)
\end{array}
\middle\|
\begin{array}{l}
\{c \rightarrowtail \overline{\mathtt{ses\_prot}}\} \\
\mathbf{let}\, (\ell, c') = \mathbf{recv}\, c\, \mathbf{in} \\
\ell \leftarrow (!\, \ell + 2); \\
\mathbf{close}\, c' \\
\\ \\ \\ \\
\end{array}\right)
\end{array}
$$

Protocol:

$$\mathtt{ses\_prot} \triangleq \mathbf{!}\, (\ell : \mathsf{Loc})\, \langle \ell \rangle \{\ell \mapsto 40\}.\, \mathbf{?end}\{\ell \mapsto 42\}$$

Functional Session Channel Example

$$\{\mathsf{True}\}$$
$$\texttt{let}\ c = \texttt{new}_{\texttt{fun}}\ ()\ \texttt{in}$$
$$\{c \rightarrowtail \texttt{ses\_prot} * c \rightarrowtail \overline{\texttt{ses\_prot}}\}$$

$$\left(\begin{array}{l} \{c \rightarrowtail \texttt{ses\_prot}\} \\ \texttt{let}\ \ell = \texttt{ref}\ 40\ \texttt{in} \\ \{c \rightarrowtail \texttt{ses\_prot} * \ell \mapsto 40\} \\ \texttt{let}\ c' = \texttt{send}\ c\ \ell\ \texttt{in} \\ \{c' \rightarrowtail \textbf{?end}\{\ell \mapsto 42\}\} \\ \texttt{wait}\ c'; \\ \{\ell \mapsto 42\} \\ \texttt{let}\ x = !\,\ell\ \texttt{in}\ \texttt{free}\ \ell; \\ \texttt{assert}(x = 42) \end{array} \middle\| \begin{array}{l} \{c \rightarrowtail \overline{\texttt{ses\_prot}}\} \\ \texttt{let}\ (\ell, c') = \texttt{recv}\ c\ \texttt{in} \\ \ell \leftarrow (!\,\ell + 2); \\ \texttt{close}\ c' \\ \\ \\ \\ \\ \\ \end{array}\right)$$

Protocol:

$$\texttt{ses\_prot} \triangleq \textbf{!}\,(\ell : \mathsf{Loc})\,\langle \ell \rangle \{\ell \mapsto 40\}.\ \textbf{?end}\{\ell \mapsto 42\}$$

Functional Session Channel Example

$$\{\mathsf{True}\}$$
$$\mathtt{let}\, c = \mathtt{new_{fun}}\,()\, \mathtt{in}$$
$$\{c \rightarrowtail \mathtt{ses\_prot} * c \rightarrowtail \overline{\mathtt{ses\_prot}}\}$$

$$\left(\begin{array}{l} \{c \rightarrowtail \mathtt{ses\_prot}\} \\ \mathtt{let}\, \ell = \mathtt{ref}\, 40\, \mathtt{in} \\ \{c \rightarrowtail \mathtt{ses\_prot} * \ell \mapsto 40\} \\ \mathtt{let}\, c' = \mathtt{send}\, c\, \ell\, \mathtt{in} \\ \{c' \rightarrowtail \mathbf{?end}\{\ell \mapsto 42\}\} \\ \mathtt{wait}\, c'; \\ \{\ell \mapsto 42\} \\ \mathtt{let}\, x = !\,\ell\, \mathtt{in}\ \mathtt{free}\, \ell; \\ \{x = 42\} \\ \mathtt{assert}(x = 42) \end{array} \;\middle\|\; \begin{array}{l} \{c \rightarrowtail \overline{\mathtt{ses\_prot}}\} \\ \mathtt{let}\, (\ell, c') = \mathtt{recv}\, c\, \mathtt{in} \\ \ell \leftarrow (!\,\ell + 2); \\ \mathtt{close}\, c' \\ \\ \\ \\ \\ \\ \\ \end{array}\right)$$

Protocol:

$$\mathtt{ses\_prot} \triangleq \mathbf{!}\,(\ell : \mathsf{Loc})\,\langle \ell \rangle\{\ell \mapsto 40\}.\,\mathbf{?end}\{\ell \mapsto 42\}$$

Functional Session Channel Example

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathtt{let}\, c = \mathtt{new}_{\mathtt{fun}}\,()\,\mathtt{in} \\
\{c \rightarrowtail \mathtt{ses\_prot} * c \rightarrowtail \overline{\mathtt{ses\_prot}}\} \\
\left(\begin{array}{l}
\{c \rightarrowtail \mathtt{ses\_prot}\} \\
\mathtt{let}\, \ell = \mathtt{ref}\, 40\, \mathtt{in} \\
\{c \rightarrowtail \mathtt{ses\_prot} * \ell \mapsto 40\} \\
\mathtt{let}\, c' = \mathtt{send}\, c\, \ell\, \mathtt{in} \\
\{c' \rightarrowtail \mathbf{?end}\{\ell \mapsto 42\}\} \\
\mathtt{wait}\, c'; \\
\{\ell \mapsto 42\} \\
\mathtt{let}\, x = !\,\ell\, \mathtt{in}\ \mathtt{free}\, \ell; \\
\{x = 42\} \\
\mathtt{assert}(x = 42) \\
\{\mathsf{True}\}
\end{array}
\middle\|
\begin{array}{l}
\{c \rightarrowtail \overline{\mathtt{ses\_prot}}\} \\
\mathtt{let}\, (\ell, c') = \mathtt{recv}\, c\, \mathtt{in} \\
\ell \leftarrow (!\,\ell + 2); \\
\mathtt{close}\, c' \\
\\
\\
\\
\\
\\
\\
\\
\end{array}\right)
\end{array}
$$

Protocol:

$$\mathtt{ses\_prot} \triangleq \mathbf{!}\,(\ell : \mathsf{Loc})\,\langle \ell \rangle \{\ell \mapsto 40\}.\, \mathbf{?end}\{\ell \mapsto 42\}$$

Functional Session Channel Example

$$\{\mathsf{True}\}$$
$$\mathtt{let}\, c = \mathbf{new}_{\mathrm{fun}}\, ()\ \mathtt{in}$$
$$\{c \rightarrowtail \mathtt{ses\_prot} * c \rightarrowtail \overline{\mathtt{ses\_prot}}\}$$

$$\left(\begin{array}{l||l}
\{c \rightarrowtail \mathtt{ses_prot}\} & \{c \rightarrowtail \overline{\mathtt{ses_prot}}\} \\
\mathtt{let}\, \ell = \mathtt{ref}\, 40\ \mathtt{in} & \mathtt{let}\, (\ell, c') = \mathbf{recv}\, c\ \mathtt{in} \\
\{c \rightarrowtail \mathtt{ses_prot} * \ell \mapsto 40\} & \{c' \rightarrowtail \mathbf{!\,end}\{\ell \mapsto 42\} * \ell \mapsto 40\} \\
\mathtt{let}\, c' = \mathbf{send}\, c\, \ell\ \mathtt{in} & \ell \leftarrow (!\,\ell + 2); \\
\{c' \rightarrowtail \mathbf{?end}\{\ell \mapsto 42\}\} & \mathtt{close}\, c' \\
\mathtt{wait}\, c'; & \\
\{\ell \mapsto 42\} & \\
\mathtt{let}\, x = !\,\ell\ \mathtt{in}\ \mathtt{free}\, \ell; & \\
\{x = 42\} & \\
\mathtt{assert}(x = 42) & \\
\{\mathsf{True}\} &
\end{array}\right)$$

Protocol:

$$\mathtt{ses\_prot} \triangleq \mathbf{!}\,(\ell : \mathsf{Loc})\,\langle \ell \rangle\{\ell \mapsto 40\}.\ \mathbf{?end}\{\ell \mapsto 42\}$$

Functional Session Channel Example

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathtt{let}\; c = \mathtt{new}_{\mathrm{fun}}\,()\;\mathtt{in} \\
\{c \rightarrowtail \mathtt{ses\_prot} * c \rightarrowtail \overline{\mathtt{ses\_prot}}\}
\end{array}
$$

$$
\left(
\begin{array}{l}
\{c \rightarrowtail \mathtt{ses\_prot}\} \\
\mathtt{let}\; \ell = \mathtt{ref}\, 40\; \mathtt{in} \\
\{c \rightarrowtail \mathtt{ses\_prot} * \ell \mapsto 40\} \\
\mathtt{let}\; c' = \mathbf{send}\, c\, \ell\; \mathtt{in} \\
\{c' \rightarrowtail \mathbf{?end}\{\ell \mapsto 42\}\} \\
\mathtt{wait}\; c'; \\
\{\ell \mapsto 42\} \\
\mathtt{let}\; x = !\,\ell\; \mathtt{in}\; \mathtt{free}\; \ell; \\
\{x = 42\} \\
\mathtt{assert}(x = 42) \\
\{\mathsf{True}\}
\end{array}
\;\middle\|\;
\begin{array}{l}
\{c \rightarrowtail \overline{\mathtt{ses\_prot}}\} \\
\mathtt{let}\; (\ell, c') = \mathbf{recv}\, c\; \mathtt{in} \\
\{c' \rightarrowtail \mathbf{!\,end}\{\ell \mapsto 42\} * \ell \mapsto 40\} \\
\ell \leftarrow (!\,\ell + 2); \\
\{c' \rightarrowtail \mathbf{!\,end}\{\ell \mapsto 42\} * \ell \mapsto 42\} \\
\mathtt{close}\; c' \\
\\
\\
\\
\\
\\
\end{array}
\right)
$$

Protocol:

$$\mathtt{ses\_prot} \triangleq \mathbf{!}\,(\ell : \mathsf{Loc})\,\langle \ell \rangle \{\ell \mapsto 40\}.\, \mathbf{?end}\{\ell \mapsto 42\}$$

Functional Session Channel Example

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\texttt{let}\ c = \texttt{new}_{\text{fun}}\ ()\ \texttt{in} \\
\{c \rightarrowtail \texttt{ses\_prot} * c \rightarrowtail \overline{\texttt{ses\_prot}}\} \\
\left(
\begin{array}{l}
\{c \rightarrowtail \texttt{ses\_prot}\} \\
\texttt{let}\ \ell = \texttt{ref}\ 40\ \texttt{in} \\
\{c \rightarrowtail \texttt{ses\_prot} * \ell \mapsto 40\} \\
\texttt{let}\ c' = \texttt{send}\ c\ \ell\ \texttt{in} \\
\{c' \rightarrowtail \mathbf{?end}\{\ell \mapsto 42\}\} \\
\texttt{wait}\ c'; \\
\{\ell \mapsto 42\} \\
\texttt{let}\ x = !\,\ell\ \texttt{in}\ \texttt{free}\ \ell; \\
\{x = 42\} \\
\texttt{assert}(x = 42) \\
\{\mathsf{True}\}
\end{array}
\middle\|
\begin{array}{l}
\{c \rightarrowtail \overline{\texttt{ses\_prot}}\} \\
\texttt{let}\ (\ell, c') = \texttt{recv}\ c\ \texttt{in} \\
\{c' \rightarrowtail \mathbf{!\,end}\{\ell \mapsto 42\} * \ell \mapsto 40\} \\
\ell \leftarrow (!\,\ell + 2); \\
\{c' \rightarrowtail \mathbf{!\,end}\{\ell \mapsto 42\} * \ell \mapsto 42\} \\
\texttt{close}\ c' \\
\{\mathsf{True}\} \\
\\ \\ \\ \\
\end{array}
\right)
\end{array}
$$

Protocol:

$$\texttt{ses\_prot} \triangleq \mathbf{!}\,(\ell : \mathsf{Loc})\,\langle \ell \rangle \{\ell \mapsto 40\}.\ \mathbf{?end}\{\ell \mapsto 42\}$$

Functional Session Channel Example

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathtt{let}\ c = \mathbf{new}_{\text{fun}}\ ()\ \mathtt{in} \\
\{c \rightarrowtail \mathtt{ses\_prot} * c \rightarrowtail \overline{\mathtt{ses\_prot}}\} \\
\left(\begin{array}{l}
\{c \rightarrowtail \mathtt{ses\_prot}\} \\
\mathtt{let}\ \ell = \mathtt{ref}\ 40\ \mathtt{in} \\
\{c \rightarrowtail \mathtt{ses\_prot} * \ell \mapsto 40\} \\
\mathtt{let}\ c' = \mathbf{send}\ c\ \ell\ \mathtt{in} \\
\{c' \rightarrowtail \mathbf{?end}\{\ell \mapsto 42\}\} \\
\mathtt{wait}\ c'; \\
\{\ell \mapsto 42\} \\
\mathtt{let}\ x = !\,\ell\ \mathtt{in}\ \mathtt{free}\ \ell; \\
\{x = 42\} \\
\mathtt{assert}(x = 42) \\
\{\mathsf{True}\}
\end{array}
\middle\|
\begin{array}{l}
\{c \rightarrowtail \overline{\mathtt{ses\_prot}}\} \\
\mathtt{let}\ (\ell, c') = \mathbf{recv}\ c\ \mathtt{in} \\
\{c' \rightarrowtail \mathbf{!\,end}\{\ell \mapsto 42\} * \ell \mapsto 40\} \\
\ell \leftarrow (!\,\ell + 2); \\
\{c' \rightarrowtail \mathbf{!\,end}\{\ell \mapsto 42\} * \ell \mapsto 42\} \\
\mathtt{close}\ c' \\
\{\mathsf{True}\}
\end{array}\right) \\
\{\mathsf{True}\}
\end{array}
$$

Protocol:

$$\mathtt{ses\_prot} \triangleq \mathbf{!}\,(\ell : \mathsf{Loc})\ \langle \ell \rangle \{\ell \mapsto 40\}.\ \mathbf{?end}\{\ell \mapsto 42\}$$

Functional Session Channel Example

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathtt{let}\ c = \mathtt{new_{fun}}\ ()\ \mathtt{in} \\
\{c \rightarrowtail \mathtt{ses\_prot} * c \rightarrowtail \overline{\mathtt{ses\_prot}}\} \\
\left(\begin{array}{l}
\{c \rightarrowtail \mathtt{ses\_prot}\} \\
\mathtt{let}\ \ell = \mathtt{ref}\ 40\ \mathtt{in} \\
\{c \rightarrowtail \mathtt{ses\_prot} * \ell \mapsto 40\} \\
\mathtt{let}\ c' = \mathtt{send}\ c\ \ell\ \mathtt{in} \\
\{c' \rightarrowtail \mathbf{?end}\{\ell \mapsto 42\}\} \\
\mathtt{wait}\ c'; \\
\{\ell \mapsto 42\} \\
\mathtt{let}\ x = !\,\ell\ \mathtt{in}\ \mathtt{free}\ \ell; \\
\{x = 42\} \\
\mathtt{assert}(x = 42) \\
\{\mathsf{True}\}
\end{array}
\middle\|
\begin{array}{l}
\{c \rightarrowtail \overline{\mathtt{ses\_prot}}\} \\
\mathtt{let}\ (\ell, c') = \mathtt{recv}\ c\ \mathtt{in} \\
\{c' \rightarrowtail \mathbf{!end}\{\ell \mapsto (x+2)\} * \ell \mapsto x\} \\
\ell \leftarrow (!\,\ell + 2); \\
\{c' \rightarrowtail \mathbf{!end}\{\ell \mapsto (x+2)\} * \ell \mapsto (x+2)\} \\
\mathtt{close}\ c' \\
\{\mathsf{True}\} \\
\\ \\ \\ \\
\end{array}\right) \\
\{\mathsf{True}\}
\end{array}
$$

Protocol:

$$\mathtt{ses\_prot} \triangleq \mathbf{!}\,(\ell : \mathsf{Loc}, x : \mathbb{Z})\,\langle \ell \rangle \{\ell \mapsto x\}.\ \mathbf{?end}\{\ell \mapsto (x+2)\}$$

Session Channels Specifications

$$\mathbf{new}\,() \triangleq \mathbf{let}\, c = \mathbf{new_{fun}}\,()\,\mathbf{in}\,(\mathbf{ref}\, c, \mathbf{ref}\, c)$$

$$c.\mathbf{send}(v) \triangleq c \leftarrow \mathbf{send}\,(!\,c)\, v \qquad c.\mathbf{close}() \triangleq \mathbf{close}\,(!\,c); \mathbf{free}\, c$$

$$c.\mathbf{recv}() \triangleq \mathbf{let}\,(v, c') = \mathbf{recv}\ !\,c\,\mathbf{in}\, c \leftarrow c'; v \qquad c.\mathbf{wait}() \triangleq \mathbf{wait}\,(!\,c); \mathbf{free}\, c$$

$$c \overset{\mathsf{imp}}{\rightarrowtail} p \triangleq \exists (c' : \mathsf{Val}).\, c \mapsto c' * c' \rightarrowtail p$$

Session Channels Specifications

$$\mathbf{new}\,() \triangleq \mathbf{let}\, c = \mathbf{new_{fun}}\,()\, \mathbf{in}\, (\mathbf{ref}\, c, \mathbf{ref}\, c)$$

$$c.\mathbf{send}(v) \triangleq c \leftarrow \mathbf{send}\,(!\,c)\, v \qquad c.\mathbf{close}() \triangleq \mathbf{close}\,(!\,c);\, \mathbf{free}\, c$$

$$c.\mathbf{recv}() \triangleq \mathbf{let}\,(v, c') = \mathbf{recv}\; !\,c\; \mathbf{in}\; c \leftarrow c';\, v \qquad c.\mathbf{wait}() \triangleq \mathbf{wait}\,(!\,c);\, \mathbf{free}\, c$$

$$c \xrightarrowtail{\mathsf{imp}} p \triangleq \exists (c' : \mathsf{Val}).\, c \mapsto c' * c' \rightarrowtail p$$

Actris specifications:

$$\{\mathsf{True}\}\ \mathbf{new}\,()\ \{w.\, \exists c_1, c_2.\, w = (c_1, c_2) * c_1 \xrightarrowtail{\mathsf{imp}} p * c_2 \xrightarrowtail{\mathsf{imp}} \overline{p}\}$$

$$\{c \xrightarrowtail{\mathsf{imp}} (\mathbf{!}\,(x : \tau)\,\langle v \rangle \{P\}.\, p) * P\, t\}\ c.\mathbf{send}(v\, t)\ \{c \xrightarrowtail{\mathsf{imp}} p\, t\}$$

$$\{c \xrightarrowtail{\mathsf{imp}} (\mathbf{?}(x : \tau)\,\langle v \rangle \{P\}.\, p)\}\ c.\mathbf{recv}()\ \{w.\, \exists (x : \tau).\, w = (v\, x) * P\, x * c \xrightarrowtail{\mathsf{imp}} p\, x\}$$

$$\{c \rightarrowtail \mathbf{!end}\{P\} * P\}\ c.\mathbf{close}()\ \{\mathsf{True}\}$$

$$\{c \rightarrowtail \mathbf{?end}\{P\}\}\ c.\mathbf{wait}()\ \{P\}$$

Session Channel Example

$$
\begin{array}{l}
\mathbf{let}\,(c_1, c_2) = \mathbf{new}\,()\,\mathbf{in} \\
\left(\begin{array}{l}
\mathbf{let}\,\ell = \mathbf{ref}\,40\,\mathbf{in} \\
c_1.\mathbf{send}(\ell); \\
c_1.\mathbf{wait}(); \\
\mathbf{let}\,x = !\,\ell\,\mathbf{in}\,\mathbf{free}\,\ell; \\
\mathbf{assert}(x = 42)
\end{array}
\;\middle\|\;
\begin{array}{l}
\mathbf{let}\,\ell = c_2.\mathbf{recv}()\,\mathbf{in} \\
\ell \leftarrow (!\,\ell + 2); \\
c_2.\mathbf{close}() \\
\\
\\
\end{array}\right)
\end{array}
$$

Session Channel Example

$$\{\mathsf{True}\}$$

$$\begin{array}{l}
\textbf{let}\,(c_1, c_2) = \textbf{new}\,()\,\textbf{in} \\
\left(\begin{array}{l|l}
\textbf{let}\,\ell = \textbf{ref}\,40\,\textbf{in} & \textbf{let}\,\ell = c_2.\textbf{recv}()\,\textbf{in} \\
c_1.\textbf{send}(\ell); & \ell \leftarrow (!\,\ell + 2); \\
c_1.\textbf{wait}(); & c_2.\textbf{close}() \\
\textbf{let}\,x = !\,\ell\,\textbf{in}\,\textbf{free}\,\ell; & \\
\textbf{assert}(x = 42) &
\end{array}\right)
\end{array}$$

Session Channel Example

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathtt{let}\,(c_1, c_2) = \mathtt{new}\,()\,\mathtt{in} \\
\left(\begin{array}{l}
\mathtt{let}\,\ell = \mathtt{ref}\,40\,\mathtt{in} \\
c_1.\mathtt{send}(\ell); \\
c_1.\mathtt{wait}(); \\
\mathtt{let}\,x = !\,\ell\,\mathtt{in}\ \mathtt{free}\,\ell; \\
\mathtt{assert}(x = 42)
\end{array}
\middle\|
\begin{array}{l}
\mathtt{let}\,\ell = c_2.\mathtt{recv}()\,\mathtt{in} \\
\ell \leftarrow (!\,\ell + 2); \\
c_2.\mathtt{close}() \\
\\
\\
\end{array}\right)
\end{array}
$$

Protocol:

$$\mathtt{ses\_prot} \triangleq \mathbf{!}\,(\ell : \mathsf{Loc}, x : \mathbb{Z})\,\langle \ell \rangle \{\ell \mapsto x\}.\,\mathbf{?end}\{\ell \mapsto (x + 2)\}$$

Session Channel Example

$$\{\mathsf{True}\}$$
$$\mathbf{let}\,(c_1, c_2) = \mathbf{new}\,()\;\mathbf{in}$$
$$\{c_1 \xrightarrowtail{\text{imp}} \texttt{ses\_prot} * c_2 \xrightarrowtail{\text{imp}} \overline{\texttt{ses\_prot}}\}$$
$$\left(\begin{array}{l} \mathbf{let}\,\ell = \mathbf{ref}\,40\;\mathbf{in} \\ c_1.\mathbf{send}(\ell); \\ c_1.\mathbf{wait}(); \\ \mathbf{let}\,x = !\,\ell\;\mathbf{in}\;\mathbf{free}\,\ell; \\ \mathbf{assert}(x = 42) \end{array} \middle\| \begin{array}{l} \mathbf{let}\,\ell = c_2.\mathbf{recv}()\;\mathbf{in} \\ \ell \leftarrow (!\,\ell + 2); \\ c_2.\mathbf{close}() \\ \\ \\ \end{array}\right)$$

Protocol:

$$\texttt{ses\_prot} \triangleq \mathbf{!}\,(\ell : \mathsf{Loc}, x : \mathbb{Z})\,\langle \ell \rangle \{\ell \mapsto x\}.\,\mathbf{?end}\{\ell \mapsto (x + 2)\}$$

Session Channel Example

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\texttt{let}\,(c_1, c_2) = \texttt{new}\,()\ \texttt{in} \\
\{c_1 \overset{\mathsf{imp}}{\rightarrowtail} \texttt{ses\_prot} * c_2 \overset{\mathsf{imp}}{\rightarrowtail} \overline{\texttt{ses\_prot}}\} \\
\left(
\begin{array}{l}
\{c_1 \overset{\mathsf{imp}}{\rightarrowtail} \texttt{ses\_prot}\} \\
\texttt{let}\,\ell = \texttt{ref}\,40\ \texttt{in} \\
c_1.\texttt{send}(\ell); \\
c_1.\texttt{wait}(); \\
\texttt{let}\,x = !\,\ell\ \texttt{in}\ \texttt{free}\,\ell; \\
\texttt{assert}(x = 42)
\end{array}
\middle\|
\begin{array}{l}
\{c_2 \overset{\mathsf{imp}}{\rightarrowtail} \overline{\texttt{ses\_prot}}\} \\
\texttt{let}\,\ell = c_2.\texttt{recv}()\ \texttt{in} \\
\ell \leftarrow (!\,\ell + 2); \\
c_2.\texttt{close}() \\
\\
\\
\end{array}
\right)
\end{array}
$$

Protocol:

$$\texttt{ses\_prot} \triangleq \mathbf{!}\,(\ell : \mathsf{Loc}, x : \mathbb{Z})\,\langle \ell \rangle \{\ell \mapsto x\}.\,\mathbf{?end}\{\ell \mapsto (x+2)\}$$

Session Channel Example

$$\{\mathsf{True}\}$$
$$\mathbf{let}\,(c_1, c_2) = \mathbf{new}\,()\ \mathbf{in}$$
$$\{c_1 \overset{\mathsf{imp}}{\rightarrowtail} \mathtt{ses\_prot} * c_2 \overset{\mathsf{imp}}{\rightarrowtail} \overline{\mathtt{ses\_prot}}\}$$

$$\left(\begin{array}{l|l}
\{c_1 \overset{\mathsf{imp}}{\rightarrowtail} \mathtt{ses_prot}\} & \{c_2 \overset{\mathsf{imp}}{\rightarrowtail} \overline{\mathtt{ses_prot}}\} \\
\mathbf{let}\,\ell = \mathbf{ref}\,40\ \mathbf{in} & \mathbf{let}\,\ell = c_2.\mathbf{recv}()\ \mathbf{in} \\
\{c_1 \overset{\mathsf{imp}}{\rightarrowtail} \mathtt{ses_prot} * \ell \mapsto 40\} & \ell \leftarrow (!\,\ell + 2); \\
c_1.\mathbf{send}(\ell); & c_2.\mathbf{close}() \\
c_1.\mathbf{wait}(); & \\
\mathbf{let}\,x = !\,\ell\ \mathbf{in}\ \mathbf{free}\,\ell; & \\
\mathbf{assert}(x = 42) &
\end{array}\right)$$

Protocol:

$$\mathtt{ses\_prot} \triangleq\ !\,(\ell : \mathsf{Loc}, x : \mathbb{Z})\,\langle \ell \rangle \{\ell \mapsto x\}.\ \mathbf{?end}\{\ell \mapsto (x+2)\}$$

Session Channel Example

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathbf{let}\,(c_1, c_2) = \mathbf{new}\,()\,\mathbf{in} \\
\{c_1 \xrightarrow{\text{imp}} \mathtt{ses\_prot} * c_2 \xrightarrow{\text{imp}} \overline{\mathtt{ses\_prot}}\} \\
\left(\begin{array}{l|l}
\{c_1 \xrightarrow{\text{imp}} \mathtt{ses\_prot}\} & \{c_2 \xrightarrow{\text{imp}} \overline{\mathtt{ses\_prot}}\} \\
\mathbf{let}\,\ell = \mathbf{ref}\,40\,\mathbf{in} & \mathbf{let}\,\ell = c_2.\mathbf{recv}()\,\mathbf{in} \\
\{c_1 \xrightarrow{\text{imp}} \mathtt{ses\_prot} * \ell \mapsto 40\} & \ell \leftarrow (!\,\ell + 2); \\
c_1.\mathbf{send}(\ell); & c_2.\mathbf{close}() \\
\{c_1 \xrightarrow{\text{imp}} \mathbf{?end}\{\ell \mapsto 42\}\} & \\
c_1.\mathbf{wait}(); & \\
\mathbf{let}\,x = !\,\ell\,\mathbf{in}\,\mathbf{free}\,\ell; & \\
\mathbf{assert}(x = 42) &
\end{array}\right)
\end{array}
$$

Protocol:

$$\mathtt{ses\_prot} \triangleq \mathbf{!}\,(\ell : \mathsf{Loc}, x : \mathbb{Z})\,\langle \ell \rangle \{\ell \mapsto x\}.\,\mathbf{?end}\{\ell \mapsto (x + 2)\}$$

Session Channel Example

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathtt{let}\,(c_1, c_2) = \mathtt{new}\,()\ \mathtt{in} \\
\{c_1 \overset{\mathsf{imp}}{\rightarrowtail} \mathtt{ses\_prot} * c_2 \overset{\mathsf{imp}}{\rightarrowtail} \overline{\mathtt{ses\_prot}}\} \\
\left(\begin{array}{l||l}
\{c_1 \overset{\mathsf{imp}}{\rightarrowtail} \mathtt{ses\_prot}\} & \{c_2 \overset{\mathsf{imp}}{\rightarrowtail} \overline{\mathtt{ses\_prot}}\} \\
\mathtt{let}\ \ell = \mathtt{ref}\,40\ \mathtt{in} & \mathtt{let}\ \ell = c_2.\mathtt{recv}()\ \mathtt{in} \\
\{c_1 \overset{\mathsf{imp}}{\rightarrowtail} \mathtt{ses\_prot} * \ell \mapsto 40\} & \ell \leftarrow (!\,\ell + 2); \\
c_1.\mathtt{send}(\ell); & c_2.\mathtt{close}() \\
\{c_1 \overset{\mathsf{imp}}{\rightarrowtail} \mathbf{?end}\{\ell \mapsto 42\}\} & \\
c_1.\mathtt{wait}(); & \\
\{\ell \mapsto 42\} & \\
\mathtt{let}\ x = !\,\ell\ \mathtt{in}\ \mathtt{free}\ \ell; & \\
\mathtt{assert}(x = 42) &
\end{array}\right)
\end{array}
$$

Protocol:

$$\mathtt{ses\_prot} \triangleq \mathbf{!}\,(\ell : \mathsf{Loc}, x : \mathbb{Z})\,\langle \ell \rangle \{\ell \mapsto x\}.\,\mathbf{?end}\{\ell \mapsto (x + 2)\}$$

Session Channel Example

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathtt{let}\,(c_1, c_2) = \mathtt{new}\,()\,\mathtt{in} \\
\{c_1 \overset{\mathsf{imp}}{\rightarrowtail} \mathtt{ses\_prot} * c_2 \overset{\mathsf{imp}}{\rightarrowtail} \overline{\mathtt{ses\_prot}}\} \\
\left(\begin{array}{l|l}
\{c_1 \overset{\mathsf{imp}}{\rightarrowtail} \mathtt{ses\_prot}\} & \{c_2 \overset{\mathsf{imp}}{\rightarrowtail} \overline{\mathtt{ses\_prot}}\} \\
\mathtt{let}\,\ell = \mathtt{ref}\,40\,\mathtt{in} & \mathtt{let}\,\ell = c_2.\mathtt{recv}()\,\mathtt{in} \\
\{c_1 \overset{\mathsf{imp}}{\rightarrowtail} \mathtt{ses\_prot} * \ell \mapsto 40\} & \ell \leftarrow (!\,\ell + 2); \\
c_1.\mathtt{send}(\ell); & c_2.\mathtt{close}() \\
\{c_1 \overset{\mathsf{imp}}{\rightarrowtail} \mathbf{?end}\{\ell \mapsto 42\}\} & \\
c_1.\mathtt{wait}(); & \\
\{\ell \mapsto 42\} & \\
\mathtt{let}\,x = !\,\ell\,\mathtt{in}\,\mathtt{free}\,\ell; & \\
\{x = 42\} & \\
\mathtt{assert}(x = 42) &
\end{array}\right)
\end{array}
$$

Protocol:

$$\mathtt{ses\_prot} \triangleq \mathbf{!}\,(\ell : \mathsf{Loc}, x : \mathbb{Z})\,\langle \ell \rangle \{\ell \mapsto x\}.\,\mathbf{?end}\{\ell \mapsto (x + 2)\}$$

Session Channel Example

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathtt{let}\,(c_1, c_2) = \mathtt{new}\,()\,\mathtt{in} \\
\{c_1 \overset{\mathsf{imp}}{\rightarrowtail} \mathtt{ses\_prot} * c_2 \overset{\mathsf{imp}}{\rightarrowtail} \overline{\mathtt{ses\_prot}}\} \\
\left(\begin{array}{l}
\{c_1 \overset{\mathsf{imp}}{\rightarrowtail} \mathtt{ses\_prot}\} \\
\mathtt{let}\,\ell = \mathtt{ref}\,40\,\mathtt{in} \\
\{c_1 \overset{\mathsf{imp}}{\rightarrowtail} \mathtt{ses\_prot} * \ell \mapsto 40\} \\
c_1.\mathtt{send}(\ell); \\
\{c_1 \overset{\mathsf{imp}}{\rightarrowtail} \mathbf{?end}\{\ell \mapsto 42\}\} \\
c_1.\mathtt{wait}(); \\
\{\ell \mapsto 42\} \\
\mathtt{let}\,x = !\,\ell\,\mathtt{in}\ \mathtt{free}\,\ell; \\
\{x = 42\} \\
\mathtt{assert}(x = 42) \\
\{\mathsf{True}\}
\end{array}
\;\middle\|\;
\begin{array}{l}
\{c_2 \overset{\mathsf{imp}}{\rightarrowtail} \overline{\mathtt{ses\_prot}}\} \\
\mathtt{let}\,\ell = c_2.\mathtt{recv}()\,\mathtt{in} \\
\ell \leftarrow (!\,\ell + 2); \\
c_2.\mathtt{close}() \\
\\ \\ \\ \\ \\ \\ \\
\end{array}\right)
\end{array}
$$

Protocol:

$$\mathtt{ses\_prot} \triangleq \mathbf{!}\,(\ell : \mathsf{Loc}, x : \mathbb{Z})\,\langle \ell \rangle \{\ell \mapsto x\}.\,\mathbf{?end}\{\ell \mapsto (x + 2)\}$$

Session Channel Example

$$\{\mathsf{True}\}$$
$$\mathbf{let}\,(c_1, c_2) = \mathbf{new}\,()\,\mathbf{in}$$
$$\{c_1 \xrightarrow{\text{imp}} \mathtt{ses\_prot} * c_2 \xrightarrow{\text{imp}} \overline{\mathtt{ses\_prot}}\}$$

$$\left(\begin{array}{l} \{c_1 \xrightarrow{\text{imp}} \mathtt{ses\_prot}\} \\ \mathbf{let}\,\ell = \mathbf{ref}\,40\,\mathbf{in} \\ \{c_1 \xrightarrow{\text{imp}} \mathtt{ses\_prot} * \ell \mapsto 40\} \\ c_1.\mathbf{send}(\ell); \\ \{c_1 \xrightarrow{\text{imp}} \mathbf{?end}\{\ell \mapsto 42\}\} \\ c_1.\mathbf{wait}(); \\ \{\ell \mapsto 42\} \\ \mathbf{let}\,x = !\,\ell\,\mathbf{in}\ \mathbf{free}\,\ell; \\ \{x = 42\} \\ \mathbf{assert}(x = 42) \\ \{\mathsf{True}\} \end{array} \middle\| \begin{array}{l} \{c_2 \xrightarrow{\text{imp}} \overline{\mathtt{ses\_prot}}\} \\ \mathbf{let}\,\ell = c_2.\mathbf{recv}()\,\mathbf{in} \\ \{c_2 \xrightarrow{\text{imp}} \mathbf{!end}\{\ell \mapsto (x + 2)\} * \ell \mapsto x\} \\ \ell \leftarrow (!\,\ell + 2); \\ c_2.\mathbf{close}() \\ \\ \\ \\ \\ \\ \\ \end{array}\right)$$

Protocol:

$$\mathtt{ses\_prot} \triangleq \mathbf{!}\,(\ell : \mathsf{Loc}, x : \mathbb{Z})\,\langle \ell \rangle \{\ell \mapsto x\}.\,\mathbf{?end}\{\ell \mapsto (x + 2)\}$$

Session Channel Example

$$
\begin{array}{l}
\{\mathsf{True}\} \\
\mathbf{let}\,(c_1, c_2) = \mathbf{new}\,()\,\mathbf{in} \\
\{c_1 \xrightarrowtail{\mathsf{imp}} \mathtt{ses\_prot} * c_2 \xrightarrowtail{\mathsf{imp}} \overline{\mathtt{ses\_prot}}\} \\
\left(\begin{array}{l||l}
\{c_1 \xrightarrowtail{\mathsf{imp}} \mathtt{ses\_prot}\} & \{c_2 \xrightarrowtail{\mathsf{imp}} \overline{\mathtt{ses\_prot}}\} \\
\mathbf{let}\,\ell = \mathbf{ref}\,40\,\mathbf{in} & \mathbf{let}\,\ell = c_2.\mathbf{recv}()\,\mathbf{in} \\
\{c_1 \xrightarrowtail{\mathsf{imp}} \mathtt{ses\_prot} * \ell \mapsto 40\} & \{c_2 \xrightarrowtail{\mathsf{imp}} \mathbf{!\,end}\{\ell \mapsto (x+2)\} * \ell \mapsto x\} \\
c_1.\mathbf{send}(\ell); & \ell \leftarrow (!\,\ell + 2); \\
\{c_1 \xrightarrowtail{\mathsf{imp}} \mathbf{?end}\{\ell \mapsto 42\}\} & \{c_2 \xrightarrowtail{\mathsf{imp}} \mathbf{!\,end}\{\ell \mapsto (x+2)\} * \ell \mapsto (x+2)\} \\
c_1.\mathbf{wait}(); & c_2.\mathbf{close}() \\
\{\ell \mapsto 42\} & \\
\mathbf{let}\,x = !\,\ell\,\mathbf{in}\ \mathbf{free}\,\ell; & \\
\{x = 42\} & \\
\mathbf{assert}(x = 42) & \\
\{\mathsf{True}\} &
\end{array}\right)
\end{array}
$$

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$$\mathtt{ses\_prot} \triangleq \mathbf{!}\,(\ell : \mathsf{Loc}, x : \mathbb{Z})\,\langle \ell \rangle \{\ell \mapsto x\}.\,\mathbf{?end}\{\ell \mapsto (x+2)\}$$

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\{c_1 \overset{\text{imp}}{\rightarrowtail} \mathtt{ses\_prot} * \ell \mapsto 40\} & \{c_2 \overset{\text{imp}}{\rightarrowtail} \mathbf{!\,end}\{\ell \mapsto (x+2)\} * \ell \mapsto x\}\\
c_1.\mathtt{send}(\ell); & \ell \leftarrow (!\,\ell + 2);\\
\{c_1 \overset{\text{imp}}{\rightarrowtail} \mathbf{?end}\{\ell \mapsto 42\}\} & \{c_2 \overset{\text{imp}}{\rightarrowtail} \mathbf{!\,end}\{\ell \mapsto (x+2)\} * \ell \mapsto (x+2)\}\\
c_1.\mathtt{wait}(); & c_2.\mathtt{close}()\\
\{\ell \mapsto 42\} & \{\mathsf{True}\}\\
\mathtt{let}\,x = !\,\ell\,\mathtt{in}\,\mathtt{free}\,\ell; & \\
\{x = 42\} & \\
\mathtt{assert}(x = 42) & \\
\{\mathsf{True}\} &
\end{array}\right)
\end{array}
$$

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\end{array}
\middle\|
\begin{array}{l}
\{c_2 \overset{\text{imp}}{\rightarrowtail} \overline{\mathtt{ses\_prot}}\}\\
\mathbf{let}\,\ell = c_2.\mathbf{recv}()\,\mathbf{in}\\
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\\ \\ \\ \\
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$$

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Break (30 min!)
We start again at 16:00!

If you attend the Coq hacking session please pull
`https://gitlab.mpi-sws.org/iris/tutorial-popl24`
and follow the installation instructions!

Iris invariants and ghost state

Tutorial Timeline

Part 1: 14:00 – 15:30

- ~~Introduction (10 min)~~
- ~~Layered implementation of session channels (10 min)~~
- ~~Basic concurrent separation logic and one-shot protocols (30 min)~~
- ~~**Break** (10 min)~~
- ~~Dependent separation protocols (30 min)~~

~~**Break** (30 min)~~

Part 2: 16:00 – 17:30

- Iris invariants and ghost state (30 min)
- **Break** (10 min)
- Supervised Coq hacking (50 min)

Overview of Abstraction Layers

Layer	Reasoning principles / specifications
#1 Iris's HeapLang	Basic concurrent separation logic Iris invariants and ghost state
#2 One-shot channels	One-shot protocols
#3 Functional session channels	Dependent separation protocols
#4 Session channels	Dependent separation protocols

One-Shot Channels Recap

One-shot channel implementations:

$$\begin{aligned}
\texttt{new1}\,() &\triangleq \texttt{ref None} \\
\texttt{send1}\, c\, v &\triangleq c \leftarrow \texttt{Some}\, v \\
\texttt{recv1}\, c &\triangleq \texttt{let}\, x = {!c}\, \texttt{in} \\
&\quad\ \texttt{match}\, x\, \texttt{with} \\
&\quad\quad\ \texttt{None} \Rightarrow \texttt{recv1}\, c \\
&\quad\ \mid \texttt{Some}\, v \Rightarrow \texttt{free}\, c;\ v \\
&\quad\ \texttt{end}
\end{aligned}$$

One-shot channel specifications:

$$\{\mathsf{True}\}\ \texttt{new1}\,()\ \{w.\, \exists c.\, w = c * c \rightarrowtail p * c \rightarrowtail \overline{p}\}$$

$$\{c \rightarrowtail (\mathsf{Send}, \Phi) * \Phi\, v\}\ \texttt{send1}\, c\, v\ \{\mathsf{True}\}$$

$$\{c \rightarrowtail (\mathsf{Recv}, \Phi)\}\ \texttt{recv1}\, c\ \{w.\, \Phi\, w\}$$

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$$

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$$
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\{\mathsf{True}\}\ &\mathtt{new1}\,()\ \{w.\,\exists c.\, w = c * c \rightarrowtail p * c \rightarrowtail \overline{p}\} \\
\{c \rightarrowtail (\mathsf{Send}, \Phi) * \Phi\, v\}\ &\mathtt{send1}\,c\,v\ \{\mathsf{True}\} \\
\{c \rightarrowtail (\mathsf{Recv}, \Phi)\}\ &\mathtt{recv1}\,c\ \{w.\,\Phi\, w\}
\end{aligned}
$$

Crux: Definition of $c \rightarrowtail p$

Definition of One-Shot Channel Resource

One-shot channel ownership defined using standard Iris methodology

$$c \rightarrowtail (tag, \Phi) \triangleq \ldots$$

Definition of One-Shot Channel Resource

One-shot channel ownership defined using standard Iris methodology:

1. Model abstraction as a state transition system (STS)

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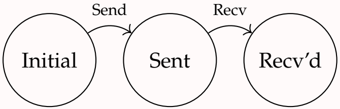

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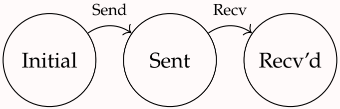
Definition of One-Shot Channel Resource

One-shot channel ownership defined using standard Iris methodology:

1. Model abstraction as a state transition system (STS)
2. Define an invariant as a disjunction of the states



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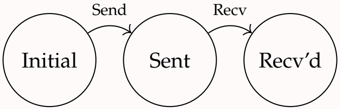

$$\mathsf{chan\_inv} \triangleq (\underbrace{\qquad\qquad}_{\text{(1) initial state}}) \lor (\underbrace{\qquad\qquad\qquad\qquad\qquad\qquad}_{\text{(2) message sent, but not yet received}}) \lor (\underbrace{\qquad\qquad\quad}_{\text{(3) final state}})$$

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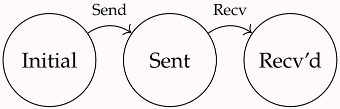
Definition of One-Shot Channel Resource

One-shot channel ownership defined using standard Iris methodology:

1. Model abstraction as a state transition system (STS)
2. Define an invariant as a disjunction of the states
3. Determine resource ownership of each state



$$\mathsf{chan\_inv} \triangleq (\underbrace{\qquad\qquad}_{\text{(1) initial state}}) \vee (\underbrace{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}_{\text{(2) message sent, but not yet received}}) \vee (\underbrace{\qquad\qquad\qquad}_{\text{(3) final state}})$$

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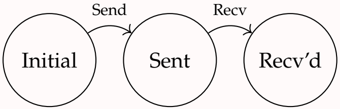

$$\mathsf{chan\_inv}\quad c \quad \triangleq \Big(\underbrace{c \mapsto \texttt{None}}_{\text{(1) initial state}}\Big) \lor \Big(\underbrace{\exists v.\, c \mapsto \texttt{Some}\, v \qquad\qquad}_{\text{(2) message sent, but not yet received}}\Big) \lor \Big(\underbrace{\qquad\qquad}_{\text{(3) final state}}\Big)$$

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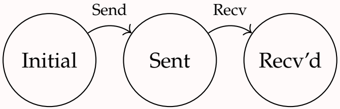

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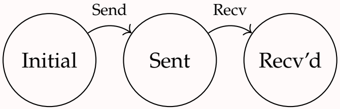
One-shot channel ownership defined using standard Iris methodology:

1. Model abstraction as a state transition system (STS)
2. Define an invariant as a disjunction of the states
3. Determine resource ownership of each state
4. Encode STS transition permissions with ghost state



$$\text{chan\_inv} \quad c\ \Phi \triangleq (\underbrace{c \mapsto \texttt{None}}_{\text{(1) initial state}}) \vee (\underbrace{\exists v.\, c \mapsto \texttt{Some}\, v * \Phi\, v \qquad}_{\text{(2) message sent, but not yet received}}) \vee (\underbrace{\qquad\qquad}_{\text{(3) final state}})$$

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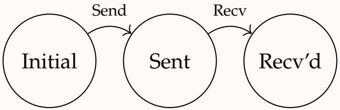

$$\mathsf{chan\_inv}\ \gamma_s \quad c\ \Phi \triangleq (\underbrace{c \mapsto \mathtt{None}}_{\text{(1) initial state}}) \lor (\underbrace{\exists v.\, c \mapsto \mathtt{Some}\, v * \Phi\ v * \mathsf{tok}\ \gamma_s}_{\text{(2) message sent, but not yet received}}) \lor (\underbrace{\qquad\qquad}_{\text{(3) final state}})$$

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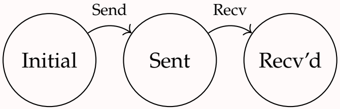

$$\mathsf{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi \triangleq (\underbrace{c \mapsto \texttt{None}}_{\text{(1) initial state}}) \vee (\underbrace{\exists v.\, c \mapsto \texttt{Some}\, v * \Phi\, v * \mathsf{tok}\ \gamma_s}_{\text{(2) message sent, but not yet received}}) \vee (\underbrace{\mathsf{tok}\ \gamma_s * \mathsf{tok}\ \gamma_r}_{\text{(3) final state}})$$

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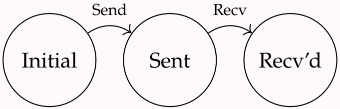

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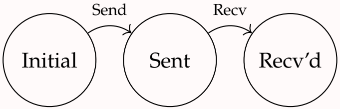

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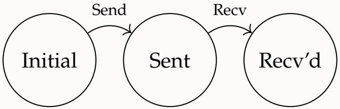

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$$c \rightarrowtail (tag, \Phi) \triangleq \exists \gamma_s, \gamma_r.\ \boxed{\mathsf{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi} * \begin{cases} \mathsf{tok}\ \gamma_s & \text{if } tag = \mathsf{Send} \\ \mathsf{tok}\ \gamma_r & \text{if } tag = \mathsf{Recv} \end{cases}$$

Invariants

The invariant assertion $\boxed{R}$ expresses that R is maintained as an invariant on the state

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Invariant opening:

$$\frac{\{R * P\}\; e\; \{R * Q\} \qquad e \text{ atomic}}{\left\{\boxed{R} * P\right\}\; e\; \left\{\boxed{R} * Q\right\}}$$

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Invariant duplication: $\boxed{R} \vdash \boxed{R} * \boxed{R}$

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The invariant assertion $\boxed{R}^{\mathcal{N}}$ expresses that R is maintained as an invariant on the state

Invariant opening:

$$\frac{\{R * P\}\ e\ \{R * Q\}_{\mathcal{E}} \qquad e \text{ atomic}}{\left\{\boxed{R}^{\mathcal{N}} * P\right\}\ e\ \left\{\boxed{R}^{\mathcal{N}} * Q\right\}_{\mathcal{E} \uplus \mathcal{N}}}$$

Invariant allocation:

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Technicalities: names prevent opening the same invariant twice

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Invariant duplication: $\boxed{R}^{\mathcal{N}} \vdash \boxed{R}^{\mathcal{N}} * \boxed{R}^{\mathcal{N}}$

Technicalities: names prevent opening the same invariant twice and the later $\triangleright$ is needed for impredicativity, i.e., $\boxed{\ldots \boxed{R}^{\mathcal{N}_2} \ldots}^{\mathcal{N}_1}$

Ghost Tokens

Consider the invariant:

$$(\underbrace{c \mapsto \mathtt{None}}_{\text{(1) initial state}}) \vee (\underbrace{\exists v.\, c \mapsto \mathtt{Some}\, v * \Phi\, v \ldots}_{\text{(2) message sent, but not yet received}}) \vee (\underbrace{\ldots}_{\text{(3) final state}})$$

How to determine which state the one-shot channel is in?

Ghost Tokens

Consider the invariant:

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Ghost tokens allow deriving contradictions:

$$\mathsf{True} \Rrightarrow \exists\gamma.\, \mathsf{tok}\,\gamma \qquad \mathsf{tok}\,\gamma * \mathsf{tok}\,\gamma \vdash \mathsf{False}$$

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Consider the invariant:

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Ghost tokens allow deriving contradictions:

$$\mathsf{True} \Rrightarrow \exists \gamma.\, \mathsf{tok}\, \gamma \qquad \mathsf{tok}\, \gamma * \mathsf{tok}\, \gamma \vdash \mathsf{False}$$

You can exclude cases which would end up in duplicate tokens:

$$\frac{((c \mapsto \texttt{None}) \lor (\exists v.\, c \mapsto \texttt{Some}\, v * \Phi\, v * \mathsf{tok}\, \gamma_s) \lor (\mathsf{tok}\, \gamma_s * \mathsf{tok}\, \gamma_r)) * \mathsf{tok}\, \gamma_s}{c \mapsto \texttt{None} * \mathsf{tok}\, \gamma_s}$$

Proof of New

$$\mathsf{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi \triangleq (\underbrace{c \mapsto \mathtt{None}}_{\text{(1) initial state}}) \vee (\underbrace{\exists v.\, c \mapsto \mathtt{Some}\, v * \Phi\, v * \mathsf{tok}\ \gamma_s}_{\text{(2) message sent, but not yet received}}) \vee (\underbrace{\mathsf{tok}\ \gamma_s * \mathsf{tok}\ \gamma_r}_{\text{(3) final state}})$$

$$c \rightarrowtail (tag, \Phi) \triangleq \exists \gamma_s, \gamma_r.\, \boxed{\mathsf{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi} * \begin{cases} \mathsf{tok}\ \gamma_s & \text{if } tag = \mathsf{Send} \\ \mathsf{tok}\ \gamma_r & \text{if } tag = \mathsf{Recv} \end{cases}$$

`ref None`

Proof of New

$$\mathsf{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi \triangleq (\underbrace{c \mapsto \mathtt{None}}_{\text{(1) initial state}}) \vee (\underbrace{\exists v.\, c \mapsto \mathtt{Some}\, v * \Phi\ v * \mathsf{tok}\ \gamma_s}_{\text{(2) message sent, but not yet received}}) \vee (\underbrace{\mathsf{tok}\ \gamma_s * \mathsf{tok}\ \gamma_r}_{\text{(3) final state}})$$

$$c \rightarrowtail (tag, \Phi) \triangleq \exists \gamma_s, \gamma_r.\, \boxed{\mathsf{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi} * \begin{cases} \mathsf{tok}\ \gamma_s & \text{if } tag = \mathsf{Send} \\ \mathsf{tok}\ \gamma_r & \text{if } tag = \mathsf{Recv} \end{cases}$$

$\{\mathsf{True}\}$
ref None
$\{w.\, \exists c.\, w = c * c \rightarrowtail p * c \rightarrowtail \overline{p}\}$

Proof of New

$$\mathsf{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi \triangleq (\underbrace{c \mapsto \mathbf{None}}_{\text{(1) initial state}}) \vee (\underbrace{\exists v.\, c \mapsto \mathbf{Some}\, v * \Phi\, v * \mathsf{tok}\ \gamma_s}_{\text{(2) message sent, but not yet received}}) \vee (\underbrace{\mathsf{tok}\ \gamma_s * \mathsf{tok}\ \gamma_r}_{\text{(3) final state}})$$

$$c \rightarrowtail (tag, \Phi) \triangleq \exists \gamma_s, \gamma_r.\, \boxed{\mathsf{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi} * \begin{cases} \mathsf{tok}\ \gamma_s & \text{if } tag = \mathsf{Send} \\ \mathsf{tok}\ \gamma_r & \text{if } tag = \mathsf{Recv} \end{cases}$$

$\{\mathsf{True}\}$
$\mathbf{ref}\,\mathbf{None}$
$\{w.\, \exists c.\, w = c * c \mapsto \mathbf{None}\}$
$\{w.\, \exists c.\, w = c * c \rightarrowtail p * c \rightarrowtail \overline{p}\}$

Proof of New

$$\mathsf{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi \triangleq (\underbrace{c \mapsto \texttt{None}}_{\text{(1) initial state}}) \vee (\underbrace{\exists v.\, c \mapsto \texttt{Some}\, v * \Phi\ v * \mathsf{tok}\ \gamma_s}_{\text{(2) message sent, but not yet received}}) \vee (\underbrace{\mathsf{tok}\ \gamma_s * \mathsf{tok}\ \gamma_r}_{\text{(3) final state}})$$

$$c \rightarrowtail (tag, \Phi) \triangleq \exists \gamma_s, \gamma_r.\, \boxed{\mathsf{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi} * \begin{cases} \mathsf{tok}\ \gamma_s & \text{if } tag = \mathsf{Send} \\ \mathsf{tok}\ \gamma_r & \text{if } tag = \mathsf{Recv} \end{cases}$$

$\{\mathsf{True}\}$

ref None

$\{w.\, \exists c.\, w = c * c \mapsto \texttt{None}\}$

$\{w.\, \exists c.\, w = c * c \mapsto \texttt{None} * \mathsf{tok}\ \gamma_s * \mathsf{tok}\ \gamma_r\}$

$\{w.\, \exists c.\, w = c * c \rightarrowtail p * c \rightarrowtail \overline{p}\}$

Proof of New

$$\mathsf{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi \triangleq (\underbrace{c \mapsto \mathtt{None}}_{\text{(1) initial state}}) \vee (\underbrace{\exists v.\, c \mapsto \mathtt{Some}\ v * \Phi\ v * \mathsf{tok}\ \gamma_s}_{\text{(2) message sent, but not yet received}}) \vee (\underbrace{\mathsf{tok}\ \gamma_s * \mathsf{tok}\ \gamma_r}_{\text{(3) final state}})$$

$$c \rightarrowtail (tag, \Phi) \triangleq \exists \gamma_s, \gamma_r.\ \boxed{\mathsf{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi} * \begin{cases} \mathsf{tok}\ \gamma_s & \text{if } tag = \mathsf{Send} \\ \mathsf{tok}\ \gamma_r & \text{if } tag = \mathsf{Recv} \end{cases}$$

$\{\mathsf{True}\}$

ref None

$\{w.\, \exists c.\, w = c * c \mapsto \mathtt{None}\}$

$\{w.\, \exists c.\, w = c * c \mapsto \mathtt{None} * \mathsf{tok}\ \gamma_s * \mathsf{tok}\ \gamma_r\}$

$\{w.\, \exists c.\, w = c * \boxed{\mathtt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi} * \mathsf{tok}\ \gamma_s * \mathsf{tok}\ \gamma_r\}$ $\quad // p = (tag, \Phi)$

$\{w.\, \exists c.\, w = c * c \rightarrowtail p * c \rightarrowtail \overline{p}\}$

Proof of New

$$\mathsf{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi \triangleq (\underbrace{c \mapsto \texttt{None}}_{\text{(1) initial state}}) \vee (\underbrace{\exists v.\, c \mapsto \texttt{Some}\ v * \Phi\ v * \mathsf{tok}\ \gamma_s}_{\text{(2) message sent, but not yet received}}) \vee (\underbrace{\mathsf{tok}\ \gamma_s * \mathsf{tok}\ \gamma_r}_{\text{(3) final state}})$$

$$c \rightarrowtail (tag, \Phi) \triangleq \exists \gamma_s, \gamma_r.\ \boxed{\mathsf{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi} * \begin{cases} \mathsf{tok}\ \gamma_s & \text{if } tag = \mathsf{Send} \\ \mathsf{tok}\ \gamma_r & \text{if } tag = \mathsf{Recv} \end{cases}$$

$\{\mathsf{True}\}$

$\texttt{ref None}$

$\{w.\ \exists c.\ w = c * c \mapsto \texttt{None}\}$

$\{w.\ \exists c.\ w = c * c \mapsto \texttt{None} * \mathsf{tok}\ \gamma_s * \mathsf{tok}\ \gamma_r\}$

$\{w.\ \exists c.\ w = c * \boxed{\mathtt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi} * \mathsf{tok}\ \gamma_s * \mathsf{tok}\ \gamma_r\}$ $\quad // p = (tag, \Phi)$

$\{w.\ \exists c.\ w = c * \boxed{\mathtt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi} * \mathsf{tok}\ \gamma_s * \boxed{\mathtt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi} * \mathsf{tok}\ \gamma_r\}$

$\{w.\ \exists c.\ w = c * c \rightarrowtail p * c \rightarrowtail \overline{p}\}$

Proof of New

$$\mathsf{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi \triangleq (\underbrace{c \mapsto \mathtt{None}}_{\text{(1) initial state}}) \vee (\underbrace{\exists v.\, c \mapsto \mathtt{Some}\, v * \Phi\ v * \mathsf{tok}\ \gamma_s}_{\text{(2) message sent, but not yet received}}) \vee (\underbrace{\mathsf{tok}\ \gamma_s * \mathsf{tok}\ \gamma_r}_{\text{(3) final state}})$$

$$c \rightarrowtail (tag, \Phi) \triangleq \exists \gamma_s, \gamma_r.\, \boxed{\mathsf{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi} * \begin{cases} \mathsf{tok}\ \gamma_s & \text{if } tag = \mathsf{Send} \\ \mathsf{tok}\ \gamma_r & \text{if } tag = \mathsf{Recv} \end{cases}$$

$\{\mathsf{True}\}$

$\mathtt{ref\,None}$

$\{w.\, \exists c.\, w = c * c \mapsto \mathtt{None}\}$

$\{w.\, \exists c.\, w = c * c \mapsto \mathtt{None} * \mathsf{tok}\ \gamma_s * \mathsf{tok}\ \gamma_r\}$

$\{w.\, \exists c.\, w = c * \boxed{\mathtt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi} * \mathsf{tok}\ \gamma_s * \mathsf{tok}\ \gamma_r\}$ $\quad // p = (tag, \Phi)$

$\{w.\, \exists c.\, w = c * \boxed{\mathtt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi} * \mathsf{tok}\ \gamma_s * \boxed{\mathtt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi} * \mathsf{tok}\ \gamma_r\}$

$\{w.\, \exists c.\, w = c * c \rightarrowtail (\mathsf{Send}, \Phi) * c \rightarrowtail (\mathsf{Recv}, \Phi)\}$

$\{w.\, \exists c.\, w = c * c \rightarrowtail p * c \rightarrowtail \overline{p}\}$

Proof of Send

$$\mathsf{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi \triangleq (\underbrace{c \mapsto \mathtt{None}}_{\text{(1) initial state}}) \lor (\underbrace{\exists v.\, c \mapsto \mathtt{Some}\, v * \Phi\, v * \mathsf{tok}\, \gamma_s}_{\text{(2) message sent, but not yet received}}) \lor (\underbrace{\mathsf{tok}\, \gamma_s * \mathsf{tok}\, \gamma_r}_{\text{(3) final state}})$$

$$c \rightarrowtail (\mathit{tag}, \Phi) \triangleq \exists \gamma_s, \gamma_r.\, \boxed{\mathsf{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi} * \begin{cases} \mathsf{tok}\, \gamma_s & \text{if } \mathit{tag} = \mathsf{Send} \\ \mathsf{tok}\, \gamma_r & \text{if } \mathit{tag} = \mathsf{Recv} \end{cases}$$

$c \leftarrow \mathtt{Some}\ v$

Proof of Send

$$\mathsf{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi \triangleq (\underbrace{c \mapsto \mathtt{None}}_{\text{(1) initial state}}) \vee (\underbrace{\exists v.\, c \mapsto \mathtt{Some}\, v * \Phi\ v * \mathsf{tok}\ \gamma_s}_{\text{(2) message sent, but not yet received}}) \vee (\underbrace{\mathsf{tok}\ \gamma_s * \mathsf{tok}\ \gamma_r}_{\text{(3) final state}})$$

$$c \rightarrowtail (tag, \Phi) \triangleq \exists \gamma_s, \gamma_r.\, \boxed{\mathsf{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi} * \begin{cases} \mathsf{tok}\ \gamma_s & \text{if } tag = \mathsf{Send} \\ \mathsf{tok}\ \gamma_r & \text{if } tag = \mathsf{Recv} \end{cases}$$

$\{c \rightarrowtail (\mathsf{Send}, \Phi) * \Phi\ v\}$

$c \leftarrow \mathtt{Some}\ v$

$\{\mathsf{True}\}$

Proof of Send

$$\mathsf{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi \triangleq (\underbrace{c \mapsto \mathtt{None}}_{\text{(1) initial state}}) \lor (\underbrace{\exists v.\, c \mapsto \mathtt{Some}\, v * \Phi\ v * \mathsf{tok}\ \gamma_s}_{\text{(2) message sent, but not yet received}}) \lor (\underbrace{\mathsf{tok}\ \gamma_s * \mathsf{tok}\ \gamma_r}_{\text{(3) final state}})$$

$$c \rightarrowtail (tag, \Phi) \triangleq \exists \gamma_s, \gamma_r.\, \boxed{\mathsf{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi} * \begin{cases} \mathsf{tok}\ \gamma_s & \text{if } tag = \mathsf{Send} \\ \mathsf{tok}\ \gamma_r & \text{if } tag = \mathsf{Recv} \end{cases}$$

$\{c \rightarrowtail (\mathsf{Send}, \Phi) * \Phi\ v\}$
$\{\boxed{\mathtt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi} * \mathsf{tok}\ \gamma_s * \Phi\ v\}$
$c \leftarrow \mathtt{Some}\ v$
$\{\mathsf{True}\}$

Proof of Send

$$\mathsf{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi \triangleq (\underbrace{c \mapsto \mathbf{None}}_{\text{(1) initial state}}) \vee (\underbrace{\exists v.\, c \mapsto \mathbf{Some}\, v * \Phi\ v * \mathsf{tok}\ \gamma_s}_{\text{(2) message sent, but not yet received}}) \vee (\underbrace{\mathsf{tok}\ \gamma_s * \mathsf{tok}\ \gamma_r}_{\text{(3) final state}})$$

$$c \rightarrowtail (tag, \Phi) \triangleq \exists \gamma_s, \gamma_r.\, \boxed{\mathsf{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi} * \begin{cases} \mathsf{tok}\ \gamma_s & \text{if } tag = \mathsf{Send} \\ \mathsf{tok}\ \gamma_r & \text{if } tag = \mathsf{Recv} \end{cases}$$

$\{c \rightarrowtail (\mathsf{Send}, \Phi) * \Phi\ v\}$

$\{\boxed{\mathtt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi} * \mathsf{tok}\ \gamma_s * \Phi\ v\}$

$\quad\{\mathtt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * \mathsf{tok}\ \gamma_s * \Phi\ v\}$

$c \leftarrow \mathbf{Some}\ v$

$\{\mathsf{True}\}$

Proof of Send

$$\mathsf{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi \triangleq (\underbrace{c \mapsto \texttt{None}}_{\text{(1) initial state}}) \vee (\underbrace{\exists v.\, c \mapsto \texttt{Some}\ v * \Phi\ v * \mathsf{tok}\ \gamma_s}_{\text{(2) message sent, but not yet received}}) \vee (\underbrace{\mathsf{tok}\ \gamma_s * \mathsf{tok}\ \gamma_r}_{\text{(3) final state}})$$

$$c \rightarrowtail (tag, \Phi) \triangleq \exists \gamma_s, \gamma_r.\ \boxed{\mathsf{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi} * \begin{cases} \mathsf{tok}\ \gamma_s & \text{if } tag = \mathsf{Send} \\ \mathsf{tok}\ \gamma_r & \text{if } tag = \mathsf{Recv} \end{cases}$$

$\{c \rightarrowtail (\mathsf{Send}, \Phi) * \Phi\ v\}$

$\{\boxed{\texttt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi} * \mathsf{tok}\ \gamma_s * \Phi\ v\}$

$\quad \{\texttt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * \mathsf{tok}\ \gamma_s * \Phi\ v\}$

$\quad \{c \mapsto \texttt{None} * \mathsf{tok}\ \gamma_s * \Phi\ v\}$

$c \leftarrow \texttt{Some}\ v$

$\{\mathsf{True}\}$

Proof of Send

$$\mathsf{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi \triangleq (\underbrace{c \mapsto \mathtt{None}}_{\text{(1) initial state}}) \vee (\underbrace{\exists v.\, c \mapsto \mathtt{Some}\, v * \Phi\ v * \mathsf{tok}\ \gamma_s}_{\text{(2) message sent, but not yet received}}) \vee (\underbrace{\mathsf{tok}\ \gamma_s * \mathsf{tok}\ \gamma_r}_{\text{(3) final state}})$$

$$c \rightarrowtail (tag, \Phi) \triangleq \exists \gamma_s, \gamma_r.\ \boxed{\mathsf{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi} * \begin{cases} \mathsf{tok}\ \gamma_s & \text{if } tag = \mathsf{Send} \\ \mathsf{tok}\ \gamma_r & \text{if } tag = \mathsf{Recv} \end{cases}$$

$\{c \rightarrowtail (\mathsf{Send}, \Phi) * \Phi\ v\}$

$\{\boxed{\mathtt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi} * \mathsf{tok}\ \gamma_s * \Phi\ v\}$

$\quad\{\mathtt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * \mathsf{tok}\ \gamma_s * \Phi\ v\}$

$\quad\{c \mapsto \mathtt{None} * \mathsf{tok}\ \gamma_s * \Phi\ v\}$

$c \leftarrow \mathtt{Some}\ v$

$\quad\{c \mapsto \mathtt{Some}\ v * \mathsf{tok}\ \gamma_s * \Phi\ v\}$

$\{\mathsf{True}\}$

Proof of Send

$$\mathsf{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi \triangleq (\underbrace{c \mapsto \mathtt{None}}_{\text{(1) initial state}}) \vee (\underbrace{\exists v.\, c \mapsto \mathtt{Some}\, v * \Phi\ v * \mathsf{tok}\ \gamma_s}_{\text{(2) message sent, but not yet received}}) \vee (\underbrace{\mathsf{tok}\ \gamma_s * \mathsf{tok}\ \gamma_r}_{\text{(3) final state}})$$

$$c \rightarrowtail (tag, \Phi) \triangleq \exists \gamma_s, \gamma_r.\, \boxed{\mathsf{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi} * \begin{cases} \mathsf{tok}\ \gamma_s & \text{if } tag = \mathsf{Send} \\ \mathsf{tok}\ \gamma_r & \text{if } tag = \mathsf{Recv} \end{cases}$$

$\{c \rightarrowtail (\mathsf{Send}, \Phi) * \Phi\ v\}$

$\{\boxed{\mathtt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi} * \mathsf{tok}\ \gamma_s * \Phi\ v\}$

$\quad\{\mathtt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * \mathsf{tok}\ \gamma_s * \Phi\ v\}$

$\quad\{c \mapsto \mathtt{None} * \mathsf{tok}\ \gamma_s * \Phi\ v\}$

$c \leftarrow \mathtt{Some}\ v$

$\quad\{c \mapsto \mathtt{Some}\ v * \mathsf{tok}\ \gamma_s * \Phi\ v\}$

$\quad\{\mathtt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi\}$

$\{\mathsf{True}\}$

Proof of Receive

```
let w = !c in
match w with
  None   ⇒ recv1 c
| Some v ⇒ free c; v
end
```

Proof of Receive

$\{c \rightarrowtail (\mathsf{Recv}, \Phi)\}$

$\texttt{let}\ w = !\, c\ \texttt{in}$
$\texttt{match}\ w\ \texttt{with}$
$\quad \texttt{None} \quad \Rightarrow \texttt{recv1}\ c$
$\mid \texttt{Some}\ v \Rightarrow \texttt{free}\ c;\ v$
$\texttt{end}$

$\{w.\, \Phi\ w\}$

Proof of Receive

$\{c \rightarrowtail (\mathsf{Recv}, \Phi)\}$
$\{\mathsf{tok}\ \gamma_r * \boxed{\texttt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi}\}$

$\mathbf{let}\ w = !\,c\ \mathbf{in}$
$\mathbf{match}\ w\ \mathbf{with}$
$\quad \mathbf{None} \Rightarrow \mathbf{recv1}\ c$
$\mid \mathbf{Some}\ v \Rightarrow \mathbf{free}\ c;\ v$
$\mathbf{end}$

$\{w.\ \Phi\ w\}$

Proof of Receive

$\{c \rightarrowtail (\mathsf{Recv}, \Phi)\}$
$\{\mathsf{tok}\ \gamma_r\}$

```
let w = !c in
match w with
  None   ⇒ recv1 c
| Some v ⇒ free c; v
end
```

$\{w.\ \Phi\ w\}$

Duplicable propositions:

$\boxed{\texttt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi}$

Proof of Receive

$\{c \rightarrowtail (\mathsf{Recv}, \Phi)\}$
$\{\mathsf{tok}\ \gamma_r\}$
$\quad\{\mathtt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * \mathsf{tok}\ \gamma_r\}$

```
let w = !c in
match w with
  None   ⇒ recv1 c
| Some v ⇒ free c; v
end
```

$\{w.\ \Phi\ w\}$

Duplicable propositions:

$\boxed{\mathtt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi}$

Proof of Receive

$\{c \rightarrowtail (\mathsf{Recv}, \Phi)\}$

$\{\mathsf{tok}\ \gamma_r\}$

$\quad\{\mathtt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * \mathsf{tok}\ \gamma_r\}$

$\quad\{(c \mapsto \mathtt{None}) \vee (\exists v.\, c \mapsto \mathtt{Some}\ v * \Phi\ v * \mathsf{tok}\ \gamma_s) * \mathsf{tok}\ \gamma_r\}$

```
let w = !c in
match w with
  None   ⇒ recv1 c
| Some v ⇒ free c; v
end
```

$\{w.\, \Phi\ w\}$

Duplicable propositions:

$\boxed{\mathtt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi}$

Proof of Receive

$\{c \rightarrowtail (\mathsf{Recv}, \Phi)\}$
$\{\mathsf{tok}\ \gamma_r\}$
$\{\mathtt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * \mathsf{tok}\ \gamma_r\}$
$\{(c \mapsto \mathbf{None}) \vee (\exists v.\, c \mapsto \mathbf{Some}\, v * \Phi\ v * \mathsf{tok}\ \gamma_s) * \mathsf{tok}\ \gamma_r\}$
$\{c \mapsto \mathbf{None} * \mathsf{tok}\ \gamma_r\}\{c \mapsto \mathbf{Some}\, v * \Phi\ v * \mathsf{tok}\ \gamma_s * \mathsf{tok}\ \gamma_r\}$

```
let w = !c in
match w with
  None   ⇒ recv1 c
| Some v ⇒ free c; v
end
```

$\{w.\, \Phi\ w\}$

Duplicable propositions:

$\boxed{\mathtt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi}$

Proof of Receive

$\{c \rightarrowtail (\mathsf{Recv}, \Phi)\}$
$\{\mathsf{tok}\ \gamma_r\}$
$\{\texttt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * \mathsf{tok}\ \gamma_r\}$
$\{(c \mapsto \texttt{None}) \vee (\exists v.\ c \mapsto \texttt{Some}\ v * \Phi\ v * \mathsf{tok}\ \gamma_s) * \mathsf{tok}\ \gamma_r\}$
$\{c \mapsto \texttt{None} * \mathsf{tok}\ \gamma_r\}\{c \mapsto \texttt{Some}\ v * \Phi\ v * \mathsf{tok}\ \gamma_s * \mathsf{tok}\ \gamma_r\}$
$\texttt{let}\ w = !c\ \texttt{in}$
$\{w = \texttt{None} * c \mapsto \texttt{None} * \mathsf{tok}\ \gamma_r\}$
$\texttt{match}\ w\ \texttt{with}$
$\quad \texttt{None} \Rightarrow \texttt{recv1}\ c$
$\mid \texttt{Some}\ v \Rightarrow \texttt{free}\ c;\ v$
$\texttt{end}$
$\{w.\ \Phi\ w\}$

Duplicable propositions:

$\boxed{\texttt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi}$

Proof of Receive

$\{c \rightarrowtail (\mathsf{Recv}, \Phi)\}$
$\{\mathsf{tok}\ \gamma_r\}$
$\{\mathtt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * \mathsf{tok}\ \gamma_r\}$
$\{(c \mapsto \mathtt{None}) \vee (\exists v.\, c \mapsto \mathtt{Some}\, v * \Phi\ v * \mathsf{tok}\ \gamma_s) * \mathsf{tok}\ \gamma_r\}$
$\{c \mapsto \mathtt{None} * \mathsf{tok}\ \gamma_r\}\{c \mapsto \mathtt{Some}\, v * \Phi\ v * \mathsf{tok}\ \gamma_s * \mathsf{tok}\ \gamma_r\}$
$\mathtt{let}\ w = {!}c\ \mathtt{in}$
$\{w = \mathtt{None} * c \mapsto \mathtt{None} * \mathsf{tok}\ \gamma_r\}$
$\{w = \mathtt{None} * \mathtt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * \mathsf{tok}\ \gamma_r\}$
$\mathtt{match}\ w\ \mathtt{with}$
$\quad \mathtt{None} \Rightarrow \mathtt{recv1}\ c$
$\mid \mathtt{Some}\ v \Rightarrow \mathtt{free}\ c;\ v$
$\mathtt{end}$
$\{w.\, \Phi\ w\}$

Duplicable propositions:

$\boxed{\mathtt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi}$

Proof of Receive

Duplicable propositions:

$\boxed{\texttt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi}$

$\{c \rightarrowtail (\mathsf{Recv}, \Phi)\}$
$\{\mathsf{tok}\ \gamma_r\}$
$\quad\{\texttt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * \mathsf{tok}\ \gamma_r\}$
$\quad\{(c \mapsto \mathbf{None}) \lor (\exists v.\, c \mapsto \mathbf{Some}\ v * \Phi\ v * \mathsf{tok}\ \gamma_s) * \mathsf{tok}\ \gamma_r\}$
$\quad\{c \mapsto \mathbf{None} * \mathsf{tok}\ \gamma_r\}\{c \mapsto \mathbf{Some}\ v * \Phi\ v * \mathsf{tok}\ \gamma_s * \mathsf{tok}\ \gamma_r\}$
$\mathbf{let}\ w = {!c}\ \mathbf{in}$
$\quad\{w = \mathbf{None} * c \mapsto \mathbf{None} * \mathsf{tok}\ \gamma_r\}\{w = \mathbf{Some}\ v * c \mapsto \mathbf{Some}\ v * \Phi\ v * \mathsf{tok}\ \gamma_s * \mathsf{tok}\ \gamma_r\}$
$\quad\{w = \mathbf{None} * \texttt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * \mathsf{tok}\ \gamma_r\}$
$\mathbf{match}\ w\ \mathbf{with}$
$\quad \mathbf{None} \Rightarrow \mathbf{recv1}\ c$
$\mid \mathbf{Some}\ v \Rightarrow \mathbf{free}\ c;\ v$
$\mathbf{end}$
$\{w.\, \Phi\ w\}$

Proof of Receive

Duplicable propositions:

$\boxed{\mathtt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi}$

$\{c \rightarrowtail (\mathsf{Recv}, \Phi)\}$

$\{\mathsf{tok}\ \gamma_r\}$

$\quad\{\mathtt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * \mathsf{tok}\ \gamma_r\}$

$\quad\{(c \mapsto \mathbf{None}) \vee (\exists v.\ c \mapsto \mathbf{Some}\ v * \Phi\ v * \mathsf{tok}\ \gamma_s) * \mathsf{tok}\ \gamma_r\}$

$\quad\{c \mapsto \mathbf{None} * \mathsf{tok}\ \gamma_r\}\{c \mapsto \mathbf{Some}\ v * \Phi\ v * \mathsf{tok}\ \gamma_s * \mathsf{tok}\ \gamma_r\}$

$\mathbf{let}\ w = !c\ \mathbf{in}$

$\quad\{w = \mathbf{None} * c \mapsto \mathbf{None} * \mathsf{tok}\ \gamma_r\}\{w = \mathbf{Some}\ v * c \mapsto \mathbf{Some}\ v * \Phi\ v * \mathsf{tok}\ \gamma_s * \mathsf{tok}\ \gamma_r\}$

$\quad\{w = \mathbf{None} * \mathtt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * \mathsf{tok}\ \gamma_r\}\{w = \mathbf{Some}\ v * \mathtt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * c \mapsto \mathbf{Some}\ v * \Phi\ v\}$

$\mathbf{match}\ w\ \mathbf{with}$

$\quad \mathbf{None} \Rightarrow \mathbf{recv1}\ c$

$\mid \mathbf{Some}\ v \Rightarrow \mathbf{free}\ c;\ v$

$\mathbf{end}$

$\{w.\ \Phi\ w\}$

Proof of Receive

Duplicable propositions:

$\boxed{\texttt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi}$

$\{c \rightarrowtail (\mathsf{Recv}, \Phi)\}$
$\{\mathsf{tok}\ \gamma_r\}$
$\quad\{\texttt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * \mathsf{tok}\ \gamma_r\}$
$\quad\{(c \mapsto \textbf{None}) \vee (\exists v.\, c \mapsto \textbf{Some}\ v * \Phi\ v * \mathsf{tok}\ \gamma_s) * \mathsf{tok}\ \gamma_r\}$
$\quad\{c \mapsto \textbf{None} * \mathsf{tok}\ \gamma_r\}\{c \mapsto \textbf{Some}\ v * \Phi\ v * \mathsf{tok}\ \gamma_s * \mathsf{tok}\ \gamma_r\}$
$\textbf{let}\ w = !c\ \textbf{in}$
$\quad\{w = \textbf{None} * c \mapsto \textbf{None} * \mathsf{tok}\ \gamma_r\}\{w = \textbf{Some}\ v * c \mapsto \textbf{Some}\ v * \Phi\ v * \mathsf{tok}\ \gamma_s * \mathsf{tok}\ \gamma_r\}$
$\quad\{w = \textbf{None} * \texttt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * \mathsf{tok}\ \gamma_r\}\{w = \textbf{Some}\ v * \texttt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * c \mapsto \textbf{Some}\ v * \Phi\ v\}$
$\quad\{\texttt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * (w = \textbf{None} * \mathsf{tok}\ \gamma_r) \vee (\exists v.\, w = \textbf{Some}\ v * c \mapsto \textbf{Some}\ v * \Phi\ v)\}$
$\textbf{match}\ w\ \textbf{with}$
$\quad\textbf{None} \Rightarrow \textbf{recv1}\ c$
$\mid \textbf{Some}\ v \Rightarrow \textbf{free}\ c;\ v$
$\textbf{end}$
$\{w.\, \Phi\ w\}$

Proof of Receive

Duplicable propositions:

$\boxed{\texttt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi}$

$\{c \rightarrowtail (\mathsf{Recv}, \Phi)\}$
$\{\mathsf{tok}\ \gamma_r\}$
$\quad \{\texttt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * \mathsf{tok}\ \gamma_r\}$
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$\quad \{w = \textbf{None} * c \mapsto \textbf{None} * \mathsf{tok}\ \gamma_r\}\{w = \textbf{Some}\ v * c \mapsto \textbf{Some}\ v * \Phi\ v * \mathsf{tok}\ \gamma_s * \mathsf{tok}\ \gamma_r\}$
$\quad \{w = \textbf{None} * \texttt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * \mathsf{tok}\ \gamma_r\}\{w = \textbf{Some}\ v * \texttt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * c \mapsto \textbf{Some}\ v * \Phi\ v\}$
$\quad \{\texttt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * (w = \textbf{None} * \mathsf{tok}\ \gamma_r) \vee (\exists v.\ w = \textbf{Some}\ v * c \mapsto \textbf{Some}\ v * \Phi\ v)\}$
$\{(w = \textbf{None} * \mathsf{tok}\ \gamma_r) \vee (\exists v.\ w = \textbf{Some}\ v * c \mapsto \textbf{Some}\ v * \Phi\ v)\}$

```
match w with
  None   ⇒ recv1 c
| Some v ⇒ free c; v
end
```

$\{w.\ \Phi\ w\}$

Proof of Receive

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Proof of Receive

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$\mathtt{let}\ w = !c\ \mathtt{in}$
$\{w = \mathbf{None} * c \mapsto \mathbf{None} * \mathsf{tok}\ \gamma_r\}\{w = \mathbf{Some}\, v * c \mapsto \mathbf{Some}\, v * \Phi\ v * \mathsf{tok}\ \gamma_s * \mathsf{tok}\ \gamma_r\}$
$\{w = \mathbf{None} * \mathtt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * \mathsf{tok}\ \gamma_r\}\{w = \mathbf{Some}\, v * \mathtt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * c \mapsto \mathbf{Some}\, v * \Phi\ v\}$
$\{\mathtt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * (w = \mathbf{None} * \mathsf{tok}\ \gamma_r) \vee (\exists v.\, w = \mathbf{Some}\, v * c \mapsto \mathbf{Some}\, v * \Phi\ v)\}$
$\{(w = \mathbf{None} * \mathsf{tok}\ \gamma_r) \vee (\exists v.\, w = \mathbf{Some}\, v * c \mapsto \mathbf{Some}\, v * \Phi\ v)\}$
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$\mathtt{end}$
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Proof of Receive

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$\quad\{c \mapsto \mathbf{None} * \mathsf{tok}\ \gamma_r\}\{c \mapsto \mathbf{Some}\ v * \Phi\ v * \mathsf{tok}\ \gamma_s * \mathsf{tok}\ \gamma_r\}$
$\mathbf{let}\ w = !c\ \mathbf{in}$
$\quad\{w = \mathbf{None} * c \mapsto \mathbf{None} * \mathsf{tok}\ \gamma_r\}\{w = \mathbf{Some}\ v * c \mapsto \mathbf{Some}\ v * \Phi\ v * \mathsf{tok}\ \gamma_s * \mathsf{tok}\ \gamma_r\}$
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Proof of Receive

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$\{\mathsf{tok}\ \gamma_r\}$

$\{\texttt{chan\_inv}\ \gamma_s\ \gamma_r\ c\ \Phi * \mathsf{tok}\ \gamma_r\}$

$\{(c \mapsto \mathbf{None}) \vee (\exists v.\ c \mapsto \mathbf{Some}\ v * \Phi\ v * \mathsf{tok}\ \gamma_s) * \mathsf{tok}\ \gamma_r\}$

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$\{w = \mathbf{None} * \mathsf{tok}\ \gamma_r\}\{w = \mathbf{Some}\ v * c \mapsto \mathbf{Some}\ v * \Phi\ v\}$

$\mathtt{match}\ w\ \mathtt{with}$

$\quad \mathtt{None} \Rightarrow \{\mathsf{tok}\ \gamma_r\}\{c \rightarrowtail (\mathsf{Recv}, \Phi)\}\mathtt{recv1}\ c\{w.\ \Phi\ w\}$

$\mid \mathtt{Some}\ v \Rightarrow \mathtt{free}\ c;\ v$

$\mathtt{end}$

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Proof of Receive

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Questions?

Iris and Actris Beyond This Tutorial

Iris

- **Features:** Custom ghost state, persistent modality, Löb induction, . . .
- **Technicalities:** Later modality, invariant masks, ghost updates, . . .
- Website: `https://iris-project.org`

Iris and Actris Beyond This Tutorial

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Actris

- Recursive protocols (POPL'20)
- Semantic Session Type System (CPP'21)
- Subprotocols (cf. subtyping) (LMCS'22)
- Dependent separation protocol ghost state and rules (LMCS'22)
- Application to distributed systems (ICFP'23)
- Deadlock-freedom (POPL'24 on Thursday: 14:40)
- Website: `https://iris-project.org/actris`

Break (10 min!)

Time for Coq hacking session!