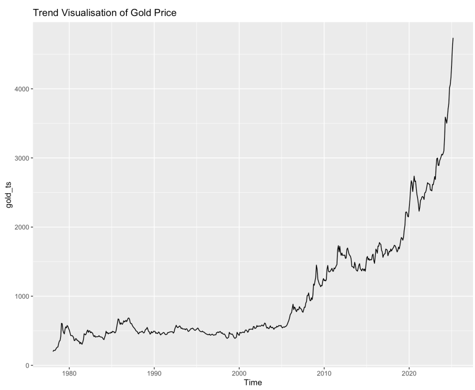
ETW3420_A1_REPORT

BYUN JIHOO_31858945

INTRODUCTION

Gold has been playing as a great hedge fund in the world. In October 2024, price of gold has reached to the new level and again renewed its higher price level in 2025 (CBS News, 2025). The research report is aiming to establish an appropriate model to forecast the gold price using exponential smoothing methods. Across the contents of the report, exploratory data analysis, model building, model selection and forecasting for next 12 months have been conducted. All analysis has been conducted for gold price in Australian Dollar (AUD).

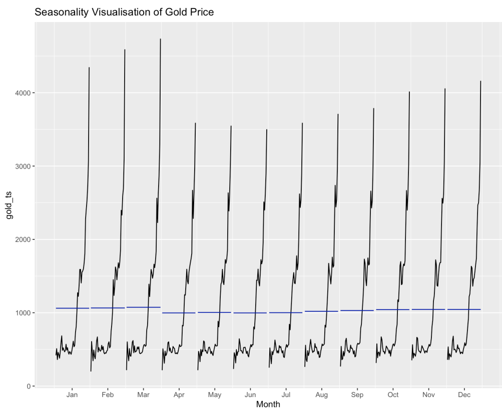
EXPLORATORY DATA ANALYSIS



First visualization is about the trend of the data. The price of gold keeps increasing as the time goes and its increasing slope is getting higher. First acceleration of price increase has started on around 2005 and second rapid increase has occurred in 2019.

There are two reasons for those rapid increase. First, in 2003, Australia launched its first Gold ETF (Zaychuk, 2019). This event which created demands and investment in the gold price was the first trigger to rapidly increase the gold price.

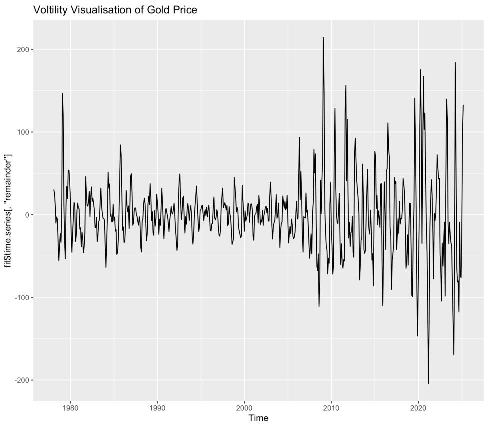
Secondly, COVID-19 pandemic lifted gold price to extreme higher level due to social anxiety. Referring to the characteristic of the hedge fund, its price has positive proportion with the level of social anxiety. When COVID-19 has outbroken, it impact the gold price level to increase (KitYeg, 2019).



The second exploratory analysis visualization is about seasonality. Especially, gold price increases dramatically in January, February, and March. During these seasons, there are Chinese New Year and Indian wedding season which both event cultures are exchanging gold presents. Besides, these two countries are the largest gold consumers in the world (Gupta, 2025). Also, there is Federal Reserve Board

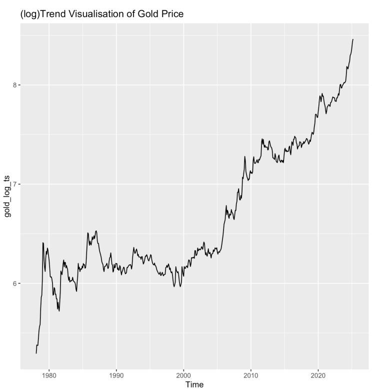
(Fed) has its annual announcement in the beginning of the years. Effect from authoritative organization could impact investor confidence and make changes in the gold price.

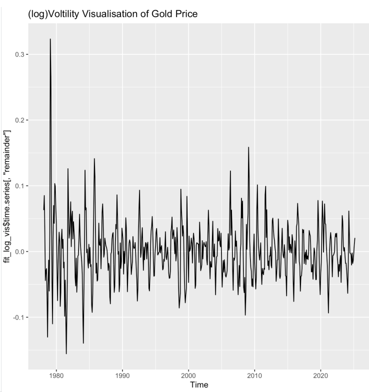
Therefore, an obvious seasonality has been detected across the years.

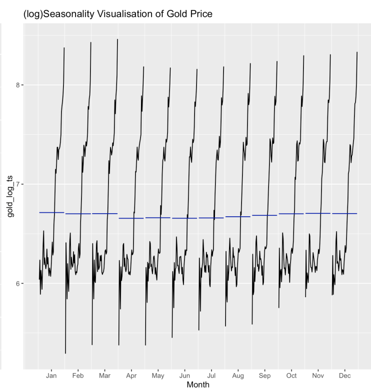


The third exploratory visualization is about the volatility of the data. From the past to present, level of data is not equal but is increasing. Through the visualization, the data has high volatility. However, different result could come out when convert the data into box-cox data. Meaning that, although only 10% of price is fluctuating every years, that 10% could vary depending on the actual price of gold. Therefore, box-cox transformation of data will be conducted if it is needed later.

However, trend, seasonality and volatility issue could be arranged via box-cox transformation which smoothens trend, seasonality and volatility. The result after box-cox transformation is provided below:







MODEL DEVELOPMENT

Bench Mark Model: Seasonal Naïve model: $\hat{y}_{t+h} = y_{t+h-k*m} = y_{t+1-1*12}$

```
Forecast method: Seasonal naive method

Model Information:
Call: snaive(y = gold_ts)

Residual sd: 222.7538

Error measures:

        Training set
ME        80.05144
RMSE     222.7538
MAE      132.7531
MPE        4.221985
MAPE      12.49821
MASE        1
ACF1       0.9166677
```

The bench mark model is Seasonal Naïve model which predicts the gold price of next month based on previous year data. Since the data set is monthly data which has 12 month in one year, m= 12 and h=1=k means the model tries to predict one month later gold price.

Therefore, based on the Seasonal Naïve model, Exponential Smoothing Models will be evaluated about how prediction of ETS models are improved from the Seasonal Naïve model.

Exponential Smoothing Method model from Visualizations:

$$\hat{y}_{t+h|t} = \ell_t + hb_t + s_{t-m+h_m}$$

$$level = \ell_t = \alpha(y_t - s_{t-m}) + (1 + a)(\ell_{t-1} + \mathrm{b}_{t-1})$$

$$trend = \mathrm{b}_t = \beta(\ell_t - \ell_{t-1}) + (1 - \beta)\mathrm{b}_{t-1}$$

$$seasonality = \mathrm{s}_t = \gamma\left(\mathrm{y}_t - \ell_{t-1} - \mathrm{b}_{t-1}\right) + (1 - \gamma)\mathrm{s}_{t-m}$$

```
ets(y = gold_boxcox,
    model = "AAA",
    damped = FALSE,
    additive.only = TRUE,
    lambda = NULL,
    biasadj = TRUE,
    opt.crit = "lik",
    bounds = “both",
    ic = "aicc",
    restrict = TRUE,
    allow.multiplicative.trend = TRUE)
```

Smoothing parameters:

alpha = 0.9879

beta = 0.0627

gamma = 0.0121

Initial states:

l = 19.9063

b = 0.6651

s = -0.0808 -0.1459 -0.0109 -5e-04 0.0436 -0.0097
-0.0621 -0.2087 -0.0102 0.1084 0.2294 0.1475

sigma: 0.7738

AIC	AICc	BIC
3315.161	3316.278	3388.917

Training set error measures:

	Training set
ME	0.002178072
RMSE	0.7628112
MAE	0.5508721
MPE	-0.03577398
MAPE	1.430439
MASE	0.2312221
ACF1	0.194782

The manual ETS model has been developed from the visualizations of the data set. Before the model development, box-cox transformation has been conducted to decrease volatility issue.

The model has “AAA” structure. The model had increasing volatility which Error should be multiplicative. However, box-cox transformation decreased volatility and allowed Error to be Additive.

After box-cox transformation, Trend has converted into linear structure which is additive trend and volatility got stabilized level. Besides box-cox transformed data has stable seasonal fluctuation. Taking three exploratory features of box-cox transformed data, the ETS model structure would be AAA structure.

The Trend of data is not damping but is additive on box-cox transformed data. Therefore, “damped” has been set ‘FALSE’ to not allow the damped trend.

By setting “additive.only” as “TRUE” value, it allows additive structure because the volatility maintains stable level after box-cox transformation.

Lamda value has been set “NULL” because of box-cox transformation stabilized the volatility and lambda does not need to play a role in the model. If Lambda is not NULL, there might be unnecessary extra transformation and make result incorrect.

Since box-cox transformation has been conducted, biased adjustment needs to play in the model to eliminate the side effect of box-cox transformation. Therefore, ‘biasadj’ has been set “TRUE” in the model.

The model implemented box-cox -likelihood as optimization criterion therefore the ‘opt.crit’ option has been set ‘lik’.

By setting ‘bounds’ option for ‘both’, the model is allowing conditions which the model could be mathematically admissible. It prevents the model goes for mathematically unrealistic structure.

Setting ‘ic’ option for ‘aicc’, model is selecting ‘Corrected Akaike Information Criterion’ as its information criterion.

‘restrict = TRUE’ option prevents model from being impossible model structure such as MMM or MAA.

The smoothing parameters has meaning in the model. When alpha=0.8545, the model is considerably sensitive to the recent data. Beta and Gamma value close to 0 means the model has smooth trend and has fixed seasonality.

Referring to the initial states, initial level is 3.0381, trend started as increasing trend at initial stage with b=0.0025 and every 12 values of initial seasonal values are set to be mean equals to zero.

$$\hat{y}_{t+h|t} = \ell_t + hb_t$$

$$level = \ell_t = \alpha y_t + (1 - a)(\ell_{t-1} + \mathrm{b}_{t-1})$$

$$trend = \mathrm{b}_t = \beta(\ell_t - \ell_{t-1}) + (1 - \beta)\mathrm{b}_{t-1}$$

```
Call:
ets(y = train_au_bc,
         model = "AAN",
         damped = FALSE,
         additive.only = TRUE,
         lambda = NULL,
         biasadj = TRUE,
         opt.crit = "lik",
         bounds = "both",
         ic = "aicc",
         restrict = TRUE,
         allow.multiplicative.trend = TRUE)

  Smoothing parameters:
    alpha = 0.9999
    beta  = 1e-04

  Initial states:
    l = 20.7124
    b = 0.0673

  sigma:  0.7395

     AIC       AICc       BIC
2752.121  2752.245  2773.103

Training set error measures:

              Training set
ME             0.001577536
RMSE           0.7365059
MAE            0.5160055
MPE           -0.02067033
MAPE           1.455009
MASE           0.2534742
ACF1           0.1864622
```

Additionally, one more manual ETS model has been developed. The model structure has changes from “AAA” to “AAN” in case which taking seasonality into the model may decrease the forecasting accuracy. The difference in forecasting functionality will be compare later in the report.

Automatic ETS model using ets() function:

$$\hat{y}_{t+h|t} = \ell_t + \phi * b_t + \phi^2 * b_t + \cdots + \phi^h * b_t = \ell_t + b_t * \frac{1-\phi^h}{1-\phi}$$

$$level = \ell_t = \alpha y_t + (1-a)(\ell_{t-1} + \phi \mathrm{b}_{t-1})$$

$$trend = \mathrm{b}_t = \beta(\ell_t - \ell_{t-1}) + (1-\beta)\phi \mathrm{b}_{t-1}$$

```
ETS(A,Ad,N)

Call:
ets(y = gold_boxcox)

 Smoothing parameters:
   alpha = 0.9999
   beta  = 0.1458
   phi   = 0.8004

 Initial states:
   l = 19.6159
   b = 0.4396

 sigma:  0.7673

     AIC     AICc      BIC
3294.818 3294.968 3320.849

Training set error measures:

             Training set
ME           0.07361221
RMSE         0.7639282
MAE          0.5435534
MPE          0.1480951
MAPE         1.400027
MASE         0.2281502
ACF1         0.130703
```

Third model developed is automatic Exponential Smoothing Method model.

The automatic ETS model has several differences from the manually established ETS model. The automatic ETS model structure has “A,Ad,N” structure which Error is additive, Damped Additive Trend without seasonality. Despite of box-cox transformation, seasonality for January, February, and March still remains in the data. However, the model algorithm did not take it into account.

Evaluating the error measures, Mean Error and RMSE has increased in the automatic ETS model. However, rest of error measure has lower value in the second automatic model.

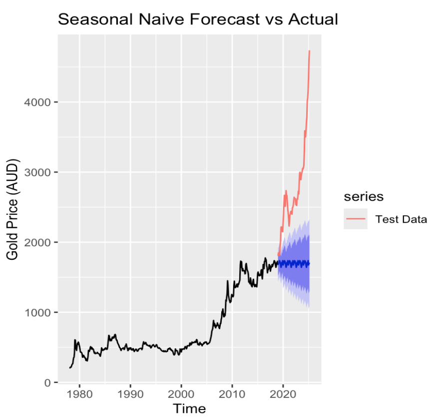
Considering the model complexity using AIC, AICc, and BIC, automatic ETS model has better condition.

MODEL EVALUATION & SELECTION:

- Seasonal Naïve Model

Seasonal Naive Model

	Training set	Test set
ME	35.61335	1062.39880
RMSE	119.6467	1238.7869
MAE	87.45931	1062.39880
MPE	3.013563	35.590774
MAPE	12.20010	35.59077
MASE	1.00000	12.14735
ACF1	0.8969967	0.9096595
Theil's U	NA	11.53142



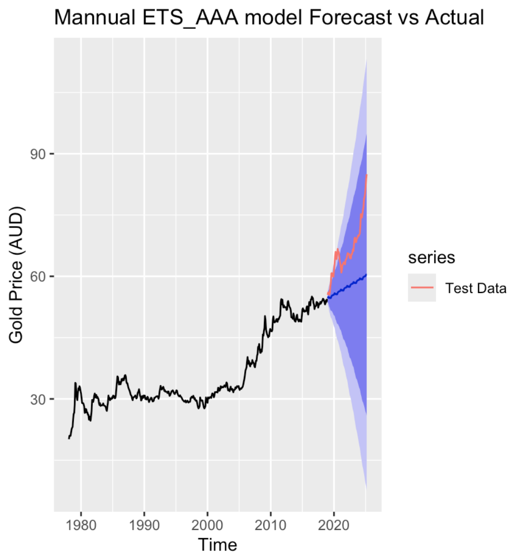
Comparing error measures of Seasonal Naïve Model in training data set and test data set, error measures are largely different in training and test data.

In the visualization graph, neither of 90 percents nor 95 percent confidence interval forecast did not cover test data value. Therefore, accuracy of Seasonal Naïve model is considerably low.

- Manual ETS AAA model:

ETS AAA Model

	ME	RMSE	MAE	MPE	MAPE	MASE	ACF1	Theil's U
Training set	-0.005974238	0.7428866	0.5370429	-0.02068693	1.517659	0.2638082	0.1550676	NA
Test set	9.147150979	10.4616174	9.1471510	13.17575040	13.175750	4.4932978	0.9025324	9.92838



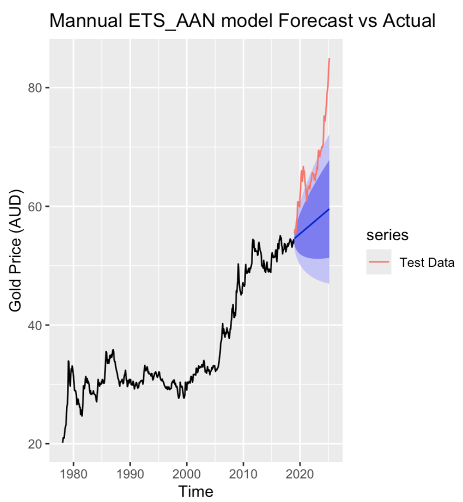
The ETS AAA model seems it successfully forecasted test data based on train data set. The error measures has significantly decreased compared to the bench mark model.

Observing the visualization plot, forecasted result seems it has similar linear shape with the test data while its confidence interval is covering the actual test data set.

- Manual ETS AAN model:

ETS AAN Model

	ME	RMSE	MAE	MPE	MAPE	MASE	ACF1	Theil's U
Training set	0.001577536	0.7365059	0.5160055	-0.02067033	1.455009	0.2534742	0.1864622	NA
Test set	9.469096565	10.8249782	9.4690966	13.63814544	13.638145	4.6514451	0.9039358	10.2728



ETS AAN model which does not take seasonality into consideration.

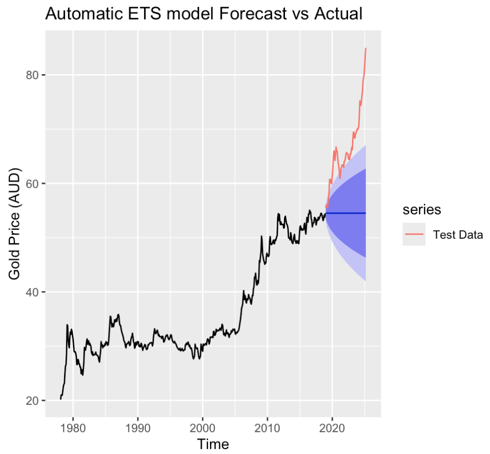
The error measures has increased from AAA model which implies that AAN model has lower functionality.

It may provide result with easier interpretation but is not reliable much as the AAA model.

- Automatic ETS model:

ETS auto Model

	ME	RMSE	MAE	MPE	MAPE	MASE	ACF1	Theil's U
Training set	0.06999013	0.7392323	0.5107489	0.1805849	1.435332	0.250892	0.1884129	NA
Test set	12.03129485	13.6805801	12.0312949	17.3364673	17.336467	5.910058	0.9158912	10.2728

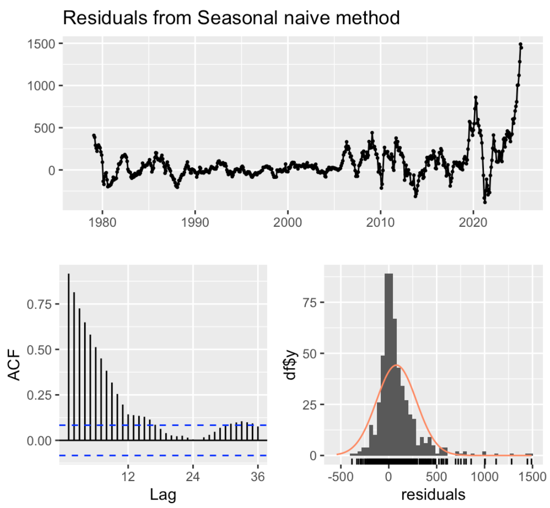


Comparing error measures of model from the measures of manual model, automatic ETS model has higher error measures which implies automatic ETS model has lower accuracy than manual models.

Reviewing the forecast visualization plot, the result data line does not follow the test data line. Inaccurate result has been made out from the automatic ETS model.

Residual diagnostics:

- Seasonal Naïve Method model



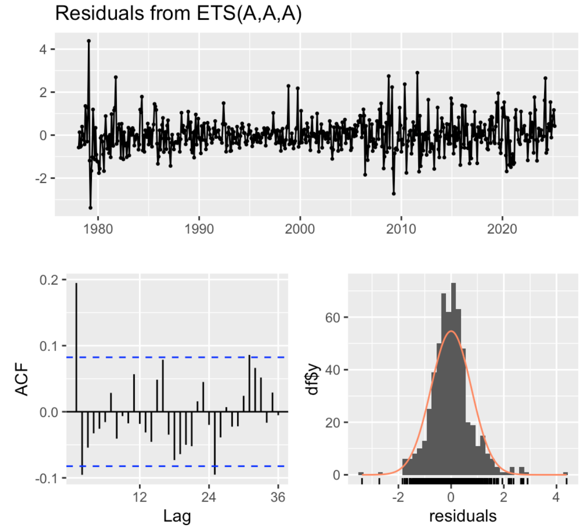
Seasonal Naïve method model has abnormal residual diagnostic result.

Firstly, the residual is not white noise, it is not mean reverting but it getting larger as time goes by.

Secondly, there is statistically significant autocorrelation function which implies that there is seasonality which the model could not predict.

Finally, the histogram of residual is not Normally Distributed. Since there is longer tail area, residual is getting larger in specific prediction.

- Manual ETS AAA model



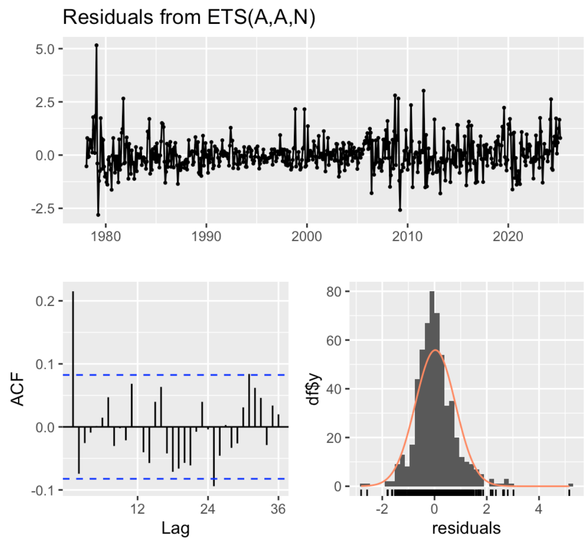
The residual diagnostic of ETS model in AAA structure has developed from the bench mark model.

Firstly, the residual over time seems white noise with mean reverting. Although there are some time region which residual gets larger, the overall residual is reverting to the mean after temporary increase.

Secondly, autocorrelation function plot shows that almost every line is staying within the blue range. It implies that the model is covering the data at least better than the bench mark model does.

Lastly, the residual plot seems it is Normally Distributed without significant skewedness.

- Manual ETS AAN model

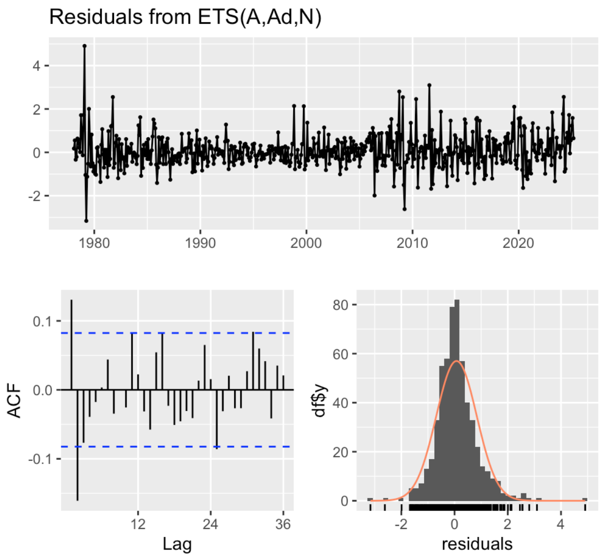


The ETS model with AAN structure has better white noise condition in residual plot than the bench mark model.

Compared to the bench mark model, the autocorrelation has improved and even compared to AAA structure model, AAN structure model has better autocorrelation plot.

However, residual plot is showing skewedness in the residual plot even the overall shape is in normal distribution form.

- Automatic ETS model



Actually, ETS models are sharing similar residual diagnostic results. Automatic ETS model has white noise residual, acceptable autocorrelation function plot and normally distributed residual histogram.

Therefore, the forecasting ability of models will be evaluated using cross-validation to select the ultimate champion model.

Cross-validation:

	h=1	h=2	h=3	h=4	h=5	h=6	h=7	h=8	h=9	h=10	h=11	h=12
Seasonal Naive	10.818751	10.494461	10.216107	10.042216	9.903237	9.805722	9.672558	9.519810	9.388223	9.286000	9.226701	9.189916
ETS_AAA	0.8229091	2.2059607	3.6762969	5.3516256	6.9084576	8.7014227	10.8658906	13.1620721	15.6410400	18.1439026	20.6021393	23.7734808
ETS_AAN	0.6876803	1.9084939	3.2512721	4.6784018	6.1144706	7.8134653	9.7420634	11.9457786	14.5445690	17.3293744	20.2781321	23.8727571
ETS_auto_AAdN	0.5860178	1.5029290	2.4195803	3.3435550	4.3203084	5.2634127	6.1797217	7.2978366	8.3089349	9.3525363	10.4600461	11.6088939

According to cross validation model, AAdN structure model which is automatic ETS model shows the lowest and the most stable MSE/RMSE value across all time. AAA and AAN model seems they are maintaining reasonable error values however, their accuracy rate gets rapid decrease in longer time. The chart might support automatic model as the best model however, considering the characteristic of hedge fund gold price, this price gets large effect from external events which model is not able to predict. Therefore, predicting price of hedge fund in long term would be unrealistic. Hence, ETS_AAA and ETS_AAN model would be considered as champion model candidate.

Champion model selection:

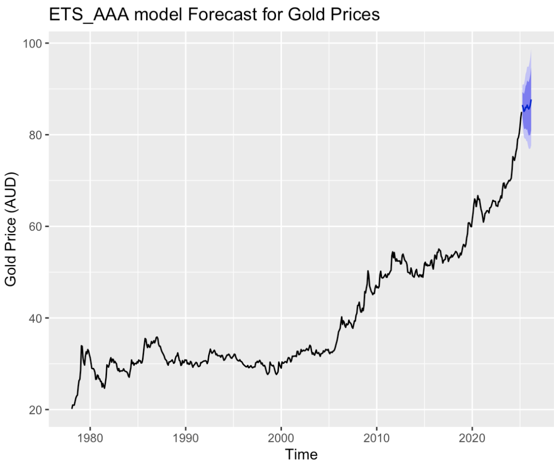
Reviewing the accuracy rate numerical chart and its visualization plot, ETS_AAA model would be the champion model.

It has the strongest accuracy in forecasting the gold price in short term. Although the model has theoretical accuracy of long term forecasting, any external events may happen and impact the gold price severely.

Therefore, ETS_AAA model is selected as the Champion model. It has reasonable residual diagnostic and acceptable confidence interval. Although ETS_AAN model has almost same feature of ETS_AAA model, it does not consider seasonality and its narrow confidence interval decreases its reliability.

Gold Price Forecasting:

		Forecast	Lo 80	Hi 80	Lo 95	Hi 95
Apr	2025	86.49751	83.72976	89.30034	82.31439	90.83583
May	2025	85.81573	82.52410	89.15751	80.85250	91.00081
Jun	2025	85.10284	81.37666	88.89398	79.49563	90.99739
Jul	2025	85.35027	81.20260	89.57835	79.11989	91.93637
Aug	2025	85.98158	81.42795	90.63159	79.15239	93.23716
Sep	2025	85.93107	81.03452	90.93931	78.59850	93.75781
Oct	2025	86.46632	81.21524	91.84528	78.61371	94.88467
Nov	2025	85.90939	80.38956	91.57164	77.66562	94.78331
Dec	2025	85.60865	79.82291	91.55166	76.97836	94.93473
Jan	2026	85.96713	79.88472	92.22286	76.90492	95.79619
Feb	2026	86.53412	80.14960	93.10860	77.03241	96.87633
Mar	2026	87.71401	80.98780	94.64849	77.71450	98.63505



Based on ETS_AAA model, gold price forecasting for next 12 months have been generated.

Point forecasting result would be useful for detailed planning for budget or financial allocation problem solving and visualized plot could play a crucial role in the risk management since its confidence interval is suggesting the worst and best scenarios or forecasting.

For example, the gold price is expected to increase in next 12 months. For stakeholders, they will be recommended to invest for the price of gold expecting its increasing price.

However, the forecasting does not take external events into the account. If any events happen and impact the price, the actual result will not be covered. Considering the characteristic of the hedge fund, what happens and how it impacts the social anxiety has considerable coefficient to the hedge fund price such as gold.

But gold is not normal hedge fund, but its supply is limited. Amount of gold in the Earth is limited, and its demand will increase in the future as long as the global society maintains. Therefore, there could be decrease in the gold price which the forecasting model is not able to consider but it will rise again eventually.

REFERNECES

- https://www.cbsnews.com/news/hedge-funds-increasing-gold-holdings-investors/
- https://www.morningstar.com.au/etfs/what-is-behind-the-etf-gold-rush
- https://www.researchgate.net/publication/354544572_Does_the_Factors_Affecting_Gold_Price_Changed_During_Covid-19_Period_in_Malaysia
- https://www.researchgate.net/publication/354544572_Does_the_Factors_Affecting_Gold_Price_Changed_During_Covid-19_Period_in_Malaysia