1 Code function tables

1.1 `Sausage Conjecture calculations.jl` function tables

Distinct functions are separated by semicolons. The "Description/Explanation" column provides additional details in cases where the code and mathematical functions diverge.

Function in code	Corresponding function in Part II	Description/Explanation
`I(x, y)`	$[x, y]$	Outputs an interval $I(x, y) \supseteq [x, y]$ whose endpoints are floating-point numbers.
`SQ(X)`	$X \cdot X$	
`K(d)`	κ_d	The volume of B^d using interval arithmetic.
`κ_big(d)`	κ_d	The volume of B^d using the `big` command instead of interval arithmetic. Intended for large d.
`B(A)`	$\beta(\alpha)$	Here, `A` is the interval version of α, not $\boldsymbol{\alpha}$.
`A`	$\boldsymbol{\alpha}$	
`△(A)`	[N/A]	
`A★`	$\boldsymbol{\alpha}^*$	
`H_min_2(A★)`	$\eta_{(2)}^{\min}(\boldsymbol{\alpha}^*)$	
`H123(A)`	$\eta_{1,2,3}(\boldsymbol{\alpha})$	
`H123_min(A★)`	$\eta_{1,2,3}^{\min}(\boldsymbol{\alpha}^*)$	
`Y31(A)`; `Y32(A)`; `Y33(A)`	$\mathbf{y}^1(\boldsymbol{\alpha}^3)$ $\mathbf{y}^2(\boldsymbol{\alpha}^3)$ $\mathbf{y}^3(\boldsymbol{\alpha}^3)$	
`F12_3_t(A)`; `F13_3_t(A)`; `F23_3_t(A)`	$f_{i,j}(\boldsymbol{\alpha}^3)$	The "`_t`" is for trigonometric functions.
`G23_3_t(A)`; `G13_3_t(A)`; `G12_3_t(A)`	$g_{j,k}^{(3)}(\boldsymbol{\alpha}^3)$	
`F123_3(A)`	$f_{1,2,3}(\boldsymbol{\alpha}^3)$	
`G123_3(A)`	$g_{1,2,3}^{(3)}(\boldsymbol{\alpha}^3)$	

Table 1.1: Functions in `Sausage Conjecture calculations.jl`:
Preliminaries and vertex/external angles.

Function in code	Corresponding function in Part II	Description/Explanation
`Integralequation(f, a, b)`	$[v(f,a,b) - e(f,a,b),\, v(f,a,b) + e(f,a,b)]$	
`IntegralEquation(F, A, B)`	$\int_A^B F(x)\,dx$ $= \left[v\left(\underline{F(x)}, \overline{A}, \underline{B}\right) - e\left(\underline{F(x)}, \overline{A}, \underline{B}\right),\ v\left(\overline{F(x)}, \underline{A}, \overline{B}\right) + e\left(\overline{F(x)}, \underline{A}, \overline{B}\right)\right]$.	
`_k(m, r);` `_K(m, r)`	$k(m,r)$	Some single-letter function names give an error; the "`_`" resolves this issue.
`_a(r);` `_A(r)`	$a(r)$	
`_b(k, m);` `_B(k, m)`	$b(k,m)$	
`μ0(k, m, r);` `MU0(k, m, r)`	$\mu_0(k,m,r)$	We use "`MU`" for capital μ.
`integrand_m(d, l, k, m, r, μ);` `Integrand_M(d, l, k, m, r, μ)`	The integrand $\frac{1}{\sqrt{(a(r))^2 + x^2 (b(k,m))^2}^{\,d-l+m}} x^{(d-l+m)-(k+2)} (1-x)^k$ of m_0 and m_1.	
`m0(d, l, k, m, r);` `M0(d, l, k, m, r)`	$m_0(d,l,k,m,r)$	
`m1(d, l, k, m, r);` `M1(d, l, k, m, r)`	$m_1(d,l,k,m,r)$	
`q_UB(d, l, m, r);` `Q_UB(d, l, m, r)`	$q(d,l,m,r)$	The suffix "`_UB`" is because this quantity is an upper bound for q. $R(d,l,m,r)$ is included in the formula for `q_UB`.
`Integrand_Ω(d, l, m, r, p)`	$\frac{1}{1+q(d,l,m,r)}$	The integrand of ω. We use $p = l - m$ in the integral ω_{l-m}.
`Ω(d, l, m, p, lb, ub)`	$\int_{\mathtt{lb}}^{\mathtt{ub}} \frac{1}{1+q(d,l,m,r)}\,dr$	The integral ω with customizable bounds. We use $p = l - m$.
`u(d);` `U(d)`	$u(d) = \min\left\{0.95, 1 - \frac{9}{d^{\frac{3}{2}}}\right\}$	
`Ω1_2(d)`	$\omega_1^{(2)}(d)$	The upper bounds are different, otherwise errors will appear.
`Ω1_3(d)`	$\omega_1^{(3)}(d)$	
`Ω2_3_min(α★, d)`	$\omega_2^{(3)}(\alpha_3^*, d)$	
`Ω3_3_min(α★, d)`	$\omega_3^{(3)}(\alpha_3^*, d)$	

Table 1.2: Functions in `Sausage Conjecture calculations.jl`:
Integrals for the volume lower bounds.

Function in code	Corresponding function in Part II	Description/Explanation
`μ(α, ζ);` `MU(α, ζ)`	$\mu\left(\alpha,\zeta\right)$	We use “`MU`” for capital μ.
`integrand_g0(m, r, ζ, d);` `Integrand_G0(m, r, ζ, d)`	The integrand $r^{m-2}\left(r\frac{\zeta}{\sqrt{4-\zeta^2}}+\frac{2}{\sqrt{4-\zeta^2}}\right)^{d-(m-1)}$ of $g_m^{(0)}\left(\alpha_{m,1},\zeta,d\right)$.	
`integrand_g1(m, r, α, ζ, d);` `Integrand_G1(m, r, α, ζ, d)`	The integrand $r^{m-2}\left(r\frac{\sin\left(\alpha_{m,1}\right)-1}{\sin\left(\alpha_{m,1}\right)}\sqrt{\frac{2+\zeta}{2-\zeta}}+\frac{1}{\sin\left(\alpha_{m,1}\right)}\right)^{d-(m-1)}$ of $g_m^{(1)}\left(\alpha_{m,1},\zeta,d\right)$.	
`g0(m, α, ζ, d);` `G0(m, α, ζ, d)`	$g_m^{(0)}\left(\alpha_{m,1},\zeta,d\right)$	
`g1(m, α, ζ, d);` `G1(m, α, ζ, d)`	$g_m^{(1)}\left(\alpha_{m,1},\zeta,d\right)$	
`g(m, α, ζ, d);` `G(m, α, ζ, d)`	$g_m\left(\alpha_{m,1},\zeta,d\right)$	
`G_min(m, α, d)`	$g_m^{\min}\left(\alpha_{m,1},d\right)$	
`integrand_p(α, d, r, m);` `Integrand_P(α, d, r, m)`	The integrand $r^{m-2}\left(-r\frac{\cos\left(\alpha\right)}{\sin\left(\alpha\right)}+\frac{1}{\sin\left(\alpha\right)}\right)^{d-(m-1)}$ of $p_m\left(\alpha,d\right)$.	
`p(m, α, d);` `P(m, α, d)`	$p_m\left(\alpha,d\right)$	
`Q(m, α, d)`	$q_m\left(\alpha,d\right)$	
`Q̂(m, α, d)`	$\widehat{q}_m\left(\alpha,d\right)$	
`ν(θ);` `N(Θ)`	$\nu\left(\theta\right)$	
`Ω20(α★, d)`	$\omega_{2,0}\left(\boldsymbol{\alpha}^*,d\right)$	
`Ω31(α★, d)`	$\omega_{3,1}\left(\boldsymbol{\alpha}^*,d\right)$	

Table 1.3: Functions in `Sausage Conjecture calculations.jl`:
More integrals for the volume lower bounds.

Function in code	Corresponding function in Part II	Description/Explanation
`Λ̂_endpoint_BHW1994(d)`	$\widehat{\lambda}_{\mathrm{e}}^{\mathrm{BHW1994}}(d)$	
`Λ̂_A_BHW1994(d)`	$\widehat{\lambda}_{\mathrm{A}}^{\mathrm{BHW1994}}(d)$	
`Λ_M_BHW1994(α★, d)`	$\widehat{\lambda}_{\mathrm{M}}^{\mathrm{BHW1994}}(\widehat{\alpha}_{2,1}, d)$	
`Λ_L_BHW1994(α★, ε, d)`	$\lambda_{\mathrm{L}}^{\mathrm{BHW1994}}(\widehat{\alpha}_{2,1}, \varepsilon, d)$	
`Λ_endpoint(α★, d)`	$\widehat{\lambda}_{\mathrm{e}}(\widehat{\alpha}_{2,1}, d)$	
`Λ_A(α★, d)`	$\lambda_{\mathrm{A}}(\widehat{\alpha}_{2,1}, d)$	
`Λ_S(A★, d)`	$\lambda_{\mathrm{S}}(\widehat{\alpha}_{2,1}, \widehat{\alpha}_{3,1}, d)$	
`Λ_MS(A★, d)`	$\lambda_{\mathrm{MS}}(\widehat{\alpha}_{2,1}, \widehat{\alpha}_{3,1}, d)$	
`Λ_LS(α★, d)`	$\lambda_{\mathrm{LS}}(\widehat{\alpha}_{2,1}, \widehat{\alpha}_{3,1}, d)$	
`Λ0_3(A, d)`	$\lambda_0^{(3)}(\boldsymbol{\alpha}, d)$	
`Λ1_3(A, d)`	$\lambda_1^{(3)}(\boldsymbol{\alpha}, d)$	
`Λ2_3(A, α★, d)`	$\lambda_2^{(3)}(\boldsymbol{\alpha}, \alpha_3^*, d)$	
`Λ3_3(A, α★, d)`	$\lambda_3^{(3)}(\boldsymbol{\alpha}, \alpha_3^*, d)$	
`Λ123_3(A, α★, d)`	$\lambda_{1,2,3}^{(3)}(\boldsymbol{\alpha}, \alpha_3^*, d)$	
`Λ0123_3(A, α★, d)`	$\lambda_{0,1,2,3}^{(3)}(\boldsymbol{\alpha}, \alpha_3^*, d)$	
`Λ_LL(α★, d)`	$\lambda_{\mathrm{LL}}(\boldsymbol{\alpha}, \alpha_3^*, d)$	
`Λ_LL0(α★, d)`	$\lambda_{\mathrm{LL0}}(\boldsymbol{\alpha}, \alpha_3^*, d)$	
`Λ_min_BHW1994(α★, d)`	$\min\left\{\lambda_{\mathrm{M}}^{\mathrm{BHW1994}}(\alpha_2^*, d),\ \lambda_{\mathrm{L}}^{\mathrm{BHW1994}}(\alpha_2^*, d)\right\}$	
`Λ_min_BH1998(α★, d)`	$\min\left\{\lambda_{\mathrm{A}}(\widehat{\alpha}_{2,1}, d),\ \lambda_{\mathrm{S}}\left(\frac{\pi}{3}, \frac{\pi}{3}, d\right),\ \lambda_{\mathrm{MS}}(\alpha_2^*, \alpha_3^*, d),\ \lambda_{\mathrm{LS}}(\alpha_2^*, \alpha_3^*, d),\ \lambda_{\mathrm{LL}}(\alpha_3^*, d)\right\}$	
`Λ_min(α★, d)`	$\widehat{\lambda}_{\min}(\boldsymbol{\alpha}^*, d)$	

`endpoints_2(α★, d);`
`endpoints_1(α★, d);`
`endpoints_0(α★, d)`
	$n_d(\boldsymbol{\alpha}^*(d), k)$	For $k=2$, $k=1$, and $k=0$ respectively.
		The corresponding functions for `BHW1994` and `BH1998` also exist.

`volumeLowerBoundList` `(paper, d, ε, α★)`	[N/A]	Outputs the excess volumes, $\lambda_k^{(3)}$ components, and upper bounds for $n_d(\boldsymbol{\alpha}^*(d), k)$. "`paper`" can be "`BHW1994`", "`BH1998`", or "`Thesis`".

Table 1.4: Functions in `Sausage Conjecture calculations.jl`: The volume lower bounds (finally).

1.2 `Sausage Catastrophe calculations.jl` function tables

Function in code	Corresponding function in Part III	Description/Explanation
`I(x)`	$I(x)$	Converts a floating-point number x to a thin interval $I(x)$ containing x.
`lo(x)`	$\underline{I(x)}$	The left endpoint of $I(x)$.
`hi(x)`	$\overline{I(x)}$	The right endpoint of $I(x)$. We always have `lo(x)` $\leq x \leq$ `hi(x)`.
`P(X, power)`	$X \cdot X \cdot \cdots \cdot X$	`power` should be a natural number.
`vol_sausage(n);` `Vol_sausage(N)`	$\mathrm{vol}\left(S_n^4\right)$	
`g_Y(m);` `G_Y(M)`	$g_{D_4}(Y_m)$	The "D_4" is omitted because every use of `G_Y` in the code is for D_4.
`h_(h);` `H_(H)`	$\left(\lVert\mathbf{h}\rVert_1, \lVert\mathbf{h}\rVert_2^2, \lVert\mathbf{h}\rVert_3^3, \lVert\mathbf{h}\rVert_4^4\right)$	Coded in Julia as an array.
`vol_Y(m);` `Vol_Y(m)`	$\mathrm{vol}(Y_m)$	
`g_t3h_Y(m, h);` `G_t3h_Y(m, h)`	$g_{D_4}\left(t_{\mathbf{h}}^3(Y_m)\right)$	
`vol_t3h_Y(m, h);` `Vol_t3h_Y(m, h)`	$\mathrm{vol}\left(t_{\mathbf{h}}^3(Y_m) + B^d\right)$	
`δ4_sausage(n)`	$\delta\left(S_n^4\right)$	The "4" in `δ4` is from the dimension.
`δ4_t3h_Y(m, h)`	$\delta\left(t_{\mathbf{h}}^3(Y_m) \cap D_4\right)$	
`δ̃4_t3h_Y(m, h, k)`	$\widetilde{\delta}\left(p_k\left(t_{\mathbf{h}}^3(Y_m) \cap D_4\right)\right)$	
`ṽ(m, h);` `Ṽ(m, h)`	$\widetilde{v}\left(t_{\mathbf{h}}^3(Y_m), \mathbf{h}\right)$	
`r̃(m, h);` `R̃(M, H)`	$\widetilde{r}\left(t_{\mathbf{h}}^3(Y_m), \mathbf{h}\right)$	
`L̃(m, h);` `L̃_IA(M, H)`	$\widetilde{L}(m, \mathbf{h})$	The minimum and maximum values of $\widetilde{L}\left(t_{\mathbf{h}}^3(Y_m) \cap D_4\right)$. "`IA`" = `IntervalArithmetic`.

Table 1.5: Functions in `Sausage Catastrophe calculations.jl`: Preliminaries.

Function in code	Corresponding function in Part III	Description/Explanation
`check_n(n, ...)`; `Check_n(n, ...)`	[N/A]	
`h_max(m)`	$\left\lfloor \frac{m-1}{2} \right\rfloor$	The maximum values of h_i in the triple facet truncation.
`check_n_range (n_start, n_end, ...)`; `Check_n_range (n_start, n_end, ...)`	[N/A]	Applies `check_n` to all integers n from n_{start} to n_{end}.

Table 1.6: Functions in `Sausage Catastrophe calculations.jl`: Checking values of n.