ECON 299 Cheatsheet

GDP

Nominal GDP

$$\sum_{i=1}^{n} = p_i \times q_i \tag{1}$$

Real GDP

$$\sum_{i=1}^{n} = p_{baseyear} \times q_i \tag{2}$$

Real, Nominal, Price Index

Real Variable

$$\frac{NominalVariable}{Price\ Index} \times 100 \tag{3}$$

Nominal Variable

$$\frac{Real\ Variable}{100} \times Price\ Index \tag{4}$$

Price Index

$$\frac{Nominal\ Variable}{RealVariable} \times 100 \tag{5}$$

Normalized Price Index$_t$

$$\frac{\text{Raw Price Index}_t}{\text{Raw Price Price}_{BaseYear}} \times 100 \tag{6}$$

LPI

LPI$_t$

$$\frac{\sum_{i=0}^{m} P_{it} \times Q_{ib}}{\sum_{i=0}^{m} P_{ib} \times Q_{ib}} \tag{7}$$

Where:
m - is the basket of goods
t - is the current time period
b - is the base time period

LCL$_{t-1,t}$

$$\frac{\sum_{i=0}^{m} P_{it} \times Q_{it-1}}{\sum_{i=0}^{m} P_{it-1} \times Q_{it-1}} \tag{8}$$

LCPI$_t$

$$LCPI_{t-1} \times LCL_{t-1,t} \tag{9}$$

Where LCPI$_1$ = 100

Joining Price Indexes

Conversion factor

The price index must overlap

$$\frac{\text{PI value from new base year}}{\text{PI value from old base year}} \tag{10}$$

Conversion

Multiply the old PI with the conversion factor to get the new PI.

Growth Rates

$$\left(\frac{X_t}{X_{t-1}} - 1\right) \cdot 100 \tag{11}$$

$$\ln\left(\frac{X_t}{X_{t-1}}\right) \cdot 100 \tag{12}$$

Only for small growth rates $< 5\%$.

Financial Calculation

Real Interest Rate

$$r_{norm} - \text{inflation rate} \tag{13}$$

Compound Interest

$$S = P(1+r)^t \tag{14}$$

$$S = P\left(1 + \left(\frac{r}{m}\right)\right)^{m \cdot p} \tag{15}$$

Where:
S: Value of Asset
P: Principle Amount
r: Interest Rate
t: # of periods
m: The frequency of the yearly rate

Effective Rate r_e

Normalize to the annual rate
The first is only for the annual rate

$$\left(1 + \frac{r}{m}\right)^m \tag{16}$$

For all other periods of rates

$$(1+r)^m - 1 \tag{17}$$

Present Value

$$\frac{S}{(1+r)^t} \tag{18}$$

Future Payments

Receive the money at the beginning of each year.

$$\frac{A\left(1 - \frac{1}{1+r}^n\right)}{1 - \left(\frac{1}{1+r}\right)} \tag{19}$$

Receive the money at the end of each year.

$$\frac{A\left(1 - \frac{1}{1+r}^n\right)}{r} \tag{20}$$

Where:
A: is the annual value of Payments
n: is the number of payments
r: is the annual interest rate

Arithmetic μ

$$\frac{\sum_{i=1}^{n} x_i}{n} \tag{21}$$

Geometric μ

$$(\Pi_{i=1}^{n} x_i)^{1/n} \tag{22}$$

Geometric Interest rate

subtract the geometric μ by 1

UCC$_t$

$$P_{kt}\left(d_t + r_t - \left(\frac{P_{kt+1}}{P_{kt}} - 1\right)\right) \tag{23}$$

Where:
P_{tk}: Purchase price of capital in period t
d_t: rate of deprecation (annual)
r_t: interest rate or RoR on alternative asset
$\left(\frac{P_{kt+1}}{P_{kt}} - 1\right)$: Capital gain or loss

Models

Growth Formula

$$g = \left(\frac{X_t}{X_{t-1}} - 1\right) \tag{24}$$

$$X_t = X_0(1+g)^t \tag{25}$$

$$\ln(X_t) = \ln(X_0) + g \cdot t \tag{26}$$

Linear time trend model

$$X_t = \beta_1 + \beta_2 t \tag{27}$$

Quadratic time trend model

$$X_t = \beta_1 + \beta_2 t + \beta_3 t^2 \tag{28}$$

Lin-Log model

$$X_t = \beta_1 + \beta_2 \ln(t) \tag{29}$$

Reciprocal model

$$X_t = \beta_1 + \frac{\beta_2}{t} \tag{30}$$

Log-Log model

$$\ln|X_t| = \beta_1 + \beta_2 \ln|t| \tag{31}$$

Cyclical model

$$X_t = \beta_1 + \beta_2 \sin\left(\frac{2\pi}{t}\right) + \beta_3 \cos\left(\frac{2\pi}{t}\right) \tag{32}$$

Stat

Economic model/Population Regression Function

$$Y_i = \beta_1 + \beta_2 X_i + \varepsilon_i \tag{33}$$

Sample Regression Function (OLS)

$$\hat{Y_i} = \hat{\beta_1} + \hat{\beta_2} X_i \tag{34}$$

$$\hat{\beta_2} = \frac{\sum(X_i - \bar{X})(Y_i - \bar{Y})}{\sum(X_i - \bar{X})^2} \tag{35}$$

$$\hat{\beta_1} = \bar{Y} - \hat{\beta_2}\bar{X} \tag{36}$$

Expected Value / Mean

Population:

$$\mu_Y = E(x) = \sum_{i=1}^{n} y_i \cdot pdf(y_i) \tag{37}$$

$$\mu_Y = E(k) = k \tag{38}$$

$$\mu_Y = E(a + bX) = a + bE(x) \tag{39}$$

$$\mu_Y = E(x^a) = \sum_{i=1}^{n} x_i^a \cdot f(x) \tag{40}$$

Sample:

$$\bar{Y} = \frac{\sum_{i=1}^{N} Y_i}{N} \tag{41}$$

Variance

Population:

$$\sigma_y^2 = \sum_{i=1}^{n} (y_i - E(y))^2 \cdot f(y_i) \tag{42}$$

$$\sigma_k^2 = 0 \tag{43}$$

$$\sigma_{a+bW}^2 = b^2 \sigma_W^2 \tag{44}$$

$$\sigma_{a+bW+cV}^2 = b^2 \sigma_W^2 + c^2 \sigma_V^2 + 2bc\ Cov(W, V)$$

Sample:

$$S_Y^2 = \frac{\sum_{i=1}^{N} (Y_i = \hat{Y})^2}{N - 1} \tag{45}$$

Std Dev

Population:

$$\sigma_Y = \sqrt{\sigma_Y^2} \tag{46}$$

Sample:

$$S_Y = \sqrt{S_Y^2} \tag{47}$$

Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \tag{48}$$

Co-variance(W,V)

Population:

$$\sum_{i=1}^{N} \sum_{j=1}^{N} (V_i - E(V))(W_j - E(W)) f(V_i, W_j) \tag{49}$$

Sample:

$$\frac{\sum_{i=1}^{N} (V_i - \bar{V})(W_i - \bar{W})}{N - 1} \tag{50}$$

Correlation(V,W)

Population:

$$\sigma_{wv} = \frac{Cov(V, W)}{\sigma_V \sigma_W} \tag{51}$$

Sample:

$$r_{wv} = \frac{Cov(V, W)}{S_V S_W} \tag{52}$$

Statistical Independence

$$f(w|any\ v) = f(w) \wedge f(v|any\ w) = f(v) \tag{53}$$

$$\Downarrow$$

$$Cov(v, w) = 0 \wedge Corr(v, w) = 0$$

Std Err

$$\frac{S_x}{\sqrt{N}} \tag{54}$$

CI

z-score

$$\frac{x - \mu_x}{\sigma} \tag{55}$$

t-score

$$\frac{x - \mu_x}{Std\ err} \tag{56}$$

With $df(n - 1)$

CI to a %

$$\bar{X} \pm t^* \cdot SE \tag{57}$$

Calculus

Optimization

3 steps for Optimization

1. FOC: find the roots of y'.
2. SOC: eval y" to confirm max/min point.
3. find coordinates.

Multi-variable Calculus

Second Cross Partial Derivatives

$\frac{\partial^2 y}{\partial a \partial b}$ means we first calculate $\frac{\partial y}{\partial a}$ then calculate $\frac{\partial y}{\partial b}$

Elasticity

$$\frac{\partial Q}{\partial P} \frac{P}{Q} = \text{Price slope} \times \frac{P}{Q} \tag{58}$$

Replace P with other variables to find the elasticity for that variable.

Marginal Variables

$$Q = AL^\beta K^\alpha \tag{59}$$

$$\frac{\beta(AL^\beta K^\alpha)}{L} = \beta \frac{Q}{L} = \beta \times AvgPL = MPL \tag{60}$$

$$\frac{\alpha(AL^\beta K^\alpha)}{K} = \alpha \frac{Q}{K} = \alpha \times AvgPK = MPK \tag{61}$$

Optimization

Unconstrained

If all but one is a constant then it just a single variable optimization.
If multiple variable then:
- FOC: Find the roots for all partial derivatives. Use linear algebra to find the parameter values for the roots.
-SOC: Make sure that the second-order partial derivatives are either all positive or all negative and that there is no saddle point.

$$\left(\frac{\partial^2 z}{\partial x^2}\right)\left(\frac{\partial^2 z}{\partial y^2}\right) - \left(\frac{\partial^2 z}{\partial x \partial y}\right)^2 > 0 \tag{62}$$

-Find the coordinates of the optimization using the values found from the FOC.

Constraint

Internalize the constraint. i.e. use algebra to have a single variable on one side and all others and constants on the other. Then substitute it into the function that is being optimized. From there its the same as optimizing for a single variable function (For the most part).

Quadratic Equation

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{63}$$