1-(a)

Because of degrees of polynomials (d)

size of w is $\langle 1 \times d \rangle$.

And because of numbers of data points (n),

size of y is $\langle 1 \times n \rangle$.

1-(b)

$$A = \begin{pmatrix} 1, & x_1, & x_1^2, & x_1^3 & \cdots, & x_1^d \\ 1, & x_2, & & & \cdots & x_2^d \\ \vdots & & & & & \\ 1, & x_n, & & & \cdots & x_n^d \end{pmatrix}$$

A's number of row is n, and

number of column is $d+1$.

Therefore, size of matrix A is

$\langle n \times d+1 \rangle$.

1-(c)

Assume the matrix B

$$B = \begin{pmatrix} 1 & 1 & \cdots & 1 & 1 \\ a_1 & a_2 & \cdots & a_n & a_{n+1} \\ \vdots & & & & \\ a_1^{n-1} & a_2^{n-1} & \cdots & a_n^{n-1} & a_{n+1}^{n-1} \\ a_1^n & a_2^n & & a_n^n & a_{n+1}^n \end{pmatrix}$$

subtracting a_1 times the ith row to the $i+1$-th row, can get matrix

$$\begin{pmatrix} 1 & 1 & \cdots & 1 & 1 \\ 0 & a_2 - a_1 & \cdots & a_n - a_1 & a_{n+1} - a_1 \\ \vdots & & \ddots & \vdots & \vdots \\ 0 & a_2^n - a_1 a_2^{n-1} & \cdots & a_n^n - a_1 a_n^{n-1} & \end{pmatrix}$$

Expanding by the first column and factoring $a_i - a_1$ from the i-th column for $i = 2 \cdots n+1$, can get determinant

$$= \prod_{j=2}^{n+1} (a_j - a_1) \det \begin{pmatrix} 1 & 1 & \cdots & 1 \\ a_2 & a_3 & \cdots & a_{n+1} \\ \vdots & \vdots & & \vdots \\ a_2^{n-1} & a_3^{n-1} & \cdots & a_{n+1}^{n-1} \end{pmatrix}$$

Due to inductive hypothesis

$$= \prod_{j=2}^{n+1} (a_j - a_1) \prod_{2 \le i < j \le n} (a_j - a_i)$$

$$= \prod_{1 \le i < j \le n+1} (a_j - a_i)$$

So the $\det B = \prod_{1 \le i < j \le n} (a_j - a_i)$

Therefore, $\det A = \prod_{1 \le i < j \le d+1} (x_j - x_i)$

1-(d)

For all of i, j that meet $1 \le i < j \le d+1$,

$\lambda_j \neq \lambda_i$.

1-(e)

$\det A$ is nonzero, so A is invertible;

the system $Aw = y$ has the unique solution

$$w = A^{-1} y.$$

2.

Because A is not square matrix,

we can use pseudo inverse.

A has linearly independent columns, so

$A^T A$ is invertible.

We know that $A^{\dagger} = (A^T A)^{-1} A^T$.

In $Aw = y$,

$$A A^T w = A^T y$$

$$w = (A^T A)^{-1} A^T y$$

$$\langle w = A^{\dagger} y \rangle$$