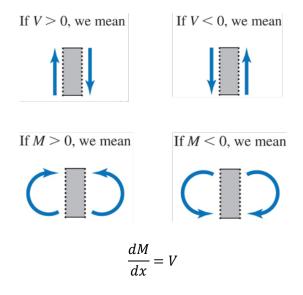
Loads on Machines

Force Equilibrium	$\sum F_x = 0$,	$\sum F_y = 0, \qquad \sum F_z = 0$
Moment Equilibrium	$\sum M_{\chi}=0$,	$\sum M_y = 0, \qquad \sum M_z = 0$
Rotational Velocity	$V = R\omega = 2\pi Rn$	 V: linear velocity R: distance from rotation center ω: angular velocity, radians per unit time n: rotational velocity, revolutions per unit time
Power (General)	$\dot{W} = FV$	F: Applied force V: linear velocity in direction of applied force
Power (Imperial units)	$\dot{W} = \frac{FV}{33,000}$ $= \frac{2\pi Tn}{33,000} = \frac{Tn}{5252}$	 W: horsepower (hp) F: pounds (lbs) V: velocity (feet per minute, fpm) T: torque (lb-ft) n: rotational speed, revs per minute (rpm)
Power (SI units)		 W: kilowatts (kW) F: Newtons (N) V: velocity (m/s) T: torque (N-m) ω: angular speed (radians/sec) n: rotational speed, revs per minute (rpm)

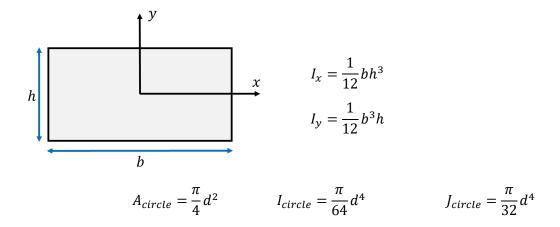
Shear and moment relations in beams



Clutches & Brakes

Torque in a clutch with constant wear rate approach	$T = \pi p_{max} r_i f(r_0^2 - r_i^2)$ or $T = F f(\frac{r_o + r_i}{2}) N$	
Annulus dimensions for highest torque, constant wear rate approach	$r_i = 0.58r_o$	
General dimensions for short- shoe drum brakes	Shoe (block) Direction of rotation	
Force and torque relations for self-energizing short shoe drum brakes	$N = \frac{Fc}{b - fa}$	$T = \frac{fFcr}{b - fa}$
Force and torque relations for de-energizing short shoe drum brakes	$N = \frac{Fc}{b + fa}$	$T = \frac{fFcr}{b + fa}$
Condition for self-locking of a short-shoe drum brake	b :	≤ fa

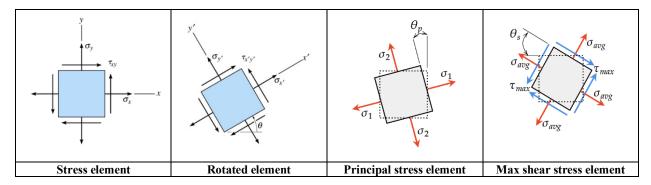
Geometry and moments of inertia



Common stress formulas

Normal stress in axially loaded member	$\sigma = \frac{P}{A}$
Bending stress in beam with bending moment	$\sigma = -\frac{My}{I}$ $ \sigma_{\max} = \frac{M}{z}$ $z = \frac{I}{c}$
Shear stress in cylindrical shaft with applied torque	$\tau = \frac{Tr}{J} \qquad \qquad \tau_{max} = \frac{16T}{\pi d^3}$
Transverse shear stress in beams (max shear stress for a solid circular and solid rectangular cross-section)	$\tau_{av} = V/A$ $\tau_{av} = V/A$ V $N.A.$ $\tau_{max} = \frac{4}{3}(V/A)$ $\tau_{max} = \frac{3}{2}(V/A)$

Stress transformations and principal stresses for a 2D stress state



Principal Stress/Stress transformation equations

Principal stresses	$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\tau_{xy}^2 + \left(\frac{\sigma_x - \sigma_y}{2}\right)^2}$ $\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\tau_{xy}^2 + \left(\frac{\sigma_x - \sigma_y}{2}\right)^2}$
Maximum in-plane shear stress	$ au_{max} = \sqrt{ au_{xy}^2 + \left(\frac{\sigma_x - \sigma_y}{2}\right)^2}$
Principal and maximum shear stress angles	$\theta_p = \frac{1}{2} \tan^{-1} \left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y} \right)$ $\theta_s = \theta_p \pm 45^o$

Mohr's Circle

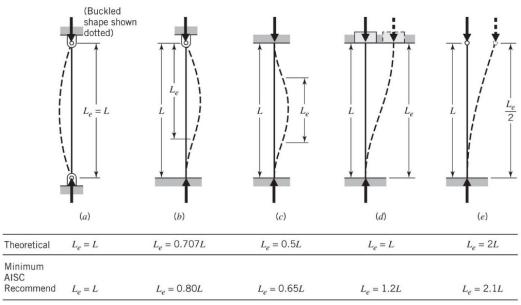
Mohr's circle center location	$C = \sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{\sigma_1 + \sigma_2}{2}$
Mohr's circle radius	$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$
Principal stresses	$\sigma_1 = \sigma_{avg} + R$ $\sigma_2 = \sigma_{avg} - R$
Maximum in-plane shear stress	$ au_{max} = R$

Stress concentrations

Maximum stress at the site of the notch	$\sigma_{max} = K_t \sigma_{nom}$
Residual stress	$\sigma_{residual} = \sigma_{load} - \sigma_{elastic}$

Buckling

Radius of gyration of a column	$\rho = \sqrt{\frac{I}{A}}$
Euler critical buckling stress Applicable when $\frac{L_e}{\rho} > \sqrt{\frac{2\pi^2 E}{S_V}}$	$S_{cr} = \frac{\pi^2 E}{\left(\frac{L_e}{2}\right)^2}$
Johnson critical buckling stress	(ρ)
Applicable when $\frac{L_e}{\rho} < \sqrt{\frac{2\pi^2 E}{S_y}}$	$S_{cr} = S_{y} - \left(\frac{S_{y}^{2}}{4\pi^{2}E}\right) \left(\frac{L_{e}}{\rho}\right)^{2}$



Source: From Manual of Steel Construction, 7th ed., American Institute of Steel Construction, Inc., New York, 1970, pp. 5-138.

Impact

Static deflection	$\delta_{st} = \frac{W}{k}$
Impact factor	$\text{IF} = 1 + \sqrt{1 + \frac{2h}{\delta_{st}}} = 1 + \sqrt{1 + \frac{v^2}{g\delta_{st}}}$
Deflection due to impact	$\delta = \delta_{st} imes ext{IF}$
Equivalent static force due to impact	$F_e = W imes ext{IF}$
Kinetic energy of impact	$U = \frac{1}{2}mv^2 = \frac{Wv^2}{2g}$
Effective spring constant for an axially loaded elastic bar	$k = \frac{EA}{L}$

Failure Theories

To avoid failure:

- Maximum normal stress theory: $\sigma_1 < S_{ut}$ and $\sigma_3 > -S_{uc}$
- Maximum shear stress theory (Tresca): $|\sigma_1 \sigma_3| < S_{yt}$
- Maximum distortion energy theory (von Mises): $\sigma_{VM} < S_{yt}$

Three-dimensional stress state:

$$\sigma_{VM} = \sqrt{\frac{\left(\sigma_x - \sigma_y\right)^2 + \left(\sigma_y - \sigma_z\right)^2 + \left(\sigma_x - \sigma_z\right)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2)}{2}}$$

$$\sigma_{VM} = \sqrt{\frac{\left(\sigma_1 - \sigma_2\right)^2 + \left(\sigma_2 - \sigma_3\right)^2 + (\sigma_3 - \sigma_1)^2}{2}}$$

Two-dimensional stress state:

$$\sigma_{VM} = \sqrt{\sigma_x^2 - \sigma_x \sigma_y + \sigma_y^2 + 3\tau_{xy}^2} = \sqrt{\sigma_1^2 - \sigma_1 \sigma_2 + \sigma_2^2}$$

Fracture mechanics

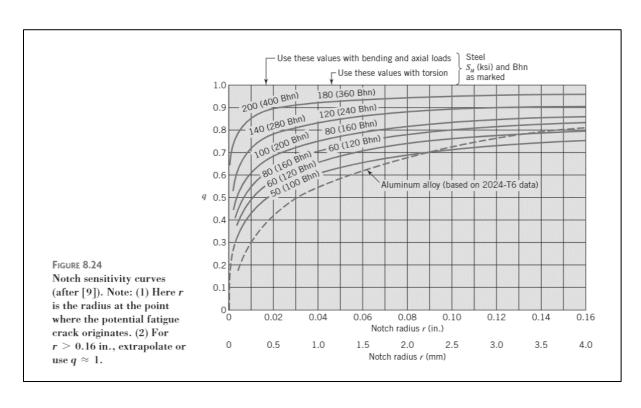
Stress intensity factor *K*:

$$K = Y\sigma\sqrt{\pi c}$$

Failure when $K > K_c$

Fatigue

Stress amplitude	$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$
Mean stress	$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$
Fatigue stress concentration factor	$K_f = 1 + (K_t - 1)q$ $\sigma_a = K_f \sigma_{a,nom}$ $\sigma_m = K_f \sigma_{m,nom}$
To avoid fatigue (zero mean stress)	$\sigma_a < S_n$
To avoid fatigue (non-zero mean stress)	$\frac{\sigma_a}{S_n} + \frac{\sigma_m}{S_u} \le 1$



Fatigue (continued)

Table 8.1 Generalized Fatigue Strength Factors for Ductile Materials (S-N curves)

a. 106-cycle strength (endurance limit)^a

Bending loads: $S_n = S'_n C_L C_G C_S C_T C_R$

Axial loads: $S_n = S'_n C_L C_G C_S C_T C_R$

Torsional loads: $S_n = S'_n C_L C_G C_S C_T C_R$

where S'_n is the R.R. Moore, endurance limit, and

		Bending	Axial	Torsion
C_L	(load factor)	1.0	1.0	0.58
$\overline{C_G}$	(gradient factor): diameter < (0.4 in. or 10 mm)	1.0	0.7 to 0.9	1.0
	$(0.4 \text{ in. or } 10 \text{ mm}) < \text{diameter} < (2 \text{ in. or } 50 \text{ mm})^c$	0.9	0.7 to 0.9	0.9
$\overline{C_S}$	(surface factor)		see Figure 8.13	
$\overline{C_T}$	(temperature factor)	Values are only for steel		steel
	T ≤ 840 °F	1.0	1.0	1.0
	840 °F < T ≤ 1020 °F	1 -	-(0.0032T - 2.69)	88)
$\overline{C_R}$	(reliability factor):d			
	50% reliability	1.000	"	"
	90% "	0.897	"	"
	95% "	0.868	"	"
	99% "	0.814	"	"
	99.9% "	0.753	"	"

b. 10³-cycle strength^{e, f, g}

Bending loads: $S_f = 0.9 S_u C_T$

Axial loads: $S_f = 0.75 S_u C_T$

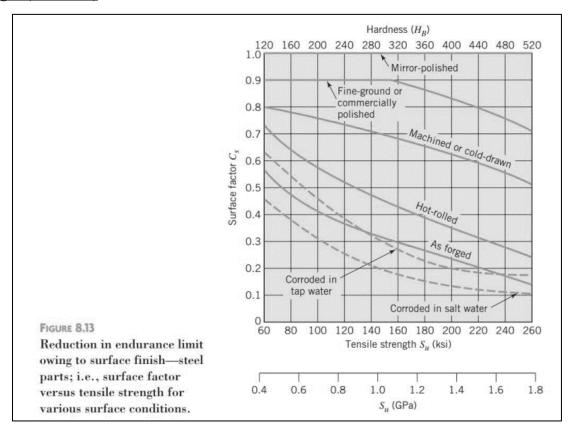
Torsional loads: $S_f = 0.9 S_{us} C_T$

where S_u is the ultimate tensile strength and S_{us} is the ultimate shear strength.

$$S_n' = 0.5 S_u$$

(in ksi, $S_n' \approx 0.25 \cdot \text{Bhn}$;
in MPa, $S_n' \approx 1.73 \cdot \text{Bhn}$)

Fatigue (continued)



Threaded components (General)

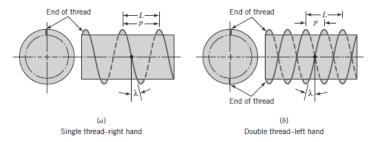


FIGURE 10.1 Helical threads of pitch p, lead L, and lead angle λ .

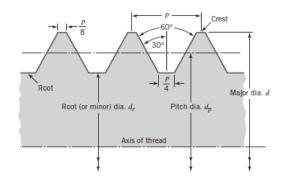


FIGURE 10.2 Unified and ISO thread geometry. The basic profile of the external thread is shown.

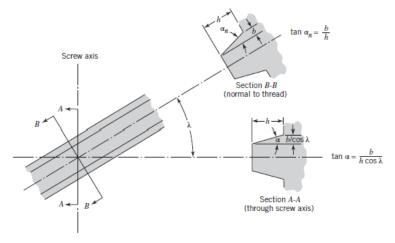


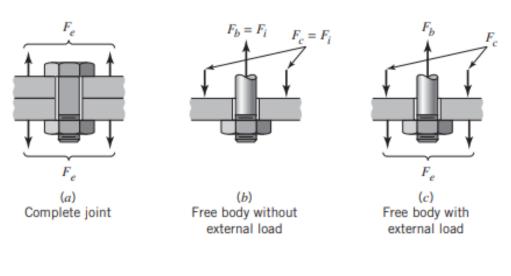
FIGURE 10.7 Comparison of thread angles measured in axial and normal planes (α and α_n).

Thread geometry	$\tan \lambda = \frac{L}{\pi d_m}$
Value of thread angle in the normal plane	$\tan \alpha_n = \tan \alpha \cos \lambda$

Threaded components (nuts and bolts)

Condition for tensile yield of a bolt	$F_{bolt} = A_t S_y$
Condition for shear yield of threads in a nut or bolt	$F_{nut} = \pi d(0.75t)S_{sy} \approx \pi d(0.75t)(0.58S_y)$

Force relationships for bolted connections:



$$F_b = F_i + \frac{k_b}{k_b + k_c} F_e \qquad F_c = F_i - \frac{k_c}{k_b + k_c} F_e$$

Threaded components (nuts and bolts) - continued

Table 10.1 Basic Dimensions of Unified Screw Threads

		C	oarse Threads—	UNC	Fi	ne Threads—Ul	NF
	Major Diameter	Threads	Minor Diameter of External Thread	Tensile Stress Area	Threads	Minor Diameter of External Thread	Tensile Stress Area
Size	d (in.)	per Inch	d_r (in.)	A_t (in. ²)	per Inch	d_r (in.)	A_t (in. ²)
0(.060)	0.0600	_	_	_	80	0.0447	0.00180
1(.073)	0.0730	64	0.0538	0.00263	72	0.0560	0.00278
2(.086)	0.0860	56	0.0641	0.00370	64	0.0668	0.00394
3(.099)	0.0990	48	0.0734	0.00487	56	0.0771	0.00523
4(.112)	0.1120	40	0.0813	0.00604	48	0.0864	0.00661
5(.125)	0.1250	40	0.0943	0.00796	44	0.0971	0.00830
6(.138)	0.1380	32	0.0997	0.00909	40	0.1073	0.01015
8(.164)	0.1640	32	0.1257	0.0140	36	0.1299	0.01474
10(.190)	0.1900	24	0.1389	0.0175	32	0.1517	0.0200
12(.216)	0.2160	24	0.1649	0.0242	28	0.1722	0.0258
1 4	0.2500	20	0.1887	0.0318	28	0.2062	0.0364
5	0.3125	18	0.2443	0.0524	24	0.2614	0.0580
3 8	0.3750	16	0.2983	0.0775	24	0.3239	0.0878
716	0.4375	14	0.3499	0.1063	20	0.3762	0.1187
1 5 16 3 8 7 16 1 2	0.5000	13	0.4056	0.1419	20	0.4387	0.1599
9 16	0.5625	12	0.4603	0.182	18	0.4943	0.203
<u>5</u>	0.6250	11	0.5135	0.226	18	0.5568	0.256
9 16 5 8 3 4 7	0.7500	10	0.6273	0.334	16	0.6733	0.373
7 8	0.8750	9	0.7387	0.462	14	0.7874	0.509
1	1.0000	8	0.8466	0.606	12	0.8978	0.663
$1\frac{1}{8}$	1.1250	7	0.9497	0.763	12	1.0228	0.856
$1\frac{1}{4}$	1.2500	7	1.0747	0.969	12	1.1478	1.073
$1\frac{3}{8}$	1.3750	6	1.1705	1.155	12	1.2728	1.315
$1\frac{1}{2}$	1.5000	6	1.2955	1.405	12	1.3978	1.581
$1\frac{1}{8}$ $1\frac{1}{4}$ $1\frac{3}{8}$ $1\frac{1}{2}$ $1\frac{3}{4}$	1.7500	5	1.5046	1.90			
2	2.0000	$4\frac{1}{2}$ $4\frac{1}{2}$	1.7274	2.50			
$2\frac{1}{4}$	2.2500	$4\frac{1}{2}$	1.9774	3.25			
$2\frac{1}{2}$	2.5000	4	2.1933	4.00			
$2\frac{1}{4}$ $2\frac{1}{2}$ $2\frac{3}{4}$	2.7500	4	2.4433	4.93			
3	3.0000	4	2.6933	5.97			
$3\frac{1}{4}$	3.2500	4	2.9433	7.10			
$3\frac{1}{2}$	3.5000	4	3.1933	8.33			
$3\frac{1}{4}$ $3\frac{1}{2}$ $3\frac{3}{4}$	3.7500	4	3.4433	9.66			
4	4.0000	4	3.6933	11.08			

Threaded components (nuts and bolts) - continued

Table 10.4 Specifications for Steel Used in Inch Series Screws and Bolts

SAE Grade	Diameter d (in.)	Proof Load (Strength) ^a	Yield Strength ^b	Tensile Strength	Elongation, Minimum	Reduction of Area, Minimum (%)	Har	ore dness, kwell Max	Grade Identification Marking on Bolt Head
		S_p (ksi)	S _y (ksi)	S _u (ksi)	(%)				
1	$\frac{1}{4}$ thru $1\frac{1}{2}$	33	36	60	18	35	B70	B100	None
2	$\frac{1}{4}$ thru $\frac{3}{4}$	55	57	74	18	35	B80	B100	None
2	Over $\frac{3}{4}$ to $1\frac{1}{2}$	33	36	60	18	35	B70	B100	None
5	$\frac{1}{4}$ thru 1	85	92	120	14	35	C25	C34	
5	Over 1 to $1\frac{1}{2}$	74	81	105	14	35	C19	C30 }	
5.2	$\frac{1}{4}$ thru 1	85	92	120	14	35	C26	C36	
7	$\frac{1}{4}$ thru $1\frac{1}{2}$	105	115	133	12	35	C28	C34	
8	$\frac{1}{4}$ thru $1\frac{1}{2}$	120	130	150	12	35	C33	C39	

Threaded components (Power Screws)

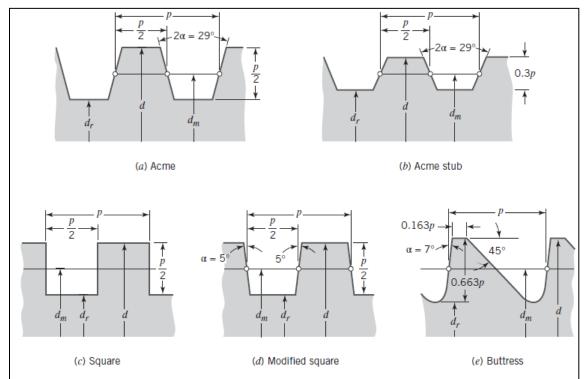


FIGURE 10.4 Power screw thread forms. [Note: All threads shown are external (i.e., on the screw, not on the nut); d_m is the mean diameter of the thread contact and is approximately equal to $(d+d_r)/2$].

	Acme & modified square threads	Square threads $(\alpha_n=0)$
Torque required to lift load W	$T = \frac{Wd_m}{2} \frac{f\pi d_m + L\cos\alpha_n}{\pi d_m\cos\alpha_n - fL} + \frac{Wf_c d_c}{2}$	$T = \frac{Wd_m}{2} \frac{f\pi d_m + L}{\pi d_m - fL} + \frac{Wf_c d_c}{2}$
Friction coefficient required for self-locking	$f \ge \frac{L\cos\alpha_n}{\pi d_m}$	$f \ge \frac{L}{\pi d_m}$
Efficiency	$e = \frac{\cos \alpha_n - f \tan \lambda}{\cos \alpha_n + f \cot \lambda}$	$e = \frac{1 - f \tan \lambda}{1 + f \cot \lambda}$

Threaded components (Power Screws) - continued

Table 10.3 Standard Sizes of Power Screw Threads

		Threads per Inch	
Major Diameter	Acme and	Square and	
d (in.)	Acme Stub ^a	Modified Square	Buttress
1/4	16	10	
5	14		
3 8	12		
3 8	10	8	
$ \begin{array}{r} \frac{1}{4} \\ \frac{5}{16} \\ \frac{3}{8} \\ \frac{3}{8} \\ \frac{7}{16} \end{array} $	12		
7 16 1 2 5 8 3 4 7 8	10		
$\frac{1}{2}$	10	$6\frac{1}{2}$	16
5 8	8	$6\frac{1}{2}$ $5\frac{1}{2}$	16
$\frac{3}{4}$	6	5	16
$\frac{7}{8}$	6	$4\frac{1}{2}$	12
1	5	4	12
$1\frac{1}{8}$	5		
$1\frac{1}{4}$	5	$3\frac{1}{2}$	10
$ \begin{array}{r} 1\frac{1}{8} \\ 1\frac{1}{4} \\ 1\frac{3}{8} \\ 1\frac{1}{2} \\ 1\frac{3}{4} \end{array} $	4		10
$1\frac{1}{2}$	4	3	10
$1\frac{3}{4}$	4	$2\frac{1}{2}$	8
2	4	$2\frac{1}{4}$ $2\frac{1}{4}$	8
$ \begin{array}{r} 2\frac{1}{4} \\ 2\frac{1}{2} \\ 2\frac{3}{4} \end{array} $	3	$2\frac{1}{4}$	8
$2\frac{1}{2}$	3	2	8
$2\frac{3}{4}$	3	2	6
3	2	$1\frac{3}{4}$	6
$3\frac{1}{2}$	2	$1\frac{3}{4}$ $1\frac{5}{8}$	6
4	2	$1\frac{1}{2}$	6
$4\frac{1}{2}$	2		5
5	2		5

Springs

Spring index	$C = \frac{D}{d}$				
Conditions to avoid 2% set under static loading:	$\begin{split} &\tau_s \leq 0.45S_u \text{(ferrouswithout presetting)} \\ &\tau_s \leq 0.35S_u \text{(nonferrous and austenitic stainlesswithout presetting)} \\ &\tau_s \leq 0.65S_u \text{(ferrouswith presetting)} \\ &\tau_s \leq 0.55S_u \text{(nonferrous and austenitic stainlesswith presetting)} \end{split}$				
Spring static loading	$\tau = \frac{8FD}{\pi d^3} K_S = \frac{8F}{\pi d^2} CK_S \text{ where } K_S = 1 + \frac{0.5}{C}$				
Spring fatigue loading	$\tau = \frac{8FD}{\pi d^3} K_w = \frac{8F}{\pi d^2} C K_w$ $K_w = \frac{4C - 1}{4C - 4} + \frac{0.615}{C}$				
Spring stiffness/rate	$k = \frac{F}{\delta} = \frac{d^4G}{8D^3N} = \frac{dG}{8NC^3}$				
Active vs Total turns	$N_t = N + 2$				
Spring ends and solid length	unground ends ground ends $L_S = (N_t + 1)d$ $L_S = N_t d$ $L_S = N_t d$				

Spur Gears

Gear ratio	$\frac{\omega_p}{\omega_g} = -\frac{N_g}{N_p} = -\frac{d_g}{d_p}$			
Diametral pitch	F	$P = \frac{N}{d}$		
Force vectors	$F_r = F_t \tan \phi$	(Driving pinion rotates clockwise) ω_p Pitch circle (pinion) F_t F_r Pitch circle (gear)		
Gear pitch line velocity [English units]	$V = \frac{\pi dn}{12}$	V: [ft/min]d: pitch diameter [in]n: gear rotational speed [rpm]		
	$\dot{W} = \frac{F_t V}{33,000}$	\dot{W} [hp] F_t [lb] V [ft/min]		
Stress in a gear tooth fillet due to bending	$\sigma = \frac{F_t P}{bJ} K_v K_o K_m$			
Endurance limit for infinite life of a gear tooth	$S_n = S'_n C_L C_G C_S k_r k_t k_{ms}$			

Helical gears

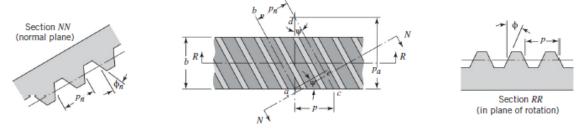
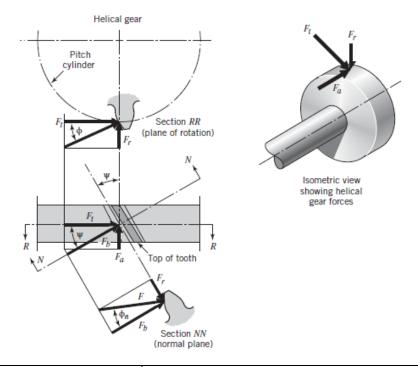
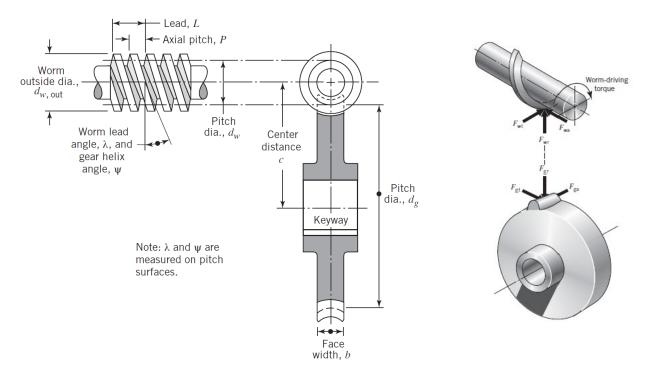


FIGURE 16.4 Portion of a helical rack.



Gear ratio	$rac{\omega_p}{\omega_g} = -rac{N_g}{N_p} = -rac{d_g}{d_p}$
Helical gear angles	$ an\phi_n= an\phi\cos\psi$
Pitch diameter of a helical gear	$d = \frac{N}{P} = \frac{N}{P_n \cos \psi}$
Helical gear axial force	$F_a = F_t \tan \psi$
Helical gear radial force	$F_r = F_t \tan \phi$

Worm gears



Gear pitch diameter	$d_g = \frac{N_g p}{\pi}$ $\omega_w N_g$
Speed ratio	$rac{\omega_w}{\omega_g} = rac{N_g}{N_w}$
Worm gear forces	$F_{g.t} = F_{w,a} = F_n \cos \phi_n \cos \lambda - f \cdot F_n \sin \lambda$ $F_{w.t} = F_{g,a} = F_n \cos \phi_n \sin \lambda - f \cdot F_n \cos \lambda$ $F_{g.r} = F_{w,r} = F_n \sin \phi_n$ $F_{g.r} = F_{g,t} \frac{\sin \phi_n}{\cos \phi_n \cos \lambda - f \sin \lambda}$ $F_{w.r} = F_{w,t} \frac{\sin \phi_n}{\cos \phi_n \sin \lambda + f \cos \lambda}$
Sliding velocity	$V_s = \frac{V_w}{\cos \lambda} = \frac{V_g}{\sin \lambda}$
Efficiency	$e = \frac{F_{g,t}V_g}{F_{w,t}V_w} = \frac{\cos\phi_n\cos\lambda - f\sin\lambda}{\cos\phi_n\sin\lambda + f\cos\lambda}\tan\lambda$ $= \frac{\cos\phi_n - f\tan\lambda}{\cos\phi_n + f\cot\lambda}$

Rolling Element Bearings

Bearing Life <i>L</i> in terms of load capacity <i>C</i>	$L = K_r L_R \left(\frac{C}{F_e K_a}\right)^{3.33}; L_R = 90 \times 10^6$
Effective load F_e for radial bearings, $\alpha = 0^0$	For $0 < \frac{F_t}{F_r} < 0.35$, $F_e = F_r$ For $0.35 < \frac{F_t}{F_r} < 10$, $F_e = F_r \left[1 + 1.115 \left(\frac{F_t}{F_r} - 0.35 \right) \right]$ For $\frac{F_t}{F_r} > 10$, $F_e = 1.176F_t$
Effective load F_e for angular bearings, $\alpha = 25^0$	For $0 < \frac{F_t}{F_r} < 0.68$, $F_e = F_r$ For $0.68 < \frac{F_t}{F_r} < 10$, $F_e = F_r \left[1 + 0.870 \left(\frac{F_t}{F_r} - 0.68 \right) \right]$ For $\frac{F_t}{F_r} > 10$, $F_e = 0.911F_t$

Sliding bearings

