Collective Communication

High-Performance Computing

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MPI Collective Communications

- **Point-to-Point (P2P) Communication** sends data from one point to another, one task sends while another receives.
  - **Blocking** (MPI_Send() and MPI_MPI_Recv()) send routing will only return after it is safe to modify the buffer.
  - **Non-blocking** (MPI_Isend() and MPI_Irecv()) send/receive routines return immediately.
  - Improper use of blocking receive/send will result in **deadlock**, where two processors can't progress because each of them is waiting on the other.
- **Collective communication** is defined as communication between more than two (usually many) processors. A few forms:
  - One-to-many
  - Many-to-one
  - Many-to-many

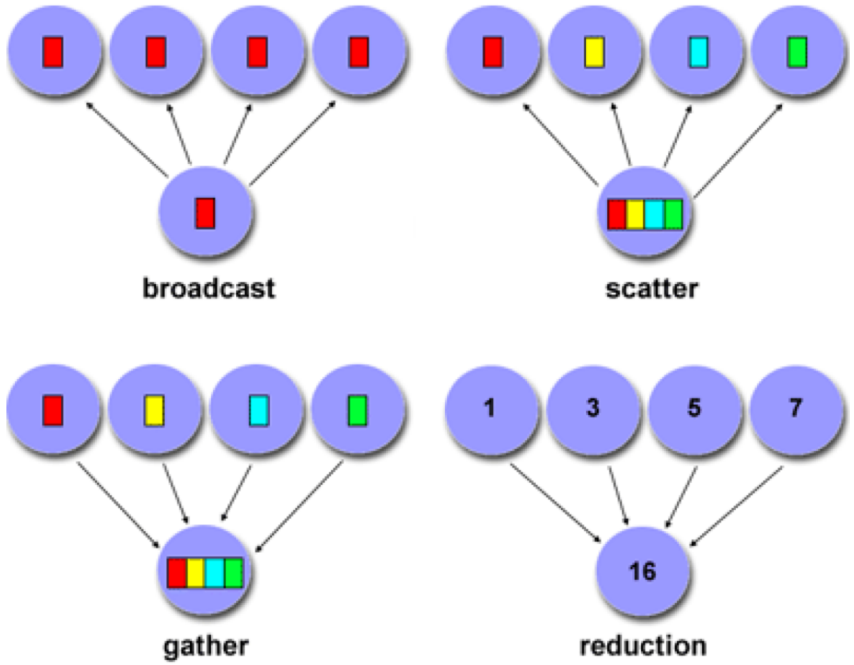
Scope

- Collective communication routines must involve **all** processes within the scope of a communicator.
  - All processes are by default, members in the communicator MPI_COMM_WORLD.
  - Additional communicators can be defined by the programmer.
- Unexpected behavior, including program failure, can occur if even one task in the communicator doesn't participate.
- It is the programmer's responsibility to ensure that all processes within a communicator participate in any collective operations.

Types of Collective Operations

- **Synchronization** - processes wait until all members of the group have reached the synchronization point.

- **Data Movement** - broadcast, scatter/gather, all to all.
- **Collective computation** (reductions) - one member of the group collects data from the other members and performs an operation (min, max, add, multiply, etc.) on that data.



Barrier for Synchronization

- MPI has a special function that is dedicated to synchronizing processes:

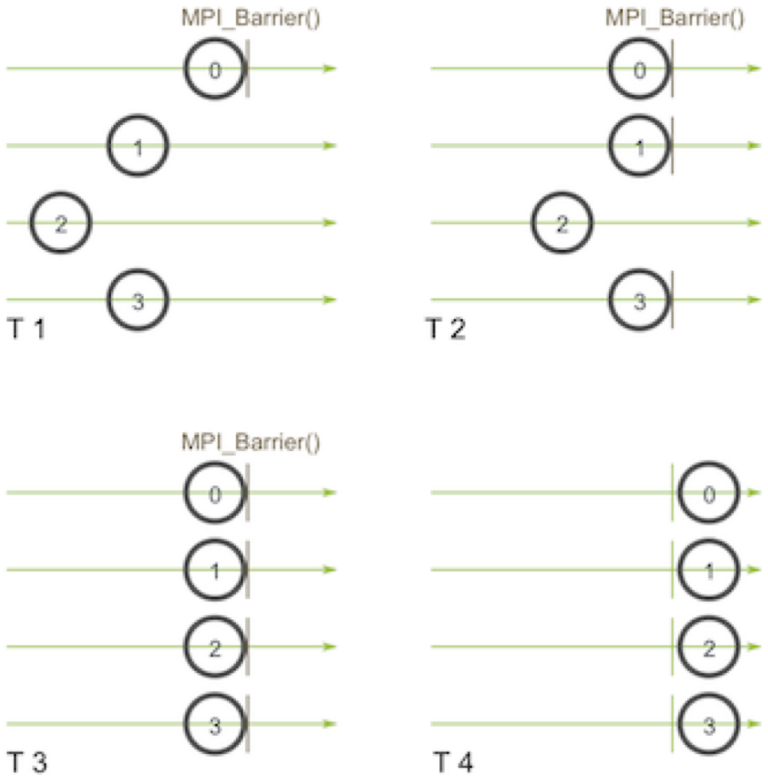
  ```
  MPI_Barrier( MPI_COMM comm )
  ```

  comm: the group of processes

- Blocks until all processes in the group call it.
- The name of the function is quite descriptive - the function forms a barrier, and no processes in the communicator can pass the barrier until all of them call the function.

Barrier for Synchronization

- Process zero first calls MPI_Barrier at the first time snapshot (T 1).
- While process zero is hung up at the barrier, process one and three eventually make it (T 2).
- When process two finally makes it to the barrier (T 3), all of the processes then begin execution again (T 4).

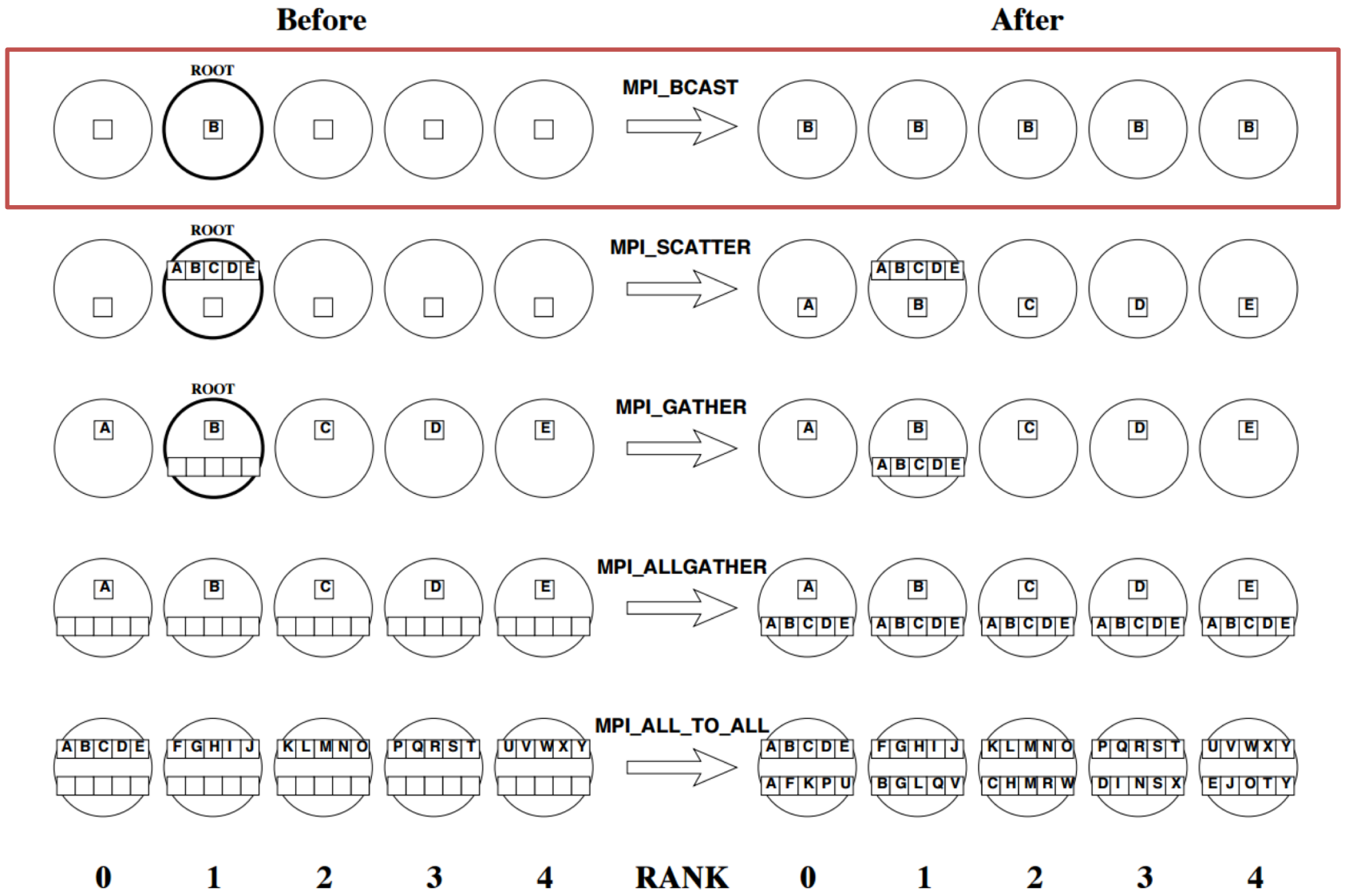


- MPI_Barrier can be useful for many things. One of the primary uses of MPI_Barrier is to synchronize a program so that portions of the parallel code can be timed accurately.

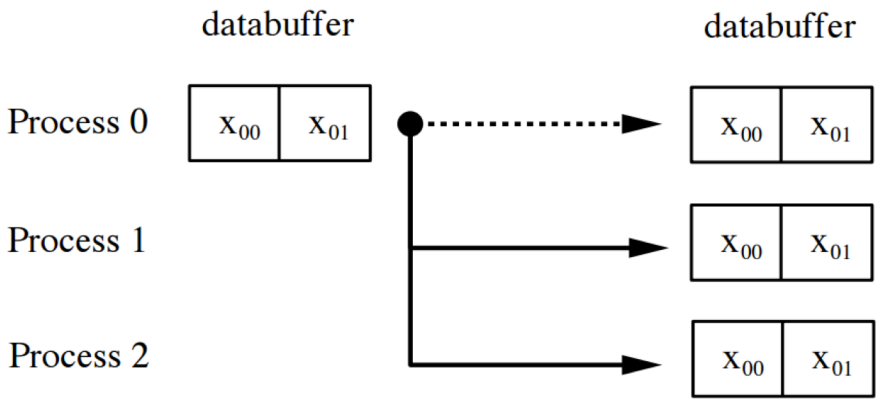
Barrier for Synchronization

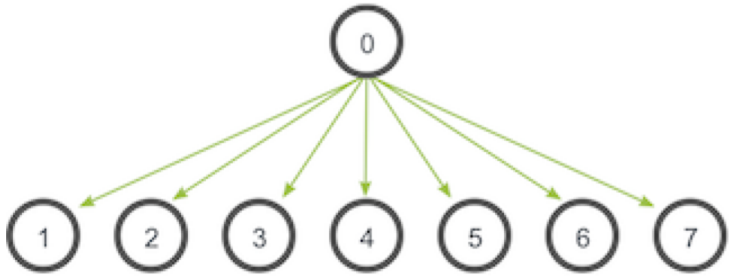
- All collective operations in MPI are blocking, which means that it is safe to use all buffers passed to them after they return.
  - In particular, this means that all data was received when one of these functions returns. (However, it does not imply that all data was sent!) So MPI_Barrier is not necessary (or very helpful) before/after collective operations, if all buffers are valid already.
- Please also note, that MPI_Barrier does not magically wait for non-blocking calls.
  - If you use a non-blocking send/recv and both processes wait at an MPI_Barrier after the send/recv pair, it is not guaranteed that the processes sent/received all data after the MPI_Barrier. Use MPI_Wait (and friends) instead.

Data Movement



Broadcast





- A **broadcast** is one of the standard collective communication techniques.
- During a broadcast, one process sends the same data to all processes in a communicator.
- One of the main uses of broadcasting is to send out user input to a parallel program, or send out configuration parameters to all processes.

Broadcast: MPI_Bcast

- C Syntax: MPI_Bcast(data, count, datatype, root, comm)
- C++ Syntax: MPI::Comm.Bcast(data, count, datatype, root)
- No tag!!
- A broadcast has a specified root process and every process receives one copy of the message from the root.
- All processes must specify the same root (and communicator).
- The root argument is the rank of the root process.
- The buffer, count and datatype arguments are treated as in a point-to-point send on the root and as in a point-to-point receive elsewhere.

Lab: mpi_collective_bcast.cpp

```
# include <iostream>
# include <cstdlib> // has exit(), etc.
# include <ctime>
# include "mpi.h"   // MPI header file

using namespace std;

//*************************************************************************80
int main (int argc, char **argv)
//*************************************************************************80
{
  int nprocs, rank;
  MPI_Init(&argc, &argv);
  MPI_Comm_size(MPI_COMM_WORLD, &nprocs);
  MPI_Comm_rank(MPI_COMM_WORLD, &rank);

  int data = 0;
  cout << "Before Bcast, data = " << data << " in rank = " << rank << endl;

  if (rank == 0) {  // root
    data = 100;
  }

  MPI_Bcast(&data, 1, MPI_INT, 0, MPI_COMM_WORLD);     // MPI_Bcast
  cout << "After Bcast, data = " << data << " in rank = " << rank << endl;

  MPI_Finalize();
  return 0;
}
```

Collective vs. P2P?

- Broadcasting with MPI_Send and MPI_Recv

```
if (rank_id == 0) {
    data = 100;
    for (int i = 1; i < num_proc; i++) {// send our data to everyone
        MPI_Send(&data, count, datatype, i, tag, MPI_COMM_WORLD);
    }
} else {
    // If we are a receiver process, receive the data from the root
    MPI_Recv(&data, count, datatype, 0, tag, MPI_COMM_WORLD status);
}
```

- Broadcasting with MPI_Bcast

```
if (rank_id == 0) {
    data = 100;
}
MPI_Bcast(&data, 1, MPI_INT, 0, MPI_COMM_WORLD);
```

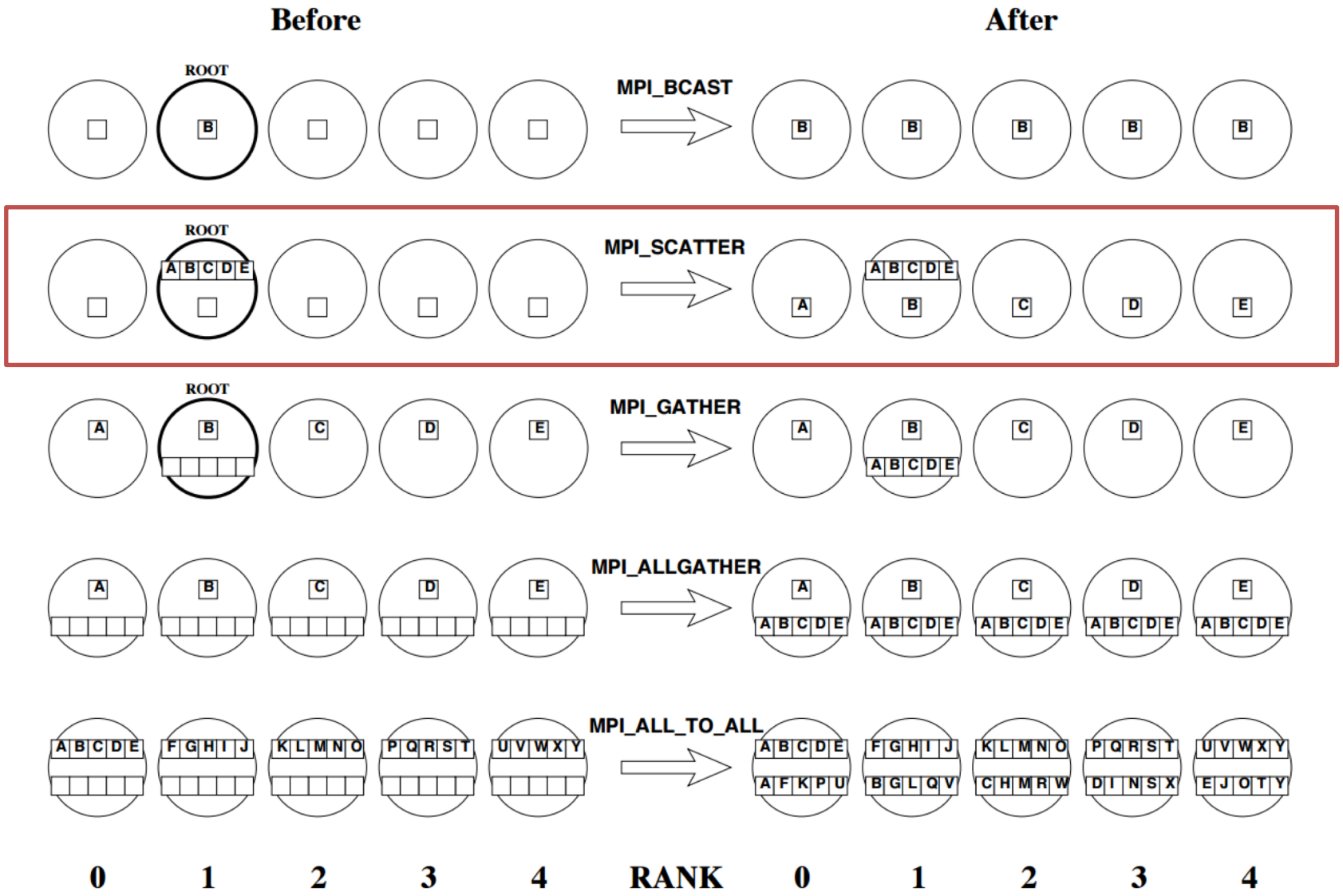
- Time comparison?

Collective vs. P2P?

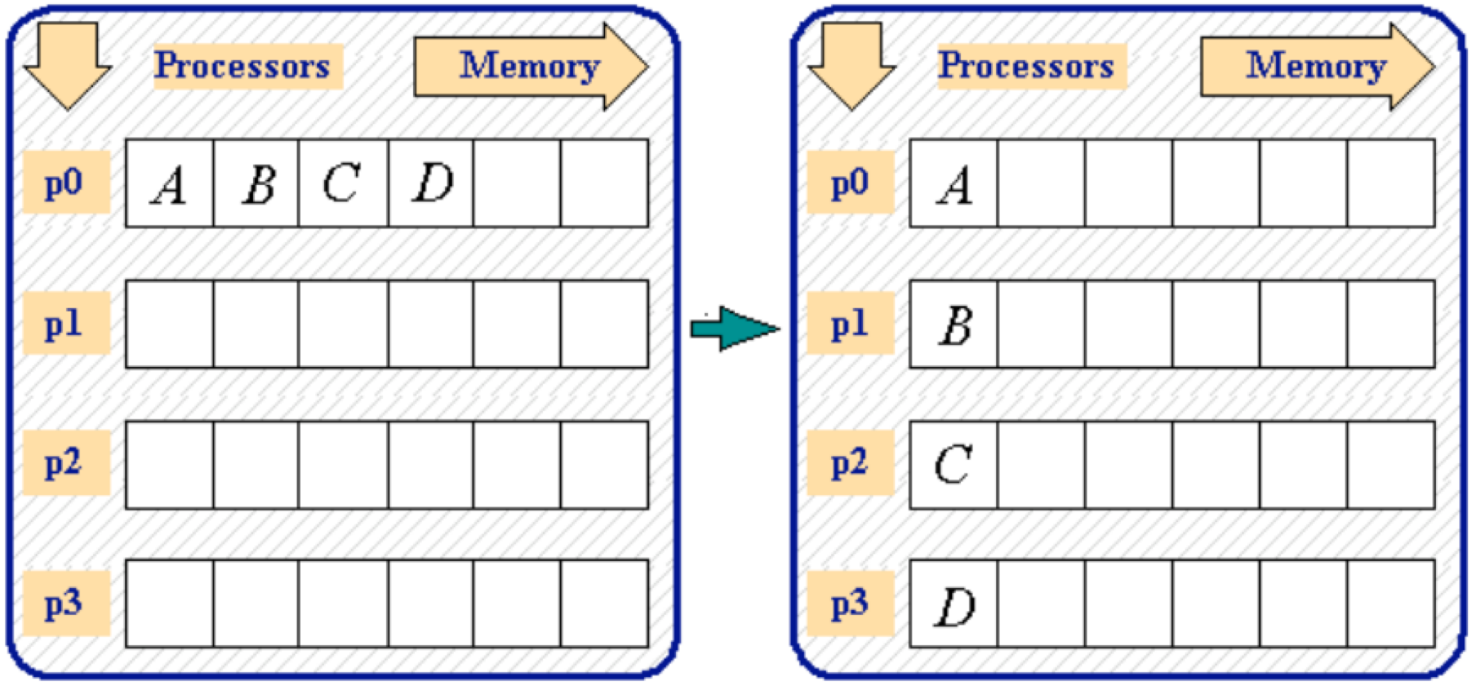
- Time comparison example:

Processors	P2P broadcast	MPI_Bcast
2	0.0344	0.0344
4	0.1025	0.0817
8	0.2385	0.1084
16	0.5109	0.1296

Data Movement

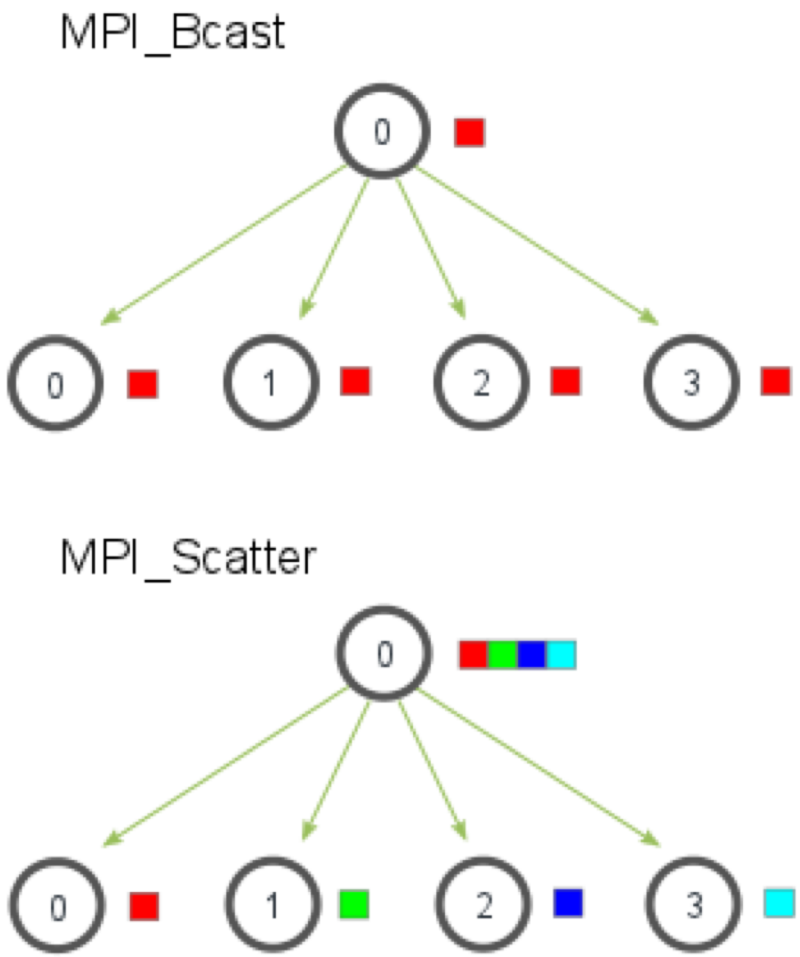


Scatter: MPI_Scatter



Scatter: MPI_Scatter

- MPI_Scatter involves a designated root process sending data to all processes in a communicator.
- MPI_Scatter sends *chunks of an array* to different processes.
- Check out the illustration below for further clarification.
- The main difference from MPI_Bcast is that the send and receive details are in general different and so must both be specified in the argument lists.



Scatter: MPI_Scatter

- C Syntax: MPI_Scatter(sendbuffer, sendcount, sendtype, recvbuffer, recvcount, recvtype, root, comm)
- C++ Syntax: MPI::Comm.Scatter(sendbuffer, sendcount, sendtype, recvbuffer, recvcount, recvtype, root)
- No tag!!
- Note that the sendcount (at the root) is the number of elements to send to each process, not to send in total. Therefore if sendtype = recvtype, sendcount = recvcount.
- The sendbuf, sendcount, sendtype arguments are significant only at the root The buffer, count and datatype arguments are treated as in a point-to-point send on the root and as in a point-to-point receive elsewhere.

Lab: MPI_Scatter()

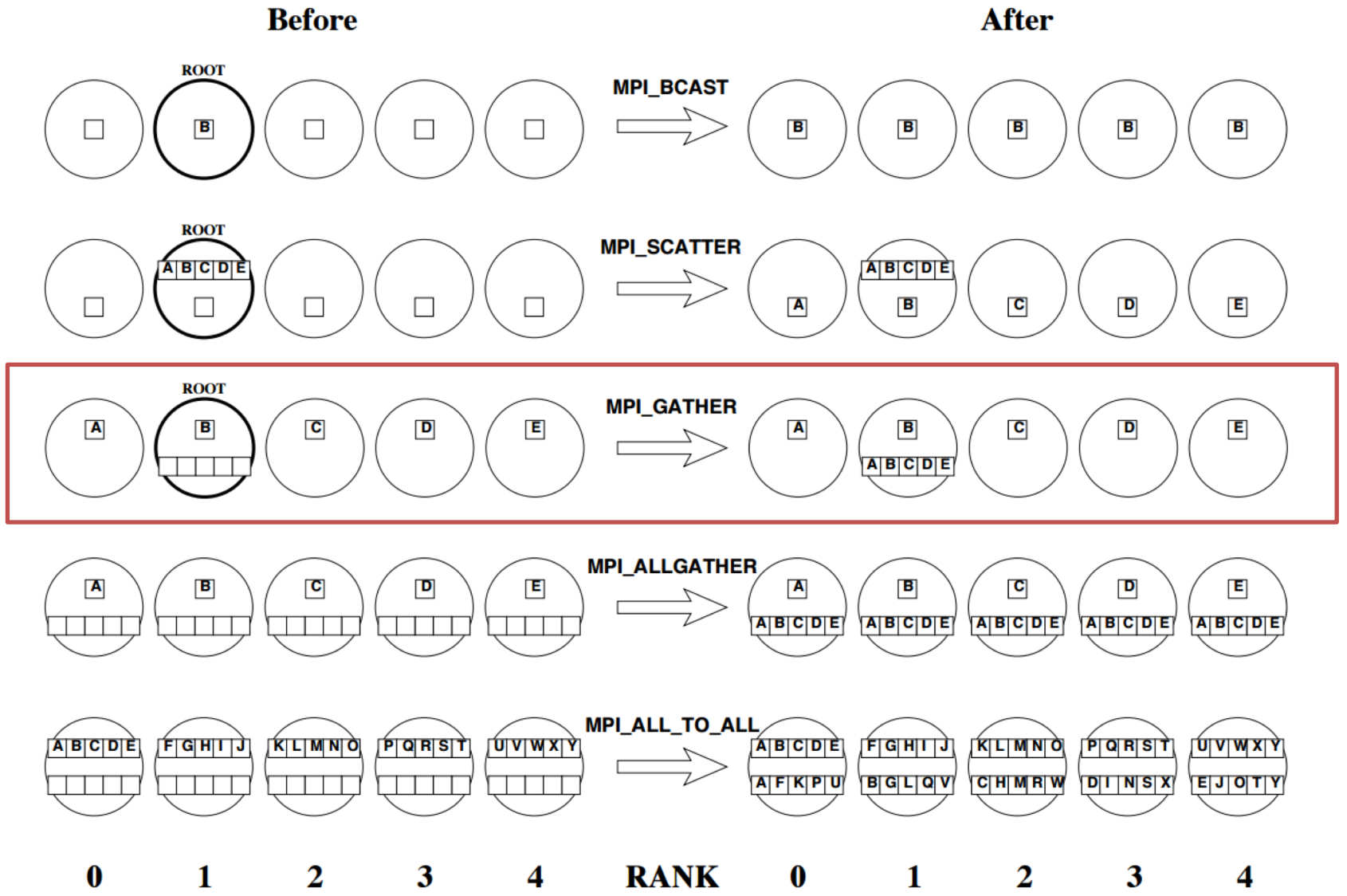
- Scattering with MPI_Scatter()
  - `int data_array[8] = {};`
  - `int recv_buffer[8] = {};`
  - `data_array is assigned to 100,...107 at rank_id = 0`
  - `scattering two elements to other processes that will be saved to recv_buffer.`

Lab: mpi_collective_scatter.cpp

```cpp
# include <iostream>
# include <cstdlib> // has exit(), etc.
# include <ctime>
# include "mpi.h"   // MPI header file

using namespace std;

int main (int argc, char **argv)
{
  int data_array[8] = {};
  int recv_buffer[8] = {};
  int nprocs, rank;

  MPI_Init(&argc, &argv);
  MPI_Comm_size(MPI_COMM_WORLD, &nprocs);
  MPI_Comm_rank(MPI_COMM_WORLD, &rank);

  if (rank == 0) {  // root
    for (int i=0; i<8; i++) {
      data_array[i] = 100+i;
    }
  }

  int send_count = 2;
  int recv_count = 2;

  cout << "Before Scatter, data_array = ";
  for (int j=0; j<8; j++) {cout << data_array[j] << " ";}
  cout << " in rank = " << rank << endl;
  cout << "Before Scatter, recv_buffer = ";
  for (int j=0; j<8; j++) {cout << recv_buffer[j] << " ";}
  cout << " in rank = " << rank << endl;

  MPI_Scatter(&data_array, send_count, MPI_INT, &recv_buffer,
recv_count, MPI_INT, 0, MPI_COMM_WORLD);

  cout << "After Scatter, data_array = ";
  for (int j=0; j<8; j++) {cout << data_array[j] << " ";}
  cout << " in rank = " << rank << endl;
  cout << "After Scatter, recv_buffer = ";
  for (int j=0; j<8; j++) {cout << recv_buffer[j] << " ";}
  cout << " in rank = " << rank << endl;

  MPI_Finalize();
  return 0;
}
```

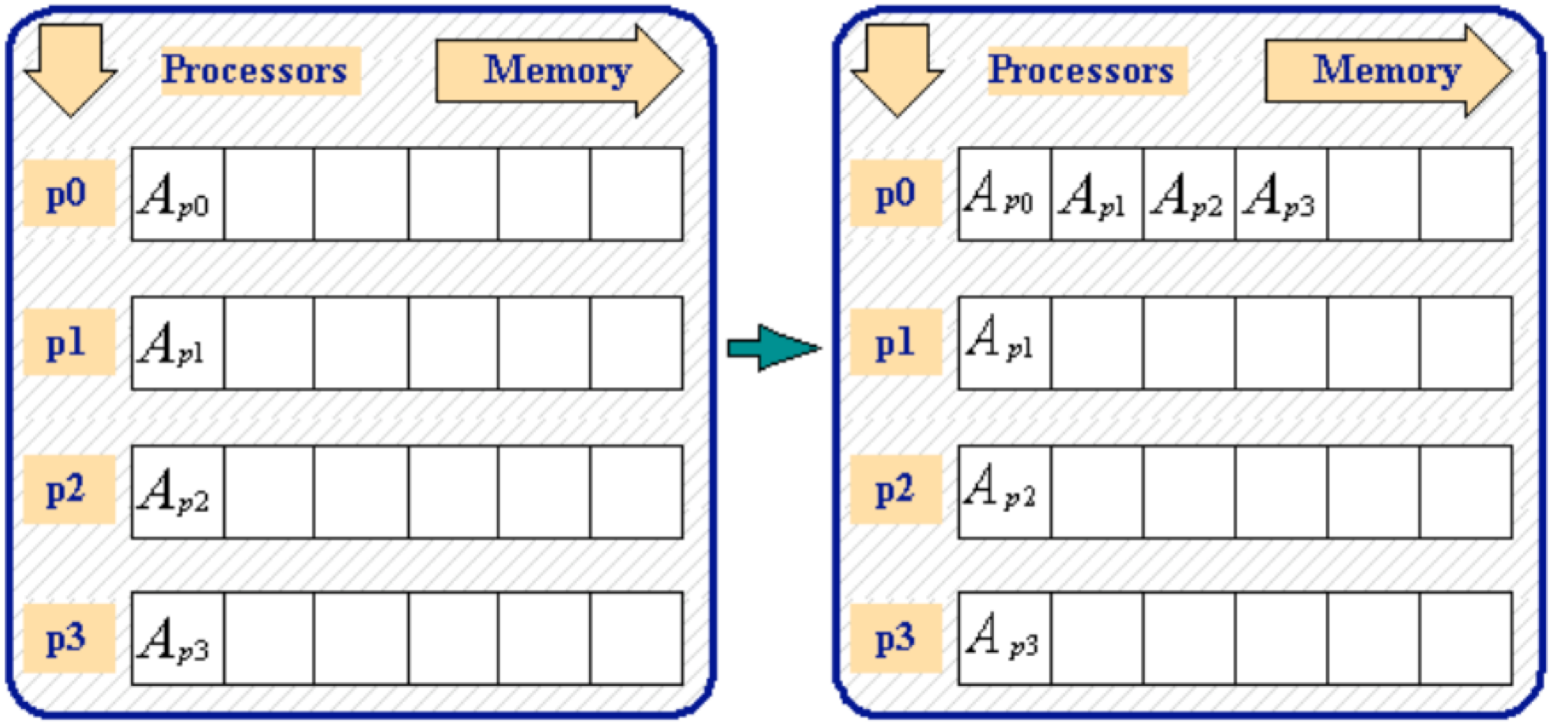
- Let us check the results together!

Data Movement



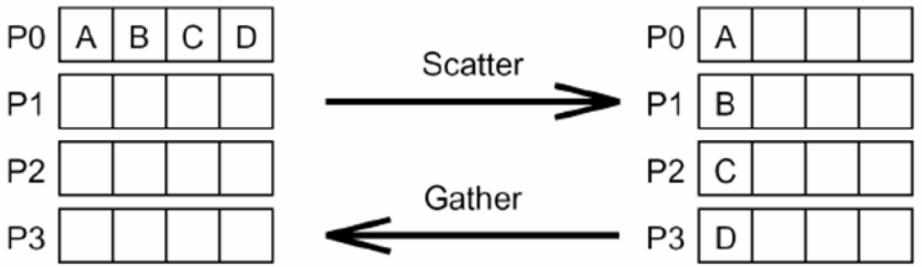
Gather: MPI_Gather

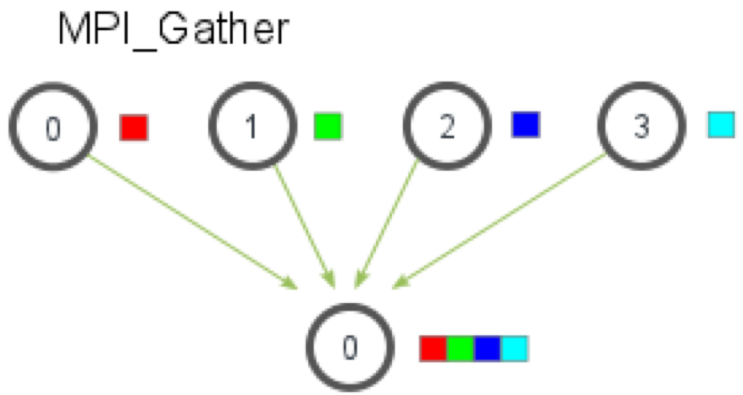
MPI_Gather



Gather: MPI_Gather

- MPI_Gather is the inverse of MPI_Scatter.
- MPI_Gather takes elements from many processes and gathers them to one single process.
- This routine is highly useful to many parallel algorithms, such as parallel sorting and searching.





Gather: MPI_Gather

- C Syntax: MPI_Gather(sendbuffer, sendcount, sendtype, recvbuffer, recvcount, recvtype, root, comm)
- C++ Syntax: MPI::Comm.Gather(sendbuffer, sendcount, sendtype, recvbuffer, recvcount, recvtype, root)
- No tag!!
- Similar to MPI_Scatter, MPI_Gather takes elements from each process and gathers them to the root process.
- The elements are ordered by the rank of the process from which they were received.

Lab: MPI_Gather( )

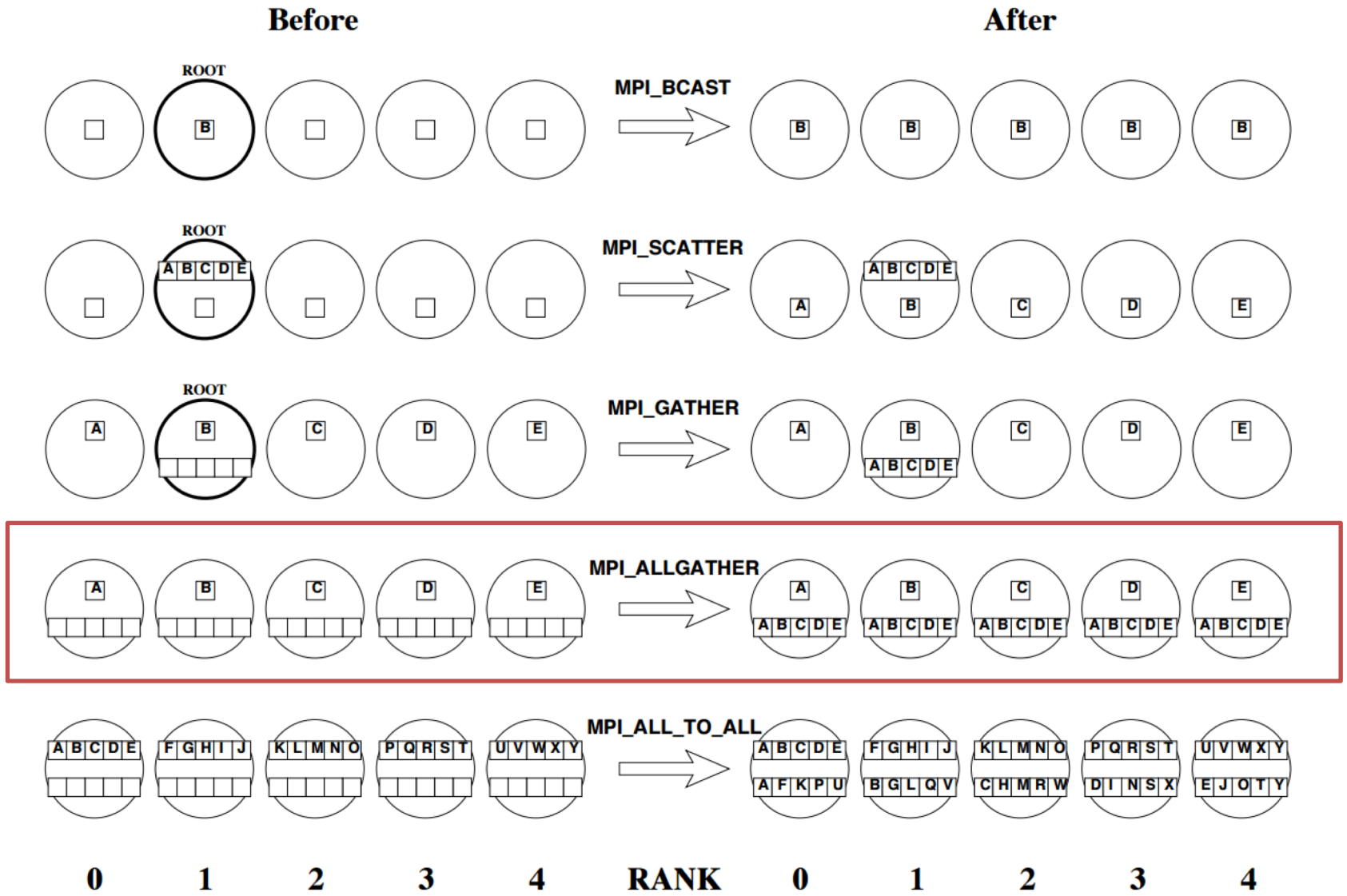
- Gathering with MPI_Gather()
  - `int data_array[8] = {};`
  - `int recv_buffer[8] = {};`
  - `data_array is assigned to 100,101 at rank_id = 0, 102, 103 at rank_id = 1, ..., 106, 107 at rank_id = 3.`
  - `Gathering all elements to root (rank_id = 0) and compute average of them.`

Lab: mpi_collective_gather.cpp

```
10 /*****************************************************************************80
11 {
12   int data_array[8] = {};
13   int recv_buffer[8] = {};
14   int nprocs, rank;
15
16   MPI_Init(&argc, &argv);
17   MPI_Comm_size(MPI_COMM_WORLD, &nprocs);
18   MPI_Comm_rank(MPI_COMM_WORLD, &rank);
19
20   // assign data_array
21   for (int i=0; i<2; i++) {
22     data_array[i] = 100+i+rank*2;
23   }
24
25   int send_count = 2;
26   int recv_count = 2;
27
28   cout << "Before Gather, data_array = ";
29   for (int j=0; j<8; j++) {cout << data_array[j] << " ";}
30   cout << " in rank = " << rank << endl;
31   cout << "Before Gather, recv_buffer = ";
32   for (int j=0; j<8; j++) {cout << recv_buffer[j] << " ";}
33   cout << " in rank = " << rank << endl;
34
35 MPI_Gather(&data_array, send_count, MPI_INT, &recv_buffer, recv_count, MPI_INT, 0, MPI_COMM_WORLD);
36
37   cout << "After Gather, data_array = ";
38   for (int j=0; j<8; j++) {cout << data_array[j] << " ";}
39   cout << " in rank = " << rank << endl;
40   cout << "After Gather, recv_buffer = ";
41   for (int j=0; j<8; j++) {cout << recv_buffer[j] << " ";}
42   cout << " in rank = " << rank << endl;
43
44   // get average
45   if (rank == 0) {
46     float avg = 0;
47     float sum = 0;
48     for (int i=0; i<8; i++) {
49       sum += recv_buffer[i];
50     }
51     avg = sum / 8;
52     cout << "Avg = " << avg << endl;
53   }
54
55   MPI_Finalize();
56   return 0;
57 }
58
```

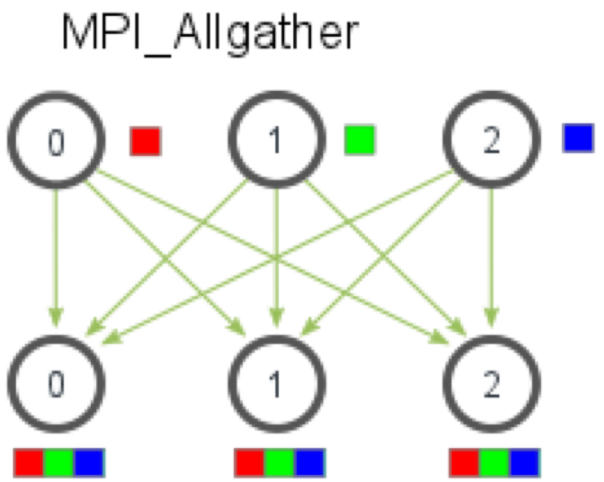
- Let us check the results together!

Data Movement



Gather: MPI_Allgather

- So far, we have covered two MPI routines that perform *many-to-one* or *one-to-many* communication patterns, which simply means that many processes send/receive to one process.
- Oftentimes it is useful to be able to send many elements to many processes (i.e. a *many-to-many* communication pattern).
- MPI_Allgather has this characteristic.
- Given a set of elements distributed across all processes, MPI_Allgather will gather all of the elements to all the processes.



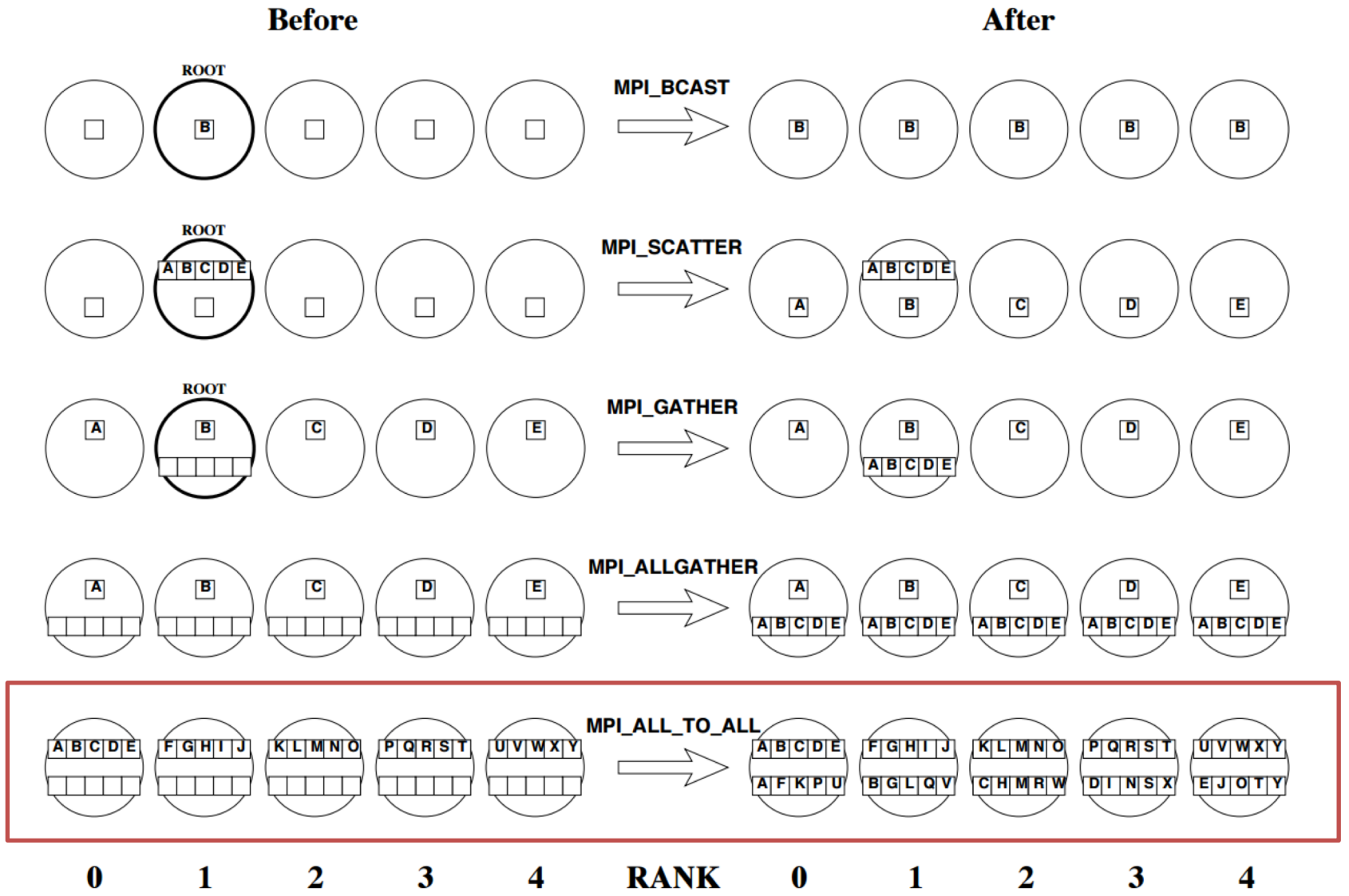
Gather: MPI_Allgather

- C Syntax: MPI_Allgather(sendbuffer, sendcount, sendtype, recvbuffer, recvcount, recvtype, comm)
- C++ Syntax: MPI::Comm.Allgather(sendbuffer, sendcount, sendtype, recvbuffer, recvcount, recvtype)
- Just like MPI_Gather, the elements from each process are gathered in order of their rank, except this time the elements are gathered to all processes.
- No root, No tag!!
- In the most basic sense,MPI_Allgather is an MPI_Gather followed by an MPI_Bcast.

Lab: MPI_Allgather( )

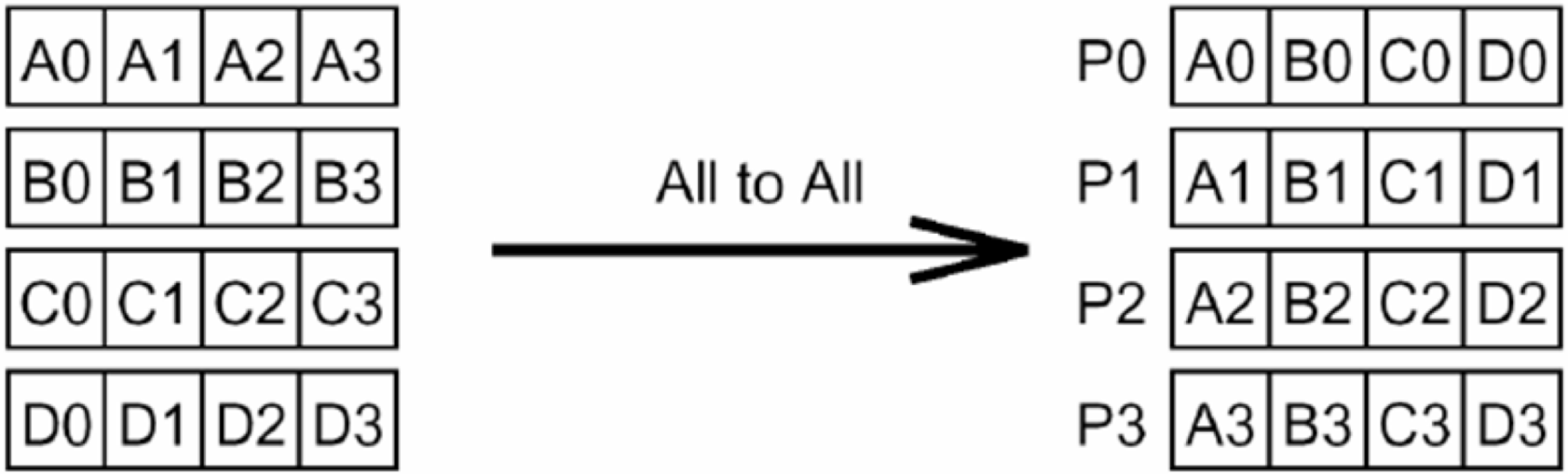
- Gathering with MPI_Allgather()
  - `int data_array[8] = {};`
  - `int recv_buffer[8] = {};`
  - `data_array is assigned to 100,101 at rank_id = 0, 102, 103 at rank_id = 1, ..., 106, 107 at rank_id = 3.`
  - `Gathering all elements to all processes and compute average of them at rank_id = 1.`

Data Movement



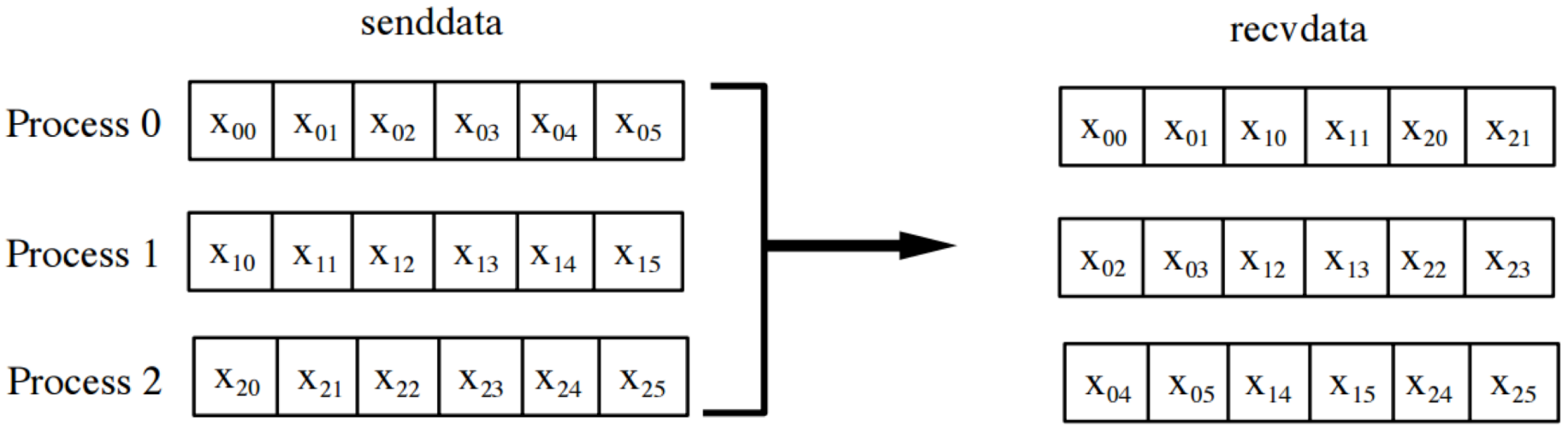
MPI_Alltoall

- Data movement operation.
- Each task in a group performs a scatter operation, sending a distinct message to all the tasks in the group in order by index.



MPI_Alltoall

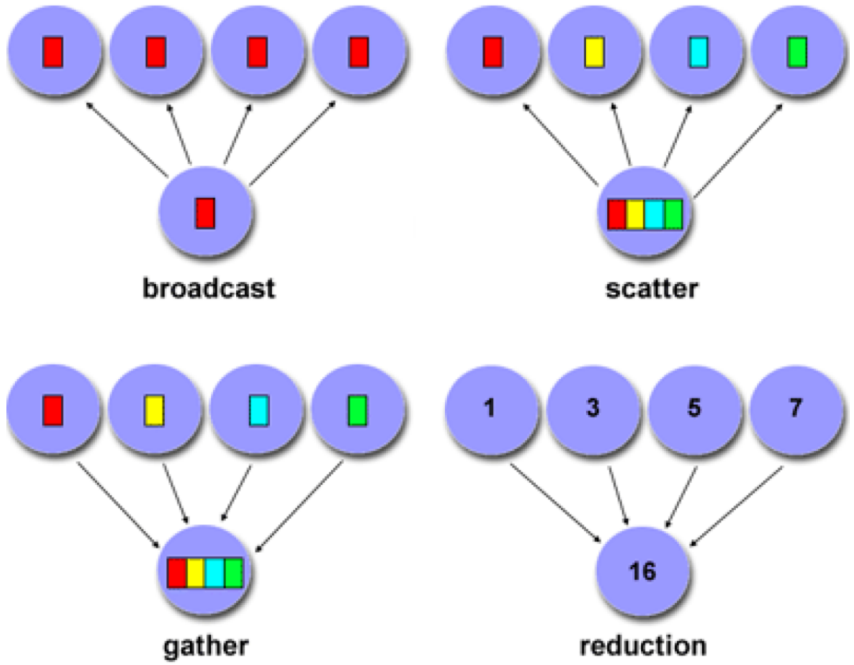
- C Syntax: MPI_Alltoall(sendbuffer, sendcount, sendtype, recvbuffer, recvcount, recvtype, comm)
- C++ Syntax: MPI::Comm.Alltoall(sendbuffer, sendcount, sendtype, recvbuffer, recvcount, recvtype)
- No root, No tag!!



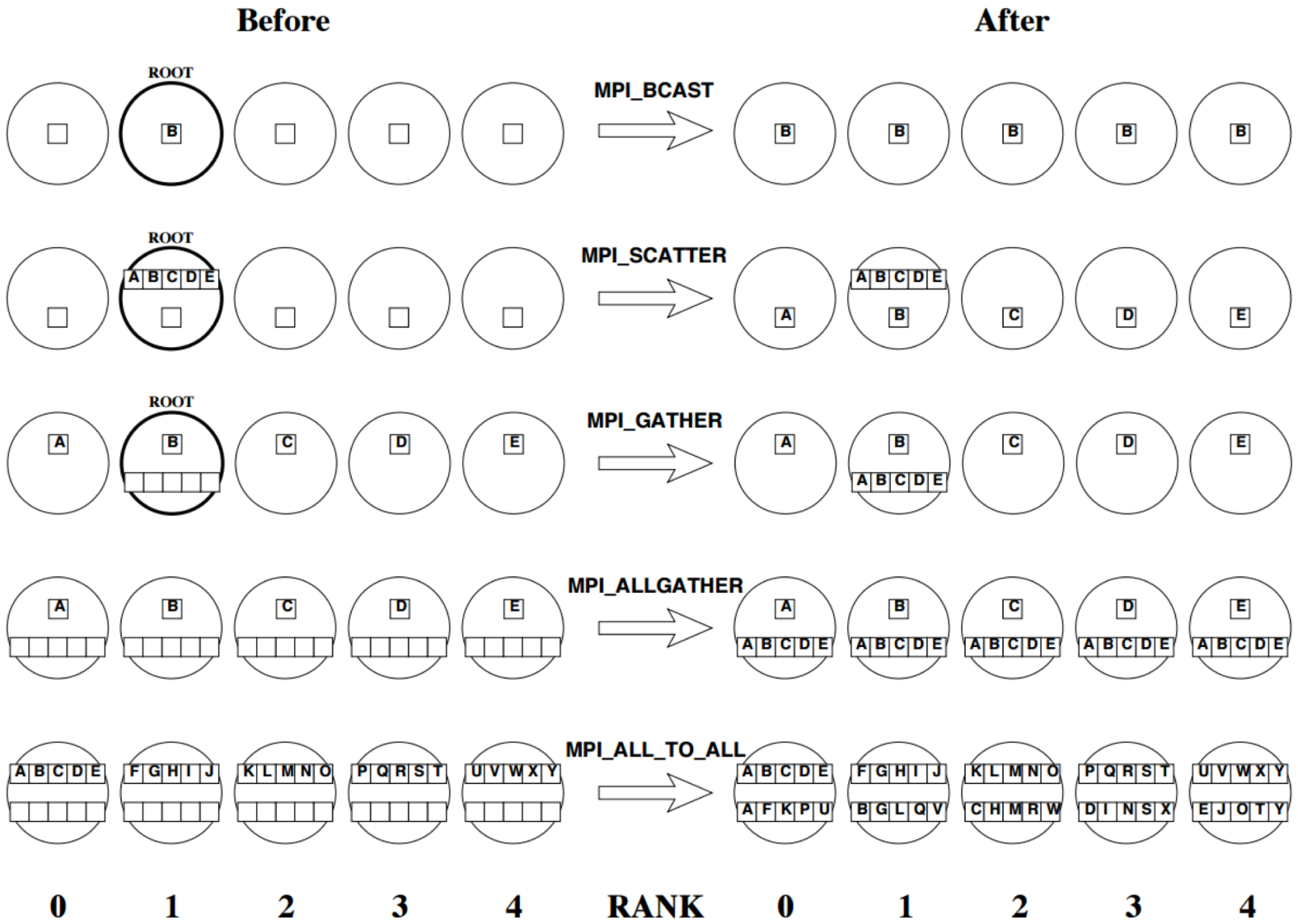
Types of Collective Operations

- **Synchronization** - processes wait until all members of the group have reached the synchronization point.

- **Data Movement** - broadcast, scatter/gather, all to all.
- **Collective computation** (reductions) - one member of the group collects data from the other members and performs an operation (min, max, add, multiply, etc.) on that data.

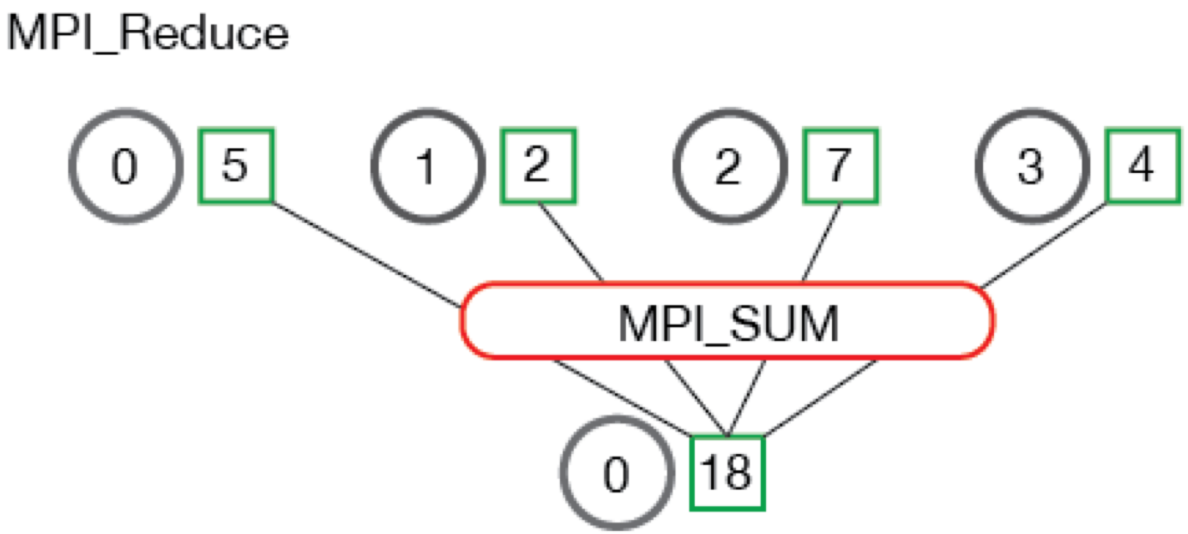


Recall: Data Movement

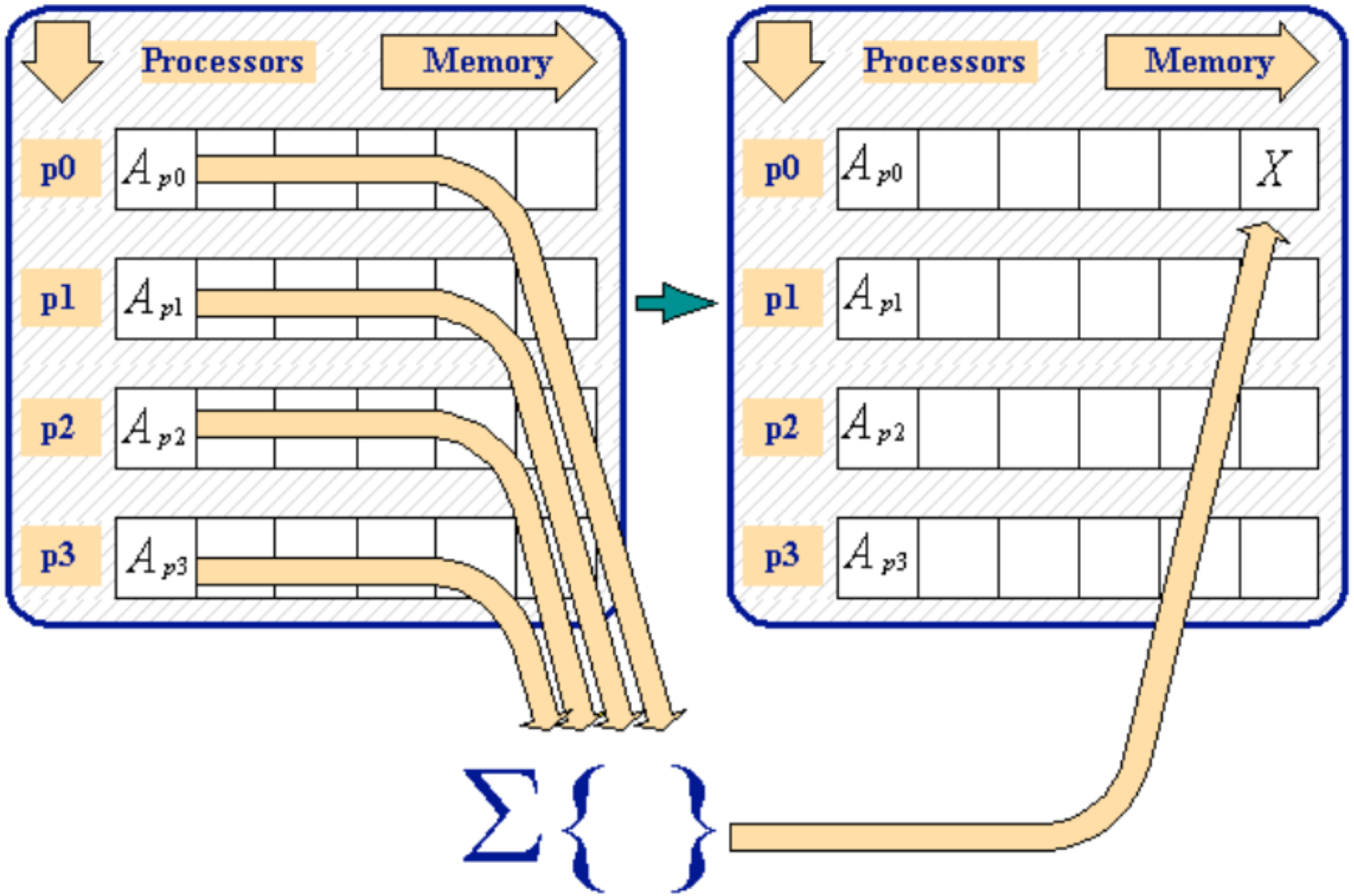


MPI_Reduce

- Collective computation operation.
- Applies a reduction operation on all tasks in the group and places the result in one task.



MPI_Reduce


Processors
Memory
p0
p1
p2
p3
A_{p0}
A_{p1}
A_{p2}
A_{p3}
Processors
Memory
p0
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p2
p3
A_{p0}
A_{p1}
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A_{p3}
X
Σ{ }

MPI_Reduce

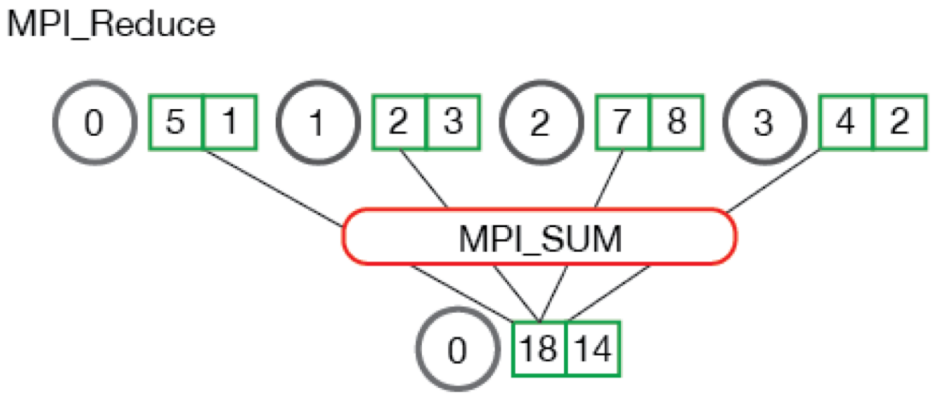
- C Syntax: MPI_Reduce(sendbuffer, recvbuffer, count, datatype, operation, root, comm)
- C++ Syntax: MPI::Reduce (sendbuffer, recvbuffer, count, datatype, operation, root)
- No tag!!
- All processes must specify the same root (and communicator).
- The sendbuffer parameter is an array of elements of type datatype that each process wants to reduce.
- The recvbuffer is only relevant on the process with a rank of root.

MPI Reduction Operations

- MPI_MAX - Returns the maximum element.
- MPI_MIN - Returns the minimum element.
- MPI_SUM - Sums the elements.
- MPI_PROD - Multiplies all elements.
- MPI_LAND - Performs a logical *and* across the elements.
- MPI_LOR - Performs a logical *or* across the elements.
- MPI_BAND - Performs a bitwise *and* across the bits of the elements.
- MPI_BOR - Performs a bitwise *or* across the bits of the elements.
- MPI_MAXLOC - Returns the maximum value and the rank of the process that owns it.
- MPI_MINLOC - Returns the minimum value and the rank of the process that owns it.

MPI_Reduce

- It is also useful to see what happens when processes contain multiple elements.
- The illustration below shows reduction of multiple numbers per process.



- The processes from the above illustration each have two elements.
- The resulting summation happens on a per-element basis. In other words, instead of summing all of the elements from all the arrays into one element, the i^{th} element from each array are summed into the i^{th} element in result array of process 0.

Lab: Sum of the N (100000000) using MPI P2P Block

```cpp
#include <iostream>
#include <mpi.h>        // MPI header file
#define MASTER 0
#define N 100000000

using namespace std;

int main(int argc, char **argv) {
  int nprocs, rank;
  long long int start_val, end_val, total_sum = 0, partial_sum = 0;

  // initialize for MPI
  MPI_Init(&argc, &argv);
  // get number of processes
  MPI_Comm_size(MPI_COMM_WORLD, &nprocs);
  // get the rank = this process's number (ranges from 0 to nprocs - 1)
  MPI_Comm_rank(MPI_COMM_WORLD, &rank);
  // MPI status
  MPI_Status status;

  // each process do "partial sum" with
  start_val = rank * (N/nprocs) + 1;
  end_val = (rank+1) * (N/nprocs);
  for (int val=start_val; val<=end_val; val++)
    partial_sum += val;
  cout << "rank " << rank << " calculated partial_sum " << partial_sum << endl;

  // master
  if (rank == MASTER)
  {
    // save current partial_sum into total_sum
    total_sum = partial_sum;

    // master receives each partial sum from a worker and add it to total_sum
    for (int src_rank = 1; src_rank < nprocs; src_rank++) {
      MPI_Recv(&partial_sum, 1, MPI_LONG_LONG_INT, src_rank, 0, MPI_COMM_WORLD, &status);
      cout << "master received partial_sum " << partial_sum << " from a source rank " << src_rank << endl;
      total_sum += partial_sum;
    }

    // print out total sum
    cout << "Total sum of the array elements is " << total_sum << endl;
  }
  // workers
  else
  {
    // each worker send the partial sum to master destination
    MPI_Send(&partial_sum, 1, MPI_LONG_LONG_INT, MASTER, 0, MPI_COMM_WORLD);
  }

  // clean up for MPI
  MPI_Finalize();

  return 0;
}
```

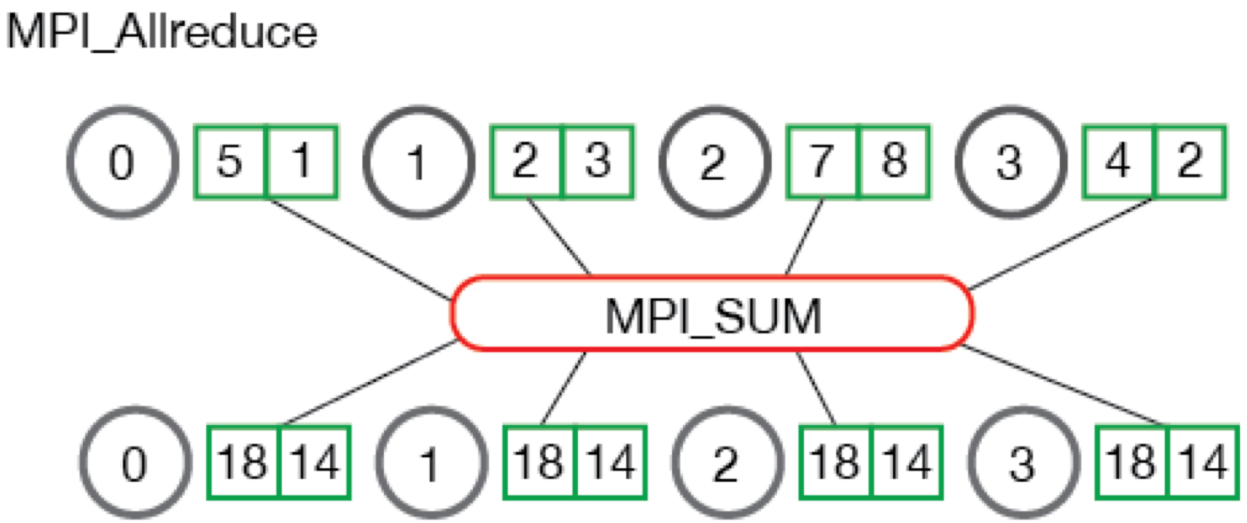
Lab: Sum of the N (100000000) using MPI_Reduce

```cpp
#include <iostream>
#include <mpi.h>        // MPI header file
#define MASTER 0
#define N 100000000

using namespace std;

int main(int argc, char **argv) {
  int nprocs, rank;
  long long int start_val, end_val, total_sum = 0, partial_sum = 0;

  // initialize for MPI
  MPI_Init(&argc, &argv);
  // get number of processes
  MPI_Comm_size(MPI_COMM_WORLD, &nprocs);
  // get the rank = this process's number (ranges from 0 to nprocs - 1)
  MPI_Comm_rank(MPI_COMM_WORLD, &rank);
  // MPI status
  MPI_Status status;

  // each process do "partial sum" with
  start_val = rank * (N/nprocs) + 1;
  end_val = (rank+1) * (N/nprocs);
  for (int val=start_val; val<=end_val; val++)
    partial_sum += val;
  cout << "rank " << rank << " calculated partial_sum " << partial_sum << endl;
```

```cpp
  // Your work

  MPI_Finalize();
}
```

MPI_Allreduce

- Many parallel applications will require accessing the reduced results across all processes rather than the root process.
- In a similar complementary style of MPI_Allgather to MPI_Gather, MPI_Allreduce will reduce the values and distribute the results to all processes.



MPI_Allreduce

- C Syntax: MPI_Allreduce(sendbuffer, recvbuffer, count, datatype, operation, comm)
- C++ Syntax: MPI:: COMM.Allreduce(sendbuffer, recvbuffer, count, datatype, operation)
- No tag, no root!!
- MPI_Allreduce is identical to MPI_Reduce with the exception that it does not need a root process id (since the results are distributed to all processes).

Lab: Sum of the first N Integers using MPI_Allreduce and Print at Rank 1

```cpp
#include <iostream>
#include <mpi.h>        // MPI header file
#define MASTER 0
#define N 100000000

using namespace std;

int main(int argc, char **argv) {
  int nprocs, rank;
  long long int start_val, end_val, total_sum = 0, partial_sum = 0;

  // initialize for MPI
  MPI_Init(&argc, &argv);
  // get number of processes
  MPI_Comm_size(MPI_COMM_WORLD, &nprocs);
  // get the rank = this process's number (ranges from 0 to nprocs - 1)
  MPI_Comm_rank(MPI_COMM_WORLD, &rank);
  // MPI status
  MPI_Status status;

  // each process do "partial sum" with
  start_val = rank * (N/nprocs) + 1;
  end_val = (rank+1) * (N/nprocs);
  for (int val=start_val; val<=end_val; val++)
    partial_sum += val;
  cout << "rank " << rank << " calculated partial_sum " << partial_sum << endl;
```

```cpp
  // Your work

  MPI_Finalize();
}
```

Collective vs. Point-to-Point Communications

- All the processes in the communicator must call the same collective function.

- For example, a program that attempts to match a call to MPI_Reduce on one process with a call to MPI_Recv on another process is erroneous, and, in all likelihood, the program will hang or crash.

Collective vs. Point-to-Point Communications

- The arguments passed by each process to an MPI collective communication must be "compatible."

- For example, if one process passes in 0 as the dest_process and another passes in 1, then the outcome of a call to MPI_Reduce is erroneous, and, once again, the program is likely to hang or crash.

Collective vs. Point-to-Point Communications

- Point-to-point communications are matched on the basis of tags and communicators.
- Collective communications don't use tags.
- They're matched solely on the basis of the communicator and the order in which they're called.

Matrix-vector multiplication

$A = (a_{ij})$ is an $m \times n$ matrix

$\mathbf{x}$ is a vector with n components

$\mathbf{y} = A\mathbf{x}$ is a vector with m components

$$y_i = a_{i0}x_0 + a_{i1}x_1 + a_{i2}x_2 + \cdots a_{i,n-1}x_{n-1}$$

i-th component of y

Dot product of the ith row of A with x.

Matrix-vector multiplication

$$\begin{bmatrix} a_{00} & a_{01} & \cdots & a_{0,n-1} \\ a_{10} & a_{11} & \cdots & a_{1,n-1} \\ \vdots & \vdots & & \vdots \\ a_{i0} & a_{i1} & \cdots & a_{i,n-1} \\ \vdots & \vdots & & \vdots \\ a_{m-1,0} & a_{m-1,1} & \cdots & a_{m-1,n-1} \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_{n-1} \end{bmatrix} = \begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_i = a_{i0}x_0 + a_{i1}x_1 + \cdots a_{i,n-1}x_{n-1} \\ \vdots \\ y_{m-1} \end{bmatrix}$$

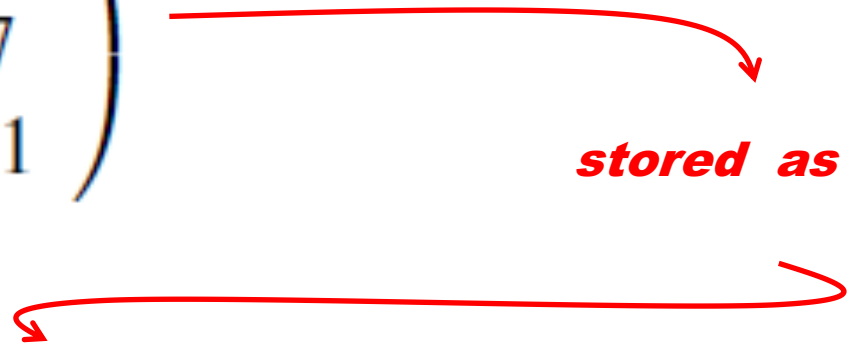
Multiply a matrix by a vector

```
/* For each row of A */
for (i = 0; i < m; i++) {
    /* Form dot product of ith row with x */
    y[i] = 0.0;

    for (j = 0; j < n; j++)
        y[i] += A[i][j]*x[j];
}
```

Serial pseudo-code

C style arrays

$$\begin{pmatrix} 0 & 1 & 2 & 3 \\ 4 & 5 & 6 & 7 \\ 8 & 9 & 10 & 11 \end{pmatrix}$$



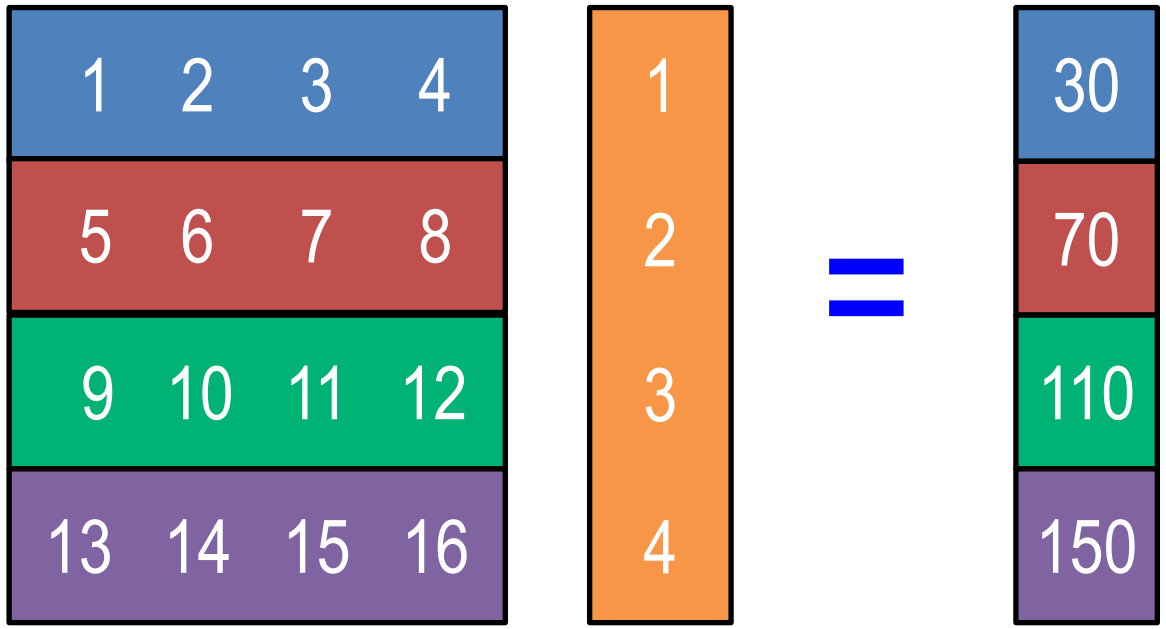
0 1 2 3 4 5 6 7 8 9 10 11

Serial matrix-vector multiplication

```
void Mat_vect_mult(
      double  A[]  /* in  */,
      double  x[]  /* in  */,
      double  y[]  /* out */,
      int     m    /* in  */,
      int     n    /* in  */) {
   int i, j;

   for (i = 0; i < m; i++) {
      y[i] = 0.0;
      for (j = 0; j < n; j++)
         y[i] += A[i*n+j]*x[j];
   }
}  /* Mat_vect_mult */
```

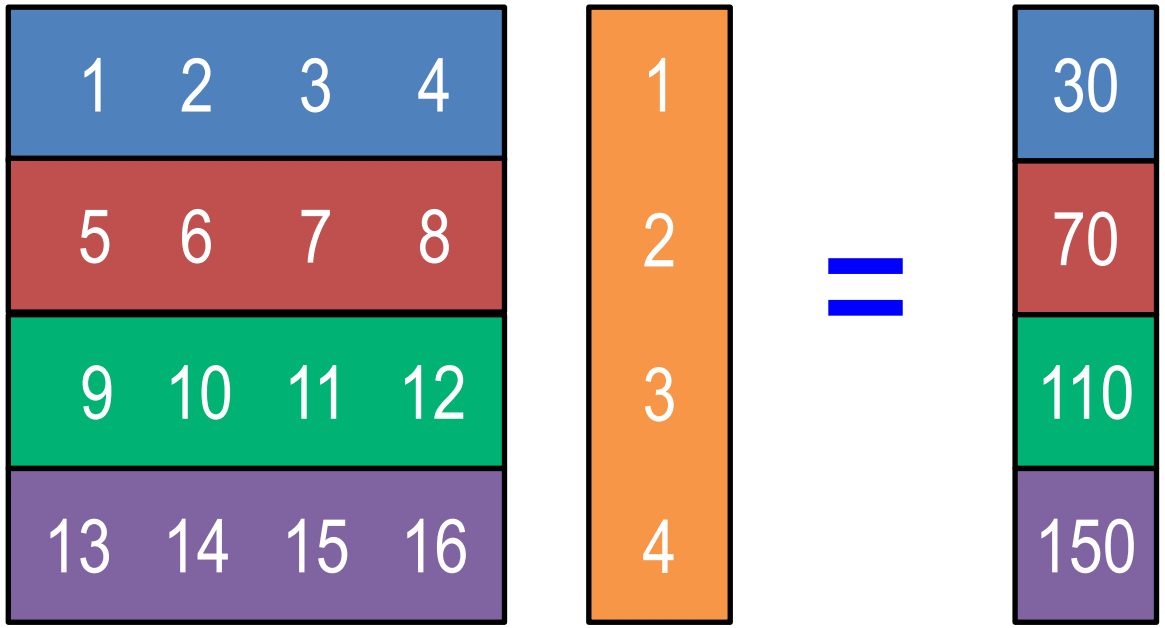
Matrix Vector Multiplication (Row Wise)

$$A \times B = X$$



Matrix Vector Multiplication (Row Wise)

- Scatter the rows of *A* to every process.
- Broadcast *B* to all the processes.
- Calculate each scalar using dot product of row *A* and *B*.
- Gather each scalar onto the root.

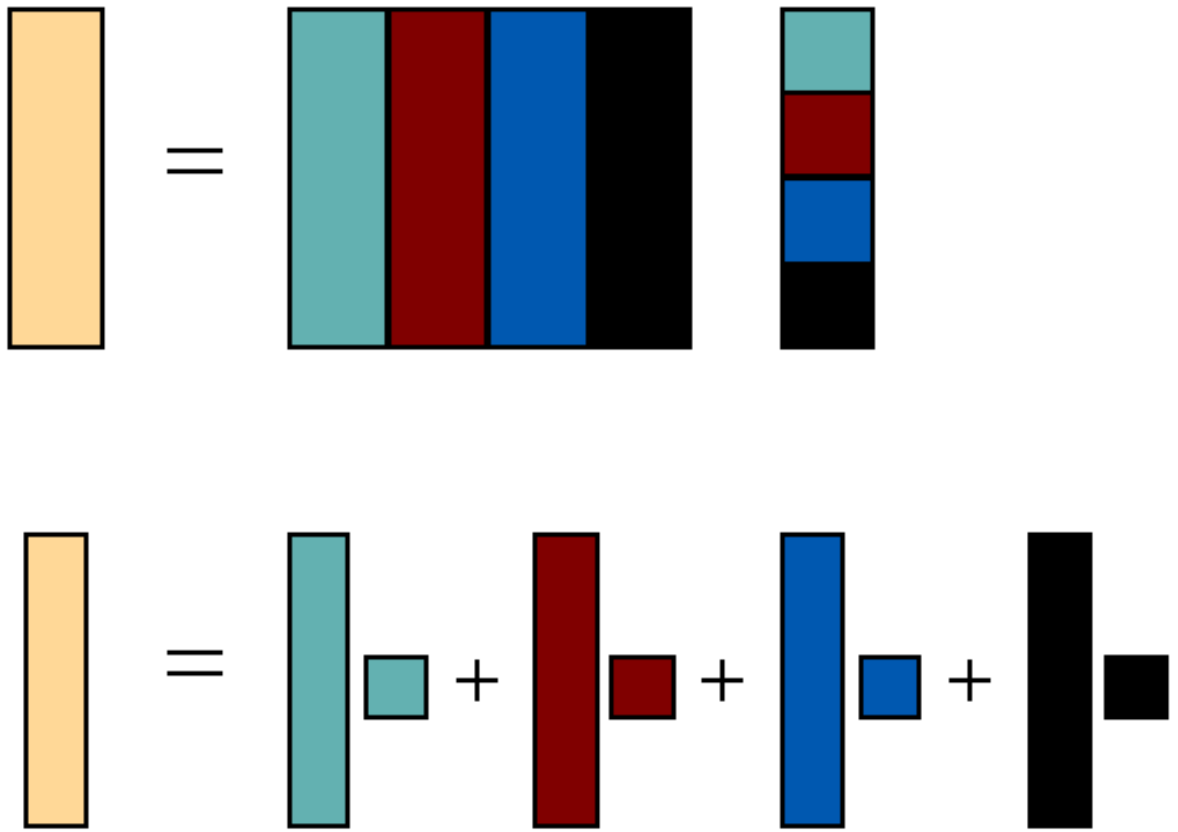


HW: Matrix Vector Multiplication (Row Wise)

```
#include <iostream>
#include <cstdlib>     // has exit(), etc.
#include <ctime>
#include <mpi.h>
#define MASTER 0

using namespace std;

int main(int argc, char **argv)
{
  // Just simply set N(4) X M(4) matrix and M(4) X 1 vector
  int N=4, M=4;
  int A[N][M], Apart[M], B[M], X[N];
  int Xpart = 0, root=0;
  int nproc, rank;

  // MPI init
  MPI_Init(&argc, &argv);
  MPI_Comm_size(MPI_COMM_WORLD, &nprocs);
  MPI_Comm_rank(MPI_COMM_WORLD, &rank);
```

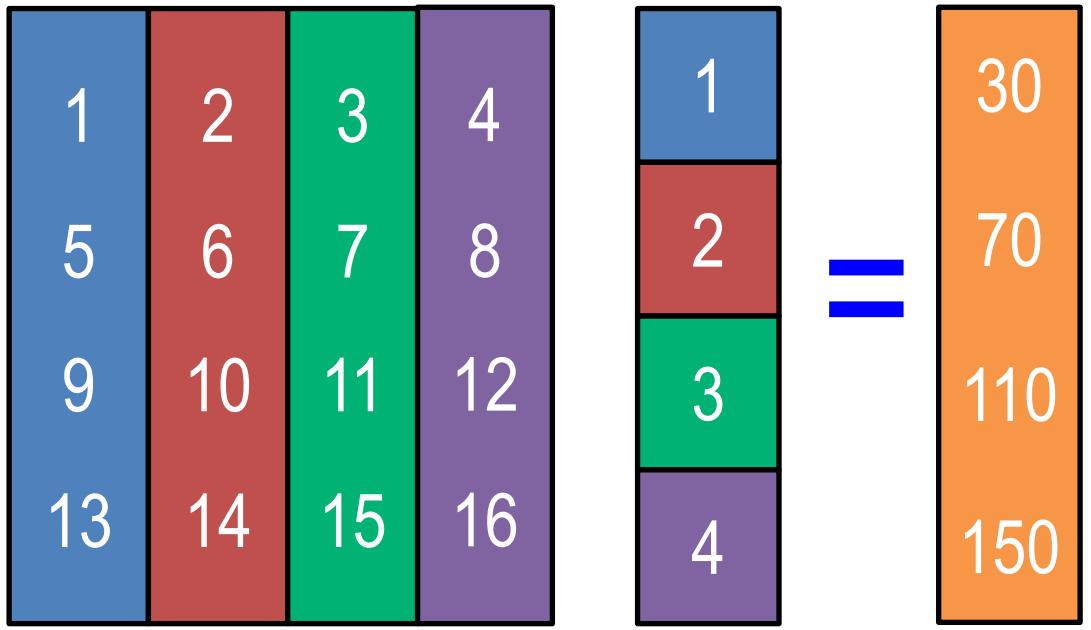
```
  // Initialize matrix
  if (rank == MASTER) {
    ...
  }

  // MPI Scatter the A Matrix
  ...

  // Broadcast the B vector
  ...

  // Calculate Xpart
  ...

  // MPI Gather
  ...

  // Print results
  if (rank == MASTER) {
    for(int i=0; i<N; i++) {
      cout << "X[" << i << "]=" << X[i] << endl;
  } }

  MPI_Finalize();  // MPI finalize
  return 0;      // Exit
}
```

Matrix-vector Multiplication (Column wise)

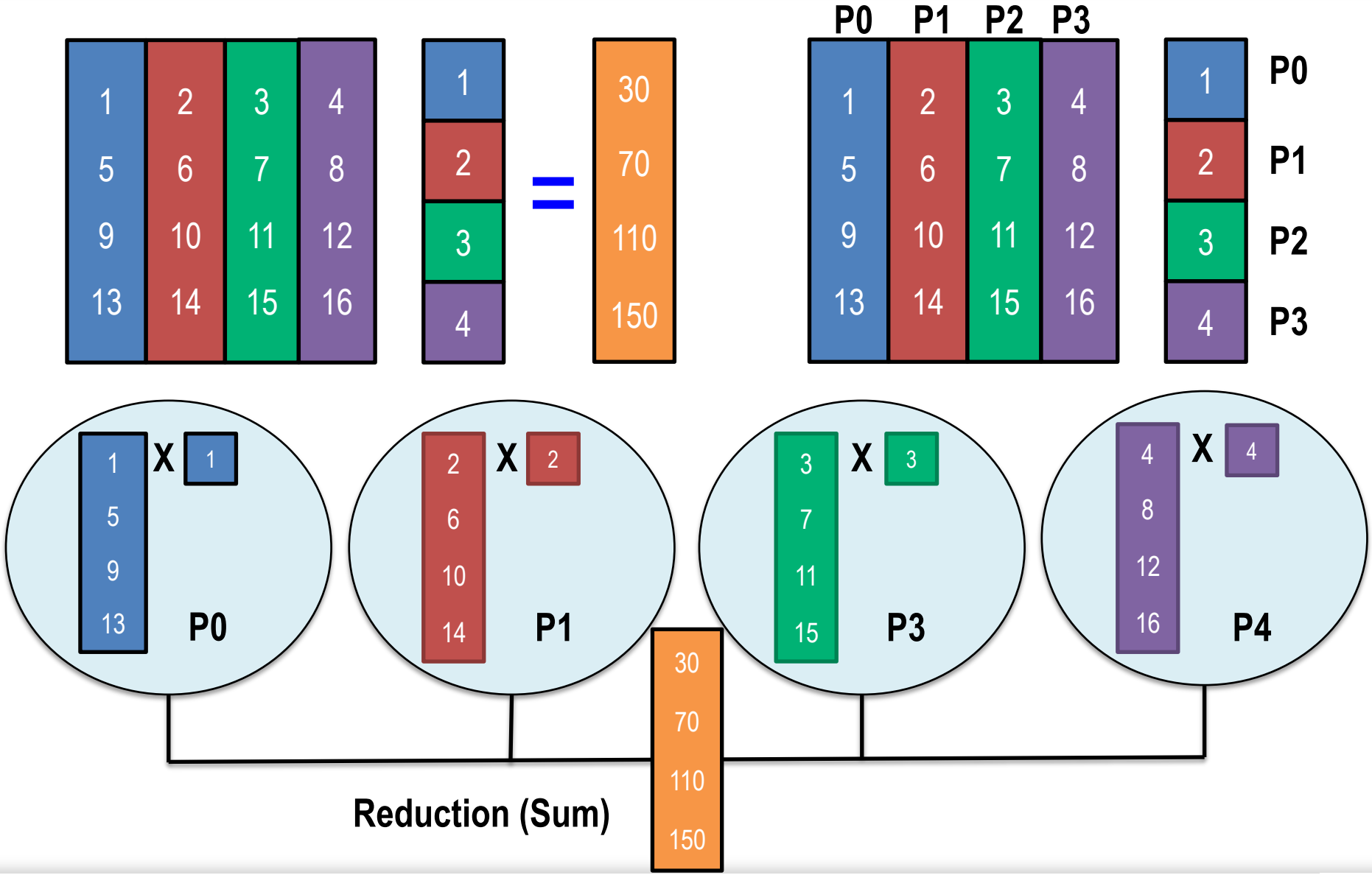


Schematic of parallel decomposition for vector-matrix multiplication, **A=B*C**. The vector **A** is depicted in yellow. The matrix **B** and vector **C** are depicted in multiple colors representing the portions, columns, and elements assigned to each processor, respectively.

Matrix Vector Multiplication (Column Wise)

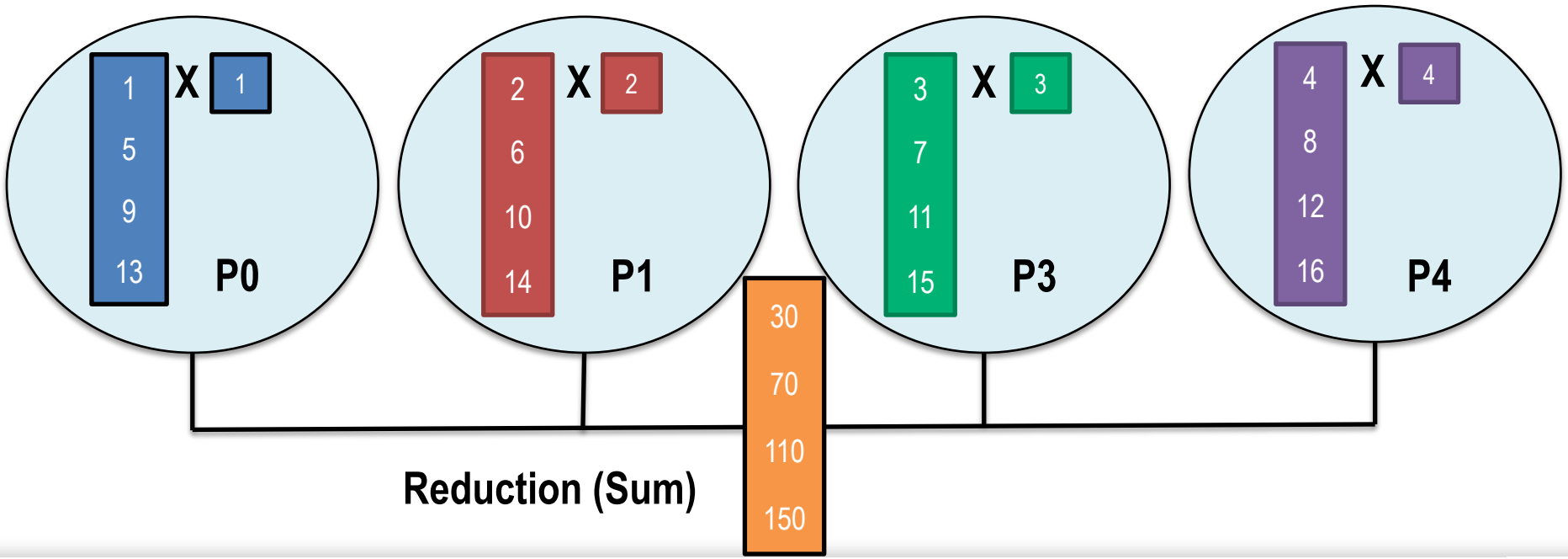


Matrix Vector Multiplication (Column Wise)



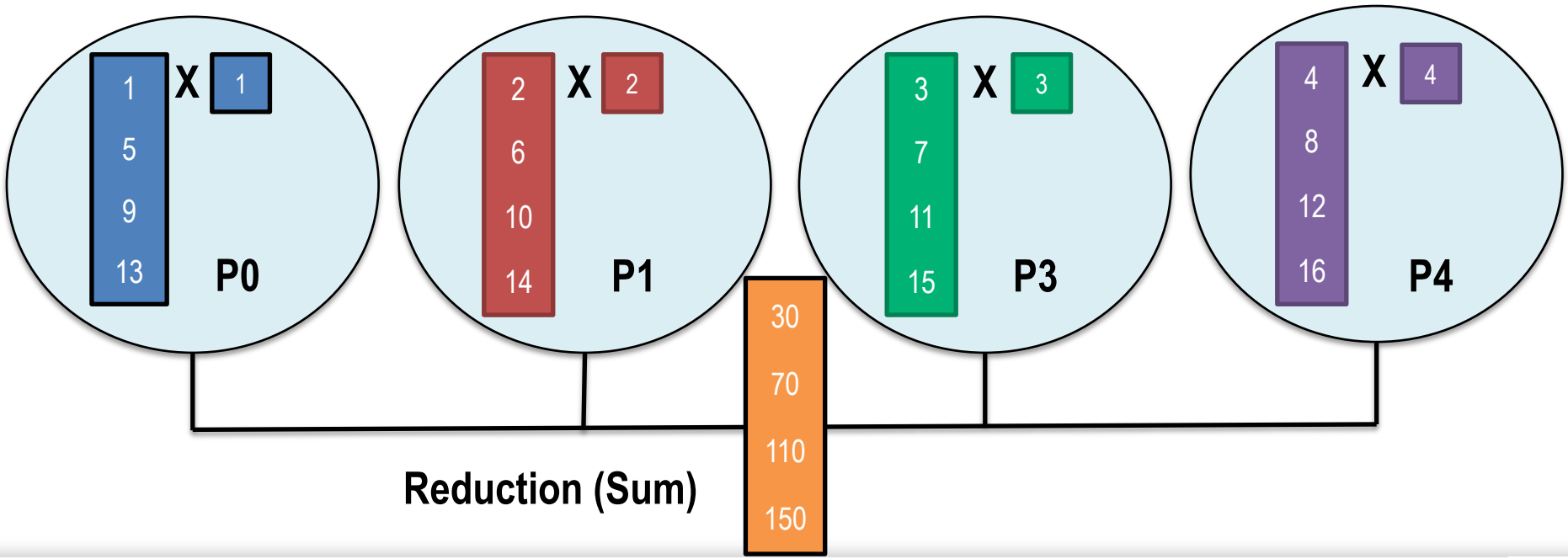
Matrix Vector Multiplication (Column Wise)

- Scatter the columns of *A* to every process.
- Scatter *B* to every process.
- Perform independent vector-scalar multiplication of row *A* and *B* element.
- Use “add” reduction operation to sum all vectors.



Matrix Vector Multiplication (Column Wise)

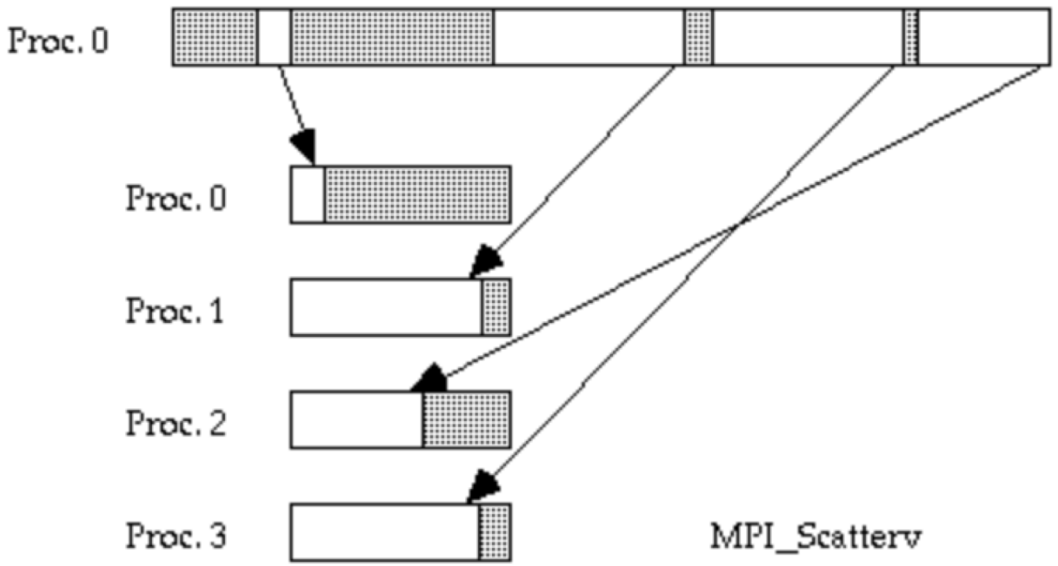
- Scatter the columns of *A* to every process. ← Can we do it?
- Scatter *B* to every process.
- Perform independent vector-scalar multiplication of row *A* and *B* element.
- Use “add” reduction operation to sum all vectors.



Scatterv

- Gaps are allowed between messages in source data

  (but the individual messages must still be contiguous)
- Irregular message sizes are allowed
- Data can be distributed to processes in any order



https://cvw.cac.cornell.edu/mpicc/scatterv

MPI_Scatterv

Synopsis

```
int MPI_Scatterv(const void *sendbuf, const int *sendcounts, const int *displs,
                 MPI_Datatype sendtype, void *recvbuf, int recvcount,
                 MPI_Datatype recvtype,
                 int root, MPI_Comm comm)
```

Input Parameters

sendbuf
address of send buffer (choice, significant only at `root`)
sendcounts
integer array (of length group size) specifying the number of elements to send to each processor
displs
integer array (of length group size). Entry `i` specifies the displacement (relative to sendbuf from which to take the outgoing data to process `i`
sendtype
data type of send buffer elements (handle)
recvcount
number of elements in receive buffer (integer)
recvtype
data type of receive buffer elements (handle)
root
rank of sending process (integer)
comm
communicator (handle)

Output Parameters

recvbuf
address of receive buffer (choice)

Lab: Scatterv example (mpi_collective_scatterv.cpp)

- Data has

```
char data[SIZE][SIZE] = {
   {'a', 'b', 'c', 'd'},
   {'e', 'f', 'g', 'h'},
   {'i', 'j', 'k', 'l'},
   {'m', 'n', 'o', 'p'}
```

- Set sendcounts and displacements as

```
sendcounts[0] = 1   displs[0] = 0
sendcounts[1] = 2   displs[1] = 4
sendcounts[2] = 3   displs[2] = 8
sendcounts[3] = 4   displs[3] = 12
```

- After scatterv, it will scatter and print out as

```
0: a

1: e f

2: i j k

3: mn o p
```

Lab: Scatterv example (mpi_collective_scatterv.cpp)

```
# include <iostream>
# include <cstdlib> // has exit(), etc.
# include <ctime>
# include <stdio.h>
# include "mpi.h"   // MPI header file

#define SIZE 4
using namespace std;

int main (int argc, char **argv)
{
  int nprocs, rank;

  MPI_Init(&argc, &argv);
  MPI_Comm_size(MPI_COMM_WORLD, &nprocs);
  MPI_Comm_rank(MPI_COMM_WORLD, &rank);

  // the data to be distributed
  char data[SIZE][SIZE] = {
    {'a', 'b', 'c', 'd'},
    {'e', 'f', 'g', 'h'},
    {'i', 'j', 'k', 'l'},
    {'m', 'n', 'o', 'p'}
  };
```

```
 char rec_buf[100];
  int sendcounts[nprocs];
  int displs[nprocs];

  // calculate send counts and displacements
  // Your code

  // print calculated send counts and displacements for each process
  if (rank == 0) {
    for (int i = 0; i < nprocs; i++) {
      printf("sendcounts[%d] = %d\tdispls[%d] = %d\n", i, sendcounts[i], i, displs[i]);
    }
  }

  // MPI Scatterv
  // Your code

  printf("%d: ", rank);
  for (int i = 0; i < sendcounts[rank]; i++) {
    printf("%c\t", rec_buf[i]);
  }
  printf("\n");

  MPI_Finalize();
  return 0;
}
```