R E P O R T

[응용수학]

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목 차

1. 미방 3

2. 라플라스변환 7

1 Each of the following equations is exact. Solve them.

(a) $x^2\frac{dy}{dx}+2xy=x^3$

(b) $\frac{1}{x^2}\frac{dy}{dx}-\frac{2}{x^3}y=5x^3$

(c) $e^x\left(y+\frac{dy}{dx}\right)=\cos x$

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① 미분의 곱 규칙 – $(uv)'=uv'+u'v$

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a) $x^2\frac{dy}{dx}+2xy=x^3$

①과 같은 꼴, $uy'+u'y=x^3$,

$$\frac{d}{dx}(x^2y)=x^3,\ (x^2y)=\int x^3dx=\frac{x^4}{4}+C$$

$$y=\frac{x^4}{4x^2}+\frac{C}{x^2},$$

$$\therefore y=\frac{x^2}{4}+\frac{C}{x^2}$$

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b) $\frac{1}{x^2}\frac{dy}{dx}-\frac{2}{x^3}y=5x^3$

①과 같은 꼴, $uy'+u'y=5x^3$,

$$\frac{d}{dx}\left(\frac{1}{x^2}y\right)=x^3,\ \left(\frac{1}{x^2}y\right)=\int 5x^3dx=\frac{5x^4}{4}+C$$

$$y=\frac{5x^6}{4}+Cx^2,$$

$$\therefore y=\frac{5x^6}{4}+Cx^2$$

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c) $e^x(y+\frac{dy}{dx})=\cos x$

①과 같은 꼴, $uy'+u'y=\cos x$,

$$\frac{d}{dx}(e^xy)=\cos x,\ (e^xy)=\int\cos x dx=\sin x+C$$

$$y=\frac{\sin x}{e^x}+\frac{C}{e^x},$$

$$\therefore y=e^{-x}\sin x+Ce^{-x}$$

1 Find the general solution of the following equations:

(a) $\frac{d^2x}{dt^2} - 2\frac{dx}{dt} - 3x = 6$

(b) $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 4y = 8$

비제차 방정식 풀이 : 1. 보조방정식 구하기, 2. 일반해 구하기, 3. 특수적분 구하기, 4. 2+3꼴

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a) $\frac{d^2x}{dt^2} - 2\frac{dx}{dt} - 3x = 6$

1. 보조 방정식 구하기, $\frac{d^2x}{dt^2} - 2\frac{dx}{dt} - 3x = 6,\ k - 2k - 3$

$=> \ (k+1)(k-3), k = \ -1, 3$

2. 일반해 구하기, $k = -1, k = 3,$ 서로 다른 실근 $\rightarrow Ae^{k_1 t} + Be^{k_2 t}$

$x_H = Ae^{-t} + Be^{3t}$

3. 특수적분 구하기, $x_p = 6\alpha, \frac{dx_p}{dt} = 0, \ \frac{d^2x_p}{dt^2} = 0$

$0 - 0 - 3x_p = 6, \ -18\alpha = 6, \alpha = -\frac{1}{3}$

$x_p = 6\alpha \ = \ -2$

4. 2+3꼴로 재차 방정식 해 구하기, $x_H + x_p = Ae^{-t} + Be^{3t} - 2$

$\therefore x = Ae^{-t} + Be^{3t} - 2$

b) $\frac{d^2y}{dx^2}+5\frac{dy}{dx}+4y=8$

1. 보조 방정식 구하기, $\frac{d^2y}{dx^2}+5\frac{dy}{dx}+4y=8,\ k+5k+4$
$=>\ (k+1)(k+4),\ k=\ -1,\ -4$

2. 일반해 구하기, $k=-1, k=-4,$ 서로 다른 실근 $\rightarrow Ae^{k_1x}+Be^{k_2x}$
$y_H=Ae^{-x}+Be^{-4x}$

3. 특수적분 구하기, $y_p=8\alpha,\ \frac{dy_p}{dx}=0,\ \frac{d^2y_p}{dx^2}=0$

$$0+0+32\alpha=8,\ \alpha=\frac{1}{4}$$

$$y_p=8\alpha\ =\ 2$$

4. 2+3꼴로 재차 방정식 해 구하기, $y_H+y_p=Ae^{-x}+Be^{-4x}+2$
$\therefore y=Ae^{-x}+Be^{-4x}+2$

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2 Find a particular integral for the equation

$$\frac{d^2x}{dt^2}-3\frac{dx}{dt}+2x=5\,e^{3t}$$

$\frac{d^2x}{dt^2}-3\frac{dx}{dt}+2=5e^{3t}$

특수적분 구하기, $x_p=Ce^{3t},\ \frac{dx_p}{dt}=3Ce^{3t},\ \frac{d^2x_p}{dt^2}=9Ce^{3t}$

$$x=9Ce^{3t}-3(3Ce^{3t})+2(Ce^{3t})=5e^{3t}$$

$$=2Ce^{3t}=5e^{3t},\ C=\frac{5}{2}$$

$$\therefore x_p=Ce^{3t}=\frac{5}{2}e^{3t}$$

Find a particular integral for the equation

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 3\cos x$$

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 3\cos x$$

특수적분 구하기.

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 3\cos x$$
$y_p = \alpha\cos x + \beta\sin x$,
$\frac{dy_p}{dx} = -\alpha\sin x + \beta\cos x$,
$\frac{d^2y_p}{dx^2} = -\alpha\cos x - \beta\sin x$

$y = -\alpha\cos x - \beta\sin x - 6(-\alpha\sin x + \beta\cos x) + 8(\alpha\cos x + \beta\sin x) = 3\cos x$
$= -\alpha\cos x - \beta\sin x + 6\alpha\sin x - 6\beta\cos x + 8\alpha\cos x + 8\beta\sin x = 3\cos x$

$\cos x$, $-\alpha - 6\beta + 8\alpha = 7\alpha - 6\beta = 3$
$\sin x$, $-\beta + 6\alpha + 8\beta = 6\alpha + 7\beta = 0$

$\cos x$에 $-\frac{6}{7}$ 곱하기, $-6a + \frac{36}{7}\beta = -\frac{18}{7}$
$\cos x + \sin x$, $\frac{36+49}{7}\beta = -\frac{18}{7}$, $\beta = -\frac{18}{85}$
$\sin x$에 대입, 6α

1 Find the Laplace transforms of the following functions:

(a) $3t^2 - 4$

(b) $2\sin 4t + 11 - t$

(c) $2 - t^2 + 2t^4$

(d) $3e^{2t} + 4\sin t$

(e) $\frac{1}{3}\sin 3t - 4\cos\left(\frac{t}{2}\right)$

(f) $3t^4 e^{5t} + t$

Laplace 변환, $F(s) = \int_0^{\infty} e^{-st} f(t)dt$ 를 이용한 표

Function, $f(t)$	Laplace transform, $F(s)$	Function, $f(t)$	Laplace transform, $F(s)$
1	$\frac{1}{s}$	$e^{-at}\cos bt$	$\frac{s+a}{(s+a)^2+b^2}$
t	$\frac{1}{s^2}$	$\sinh bt$	$\frac{b}{s^2-b^2}$
t^2	$\frac{2}{s^3}$	$\cosh bt$	$\frac{s}{s^2-b^2}$
t^n	$\frac{n!}{s^{n+1}}$	$e^{-at}\sinh bt$	$\frac{b}{(s+a)^2-b^2}$
e^{at}	$\frac{1}{s-a}$	$e^{-at}\cosh bt$	$\frac{s+a}{(s+a)^2-b^2}$
e^{-at}	$\frac{1}{s+a}$	$t\sin bt$	$\frac{2bs}{(s^2+b^2)^2}$
$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$	$t\cos bt$	$\frac{s^2-b^2}{(s^2+b^2)^2}$
$\sin bt$	$\frac{b}{s^2+b^2}$	$u(t)$ unit step	$\frac{1}{s}$
$\cos bt$	$\frac{s}{s^2+b^2}$	$u(t-d)$	$\frac{e^{-sd}}{s}$
$e^{-at}\sin bt$	$\frac{b}{(s+a)^2+b^2}$	$\delta(t)$	1
		$\delta(t-d)$	e^{-sd}

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a) $3t^2-4$

풀이. $L(t^n)=\dfrac{n!}{s^{n+1}}$, $L(3t^2)=3\dfrac{2!}{s^3}=\dfrac{6}{s^3}$,

$L(C)=\dfrac{C}{s}$, $L(4)=\dfrac{4}{s}$

$\therefore L(3t^2-4)=\dfrac{6}{s^3}-\dfrac{4}{s}$

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b) $2\sin 4t+11-t$

풀이. $L(\sin t)=\dfrac{\omega}{s^2+w^2}$, $L(2\sin 4t)=2\dfrac{4}{s^2+16}=\dfrac{8}{s^2+16}$,

$L(C)=\dfrac{C}{s}$, $L(11)=\dfrac{11}{s}$,

$L(t^n)=\dfrac{n!}{s^{n+1}}$, $L(t)=\dfrac{1}{s^2}$

$\therefore L(2\sin 4t+11-t)=\dfrac{8}{s^2+16}+\dfrac{11}{s}-\dfrac{1}{s^2}$

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c) $2-t^2+2t^4$

풀이. $L(C)=\dfrac{C}{s}$, $L(2)=\dfrac{2}{s}$,

$L(t^n)=\dfrac{n!}{s^{n+1}}$, $L(t^2)=\dfrac{2}{s^3}$, $L(2t^4)=2\dfrac{4!}{s^5}=\dfrac{48}{s^5}$

$\therefore L(2-t^2+2t^4)=\dfrac{2}{s}-\dfrac{2}{s^3}+\dfrac{48}{s^5}$

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d) $3e^{2t}+4\sin t$

풀이. $L(e^{at})=\dfrac{1}{s-a}$, $L(3e^{2t})=\dfrac{3}{s-2}$,

$L(\sin t)=\dfrac{\omega}{s^2+\omega^2}$, $L(4\sin t)=\dfrac{4}{s^2+1}$,

$\therefore L(3e^{2t}+4\sin t)=\dfrac{3}{s-2}+\dfrac{4}{s^2+1}$

e) $\frac{1}{3}sin3t-4\cos(\frac{t}{2})$

풀이, $L(\sin t)=\frac{\omega}{s^2+\omega^2}$, $L(\frac{1}{3}sin3t)=\frac{3}{3(s^2+9)}=\frac{1}{s^2+9}$,

$L(\cos t)=\frac{s}{s^2+\omega^2}$, $L(4\cos(\frac{1}{2}t))=\frac{4s}{s^2+\frac{1}{4}}=\frac{16s}{4s^2+1}$,

$\therefore L(\frac{1}{3}sin3t+4\cos(\frac{t}{2}))=\frac{1}{s^2+9}+\frac{16s}{4s^2+1}$

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f) $3t^4e^{5t}+t$

풀이, $L(t^ne^{at})=\frac{n!}{(s-a)^{n+1}}$, $L(3t^4e^{5t})=3\frac{4!}{(s-5)^5}=\frac{72}{(s-5)^5}$

$L(t^n)=\frac{n!}{s^{n+1}}$, $L(t)=\frac{1}{s^2}$

$\therefore\ L(3t^4e^{5t}+t)=\frac{72}{(s+5)^5}+\frac{1}{s^2}$

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2 The Laplace transform of $f(t)$ is given as

$$F(s) = \frac{3s^2 - 1}{s^2 + s + 1}$$

Find the Laplace transform of

(a) $e^{-t} f(t)$ (b) $e^{3t} f(t)$ (c) $e^{-t/2} f(t)$

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a) $e^{-t} f(t)$

풀이, $L(e^{-t} f(t)) = F(s+1)$,

$$F(s) = \frac{3s^2 - 1}{s^2 + s + 1}, \ F(s)\text{에 대입}$$

$$F(s+1) = \frac{3(s+1)^2 - 1}{(s+1)^2 + (s+1) + 1} = \frac{3s^2 + 6s + 2}{s^2 + 3s + 3},$$

$$\therefore L(e^{-t} f(t)) = F(s+1) = \frac{3s^2 + 6s + 2}{s^2 + 3s + 3}$$

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b) $e^{3t} f(t)$

풀이, $L(e^{3t} f(t)) = F(s-3)$,

$$F(s) = \frac{3s^2 - 1}{s^2 + s + 1}, \ F(s)\text{에 대입}$$

$$F(s-3) = \frac{3(s-3)^2 - 1}{(s-3)^2 + (s-3) + 1} = \frac{3s^2 - 18s + 26}{s^2 - 5s + 7},$$

$$\therefore L(e^{-t} f(t)) = F(s-3) = \frac{3s^2 - 18s + 26}{s^2 - 5s + 7}$$

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c) $e^{-t/2} f(t)$

풀이,

$$L(e^{-t/2} f(t)) = F(s + \frac{1}{2}),$$

$$F(s) = \frac{3s^2 - 1}{s^2 + s + 1}, \ F(s)\text{에 대입}$$

$$F(s + \frac{1}{2}) = \frac{3(s + \frac{1}{2})^2 - 1}{(s + \frac{1}{2})^2 + (s + \frac{1}{2}) + 1} = \frac{3s^2 + 3s - \frac{1}{4}}{s^2 + 2s + \frac{7}{4}} = \frac{12s^2 + 12s - 1}{4s^2 + 8s + 7}$$

$$\therefore L(e^{-t/2} f(t)) = F(s + \frac{1}{2}) = \frac{12s^2 + 12s - 1}{4s^2 + 8s + 7}$$

Express the following fractions as partial fractions and hence find their inverse Laplace transforms:

(a) $\dfrac{3s+3}{(s-1)(s+2)}$

(b) $\dfrac{5s}{(s+1)(2s-3)}$

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a) $\dfrac{3s+3}{(s-1)(s+2)}$

풀이 , $\dfrac{A}{(s-1)}+\dfrac{B}{(s+2)}=\dfrac{A(s+2)}{s-1}+\dfrac{B(s-1)}{s+2}$

$$sA+2A+sB-B=3s+3$$
$$s(A+B)=3$$
$$2A-B=3$$
$$-----------$$
$$3A=6,\quad A=2, B=1$$
$$L\left\{\frac{2}{s-1}+\frac{1}{s+2}\right\}=2e^{t}+e^{-2t}$$

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b) $\dfrac{5s}{(s+1)(2s-3)}$

풀이 , $\dfrac{A}{(s+1)}+\dfrac{B}{(2s-3)}=\dfrac{A(2s-3)}{s+1}+\dfrac{B(s+1)}{2s-3}$

$$s2A-3A+sB+B=5s$$
$$s(2A+B)=5$$
$$-3A+B=0$$
$$-----------$$
$$5A=5,\quad A=1, B=3$$
$$L\left\{\frac{1}{s+1}+\frac{3}{2s-3}\right\}=L\left\{\frac{1}{s+1}+\frac{3}{2}\frac{1}{s-\frac{3}{2}}\right\}=e^{-t}+\frac{3}{2}e^{\frac{3}{2}t}$$

1 Find

(a) $e^{-2t} * e^{-t}$

(b) $t^2 * e^{-3t}$

$$(f * g)(t) = \int_0^t f(t-v)g(v)dv$$

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a) $e^{-2t} * e^{-t}$

풀이, $e^{-2t} * e^{-t}, f(t) = e^{-2t}, g(t) = e^{-t}$

$f(t-v) = e^{-2(t-v)}, g(v) = e^{-v}$

①, $\int_0^t e^{-2t+2v}e^{-v}dv = \int_0^t e^{v-2t}dv$

$= \left[e^v - e^{2t}\right]_0^t = e^{-t} - e^{-2t}$

$\therefore e^{-2t} * e^{-t} = e^{-t} - e^{-2t}$

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b) $t^2 * e^{-3t}$

풀이,

$t^2 * e^{-3t}, f(t) = t^2, g(t) = e^{-3t}$

$f(t-v) = (t-v)^2, g(v) = e^{-3v}$

①, $\int_0^t (t-v)^2 e^{-3v}dv$

$$= \left[\frac{(v-t)^2e^{-3v}}{3} - \frac{2(v-t)e^{-3v}}{9} - \frac{2e^{-3v}}{27}\right]_0^t = \left[\frac{(3(v-t)(3v-3t+2)+2)e^{-3v}}{27}\right]_0^t$$

$$= \left[\frac{3(v-t)(3v-3t+2)+2)e^{-3v}}{27}\right]_0^t = \left[\frac{(9v^2-(18t-6)v+9t^2-6t+2)e^{-3v}}{27}\right]_0^t$$

$$= \frac{0-0+9t^2-6t+2}{27} - \frac{(9t^2+6t-18t^2+9t^2-6t+2)e^{-3t}}{27}$$

$$= \frac{9t^2-6t+2}{27} - \frac{e^{-3t}}{27}$$

$$\therefore t^2 * e^{-3t} = \frac{t^2}{3} - \frac{2t}{9} + \frac{2}{27} - \frac{e^{-3t}}{27}$$

2 Use Laplace transforms to solve

(a) $x'' + x = 2t$,

$x(0) = 0, x'(0) = 5$

(b) $2x'' + x' - x = 27\cos 2t + 6\sin 2t$,

$x(0) = -1, x'(0) = -2$

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a) $x'' + x = 2t$,
$x(0) = 0,\ x'(0) = 5$

풀이, $x'' = s^2X(s) - sx(0) - x'(0) = s^2X(s) - 0 - 5$

$x = X(s)$,

$2t = \dfrac{2}{s^2}$

$X(s)(s^2+1) = \dfrac{2}{s^2} + 5 = \dfrac{2+5s^2}{s^2}$

$X(s) = \dfrac{2+5s^2}{s^2(s^2+1)}$,

$\dfrac{A(s^2+1)}{s^2} + \dfrac{B(s^2)}{s^2+1}$,

$As^2 + A + Bs^2 = 2 + 5s^2$,

$s^2(A+B=5)$

$A=2,\ B=3$

$\therefore\ L\left\{\dfrac{2}{s^2} + \dfrac{3}{s^2+1}\right\} = 2t + 3\sin t$

b) $2x'' + x' - x = 27\cos 2t + 6\sin 2t,$
$x(0) = -1,\ x'(0) = -2$

풀이,

$x'' = s^2X(s) - sx(0) - x'(0) = s^2X(s) + s + 2$
$x' = sX(s) - x(0),\ sX(s) + 1$
$x \ = X(s)$

$$27\cos 2t = \frac{27s}{s^2+4},\ \ 6\sin 2t = \frac{12}{s^2+4}$$

$2x'' + x' - x = 2s^2X(s) + 2s + 4 + sX(s) + 1 - X(s)$
$= 2s^2X(s) + sX(s) - X(s) + 2s + 5$

$$X(s)(2s^s + s - 1) = \frac{27s+12}{s^2+4} - 2s - 5 = \frac{27s + 12 - 2s(s^2+4) - 5(s^2+4)}{s^2+4}$$

$$= \frac{27s + 12 - 2s^3 - 8s - 5s^2 - 20}{s^2+4} = \frac{-2s^3 - 5s^2 + 19s - 8}{s^2+4}$$

$$X(s) = \frac{-2s^3 - 5s^2 + 19s - 8}{(s^2+4)(2s^2+s-1)} = \frac{-(2s-1)(s^2+3s-8)}{(s^2+4)(2s-1)(s+1)} = \frac{-(s^2+3s-8)}{(s^2+4)(s+1)}$$

$$-\left(\frac{As(s+1)}{(s^2+4)} + \frac{B(s^2+4)}{(s+1)}\right),\ -(As^2 + As + Bs^2 + 4B)$$

$-(A+B) = 1,\ s^2$
$-(A) = -3,\ s$
$-(B) = 2,\ C$
$\therefore L\{$