R E P O R T

[응용수학 - fourier series]

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목 차

Find the Fourier series representation of the function

$$f(t) = \begin{cases} -4 & -\pi < t \leqslant 0 \quad \text{period } 2\pi \\ 4 & 0 < t < \pi \end{cases}$$

푸리에 급수 풀이

① $f(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos(n\pi t/T) + b_n \sin(n\pi t/T))$

② $a_0 = \dfrac{1}{2T}\displaystyle\int_{-T}^{T} f(t)\,dt$

③ $a_n = \dfrac{1}{T}\displaystyle\int_{-T}^{T} f(t)\cos n\pi t/T dt$

④ $b_n = \dfrac{1}{T}\displaystyle\int_{-T}^{T} f(t)\sin n\pi t/T dt$

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② 에 대입, $a_0 = \dfrac{1}{2\pi}\displaystyle\int_{-\pi}^{\pi} f(t)dt = \dfrac{1}{2\pi}\left\{\int_{-\pi}^{0} -4dt + \int_{0}^{\pi} 4dt\right\}$

$$= \frac{1}{2\pi}\left\{[-4t]_{-\pi}^{0} + [4t]_{0}^{\pi}\right\} = 0$$

③ 에 대입,

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cos n\pi t/\pi\,dt = \frac{1}{\pi}\left\{\int_{-\pi}^{0} -4\cos nt dt + \int_{0}^{\pi} 4\cos nt dt\right\}$$

$$= \frac{1}{\pi}\left\{-\left[\frac{4}{n}sinnt\right]_{-\pi}^{0} + \left[\frac{4}{n}sinnt\right]_{0}^{\pi}\right\} = 0$$

④ 에 대입,

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\sin n\pi t/\pi\, dt = \frac{1}{\pi}\left\{\int_{-\pi}^{0} -4\sin ntdt + \int_{0}^{\pi} 4\sin ntdt\right\}$$

$$= \frac{1}{\pi}\left\{\int_{0}^{\pi} 4\sin ntdt + \int_{0}^{\pi} 4\sin ntdt\right\} = \frac{2}{\pi}\left\{-\left[\frac{4}{n}cosnt\right]_{0}^{\pi}\right\}$$

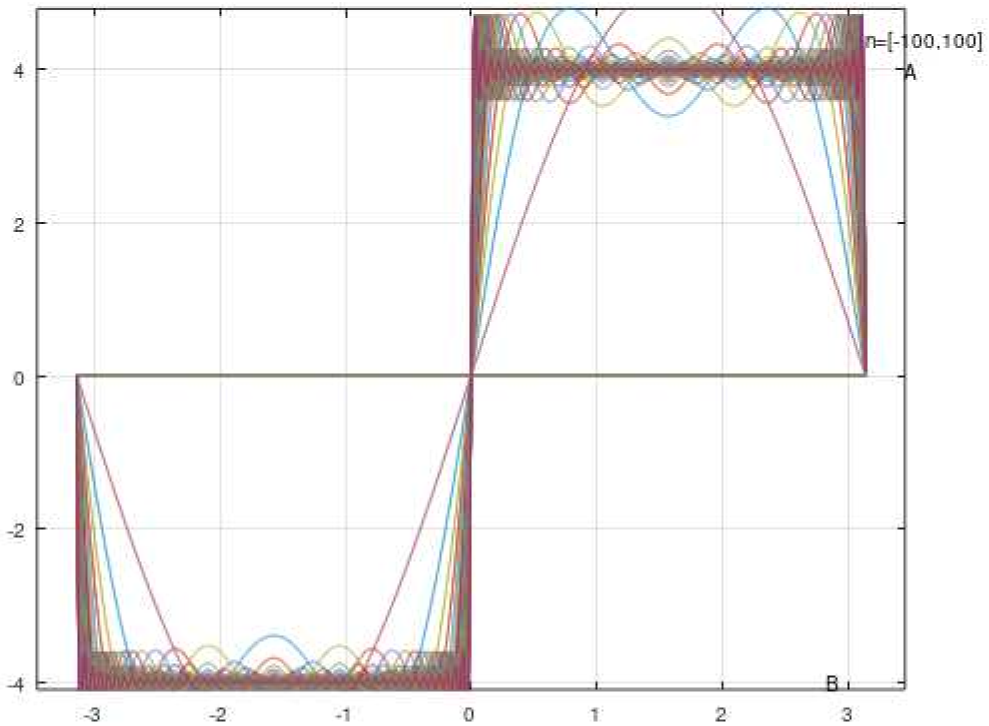
$$= \frac{2}{\pi}\left\{\frac{4-4\cos n\pi}{n}\right\} = \frac{8}{n\pi}(1-\cos n\pi)$$

① 에 대입, $f(t) = \sum_{n=1}^{\infty}\left(\frac{8}{n\pi}(1-\cos n\pi)\sin nt\right)$

$\cos n\pi = (-1)^n$,

if,
$n \equiv even,\ (1-\cos n\pi) = 0,$
$n \equiv odd,\ (1-\cos n\pi) = 2$
$n = 1 \rightarrow (\frac{8}{\pi}2\sin t),\ n = 3 \rightarrow (\frac{8}{3\pi}2\sin 3t),\ n = 5 \rightarrow (\frac{8}{5\pi}2\sin 5t)$

$$\therefore f(t) = \frac{8\left(2\sin t + \frac{2}{3}sin3t + \frac{2}{5}sin5t + \ldots\right)}{\pi}$$



octave
구하고자 하는 값에 수렴한다.

Find the Fourier series representation of the function

$$f(t) = \begin{cases} 2(1+t) & -1 < t \leqslant 0 \quad \text{period 2} \\ 0 & 0 < t < 1 \end{cases}$$

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② 에 대입, $a_0 = \frac{1}{2}\int_{-1}^{1} f(t)dt = \frac{1}{2}\int_{-1}^{0} 2(1+t)dt$

$$= \frac{1}{2}\left[t^2 + 2t\right]_{-1}^{0} = \frac{1}{2}$$

③ 에 대입,

$$a_n = \int_{-1}^{1} f(t)\cos n\pi t dt = \int_{-1}^{0} 2(1+t)\cos n\pi t dt$$

$$= 2\left\{\left[\frac{(1+t)\sin n\pi t}{n\pi}\right]_{-1}^{0} - \int_{-1}^{0}\frac{\sin n\pi t}{n\pi}dt\right\} = 2\left\{0 + \left[\frac{\cos n\pi t}{n^2\pi^2}\right]_{-1}^{0}\right\}$$

$$= 2\left\{\frac{1-\cos n\pi}{n^2\pi^2}\right\} = \frac{2-2\cos n\pi}{n^2\pi^2}$$

④ 에 대입,

$$b_n = \int_{-1}^{1} f(t)\sin n\pi t dt = \int_{-1}^{0} 2(1+t)\sin n\pi t dt$$

$$= 2\left\{-\left[\frac{(1+t)\cos n\pi t}{n\pi}\right]_{-1}^{0} - \int_{-1}^{0}\frac{\cos n\pi t}{n\pi}dt\right\} = 2\left\{-\frac{1}{n\pi} - \left[\frac{\sin n\pi t}{n^2\pi^2}\right]_{-1}^{0}\right\}$$

$$= 2\left\{-\frac{1}{n\pi} + \frac{\sin n\pi}{n^2\pi^2}\right\} = \frac{2\sin n\pi - 2n\pi}{n^2\pi^2}$$

① 에 대입,

$$f(t)=\frac{1}{2}+\sum_{n=1}^{\infty}\left(\frac{2-2\cos(n\pi)}{n^2\pi^2}cos(n\pi t)+\frac{2\sin(n\pi)-2n\pi}{n^2\pi^2}sin(n\pi t)\right)$$

$\cos n\pi=(-1)^n, \sin n\pi=0$

if ,

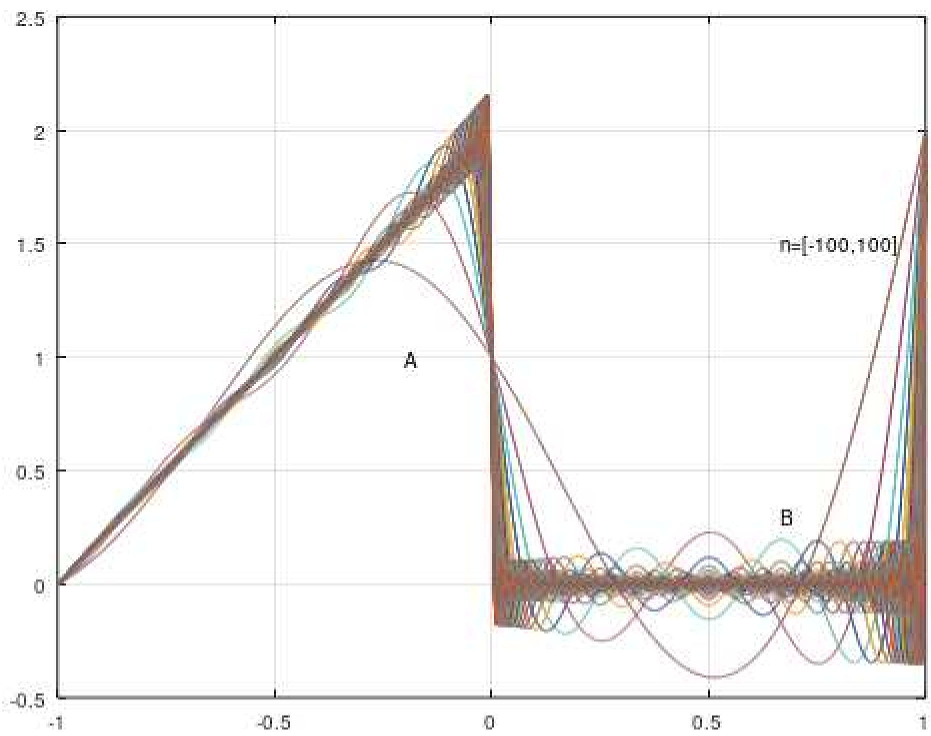
$n\equiv even,\ \frac{2-2\cos(n\pi)}{n^2\pi^2}=0,$

$n\equiv odd, \frac{2-2\cos(n\pi)}{n^2\pi^2}=\frac{4}{n^2\pi^2}$

$n=1\rightarrow\frac{4}{\pi^2}cos\,\pi t-\frac{2}{\pi}sin\,\pi t,\ n=2\rightarrow-\frac{1}{\pi}sin\,2\pi t,$

$n=3\rightarrow\frac{4}{9\pi^2}cos\,\pi t-\frac{2}{3\pi}sin\,3\pi t, n=4\rightarrow-\frac{1}{2\pi}sin\,4\pi t$

$\therefore f(t)=\frac{1}{2}+2\left(\frac{2}{\pi}cos\,\pi t-\sin\pi t-\frac{1}{2}sin\,2\pi t+\frac{2}{}\right.$



(a) $f(t)=\begin{cases}1 & 0<t<2\\ 0 & 2<t<4\end{cases}$ period 4

(b) $f(t)=e^{t}$ $-1<t<1$ period 2

복소 푸리에 급수 풀이 ① $f(t)=\sum_{-\infty}^{\infty} c_n e^{j\pi nt/T}$

② $c_n=\dfrac{1}{2T}\displaystyle\int_{-T}^{T} f(t)e^{-j\pi nt/T}dt$

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a) $f(t)=\begin{cases}1 & 0<t<2\\ 0 & 2<t<4\end{cases}$ $period\,4$

② 에 대입, $c_n=\dfrac{1}{4}\displaystyle\int_{-2}^{2} f(t)e^{-j\pi nt/2}dt=\dfrac{1}{4}\displaystyle\int_{0}^{2} e^{-j\pi nt/2}dt$

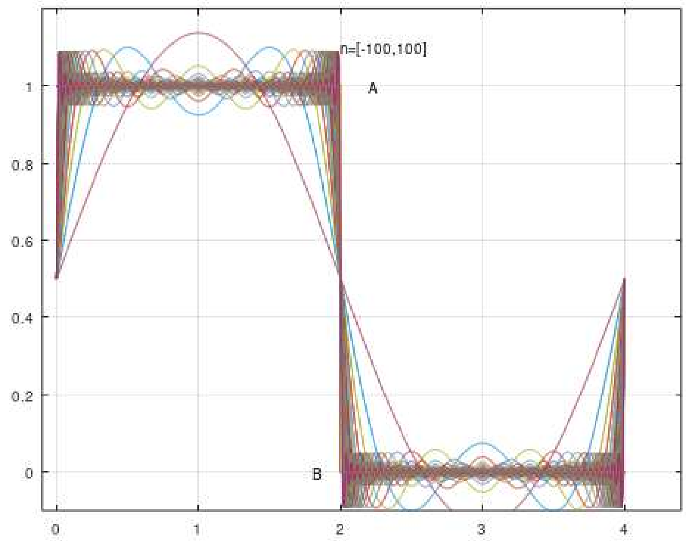
$$=\frac{1}{4}\left[\frac{e^{-j\pi nt/2}}{-j\pi n/2}\right]_0^2=\frac{e^{-j\pi n}-1}{-2j\pi n}=\frac{j(\cos n\pi-j\sin n\pi-1)}{2\pi n}$$

$,\sin\pi=0,\cos n\pi=(-1)^n$

$\therefore c_n=\dfrac{j(\cos n\pi-1)}{2n\pi}$

① 에 대입,

$$f(t)=\sum_{-\infty}^{\infty}\frac{j(\cos n\pi-1)}{2n\pi}e^{jn\pi t/2}$$



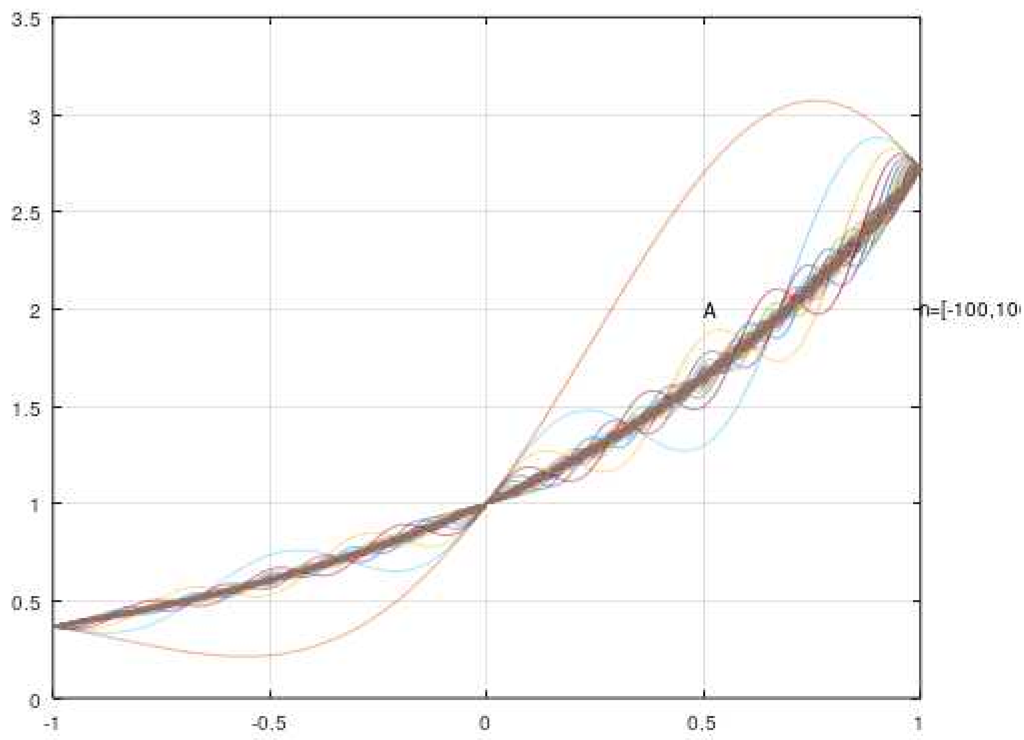
b) $f(t) = e^t \qquad -1 < t < 1 \quad period\,2$

② 에 대입, $c_n = \dfrac{1}{2}\displaystyle\int_{-1}^{1} f(t)e^{-j\pi nt}dt = \dfrac{1}{2}\int_{-1}^{1} e^t e^{-j\pi nt}dt = \dfrac{1}{2}\int_{-1}^{1} e^{(1-j\pi n)t}dt$

$$= \frac{1}{2}\left[\frac{e^{(1-j\pi n)t}}{1-j\pi n}\right]_{-1}^{1} = \frac{1}{2}\left(\frac{e^{1-j\pi n}-e^{-1+j\pi n}}{1-j\pi n}\right)$$

① 에 대입,

$$f(t) = \sum_{-\infty}^{\infty}\frac{1}{2}\left(\frac{e^{1-j\pi n}-e^{-1+j\pi n}}{1-j\pi n}\right)e^{jn\pi t}$$



Use the definition to find the d.f.t. of the sequences
$f[n] = 1, 2, 0, -1$ and $g[n] = 3, 1, -1, 1$.

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d.f.t $\quad F[k] = \sum_{n=0}^{N-1} f[n] e^{-2jnk\pi/N}$

$$F[k] = \sum_{n=0}^{3} f[n] e^{-jnk\pi/2}, \quad G[k] = \sum_{n=0}^{3} g[n] e^{-jnk\pi/2}$$

$nk = 0,1,\ldots,9, \;\; e^{-jx} = \cos x - jsinx$
$nk = 0 \to e^0 = 1, \;\; nk = 1 \to e^{-j\pi/2} = -j, \;\; nk = 2 \to e^{-j\pi} = -1$
$nk = 3 \to e^{-j3\pi/2} = j, \;\; nk = 4 \to e^{-j2\pi} = 1, \;\; nk = 5 \to -j..loop$

$k = 0,$

$$F[0] = \sum_{n=0}^{3} f[n] e^0 = 1 + 2 + 0 - 1 = 2,$$

$$G[0] = \sum_{n=0}^{3} g[n] e^0 = 3 + 1 - 1 + 1 = 4$$

$k = 1,$

$$F[1] = \sum_{n=0}^{3} f[n] e^{-jn\pi/2} - j\pi/2 = 1 + 2e^{-j\pi/2} + 0 - 1e^{-3j\pi/2} = 1 - 3j,$$

$$G[1] = \sum_{n=0}^{3} g[n] e^{-jn\pi/2} = 3 + 1e^{-j\pi/2} - 1e^{-j\pi} + 1e^{-3j\pi/2} = 4$$

$k = 2,$

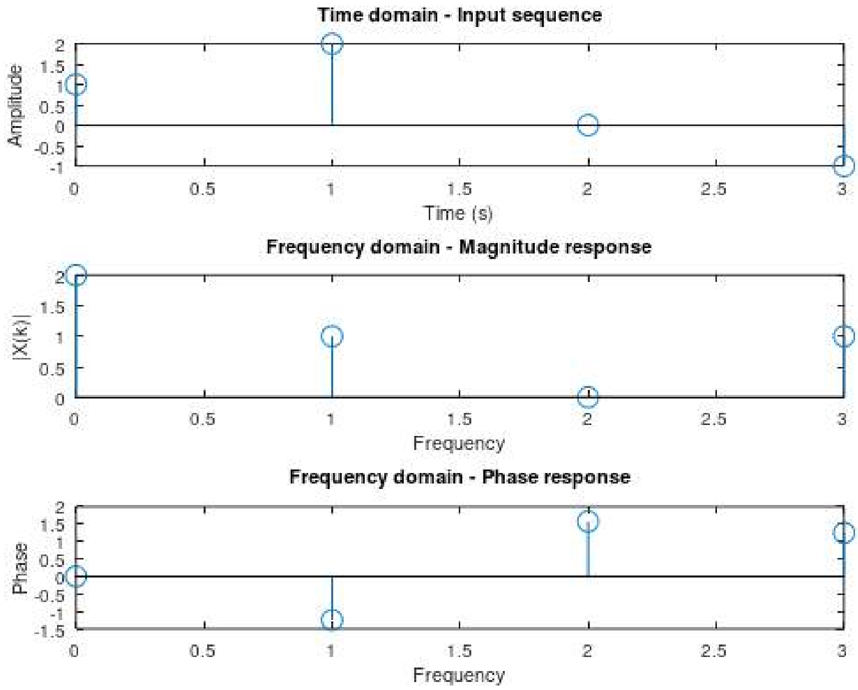
$$F[2] = \sum_{n=0}^{3} f[n] e^{-jn\pi} = 1 + 2e^{-j\pi} + 0 - 1e^{-j3\pi} = 0,$$

$$G[2] = \sum_{n=0}^{3} g[n] e^{-jn\pi} = 3 + 1e^{-j\pi} - 1e^{-j2\pi} + 1e^{-j3\pi} = 0$$

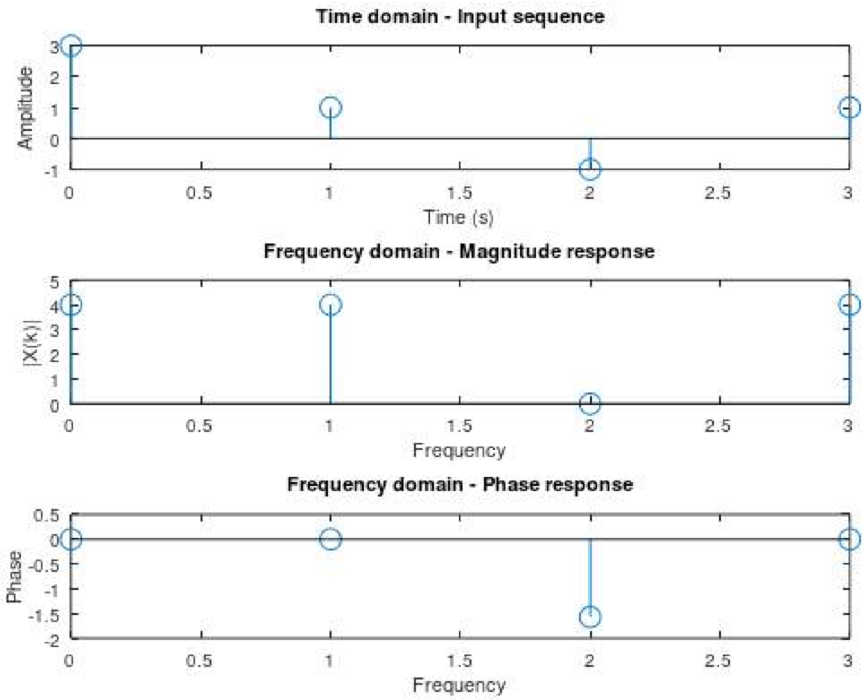
$k = 3,$

```
>> fft([1 2 0 -1])
ans =

   2 + 0i   1 - 3i   0 + 0i   1 + 3i
```

```
>> fft([3 1 -1 1])
ans =

   4   4   0   4
```


Time domain - Input sequence
Amplitude
Time (s)
Frequency domain - Magnitude response
|X(k)|
Frequency
Frequency domain - Phase response
Phase
Frequency

--- f[n] ---


Time domain - Input sequence
Amplitude
Time (s)
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|X(k)|
Frequency
Frequency domain - Phase response
Phase
Frequency

--- g[n] ---

Calculate the d.f.t. of the sequence $f[n] = 5, -1, 2$.

$$F[k] = \sum_{n=0}^{2} f[n] e^{-2jnk\pi/3}$$

$$nk = 0,1,2,4$$

$$nk = 0 \rightarrow e^0 = 1,\ \ nk = 1 \rightarrow e^{-j2\pi/3} = -\frac{1}{2} - j\frac{\sqrt{3}}{2},$$

$$nk = 2 \rightarrow e^{-j4\pi/3} = -\frac{1}{2} + j\frac{\sqrt{3}}{2},\ \ nk = 4 \rightarrow e^{-j8\pi/3} = -\frac{1}{2} - j\frac{\sqrt{3}}{2}$$

$k = 0,$

$$F[0] = \sum_{n=0}^{2} f[n] e^0 = 5 - 1 + 2 = 6,$$

$k = 1,$

$$F[1] = \sum_{n=0}^{2} f[n] e^{-2jn\pi/3} = 5e^0 - 1e^{-2j\pi/3} + 2e^{-4j\pi/3}$$

$$= 5 - (-\frac{1}{2} - j\frac{\sqrt{3}}{2}) + 2(-\frac{1}{2} + j\frac{\sqrt{3}}{2}) = \frac{9}{2} + j\frac{3\sqrt{3}}{2}$$

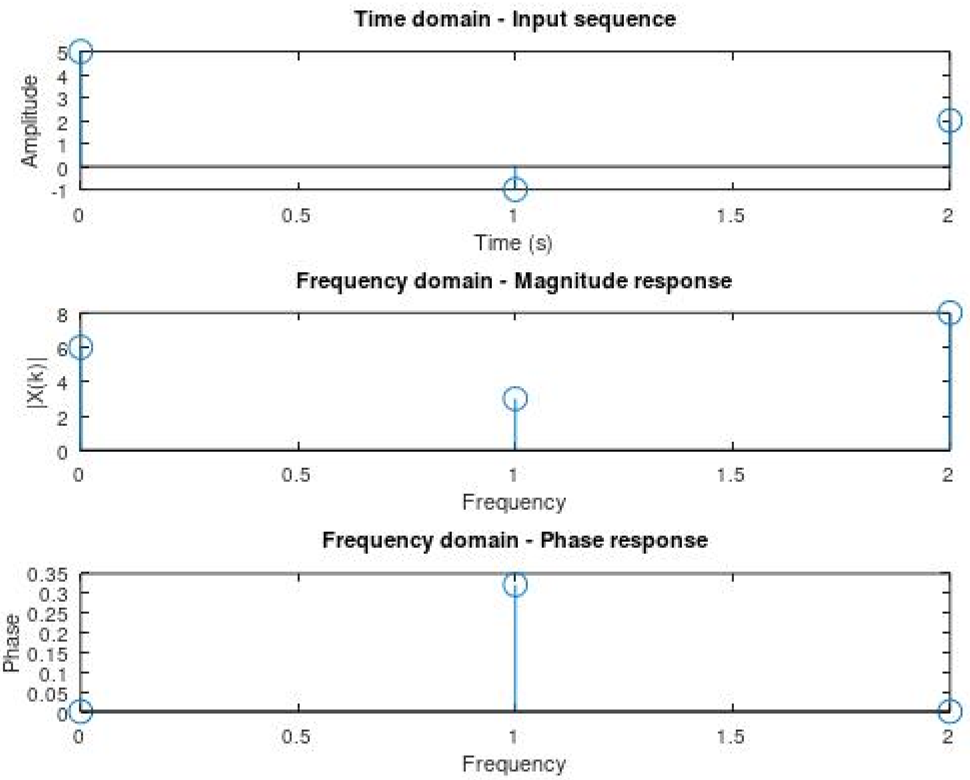
$k = 2,$

$$F[2] = \sum_{n=0}^{2} f[n] e^{-j4n\pi/3} = 5e^0 - 1e^{-j4\pi/3} + 2e^{-j8\pi/3}$$

$$= 5 + (\frac{1}{2} - j\frac{\sqrt{3}}{2}) - 2($$

```
>> fft([5 -1 2])
ans =

   6.0000 +      0i   4.5000 + 2.5981i   4.5000 - 2.5981i
```


Time domain - Input sequence
Amplitude
Time (s)
Frequency domain - Magnitude response
|X(k)|
Frequency
Frequency domain - Phase response
Phase
Frequency

푸리에 급수 Octave 코드

```
clear all;
close all;
clc;

T0 = 2.0; %주기
f0 = 1/T0; %주파수
Ts = 0.001; %샘플링레이트

t = -T0/2:Ts:T0/2; %시간
A = 0; %조건1 범위가 더 큰 값
B = 2*(t.+1); %조건2 범위가 더 작은 값

F0 = 0.5 * (A + B); % F0는 A와 B의 중간값(미분)
f2_t = F0;

figure()
for n=1:100
    Fn = (A-B) * (1-exp(-1i*pi*n)) / (1i*2*pi*n);
    fn = F_n .* exp(1i*2*pi*n*f0*t); % 푸리에 급수식 = 푸리에 복소 급수식
    f2 = f2_t + 2 * real(fn); % 급수

    plot(t, f2_t);
    hold on
    grid on

    pause(0.00001)
end

% N값 보여줌. 임의로 위치 설정
strN = num2str(n);
str = strcat('n=[-', strN, ',', strN, ']');
text(T_0/3, 1.5, str);
text(-0.2, 1, 'A');
text(T_0/3, 0.3, 'B');
```

-> A, B, T0, t 값만 변경하면 된다.

이산 푸리에 변환(DFT) 코드

```
x = [5 -1 2]; %x값 입력
N = length(x);
X = zeros(N,1)
for k = 0:N-1   %DFT 과정 FFT(x)
    for n = 0:N-1
        X(k+1) = X(k+1) + x(n+1)*exp(-j*pi/2*n*k)
    end
end
t = 0:N-1
subplot(311)
stem(t,x);
xlabel('Time (s)');
ylabel('Amplitude');

subplot(312)
stem(t,X)
xlabel('Frequency');
ylabel('|X(k)|');

subplot(313)
stem(t,angle(X))
xlabel('Frequency');
ylabel('Phase');
```