R E P O R T

[응용수학 - 미분 풀이]

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목 차

Use the quotient rule to find the derivatives of the following:

(a) $\dfrac{\cos x}{\sin x}$ (b) $\dfrac{\tan t}{\ln t}$

(c) $\dfrac{e^{2t}}{t^3+1}$ (d) $\dfrac{3x^2+2x-9}{x^3+1}$

--분수 함수의 미분--

① $f(x)=\dfrac{g(x)}{h(x)}$

② $f'(x)=\dfrac{h(x)g'(x)-g(x)h'(x)}{h^2(x)}$

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a) $\dfrac{\cos(x)}{\sin(x)}$

①에 대입, $f(x)=\dfrac{\cos(x)}{\sin(x)}$

$g(x)=\cos(x), h(x)=\sin(x)$

$g'(x)=-\sin(x), h'(x)=\cos(x)$

②에 대입, $f'(x)=\dfrac{\sin(x)\cdot(-\sin(x))-\cos(x)\cos(x)}{\sin^2(x)}$

$$=\frac{-\sin^2(x)-\cos^2(x)}{\sin^2(x)}$$

$$=-\frac{\sin^2(x)+\cos^2(x)}{\sin^2(x)}$$

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b) $\dfrac{\tan(t)}{\ln(t)}$

①에 대입, $f(t)=\dfrac{\tan(t)}{\ln(t)}$

$g(t)=\tan(t), h(t)=\ln(t)$

$g'(t)=\sec^2(t), h'(t)=\dfrac{1}{t}$

②에 대입, $f'(t)=\dfrac{\ln(t)\sec^2(t)-\tan(t)\dfrac{1}{t}}{\ln^2(t)}$

$$=\frac{\sec^2(t)}{\ln(t)}-\frac{\tan(t)}{t\cdot\ln^2(t)}$$

c) $\dfrac{e^2}{t^3+1}$

①에 대입, $f(t) = \dfrac{e^{2t}}{t^3+1}$

$g(t) = e^{2t}, h(t) = t^3+1$

$g'(t) = 2e^{2t}(t), h'(t) = 3t^2$

②에 대입, $f'(t) = \dfrac{2e^{2t}(t^3+1) - e^{2t}3t^2}{(t^3+1)^2}$

$= \dfrac{e^{2t}(2t^3+2-3t^2)}{(t^3+1)^2} = \dfrac{e^{2t}(2t^3-3t^2+2)}{(t^3+1)^2}$

$= \dfrac{2e^{2t}}{t^3+1} - \dfrac{3t^2e^{2t}}{(t^3+1)^2}$

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d) $\dfrac{3x^2+2x-9}{x^3+1}$

①에 대입, $f(x) = \dfrac{3x^2+2x-9}{x^3+1}$

$g(x) = 3x^2+2x-9, h(x) = x^3+1$

$g'(x) = 6x+2, h'(x) = 3x^2$

②에 대입, $f'(x) = \dfrac{(x^3+1)(6x+2) - (3x^2+2x-9)(3x^2)}{(x^3+1)^2}$

$= \dfrac{(6x^4+2x^3+6x+2) - (9x^4+6x^3-27x^2)}{(x^3+1)^2}$

$= \dfrac{-3x^4-4x^3+27x^2+6x+2}{(x^3+1)^2}$

Use logarithmic differentiation to find the derivatives of the following functions:

(a) $y = x^4 e^x$ (b) $y = \frac{1}{x} e^{-x}$

(c) $z = t^3 (1+t)^9$ (d) $y = e^x \sin x$

로그 미분 사용

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a) $y = x^4 e^x,\ \frac{dy}{dx} = ?$

$$\ln y = \ln 4x + \ln x$$

$$\frac{d}{dx}(\ln y) = \frac{d}{dx}(4\ln x) + \frac{d}{dx}(\ln x)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{4}{x} + 1$$

$$\frac{dy}{dx} = y\left(\frac{4}{x} + 1\right) = x^4 e^x \left(\frac{4}{x} + 1\right)$$
$$= 4x^3 e^x + x^4 e^x$$

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b) $y = \frac{1}{x} e^{-x},\ \frac{dy}{dx} = ?$

$$\ln y = \ln x^{-1} + \ln e^{-x} = -\ln x - x$$

$$\frac{d}{dx}(\ln y) = \frac{d}{dx}(-\ln x) + \frac{d}{dx}(-x)$$

$$\frac{1}{y}\frac{dy}{dx} = -\frac{1}{x} - 1$$

$$\frac{dy}{dx} = y\left(-\frac{1}{x} - 1\right) = \frac{1}{x} e^{-x}\left(-\frac{1}{x} - 1\right)$$
$$= -e^{-x}\left(\frac{1}{x^2} + \frac{1}{x}\right)$$

c) $z = t^3(1+t)^9,\ \frac{dz}{dt} = ?$

$$\ln z = \ln t^3 + \ln (1+t)^9 = 3\ln t + 9\ln (1+t)$$

$$\frac{d}{dt}(\ln z) = \frac{d}{dt}3\ln t + \frac{d}{dt}9\ln (1+t)$$

$$\frac{1}{z}\frac{dz}{dt} = \frac{3}{t} + \frac{9}{1+t}$$

$$\frac{dz}{dt} = z(\frac{3}{t} + \frac{9}{1+t}) = t^3(1+t)^9(\frac{3}{t} + \frac{9}{1+t})$$

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d) $y = e^x \sin x,\ \frac{dy}{dx} = ?$

$$\ln y = \ln e^x + \ln \sin (x) = x + \cos(x)\ln (\sin (x))$$

$$\frac{d}{dx}(\ln y) = 1 + \frac{d}{dx}cos(x)\ln (\sin (x))$$

$$\frac{1}{y}\frac{dy}{dt} = 1 + \frac{\cos(x)}{\sin (x)} = 1 + \cot(x)$$

$$\frac{dz}{dt} = y(1 + \cot(x)) = e^x \sin x(1 + \cot(x))$$

Use the Newton–Raphson technique to find the value of a root of the following equations correct to two decimal places. An approximate root, x_1, is given in each case.

(a) $2\cos x = x^2 \qquad x_1 = 0.8$

(b) $3x^3 - 4x^2 + 2x - 9 = 0 \qquad x_1 = 2$

뉴턴-랩슨 기법 사용, 옥타브 증명

①$f(x) = g(x),\ f'(x) = g'(x)$

②$x_{n+1} = x_n - \dfrac{f(x)}{f'(x)}$ 반복

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a) $2\cos(x) = x^2,\ x_1 = 0.8$

① $f(x) = 2\cos(x) - x^2$

$f'(x) = -2\sin(x) - 2x$

②$x_{n+1} = x_n - \dfrac{f(x)}{f'(x)}$ 반복

$x_1 = 0.8,$

$$x_2 = 0.8 - \frac{2\cos(0.8) - (0.8)^2}{-2\sin(0.8) - 2(0.8)}$$
$$= 0.8 - \frac{0.7534}{-3.0347} = 0.8 - (-0.2483) = 1.0483,$$

$$x_3 = 1.0483 - \frac{2\cos(1.0483) - (1.0483)^2}{-2\sin(1.0483) - 2(1.0483)}$$
$$= 1.0483 - \frac{-0.1007}{-3.8296} = 1.0483 - (0.026297) = 1.022,$$

$$x_4 = 1.022 - \frac{2\cos(1.022) - (1.022)^2}{-2\sin(1.022) - 2(1.022)}$$
$$= 1.022 - \frac{-0.0010419}{-3.7502} = 1.022 - (0.00027783) = 1.0217$$

-octave-

```
---1---
x_1 = 0.8000
x_2 = 1.0483

---2---
x_2 = 1.0483
x_3 = 1.0220

---3---
x_3 = 1.0220
x_4 = 1.0217

---4---
x_4 = 1.0217
x_5 = 1.0217

---5---
x_5 = 1.0217
x_6 = 1.0217
```

이 방식대로 계산하면 octave에서 x_5또한 1.0217임을 알 수 있고 1.0217에 수렴한다.

b) $3x^3 - 4x^2 + 2x - 9 = 0,\ x_1 = 2$

① $f(x) = 3x^3 - 4x^2 + 2x - 9$

$f'(x) = 9x^2 - 8x + 2$

② $x_{n+1} = x_n - \dfrac{f(x)}{f'(x)}$ 반복

$x_1 = 2,$

$$x_2 = 2 - \frac{3(2^3) - 4(2^2) + 2(2) - 9}{9(2^2) - 8(2) + 2}$$
$$= 2 - \frac{24 - 16 + 4 - 9}{36 - 16 + 2} = 2 - \frac{3}{10} = 1.8636,$$

$$x_3 = 1.8636 - \frac{3(1.8636^3) - 4(1.8636^2) + 2(1.8636) - 9}{9(1.8636^2) - 8(1.8636) + 2}$$
$$= 2 - \frac{19.418 - 13.893 + 3.7272 - 9}{31.258 - 14.909 + 2} = 1.8636 - \frac{0.2522}{18.349} = 1.8499,$$

$$x_4 = 1.8499 - \frac{3(1.8499^3) - 4(1.8499^2) + 2(1.8499) - 9}{9(1.8499^2) - 8(1.8499) + 2}$$
$$= 2 - \frac{18.992 - 13.689 + 3.6998 - 9}{30.799 - 14.799 + 2} = 1.8499 - \frac{0.0028}{18} = 1.8497$$

```
---1---
x_1 = 2
x_2 = 1.8636

---2---
x_2 = 1.8636
x_3 = 1.8499

---3---
x_3 = 1.8499
x_4 = 1.8497

---4---
x_4 = 1.8497
x_5 = 1.8497
```

이 방식대로 계산하게 되면 x_5또한 1.8497임을 알 수 있고 1.8497에 수렴한다.

Find the Maclaurin series for
$y(x) = x^2 + \sin x$.

$$p(x) = y(0) + y'(0)(x) + y''(0)\frac{x^2}{2!} + y^3(0)\frac{x^3}{3!} + ... + y^n(0)\frac{x^n}{n!}$$

풀이

$y(x) = x^2 + \sin(x)$ and $x = 0 \rightarrow Maclaurin$

$If,\ y(x) = \sin(x), y(0) = 0$

$y'(x) = \cos(x), y'(0) = 1$
$y''(x) = -\sin(x),\ y''(0) = 0$
$y^3(x) = -\cos(x), y^3(0) = -1$

$y^4(x) = \sin(x) ...$ 이후로 반복

$$\begin{aligned} p(x) &= y(0) + y'(0)(x) + y''(0)\frac{x^2}{2!} + y^3(0)\frac{x^3}{3!} + ... + y^n(0)\frac{x^n}{n!} \\ &= 0 + x + 0 + (-\frac{x^3}{3!}) + 0 + \frac{x^5}{5!} + ... \\ &= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ... \\ &= \sum_{i=0}^{\infty} \frac{(-1)^i x^{2i+1}}{(2i+1)!} \end{aligned}$$

$Then, y(x) = x^2 + \sin(x)$

$$\begin{aligned} \therefore p(x) &= x^2 + (x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ...) \\ &= x^2 + \sum_{i=0}^{\infty} \frac{(-1)^i x^{2i+1}}{(2i+1)!} \end{aligned}$$

Find the Taylor series for $y(x) = x + e^x$ about $x = 1$.

$$p(x) = y(a) + y'(a)(x-a) + y''(a)\frac{(x-a)^2}{2!} + y^3(a)\frac{(x-a)^3}{3!} + \dots + y^n(a)\frac{(x-a)^n}{n!}$$

풀이

$y(x) = x + e^x$ and $x = 1 \rightarrow Taylor$

$If,\ y(x) = e^x, y(1) = e^1$

$y'(x) = e^x, y'(1) = e^1$

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$y^n(x) = e^1$

$$p(x) = y(1) + y'(1)(x-1) + y''(1)\frac{(x-1)^2}{2!} + y^3(1)\frac{(x-1)^3}{3!} + \dots + y^n(1)\frac{(x-1)^n}{n!}$$

$$= e + e(x-1) + e\frac{(x-1)^2}{2!} + e\frac{(x-1)^3}{3!}\dots$$

$$= e\sum_{i=0}^{\infty}\frac{(x-1)^i}{i!}$$

$Then, y(x) = x + e^x$

$$\therefore p(x) = x + \left(e + e(x-1) + e\frac{(x-1)^2}{2!} + e\frac{(x-1)^3}{3!}\dots\right)$$

$$= x + e + ex - e + e\left(\frac{(x-1)^2}{2!} + e\frac{(x-1)^3}{3!}\dots\right)$$

$$= (1+e)x + e\left(\frac{(x-1)^2}{2!} + e\frac{(x-1)^3}{3!}\dots\right)$$

$$= x + e\sum_{i=0}^{\infty}\frac{(x-1)^i}{i!}$$

Given that $y(x) = x^2$,

(a) Calculate the Taylor series of $y(x)$ about $x = a$.

(b) Calculate the Maclaurin series of $y(x)$.

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①$p(x) = y(0) + y'(0)(x) + y''(0)\dfrac{x^2}{2!} + y^3(0)\dfrac{x^3}{3!} + \ldots + y^n(0)\dfrac{x^n}{n!}$

②$p(x) = y(a) + y'(a)(x-a) + y''(a)\dfrac{(x-a)^2}{2!} + y^3(a)\dfrac{(x-a)^3}{3!} + \ldots + y^n(a)\dfrac{(x-a)^n}{n!}$

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a) $y(x) = x^2$ and $x = a$

$y(a) = a^2,\ y'(a) = 2a,\ y''(a) = 2,\ y^3(a) = 0$

②적용

$$p(x) = y(a) + y'(a)(x-a) + y''(a)\frac{(x-a)^2}{2!} + y^3(a)\frac{(x-a)^3}{3!} + \ldots + y^n(a)\frac{(x-a)^n}{n!}$$

$$= a^2 + 2a(x-a) + 2\frac{(x-a)^2}{2!} + 0 + \ldots + 0$$

$$= a^2 + 2ax - 2a^2 + x^2 - 2ax + a^2$$

$\therefore\ = x^2$

b) $y(x) = x^2$

$y(0) = x^2 = 0, y'(0) = 2x = 0, y''(0) = 2,\ y^3(0) = 0$

①적용

$$p(x) = y(0) + y'(0)(x) + y''(0)\frac{x^2}{2!} + y^3(0)\frac{x^3}{3!} + \ldots + y^n(0)\frac{x^n}{n!}$$

$$= 0 + 0 + 2\frac{x^2}{2!} + 0 + \ldots + 0$$

$\therefore\ = x^2$