

### **$\Theta(n+m)$ algorithm to print in-degree and out-degree**

Pseudocode

```
ALGORITHM IN-DEGREE-OUT-DEGREE( $G$ )
1 // inDegreeCount will be the counter for inDegrees of each vertex
2 // outDegreeCount will be the counter for outDegrees of each vertex
3
4 let inDegreeCount[1.. $G.V.length$ ] and outDegreeCount[1..  $G.V.length$ ] be new arrays
5
6
7 // initialize inDegreeCount and outDegreeCount to 0
8 //  $\Theta(n)$  time
9 for each vertex  $u \in G.V$ 
10 inDegreeCount[ $u$ ] = 0
11 outDegreeCount[ $u$ ] = 0
12
13 // iterate over each edge so inDegreeCount outDegreeCount will be incremented
14 //  $\Theta(m)$  time - where  $u$  is the Vertex and  $v$  is the Edge connected to Vertex
15 for each vertex  $u$ , edge  $v \in G.E$ 
16 outDegreeCount[ $u$ ] += 1
17 inDegreeCount[ $v$ ] += 1
18
19 // iterate over each vertex and print in-degree and out-degree of every vertex in an m-edge
20 // n-vertex directed graph
21 //  $\Theta(n)$  time
22 for each vertex  $u \in G.V$ 
23 PRINT "out-degree({0}): ".format( $u$ ), outDegreeCount[ $u$ ]
24 PRINT "in-degree({0}): ".format( $u$ ), inDegreeCount[ $u$ ]
```

Analysis of algorithm  **$\Theta(n+m)$**

$$\Theta(n) + \Theta(m) + \Theta(n) = \Theta(n+m)$$

Lines 9 through 11 will initialize values of 0 for the counter representation in each in-degree and out-degree array. The length of each in-degree and out-degree array will be the length of the vertex array V. As we are iterating over all  $n$ -elements of the V array, this will be an  $\Theta(n)$  time operation. Lines 15 through 17 will iterate over all the  $m$ -edges in the array E which will be an  $\Theta(m)$  operation. For each out-degree and for each in-degree vertex of each edge, the counter will be incremented by 1 each time. Lines 22 through 24 will output the out-degree counts and in-degree counts for every vertex. This will be a Theta -  $\Theta$  time operation. The  $\Theta(n+m)$  represents the bounding of a function from both above and below, an asymptotically tight bound – which represents the exact asymptotic complexity of an algorithm. Theta notation  $\Theta$  is used to denote that the rate of growth is equivalent to the specified values of V and E, and since we are iterating over every single vertex and edge presented, this will be a  $\Theta(n+m)$  time complexity.