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Discussion 10

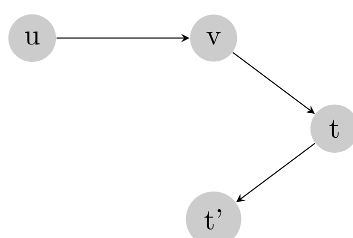
1. Modify MST or Shortest Path

Example 1. Homework 7 Problem 4 (MST) and 8 (Dijkstra's)

Example 2. Suppose you are given a directed graph $G = (V, E)$ with weights on the edges $w(u, v)$ for $(u, v) \in E$ and a destination vertex (sink) t (weights may be negative). For each vertex $v \in V$, $d(v)$ is the shortest weighted path from vertex v to the sink t.

(a). How can we verify $d(v)$ is true in $O(|E|)$ time? (Hint: try to modify the edges!)

(b). Now you need to compute shortest weighted path to a different sink t'. Give an $O(|E| \log |V|)$ algorithm for computing $d'(v)$ for all vertices $v \in V$ to the new sink vertex t'. (Hint: It is useful to consider a new weight function defined as follows: for edge (u, v), let $w'(u, v) = w(u, v) - d(u) + d(v)$. Can we use some known algorithm?)



2. NP: Decision Problem to Optimization Problem

Example 1. Homework 8 Problem 1: Bin packing

Example 2. A decision problem related to the Shortest path problem is : Given a graph $G = (V, E)$, two vertices $u, v \in V$, and a non-negative integer k, does a path exist in G between u and v whose length is at most k? The answer is Yes or No.

The corresponding optimization problem is to find such a path between u and v. How to use the decision problem as a subroutine to solve the optimization problem in polynomial time, namely $O(|E| \cdot O(\text{SP-Decision}))$?