gsvd

Generalized singular value decomposition

Syntax

```
[U,V,X,C,S] = gsvd(A,B)
[U,V,X,C,S] = gsvd(A,B,0)
sigma = gsvd(A,B)
```

Description

`[U,V,X,C,S] = gsvd(A,B)` returns unitary matrices `U` and `V`, a (usually) square matrix `X`, and nonnegative diagonal matrices `C` and `S` so that

```
A = U*C*X'
B = V*S*X'
C'*C + S'*S = I
```

`A` and `B` must have the same number of columns, but may have different numbers of rows. If `A` is `m`-by-`p` and `B` is `n`-by-`p`, then `U` is `m`-by-`m`, `V` is `n`-by-`n`, `X` is `p`-by-`q`, `C` is `m`-by-`q` and `S` is `n`-by-`q`, where `q = min(m+n,p)`.

The nonzero elements of `S` are always on its main diagonal. The nonzero elements of `C` are on the diagonal `diag(C,max(0,q-m))`. If `m >= q`, this is the main diagonal of `C`.

`[U,V,X,C,S] = gsvd(A,B,0)`, where `A` is `m`-by-`p` and `B` is `n`-by-`p`, produces the "economy-sized" decomposition where the resulting `U` and `V` have at most `p` columns, and `C` and `S` have at most `p` rows. The generalized singular values are `diag(C)./diag(S)` so long as `m >= p` and `n >= p`.

If `A` is `m`-by-`p` and `B` is `n`-by-`p`, then `U` is `m`-by-`min(q,m)`, `V` is `n`-by-`min(q,n)`, `X` is `p`-by-`q`, `C` is `min(q,m)`-by-`q` and `S` is `min(q,n)`-by-`q`, where `q = min(m+n,p)`.

`sigma = gsvd(A,B)` returns the vector of generalized singular values, `sqrt(diag(C'*C)./diag(S'*S))`. When `B` is square and nonsingular, the generalized singular values, `gsvd(A,B)`, correspond to the ordinary singular values, `svd(A/B)`, but they are sorted in the opposite order. Their reciprocals are `gsvd(B,A)`.

The vector `sigma` has length `q` and is in non-decreasing order.

Examples

Example 1

The matrices have at least as many rows as columns.

```
A = reshape(1:15,5,3)
B = magic(3)
A =
         1     6    11
         2     7    12
         3     8    13
         4     9    14
         5    10    15
B =
         8     1     6
         3     5     7
         4     9     2
```

The statement

```
[U,V,X,C,S] = gsvd(A,B)
```

produces a 5-by-5 orthogonal `U`, a 3-by-3 orthogonal `V`, a 3-by-3 nonsingular `X`,

```
X =
        2.8284   -9.3761   -6.9346
       -5.6569   -8.3071  -18.3301
        2.8284   -7.2381  -29.7256
```

and

```
C =
        0.0000         0         0
             0    0.3155         0
             0         0    0.9807
             0         0         0
             0         0         0
S =
        1.0000         0         0
             0    0.9489         0
             0         0    0.1957
```

Since `A` is rank deficient, the first diagonal element of `C` is zero.

The economy sized decomposition,

```
[U,V,X,C,S] = gsvd(A,B,0)
```

produces a 5-by-3 matrix `U` and a 3-by-3 matrix `C`.

```
U =
        0.5700   -0.6457   -0.4279
       -0.7455   -0.3296   -0.4375
       -0.1702   -0.0135   -0.4470
        0.2966    0.3026   -0.4566
        0.0490    0.6187   -0.4661

C =
        0.0000         0         0
             0    0.3155         0
             0         0    0.9807
```

The other three matrices, `V`, `X`, and `S` are the same as those obtained with the full decomposition.

The generalized singular values are the ratios of the diagonal elements of `C` and `S`.

```
sigma = gsvd(A,B)
sigma =
        0.0000
        0.3325
        5.0123
```

These values are a reordering of the ordinary singular values

```
svd(A/B)
ans =
        5.0123
        0.3325
        0.0000
```

Example 2

The matrices have at least as many columns as rows.

```
A = reshape(1:15,3,5)
B = magic(5)
A =

     1     4     7    10    13
     2     5     8    11    14
     3     6     9    12    15

B =

    17    24     1     8    15
    23     5     7    14    16
     4     6    13    20    22
    10    12    19    21     3
    11    18    25     2     9
```

The statement

```
[U,V,X,C,S] = gsvd(A,B)
```

produces a 3-by-3 orthogonal `U`, a 5-by-5 orthogonal `V`, a 5-by-5 nonsingular `X` and

```
C =

         0         0    0.0000         0         0
         0         0         0    0.0439         0
         0         0         0         0    0.7432

S =

    1.0000         0         0         0         0
         0    1.0000         0         0         0
         0         0    1.0000         0         0
         0         0         0    0.9990         0
         0         0         0         0    0.6690
```

In this situation, the nonzero diagonal of `C` is `diag(C,2)`. The generalized singular values include three zeros.

```
sigma = gsvd(A,B)
sigma =

         0
         0
    0.0000
    0.0439
    1.1109
```

Reversing the roles of `A` and `B` reciprocates these values, producing two infinities.

```
gsvd(B,A)
ans =

   1.0e+16 *

    0.0000
    0.0000
    8.8252
       Inf
       Inf
```

Tips

- In this formulation of the `gsvd`, no assumptions are made about the individual ranks of `A` or `B`. The matrix `X` has full rank if and only if the matrix `[A;B]` has full rank. In fact, `svd(X)` and `cond(X)` are equal to `svd([A;B])` and `cond([A;B])`. Other formulations, eg. G. Golub and C. Van Loan [1], require that `null(A)` and `null(B)` do not overlap and replace `X` by `inv(X)` or `inv(X')`.

  Note, however, that when `null(A)` and `null(B)` do overlap, the nonzero elements of `C` and `S` are not uniquely determined.

Algorithms

The generalized singular value decomposition uses the C-S decomposition described in [1], as well as the built-in `svd` and `qr` functions. The C-S decomposition is implemented in a local function in the `gsvd` program file.

References

[1] Golub, Gene H. and Charles Van Loan, *Matrix Computations*, Third Edition, Johns Hopkins University Press, Baltimore, 1996

See Also

`qr` | `svd`

Introduced before R2006a