LEVEL II SCHWESER'S QuickSheet

CRITICAL CONCEPTS FOR THE 2020 CFA® EXAM

ETHICAL AND PROFESSIONAL STANDARDS

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- I (A) Knowledge of the Law
- I (B) Independence and Objectivity
- I (C) Misrepresentation
- I (D) Misconduct
- II Integrity of Capital Markets
- II (A) Material Nonpublic Information
- II (B) Market Manipulation
- III Duties to Clients
- III (A) Loyalty, Prudence, and Care
- III (B) Fair Dealing
- III (C) Suitability
- III (D) Performance Presentation
- III (E) Preservation of Confidentiality
- IV Duties to Employers
- IV (A) Loyalty
- IV (B) Additional Compensation Arrangements
- IV (C) Responsibilities of Supervisors
- V Investment Analysis, Recommendations, and Action
- V (A) Diligence and Reasonable Basis
- V (B) Communication with Clients and Prospective Clients
- V (C) Record Retention
- VI Conflicts of Interest
- VI (A) Disclosure of Conflicts
- VI (B) Priority of Transactions
- VI (C) Referral Fees
- VII Responsibilities as a CFA Institute Member or CFA Candidate
- VII (A) Conduct in the CFA Program
- VII (B) Reference to CFA Institute, CFA Designation, and CFA Program

QUANTITATIVE METHODS

Simple Linear Regression

Confidence interval for predicted Y-value:

 $\hat{Y} \pm t_c \times SE$ of forecast

Multiple Regression

$$Y_{i} = b_{0} + (b_{1} \times X_{1i}) + (b_{2} \times X_{2i}) + (b_{3} \times X_{3i}) + \varepsilon_{i}$$

• Test statistical significance of b; H_0 : b = 0.

$$t = \hat{b}_{s_{\hat{k}}}, n-k-1 df$$

- Reject if |t| > critical t or p-value < α .
- Confidence Interval: $\hat{b}_j \pm (t_c \times s_{\hat{b}_j})$.
- SST = RSS + SSE.
- MSR = RSS / k.
- MSE = SSE / (n k 1).
- Test statistical significance of regression:
 F = MSR / MSE with k and n k 1 df (1-tail).
- Standard error of estimate (SEE = \sqrt{MSE}). Smaller SEE means better fit.
- Coefficient of determination (R² = RSS / SST).
 % of variability of Y explained by Xs; higher R² means better fit.

Regression Analysis—Problems

- Heteroskedasticity. Non-constant error variance.
 Detect with Breusch-Pagan test. Correct with White-corrected standard errors.
- Autocorrelation. Correlation among error terms. Detect with Durbin-Watson test; positive

- autocorrelation if DW < dl. Correct by adjusting standard errors using Hansen method.
- Multicollinearity. High correlation among Xs.
 Detect if F-test significant, t-tests insignificant.
 Correct by dropping X variables.

Model Misspecification

- Omitting a variable.
- Variable should be transformed.
- Incorrectly pooling data.
- Using lagged dependent vbl. as independent vbl.
- Forecasting the past.
- Measuring independent variables with error.

Effects of Misspecification

Regression coefficients are biased and inconsistent, lack of confidence in hypothesis tests of the coefficients or in the model predictions.

Linear trend model:
$$y_t = b_0 + b_1 t + \varepsilon_t$$

Log-linear trend model: $ln(y_t) = b_0 + b_1 t + \epsilon_t$

Covariance stationary: mean and variance don't change over time. To determine if a time series is covariance stationary, (1) plot data, (2) run an AR model and test correlations, and/or (3) perform Dickey Fuller test.

Unit root: coefficient on lagged dep. vbl. = 1. Series with unit root is not covariance stationary. First differencing will often eliminate the unit root.

Autoregressive (AR) model: specified correctly if autocorrelation of residuals not significant.

Mean reverting level for AR(1):

$$\frac{b_0}{(1-b_1)}$$

RMSE: square root of average squared error. Random Walk Time Series:

$$\mathbf{x}_{t} = \mathbf{x}_{t-1} + \mathbf{\varepsilon}_{t}$$

Seasonality: indicated by statistically significant lagged err. term. Correct by adding lagged term. ARCH: detected by estimating:

$$\hat{\epsilon}_{r}^{2} = a_{0} + a_{1}\hat{\epsilon}_{r-1}^{2} + \mu_{r}$$

Variance of ARCH series:

$$\hat{\sigma}_{r+1}^2 = \hat{a}_0 + \hat{a}_1 \hat{\epsilon}_r^2$$

MACHINE LEARNING

Supervised learning: Algorithm uses labeled training data (inputs and outputs identified) to model relationships.

Unsupervised learning: Algorithm uses unlabeled data to determine the structure of the data.

Deep learning algorithms: Algorithms such as neural networks and reinforced learning learn from their own prediction errors and are used for complex tasks such as image recognition and natural language processing.

PREPARING DATA

Normalization: Scales values between 0 and 1.

$$Normalized \ X_i = \frac{X_i - X_{min}}{X_{max} - X_{min}}$$

Standardization: Centered at 0; scaled as standard deviations from mean.

Standardized
$$X_i = \frac{X_i - \mu}{\sigma}$$

FIT OF A MACHINE LEARNING ALGORITHM

precision (P) = true positives / (false positives + true positives)

recall (R) = true positives / (true positives + false negatives)

accuracy = (true positives + true negatives) / (all
positives and negatives)

F1 Score = $(2 \times P \times R) / (P + R)$

Receiver operating characteristic (ROC): Shows tradeoff between false positives and true positives.

Root mean square error (RMSE): Used when the target variable is continuous.

$$RMSE = \sqrt{\sum_{i=1}^{n} (Predicted_{i_i} - Actual_{i_j})^2}$$

Risk Types:

Appropriate method	Distribution of risk	Sequential?	Accommodates Correlated Variables?
Simulations	Continuous	Does not matter	Yes
Scenario analysis	Discrete	No	Yes
Decision trees	Discrete	Yes	No

ECONOMICS

bid-ask spread = ask quote - bid quote
Cross rates with bid-ask spreads:

Currency arbitrage: "Up the bid and down the ask." Forward premium = (forward price) – (spot price) Value of fwd currency contract prior to expiration:

$$V_{t} = \frac{(FP_{t} - FP)(contract \ size)}{\left[1 + R_{A} \left(\frac{days}{360}\right)\right]}$$

Covered interest rate parity:

$$F = \frac{\left| 1 + R_A \left(\frac{\text{days}}{360} \right) \right| S_0}{\left| 1 + R_B \left(\frac{\text{days}}{360} \right) \right|}$$

Uncovered interest rate parity:

$$E(\%\Delta S)_{(A/B)} = R_A - R_B$$

Fisher relation:

$$R_{\text{nominal}} = R_{\text{real}} + E(\text{inflation})$$

$$R_{\text{nominal A}} - R_{\text{nominal B}} = E(\text{inflation}_{A}) - E(\text{inflation}_{B})$$

Relative Purchasing Power Parity: High inflation rates leads to currency depreciation.

 $\%\Delta S(A/B) = inflation_{(A)} - inflation_{(B)}$ where: $\%\Delta S(A/B) = change in spot price (A/B)$

Profit on FX Carry Trade = interest differential – change in the spot rate of investment currency. Mundell-Fleming model: Impact of monetary and fiscal policies on interest rates & exchange rates. Under high capital mobility, expansionary monetary policy/restrictive fiscal policy → low interest rates → currency depreciation. Under low capital mobility, expansionary monetary

policy/expansionary fiscal policy \rightarrow current account deficits \rightarrow currency depreciation. Dornbusch overshooting model: Restrictive monetary policy \rightarrow short-term appreciation of currency, then slow depreciation to PPP value. Labor Productivity:

output per worker $Y/L = T(K/L)^{\alpha}$

Growth Accounting:

growth rate in potential GDP

- = long-term growth rate of technology
 - + α (long-term growth rate of capital)
- + (1α) (long-term growth rate of labor) growth rate in potential GDP
 - = long-term growth rate of labor force
 - + long-term growth rate in labor productivity

Classical Growth Theory

• Real GDP/person reverts to subsistence level.

Neoclassical Growth Theory

- Sustainable growth rate is a function of population growth, labor's share of income, and the rate of technological advancement.
- Growth rate in labor productivity driven only by improvement in technology.
- Assumes diminishing returns to capital.

$$g^* = \frac{\theta}{(1-\alpha)} \qquad G^* = \frac{\theta}{(1-\alpha)} + \Delta L$$

Endogenous Growth Theory

- Investment in capital can have constant returns.
- \uparrow in savings rate \rightarrow permanent \uparrow in growth rate.
- R&D expenditures ↑ technological progress.

Classifications of Regulations

- Statutes: Laws made by legislative bodies.
- Administrative regulations: Issued by government.
- Judicial law: Findings of the court.

Classifications of Regulators

- · Can be government agencies or independent.
- Independent regulator can be SRO or SRB. SRBs are given government recognition.

Self-Regulation in Financial Markets

 SROs are more prevalent in common-law countries than in civil-law countries.

Economic Rationale for Regulatory Intervention

- *Informational frictions* arise in the presence of information asymmetry.
- Externalities deal with provision of public goods.
- Weak competition can lead to lack of innovation and higher prices.
- Social objectives such as provision of public goods.

Regulatory Interdependencies and Their Effects Regulatory capture theory: Regulatory body is influenced or controlled by industry being regulated. Regulatory arbitrage: Exploiting regulatory differences between jurisdictions, or difference between substance and interpretation of a regulation.

Tools of Regulatory Intervention

 Price mechanisms, restricting or requiring certain activities, and provision of public goods or financing of private projects.

Financial market regulations: Seek to protect investors and to ensure stability of financial system. Securities market regulations: Include disclosure requirements, regulations to mitigate agency conflicts, and regulations to protect small investors. Prudential supervision: Monitoring institutions to reduce system-wide risks and protect investors.

Anticompetitive Behaviors and Antitrust Laws

- Discriminatory pricing, bundling, exclusive dealing.
- Mergers leading to excessive market share blocked.
 Net regulatory burden: Costs to the regulated
 entities minus the private benefits of regulation.
 Sunset clauses: Require a cost-benefit analysis to be
 revisited before the regulation is renewed.

FINANCIAL STATEMENT ANALYSIS

Accounting for Intercorporate Investments Investment in Financial Assets: <20% owned, no significant influence.

- Amortized cost on balance sheet; interest and realized gain/loss on income statement.
- Fair value through OCI at FMV with unrealized gains/losses in equity on B/S; dividends, interest, realized gains/losses on I/S.
- Fair value through profit or loss at FMV; dividends, interest, realized and unrealized gains/losses on I/S.

 Investments in Associates: 20–50% owned, significant

influence. With equity method, pro-rata share of the investee's earnings incr. B/S inv. acct., also in I/S. Div. received decrease investment account (div. not in I/S). Business Combinations: >50% owned, control. Acquisition method required under U.S. GAAP and IFRS. Goodwill not amortized, subject to annual impairment test. All assets, liabilities, revenue, and expenses of subsidiary are combined with parent, excluding intercomp. trans. If <100%, minority interest acct. for share not owned.

Joint Venture: 50% shared control. Equity method. Financial Effect of Choice of Method

Equity, acquisition, & proportionate consolidation:

- All three methods report same net income.
- Assets, liabilities, equity, revenues, and expenses are higher under acquisition compared to the equity method.

Pension Accounting

• PBO components: current service cost, interest cost, actuarial gains/losses, benefits paid.

Balance Sheet

• Funded status = plan assets – PBO = balance sheet asset (liability) under GAAP and IFRS.

Income Statement

- Total periodic pension cost (under both IFRS and GAAP) = contributions Δ funded status.
- IFRS and GAAP differ on where the total periodic pension cost (TPPC) is reflected (Income statement vs. OCI).
- Under GAAP, periodic pension cost in P&L
 = service cost + interest cost ± amortization of actuarial (gains) and losses + amortization of past service cost expected return on plan assets.
- Under IFRS, reported pension expense = service cost + past service cost + net interest expense.
- Under IFRS, discount rate = expected rate of return on plan assets. Net interest expense = discount rate × beginning funded status. If funded status was positive, a net interest income would be recognized.

Total Periodic Pension Cost

TPPC = ending PBO – beginning PBO + benefits paid – actual return on plan assets

TPPC = contributions - (ending funded status - beginning funded status)

Cash Flow Adjustment

If TPPC < firm contribution, difference = Δ in PBO (reclassify difference from CFF to CFO aftertax). If TPPC > firm contribution, diff = borrowing (reclassify difference from CFO to CFF after-tax).

Multinational Operations: Choice of Method For self-contained sub, functional ≠ presentation currency; use current rate method:

- Assets/liabilities at current rate.
- Common stock at historical rate.
- Income statement at average rate.
- Exposure = shareholders' equity.
- Dividends at rate when paid.

For integrated sub., functional = presentation currency, use temporal method:

- Monetary assets/liabilities at current rate.
- Nonmonetary assets/liabilities at historical rate.
- Sales, SGA at average rate.

- · COGS, depreciation at historical rate.
- Exposure = monetary assets monetary liabilities. Net asset position & depr. foreign currency = loss. Net liab. position & depr. foreign currency = gain.

Original Financial Statements vs. All-Current

- Pure balance sheet and income statement ratios unchanged.
- If LC depreciating (appreciating), translated mixed ratios will be larger (smaller).

Hyperinflation: GAAP vs. IFRS

Hyperinfl. = cumul. infl. > 100% over 3 yrs. GAAP: use temporal method. IFRS: 1st, restate foreign curr. st. for infl. 2nd, translate with current rates. Net purch. power gain/loss reported in income.

Beneish model: Used to detect earnings manipulation based on eight variables.

High-quality earnings are:

- 1. Sustainable: Expected to recur in future.
- 2. Adequate: Cover company's cost of capital.

IFRS AND U.S. GAAP DIFFERENCES

Fair value accounting, investment in associates: IFRS – Only for venture capital, mutual funds, etc. U.S. GAAP – Fair value accounting allowed for all.

 IFRS permits either the "partial goodwill" or "full goodwill" methods to value goodwill and noncontrolling interest. U.S. GAAP requires the full goodwill method.

Goodwill impairment processes:

IFRS – 1 step (recoverable amount vs. carrying value)U.S. GAAP – 2 steps (identify; measure amount)

Acquisition method contingent asset recognition: IFRS – Contingent assets are not recognized. U.S. GAAP – Recognized; recorded at fair value.

Prior service cost:

IFRS – Recognized as an expense in P&L. U.S. GAAP – Reported in OCI; amortized to P&L.

Actuarial gains/losses:

IFRS – Remeasurements in OCI and not amortized. U.S. GAAP – OCI, amortized with corridor approach.

Dividend/interest income and interest expense: IFRS – Either operating or financing cash flows. U.S. GAAP – Must classify as operating cash flow.

ROE decomposed (extended DuPont equation)

Tax Interest EBIT Burden Burden Margin

$$ROE = \frac{NI}{EBT} \times \frac{EBT}{EBIT} \times \frac{EBIT}{revenue} \times$$

Total Asset
Turnover

revenue
average assets

average assets
average equity

Accruals Ratio (balance sheet approach)

accruals ratio^{BS} =
$$\frac{(NOA_{END} - NOA_{BEG})}{(NOA_{END} + NOA_{BEG})/2}$$

Accruals Ratio (cash flow statement approach)

accruals ratio^{CF} =
$$\frac{(NI - CFO - CFI)}{(NOA_{END} + NOA_{BEG})/2}$$

Financial institutions differ from other companies due to systemic importance and regulated status.

Basel III: Minimum levels of capital and liquidity.

CAMELS: Capital adequacy, Asset quality,

Management, Earnings, Liquidity, and Sensitivity.

$$\label{eq:Liquidity} \text{Liquidity coverage ratio} = \frac{\text{highly liquid assets}}{\text{expected cash outflows}}$$

Net stable funding ratio = $\frac{\text{available stable funding}}{\text{required stable funding}}$

INSURANCE COMPANY KEY RATIOS Underwriting loss ratio

 $= \frac{\text{claims paid} + \Delta \text{ loss reserves}}{\text{net premium earned}}$

Expense ratio

 $= \frac{\text{underwriting expenses incl. commissions}}{\text{net premium written}}$

Loss and loss adjustment expense ratio

 $= \frac{\text{loss expense} + \text{loss adjustment expense}}{\text{net premiums earned}}$

Dividends to policyholders ratio

 $= \frac{\text{dividends to policyholders}}{\text{net premiums earned}}$

Combined ratio after dividends

= combined ratio + divs to policyholders ratio Total investment return ratio

= total investment income / invested assets

Life and health insurers' ratios

total benefits paid / (net premiums written and deposits)

commissions + expenses / (net premiums written + deposits)

CORPORATE FINANCE

Capital Budgeting Expansion

• Initial outlay = FCInv + WCInv

• CF = (S - C - D)(1 - T) + D = (S - C)(1 - T) + DT

• TNOCF = Sal_T + NWCInv – $T(Sal_T - B_T)$

Capital Budgeting Replacement

 Same as expansion, except current after-tax salvage of old assets reduces initial outlay.

• Incremental depreciation is Δ in depreciation.

Evaluating Projects with Unequal Lives

· Least common multiple of lives method.

 Equivalent annual annuity (EAA) method: annuity w/ PV equal to PV of project cash flows.

Effects of Inflation

 Discount nominal (real) cash flows at nominal (real) rate; unexpected changes in inflation affect project profitability; reduces the real tax savings from depreciation; decreases value of fixed payments to bondholders; affects costs and revenues differently.

Capital Rationing

 If positive NPV projects > available capital, choose the combination with the highest NPV.

Real Options

• Timing, abandonment, expansion, flexibility, fundamental options.

Economic and Accounting Income

• Econ income = AT CF + Δ in project's MV.

• Econ dep. based on Δ in investment's MV.

• Econ income is calculated before interest expense (cost of capital is reflected in discount rate).

• Accounting income = revenues – expenses.

Acc. dep'n based on original investment cost.

 Interest (financing costs) deducted before calculating accounting income.

Valuation Models

• Economic profit = NOPAT – \$WACC

• Market Value Added = $\sum_{t=1}^{\infty} \frac{EP_t}{(1 + WACC)^t}$

 Residual income: = NI – equity charge; discounted at required return on equity.

 Claims valuation separates CFs based on equity claims (discounted at cost of equity) and debt holders (discounted at cost of debt).

MM Prop I (No Taxes): capital structure irrelevant (no taxes, transaction, or bankruptcy costs).

$$V_{L} = V_{U}$$

MM Prop II (No Taxes): increased use of cheaper debt increases cost of equity, no change in WACC.

$$\mathbf{r}_e = \mathbf{r}_0 + \frac{D}{E}(\mathbf{r}_0 - \mathbf{r}_d)$$

MM Proposition I (With Taxes): tax shield adds value, value is maximized at 100% debt.

$$V_{L} = V_{U} + (t \times d)$$

MM Proposition II (With Taxes): tax shield adds value, WACC is minimized at 100% debt.

$$r_e = r_0 + \frac{D}{E}(r_0 - r_d)(1 - T_c)$$

Investor Preference Theories

 MM's dividend irrelevance theory: In a no-tax/ no-fee world, dividend policy is irrelevant because investors can create a homemade dividend.

 Dividend preference theory says investors prefer the certainty of current cash to future capital gains.

• Tax aversion theory: Investors are tax averse to dividends; prefer companies buy back shares.

Effective Tax Rate on Dividends

Double taxation or split rate systems:

eff. rate = corp. rate + (1 - corp. rate)(indiv. rate) *Imputation* system: effective tax rate is the shareholder's individual tax rate.

Signaling Effects of Dividend Changes

Initiation: ambiguous signal.

Increase: positive signal.

Decrease: negative signal unless management sees many profitable investment opportunities.

Price change when stock goes ex-dividend:

$$\Delta P = \frac{D(1 - T_D)}{(1 - T_{CG})}$$

Target Payout Adjustment Model

expected increase in dividends =

$$\left[\begin{pmatrix} expected & target \\ earnings \times payout \\ ratio \end{pmatrix} - \begin{array}{c} previous \\ dividend \\ \end{bmatrix} \times \begin{array}{c} adjustment \\ factor \\ \end{array} \right.$$

Dividend Coverage Ratios

dividend coverage ratio = net income / dividends

FCFE coverage ratio

= FCFE / (dividends + share repurchases)

Share Repurchases

• Share repurchase is equivalent to cash dividend, assuming equal tax treatment.

· Unexpected share repurchase is good news.

Rationale for: (1) potential tax advantages, (2) share price support/signaling, (3) added flexibility,
 (4) offsetting dilution from employee stock options, and (5) increasing financial leverage.

Dividend Policy Approaches

• Residual dividend: dividends based on earnings less funds retained to finance capital budget.

• Longer-term residual dividend: forecast capital budget, smooth dividend payout.

• Dividend stability: dividend growth aligned with sustainable growth rate.

• Target payout ratio: long-term payout ratio target.

ESG CONSIDERATIONS

The board of directors of a company can be structured either as a single tier board of internal (executive) and external (non-executive) directors,

or a two-tiered board where the management board is overseen by a supervisory board.

CEO duality: When the CEO is also the company's chairperson of the board.

ESG-Related Risk Exposures: In fixed-income analysis, ESG considerations are primarily concerned with downside risk. In equity analysis, ESG is considered both in regards to upside opportunities and downside risk.

MERGERS

Merger Types: horizontal, vertical, conglomerate. Merger Motivations: achieve synergies, more rapid growth, increased market power, gain access to unique capabilities, diversify, personal benefits for managers, tax benefits, unlock hidden value, international goals, and bootstrapping earnings.

Pre-Offer Defense Mechanisms: poison pills and puts, reincorporate in a state w/ restrictive takeover laws, staggered board elections, restricted voting rights, supermajority voting, fair price amendments, and golden parachutes.

Post-Offer Defense Mechanisms: litigation, greenmail, share repurch, leveraged recap, the "crown jewel," "Pac-Man," and "just say no" defenses, and white knight/white squire.

The Herfindahl-Hirschman Index (HHI): market power = sum of squared market shares for all industry firms. In a moderately-concentrated industry (HHI 1,000 to 1,800), a merger is likely to be challenged if HHI increases 100 points (or increases 50 points for HHI >1,800).

$$HHI = \sum_{i=1}^{n} (MS_i \times 100)^{2}$$

Methods to Determine Target Value

DCF method: target pro forma FCF discounted at adjusted WACC.

Comparable company analysis: based on relative valuation vs. similar firms + takeover premium.

Comparable transaction analysis: target value from takeover transaction; takeover premium included.

Merger Valuations

Combined firm: $V_{AT} = V_A + V_T + S - C$

Takeover premium (to target): $Gain_{T} = TP = P_{T} - V_{T}$

Synergies (to acquirer): $Gain_A = S - TP = S - (P_T - V_T)$

Merger Risk & Reward

Cash offer: acquirer assumes risk & receives reward. Stock offer: some of risks & rewards shift to target. If higher confidence in synergies; acquirer prefers cash & target prefers stock.

Forms of divestitures: equity carve-outs, spin-offs, split-offs, and liquidations.

EQUITY

Holding period return:

$$= r = \frac{P_1 - P_0 + CF_1}{P_0} = \frac{P_1 + CF_1}{P_0} - 1$$

Required return: Minimum expected return an investor requires given an asset's characteristics. Internal rate of return (IRR): Equates discounted cash flows to the current price.

Equity risk premium:

required return = risk-free rate + $(\beta \times ERP)$ Gordon growth model equity risk premium:

= 1-yr forecasted dividend yield on market index

+ consensus long-term earnings growth rate

- long-term government bond yield

Ibbotson-Chen equity risk premium

 $[1+\hat{i}]\times[1+\widehat{rEg}]\times[1+\widehat{PEg}]-1+\widehat{Y}-\widehat{RF}$

Models of required equity return:

- *CAPM*: $r_i = RF + (equity risk premium <math>\times \beta_i$)
- Multifactor model: required return = RF + (risk $premium)_1 + ... + (risk premium)_n$

Fama-French: $r_i = RF + \beta_{mkt,j} \times (R_{mkt} - RF)$

- $\begin{array}{l} +\beta_{\text{SMB,j}}\times(R_{\text{small}}-R_{\text{big}})+\beta_{\text{HML,j}}\times(R_{\text{HBM}}-R_{\text{LBM}})\\ \bullet \ \textit{Pastor-Stambaugh model} \text{:} \ \text{Adds a liquidity factor to} \end{array}$
- the Fama-French model.
- Macroeconomic multifactor models: Uses factors associated with economic variables.
- Build-up method: r = RF + equity risk premium + size premium + specific-company premium

Blume adjustment:

adjusted beta = $(2/3 \times \text{raw beta}) + (1/3 \times 1.0)$

WACC = weighted average cost of capital

$$= \frac{MV_{debt}}{MV_{debt+equity}} r_d \left(1 - T \right) + \frac{MV_{equity}}{MV_{debt+equity}} r_e$$

Discount cash flows to firm at WACC, and cash flows to equity at the required return on equity.

Discounted Cash Flow (DCF) Methods

Use dividend discount models (DDM) when:

- Firm has dividend history.
- Dividend policy is related to earnings.
- Minority shareholder perspective.

Use free cash flow (FCF) models when:

- Firm lacks stable dividend policy.
- Dividend policy not related to earnings.
- FCF is related to profitability.
- · Controlling shareholder perspective.
- Use residual income (RI) when:
- Firm lacks dividend history.
- Expected FCF is negative.

Gordon Growth Model (GGM)

Assumes perpetual dividend growth rate:

$$V_0 = \frac{D_1}{r - g}$$

Most appropriate for mature, stable firms. Limitations are:

- Very sensitive to estimates of r and g.
- Difficult with non-dividend stocks.
- Difficult with unpredictable growth patterns (use multi-stage model).

Present Value of Growth Opportunities

$$V_0 = \frac{E_1}{r} + PVGO$$

H-Model
$$V_0 = \frac{\left[D_0 \times (1 + g_L)\right]}{r - g_L} + \frac{\left[D_0 \times H \times (g_S - g_L)\right]}{r - g_L}$$
Sustainable Growth Rate: b × ROE.

Sustainable Growth Rate: b × ROE.

Required Return From Gordon Growth Model:

$$r = (D_1 / P_0) + g$$

Free Cash Flow to Firm (FCFF)

Assuming depreciation is the only NCC:

- $FCFF = NI + Dep + [Int \times (1 tax rate)] FCInv$ WCInv.
- $FCFF = [EBIT \times (1 tax rate)] + Dep FCInv$ – WCInv.
- FCFF = $[EBITDA \times (1 tax rate)] + (Dep \times tax)$ rate) – FCInv – WCInv.
- $FCFF = CFO + [Int \times (1 tax rate)] FCInv.$

Free Cash Flow to Equity (FCFE)

- $FCFE = FCFF [Int \times (1 tax rate)] + Net$ borrowing.
- FCFE = NI + Dep FCInv WCInv + Net borrowing.
- $FCFE = NI [(1 DR) \times (FCInv Dep)]$ - [(1 - DR) × WCInv]. (Used to forecast.)

Single-Stage FCFF/FCFE Models

- For FCFF valuation: $V_0 = \frac{FCFF_1}{WACC g}$
- For FCFE valuation: $V_0 = \frac{FCFE_1}{r-\sigma}$

2-Stage FCFF/FCFE Models

Step 1: Calculate FCF in high-growth period.

Step 2: Use single-stage FCF model for terminal value at end of high-growth period.

Step 3: Discount interim FCF and terminal value to time zero to find stock value; use WACC for FCFF, r for FCFE.

Price to Earnings (P/E) Ratio

Problems with P/E:

- If earnings < 0, P/E meaningless.
- Volatile, transitory portion of earnings makes interpretation difficult.
- Management discretion over accounting choices affects reported earnings.

Justified P/E

leading P/E =
$$\frac{1-b}{r-g}$$

trailing P/E = $\frac{(1-b)(1+g)}{r-g}$

Justified dividend yield:

$$\frac{D_0}{P_0} = \frac{r - g}{1 + g}$$

Normalization Methods

- · Historical average EPS.
- · Average ROE.

Price to Book (P/B) Ratio

Advantages:

- BV almost always > 0.
- · BV more stable than EPS.
- · Measures NAV of financial institutions.

Disadvantages: · Size differences cause misleading comparisons.

- · Influenced by accounting choices.
- BV ≠ MV due to inflation/technology.

justified P/B =
$$\frac{ROE - g}{r - g}$$

Price to Sales (P/S) Ratio

Advantages:

- Meaningful even for distressed firms.
- Sales revenue not easily manipulated.
- Not as volatile as P/E ratios.
- Useful for mature, cyclical, and start-up firms. Disadvantages:
- High sales ≠ imply high profits and cash flows.
- Does not capture cost structure differences.
- Revenue recognition practices still distort sales.

justified P/S =
$$\frac{PM_0 \times (1-b)(1+g)}{r-g}$$

DuPont Model

$$ROE = \left[\frac{\text{net income}}{\text{sales}}\right] \times \left[\frac{\text{sales}}{\text{total assets}}\right] \times \left[\frac{\text{total assets}}{\text{equity}}\right]$$

Price to Cash Flow Ratios

Advantages:

- Cash flow harder to manipulate than EPS.
- More stable than P/E.
- Mitigates earnings quality concerns.

Disadvantages:

- · Difficult to estimate true CFO.
- FCFE better but more volatile.

Method of Comparables

- Firm multiple > benchmark implies overvalued.
- Firm multiple < benchmark implies undervalued.
- Fundamentals that affect multiple should be similar between firm and benchmark.

Residual Income Models

- $RI = E_t (r \times B_{t-1}) = (ROE r) \times B_{t-1}$
- Single-stage RI model:

$$V_0 = B_0 + \left| \frac{(ROE - r) \times B_0}{r - g} \right|$$

• Multistage RI valuation: $V_0 = B_0 + (PV \text{ of interim})$ high-growth RI) + (PV of continuing RI)

Economic Value Added®

EVA = NOPAT - \$WACC; NOPAT = EBIT(1-t).

Private Equity Valuation

$$DLOC = 1 - \left[\frac{1}{1 + Control Premium} \right]$$

Total discount = 1 - [(1 - DLOC)(1 - DLOM)].

The DLOM varies with the following.

- An impending IPO or firm sale ↓ DLOM.
- The payment of dividends \ DLOM.
- Earlier, higher payments | DLOM.
- Restrictions on selling stock ↑ DLOM.
- A greater pool of buyers ↓ DLOM.
- Greater risk and value uncertainty ↑ DLOM.

FIXED INCOME

Price of a T-period zero-coupon bond:

$$P_{T} = \frac{1}{\left(1 + S_{T}\right)^{T}}$$

Forward price of zero-coupon bond:

$$F_{(j,k)} = \frac{1}{[1+f(j,k)]^k}$$

Forward pricing model:

$$F_{(j,k)} = \frac{P_{(j+k)}}{P_i}$$

Forward rate model:

$$[1 + f(j,k)]^k = [1 + S_{(j+k)}]^{(j+k)} / (1 + S_j)^j$$

"Riding the yield curve": Holding bonds with maturity > investment horizon, with upward sloping yield curve.

swap spread = swap rate - treasury yield

TED spread: = (3-month LIBOR rate) - (3-month T-bill rate)

Libor-OIS spread

= LIBOR rate - "overnight indexed swap" rate

Term Structure of Interest Rates

Traditional theories:

- Unbiased (pure) expectations theory.
- · Local expectations theory.
- Liquidity preference theory.
- Segmented markets theory.
- Preferred habitat theory.

Modern term structure models:

- Cox-Ingersoll-Ross: $dr = a(b-r)dt + \sigma \sqrt{r}dz$
- Vasicek model: $dr = a(b r)dt + \sigma dz$
- Ho-Lee model: $dr = \theta_i dt + \sigma dz$

Managing yield curve shape risk:

$$\begin{split} \Delta P/P \approx -D_L \Delta x_L - D_S \Delta x_S - D_C \Delta x_C \\ (L = level, S = steepness, C = curvature) \end{split}$$

Yield volatility: Long-term ← uncertainty regarding the real economy and inflation.

Short term ← uncertainty re: monetary policy. Long-term yield volatility is generally lower than volatility in short-term yields.

Value of option embedded in a bond:

$$V_{call} = V_{straight bond} - V_{callable bond}$$
 $V_{put} = V_{putable bond} - V_{straight bond}$

When interest rate volatility increases:

$$V_{\text{call option}}^{\uparrow}, V_{\text{put option}}^{\uparrow}, V_{\text{callable bond}}^{\downarrow}, V_{\text{putable bond}}^{\uparrow}$$

Upward sloping yield curve: Results in lower call value and higher put value.

When binomial tree assumed volatility increases:

- computed OAS of a callable bond decreases.
- · computed OAS of a putable bond increases.

computed OAS of a *putable* bond *inc*
effective duration =
$$\frac{BV_{-\Delta y} - BV_{+\Delta y}}{2 \times BV_0 \times \Delta y}$$

$$\text{effective convexity} = \frac{BV_{-\Delta y} + BV_{+\Delta y} - (2 \times BV_0)}{BV_0 \times \Delta y^2}$$

Effective duration:

- ED (callable bond) ≤ ED (straight bond).
- ED (putable bond) ≤ ED (straight bond).
- ED (zero-coupon) ≈ maturity of the bond.
- ED fixed-rate bond < maturity of the bond.
- ED of floater ≈ time (years) to next reset.

One-sided durations: Callables have lower downduration; putables have lower up-duration.

Value of a capped floater

= straight floater value – embedded cap value

Value of a floored floater

- = straight floater value + embedded floor value
- Minimum value of convertible bond
- = greater of conversion value or straight value

Conversion value of convertible bond

= market price of stock × conversion ratio

Market conversion price

- market price of convertible bond
 - conversion ratio

Market conversion premium per share

= market conversion price - stock's market price

Market conversion premium ratio

market conversion premium per share

market price of common stock

Premium over straight value

$$= \left(\frac{\text{market price of convertible bond}}{\text{straight value}}\right) - 1$$

Callable and putable convertible bond value

- = straight value of bond
- + value of call option on stock
- value of call option on bond
- + value of put option on bond

Expected exposure: Amount a risky bond investor stands to lose before any recovery is factored in. Loss given default = loss severity × exposure Probability of default: Likelihood in a given year. Credit valuation adjustment (CVA): Sum of the present values of expected losses for each period. Credit score/rating: Ordinal rank; higher = better. Return from bond credit rating migration: Δ %P

= –(modified duration of bond) \times (\triangle spread)

Structural models of corporate credit risk:

• value of risky debt = value of risk-free debt - value of put option on the company's assets

• equity ≈ European call on company assets

Reduced-form models: Do not explain why default occurs, but statistically model when default occurs.

Credit spread on a risky bond = YTM of risky bond - YTM of benchmark

Credit Default Swap (CDS): Upon credit event, protection buyer compensated by protection seller. Index CDS: Multiple borrowers, equally weighted.

Default: Occurrence of a credit event.

Common credit events in CDS agreements:

Bankruptcy, failure to pay, restructuring. CDS spread: Higher for a higher probability of

default and for a higher loss given default. Hazard rate = conditional probability of default.

expected loss, = (hazard rate), × (loss given default), **Upfront CDS payment** (paid by protection buyer)

- = PV(protection leg) PV(premium leg)
- ≈ (CDS spread CDS coupon) × duration × NP

Change in value for a CDS after inception

≈ chg in spread × duration × notional principal

DERIVATIVES

Forward contract price (cost-of-carry model)

$$FP = S_0 \times (1 + R_f)^T \qquad S_0 = \frac{FP}{(1 + R_f)^T}$$

Price of equity forward with discrete dividends FP(on an equity security) = $(S_0 - PVD) \times (1 + R_f)^T$

Value of forward on dividend-paying stock

$$V_t$$
(long position) = $[S_t - PVD_t] - \left[\frac{FP}{(1 + R_f)^{(T-t)}}\right]$

Forward: equity index (continuous dividend)

$$\begin{aligned} \text{FP(on an equity index)} &= S_0 \times e^{\left(R_f^c - \delta^c\right) \times T} \\ &= \left(S_0 \times e^{-\delta^c \times T}\right) \times e^{R_f^c \times T} \end{aligned}$$

 R_f^c = continuously compounded risk-free rate

 δ^c = continuously compounded dividend yield

Forward price on a coupon-paying bond:

$$\begin{aligned} \text{FP(on a fixed income security)} \\ &= \left(S_0 - \text{PVC} \right) \times \left(1 + R_f \right)^T \\ &= S_0 \times \left(1 + R_f \right)^T - \text{FVC} \end{aligned}$$

Value of a forward on a coupon-paying bond:

$$V_{t}(long) = \left[S_{t} - PVC_{t}\right] - \left[\frac{FP}{(1 + R_{f})^{(T-t)}}\right]$$

Price of a bond futures contract:

 $FP = [(full price)(1+R_c)^T - AI_T - FVC]$ full price = quoted spot price + AI₀

Quoted bond futures price:

QFP = forward price / conversion factor
=
$$\left| (\text{full price})(1+R_f)^T - \text{AI}_T - \text{FVC} \right| \left| \frac{1}{CF} \right|$$

Price of a currency forward contract:

$$F_T = S_0 \times \frac{(1 + R_{PC})^T}{(1 + R_{RC})^T}$$

Value of a currency forward contract

$$V_{t} = \frac{[FP_{t} - FP] \times (contract \ size)}{(1 + r_{DC})^{(T-t)}}$$

Currency forward price (continuous time):

$$F_{T} = S_{0} \times e^{\left(R_{PC}^{c} - R_{BC}^{c}\right) \times T}$$

Swap fixed rate:

$$C = \frac{1 - Z_4}{Z_1 + Z_2 + Z_3 + Z_4}$$

where: $Z_n = 1/(1+R_n) = price of zero-coupon $1 bond$

Value of interest rate swap to fixed payer:

$$= \sum Z \times (SFR_{New} - SFR_{Old}) \times \frac{days}{360} \times notional$$

Binomial stock tree probabilities

$$\pi_U$$
 = probability of up move = $\frac{1 + R_f - D}{U - D}$

 π_D = probability of a down move = $(1 - \pi_D)$

Put-call parity:

$$S_0 + P_0 = C_0 + PV(X)$$

Put-call parity when the stock pays dividends:

$$P_0 + S_0 e^{-8T} = C_0 + e^{-rT} X$$

Dynamic delta hedging

shares hedged # of short call options = delta of call option

shares hedged # of long put options = delta of put option

Change in option value

 $\Delta C \approx \text{call delta} \times \Delta S + \frac{1}{2} \text{ gamma} \times \Delta S^2$ $\Delta P \approx$ put delta × ΔS + ½ gamma × ΔS^2

Option value using arbitrage-free pricing

$$\begin{split} C_0 &= hS_0 + \frac{(-hS^+ + C^+)}{(1 + R_f)} = hS_0 + \frac{(-hS^- + C^-)}{(1 + R_f)} \\ P_0 &= hS_0 + \frac{(-hS^- + P^-)}{(1 + R_f)} = hS_0 + \frac{(-hS^+ + P^+)}{(1 + R_f)} \end{split}$$

 $\begin{aligned} & \textbf{Black-Scholes-Merton option valuation model} \\ & C_0 = S_0 e^{-\delta T} N(d_1) - e^{-rT} X N(d_2) \end{aligned}$

$$C_0 = S_0 e^{-\delta T} N(d_1) - e^{-rT} X N(d_2)$$

$$P_0 = e^{-rT}XN(-d_2) - S_0e^{-\delta T}N(-d_1)$$

 δ = continuously compounded dividend yield

$$d_1 = \frac{\ln(S/X) + (r - \delta + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma \sqrt{T}$$

 $S_0e^{-\delta T}$ = stock price, less PV of dividends

ALTERNATIVE INVESTMENTS

Value of property using direct capitalization:

- rental income if fully occupied
- + other income = potential gross income
- vacancy and collection loss
- = effective gross income
- operating expense
- = net operating income

$$cap rate = \frac{NOI_1}{comparable sales price}$$

$$value = V_0 = \frac{NOI_1}{cap \ rate} \ \textit{or} \ V_0 = \frac{stabilized \ NOI}{cap \ rate}$$

Property value based on "All Risks Yield": value = V_0 = rent, / ARY

gross income multiplier =
$$\frac{\text{sales price}}{\text{gross income}}$$

Term and reversion valuation approach:

total property value

= PV of term rent + PV reversion to ERV

Layer approach:

total property value

= PV of term rent + PV of incremental rent

Debt service coverage ratio:

$$DSCR = \frac{\text{first-year NOI}}{\text{debt service}}$$

Loan-to-value (LTV) ratio:

$$LTV = \frac{loan \ amount}{appraisal \ value}$$

equity dividend rate =
$$\frac{\text{first year cash flow}}{\text{equity}}$$

NAV approach to REIT share valuation:

estimated cash NOI

- ÷ assumed cap rate
- = estimated value of operating real estate
- + cash & accounts receivable
- debt and other liabilities
- = net asset value
- + shares outstanding
- = NAV/share

Price-to-FFO approach to REIT share valuation:

funds from operations (FFO)

- + shares outstanding
- = FFO/share
- × sector average P/FFO multiple
- = NAV/share

Price-to-AFFO approach to REIT share valuation:

funds from operations (FFO)

- non-cash rents
- recurring maintenance-type capital expenditures
- = AFFO
- + shares outstanding
- = AFFO/share
- × property subsector average P/AFFO multiple
- = NAV/share

Discounted cash flow REIT share valuation:

- value of a REIT share
- = PV(dividends for years 1 through n)
 - + PV(terminal value at the end of year n)

Private Equity

Sources of value creation: reengineer firm, favorable debt financing; superior alignment of interests between management and PE ownership. Valuation issues (VC firms relative to Buyouts): DCF not as common; equity, not debt, financing. Key drivers of equity return:

Buyout: ↑ of multiple at exit, ↓ in debt. VC: pre-money valuation, the investment, and subsequent equity dilution.

Components of performance (LBO): earnings growth, ↑ of multiple at exit, ↓ in debt. Exit routes (in order of exit value, high to low): IPOs secondary market sales; MBO; liquidation. Performance Measurement: gross IRR = return from portfolio companies. Net IRR = relevant for LP, net of fees & carried interest.

Performance Statistics:

- PIC = % capital utilized by GP; cumulative sum of capital called down.
- Management fee: % of PIC.
- Carried interest: % carried interest × (change in NAV before distribution).
- NAV before distrib. = prior yr. NAV after distrib. + cap. called down - mgmt. fees + op. result.
- NAV after distributions = NAV before distributions - carried interest - distributions
- DPI multiple = (cumulative distributions) / PIC = LP's realized return.
- RVPI multiple = (NAV after distributions) / PIC = LP's unrealized return.
- TVPI mult. = DPI mult. + RVPI mult.

Assessing Risk: (1) adjust discount rate for prob of failure; (2) use scenario analysis for term.

Commodities

Contango: futures prices > spot prices Backwardation: futures prices < spot prices

Term Structure of Commodity Futures

- 1. Insurance theory: Contract buyers compensated for providing protection to commodity producers. Implies backwardation is normal.
- 2. Hedging pressure hypothesis: Like insurance theory, but includes both long hedgers → contango) and short hedgers (→ backwardation).
- 3. Theory of storage: Spot and futures prices related through storage costs and convenience

Total return on fully collateralized long futures

= collateral return + price return + roll return Roll return: positive in backwardation because longdated contracts are cheaper than expiring contracts.

PORTFOLIO MANAGEMENT

Creation/redemption of ETFs: Authorized participants (APs) create additional shares by delivering the creation basket to the ETF manager. Redemption is by tendering ETF shares and receiving a redemption basket.

ETF spreads: Positively related to cost of creation/ redemption, spread on the underlying securities, risk-premium for carrying trades until close of trade, and APs' normal profit margin. Negatively related to probability of completing an offsetting trade on the secondary market.

ETF premium (discount) % =

(ETF price - NAV per share) / NAV per share

Arbitrage Pricing Theory

$$E(R_p) = R_p + \beta_{P,1}(\lambda_1) + \beta_{P,2}(\lambda_2) + \ldots + \beta_{P,k}(\lambda_k)$$

Expected return = risk free rate

that the loss exceeds the VaR.

+ Σ (factor sensitivity) × (factor risk premium) Value at risk (VaR): Estimate of minimum loss with a given probability over a specified period, expressed as \$ amount or % of portfolio value.

5% annual \$VaR = (Mean annual return - 1.65 × annual standard deviation) × portfolio value Conditional VaR (CVaR): The expected loss given

Incremental VaR (IVaR): The change in VaR from a specific change in the size of a portfolio position. Marginal VaR (MVaR): Change in VaR for a small change in a portfolio position. Used as an estimate of the position's contribution to overall VaR.

Variance for W_A% fund A + W_B% fund B

$$\sigma_{\text{Portfolio}}^2 = W_A^2 \sigma_A^2 + W_B^2 \sigma_B^2 + 2W_A W_B \text{Cov}_{AB}$$

Annualized standard deviation

= $\sqrt{250}$ × (daily standard deviation)

% change in value vs. change in YTM

= -duration (ΔY) + $\frac{1}{2}$ convexity (ΔY)² for Macaulay duration, replace ΔY by $\Delta Y/(1+Y)$

Inter-temporal rate of substitution = $m_t = \frac{u_t}{u_0}$

marginal utility of consuming 1 unit in the future marginal utility of current consumption of 1 unit

Real risk-free rate of return = $\frac{1 - P_0}{P_0} = \left| \frac{1}{E(m_t)} \right| - 1$

Default-free, inflation indexed, zero coupon:

Bond price =
$$P_0 = \frac{E(P_1)}{(1+R)} + cov(P_1, m_1)$$

Nominal short term interest rate (r)

= real risk-free rate (R) + expected inflation (π)

Nominal long term interest rate = $R + \pi + \theta$ where θ = risk premium for inflation uncertainty



Break-even inflation rate (BEI)

= yield_non-inflation indexed bond - yield_inflation indexed bond

BEI for longer maturity bonds

= expected inflation (π) + infl. risk premium (θ)

Credit risky bonds required return = $R + \pi + \theta + \gamma$ where $\gamma = risk$ premium (spread) for credit risk

Discount rate for equity = $R + \pi + \theta + \gamma + \kappa$ λ = equity risk premium = $\gamma + \kappa$

 γ = risk premium for equity vs. risky debt

Discount rate for commercial real estate

$$= R + \pi + \theta + \gamma + \kappa + \phi$$

 κ = terminal value risk, ϕ = illiquidity premium Multifactor model return attribution:

$$\text{factor return} = \sum_{i=1}^k (\beta_{pi} - \beta_{bi}) \times (\lambda_i)$$

Active return

= factor return + security selection return

Active risk squared

= active factor risk + active specific risk

$$\text{Active specific risk} = \sum_{i=1}^n (w_{pi} - w_{bi})^2 {\sigma_{\tilde{\epsilon}i}}^2$$

Active return = portfolio return - benchmark return

$$R_A = R_P - R_B$$

Portfolio return = $R_P = \sum_{i=1}^{n} w_{P,i} R_i$

Benchmark return =
$$R_B = \sum_{i=1}^{n} w_{B,i} R_i$$

Information ratio

$$= \frac{R_P - R_B}{\sigma_{(R_P - R_B)}} = \frac{R_A}{\sigma_A} = \frac{\text{active return}}{\text{active risk}}$$

Portfolio Sharpe ratio = $SR_P = \frac{R_P - R_F}{STD(R_P)}$

Optimal level of active risk:

Sharpe ratio =
$$\sqrt{SR_B^2 + IR_P^2}$$

Total portfolio risk: $\sigma_{\rm p}^2 = \sigma_{\rm p}^2 + \sigma_{\rm a}^2$

Information ratio: $IR = TC \times IC \times \sqrt{BR}$

Expected active return: $E(R_A) = IR \times \sigma_A$

"Full" fundamental law of active management:

$$E(R_A) = (TC)(IC)\sqrt{BR}\sigma_A$$

Sharpe-ratio-maximizing aggressiveness level:

$$STD(R_A) = \frac{IR}{SR_B}STD(R_B)$$

TRANSACTION COST ESTIMATES

Per share effective spread transaction cost = (side) × (transaction price – midquote price) **Effective spread** = $2 \times$ (per share effective spread transaction cost)

VWAP transaction cost for *sell* orders = trade size × (benchmark VWAP benchmark – trade VWAP) VWAP transaction cost for buy orders = trade size × (trade VWAP benchmark – benchmark VWAP) Implementation shortfall: Difference in value between a hypothetical "paper" portfolio and the actual portfolio.

Electronic markets enable: Hidden orders, leapfrogging, flickering quotes, electronic arbitrage machine learning.

Abusive trading practices include front running and market manipulation. Market manipulation: trading for price impact, rumormongering, wash trading, spoofing, bluffing, gunning the market, and squeezing/cornering.