2020 Level II Fact Sheet

Key Facts & Formulas for the CFA® Exam

Approved Prep Provider CFA Institute

Ethical and Professional Standards

I(A) Knowledge of the law: comply with the strictest law; disassociate from violations.
I(B) Independence and objectivity: do not offer, solicit or accept gifts; but small token gifts are ok.
I(C) Misrepresentation: do not guarantee performance; avoid plagiarism.
I(D) Misconduct: do not behave in a manner that affects your professional reputation or integrity.
II(A) Material nonpublic information: do not act or help others to act on this information; but mosaic theory is not a violation.
II(B) Market manipulation: do not manipulate prices/trading volumes to mislead others; do not spread false rumors.
III(A) Loyalty, prudence, and care: place client's interest before employer's or your interests.
III(B) Fair dealing: treat all client's fairly; disseminate investment recommendations and changes simultaneously.
III(C) Suitability: in advisory relationships, understand client's risk profile, develop and update an IPS periodically; in fund/index management, ensure investments are consistent with stated mandate.
III(D) Performance presentation: do not misstate performance; make detailed information available on request.
III(E) Preservation of confidentiality: maintain confidentiality of clients; unless disclosure is required by law, information concerns illegal activities, client permits the disclosure.
IV(A) Loyalty: do not harm your employer; obtain written consent before starting an independent practice; do not take confidential information when leaving.
IV(B) Additional compensation arrangements: do not accept compensation arrangements that will create a conflict of interest with your employer; but you may accept if written consent is obtained from all parties involved.
IV(C) Responsibilities of supervisors: prevent employees under your supervision from violating laws.
V(A) Diligence and reasonable basis: have a reasonable and adequate basis for any analysis, recommendation or action.
V(B) Communication with clients and prospective clients: distinguish between fact and opinion; make appropriate disclosures.
V(C) Record retention: maintain records to support your analysis.
VI(A) Disclosure of conflicts: disclose conflict of interest in plain language.
VI(B) Priority of transactions: client transactions come before employer transactions which come before personal transactions.
VI(C) Referral fees: disclose referral arrangements to clients and employers.
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VII(B) Reference to CFA Institute, the CFA designation, and the CFA program: don't brag, references to partial designation not allowed.

Quantitative Methods

Simple linear regression: regression equation

$Y_i = b_0 + b_1X_i + \varepsilon_i,\ i = 1, \ldots, n$

Confidence interval for regression coefficients

$$\widehat{b_1} \pm t_c s_{\widehat{b_1}}$$

$$t = \frac{\widehat{b_1} - b_1}{s_{\widehat{b_1}}}$$

Prediction interval for regression equation:

$\widehat{Y} \pm t_c s_f$

s_f = Standard deviation of prediction error

R-squared (coefficient of determination) measures the fraction of the total variation in the dependent variable that is explained by the independent variable.

$$R^2 = \frac{\text{explained variation}}{\text{total variation}}$$

Total variation = unexplained variation + explained variation

F-statistic tests whether all the slope coefficients in a linear regression are equal to 0.

$$F = \frac{RSS/1}{SSE/(n-2)} = \frac{\text{Mean regression sum of squares}}{\text{Mean squared error}}$$

Standard error of estimate (SSE) measures how well a given linear regression model captures the relationship between the dependent and independent variables.

$$SEE = \left(\frac{\sum_{i=1}^{n}\left(Y_i - \hat{b}_0 - \hat{b}_0X_i\right)^2}{n-2}\right)^{\frac{1}{2}} = \left(\frac{\sum_{i=1}^{n}(\hat{e}_i)^2}{n-2}\right)^{\frac{1}{2}}$$

SEE = Square root of mean square error.
Test for serial correlation: DW ≈ 2 (1 – r)

Multiple regression: regression equation

$$Y_i = b_0 + b_1X_{1i} + b_2X_{2i} + \varepsilon_i$$

Violations of regression assumptions

- Heteroskedasticity: Variance of error term is conditional on X. Solution: Robust standard errors. Detect with Breuch-Pagan test: F-test is unreliable Standard error for coefficients will be underestimated; t-stat will be inflated. Solution: Generalized least squares.
- Serial correlation: Errors correlated across observations. Solution: Hansen Method. Detect by the DW test DW ≈ 2 (1 – r): t-stat and F-stat too high Solution: Modify the regression equation
- Multicollinearity: Two or more independent variables are highly correlated with each other high R^2, significant F-stat, inflated standard error, low t-stat for coefficients. Solution: Omit one or more of the "X" variables.

Trend models

- Linear trend model: dependent variable changes at a constant rate with time. The independent variable is time: Y = b0+ b1t + εi , t = 1, 2, …, T.
- Log-linear trends work well in fitting time series that have exponential growth.
- An autoregressive model (AR) is a time series where a given variable is regressed on its own past values.

$$X_t = b_0 + b_1X_{1-t} + \varepsilon_t$$

- For AR models to work the time series must be covariance-stationary: Constant expected value, variance and covariance.
- Durbin-Watson does NOT work for AR models.
- Test whether the autocorrelations of the error term (error autocorrelations) differ significantly from 0. Test-stat = residual autocorrelation / standard error
- Compare the out-of-sample forecasting performance of forecasting models by comparing their root mean squared error (RMSE), which is the square root of the average squared error.
- Mean-reverting level is given by:

$$x_t = \frac{b_0}{(1 - b_1)}$$

- A random walk is a time series in which the value of the series in one period is the value of the series in the previous period plus an unpredictable random error.

$$x_t = x_{t-1} + E(\varepsilon)_t = 0$$

- With a random walk b1 = 1, so MRL is undefined. This is called the unit root problem. Solution is to use first differencing.

Supervised machine learning algorithms: penalized regression, support vector machine (SVM), K-nearest neighbour (KNN), classification and regression tress (CART), random forest classifier, ensemble learning

Unsupervised machine learning algorithms: principal component analysis (PCA), K-means algorithm, hierarchical clustering

Neural networks: deep learning nets (DLNs), reinforcement learning (RL)

Data prep & wrangling involves data cleansing and data pre-processing. The steps vary based on whether we are working with structured data or unstructured data.

Data exploration includes three steps: exploratory data analysis (EDA), feature selection, and feature engineering.

Model training consists of three major tasks: method selection, performance evaluation, and model tuning.

The following metrics are used to evaluate a **confusion matrix**:
Precision (P) = TP / (TP + FP)
Recall (R) = TP / (TP + FN)
Accuracy = (TP + TN) / (TP + FP + TN + FN)
F1 score = (2 * P * R) / (P + R)

Types of risk

Risk Approach	Discrete / Continuous	Correlated / Independent	Sequential / Concurrent	Complements for Risk-adjusted value	Substitutes for Risk-adjusted value
Decision tree	Discrete	Independent	Sequential	Yes	No
Scenario analysis	Discrete	Correlated	Concurrent	Yes	Yes
Simulations	Continuous	Either	Either	Yes	Yes

Economics

Bid-ask spread is impacted by:
- Currency pair
- Time of day
- Market volatility
- Size of transaction

Cross rates: A/B = 2.0000/2.0006 and B/C = 4.0000/4.0008
Implied A/C cross rate = 8.0000/8.0036

Forward exchange rates are quoted in terms of points to be added to the spot exchange rate. Forward points represent the difference between the forward rate and the spot rate. If the points are positive (negative), the base currency is trading at a forward premium (discount).

International parity conditions

Covered interest rate parity = $F_f / d = S_f / d \left(\frac{1+ i_f \left[\frac{Actual}{360}\right]}{1+ i_d \left[\frac{Actual}{360}\right]}\right)$

Uncovered interest rate parity: Expected % change in spot rate (P/B) ≈ ip – iB

- If covered interest rate parity and uncovered interest rate parity hold then forward rates are unbiased estimates of future spot rates.
- Ex ante purchasing power parity: Expected % change in spot rate (P/B) ≈ Πp – ΠB
- International Fisher effect assumes that uncovered interest rate parity and ex ante purchasing power parity hold. If so: ip - iB = Πp – ΠB

Balance of payments and exchange rates
For the most part, countries that run persistent current account deficits will see their currencies weaken over time. Similarly, countries that run persistent current account surpluses will tend to see their currencies appreciate over time.

Monetary policy and Mundell-Fleming
- In the Mundell–Fleming model, monetary policy affects the exchange rate primarily through the interest rate sensitivity of capital flows, strengthening the currency when monetary policy is tightened and weakening it when monetary policy is eased. The more sensitive capital flows are to the change in interest rates, the greater the exchange rate's responsiveness to the change in monetary policy.
- Countries that pursue overly easy monetary policies will see their currencies depreciate over time.
- Under conditions of high capital mobility, countries that simultaneously pursue expansionary fiscal policies and relatively tight monetary policies should see their currencies strengthen over time.

Economic growth and investment decisions
$P = GDP \left(\frac{E}{GDP}\right)\left(\frac{P}{E}\right)$
P represented aggregate price (value) of stocks; E represents aggregate earnings;

Growth accounting: $\frac{\Delta Y}{Y} = \frac{\Delta A}{A} = \alpha\frac{\Delta K}{K} + (1-\alpha)\frac{\Delta L}{L}$
Growth rate in potential GDP = Long-term growth rate of labor force + Long-term growth rate in labor productivity

Neo-classical model:
Growth rate of output per capita = $\frac{\theta}{1-\alpha}$
Growth rate of output = $\frac{\theta}{1-\alpha} + n$
- Θ is growth rate of TFP
- α is the share of GDP paid out to the suppliers of capital
- n is the growth rate of labor

Labor productivity:
$y = Y/L = A\,(K/L)^{\alpha}\,(L/L)^{1-\alpha} = Ak^{\alpha}$

Regulatory intervention is required because of the presence of informational frictions and externalities.

Regulators can be classified as: legislative bodies, government backed regulator bodies, and courts.

Regulations can be classified as: statutes, administrative regulations, and judicial law.

Self-regulatory bodies are private organizations that both represent and regulate their members. If a self-regulatory body is given recognition and authority by a government body or agency, it is called a **self-regulatory organization** (SRO).

Theories of regulatory interdependence include: regulatory capture, regulatory competition, and regulatory arbitrage.

Regulatory tools

- Price mechanisms: taxes, tariffs and subsidies.
- Restricting activities like insider trading and short selling.
- Mandating activities like minimum capital requirements.
- Provision of public goods and public financing of private projects.
- Ability to impose sanctions on violators.

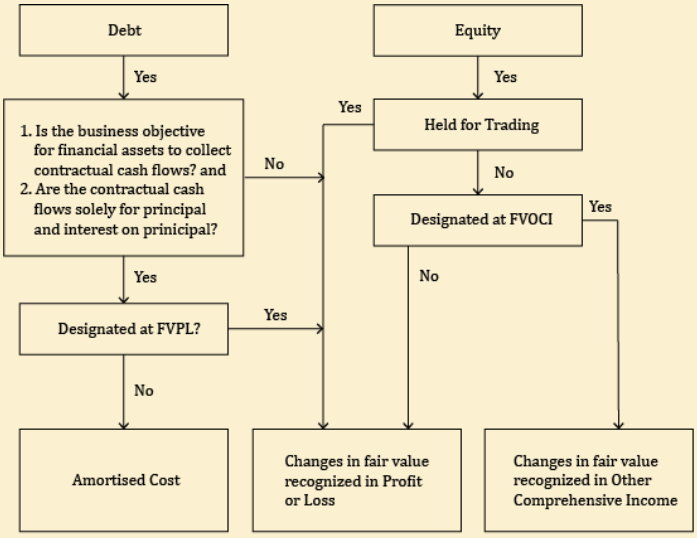
A cost-benefit analysis should be conducted before any new regulation is implemented. Regulatory burden refers to the costs of regulation for the regulated entity. Net regulatory burden is the private cost of regulation less the private benefits of regulation.

Financial Reporting and Analysis

Intercorporate investments

	Financial assets	Associates	Business combinations	Joint ventures
Influence	Not significant	Significant	Controlling	Shared control
Typical % interest	Usually < 20%	Usually 20% to 50%	Usually > 50%	Varies
Financial Reporting	Classified as: • Fair value through profit or loss • Fair value through other comprehensive income • Amortized cost	Equity Method	Consolidation	IFRS: Equity method

Investment in financial assets



Investments in associates and joint ventures

An investment is considered an "associate company" when the investor has (or can exercise) significant influence, but not control, over the investee's business activities. Equity method of accounting is required for investment in associates and joint venture

- Investment is initially recorded at cost.
- Share of income (not dividends) recorded in investor's I/S.
- Investment account reflected as single line item on B/S.
- Value of investment = beginning value + share of profit + share of dividends
- Investment classified as noncurrent asset on B/S.
- One-line consolidation.

Business combinations

Acquisition method

- Combine assets, liabilities, revenues and expenses of subsidiary with parent.
- Intercompany transactions are eliminated.
- For business combination with less than 100% acquisition show non-controlling (minority) interests on balance sheet. U.S. GAAP says use full goodwill method. IFRS says either full goodwill or partial goodwill methods can be used.

Difference between full goodwill and partial goodwill

- Small company has net assets worth 90 but the overall company is valued at 100.
- Big company buys 80% of small company.
- Under full goodwill method a goodwill of 10 will be shown
- Under partial goodwill method a goodwill of 8 will be shown.

Comparison of methods

	Equity method	Acquisition method (Partial goodwill)	Acquisition method (Full goodwill)
Assets	Net assets	All assets (partial goodwill)	All assets (full goodwill)
Liabilities		All liabilities	All liabilities
Equity	Net assets	More than net assets because of minority interest	Little more relative to partial goodwill method
Revenue / Exp.		Full consolidation	Full consolidation
N. I.	Same	Same	Same
Profit margin	More favorable	Less favorable	Less favorable
Leverage	Most favorable because liabilities are low	Less favorable than equity method	Slightly better than partial goodwill method
ROE and ROA	Most favorable	Worse than equity method but slightly better than full goodwill method	Least favorable

Defined contribution pension plans

Expense every period. Unpaid expense at end of period is shown as a liability.

Defined benefit pension plans

- Both IFRS and U.S. GAAP require funded status to be reported on the balance sheet.
- Funded status = PV of DBO – fair value of plan assets.
- Reported as net pension liability or net pension asset.

Periodic pension costs recognized in P&L vs. OCI

Total periodic pension costs = net pension liability at the end of the period – net pension liability at the start of the period + employer contribution.

IFRS component	IFRS recognition	U.S. GAAP component	U.S. GAAP recognition
Service costs	Recognized in P&L.	Current service costs Past service costs	Recognized in P&L. Recognized in OCI and subsequently amortized to P&L over the service life of employees.
Net interest income / expense	Recognized in P&L as the following amount: Net pension liability or asset × interest rate	Interest expense on pension obligation Expected return on plan assets	Recognized in P&L. Recognized in P&L as the following amount: Plan assets × expected return.
Re-measurement: Net return on plan assets and actuarial gains and losses	Recognized in OCI and not subsequently amortized to P&L. Net return on plan assets = Actual return – (Plan assets × Interest rate). Actuarial gains and losses = Changes in a company's pension obligation arising from changes in actuarial assumptions.	Actuarial gains and losses including differences between the actual and expected returns on plan assets	Recognized immediately in P&L or, more commonly, recognized in OCI and subsequently amortized to P&L using the corridor or faster recognition method. Difference between expected and actual return on assets = Actual return – (Plan assets × Expected return). Actuarial gains and losses = Changes in a Company's pension obligation arising from changes in actuarial assumptions.

- Current service cost: amount by which pension obligation increases as a result of employees' service in the current period.

- Past service cost: amount by which pension obligation relating to employees' service in prior periods changes as a result of plan amendments or a plan curtailment.

Multinational operations: **Translation methods**
Determine an entity's functional currency:
- The currency that influences sales prices.
- The currency of the country whose competitive forces and regulations mainly determine the sales prices.
- The currency that mainly influences labor, material and other costs.
- The currency in which funds for financing are generated.
- The currency in which receipts from operating activities and retained.

Current rate method: Foreign subsidiary's functional currency is DIFFERENT from parent's presentation currency. Foreign entity's foreign currency financial statements are translated into the parent's presentation currency using the following procedures:
1. All assets and liabilities are translated at the current exchange rate at the balance sheet date.
2. Stockholders' equity accounts are translated at historical exchange rates.
3. Revenues and expenses are translated at the exchange rate that existed when the transactions took place.
4. Translation adjustment is reflected in equity.

Temporal method: Foreign subsidiary's functional currency is SAME as parent's presentation currency. Monetary assets and liabilities are translated at the current exchange rate. Non-monetary assets and liabilities measured at historical cost are translated at historical exchange rates. Non-monetary assets and liabilities measured at current value are translated at the exchange rate at the date when the current value was determined.

1. Stockholders' equity accounts are translated at historical exchange rates.
2. Revenue and Expenses. Revenues and expenses, other than those expenses related to non-monetary assets, are translated at the exchange rate that existed when the transactions took place. Expenses related to non-monetary assets, such as cost of goods sold (inventory), depreciation (fixed assets), and amortization (intangible assets), are translated at the exchange rates used to translate the related assets.
3. Translation gain/loss on income statement.

If value of assets to be translated is greater than value of liabilities to be translated then we have a net asset exposure. With the current rate there will almost always be a net asset exposure.

Basel III framework
- Minimum capital requirement: A bank must have sufficient equity capital to absorb the loss in value of assets in a financial crisis.
- Minimum liquidity: A bank must hold enough high-quality liquid assets to cover its liquidity needs in a 30-day liquidity stress scenario.
- Stable funding: A bank should have adequate amount of stable funding relative to the bank's liquidity needs over a one- year horizon.

CAMELS approach
1. Capital adequacy – Does the bank have sufficient capital given its assets?
2. Asset quality – What is the quality of the bank' financial assets?
3. Management capabilities – Is the management effective? What is its track record?
4. Earnings – What is the level of earnings? What is the quality of earnings? Are earnings trending up or down?
5. Liquidity: How strong is the liquidity position of the bank? What are the sources of funding? Are the sources of funding stable?
6. Sensitivity to market risk: How sensitive are the bank' earnings to market risk?

Quality of financial reports
- The lowest-quality reports contain information that is pure fabrication.
- High-quality reports contain information that is relevant, complete, neutral, and free from error.

High quality earnings are sustainable and represent returns equal to or in excess of the company's cost of capital.

Quantitative tools to assess the likelihood of misreporting
Beneish model: Uses eight variables to detect earnings manipulation.
M-score = Score indicating probability of earnings manipulation; normally distributed random variable with a mean of 0 and a standard deviation of 1.

Translation in a hyperinflationary economy
U.S. GAAP requires use of the temporal method.
IFRS require financial statements to be first restated for inflation and then inflation adjusted financial statements be translated at the current exchange rate.
Purchasing power gains/losses that arises from holding monetary assets and monetary liabilities are included in net income (net monetary assets → purchasing power loss; net monetary liabilities → purchasing power gain)

Decomposition of ROE(DuPont analysis)
ROE = return on assets × leverage = net profit margin × asset turnover × leverage
ROE = EBIT margin × tax burden × interest burden × asset turnover × leverage

Accrual ratios
Balance sheet accruals ratio for time t = $(NOA_t - NOA_{t-1}) / [(NOA_t + NOA_{t-1})/2]$
Cash flow accruals ratio for time t = $[NI_t - (CFO_t + CFI_t)]/[(NOA_t + NOA_{t-1})/2]$

Corporate Finance

Capital budgeting: determining cash flows
- Initial outlay: for new investment →Outlay = FCInv + NWCInv
- Initial outlay for a replacement project →Outlay = FCInv + NWCInv – Sal0 + T(Sal0 – B0)
- Annual after-tax operating cash flow
  CF = (S – C – D) (1 – T) + D
  CF = (S – C)(1 – T) + TD
- Terminal year after-tax non-operating cash flow: TNOCF = SalT + NWCInv – T(SalT – BT)

Effects of inflation on capital budgeting
- (1 + nominal rate) = (1+ real rate)(1+ inflation rate)
- Higher than expected inflation reduces value of depreciation tax savings
  Annual cash flow = (S – C)(1 – T) + TD
- High inflation →High discount rate→ Low present value→ Higher real taxes
- Higher than expected inflation reduces value of payments to bondholders →Real payments to bondholders are lower than expected.

Real options: Types of real options include:
- Timing options: delay investing until you have better information
- Sizing options: growth and expansion option, abandonment option
- Flexibility options: price setting, production flexibility
- Fundamental options: the whole investment is an option

Economic and accounting income
Economic income = Cash flow + change in market value
Market value is based on present value of future cash flows.
Economic income = cash flow – economic depreciation
Accounting income is different from economic income because:
- Accounting depreciation based on original cost.
- Accounting income considers interest expense; EI does not.

Economic profit takes perspective of all investors (debt and equity).
EP = NOPAT - $WACC
NOPAT = EBIT(1 Tax rate)
$WACC = WACC x Capital
NPV = MVA = $\Sigma\ EP_t / (1 + WACC)_t$

Residual income = net income – equity charge = RI = $NI_t - reB_{t-1}$
NPV = $\Sigma\ RI_t / (1 + r_e)^t$

Claims valuation approach values the liabilities and equity, the claims against the assets, which are on the right-hand side of the balance sheet. The value of the claims should equal the value of the assets.

Value of company and cost of capital for propositions without and with taxes

	Without taxes	With taxes
Proposition I	$V_L = V_U$	$V_L = V_U + tD$
Proposition II	$r_e = r_0 + (r_0 - r_d)\frac{D}{E}$	$r_e = r_0 + (r_0 - r_d)(1 - t)\frac{D}{E}$

Divided policy theories
MM's dividend irrelevance theory: Assuming perfect capital markets (no taxes, no transaction costs and symmetric information), dividend policy does not matter

Clientele effect and impact of taxes
Clientele effect: Groups of investors are attracted by particular dividend policy. Dividend clienteles promote stability of dividend policy and do not contradict dividend policy irrelevance.
Impact of tax rates: If the tax rates applied to dividends and capital gains are the same, then (all else being equal) the expected price drop when a share goes ex-dividend is the amount of the dividend.
$Pw - Px = D\ \frac{1-T_D}{1-T_{CG}}$

Signaling content of dividends: Managers/board (insiders) know more than outside investors (asymmetric information). Board and management my use dividends to signal to investors how the company is really performing.
- Dividend initiation or increase generally seen as positive signal and stock prices will probably rise in short-run.
- If revised dividends are not sustainable, stock price will fall.
- Decrease/omission generally seen as negative signal.

Effective tax rates on dividends
Double taxation: Effective tax rate = corporate tax rate + (1 – corporate tax rate)(individual marginal tax rate on dividends)
Tax imputation system: Taxes on dividends are effectively levied only at shareholder level.
Split rate system: Different tax rate for retained earnings vs. earnings that will be distributed.

Payout policies
Stable dividend: dividends are based on long term forecast of sales. Dividends are increased when earnings have increased to a sustainably higher level.
Constant dividend payout ratio: company decides on a dividend payout ratio and then applies this ratio to current earnings to calculate dividend amount.

Residual dividend policy: dividends = internally generated funds remaining after financing the current period's capital expenditures consistent with the target capital structure.
Dividend = earnings – (capital budget × equity percent in capital structure)

Dividend vs. share repurchase decision
Factors to consider when evaluating share repurchases:
- Potential tax advantages.
- Share price support/signaling that the company considers its shares a good investment.
- Added managerial flexibility.
- Offsetting dilution from employee stock options.
- Increasing financial leverage.

Analysis of dividend safety
- Traditional measure: dividend payout ratio and its inverse, the dividend coverage ratio.
- A more comprehensive measure: FCFE coverage ratio = FCFE / (dividends + share repurchases)

Corporate governance
Corporate ownership structures can be classified as: dispersed, concentrated, and hybrid. Different kinds of conflicts exist depending on the ownership structure and voting power.

Two typical structures for a company's board of directors are: one-tier board, and two-tier board. CEO duality exists when the chief executive officer (CEO) also serves as the chairperson of the board.

To evaluate a company's board, we look at the following: board of director's structure, board independence, board skills and experience, and board composition.

ESG integration
While evaluating ESG factors, analysts first need to evaluate if an information is material. They should also consider their investment horizon.
Equity vs. fixed-income security analysis
- Equity analysis: ESG integration is used to both identify potential opportunities and mitigate downside risk. Analysts adjust financial model variables such as cost of capital to reflect ESG factors.
- Fixed-income analysis: ESG integration is generally focused on mitigating downside risk. Analysts usually do not focus on potential opportunities. The credit assessment may vary depending on maturity.

Types of mergers
Horizontal merger: economies of scale; increased market power.
Vertical merger: backward or forward integration; cost savings; greater control.
Conglomerate merger: diversification.

Motives for merger: synergy; growth; increasing market power; acquiring unique capabilities and resources; diversification; bootstrapping earnings; managers' personal incentives; tax considerations; unlocking hidden value; cross-border motivations.

Pre-offer takeover defense mechanisms
- Poison pill: Allow issuance of target company shares at substantial discount which makes it costly for acquirer to take control.
  Flip-in pill: target shareholders have right to buy its shares at a discount.
  Flip-over pill: target shareholders have right to buy shares of acquiring company at a significant discount.
- Poison puts: Target bondholders have right to sell (put) bonds at high price.
- Incorporate in 'target friendly' states.

- Staggered board of directors.
- Restricted voting rights.
- Supermajority voting provisions: Ex: More than 80% majority to approve takeover.
- Fair price amendments.
- Golden parachutes.

Post-offer takeover defense
- Just say no
- Litigation
- Greenmail
- Share repurchase: increases stock price making takeover more expensive; increased leverage.
- Leveraged recapitalization
- Crown jewel defense
- Pac-man defense
- White knight defense
- White squire defense

Regulation
Look at pre and post-merger HHI (sum of squares of market share).

Post-merger HHI	Concentration	Change in HHI	Government action
< 1,000	No	Any amount	No action
Between 1,000 and 1,800	Moderate	100 or more	Possible challenge
> 1,800	High	50 or more	Challenge

Target company valuation
Discounted cash flow method: Determine FCF till terminal year and the terminal value; then discount back. Two methods for determining terminal value:
Method 1: FCFT(1 + g) / (WACC – g)
Method 2: Apply a valuation multiple
Comparable company analysis:
- Define set of companies that are similar to target and calculate relative value measures.
- Determine price at which target company should trade.
- Determine takeover premium: PRM = (DP – SP)/SP

Comparable transaction analysis:
- Use details from takeover transactions of comparable companies.
- Study P/E, P/CF and other industry specific multiples.
- Determine take over price (takeover premium already built into numbers).

Bid evaluation:
Target shareholder's gain = premium = $P_T - V_T$
Acquirer's gain = synergies – premium
Post-merger value of a company is given by: $V_{A^*} = V_A + V_T + S - C$

Divestiture: company becomes smaller by selling, liquidating or spinning off a division/subsidiary. Types include:
- Equity carve-out: creation of new legal entity; new shares issued
- Spin-off: shareholders receive proportional shares in new company; no cash to company
- Split-off: shareholders get shares in new company in exchanged of shares from parent company
- Liquidation

Equity

Holding period return is the return earned from investing in an asset over a specified period of time.

$$r = \frac{D_H}{P_0} + \frac{P_H - P_0}{P_0}$$

If expected return > required return →asset is undervalued
Expected (ex ante) alpha = expected return – required return
Realized (ex post) alpha = actual holding-period return – contemporaneous required return

Models of required return on equity
Capital asset pricing model
- E(Ri) = RF + βi [E(RM) – RF]
- Adjusted beta = (2/3) (Unadjusted beta) + (1/3) (1.0)

Multifactor models
- ri = RF + βimkt RMRF + βisize SMB + βivalue HML
- ri = RF + βi^{mkt} RMRF + βi^{size} SMB + βi^{value} HML+ βi^{liq} LLQ

Macroeconomic multifactor model
ri = T-bill rate + (Sensitivity to confidence risk × 2.59%) (Sensitivity to time horizon risk × 0.66%) (Sensitivity to inflation risk × 4.32%) + (Sensitivity to business-cycle risk × 1.49%) + (Sensitivity to market-timing risk × 3.61%)

Discounted dividend valuation
Dividend discount model
- $V_0 = \sum_{t=1}^{n} \frac{D_t}{(1+r)^t} + \frac{P_n}{(1+r)^n}$
- $V0 = \frac{D_0 \times (1+g)}{r-g}$
- $\frac{P_0}{E_1} = \frac{D_{1/E_1}}{r-g} = \frac{1-b}{r-g}$
- $\frac{P_0}{E_0} = \frac{D_0 (1+g)/ E_0}{r-g} = \frac{(1-b)(1+g)}{r-g}$
- $V0 = \frac{E_1}{r} + PVGO$
- $\frac{P_0}{E_1} = \frac{1}{r} + \frac{PVGO}{E_1}$

The H-model
- $V_0 = \frac{D_0 (1+ g_L)}{r-g_L} + \frac{D_0 H (g_S - g_L)}{r-g_L}$

Three-stage dividend discount models
- First version: company is assumed to have 3 distinct stages of growth and the growth rate of second stage is typically constant.
- Second version: the middle stage is similar to the first stage in the H-model. Dividends grow at a constant high rate in the first stage, decline linearly in the second stage, and grow at sustainable constant growth rate in the third stage.

Free cash flow valuation
- FCFF = NI + Dep + Int(1 – tax rate) – FCInv – WCInv
  WCInv = Change in working capital, excluding cash and short term debt
  FCInv = Change in gross fixed assets
- FCFF = CFO + Int(1 – tax rate) – FCInv
  If interest is not categorized in CFO then do not add back.
- FCFF = EBIT(1 – tax rate) + dep – FCInv – WCInv
- FCFF = (EBITDA – dep)(1 – tax rate) + dep – FCInv – WCInv
  FCFF = EBITDA (1 – tax rate) + dep (tax rate) – FCInv – WCInv
- FCFE = FCFF - Int(1 – tax rate) + net borrowing
  FCFE = NI + NCC – FCInv – WCInv + net borrowing

Cases in which FCFF-based valuation is preferred to FCFE-based valuation:
- A levered company with negative FCFE
- A levered company with changing capital structure

On historic basis, FCFF growth may be more representative of fundamentals than FCFE growth. On forward looking basis, given changing capital structure, the required return on equity may be more sensitive to changes in capital structure than the changes in WACC.

Value of operating assets = $\sum_{t=1}^{\infty} \frac{FCFF_t}{(1+WACC)^t}$

Value of equity = $\sum_{t=1}^{\infty} \frac{FCFE_t}{(1+r)^t}$

Value of firm = Value of operating assets + Value of non-operating assets
Equity value = Firm value – Market value of debt
Enterprise value / EBITDA

Method based on forecasted fundamentals: Compute ratio based on company fundamentals and compare with ratio based on

actual market price. If the ratio based on actual market price is low then the stock is undervalued.

Method of comparables
Determine benchmark value:
- median industry P/E
- average industry P/E
- average of own past P/Es

If firm multiple> benchmark implies overvalued and vice versa.
Advantages and drawbacks of using P/E for valuation
- Earnings are the key driver of value.
- P/E ratio is widely recognized and used by investors.
- Differences in P/Es may be related to differences in long-run average returns.
- Negative EPS→ P/E would be meaningless.
- Earnings often contain volatile, transient components.
- Management's choice of accounting policies may taint EPS's value.

Rationales for the use of P/B
- Can be used when EPS is negative.
- BVPS is more stable than EPS (especially when EPS is volatile).
- More useful for valuing companies with liquid assets (banks, etc).
- Can be used when company is not a going concern.
- Differences in P/B may relate to differences in long run returns.

Drawbacks of using P/B
- Assets creating value (e.g., human capital) not on balance sheet.
- Can be misleading when companies have different level of assets.
- Accounting effects (e.g., R&D) can compromise BV.
- BV is generally not reflective of market value.

Rationales for the use of P/S
- Sales are less subject to accounting manipulation than EPS or BVPS.
- Sales are positive even when EPS is negative.
- Sales are more stable than EPS.
- More useful in valuing mature, cyclical and zero income companies.
- Differences in P/S may relate to differences in long-run returns.

Drawbacks of using P/S
- Might have high sales growth but low earnings and CFO.
- P/S does not capture differences in cost structures.
- Does not capture different capital structures.
- Room for manipulating revenues remains.

Rationales for the use of P/CF
- Less subject to manipulation.
- More stable than EPS.
- Addresses the issue of accounting differences.
- Differences in P/CF may relate to differences in long-run returns.

Drawbacks of using P/CF
- Earnings plus noncash charges, definition of cash flow ignores the changes in working capital; could use FCFE, but:
- FCFE is more volatile and more frequently negative.

Residual income valuation
- Residual income = Net income – (Equity capital x Cost of equity)
- Residual income = EBIT (1 – Tax rate) – (Total capital x WACC)
- Economic value added (EVA) = NOPAT – (C% x TC)
  NOPAT and TC are adjusted
- MVA = Market Value – Book Value
- $V_0 = B_0 + \sum_{t=1}^{\infty} \frac{RI_t}{(1+r)^t} = B_0 + \sum_{t=1}^{\infty} \frac{E_t - rB_{t-1}}{(1+r)^t}$
- $V_0 = B_0 + \sum_{t=1}^{\infty} \frac{(ROE_t - r)B_{t-1}}{(1+r)^t}$

If residual income continues indefinitely at a positive level:
- $V_0 = B_0 + \frac{ROE - r}{r - g} B_0$

If at the end of time horizon T, a certain premium over book value ($P_T - B_T$) exists for the company:
- $V_0 = B_0 + \sum_{t=1}^{T} \frac{(E_t - rB_{t-1})}{(1+r)^t} + \frac{P_T - B_T}{(1+r)^T}$
- $V_0 = B_0 + \sum_{t=1}^{T} \frac{(ROE_t - r)B_{t-1}}{(1+r)^t} + \frac{P_T - B_T}{(1+r)^T}$

If residual income fades over time:
- $V_0 = B_0 + \sum_{t=1}^{T-1} \frac{(E_t - rB_{t-1})}{(1+r)^t} + \frac{E - rB_{T-1}}{(1+r-\omega)(1+r)^{T-1}}$

Persistence factor, ω, is between 0 and 1
1 →residual income will not fade
0 →residual income will not continue after the initial forecast horizon.

Valuation discounts and premiums
Discount for lack of control = DLOC = 1 [1 / (1 + control premium)]
A discount for lack of marketability (DLOM) is an amount or percentage deducted from the value of an ownership interest to reflect the relative absence of marketability.

Fixed Income

Spot rate and forward rate
Forward rate: interest rate that is determined today for a loan that will be initiated in a future time period.
f(T*,T): forward rate of a loan initiated T* years from today with tenor (further maturity) of T years.
F(T*,T): forward contract price at time T* for a zero-coupon bond with maturity T years and unit principal.
Forward pricing model: P(T* + T) = P(T*)F(T*,T)
Relationship between spot rate and forward rates:
$(1+{}_xS_0)^x = (1 + {}_1s_0)(1 + {}_1f_1)(1 + {}_1f_2)\ldots(1+{}_1f_x\text{-}1)$

Forward rate model:
$[1 + r(T^* + T)]^{T^*+T} = [1 + r(T^*)^{T^*}][1 + f(T^*, T)]^T$
When the spot curve is upward (downward) sloping → the forward curve will lie above (below) the spot curve.

Riding the yield curve or rolling down the yield curve
If a trader does not believe that the yield curve will change its level and shape over an investment horizon, he will buy bonds with a maturity longer than the investment horizon. This strategy is called riding the yield curve or rolling down the yield curve.

Spreads
- The Swap spread is the spread paid by the fixed-rate payer of an interest rate swap over the rate of the "on-the-run" (most recently issued) government security with the same maturity as the swap.
- The Z-spread is the constant basis point spread that would need to be added to the implied spot yield curve so that the discounted cash flows of a bond are equal to its current market price.
- The TED spread is calculated as the difference between Libor and the yield on a T-bill of matching maturity. The TED spread is an indicator of perceived credit risk in the general economy.
- The Libor-OIS spread is an indicator of the risk and liquidity of money market securities. It is the difference between Libor and the overnight indexed swap (OIS) rate. An OIS is an interest rate swap in which the periodic floating rate of the swap is equal to the geometric average of an overnight rate (or overnight index rate) over every day of the payment period. The index rate is typically the rate for overnight unsecured lending between banks—for example, the federal funds rate for US dollars.

Traditional term structure models
Unbiased expectations
- Also called pure expectations theory.
- Forward rate is an unbiased predictor of the future spot rate.
- Implication: buying a bond with a maturity of five years and holding it for three years has the same expected return as buying a three-year bond or buying a series of three one-year bonds.
- Can be used to explain any shape of the yield curve.

- Does not consider risk.

Local expectations
- More rigorous than the unbiased expectations theory.
- Expected return for every bond over short time periods is the risk-free rate.
- Assumption: bond pricing does not allow for traders to earn arbitrage profits.
- No risk premium for short time periods.
- Theory accommodates risk premium for long time periods.

Liquidity preference theory
- Liquidity preference theory asserts that liquidity premiums exist to compensate investors for the added interest rate risk they face when lending long term and that these premiums increase with maturity.
- Given an expectation of unchanging short-term spot rates, liquidity preference theory predicts an upward-sloping yield curve.
- Forward rate provides an estimate of the expected spot rate that is biased upward by the amount of the liquidity premium.
- Existence of liquidity premiums implies that the yield curve will typically be upward sloping.

Segmented markets theory
- Yields are solely a function of the supply and demand for funds of a particular maturity.
- Each maturity sector is considered a segmented market.
- The theory is consistent with a world where there are asset/liability management constraints, either regulatory or self-imposed.

Preferred habit theory
- The preferred habitat theory is similar to the segmented markets theory in proposing that many borrowers and lenders have strong preferences for particular maturities but it does not assert that yields at different maturities are determined independently of each other.
- If the expected additional returns to be gained become large enough, institutions will be willing to deviate from their preferred maturities or habitats.
- Agents and institutions will accept additional risk in return for additional expected returns.

Modern term structure models

CIR model: $dr = a(b - r)dt + \sigma \sqrt{r}\, dz$
assumes that interest rates are mean reverting and volatility increases as rates increase.

Vasicek model: $dr = a(b - r)dt + \sigma dz$
interest rates are calculated assuming that volatility remains constant over the period of analysis. As with the CIR model, there is only one stochastic driver of the interest rate process. Both the Vasicek and CIR models assume a single factor, the short-term interest rate, r. The main disadvantage of the Vasicek model is that it is theoretically possible for the interest rate to become negative.

Ho–Lee model: $dr_t = \Theta_t dt + \sigma dz_t$
It is an arbitrage-free model and can be calibrated to closely match the observed term structure. Arbitrage-free models are frequently used to value bonds with embedded options.
A benefit of arbitrage-free term structure models is that they are calibrated to the current term structure. In contrast, equilibrium term structure models frequently generate term structures that are inconsistent with current market data.
Shaping risk is defined as the sensitivity of a bond's price to the changing shape of the yield curve.

Valuation of bonds with embedded options
- Value of callable bond = Value of straight bond – Value of issuer call option
- Value of issuer call option = Value of straight bond – Value of callable bond
- Value of putable bond = Value of straight bond + Value of investor put option
- Value of investor put option = Value of putable bond – Value of straight bond
- The value of any embedded option increases with interest rate volatility.
- The value of an embedded call option is higher if the yield curve is downward sloping.
- The value of an embedded put option is higher if the yield curve is upward sloping.

Effective duration = $\frac{(PV_-) - (PV_+)}{2 \times (\Delta Curve) \times (PV_0)}$
- Cash → effective duration: 0
- Zero-coupon bond → effective duration ≈ Maturity
- Fixed-rate bond → effective duration < Maturity
- Callable bond → effective duration ≤ Duration of straight bond
- Putable bond → effective duration ≤Duration of straight bond
- Floater (Libor flat) → effective duration ≈ Time (in years) to next reset
- The effective duration of a callable bond cannot exceed that of the straight bond.
- One-sided up-duration (One-sided down-duration) measures the sensitivity of a bond's value to an increase (decrease) in interest rates.

Effective convexity = $\frac{(PV_-) + (PV_+) - [2 \times (PV_0)]}{(\Delta Curve)^2 \times (PV_0)}$
Callable bonds: Negative convexity at low interest rates; Similar to straight bonds at high rates.
Putable bonds: Similar to straight bonds at low rates; Positive but low convexity at high rates
Value of capped floater = Value of straight bond – Value of embedded cap

Value of floored floater = Value of straight bond + Value of embedded floor

Conversion value = Underlying share price × Conversion ratio

Market conversion premium per share = Market conversion price – Underlying share price

Market conversion price = convertible bond price / conversion ratio

Market conversion premium ratio = market conversion premium per share / share price

Premium over straight value = (convertible bond price / straight value) - 1

Value of convertible bond = Value of straight bond + Value of call option on the issuer's stock

Value of callable convertible bond = Value of straight bond + Value of call option on the issuer's stock – Value of issuer call option

Value of callable putable convertible bond = Value of straight bond + Value of call option on the issuer's stock – Value of issuer call option + Value of investor put option

Expected exposure is the maximum amount of money that an investor could lose in the event of a default.
Loss given default is the expected amount that would be lost if there is a default. It is calculated using assumed recovery rates.
Probability of default is the risk-neutral default probability. (Risk-neutral default probability > Actual probability of default)
Credit valuation adjustment (CVA) is subtracted from the hypothetical value of the bond, if it were default risk-free, to get the bond's fair value given its credit risk. CVA represents compensation for bearing credit risk.

Structural models predict <u>why</u> a default may occur. They are based on insights from option pricing theory.
Reduced-Form Models predict <u>when</u> a default may occur. They are based on statistical methods. Default is treated as an exogenous variable that occurs randomly.

We can calculate the value of a bond and its credit spread using the following steps:

1. Determine value of bond assuming no default (VND)
2. Calculate credit valuation adjustment (CVA)
3. Fair value of bond = VND – CVA
4. Using fair value determine YTM.
5. Using YTM determine credit spread.

Asset backed securities
Three major factors to consider when evaluating asset-backed securities (ABS)

- Underlying collateral
- Origination and servicing of collateral
- Structure of the transaction

Three major credit analysis approaches can be used for ABS:

- Book of loans
- Portfolio
- Loan by loan

Credit default swaps

- Payout ratio = 1 – recovery rate (%) and payout amount = payout ratio x notional amount
- Expected Loss = PD x LGD
- Upfront payment = present value of protection leg – present value of premium leg
- Present value of credit spread = upfront premium + present value of fixed coupon
- Upfront premium ≈ (credit spread – fixed coupon) x duration
- Credit spread ≈ (upfront premium/duration) + fixed coupon
- Price of CDS in currency per 100 par = 100 – upfront premium %
- Upfront premium % = 100 – price of CDS in currency per 100 par
- Profit for the protection buyer = change in spread in bps x duration x notional amount
- Percentage change in CDS price = change in spread in bps x duration

Derivatives

Forward contract price

- If underlying has no cash flows: F = S × (1 + r)T
- If underlying has cash flows: F = S (1 + r)T + Future Value of Costs – Future Value Benefits
  F = Future value of underlying adjusted for carry cash flows = FV (S0 + θ0 - γ0) where θ0 represents present value of costs at time 0 and γ0 represents present value of benefits at time 0.
  Vt = Present value of difference in in forward prices
- Using continuous compounding: $F_0(T) = S_0 e^{(r_c+\theta-\gamma)T}$

FRA pricing and valuation

$$\text{FRA}(0,h,m) = \left(\frac{1+(L_0(h+m)\times t_{h+m})}{1+(L_0(h)\times t_h)} - 1\right)/t_m$$

FRA value per unit of notional principal is $V(0,h,m) = \frac{(\text{FRA}(g,h-g,m) - \text{FRA}(0,h,m))t_m}{1+D_g(h+m-g)t_{h+m-g}}$

Fixed-income forward or futures price:
$F_0(T)=FV_{0,T}(B_0(T+Y)+AI_0)-AI_T-FVCI_{0,T}$
The quoted price which includes the conversion factor is:
$QF_0(T)=F_0(T)/CFT$

$V(T)=PV_{t,T}(Ft(T)-F_0(T))$

Currency forward and futures contracts
Covered interest rate parity: $F_{P/B} = S_{P/B}(1 + i_P)^T / (1 + i_B)^T$

Interest rate swaps
Swap fixed rate = (1 – final discount factor) / (sum of all discount factors)
Value at time t = the sum of the present value of the difference in fixed swap rates x notional amount

Currency swaps: two counterparties agree to exchange future interest payments in different currencies. Notional amount of one currency will be given; notional amount of other currency will be based on the spot exchange rate at time 0.

Equity swap pricing and valuation

$$V_t = FB_t(C_0) - \left(\frac{S_t}{S_{t-}}\right)NA_E - PV(Par - NA_E)$$

Binomial model no arbitrage approach (call options)
Hedge ratio, h = $(c^+ - c^-) / (S^+ - S^-)$
$c = hS + PV(-hS^- + c^-)$
No-arbitrage approach: a long call option is equal to owning h shares of stock partially financed, where the financed amount is $PV(-hS^- + c^-)$.

Binomial model no arbitrage approach (put options)
Hedge ratio, h = $(p^+ - p^-) / (S^+ - S^-)$
$p = hS + PV(-hS^- + p^-)$
$p = hS + PV(-hS^+ + p^+)$

Binomial model: expectations approach
Risk-neutral probability of an up move:
$\pi = (1 + r - d) / (u - d)$
$E(c_1) = \pi c^+ + (1 - \pi)c^-$
$c = PV_r[E(c_1)]$

BSM model
<u>without carry benefit</u>:
$c = SN(d_1) - e^{-rT} XN(d_2)$
$p = e^{-rT} XN(-d_2) - SN(-d_1)$
$d_1 = \frac{\ln\left(\frac{S}{X}\right)+\left(r+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$
$d_2 = d_1 - \sigma\sqrt{T}$

<u>with carry benefit</u>:
$c = Se^{-\gamma T}N(d_1) - e^{-rT} XN(d_2)$
$p = e^{-rT} XN(-d_2) - Se^{-\gamma T}N(-d_1)$
$d_1 = \frac{\ln\left(\frac{S}{X}\right)+\left(r-\gamma+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$
$d_2 = d_1 - \sigma\sqrt{T}$

	Call option		Put option	
	Underlying	**Financing**	**Underlying**	**Financing**
Binomial model	hS	PV ($-hS^- + C^-$)	hS	PV ($-hS^- + p^-$)
BSM model	$N(d_1)S$	$-N(d_2)e^{-rT}X$	$-N(-d_1)S$	$N(-d_2)e^{-rT}X$

Alternative Investments

Valuation of commercial real estate
The highest and best use of a vacant site is the use that would result in the highest value for the land. Three major approaches to estimate value: income approach, cost approach and sales comparison approach.
<u>Income approaches</u>

The income approach focuses on net operating income generated from a property. NOI is roughly analogous to EBITDA
Rental income at full occupancy + other income = Potential gross income - Vacancy and collection loss = Effective gross income - Operating expenses = Net operating income
Estimate value by capitalizing NOI
Value = NOI / cap rate
Cap rate = discount rate - growth rate
Can also estimate cap rate from comparables
The term ARY stands for all risk yield and is used in the UK:
ARY = NOI / recent sales price of comparable
Stabilized NOI is the NOI assuming property is not being renovated.
Gross income multiplier = value / gross income

Cost approach

Used for unusual properties or properties for specialized use (comparable data not available)
Land + replacement cost
- Adjust for depreciation
- Physical deterioration (curable and incurable)
- Functional obsolescence
- Locational obsolescence
- Economic Obsolescence

Sales comparison approach

Subject property and comparables use comparables price as starting point and then make adjustments.

Valuation of REITS

Net asset value approach

- NAV = estimated value of operating real estate + cash and A/R - debt and other liabilities
- NAVPS = NAV / # of shares outstanding
- NAVPS = FFO/share x P/FFO multiple
- NAVPS = AFFO/share x P/AFFO multiple

Relative value (price multiple) approach

Major multiples: P/FFO, P/AFFO, EV/EBITDA

FFO = Accounting net earnings + Depreciation expense + Deferred tax expenses – Gains (losses) from sales of property and debt restructuring

AFFO = FFO - straight line adjustment – recurring maintenance type capital expenditures and leasing commissions

Venture capital investments

Valuation issues in venture capital transactions

PRE: Agreed value of a company prior to a round of financing or investment (I)
POST: Value of a company after the financing or investing round
POST = PRE + I
Proportionate ownership of the venture capital investor = I/POST

VC exit routes: returning cash to investors

Initial Public Offering (IPO):

- Highest exit value relative to other methods.
- High liquidity, access to capital, and attracts good management.
- Less flexible, more costly, and complex.
- Use when company has strong growth prospects, operating history, size.
- Timing of IPO is an important consideration.

Secondary market: sale to other financial investors or strategic investors.
Management buyout: firm sold to management.
Liquidation: sale of firm's assets.
J curve effect: Low or negative returns initially followed by increased returns.

Economic terms (most testable)

- Management fees represent a percentage of committed capital paid annually to the GP
- Carried interest represents the general partner's share of profits generated by a private equity fund.
- Hurdle rate is the internal rate of return that a private equity fund must achieve before the GP receives any carried interest.
- Vintage year is the year the private equity fund was launched. Reference to vintage year allows performance comparison of funds of the same stage and industry focus.
- Term of the fund is typically 10 years, extendable for additional shorter periods (by agreement with the investors).

Evaluating fund performance

- Gross IRRs are estimated by calculating the internal rate of return between the following cash flows: called down capital at the beginning of period and the previous year's operating results.
- Net IRRs are calculated by removing management fees and carried interest from gross IRR.
- PIC (paid in capital): the ratio of paid in capital to date divided by committed capital.
- DPI (distributed to paid in): cumulative distributions paid out to LPs as a proportion of the cumulative invested capital. DPI is presented net of management fees and carried interest.
- RVPI (residual value to paid in): value of LPs' shareholding held with the private equity fund as a proportion of the cumulative invested capital. RVPI is presented net of management fees and carried interest.
- TVPI (total value to paid in): the portfolio companies' distributed and undistributed value as a proportion of the cumulative invested capital. TVPI is the sum of DPI and RVPI. TVPI is presented net of management fees and carried interest.

Venture capital valuation method:

1. Post-money valuation POST = V/(1 + r)t
2. Pre-money valuation PRE = POST−I
3. Ownership fraction F = I/POST
4. Number of shares y = x * F / (1−F)
5. Price of shares P1 = I/y

Commodities and commodity derivatives

Spot and futures pricing

- Basis = spot price – futures price
- Backwardation (positive basis) and contango (negative basis)

Theories explaining futures returns

Insurance theory: Futures prices are impacted by commodity producers who are long the physical good are motived to sell the commodity for future delivery to hedge their production price risk exposure.
The hedging pressure hypothesis: Futures prices are impacted by producers along with consumers seek to protect themselves from commodity market price volatility by entering into price hedges to stabilize their projected profits and cash flow.
The theory of storage: Futures prices are impacted by supply and demand dynamics of commodity inventories, including the concept of "convenience yield." Convenience yield is inversely related to general available of the commodity.

Futures price = spot price + direct storage costs - convenience yield

Total return on a commodity futures position = price return + roll return + collateral return

Portfolio Management

ETFs rely on a creation/redemption process that is carried out in an OTC primary market between the ETF issuer and authorized participants. This mechanism makes the ETF structure inherently more fair as compared to traditional mutual funds. It also allows an ETF to keep capital gains in check, leading to greater tax efficiency. While evaluating how well an ETF tracks its underlying index, a rolling return assessment using periodic performance deviations (tracking differences) are more useful than the standard deviation of daily returns (tracking error).

ETF premiums and discounts refer to the differences between the exchange price of the ETF and the fund's calculated NAV. The major sources of ETF premiums and discounts are: timing differences and stale pricing.

Risks of investing in ETFs include: counterparty risk, fund closures risk, and expectation-related risks.

Primary ETF strategies include: portfolio efficiency, asset class exposure management, and active or factor investing.ETFs rely on a creation/redemption process that is carried out in an OTC primary market between the ETF issuer and authorized participants. This mechanism makes the ETF structure inherently more fair as compared to traditional mutual funds. It also allows an ETF to keep capital gains in check, leading to greater tax efficiency.

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Arbitrage Pricing Theory:

$$E(Rp) = R_F + \lambda_1\beta_{p,1} + \ldots + \lambda_K\beta_{p,K}$$

Portfolio manager's active return has two components

1. The return from factor tilts: product of the portfolio manager's factor tilts (active factor sensitivities) and the factor returns.
2. The return from asset selection: part of active return reflecting the manager's skill in individual asset selection.

Active return: $R_A = R_P - R_B$
Active weight: $\Delta w_i = w_{P,i} - w_{B,i}$
$R_A = \Sigma\Delta w_i R_{Ai}$
Active return = $\sum_{j=1}^{k}$[Portfolio sensitivityj −Benchmark sensitivity] × Factor return$_i$ + Asset selection
Information ratio = $\frac{\overline{R_p} - \bar{R}_B}{s\,(R_p - R_B)}$
Active risk squared = active factor risk + active specific risk = risk due to portfolio's different-than-benchmark exposures relative to factors specified in the risk model + risks resulting from the portfolio's active weights on individual assets. Also called asset selection risk.

Value at risk is the minimum loss that would be expected a certain percentage of the time over a certain period of time given the assumed market conditions.

Extensions of VaR

- Conditional VaR (CVaR): average loss conditional on exceeding the VaR cutoff.
  Answers the question: how much can I expect to lose if VaR is exceeded.
  Also called: expected tail loss or expected shortfall.
- Incremental VaR (IVaR): change in VaR resulting from a change in position size.
  VaR with proposed new allocation – VaR with original allocation.
  Considers diversification effect.
- Marginal VaR (Mvar): Effect of a very small change in a position.
  Some practitioners interpret 'small' as a 1% change.
  Considers diversification effect.
- Relative VaR: measure of the degree to which the performance of a given investment portfolio might deviate from its benchmark.
  Also called ex ante tracking error.

Inter-Temporal Rate of Substitution t for (real) wealth at time t + s.
A bond promises $1 at the end of the year. Inflation rate is 0. The year-end state of the economy is uncertain. Investors are willing to pay more if the economy is expected to perform poorly.
If an investor's inter-temporal rate of substitution is higher than the current bond price she will buy.
The one-period real risk-free rate: $l_{t,1} = \frac{1 - P_{t,1}}{P_{t,1}}$
If uncertainty about the state of the economy increases the expected intertemporal rate of substitution will be lower.
Investors are compensated with a higher expected return when uncertainty increases.

Default-free interest rates and economic growth
Higher trend real economic growth→higher real default-free interest rates
Higher volatility of GDP growth rate→ higher real default-free interest rates
For a real default-free bond we have:

$$P_t^i = \sum_{s=1}^{N} \frac{CF_{t+s}^i}{(1 + l_{t,s})^s}$$

Only changes in $l_{t,s}$ will affect the price of such a bond. $l_{t,s}$ will be determined by real economic growth and the volatility in economic growth over time as a result of the aggregation of the consumption and saving decisions of individual investors.

Short-term rates and the business cycle
Basic pricing formula for a default-free nominal coupon-paying bond:

$$P_t^i \sum_{s=1}^{N} \frac{CF_{t+S}^i}{(1 + l_{t,S} + \theta_{t,S} + \pi_{t,S})^S}$$

With short-term nominal interest rates inflation uncertainty can be ignored:

$$P_t^i \frac{CF_{t+S}^i}{(1 + l_{t,S} + \theta_{t,S})^S}$$

Short-term nominal interest rates are positively related to short-term real interest rates and short-term inflation expectations.
Central banks set short-term interest rates in response to the economy's position in the business cycle.

- Cut rates when economic activity and/or inflation are judged to be too slow.
- Increase rates when economic activity and/or inflation are judged to be too high.

Taylor rule: $pr_t = l_t + t_t + 0.5\,(t_t - t_t^*) + 0.5\,(y_t - y_t^*) = l_t + 1.5t_t + 0.5t_t^* + 0.5\,(y_t - y_t^*)$

Credit premiums and the business cycle

$$P_t^i \sum_{s=1}^{N} \frac{E_t[\widetilde{CF_{t+S}^i}]}{(1 + l_{t,S} + \theta_{t,S} + \pi_{t,S} + \gamma_{t,S}^i)^S}$$

Credit spread = yield on credit-sensitive bond – yield on government bond

- Credit spreads increase in times of economic weakness.
  Investors demand a return (yield) for possible losses.
  Expected loss = PD x (1 – RR).
- Credit spreads narrow as the economy strengthens.
- When spreads narrow, credit-sensitive bonds perform well.
- When spreads narrow relative to government bonds, spreads between higher- and lower-rated bonds also narrow.

Core point: Credit risky bonds perform poorly in bad economic times.

Equities and the equity risk premium

$$P_t^i \sum_{s=1}^{\infty} \frac{E_t[\widetilde{CF_{t+S}^i}]}{(1 + l_{t,S} + \theta_{t,S} + \pi_{t,S} + \gamma_{t,S}^i + K_{t,S}^i)^S}$$

$$P_t^i \sum_{s=1}^{\infty} \frac{E_t[\widetilde{CF_{t+S}^i}]}{(1 + l_{t,S} + \theta_{t,S} + \pi_{t,S} + \lambda_{t,S}^i)^S}$$

- Equities do not have good consumption hedging properties (do not pay off in bad times)
- Risk averse investors would demand a higher premium on equity holdings relative to bond holdings
- ERP and CRP are positively correlated and are influenced by the business cycle in similar ways

Commercial real estate

$$P_t^i \sum_{s=1}^{\infty} \frac{E_t[\widetilde{CF_{t+S}^i}]}{(1 + l_{t,S} + \theta_{t,S} + \pi_{t,S} + \gamma_{t,S}^i + K_{t,S}^i + \emptyset_{t,S}^i)^S}$$

- Commercial real estate has bond-like and equity like characteristics
  Rental income is like coupon payments.
  Value of property at the end of a lease term is uncertain.
- Commercial real estate is generally illiquid.
- Commercial real estate prices are pro-cyclical.
- Investors will demand a relatively high risk premium.

Decomposition of value added

$R_A = (\Delta w_{stocks} R_{B,stocks} + \Delta w_{bonds} R_{B,bonds}) + (w_{P,stocks} R_{A,stocks} + w_{P,bonds} R_{A,bonds})$ = Value added due to asset allocation + Value added due to security selection

Information Ratio (IR) is affected by the addition of cash or leverage in a portfolio. Adding cash to a portfolio of risky assets will cause the IR to decrease. The IR of an unconstrained portfolio is unaffected by the aggressiveness of active weights. If the active security weights are all multiplied by some constant, c, the information ratio of an actively managed portfolio will remain unchanged.

$$IR_C = c\,R_A / c\,STD(R_A) = R_A / STD(R_A)$$

Constructing optimal portfolios

Optimal portfolio (P) should have the highest possible Sharpe ratio
$SR_P^2 = SR_B^2 + IR^2$
Optimal amount of active risk: $STD(R_A) = STD(R_B) \times (IR / SR_B)$
Optimal portfolio (P): $STD(R_P)^2 = STD(R_B)^2 + STD(R_A)^2$

The Full Fundamental Law

$E(R_A) = (TC)(IC)\sqrt{BR}\sigma_A$
$IR = (TC)(IC)\sqrt{BR}$
$TC = COR(\mu i/\sigma i, \Delta wi\sigma i)$
$\sigma_A = TC \frac{IR^*}{SR_B} \sigma_B$
$SR_P^2 = SR_P^2 + (TC)^2(IR^*)^2$
For unconstrained portfolios IR does not change with active risk.
For constrained portfolios IR generally decreases as the active risk increases.

The **effective spread** is two times the difference between the trade price and the midquote price before the trade occurred.

The **implementation shortfall** method compares the values of the actual portfolio with that of a paper portfolio constructed on the assumption that trades could take place at the prices that prevailed when the decision to trade was made. The excess of the paper value over the actual value is the implementation shortfall.

The **volume-weighted average price (VWAP)** method of estimating transaction costs compares average fill prices to average market prices during a period surrounding the trade.

Market fragmentation occurs when the same instrument is traded across multiple venues. Electronic algorithmic trading techniques, such as liquidity aggregation and smart order routing, help traders deal with market fragmentation.

The major types of **electronic traders** include: electronic news traders, electronic dealers, electronic arbitrageurs, electronic front runners, electronic quote matchers.

Real-time surveillance of markets often can detect order front running and various **market manipulation strategies** such as: bluffing, gunning the market, and squeezing and cornering.