

Mathematics: A Discrete Introduction

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#10: Quantifiers

10.1

- a. $\forall x \in \mathbb{Z}, x$ is prime.
- b. $\exists x \in \mathbb{Z}, \neg(x \text{ is prime or composite})$.
- c. $\exists x \in \mathbb{Z}, x^2 = 2$.
- d. $\forall x \in \mathbb{Z}, 5|x$.
- e. $\exists x \in \mathbb{Z}, 7|x$.
- f. $\forall x \in \mathbb{Z}, x^2 > 0$.
- g. $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, xy = 1$.
- h. $\exists x, y \in \mathbb{Z}, x/y = 10$.
- i. $\exists x \in \mathbb{Z}, \forall y \in \mathbb{Z}, xy = 0$.
- j. $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, x < y$.

10.2

- a. There is an integer that is not prime.
 $\exists x \in \mathbb{Z}, x$ is not prime.
- b. Every integer is prime or composite.
 $\forall x \in \mathbb{Z}, x$ is prime or composite.
- c. Every integer is not an integer whose square is 2.
 $\forall x \in \mathbb{Z}, \neg(x^2 = 2)$.

- d. There is an integer that is not divisible by 5.
 $\exists x \in \mathbb{Z}, \neg(5|x).$
- e. All integers are not divisible by 7.
 $\forall x \in \mathbb{Z}, \neg(7|x).$
- f. There is an integer whose square is negative.
 $\exists x \in \mathbb{Z}, x^2 < 0.$
- g. There is an integer x for any integer y such that $xy \neq 1$.
 $\exists x \in \mathbb{Z}, \forall y \in \mathbb{Z}, xy \neq 1.$
- h. For every integer x, y , $x/y \neq 10$.
 $\forall x, y \in \mathbb{Z}, x/y \neq 10.$
- i. For every integer x there is an integer y such that $xy \neq 0$.
 $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, xy \neq 0.$
- j. There is an integer x which is equal or greater than any integer.
 $\exists x \in \mathbb{Z}, \forall y \in \mathbb{Z}, x \geq y.$

10.3

No one is invited to the party.

10.4

- a. False
- b. True
- c. True
- d. True
- e. False
- f. True
- g. True
- h. True

10.5

- a. $\exists x \in \mathbb{Z}, x \geq 0$.
 There is an integer that is greater than or equal to 0.
- b. $\forall x \in \mathbb{Z}, x \neq x + 1$.
 Every integer x is not equal to $x + 1$.

- c. $\forall x \in \mathbb{Z}, x \leq 10$.
All integers are less or equal to 10.
- d. $\exists x \in \mathbb{Z}, x + x \neq 2x$.
There is an integer x such that $x + x \neq 2x$.
- e. $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, x \leq y$.
For every integer x there is an integer y such that $x \leq y$.
- f. $\exists x \in \mathbb{Z}, \exists y \in \mathbb{Z}, x \neq y$.
There is an integer x such that there is an integer y which satisfies $x \neq y$.
- g. $\exists x \in \mathbb{Z}, \forall y \in \mathbb{Z}, x + y \neq 0$.
There is an integer x for any integer y such that $x + y \neq 0$.

10.6

$\forall x, \forall y$, assertions about x and y

$\forall y, \forall x$, assertions about x and y

These two statements mean the same thing, since we have "uninterrupted subsequences of quantifiers of the same type". Likewise, the following statements mean the same thing.

$\exists x, \exists y$, assertions about x and y

$\exists y, \exists x$, assertions about x and y

"Being able to swap the use of the variables is of course due to the fact that variables are just 'dummies', and can therefore be replaced by other variables." Check: <https://math.stackexchange.com/q/3515811>.