Theorem 3

In a graph $G = (V,E)$

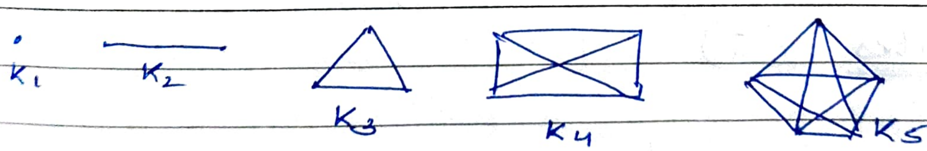
$$\sum_{v \in V} deg^-(v) = \sum_{v \in V} deg^+(v) = |E|$$

$$(v_1, v_2, \cdots, v_n)$$
$$|E| = k$$

It is given that $G = (V,E)$ is a directed graph. Let E contain k no. of edges that is $|E| = k$.
Since an edge will start from a vertex. Therefore the sum of all all out degree vertices is $|E|$.
Similarly, every edge will incident on a vertices.

Some special types of simple graphs

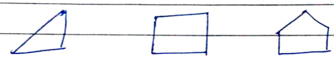
Complete graph with n no. of vertices is denoted by K_n.



It is a simple graph in which there is an edge in between every pair of vertices.

#1 Cycle C_n , $n \geq 3$

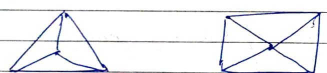
A cycle with n vertices ~~who~~ is denoted by C_n where $n \geq 3$ is a graph in which the vertices $1, 2, 3 \ldots n$ are connected as $(1,2), (2,3), (3,4), \ldots, (n,1)$



C_3 C_4 C_5

Wheel

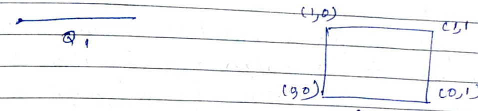
A wheel denoted by W_n is obtained from the cycle C_n by introducing a new vertex such that all the vertices of C_n are joined with the new vertex.



W_3 W_4

n-Cubes (Q_n)

The n-cube is denoted by Q_n in which there are 2^n no. of vertices and this vertices are the binary strings of the length L. There is an edge from one vertex to another vertex if ~~the~~ in their binary strings they differ from one other in exactly one place.



Q_1 Q_2

Bipartite graph

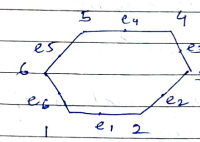
$G(V, E)$

V_1 , V_2

A simple graph $G = (V, E)$ is said to be Bipartite graph if V can be divided into 2 sets (V_1, V_2) such that the end vertices of every edge of the graph G; one will lie in V_1 & other is in V_2.

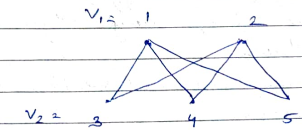
NOTE

In a bipartite graph the end vertices of any of the edge cannot lie in a single set say, V_1 or V_2.



$G = (V, E)$

$V_1 = \{1, 3, 5\}$
$V_2 = \{2, 4, 6\}$



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Theorem 4

A simple graph is bipartite if and only if (iff) it is possible to assign one of the 2 different colour to each vertex of the graph. So that no 2 adjacent vertices are assigned to the same colour.

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The proof of this theorem consist of 2 cases.

Case I Let the simple graph $G=(V,E)$ is bipartite. We have to show that one of the 2 adjacent vertices of every edge have different colour. Since, $G=(V,E)$ is a bipartite graph. So, $V=V_1 \cup V_2$. where V_1 contains one end vertex of all the edges of G and V_2 contains the other end vertex.

If the vertices of V_1 is coloured with one colour & vertices of V_2 is coloured with another colour. So, it is clear that both end vertices of every end are of different colour.

Case II

Let us assume that the end vertices of every edge are of different colour.
One end of every edge are coloured with black colour
& the other end vertices will be coloured with red colour
Let V_1 be the set of all black coloured vertices & V_2 contain all the red coloured vertices.
It is clear that every edge of the graph G start with one vertex of V_1 & end with one vertex of V_2.

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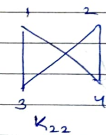
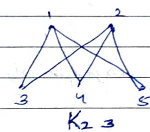
Becoz it is given that both end vertices of every edge are of diff. colour.
Hence, the set of all vertices V is divided into 2 sets V_1 & V_2 such that one end of every edge start with one vertex of V_1 & ends with one vertex of V_2. Hence, $G=(V,E)$ is a bipartite graph.

Complete Bipartite Graph

A graph $G=(V,E)$ is said to be complete bipartite graph if it will satisfy the following 2 conditions.
i) It is a bipartite graph
ii) Let $V=V_1 \cup V_2$ where cardinality of V_1 $(|V_1|)=m$ & $|V_2|=n$ & every ~~very~~ vertices of V_1 & connected with every vertices of V_2.
This complete bipartite graph is denoted by $\boxed{K_{mn}}$

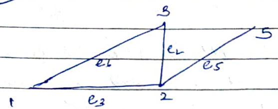
Ex

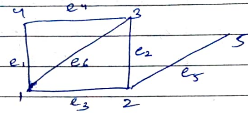




Sub-Graph

Let $G=(V,E)$ be a graph, the graph $H=(W,E')$ is called a proper sub graph of G if W is subset of V and E' is a subset of E ex





$W \subseteq V$

$E' \subseteq E$

So, H is a sub-graph G

Proper Sub-graph

A subgraph H of the graph G is called proper subgraph if H is not equal to G.

NOTE:

Every graph is a subgraph of itself.

Regular graph

A graph $G=(V,E)$ is said to regular graph if degree of all the vertices are equal.

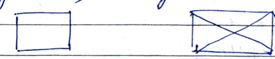
(n- Regular graph) → Next page.

n- Regular graph

If degree of all the vertices is equal to n.

ex

2-Regular, 3-Regular ...

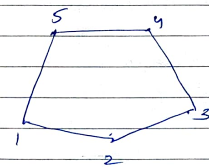


Complimentary graph

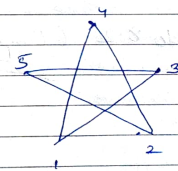
Let $G=(V,E)$ be any graph the complimentary graph denoted by $\bar{G}=(V,E')$ is denified as which contains all the vertices of G and E' is the set of all edges from the vertex q to vertex v if the graph G if there is no edge from vertex q to vertex v.

In other words, the complimentary graph of $G=(V,E)$ is ~~the complimentary~~ $\bar{G}=(V,\bar{E})$, where 2 vertices are adjacent to $\bar{G}$ if these vertices are not adjacent with each other.

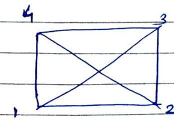
ex



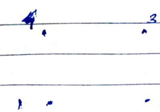
G



$\bar{G}$



K_4 →



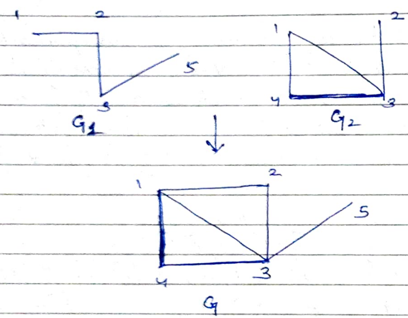
$\bar{K}_4$ (no edges)

Union of 2 Graph

If $G_1 = (V_1, E_1)$ & $G_2 = (V_2, E_2)$ be any 2 graphs. Then their union G_1, G_2 is another graph where $G_1 \cup G_2 = (V, E)$

$V = V_1 \cup V_2$

$E = E_1 \cup E_2$



Article 8.2 (Completed)

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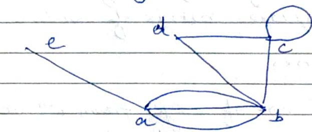
Extra Class

Article 8.3

Adjacency List

Whenever we prepare the adjacency list of an undirected graph, then basically we form 2 columns. In the first column we write the vertices & in second we write the adjacent vertices.



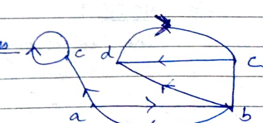
Adjacency List (Undirected Graph)

Vertices		Adjacent Vertices
a	$\longrightarrow$	b, e
b	$\longrightarrow$	c, a, d
c	$\longrightarrow$	b, d, c
d	$\longrightarrow$	b, c
e	$\longrightarrow$	a

In case of directed graph the Adjacency list can also be prepared in a table of 2 columns. But in the first column we collect the initial vertex & in the second column we collect the terminal vertex.

Adjacency List

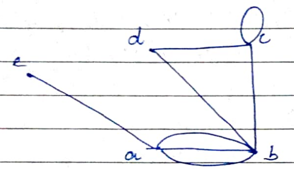
Initial Vertices	Terminal Vertices
a	b, c
b	c, d, a
c	d
d	c
e	e



Adjacency Matrix for an Undirected Graph

Adjacency Matrix is a matrix which represent the number of edges from a vertex(i) to another vertex(j). Let $G=(V,E)$ be any graph where cardinality of $V = (|V|) = n$.
Then its corresponding adjacency matrix ~~denoted~~ denoted by A_G is a square matrix of order $n \times n$.
where $A_G = (a_{ij})_{n \times n}$

& a_{ij} is the no. of edges from i^{th} vertex to j^{th} vertex.



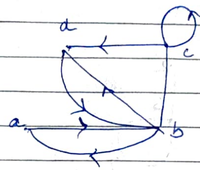
Q Draw the adjacency matrix of this graph

$$A_G = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{pmatrix} 0 & 3 & 0 & 0 & 1 \\ 3 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

NOTE :-

The adjacency matrix of a undirected graph is a symmetric Matrix.

Q Draw the adjacency matrix of following directed graph.



$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

NOTE :-

The adjacency matrix of a directed graph may or may not be symmetric.

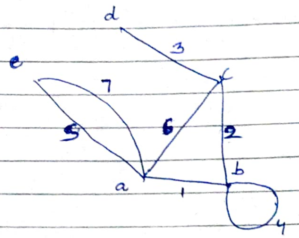
Incident Matrix of an undirected Graph

Let $G=(V,E)$ be any undirected graph where $|V| = m$ & $|E| = n$
Then its Incident Matrix A is a matrix of order $n \times n$. whose elements a_{ij} is obtained by using the following rule.

$$a_{ij} = \begin{cases} 1, & \text{if edge } j \text{ is incident on } i^{th} \text{ vertex} \\ 0, & \text{otherwise.} \end{cases}$$

In incident matrix the vertices are taken in rows & the edges are taken in column

Q



Incident Matrix

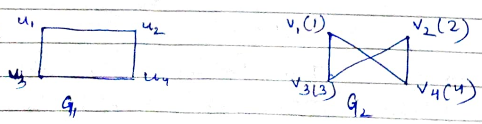
$$\begin{array}{c|ccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline a & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ b & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ c & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ d & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ e & 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{array}$$

Isomorphism of Graphs

Two graphs $G_1 = (V_1, E_1)$ & $G_2 = (V_2, E_2)$ are said to be isomorphic graph if ~~their~~ there exist a fx^n f from $V_1 \to V_2$, which is both one-one and onto and u_1 & u_2 are end vertices in V_1 then $f(u_1)$ & $f(u_2)$ are end vertices in V_2.

In this case G_1 & G_2 are called isomorphic graph & the function is called isomorphism.

Q.N



Q Verify the above graph is isomorphic or not.

$V_1 = \{u_1, u_2, u_3, u_4\}$

$V_2 = \{1, 2, 3, 4\}$

$f: V_1 \to V_2$

$f(u_1) = 1$	$u_1 \to u_3$
$f(u_2) = 3$	$u_1 \to u_2$
$f(u_3) = 4$	$u_3 \to u_4$
$f(u_4) = 2$	$u_2 \to u_4$

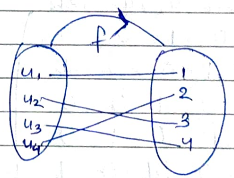
$f(u_1) \to f(u_3)$

$f(u_1) \to f(u_2)$

$f(u_3) \to f(u_4)$

$f(u_2) \to f(u_4)$

Preservation condition satisfied.



f: is one-one and onto.

$\therefore$ G_1 & G_2 are isomorphic graphs & function f is isomorphism.

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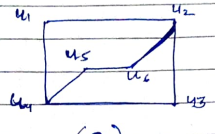
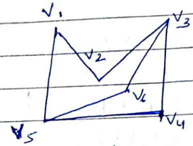
(Q) Check whether the 2 graph G_1 & G_2 are isomorphic or not by using adjacency matrix.



(G_1)



(G_2)

$$\boxed{\begin{array}{l} f(u_2) = V_3 \\ f(u_3) = V_4 \\ f(u_1) = V_6 \\ f(u_4) = V_5 \\ f(u_5) = V_1 \\ f(u_6) = V_2 \end{array}}$$

Imp.

$$A_{G_1} = \begin{array}{c|cccccc} & u_1 & u_2 & u_3 & u_4 & u_5 & u_6 \\ \hline u_1 & 0 & 1 & 0 & 1 & 0 & 0 \\ u_2 & 1 & 0 & 1 & 0 & 0 & 1 \\ u_3 & 0 & 1 & 0 & 1 & 0 & 0 \\ u_4 & 1 & 0 & 1 & 0 & 1 & 0 \\ u_5 & 0 & 0 & 0 & 1 & 0 & 1 \\ u_6 & 0 & 1 & 0 & 0 & 1 & 0 \end{array}$$

$$A_{G_2} = \begin{array}{c|cccccc} & V_6 & V_3 & V_1 & V_5 & V_1 & V_2 \\ \hline V_6 & 0 & 1 & 0 & 1 & 0 & 0 \\ V_3 & 1 & 0 & 1 & 0 & 0 & 1 \\ V_4 & 0 & 1 & 0 & 1 & 0 & 0 \\ V_5 & 1 & 0 & 1 & 0 & 1 & 0 \\ V_1 & 0 & 0 & 0 & 1 & 0 & 1 \\ V_2 & 0 & 1 & 0 & 0 & 1 & 0 \end{array}$$

Here,

$A_{G_1} = A_{G_2}$

Therefore the graph G_1 & G_2 are isomorphic and the function f is an isomorphism.

Article 8.3 (Completed)

Article 8.4

Connectivity

→ **Path / Walk**

It is a sequence of edges from one vertex u to any vertex v.

Let G be an undirected graph and n be any +ve integer. A path of length n in the graph G from the vertex u to vertex v is a sequence of n edges $(e_1, e_2, e_3 \dots e_n)$ such that $e_1 = (u_1, x_1)$

$e_2 = (x_1, x_2)$

$e_3 = (x_2, x_3)$

$\vdots$

$e_n = (x_n, v)$

If the graph G is a simple graph then the path or ~~graph~~ walk is represented in vertex sequence but if it is a pseudo graph then the path must be ~~repres~~ represented in edge sequence form.

Length of a Path

The length of a path is n if in that path n no. of edges are covered.

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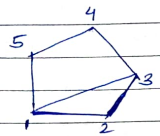
Circuit / Cycle

The path in which the starting vertex & ending vertex are same is called a circuit or cycle.



1 — 4

1, 2, 3, 4 (1-2-3-4) is a path //

1-2-3-1 is a circuit //

Simple Circuit / Simple Path

It is a path / circuit in which non of the edge will be repeated.

Ex

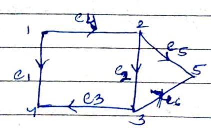
1-2-3-4 → simple path

1-2-3-1-5-4 → simple path

1-2-3-1-2-3-4 → not simple path.

NOTE :-

In case of directed graph we can also define in similar manner but only difference is in the direction of the graph we can proceed.



(G)

In the directed Graph G the path from vertex 1-4 is

1-4

1-2-3-4

In this graph there is no path from vertex 5-3.

Connected Graph

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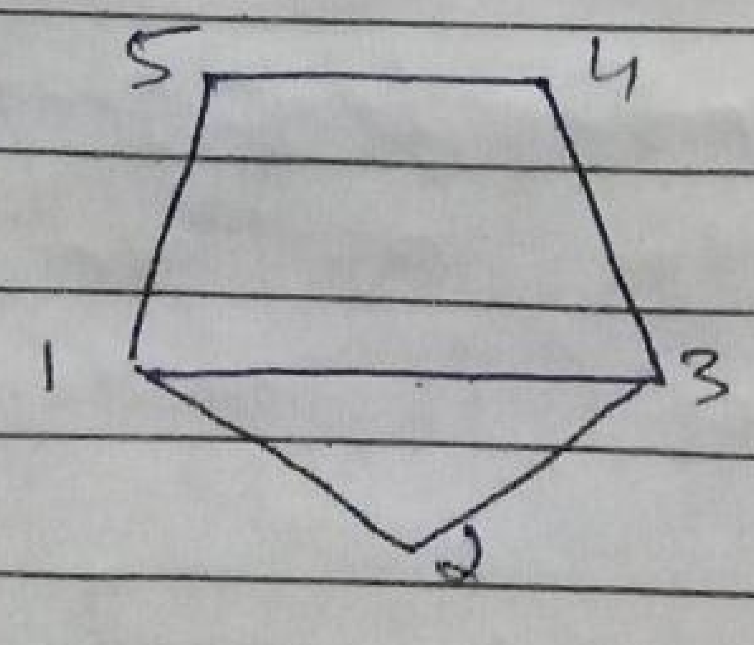
8.4 Connectivity

Path/Walk of graph: Let G be an undirected graph an n be any positive integer. A path of length n in the graph G from the vertex u to vertex v is a sequence of n edges $(e_1, e_2, \ldots, e_n)$ such that $e_1 = (u, x_1)$, $e_2 = (x_1, x_2)$ ---- $e_n = (x_n, v)$.

If the graph G is a simple graph then the path/walk is represented in terms of vertex sequence, but if it is a psudo graph then the path must be represented in edge sequence form.

Length of a path: It is $= n$ if in that path n no. of edges are covered.

Circuit/Cycle: The path in which the starting vertex and ending vertex are same is called a circuit or cycle.



1 - 2 - 3 - 4 (path)

1 - 2 - 3 - 1 (cycle)

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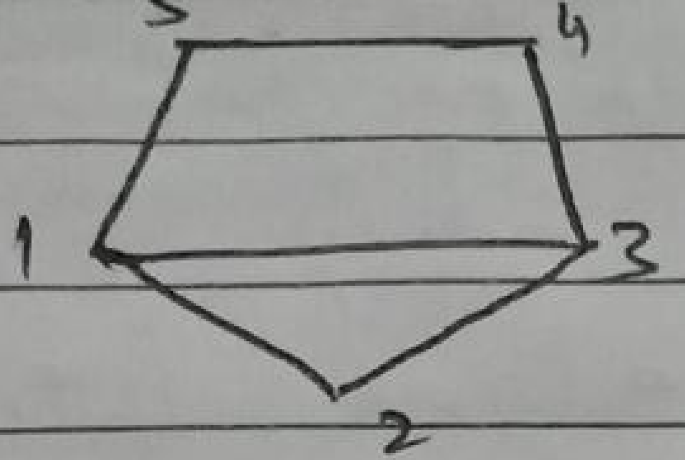
Simple ckt/path : It is a path or circuit in which none of the edge will be repeated.

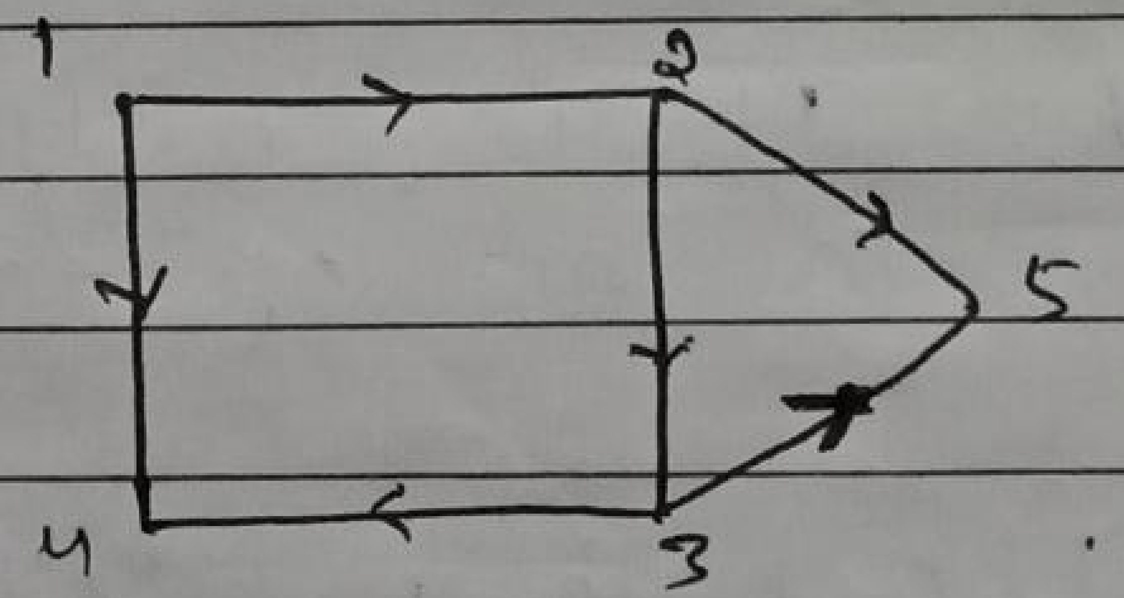
Ex:



1 - 2 - 3 - 4 (simple path)
1 - 2 - 3 - 1 - 5 - 4 - 5 (")
1 - 2 - 3 - 1 - 2 - 3 - 4 (not simple path).

NOTE: In case of directed graph we can also define in similar manner but only difference is in the dir^n of graph we can proceed.

Ex:

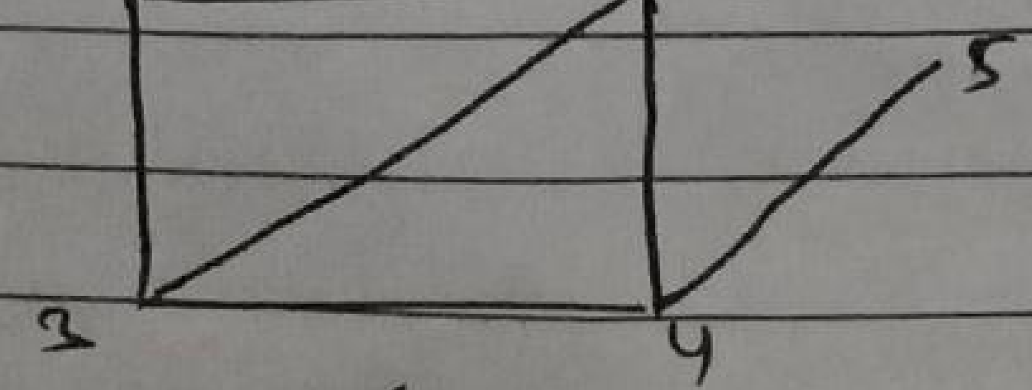


In the directed graph G:
(i) 1 - 4 :
1 - 2 - 3 - 4

But, in this graph there is no path from vertex 5 to vertex 3.

Connected graph : An undirected graph in which there is a path b/w every distinct pair of vertices is called a connected graph.

Ex.



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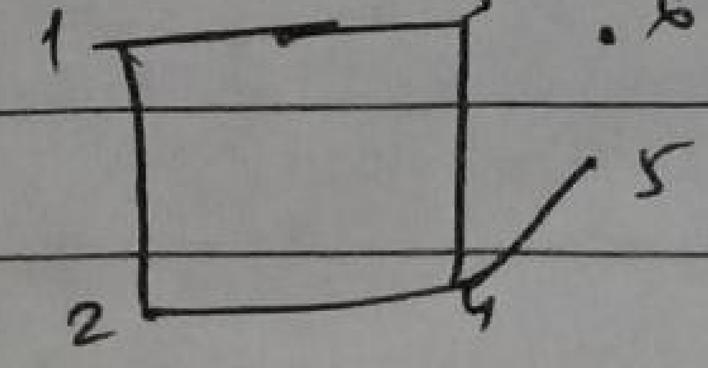

X

XTRA class :

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Thm 1 :

There is a simple path b/w every pair of distinct vertices of a connected undirected graph.

Pf : Let $G = (V, E)$ be a connected undirected graph where $V = \{x_1, x_2, \ldots, x_n\}$ is the set of all vertices.

We have to show that in b/w every pair of vertices there is a simple path.

$\because$ the graph is connected so, in b/w every pair of vertices there is a path. If this path is simple then the thm is proved.

If this path is not simple then some of the edges must be repeated. If we neglect or omit the end vertices of these repeated edges the we also get another path from ~~the~~ i^{th} vertex to j^{th} vertex.
As all the repeated edges are omitted so simple ~~graph~~ path.
Hence it is clear that in a connected undirected graph in b/w every pair of distinct vertices there is a simple path.

Connected component of a graph

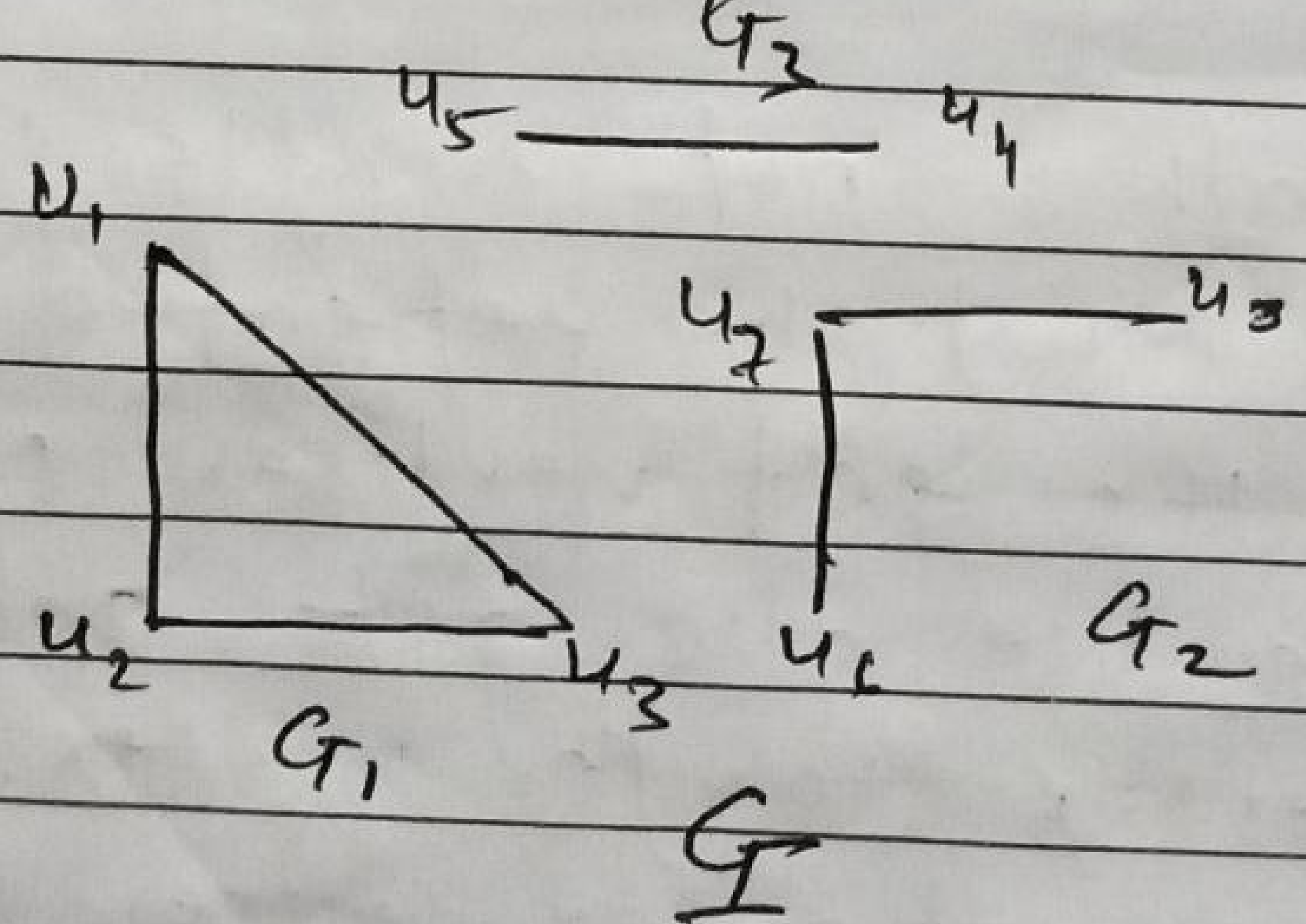
Let $G = (V, E)$ be an undirected graph, then any maximal sub graph of G which is itself connected is called a connected component of the graph G.

In other words, let $G = (V, E)$ be any graph. A subgraph H ~~of~~ is called a connected component of G, it satisfies two conditions:

(i) H is connected.

(ii) there is no sub graph of G which is itself connected and a supergraph of H.

Ex.



Here G_1, G_2 & G_3 are the connected components of the graph G.

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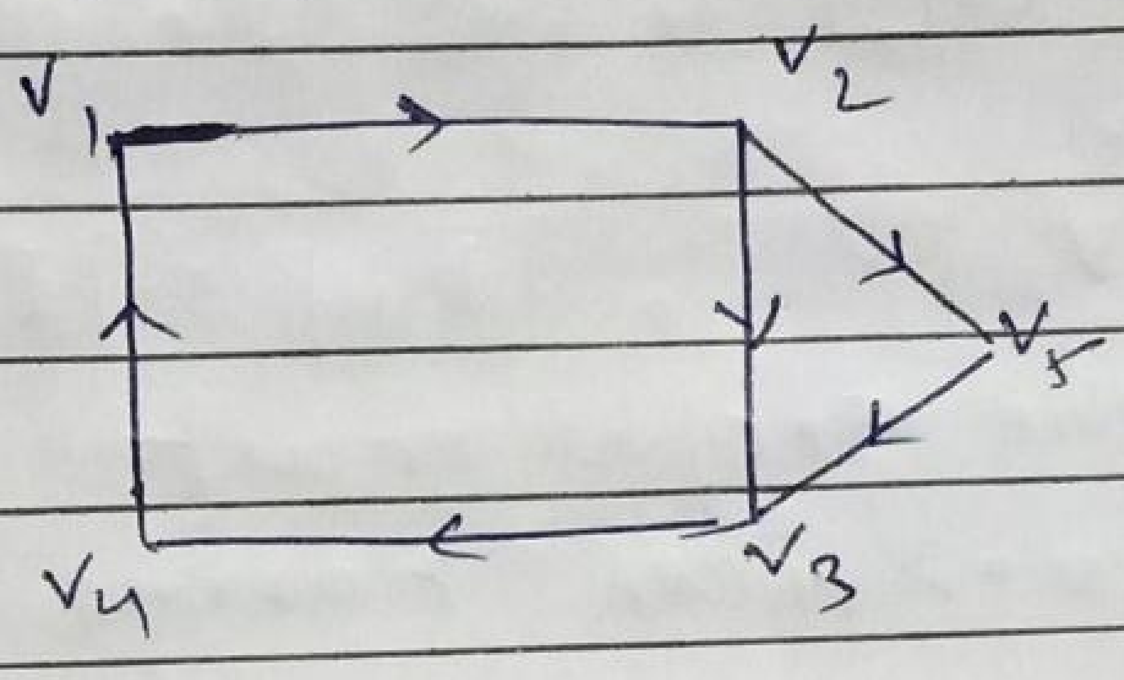
Strongly Connected Graph :

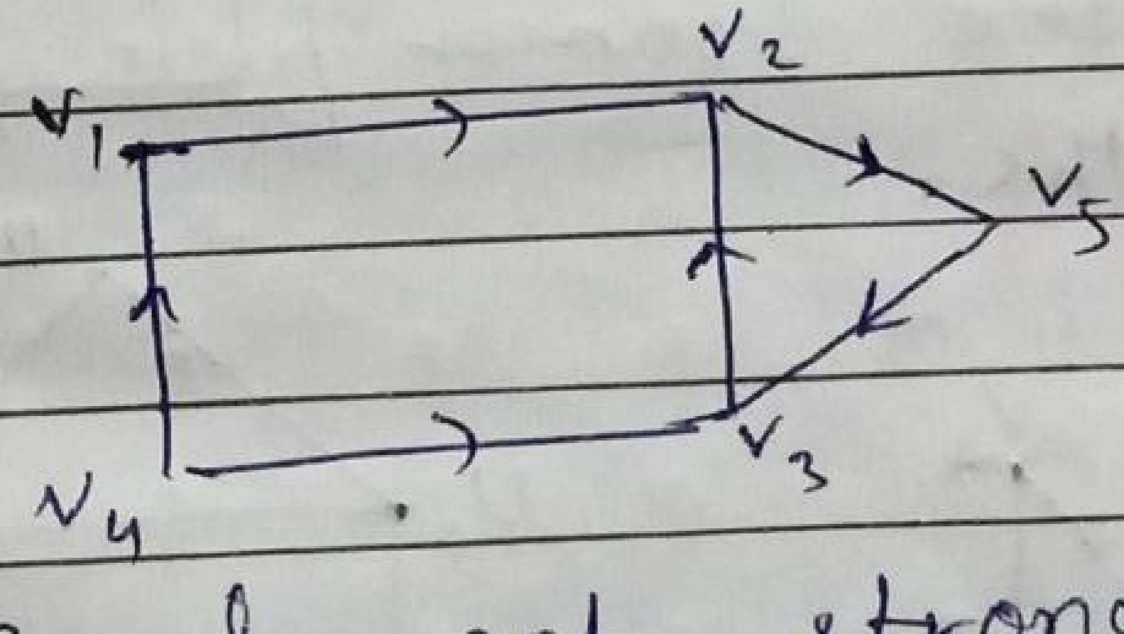
A directed graph $G = (V, E)$ is said to be strongly connected graph if there is a path b/w every pair of distinct vertices.



G_1

Weakly Connected Graph:

A directed graph $G = (V, E)$ is said to be weakly connected graph if this graph is connected without considering the dir^n.



G_1

Here, G_2 is not strongly connected becoz from v_2 to v_1 there is no path (& many other) but this is a weakly connected graph.

NOTE: Every strongly connected graph is a weakly connected graph but the converse is not true always.

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Method-3 (Isomorphic graphs).

Let,

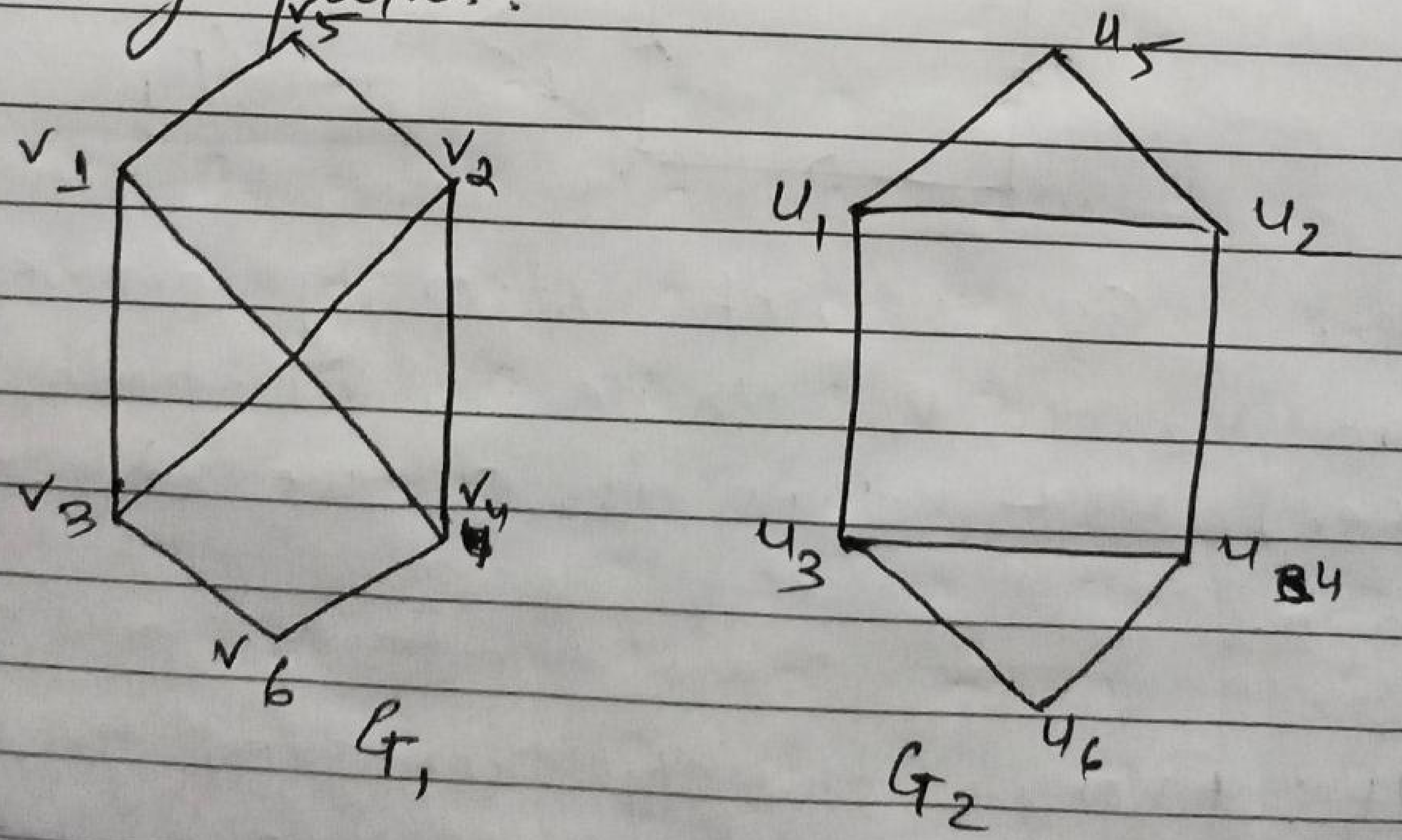
$G_1 = (V_1, E_1)$ & $G_2 = (V_2, E_2)$ be two graphs. These two graphs will be isomorphic if they will satisfy the following cond^{n}s :

(1) $|V_1| = |V_2|$, $|E_1| = |E_2|$

(2) Both the graphs must contain equal no. of vertices having same degree.

(3) Both the graphs must contain same no. of simple ckts of ~~with~~ same length.

Ex. Verify whether the graphs given by G_1 & G_2 are Isomorphic or not by using paths.



→ 1st $|V_1| = |V_2| = 6$

$|E_1| = 8 = |E_2|$

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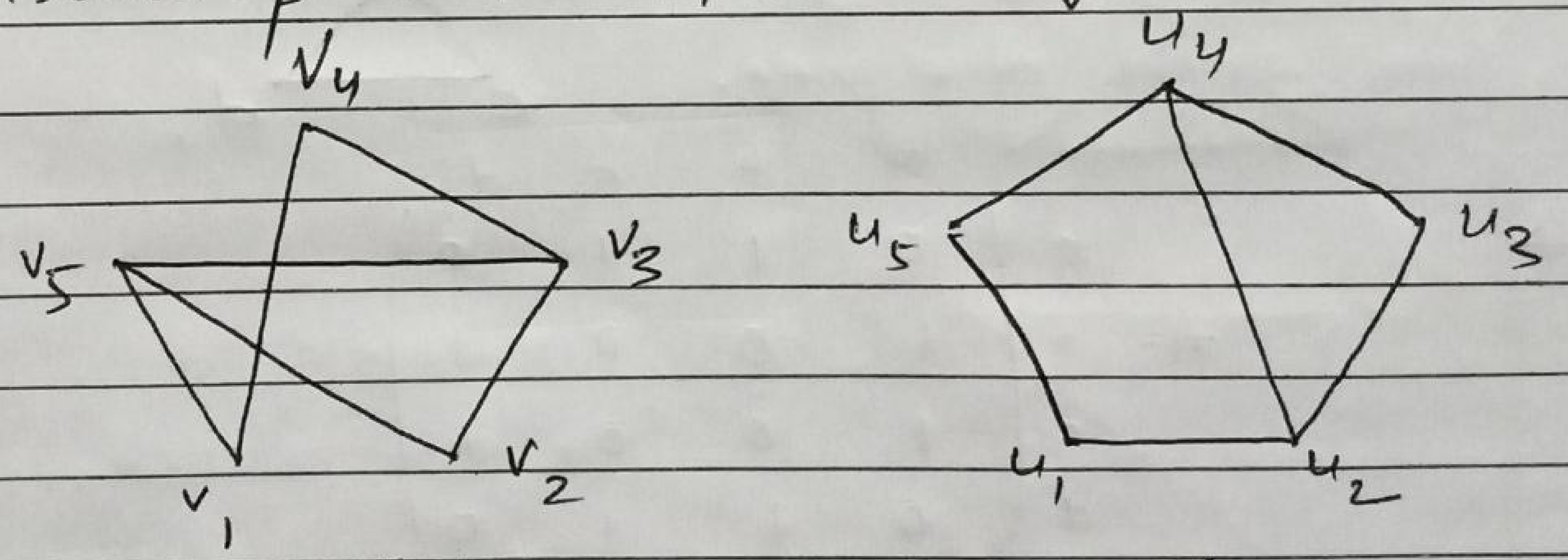
2nd deg (v_1) | deg (u_1)
(v_2) | (u_2)
(v_3) | (u_3)
(v_4) | (u_4)
(v_5) | (u_5)
(v_6) | (u_6)

3rd. In G_2 there is a simple ckt of length 3 (u_1, u_2, u_5) but in G_1 there no simple ckt of length 3. Hence, these two graphs are not isomorphic.

Q. Verify whether the following graphs are isomorphic or not.



G_1 G_2

→ 1st $|V_1| = |V_2| = 5$, $|E_1| = |E_2| = 6$.

2nd. $d(v_1) = 2$ | $d(u_1) = 2$
$d(v_2) = 2$ | $(u_2) = 3$
$d(v_3) = 3$ | $(u_3) = 2$
$(v_4) = 2$ | $(u_4) = 3$
$(v_5) = 3$ | $(u_5) = 2$
~~(v_6) =~~ | ~~(u_6)~~ =

3rd: $G_1 = 3, 4, 5$ (ckt length) G_2 : 3, 4, 5 (ckt lengths)

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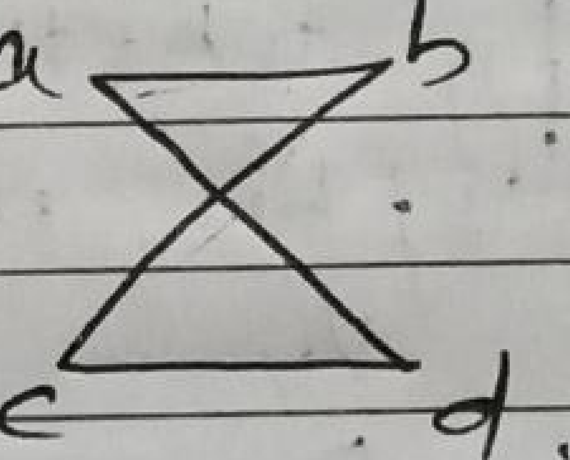
Thm 02:

Let $G = (V, E)$ be a graph whose adjacency matrix is A in which seqnce of vertices are $\{v_1, v_2, \ldots, v_n\}$
The no. of paths from v_i to v_j in the graph G with length k is = the i^{th} row and j^{th} elm (cell) of ~~of~~ the matrix A^k.

(v_i = vertex i)

// Q. How many paths of length 4 are there from V(A) to vert.(B) in the following graph:



$$A = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix} \end{array}$$

$$A^2 = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 0 & 0 & 2 \\ 0 & 2 & 2 & 0 \\ 0 & 2 & 2 & 0 \\ 2 & 0 & 0 & 2 \end{pmatrix}$$

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$$A^4 = \begin{pmatrix} 2 & 0 & 0 & 2 \\ 0 & 2 & 2 & 0 \\ 0 & 2 & 2 & 0 \\ 2 & 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} 2 & 0 & 0 & 2 \\ 0 & 2 & 2 & 0 \\ 0 & 2 & 2 & 0 \\ 2 & 0 & 0 & 2 \end{pmatrix}$$

$$= \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{pmatrix} 8 & 0 & 0 & 8 \\ 0 & 8 & 8 & 0 \\ 0 & 8 & 8 & 8 \\ 8 & 0 & 0 & 8 \end{pmatrix} \end{array}$$

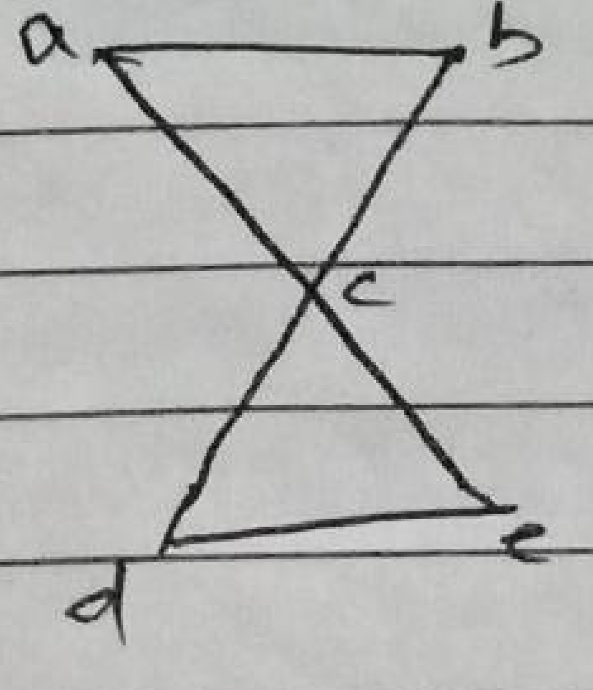
So, in the graph G there are 8 no. of path from vert(a) to vert(d) whose length is 4.

Euler Path

Let $G=(V,E)$ be a graph. Euler path ~~IF~~ of G is a simple path that contains all the edges of graph G

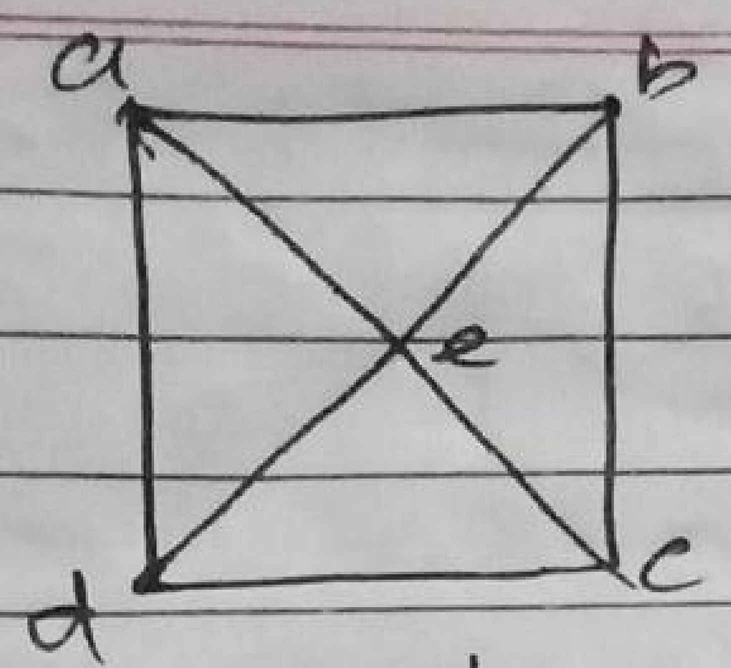
Euler Ckt

Euler ckt of a graph G is a simple ckt which contains all the edges of graph G.

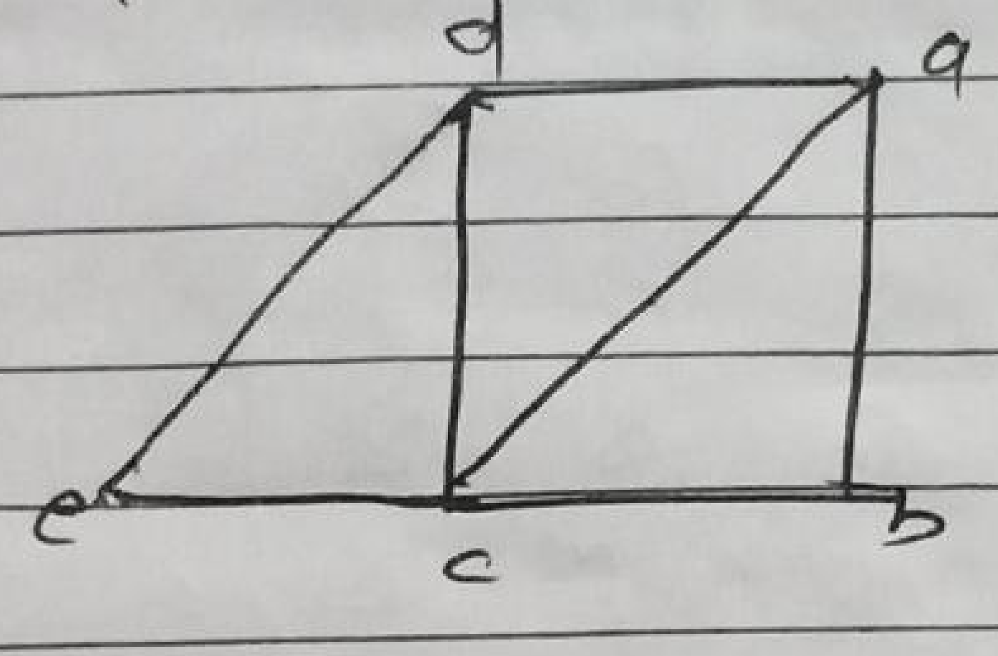


a – c – e – d – c – b – a (Euler ckt)

But it has not Euler path.



It has not Euler path &
no Euler ckt.



a – b – c – a – d – c – e – d
(Euler pat).

But it has no Euler ckt.

<u>Th^m</u> 1 :

A connected multigraph with at least two vertices has an Euler ckt if and only if each of its vertex has an even degree.

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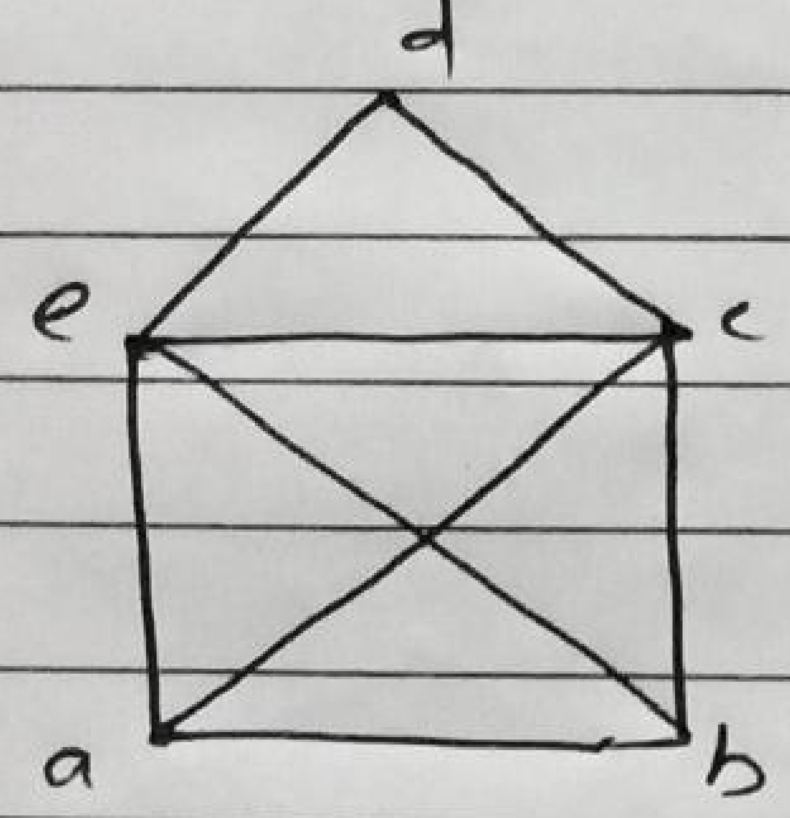
Th^{m} 2:

Hamiltonian path

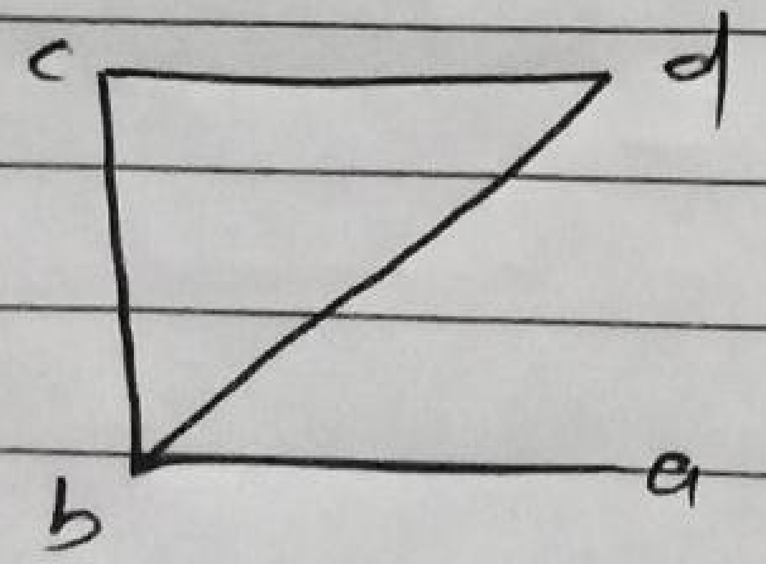
A simple path in a graph G is said to be a Hamitonian path if it covers all the vertices of the graph exactly once.

Hamiltonian ckt

It is a simple ckt in a graph which passes th. all vertices of graph exactly once (except the starting vertex).



a – b – e – d – e (H. P.)
a – b – c – d – e – a (H. ckt)



a – b – c – d (H.P.)
NO. H. ckt.

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Theorms :-

Dirac's Theorem

If G is a simple graph with n vertices where n is ≥ 3 such that the degree of each vertex of the graph is atleast $n/2$ then the graph has a hamilton circuit.

ORE's Theorem

If G be a simple graph with n vertices where $n \geq 3$ and the $deg(u) + deg(v) \geq n$, where u & v are non-adjacent vertices then the graph G has a hamilton circuit

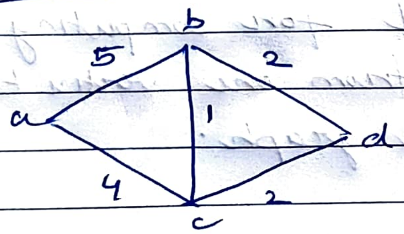
Shortest Path Problem

Weighted graph

→ Weight of an Edge

~~Given~~ If is a graph some positive quantity is assign with an edge whose end vertices are a & b, then this +ve quantity is called the weight of the edge joining a & b.

For Example



weigth of a to b is 5

→ Weighted graph

~~The~~ It is the graph in which every edge is attached with some weight. For example above graph.

→ # Length of ~~a~~ Path

It is the sum of the weights of the edges associated with these path.

For example

In the previous weighted graph the length of the path a – b – c – d is

$5 + 1 + 2 = 8$

→ Shortest Path

In a graph in between any pair of vertices there are several paths. But the length of these paths are not same. So, the shortest path in betwn a pair of vertices u & v is the path whose length is minimum in comparision to the length of other path in betwn u & v.

Determination of shortest path by using ~~Dijkstra's~~ Dijkustra's Method

It is one of the method for computing or determining the shortest path from one vertex to another vertex of a weighted graph.

Suppose,

$G = (V, E)$ be the weighted graph.

where $V = \{a, b, c, d, \ldots, z\}$

Our objective is to find the shortest path from vertex a to vertex z.

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Step 1

Let $S_0 = \{a\}$ & $S_1 = V - \{a\} = \{b, c, d, \ldots, z\}$

Let $L_1(a) = 0$

$L_1(b) = \min\{L_1(a) + w(a,b), L_1(a) + \infty\}$

when there is no path betw^n a to b.

If there is no edge from a to b then it is ∞.

$L_1(c) = \{L_1(a) + w(a,c)\}$

$L_1(d) = \{L_1(a) + w(a,d)\}$

$\vdots$

$L_1(z) = \{L_1(z) + w(a,z)\}$

After finding it, from $L_1(b), L_1(c), \ldots, L_1(z)$ find the minimum one.
Let, the min. one be $L_1(b)$

Step 2

Now,

$S_0' = \{a, b\}$ $S_1' = S_1 - \{b\} = \{c, d, e, \ldots, z\}$

(S_0')

minimum of

$L_2(c) = \{L_1(b) + w(b,c), w(a,c)\}$

$L_2(d) = \{L_1(b) + w(b,d), w(a,d)\}$

$\vdots$

$L_2(z) = L_1(b) + w(b,z), w(a,d)\}$

Select the minimum from $L_2(c), L_2(d) \ldots L_2(z)$
Let $L_2(d)$ is min.

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Step 2

$S_0' = \{a, b\}$, $S_1' = \{c, d, e, \dots z\}$

$L_2(c) = \min\{\omega(a,c), L_1(b) + \omega(b,c)\}$
$L_2(d) = \min\{\omega(a,d), L_1(b) + \omega(b,d)\}$
$\vdots$
$L_2(z) = \min\{\omega(a,z), L_1(b) + \omega(b,z)\}$

Let minimum is $L_2(d)$

Step 3

$S_0'' = \{a, b, d\}$, $S_1'' = \{c, e, f, \dots, z\}$

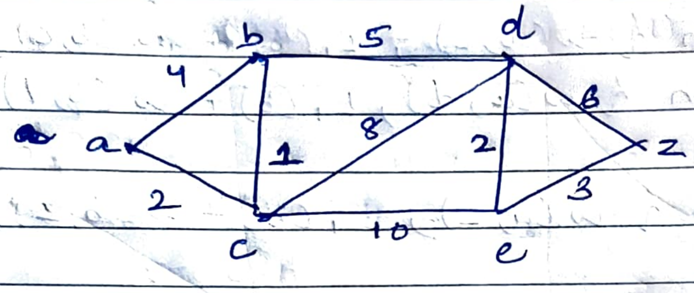
$L_3(c) = \min\{\omega(a,c), \omega(b,c), L_2(d) + \omega(d,c)\}$

$L_3(e) = \min\{\omega(a,e), \omega(b,e), L_2(d) + \omega(d,e)\}$
$\vdots$

$L_3(z) = \min\{\omega(a,z), \omega(b,z), L_2(d) + \omega(d,z)\}$

Continuing in same manner we will get S_0 & S_1 in similar manner.
whenever, the vertex z will enter in S_0, the process is terminated and the sequence in which the vertices are present in S_0 will give the required shortest path.

Q) Find the shortest path from the vertex a to vertex z using dijkustra's algorithm of the following weighted graph.



Ans

Step 1

Let $S_0 = \{a\}$ $\quad$ $S_1 = \{b, c, d, e, z\}$

$L_1(a) = 0$
$L_1(b) = w(a,b) = 4$
$L_1(c) = w(a,c) = 2$
$L_1(d) = w(a,d) = \infty$
$L_1(e) = w(a,e) = \infty$
$L_1(z) = w(a,z) = \infty$

Here, $L_1(c)$ is minimum.

Step 2

Let $S_0' = \{a, c\}$ $\quad$ $S_1' = \{b, d, e, z\}$

$L_2(b) = \min\{w(a,b), L_1(c) + w(c,b)\}$
$= \min(4, 2+1)$
$= 3$

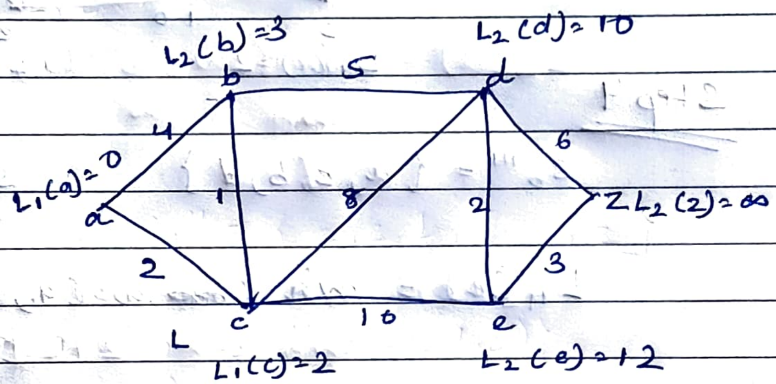
$L_2(d) = \min\{w(a,d), L_1(c) + w(c,d)\}$
$= \min(\infty, 2+8)$
$= 10$

$$L_2(e) = \min\{w(a,e),\ L_1(c) + w(c,e)\}$$
$$= \min(\infty,\ 2+10)$$
$$= 12$$

$$L_2(z) = \min\{w(a,z),\ L_1(c) + w(c,z)\}$$
$$= \min\{\infty, \infty\}$$
$$= \infty$$

Here, $L_2(b)$ is minimum.

Step3



$S_0'' = \{a, c, b\}$ $\quad$ $S_1'' = \{d, e, z\}$

$$L_3(d) = \min\{w(a,d),\ L_1(c) + w(c,d),\ L_2(b) + w(b,d)\}$$

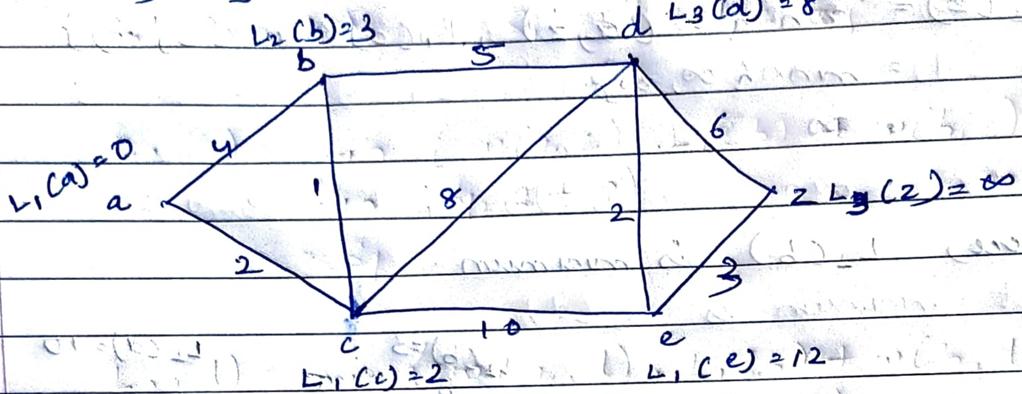
$$= \min\{\infty,\ 2+8,\ 3+5\}$$
$$= 8$$

$$L_3(e) = \min\{w(a,e),\ L_1(c) + w(c,e),\ L_2(b) + w(b,e)\}$$

$$= \min\{\infty,\ 2+10,\ 3+\infty\}$$
$$= \cancel{\min\{12\}}\ 12$$

$L_3(z) = \min\{ w(a,z), L_1(c) + w(c,z), L_2(b) + w(b,z)\}$
$= \infty$

Here, $L_3(d)$ is minimum.



Step 4

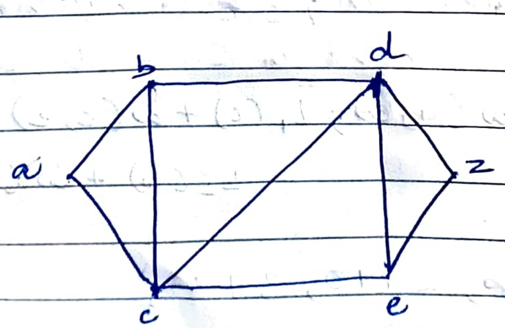
$S_0''' = \{a, c, b, d\}$ $S_1''' = \{e, z\}$

$L_4(e) = \min\{ w(a,e), L_1(c) + w(c,e), L_3(d) + w(d,e)\}$

$= \min\{\infty, 2+12, 8+2\}$
$= 10$

$L_4(z) = \min\{ w(a,z), L_3(d) + w(d,z)\}$
$= \min(\infty, 8+6)$
$= 14$

Here, $L_4(e)$ is minimum.



Step 5

$S_0^{IV} = \{a, c, b, d, e\}$ $S_1^{IV} = \{z\}$

$L_5(z) = \min(\omega(a, z), L_3(d) + \omega(d, z), L_4(e) + \omega(e, z)\}$

$= \min(\infty, 8 + 6, 10 + 3)$

$= 13$

Step 6

$S_0^{V} = \{a, c, b, d, e, z\}$

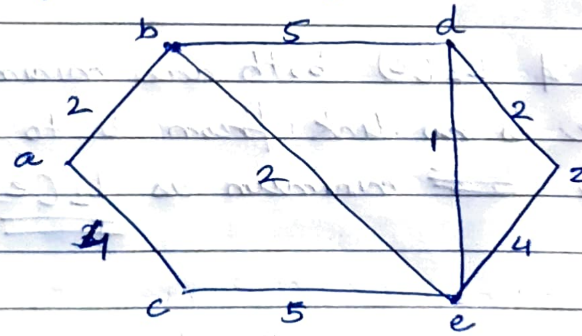
So, the shortest path is

a - c - b - d - e - z

= 2 + 1 + 5 + 2 + 3

= 13

(8) Find the shortest path from vertex a to z in the following weighted graph.



Step 1

$S_0 = \{a\}$ $S_1 = \{b, c, d, e, z\}$

$L_1(a) = 0$ $L_1(d) = \infty$

$L_1(b) = 2$ $L_1(e) = \infty$

$L_1(c) = 3$ $L_1(z) = \infty$

Here,

$L_1(b)$ is minimum.

Step 2

$S_0' = \{a, b\}$ $\quad S_1' = \{c, d, e, z\}$

$L_2(c) = \min\{w(a,c), L_1(b) + w(b,c)\}$
$= \min(4, 2+\infty)$
$= 4$

$L_2(d) = \min\{w(a,d), L_1(b) + w(b,d)\}$
$= \min(\infty, 2+5)$
$= 7$

$L_2(e) = \min\{w(a,e), L_1(b) + w(b,e)\}$
$= \min\{\infty, 2+2\}$
$= 4$

$L_2(z) = \infty$

Here, $L_2(c)$ & $L_2(e)$ both are minimum but there is no link from b to c, so we will take minimum as $L_2(e)$

Step 3

$S_0'' = \{a, b, e\}$ $\quad S_1'' = \{\cancel{c}\, d, z\}$

$L_3(d) = \min\{w(a,d), L_1(b) + w(b,d), L_2(e) + w(e,d)\}$
$= \min\{\infty, 2+5, 4+1\}$
$= 5$

$L_3(z) = \min\{w(a,z), L_1(b) + w(b,z), L_2(e) + w(e,z)\}$
$= \min\{\infty, 2+\infty, 4+4\}$
$= 8$

Here, minimum is $L_3(d)$

Step 4

$S_0''' = \{a, b, e, d\}$ $\quad S_1''' = \{z\}$

$L_4(z) = \min\{w(a,z), L_1(b) + w(b,z), L_2(e) + w(e,z), L_3(d) + w(d,z)\}$
$= \min\{\infty, 2+\infty, 4+4, 5+\infty\}$
$= 8$

Here minimum is $L_4(z)$

Now,

$S_0'''' = \{a, b, e, d, z\}$

So, the shortest path is

a - b - e - d - z
$= 2+2+5+1+2$
$= 12$