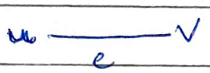
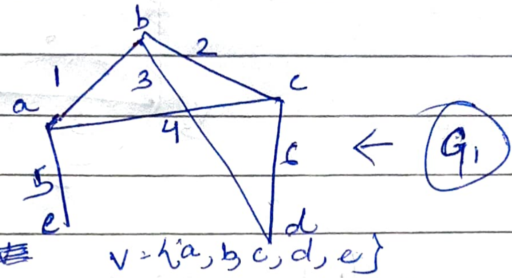
Graph

A graph G ~~defined as~~ is the set of all vertices and edges denoted by $G = (V, E)$ where V is a non-empty set which contains all the vertices of the graph and E is the set of all the edges of the graph.

$G(V, E)$





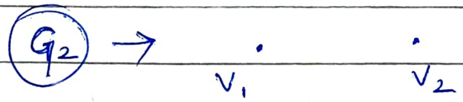
$V = \{a, b, c, d, e\}$
$E = \{1, 2, 3, 4, 5, 6\}$

Finite Graph

If the number of vertices is finite it is called finite graph.

In-finite Graph

If the number of vertices is infinite it is called an infinite graph.



$V = \{v_1, v_2\}$
$E = \{\ \ \}$

End-Vertices of an edge

Every edge must start from one vertex and ends on another vertex.
If an edge starts from vertex u & ends at v. Then u & v are called the end vertices of an edge is u & v.
In G_1 → End vertices of 1 is a & b.

Graph

A graph G ~~defined as~~ is the set of all vertices and edges denoted by $G = (V, e)$ where V is a non-empty set which contains all the vertices of the graph and e is the set of all the edges of the graph.

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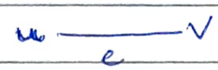

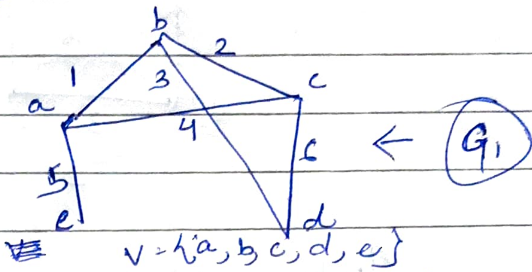

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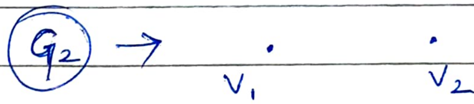

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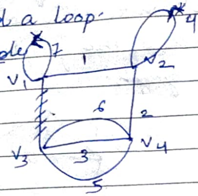
Loop

~~The edge whose~~

If the end vertices of an edge are same it is called a loop.

For example



non-directed graph

In the above diagram 4 & 7 are loop

Parallel edges / Multiple edges

If ~~in a pair of~~ there are number of edges in betw^n a pair of vertices then these are called parallel edges.
In other words, if a number of edges have same end vertices then these are called parallel edges.

For Ex

In the above diagram edge no. 3, 5 & 6 are parallel edges.

Finite Graph.

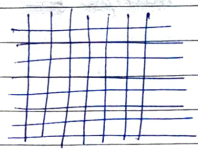
A Graph $G = (V, e)$ is called a finite graph if V contains finite no. of element that means the no. of vertices of the graph is finite.

Ex The above diagram is finite graph

Infinite Graph

The graph $G = (V, e)$ is called an infinite graph if it contains infinite no. of vertices.

For ex



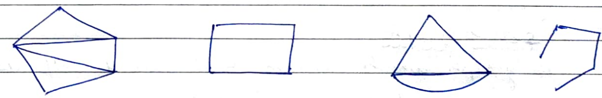
infinte no. of horizontal lines
infinite no. of vertical lines.
point of intersection are treated as vertices
So, it has infinite no. of vertices

A graph $G = (V, e)$ contains infinite graph if V contains infinite no. of vertices.

Simple Graph

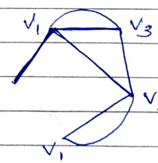
The graph in which there is no parallel edges as well as loop is called a simple graph.

Ex



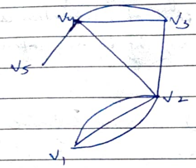
Multiplegraph (Multi-Graph)

The graph in which there is no loop but must contain parallel edges are present is called a Multiple Graph.



Multiplicity of a Multiple edge

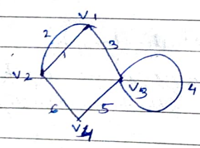
If there are m no. of edges which are associated with the same end vertices. u & v then we say that the edge {u,v} is a mutiple ~~moo~~ edge with multiplicity m.



In the above diagram {V1,V2} is a multiple edge with multiplicity 3. & {V3, V4} is a multiple edge with multiplicity 2.

Pseudo graph

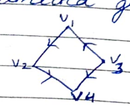
A graph which contain both loop as well as parallel edges is called a pseudo graph.



Directed Graph

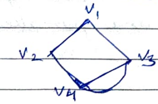
The graph in which all the edges are directed is called a directed graph.

eg



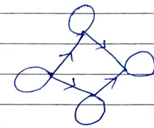
Undirected Graph

The graph in which all the edges are undirected.



Mixed Graph

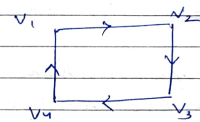
It is a graph in which some of the edges are directed and rest are undirected.



Loops are undirected

Simple directed Graph

It is simple graph in which all the edges are directed.

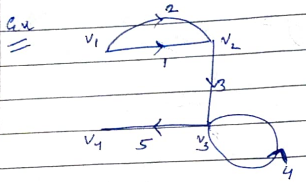


Date 19/01/22

Directed Multi-graph

It is a graph in which all edges are directed and it also contains loops and parallel edges.

Ex



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Influence Graph

It is a directed graph in which the vertices are the persons, there is an edge from vertex u to vertex v if, the person u is influenced to the person v.

In this graph there is no self loop and no parallel edges.

Collaboration Graph

It is an undirected graph in which the vertices are the persons. If u & v are ~~the~~ 2 vertices, then there is an edge from u to v; if u & v work in same project.

In this graph there is no self loop and no parallel edges.

Acquitancy Graph

It is an undirected graph; in which the vertices are persons, there is an edge from vertex u to vertex v. If the person u & v are acquiented with each other.

In this graph there is no loop as well as no parallel edges.

Article $\boxed{8.2}$

Graph Terminology & special types of graphs.

Adjacent ~~edges~~ vertices / Neighbour Vertices

If there is an edge from the vertex u to v. Then u & v are called the adjacent vertices of this edge.

Degree of ~~the~~ a vertex

The degree of a vertex in an undirected graph is the number of edges incident on that vertex.

The degree of vertex V is denoted deg(V)

Isolated Vertex:

The vertex whose degree is 0 is called an Isolated vertex

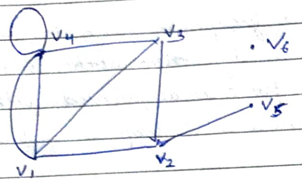
Pendant Vertex

The vertex whose degree is 1 is called a Pendant vertex

NOTE

If there is a loop at the vertex then the degree will the taken as 2.

Q Find the degree of all vertices of the following graph.



$v_1 = 4$
$v_2 = 3$
$v_3 = 3$
$v_4 = 5$
$v_5 = 1$
$v_6 = 0$

Theorem-1 (Handshaking Theorem)

Let $G = (V, E)$ is an undirected graph with V no. of vertices, then

$$\sum_{v \in V} \deg(v) = 2e$$

It is given that $G = (V, E)$ is a graph in which E contains e no. of edges. But it is clear that every edge is incident on 2 vertices. So, for that edge the degree of the vertices will be 2. Hence, it is clear that each edge contributes to the sum of the degree of the vertices as each edge is incident on 2 no. of vertices. Therefore the sum of the degree of the vertices is twice of the total edges.

Hence,

$$\sum_{v \in V} \deg(v) = 2e$$

Qn

In a graph there are 15 vertices and degree of each vertex is 4.
How many edges are there in the graph?

Ans

$$15 \times 4 = 2e$$
$$60 = 2e$$
$$e = 30$$

Total no. of edges is 30.

Theorem 2

In the undirected graph the number of odd degree vertices is always even.

It is given that $G = (V, E)$ is a graph, let $|V| = n$ & $|E| = k$.
According to Handshaking Theorem,

$$\sum_{v \in V} \deg(v) = 2e$$

$\Rightarrow \sum \deg(\text{even degree vertices}) + \sum \deg(\text{odd degree vertices}) = 2k$. — (1)

But it is clear that,
$\sum \deg(\text{even degree vertices})$ is always even.

So, it is clear that,

$\sum$ deg (odd degree vertices) = even.

But we know that,

The sum of even number of odd number is even.

Hence, the total no. of odd degree vertices is even.