Currying

Motivation

Look again at the summation functions:

```
def sumInts(a: Int, b: Int)       = sum(x => x, a, b)
def sumCubes(a: Int, b: Int)      = sum(x => x * x * x, a, b)
def sumFactorials(a: Int, b: Int) = sum(fact, a, b)
```

Question

Note that `a` and `b` get passed unchanged from `sumInts` and `sumCubes` into `sum`.

Can we be even shorter by getting rid of these parameters?

Functions Returning Functions

Let's rewrite `sum` as follows.

```
def sum(f: Int => Int): (Int, Int) => Int = {
  def sumF(a: Int, b: Int): Int =
    if (a > b) 0
    else f(a) + sumF(a + 1, b)
  sumF
}
```

`sum` is now a function that returns another function.

The returned function `sumF` applies the given function parameter `f` and sums the results.

Stepwise Applications

We can then define:

```
def sumInts       = sum(x => x)
def sumCubes      = sum(x => x * x * x)
def sumFactorials = sum(fact)
```

These functions can in turn be applied like any other function:

```
sumCubes(1, 10) + sumFactorials(10, 20)
```

Consecutive Stepwise Applications

In the previous example, can we avoid the `sumInts`, `sumCubes`, ... middlemen?

Of course:

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- `sum(cube)` is therefore equivalent to `sumCubes`.
- This function is next applied to the arguments `(1, 10)`.

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Generally, function application associates to the left:

```
sum(cube)(1, 10)   ==   (sum (cube)) (1, 10)
```

Multiple Parameter Lists

The definition of functions that return functions is so useful in functional programming that there is a special syntax for it in Scala.

For example, the following definition of `sum` is equivalent to the one with the nested `sumF` function, but shorter:

```
def sum(f: Int => Int)(a: Int, b: Int): Int =
  if (a > b) 0 else f(a) + sum(f)(a + 1, b)
```

Syntatic sugar: compare it with the following defintion
def sum(f: Int => Int): (Int, Int) => Int

Expansion of Multiple Parameter Lists

In general, a definition of a function with multiple parameter lists

$$\mathrm{def}\ f(args_1)...(args_n) = E$$

where $n > 1$, is equivalent to

$$\mathrm{def}\ f(args_1)...(args_{n-1}) = \{\mathrm{def}\ g(args_n) = E; g\}$$

where g is a fresh identifier. Or for short:

$$\mathrm{def}\ f(args_1)...(args_{n-1}) = (args_n \Rightarrow E)$$

Expansion of Multiple Parameter Lists (2)

By repeating the process *n* times

$$\texttt{def f(args}_1\texttt{)...(args}_{n-1}\texttt{)(args}_n\texttt{)} = \texttt{E}$$

looks similar to the idea of closure function

is shown to be equivalent to

$$\texttt{def f} = \texttt{(args}_1 \Rightarrow \texttt{(args}_2 \Rightarrow \texttt{...(args}_n \Rightarrow \texttt{E)...))}$$

This style of definition and function application is called *currying*, named for its instigator, Haskell Brooks Curry (1900-1982), a twentieth century logician.

In fact, the idea goes back even further to Schönfinkel and Frege, but the term "currying" has stuck.

More Function Types

Question: Given,

```
def sum(f: Int => Int)(a: Int, b: Int): Int = ...
```

What is the type of `sum` ?

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```scala
def sum(f: Int => Int)(a: Int, b: Int): Int = ...
```

What is the type of `sum` ?

Answer:

```scala
(Int => Int) => (Int, Int) => Int
```

Note that functional types associate to the right. That is to say that

```scala
Int => Int => Int
```

is equivalent to

```scala
Int => (Int => Int)
```

Exercise

The `sum` function uses linear recursion. Write a tail-recursive version by replacing the `???`s.

```
def sum(f: Int => Int)(a: Int, b: Int): Int = {
  def loop(a: Int, acc: Int): Int = {
    if (???) ???
    else loop(???, ???)
  }
  loop(???, ???)
}
```