

Homework 11

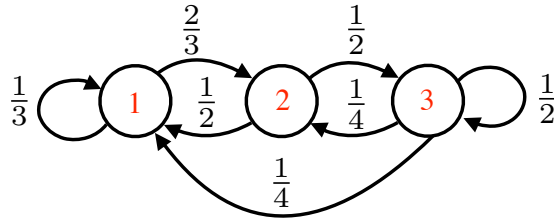
Due: 11:00pm, Wednesday, May 1 via Gradescope

Reading: Chapter 11.

Videos: 11.1 - 11.4

Problem 11.1 (Video 11.1 - 11.4, Lecture Problem)

Consider the following Markov chain with initial state probability vector $\underline{p}_0 = \begin{bmatrix} 1/5 \\ 2/5 \\ 2/5 \end{bmatrix}$.



- Write down the state transition matrix \mathbf{P} .
- Determine the 2-step state transition matrix $\mathbf{P}(2)$. You can use calculators or MATLAB for this computation.
- What are the state probability vectors \underline{p}_1 and \underline{p}_2 ?
- Evaluate $\mathbb{P}[X_0 = 1, X_1 = 2, X_2 = 2]$ and $\mathbb{P}[X_0 = 3, X_1 = 1, X_2 = 2]$.

Problem 11.2 (Video 11.1 - 11.4)

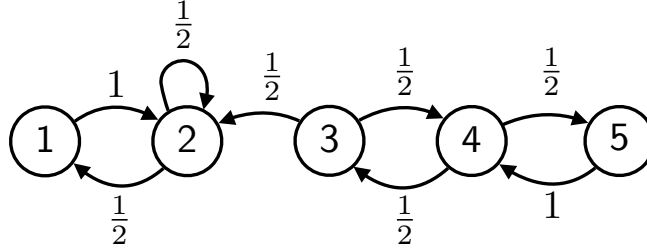
Consider a Markov chain with the following state transition matrix and initial probability state vector:

$$\mathbf{P} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1/3 & 2/3 \\ 1 & 0 & 0 \end{bmatrix} \quad \underline{p}_0 = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

- Draw the Markov chain, labeling the states as 1, 2, and 3, as well as labeling the arcs with the appropriate transition probabilities.
- What is the period of state 1?
- Determine $\mathbb{P}[X_0 = 2, X_1 = 2, X_2 = 3]$.
- Does a unique limiting state probability vector $\underline{\pi}$ exist? If so, argue why and solve for it. If not, argue why.

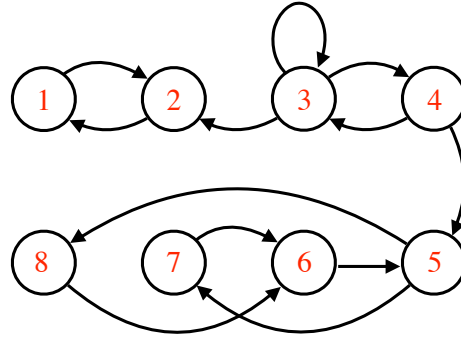
Problem 11.3 (Video 11.1 - 11.4, Lecture Problem)

Consider the following discrete-time Markov chain with initial state 3.



- What are the communicating classes?
- For each communicating class, determine the period and whether it is transient or recurrent.
- Write down the state transition matrix \mathbf{P} .
- Does a unique limiting state probability vector $\underline{\pi}$ exist? If so, argue why and solve for it. If not, argue why.

Problem 11.4 ([Video 11.3](#)) Consider the following Markov chain. For each of the parts below, you only need to know that each arc represents a positive probability.



- Determine the communicating classes.
- Determine the period for each communicating classes.
- Determine which communicating classes are recurrent and which ones are transient.

Problem 11.5 [Video 11.1- 11.4](#)

Consider a 4 state Markov chain with the transition probability matrix

$$\mathbf{P} = \begin{pmatrix} 0.1 & 0.2 & 0.3 & 0.4 \\ 0 & 0.5 & 0.2 & 0.3 \\ 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0.1 & 0.9 \end{pmatrix}$$

- Draw the state transition diagram, with the probabilities for the transitions.
- Find the transient states and recurrent states.
- Is the Markov chain irreducible? Explain.
- Is the Markov chain aperiodic? Explain.
- Find the steady state distribution of this Markov chain.