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EC330 Homework #2

1.

	A	B	O	o	θ	ω	Ω
a)	$n^3+2n+100$	$20n^3-5n+2$	yes	no	yes	no	yes
b)	2489^{200}	$\log_{2489}(n)$	yes	yes	no	no	no
c)	n^7	3^n	yes	yes	no	no	no
d)	n	$\sum_{i=1}^{n} \frac{50}{i}$	no	no	no	yes	yes
e)	$200n^9$	e^n	yes	yes	no	no	no

Note: $f(n)$ refers to the functions in the A column while $g(n)$ refers to the functions in the B column.

a)
- $\lim_{n\to\infty} \frac{n^3+2n+100}{20n^3-5n+2} = \lim_{n\to\infty} \frac{1+\frac{2}{n^2}+\frac{100}{n^3}}{20-\frac{5}{n^2}+\frac{2}{n^3}} = \frac{1}{20} \rightarrow$ finite but not zero $\rightarrow O(g(n))$ ✓
- $n^3+2n+100$ is $O(20n^3-5n+2)$: $c=1, n_0=5$: $n^3+2n+100 \le 1\cdot(20n^3-5n+2)\ \forall n \ge 5$ ✓
- $n^3+2n+100$ is $\Omega(20n^3-5n+2)$: $c=\frac{1}{20}, n_0=1$: $n^3+2n+100 \le \frac{1}{20}(20n^3-5n+2)$ ✓

b)
- $\lim_{n\to\infty} \frac{2489^{200}}{\log_{2489}(n)} = 0 \rightarrow o(g(n))$ ✓
- $c=2489^{200}, n_0=10$: $2489^{200} \ge 2489^{200}\log_{2489}(n)\ \forall n \ge 10 \rightarrow O(g(n))$ ✓
- There exists no $c>0$ and $n_0>0$ such that $f(n) \ge c\cdot g(n)$ for all $n>n_0 \rightarrow \Omega(g(n)), \omega(g(n)), \theta(g(n))$ X

c)
- $\lim_{n\to\infty} \frac{n^7}{3^n} = \frac{\infty}{\infty}$; Apply L'Hopital's Rule $\lim_{n\to\infty} \frac{7n^6}{3^n \ln 3} = \frac{\infty}{\infty}$; L.H. $\lim_{n\to\infty} \frac{42n^5}{3^n \ln^2 3}$; if we keep using L.H., the limit will eventually be zero divided by something, equaling zero $\rightarrow o(g(n))$ ✓
- n^7 is $O(3^n)$: $c=128, n_0=2$: $n^7 \le 128\cdot 3^n\ \forall n \ge 2 \rightarrow O(g(n))$ ✓
- There exists no $c>0$ and $n_0>0$ such that $n^7 \ge c\cdot 3^n$ for all $n \ge n_0 \rightarrow \Omega, \omega, \theta$ X

d)
- The nth element of $\sum_{i=1}^{n} \frac{50}{i}$ is $\ln(n)$ + constant
- There exists no $c>0$ and $n_0>0$ such that $n \le c\cdot \sum_{i=1}^{n} \frac{50}{i}$ for all $n \ge n_0 \rightarrow O, o, \theta$ X
- $c=\frac{1}{50}, n_0=1$: $n \ge \frac{1}{50}\sum_{i=1}^{n} \frac{50}{i} = \sum_{i=1}^{n} \frac{1}{i}\ \forall n \ge n_0 \rightarrow \Omega$ ✓
- $\lim_{n\to\infty} \frac{n}{\sum_{i=1}^{n} \frac{50}{i}} \rightarrow$ approaches zero, so limit goes to infinity $\rightarrow \omega$ ✓

e)
- $\lim_{n\to\infty} \frac{200n^9}{e^n} = \frac{\infty}{\infty}$; Apply L.H. $\lim_{n\to\infty} \frac{1800n^8}{e^n}$; keep applying L.H. and limit will eventually be $0 \rightarrow o, O$ ✓
- There exists no $c>0$ and $n_0>0$ such that $200n^9 \ge e^n\ \forall n \ge n_0 \rightarrow \theta, \Omega, \omega$ X

2. $\lim\limits_{n\to\infty} \frac{24^{100}}{\frac{1}{\log n}} = \infty \to 24^{100}$ is $\omega\left(\frac{1}{\log n}\right)$; $\frac{1}{\log n}$ is $o(1)$

$\lim\limits_{n\to\infty} \frac{1}{24^{100}} = \frac{1}{24^{100}} \to 1$ is $\Theta(24^{100})$

$\lim\limits_{n\to\infty} \frac{24^{100}}{\log_{540} n} = 0 \to 24^{100}$ is $o(\log_{540} n)$

$\lim\limits_{n\to\infty} \frac{\log_{540} n}{\frac{540}{n}} = \lim\limits_{n\to\infty} \log_{540} n \cdot \lim\limits_{n\to\infty} \frac{n}{540} = \infty \to \log_{540} n$ is $\omega\left(\frac{540}{n}\right)$; $\frac{540}{n}$ is $o(\log_{540} n)$

$\lim\limits_{n\to\infty} \frac{\log_{540} n}{n^{1/540}} = \frac{\infty}{\infty} \to$ L'Hopital's Rule $\lim\limits_{n\to\infty} \frac{\frac{1}{n\ln 540}}{\frac{1}{540}n^{-539/540}} = \lim\limits_{n\to\infty} \frac{1}{n\ln 540} \cdot \lim\limits_{n\to\infty} 540 n^{\frac{539}{540}} = 0$; $\log_{540} n$ is $o(n^{\frac{1}{540}})$

$\sqrt{n+540} = (n+540)^{\frac{1}{2}} \to$ higher power than $n^{\frac{1}{540}}$ & both functions are polynomial $\to \sqrt{n+540} > n^{\frac{1}{540}}$

similarly, $540n^1$ is higher polynomial than $(n+540)^{\frac{1}{2}}$

$\lim\limits_{n\to\infty} \frac{540n}{n\log n} = \lim\limits_{n\to\infty} \frac{540}{\log n} = 0$; $540n$ is $o(n\log n)$

$\lim\limits_{n\to\infty} \frac{n\log n}{n^{540}} = \lim\limits_{n\to\infty} \frac{\log n}{n^{539}} = \frac{\infty}{\infty} \to$ L'Hopital's Rule $\lim\limits_{n\to\infty} \frac{\frac{1}{n\log 10}}{539n^{538}} = \lim\limits_{n\to\infty} \frac{1}{n\log 10} \cdot \lim\limits_{n\to\infty} 539n^{538} = 0$; $n\log n$ is $o(n^{540})$

$\lim\limits_{n\to\infty} \frac{\log_n 540}{n^{540}} = \frac{0}{\infty} = 0$; $\log_n 540$ is $o(n^{540})$

$\lim\limits_{n\to\infty} \frac{\frac{540}{n}}{\log_n 540} = \frac{0}{0} \to$ L.R. $\lim\limits_{n\to\infty} \frac{-\frac{540}{n^2}}{\frac{1}{540\log n}} = \lim\limits_{n\to\infty} -\frac{540}{n^2} \cdot \lim\limits_{n\to\infty} 540\log n = 0$; $\frac{540}{n}$ is $o(\log_n 540)$

$\lim\limits_{n\to\infty} \frac{\left(\frac{540}{549}\right)^n}{1} = 0 \to \left(\frac{540}{549}\right)^n$ is $o(1)$

$\lim\limits_{n\to\infty} \frac{1/\log n}{(540/549)^n} = \lim\limits_{n\to\infty} \frac{1}{\log n} \cdot \lim\limits_{n\to\infty} \frac{1}{(540/549)^n}$

$\lim\limits_{n\to\infty} \frac{(540/549)^n}{1/\log n} = \lim\limits_{n\to\infty} \left(\frac{540}{549}\right)^n \cdot \lim\limits_{n\to\infty} \log n = 0 \to \left(\frac{540}{549}\right)^n$ is $o\left(\frac{1}{\log n}\right)$

$\lim\limits_{n\to\infty} \frac{\log_{540} n}{\log_n 540} = \lim\limits_{n\to\infty} \frac{\log n}{\log 540} \cdot \lim\limits_{n\to\infty} \frac{\log n}{\log 540} = \infty \to \log_{540} n$ is $\omega(\log_n 540)$

$\lim\limits_{n\to\infty} \frac{\log_n 540}{\log_{540} n} = \lim\limits_{n\to\infty} \frac{\log 540}{\log n} \cdot \lim\limits_{n\to\infty} \frac{\log 540}{\log n} = 0 \to \log_n 540$ is $o(\log_{540} n)$

$\lim\limits_{n\to\infty} \frac{540/n}{\log_n 540} = \lim\limits_{n\to\infty} \frac{540}{n} \cdot \lim\limits_{n\to\infty} \frac{\log n}{\log 540} = 0 \to \frac{540}{n}$ is $o(\log_n 540)$

$\lim\limits_{n\to\infty} \frac{1}{540/n} = \lim\limits_{n\to\infty} \frac{n}{540} = \infty \to 1$ is $\omega\left(\frac{540}{n}\right)$

$\lim\limits_{n\to\infty} \frac{1/\log n}{540/n} = \frac{1}{540} \lim\limits_{n\to\infty} \frac{n}{\log n} = \frac{\infty}{\infty}$; Apply L.H. $\to \lim\limits_{n\to\infty} \frac{1}{\frac{1}{n\ln 10}} = \lim\limits_{n\to\infty} n\ln 10 = \infty \to \frac{1}{\log n}$ is $\omega\left(\frac{540}{n}\right)$

$\lim\limits_{n\to\infty} \frac{(540/549)^n}{540/n} = \frac{1}{540} \lim\limits_{n\to\infty} n\left(\frac{540}{549}\right)^n = 0$ (second term approaches zero because fraction is less than one) $\to \left(\frac{540}{549}\right)^n$ is $o\left(\frac{540}{n}\right)$

if n is very very big, n^{540} just multiplies itself 540 times; however, $n!$ has to go through ALL of the numbers before it, i.e. $n(n-1)(n-2)(n-3)\ldots\ldots$, so $n!$ grows faster than $(n)^{540}$

$\left(\frac{540}{549}\right)^n < \frac{540}{n} < \frac{1}{\log n} < 1 = 24^{100} < \log_n 540 < \log_{540} n < n^{\frac{1}{540}} < \sqrt{n+540} < 540n < n\log n < (n)^{540} < n!$