Jilin Zhang //U49258796

EC330 Homework 1

1. Summations

a. $\sum_{i=11}^{37} 9^i = 9^{11} + 9^{12} + 9^{13} + \cdots + 9^{37} = 9^{11}(1 + 9 + 9^2 + \cdots + 9^{26}) = 9^{11}\left[\sum_{i=0}^{26} 9^i\right] = 9^{11}\left[\frac{9^{x+1}-1}{9-1}\right] = 9^{11}\left[\frac{9^{27}-1}{8}\right]$

explanation: factor out 9^{11}, then apply geometric series summation formula

b. $\sum_{i=0}^{\infty} \frac{6}{17^i} = 6\sum_{i=0}^{\infty}\left(\frac{1}{17}\right)^i = 6\left[\frac{1}{1-\frac{1}{17}}\right] = \frac{6}{\frac{16}{17}} = 6 \cdot \frac{17}{16} = \frac{51}{8} = 6.375$

explanation: factor out constant 6, then apply infinite geometric series summation formula

c. $\sum_{i=1}^{2N}(2i^2 - 12i + 3) = \sum_{i=1}^{2N} 2i^2 - \sum_{i=1}^{2N} 12i + \sum_{i=1}^{2N} 3 = 2\left[\frac{(2N)(2N+1)(4N+1)}{6}\right] - 12\left[\frac{2N(2N+1)}{2}\right] + 3(2N)$

explanation: apply linearity to break into 3 summations, then apply arithmetic series, sum of squares, and sum of cubes formulas

d. $\sum_{i=0}^{\infty}\frac{i-1}{2^i} = \sum_{i=0}^{\infty}\frac{i}{2^i} - \left(\sum_{i=0}^{\infty}\left(\frac{1}{2}\right)^i\right) = 2 - \frac{1}{1-\frac{1}{2}} = 2 - 1.2 = 0$

$\sum_{i=0}^{\infty}\frac{i}{2^i} = 0 + \frac{1}{2^1} + \frac{2}{2^2} + \frac{3}{2^3} + \cdots = S$

$\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{2}{2^3} + \cdots = S$

$\underbrace{\left(\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots\right)}_{\text{infinite geometric series}} + \frac{1}{2}\underbrace{\left(\frac{1}{2^1} + \frac{2}{2^2} + \frac{3}{2^3} + \cdots\right)}_{S} = S$

$\frac{\frac{1}{2}}{1-\frac{1}{2}} + \frac{1}{2}S = S$

$1 + \frac{1}{2}S = S$

$S = 2$

explanation: use linearity to break into 2 summations, then solve first summation by work on left, and other summation by infinite geometric series formula

e. $\sum_{i=111}^{1230}\frac{1}{i} = \sum_{i=1}^{1230}\frac{1}{i} - \sum_{i=1}^{110}\frac{1}{i} = \ln(1230) + \text{constant} - (\ln(110) + \text{constant}) = \ln\left(\frac{1230}{110}\right) + \text{constant}$

explanation: use finite harmonic series approximation and log rules

2. Exponents & Logs

a. $x^8 \cdot x^9 \cdot x^{10} \cdots x^N = x^{8+9+10+\cdots+N} = x^{\sum_{i=8}^{N} i} = x^{\sum_{i=1}^{N} i - \sum_{i=1}^{7} i} = x^{\frac{N(N+1)}{2} - \frac{7(7+1)}{2}} = x^{\frac{N^2+N-56}{2}} = x^{\frac{(N+8)(N-7)}{2}}$

explanation: rewrite exponents by product rule, then apply arithmetic series

b. $\log_{99}(33 \cdot 33 \cdot 33 \cdot 33 \cdot 33) = \log_{99}(33^5) = 5\log_{99}(33)$

explanation: the product of five 33's is the same as 33 raised to the power of 5; apply log rule afterwards

c. $\log_x((3x)^x) = x\log_x(3x) = x[\log_x 3 + \log_x x] = x(\log_x 3 + 1) = x\log_x(3) + x$ explanation: apply logarithmic rules

d. $44^{\log_{44} 330} = 330$ explanation: apply log rules

e. $\sum_{i=1}^{3^N}\log_{18} i = \log_{18} 1 + \log_{18} 2 + \log_{18} 3 + \cdots + \log_{18} 3^N = \log_{18}(1 \cdot 2 \cdot 3 \cdots 3^N) = \log_{18}(3^N!)$

explanation: using properties of logarithms, the summation can be rewritten to a form that resembles that of a factorial

3. Combinatorics

a. 12-digit hex (0-9, A-F) → 10^{12}

explanation: hex digits range from 0 to 9, and A-F; there are 10 numbers and 6 letters, so there are 10 options for each digit of the 12 digit hex number, omitting letters

b. 3 groups of 8 students from a class of 48 → $\binom{48}{8}\binom{40}{8}\binom{32}{8} = \frac{48!}{8!40!} \cdot \frac{40!}{8!32!} \cdot \frac{32!}{8!24!}$

explanation: choose 8 students from 48, then 8 from remaining 40, then 8 from remaining 32