Digital Technologies and Value Creation (Lecturer: Philippe Blaettchen) – Integer Programming Exercise Solutions

Situation 1

The decision variables are x_{ij}, i.e., the units of cargo i placed in compartment j (with $i = 1,2,3,4$ and $j = F, C, B$). The decision variables are binary, as we can only take whole units. The objective function is to maximize the profit

$$320(x_{1F} + x_{1C} + x_{1B}) + 400(x_{2F} + x_{2C} + x_{2B}) + 360(x_{3F} + x_{4C} + x_{5B}) + 290(x_{4F} + x_{4C} + x_{4B})$$

under the constraints

$$\begin{gathered}
2x_{1F} + 1.6x_{2F} + 2.5x_{3F} + 1.3x_{4F} \leq 12 \\
2x_{1C} + 1.6x_{2C} + 2.5x_{3C} + 1.3x_{4C} \leq 18 \\
2x_{1B} + 1.6x_{2B} + 2.5x_{3B} + 1.3x_{4B} \leq 10 \\
1{,}000x_{1F} + 1{,}150x_{2F} + 1{,}400x_{3F} + 780x_{4F} \leq 7{,}000 \\
1{,}000x_{1C} + 1{,}150x_{2C} + 1{,}400x_{3C} + 780x_{4C} \leq 9{,}000 \\
1{,}000x_{1B} + 1{,}150x_{2B} + 1{,}400x_{3B} + 780x_{4B} \leq 5{,}000 \\
x_{ij} \geq 0 \text{ for all } i, j
\end{gathered}$$

Situation 2

We let x_j be whether the VC should invest in project j or not. This is a binary variable (i.e., it can be either 0 or 1). The constraints are on the initial investment which should be less than 4M and the average failure risk which should be below 5%. The objective is to maximize total expected profit.

$$\begin{aligned}
max \quad & 0.1\,x_1 + 0.2\,x_2 + 0.2\,x_3 + 0.1\,x_4 + 0.1\,x_5 + 0.1\,x_6 \\
\text{s.t.} \quad & 0.06\,x_1 + 0.04\,x_2 + 0.06\,x_3 + 0.05\,x_4 + 0.05\,x_5 + 0.04\,x_6 \leq 0.05 \cdot (x_1 + \ldots + x_6) \\
& 1.3\,x_1 + 0.8\,x_2 + 0.6\,x_3 + 1.8\,x_4 + 1.2\,x_5 + 2.4\,x_3 \leq 4 \\
& x_1, \ldots, x_6 \in \{0,1\}
\end{aligned}$$